1638

214
3-7-80

FEBRUARY 1980

14. 801

PPPL-1638

UC-20b,g

GENERALIZATION OF CONNOR AND
HASTIE'S RIPPLE-TRAPPING
COEFFICIENT, $G(\alpha)$

MASTER

BY

R. J. GOLDSTON AND H. H. TOWNER

PLASMA PHYSICS
LABORATORY

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

This work was supported by the U. S. Department of Energy
Contract No. EY-76-C-02-3073. Reproduction, translation,
publication, use and disposal, in whole or in part, by or
for the United States Government is permitted.

GENERALIZATION OF CONNOR AND HASTIE'S RIPPLE-TRAPPING
COEFFICIENT, $G(\alpha)$

R. J. Goldston and H. H. Towner
Plasma Physics Laboratory, Princeton University
Princeton, N. J. 08544

ABSTRACT

In their calculation of transport due to ripple trapping, Connor and Hastie³ employ a factor $G(\alpha)$ to take into account the poloidal variation of the depth and shape of the local magnetic wells. They assume, however, that the toroidal field ripple strength, δ , is independent of poloidal angle. In this letter, we generalize G to treat any arbitrary poloidal variation of δ , and also high beta and noncircular plasma configurations.

DISCLAIMER

This book was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors are those of the authors and do not necessarily state or reflect those of the United States Government or any agency thereof.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

844

GENERALIZATION OF CONNOR AND HASTIE'S RIPPLE-TRAPPING
COEFFICIENT, $G(\alpha)$

R. J. Goldston and H. H. Towner
Plasma Physics Laboratory, Princeton University
Princeton, N. J. 08544

February 1980

We first note that Uckan et al.,¹ gives a result for G when $\delta = \delta(\theta)$ which has three shortcomings:

1. it assumes $\delta = \delta(r) \exp(-\beta\theta^2)$
2. it generalizes Stringer's² $J(\alpha)$, not the more accurate Connor and Hastie³ (C + H) result, which takes into account the ripple well shape as well as its depth, giving a factor of ~1.6 difference
3. it does not treat noncircular and/or high beta plasmas.

The second shortcoming might be acceptable, since the Stringer result is the more pessimistic, but the first and third problems introduce substantial uncertainty into how to evaluate the transport coefficients. We will use the approach developed in Goldston and Towner⁴ (G + T) to evaluate the C + H factor G , for arbitrary $\delta(\theta)$.

C + H have a factor "A" in each of their transport coefficients. Their Eq. 27 gives:

$$A = \frac{2\sqrt{2}N}{3\pi} \int_0^{2\pi} d\theta \sin^2\theta \int_a^b d\phi (1 - B/B_{\max})^{3/2}$$

where

$$B(a) = B(b) = B_{\max}$$

Furthermore, they define $G(\alpha)$ by:

$$A = \frac{64}{9} \delta^{3/2} G(\alpha)$$

where

$$\alpha = \epsilon / Nq\delta$$

If we define $\delta_0(r)$ as $\delta(r, \theta = 0)$, and use δ_0 in the evaluation of the transport coefficients, we may substitute a more general $G(r)$ for C + H's $G(\alpha)$.

$$G(r) = \frac{3\sqrt{2} N}{32\pi\delta_0^{3/2}} \int_0^{2\pi} d\theta \sin^2\theta \int_a^b d\phi (1 - B/B_{\max})^{3/2}$$

where the ϕ integral takes account of the explicit variation of δ with θ .

To evaluate $G(r)$, we follow G + T, and model the field near θ as:

$$B = B_0 (1 + \alpha^* \delta x + \delta \sin x)$$

where

$$x = N\phi \quad \alpha^* = \epsilon |\sin\theta| / Nq\delta$$

$B_{\max}$ is then given by

$$B_{\max} = B_0 \left(1 + \alpha^* \delta (\pi - \cos^{-1} \alpha^*) + \delta \sqrt{1 - \alpha^{*2}} \right)$$

a is given by

$$a = x_{\max} = \pi - \cos^{-1} \alpha^*$$

and $b = x_b$ is the solution to the transcendental equation:

$$\alpha^* x_b + \sin x_b = \alpha^* (\pi - \cos^{-1} \alpha^*) + \sqrt{1 - \alpha^{*2}}$$

Assuming top-bottom symmetry, we have

$$G(r) = \frac{3\sqrt{2}}{16\pi} \int_0^\pi d\theta \sin^2\theta \delta_0^{-3/2} \int_{x_{\max}}^{x_b} dx \left(1 - \frac{1 + \alpha^* \delta x + \delta \sin x}{1 + \alpha^* \delta (\pi - \cos^{-1} \alpha^*) + \delta \sqrt{1 - \alpha^{*2}}} \right)^{3/2}$$

for δ small, and $\alpha^* \delta = e |\sin \theta| / Nq$ small, we have

$$G(r) = \frac{3\sqrt{2}}{16\pi} \int_0^\pi d\theta \sin^2 \theta \left(\frac{\delta(\theta)}{\delta_0} \right)^{3/2} \underbrace{\int_{x_{\max}}^{x_b} dx \left(\alpha^* (\pi - \cos^{-1} \alpha^*) + \sqrt{1 - \alpha^{*2}} - \alpha^* x - \sin x \right)^{3/2}}_{C(\alpha^*)}.$$

The integral $C(\alpha^*)$ is performed numerically (Fig. 1), and then the θ integral must be performed numerically as well. Note that α^* varies implicitly with θ (due to the variation of δ with θ) as well as explicitly. For regions of $\alpha^* > 1$, $C = 0$. Unfortunately, due to the dependence of α^* on q , G must be evaluated at each time step in a time-dependent transport code.

To simplify the problem, if necessary, we may approximate $C(\alpha^*)$. Stringer's approximation is to assume a sinusoidal well shape, a well length of $N\Delta\phi = 2\pi$, and a well depth of δ^* . This is equivalent to:

$$C \approx \left(\frac{\delta^*}{2\delta} \right)^{3/2} \frac{16\sqrt{2}}{3}.$$

As pointed out previously, this approximation results in a $G(\alpha)$ which is typically high by a factor of ~ 1.6 . We may improve the estimate by including the variation of the length of the ripple well, setting $N\Delta\phi = 2(x_{\min} - x_{\max})$, where $x_{\min}$ is the location of the local minimum in $|B|$. $x_{\min} = \pi + \cos^{-1} \alpha^*$. We arrive at:

$$\begin{aligned} C &\approx \left(\frac{\delta^*}{2\delta} \right)^{3/2} \left(\frac{2 \cos^{-1} \alpha^*}{\pi} \right) \frac{16\sqrt{2}}{3} \\ &= \frac{32\sqrt{2}}{3\pi} \cos^{-1} \alpha^* \left(\sqrt{1 - \alpha^{*2}} - \alpha^* \cos^{-1} \alpha^* \right)^{3/2}. \end{aligned}$$

This analytic approximation is also shown in Fig. 1.

Finally, the generalization to noncircularity and high beta is quite straightforward. The θ integral in the definition of G represents a flux surface average, and the $\sin^2 \theta$ factor arises from the cosine of the angle between $\vec{v}_d$ and $\vec{v}_\psi$.

Thus the more general form of G is

$$G(\psi) = \frac{3\sqrt{2}}{16} \int d\chi \frac{[(\vec{B} \times \vec{\nabla}\psi) \cdot \vec{\nabla}\psi]^2}{(B^3/RB_{vac})^2 (\vec{\nabla}\psi)^2} \left(\frac{\delta(\chi)}{\delta_0}\right)^{3/2} C(\alpha^*) / \int d\chi$$

where χ is the poloidal variable with unit Jacobian, and B_{vac} is the vacuum toroidal field. α^* is now given by

$$\alpha^* = \frac{|\vec{B}_p \cdot \vec{\nabla}B|_{sym}}{N\delta B_T^2}.$$

References

1. N. A. Uckan, K. T. Tsang, J. D. Callen, ORNL/TM-5438 (1976).
2. T. E. Stringer, Nucl. Fusion 12, 289 (1972).
3. J. W. Connor, R. J. Hastie, Nucl. Fus. 13, 221 (1973).
4. R. J. Goldston, H. H. Towner, PPPL-1637 (1980).

Acknowledgment

This work supported by the U.S. Department of Energy Contract No. EY-76-C-02-3073.

Figure 1. Dependence of the integral $C(\alpha^*)$ on α^* .
803139