TITLE: FRC TRANSLATION INTO A COMPRESSION COIL

AUTHOR(S): ROBERT E. CHRIEN

SUBMITTED TO: Seventh U.S. Symposium on Compact Toroid Research, Santa Fe, NM, May 21-23, 1985

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, representation, or guaranty, express or implied, or assumes any liability whatever, for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademarker, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

Los Alamos National Laboratory is operated by the University of California for the United States Department of Energy under contract W-7405-ENG-36.
FRC TRANSLATION INTO A COMPRESSION COIL

Robert E. Chrien
Los Alamos National Laboratory, Los Alamos NM 87545

I. Introduction: The equilibrium and translational kinematics of Field-Reversed Configurations (FRCs) in a cylindrical coil which does not conserve flux are problems that arise in connection with adiabatic compressional heating. The usual treatments of the equilibrium problem for FRCs specify a fixed amount of open flux within a conducting boundary, and in previous work the kinematics of translation between two flux conservers have been discussed. However, in order to most efficiently heat the FRC plasma, it is desirable to translate the FRC after it is formed, into a close-fitting compression coil and then to adiabatically compress the plasma by increasing the open flux. The compression coil, or "flux shaper", must be a flux surface on the time scales for translation (t_{trans}) and compression (t_{comp}) in order to maintain the axial equilibrium of the FRC, but the amount of open flux is influenced by the external circuit. An additional problem is that, unless the energy confinement time T_{E} is so long that t_{trans} \ll t_{comp} \ll T_{E} can be satisfied, a separate guide field in addition to the compression field is needed in the compression coil to confine the FRC prior to compression. Coupling between the guide field and compression field systems must then be considered.

In this paper, we consider several features of the problem of FRC translation into a compression coil. First, the magnitude of the guide field is calculated and found to exceed that which would be applied to a flux conserving coil. Second, energy conservation is applied to FRC translation from a flux conserving coil into a compression coil. It is found that a significant temperature decrease is required for translation to be energetically possible. The temperature change depends on the external inductance in the compression circuit. An analogous case is that of a compression region composed of a compound magnet; in this case the temperature change depends on the ratio of inner and outer coil radii. Finally, the kinematics of intermediate translation states are calculated using an "abrupt transition" model. It is found, in this model, that the FRC must overcome a potential hill during translation, which requires a small initial velocity.

II. Translation into a Compression Coil: We consider the situation in which a guide field is present in the compression coil to confine the translating FRC prior to compression. The guide field can be applied either by switching a capacitor bank C_{H} into the compression coil of inductance L_{c} through an isolating inductor L_{B} (Fig. 1) or by energizing a separate coil outside the compression coil. Considering the former case first, we assume that the guide field bank is crowbarred at peak current to provide the guide field. By equating the flux in the circuit before and after the FRC enters the compression coil \((I_{B0} + L_{c})I_{0} = (L_{B} + L_{c})I_{1}\), we find (neglecting finite length corrections) that

\[ B_{o} = (1 - x_{y}^{2})B_{w} \frac{1 + L_{B}/L_{c}}{1 + L_{H}/L_{c}} \tag{1} \]

In this expression, B_{o} is the guide field, B_{w} is the confining field at the wall after the FRC has entered, x_{y} is the ratio of FRC separatrix and coil radii, and L_{c} < L_{c0} is the coil inductance in the presence of the FRC.
Using long-coil and cylindrical FRC assumptions for $L_c$ ($L_c = L_{co} (1 - x^2) / (z_g + (1 - x^2) (1 - z_g))$ where $z_g$ is the ratio of FRC length $L_c$ to coil length $f_c$) the guide field can be written as

$$B_0 = B_{w}(1 - x^2 + f_L z_g x^2)$$

(2)

where $f_L = L_R / (L_B + L_{co})$. The guide field required in the compression coil for given $x_g$ and $B_0$ is larger than that needed in a flux conservor where $L_B = 0$ and can approach $B_w$ if $L_R > L_{co}$ and the FRC fills the length of the coil. The external inductance has the effect of reducing the increase in total current when the FRC enters, so more current is needed initially.

The axial kinetic energy of the FRC can be determined by considering the total energy of the system consisting of FRC, formation coil, and compression coil. The formation coil is assumed to be crowbarred and acting as a flux conservor. Before translation, the total energy is

$$\frac{5}{2}NT_1 + U_{1,\text{vac}} + \frac{1}{2}Nm_1 v_1^2 + U_{2,\text{vac}}$$

(3)

where $N$ and $T_1$ are the particle inventory and temperature in the formation region, $U_{1,\text{vac}}$ and $U_{2,\text{vac}}$ are the magnetic energies in the formation and compression circuits in the absence of plasma, $m_1$ is the ion mass, and $v_1$ is the initial axial velocity. After translation, the total energy is

$$U_{1,\text{vac}} + \frac{3}{2}NT_2 + \frac{1}{2}Nm_1 v_2^2 + U_2$$

(4)

where $T_2$ and $v_2$ are the temperature and velocity following translation, $U_2$ is the magnetic energy in the compression circuit and the FRC, and where losses are ignored.

The change of magnetic energy in the compression region, $\Delta U_2 = U_2 - U_{2,\text{vac}}$, can be written as

$$\Delta U_2 = \frac{1}{2}\beta (T_2^2 - T_0^2) + \frac{1}{2}c_1 T_2^2 - \frac{1}{2}c_0 T_0^2 + v_{\text{FRC}}(1 - \langle \beta \rangle) \frac{B_w^2}{8\pi}$$

(5)

where $\langle \beta \rangle$ is the volume-averaged beta within the separatrix. The first two terms represent the energy changes in the external inductor and the coil, respectively, and the third term is the magnetic energy within the separatrix. After some algebra, this can be written as

$$\Delta U_2 = NT_2 (1 + \frac{z_g x_g^2 f_L}{\langle \beta \rangle})$$

(6)

In deriving this expression, Eq. 2 and the approximate expression for coil inductance were used.

By equating (3) and (4) and using (6), we find that

$$T_2 = \frac{T_1 + \frac{1}{2} m_1 (v_1^2 - v_2^2)}{1 + \frac{2}{5} \frac{z_g x_g^2 f_L}{\langle \beta \rangle}}$$

(7)

This equation reduces to the result for translation between flux conservors when $f_L = 0$ and shows that $T_2$ is lower when $f_L > 0$. The magnitude or this decrease can be important. In the case of the proposed RHX-D experiment, $x_g = 0.8$, $f_L = 0.7$, $z_g = 1.0$, and $\langle \beta \rangle = 0.68$ so that $T_2$ is reduced by 21%. Thus either the formation region must be designed to form the FRC at higher temperature to yield the desired temperature before
compression, or the compression bank must be increased to reach the desired final temperature.

We now consider the case in which the compression region is a compound magnet. The guide field is then provided by dc magnet coils located outside the compression coil. In general, the guide field coils must be protected from the voltages induced by the translation and compression of the FRC. Thus their protective cladding is a flux conservor on the translation and compression time scales. The compression coil is a flux surface on these time scales and carries the azimuthal currents needed to maintain the axial equilibrium of the FRC. However, the net azimuthal current must be zero since the compressional coil circuit is open before compression. These constraints result in the relation between the guide and confining fields

\[
B_0 = B_0 \left[1 - x_0^2 + (1 - y^2)z_g x_g^2\right]
\]

(8)

where \( y \) is the ratio of flux shaper and flux conservor radii. This equation is similar to (2) except that \( (1 - y^2) \) replaces \( f_L \). A calculation of the total magnetic energy in the compound magnet compression region yields a result which differs from (6) in the same way. Thus a lower final temperature is necessary in either case. Of course, either \( (1 - y^2) \) or \( f_L \) must be kept close to unity to prevent excessive coupling between the compression flux and coil cladding or between the compression bank and guide field inductor, respectively.

III. Kinematics of the Translation: We have seen that translation into a compression coil requires a larger guide field than translation into a flux conservor. However, when only a fraction of the FRC length has entered the compression coil, Eqs. (2) and (8) show that it encounters a larger confining field \( B_0 \) than when its full final length is within the coil. Thus the FRC may encounter a potential hill between the initial and final states. In practice, this effect may be mitigated by a transient modification of the FRC equilibrium or by designing a tapered compression coil to soften the transition in radius. In order to estimate the magnitude of this effect, we have calculated the total energy of the intermediate states using an "abrupt transition" model which may provide an upper bound on the size of the potential hill.

In this model, we assume that a cylindrical FRC equilibrium encounters an abrupt change in wall radius from the formation coil to the compression coil. We further assume that the response of the portion of the FRC which has entered the compression coil is governed by the adiabatic laws for FRC equilibria and by Eq. (2). These adiabatic laws were found to be in reasonable agreement with FRC translation experiments in which FRCs traversed a change in wall radius of 0.74 at axial speeds approaching the ion thermal speed; thus it may be reasonable to use them in this model.

The internal flux of the FRC, assuming a typical diffuse profile, is given by \( \phi = r_w \phi_0 r_x x_g \). Applying conservation of flux across the transition, we have

\[
(1 - x_2^2 + f_L x_2^2) \left(\frac{r_4}{r_3}\right)^2 \left(\frac{x_4}{x_3}\right)^{3.25} = 1.
\]

(9)
The FRC length $L_2$ within the compression coil is estimated using the adiabatic laws for the portion $\nu t_1$ of the initial FRC length which crossed the transition:

$$L_2 = x_2 l_c = \nu t_1 \left( \frac{r_{w2} - 0.4x_{b2} - 1.9\langle \beta_2 \rangle - 0.7}{r_{w1} - x_{b1} \langle \beta_1 \rangle} \right).$$

Equations 10 and 11 can be numerically solved for $x_{a2}$ as a function of $\nu$. This permits the total energy of the intermediate state to be calculated as a function of $\nu$ using the adiabatic law to estimate $T_2$:

$$T_2 = T_1 \left( \frac{r_{w2} - 1.6x_{b2} - 2.6\langle \beta_1 \rangle}{r_{w1} - x_{b1}} \right).$$

Comparison of the total energies of the initial and intermediate states yields

$$\frac{1}{2}m_1 v_1^2 + \frac{5}{2}T_1 = \frac{1}{2}m_1 v_2^2 + \frac{5}{2}(1 - \nu)T_1 + \frac{5}{2}\nu T_2 + \nu T_2 ^{0.2}L_{\langle \beta \rangle}. \tag{12}$$

The results of this calculation for the parameters of FRX-D are illustrated in Fig. 2. A potential hill is observed with a magnitude of about 18 eV/particle. This requires an initial velocity of about 4 cm/µs to overcome, which is relatively easy to produce in present FRC translation experiments. Thus this effect does not appear to be a serious obstacle to a translation and compression experiment.

IV. Conclusions: FRC translation into a compression coil in an adiabatic compressional heating experiment differs significantly from translation between flux conservers. It is shown that translation into a compression coil requires more guide field and a lower final temperature because of the interaction between the compression circuit and the two possible methods by which guide field can be applied. These effects are of sufficient magnitude to influence the design of a compressional heating experiment. An additional potential hill may be present between the initial and final states, but it can be overcome by a modest initial translation velocity.

References

6. LA-UR 84-3371
Figure 1. Compression coil circuit for FRX-D, showing the guide field (C_B) and compression field (C_c) capacitor banks and the isolation (L_b) and load coil (L_co) inductances.

Figure 2. Prediction of the "abrupt transition" model for the translational potential energy per ion as a function of the friction $\nu$ of the FRC which has crossed the transition.