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WFPS:TME-79-032
DECEMBER 1979
UC-20D

CONCEPTUAL DESIGN OF A
COMMERCIAL TOKAMAK HYBRID REACTOR FUELING SYSTEM

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COOPERATIVE GRADUATE EDUCATION PROGRAM IN FUSION TECHNOLOGY
ADMINISTERED FOR THE U.S. DEPARTMENT OF ENERGY
BY THE WESTINGHOUSE ELECTRIC CORPORATION
CONTRACT EG-77-C-02-4231-A000

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Special personal thanks are being extended to Dr. Edwin E. Kintner of the U.S. Department of Energy and to Drs. Zalman Shapiro and Daniel Klein of Westinghouse Electric Corporation for their contributions in making this superb program a viable and continuing reality.

During the course of this study, the authors have become deeply indebted to Dr. Z. Shapiro, Dr. D. Klein, Dr. G. Gibson, Dr. E. W. Sucov, Mr. G. Collier, Mrs. E. Iwinski, Mr. J. Easoz, Mr. J. Selembo, and Mrs. Yvonne Harlow of Westinghouse, as well as Ms. T. Gleue, Mr. C. O'Brian, and Mrs. J. Findley of Kansas State University; their contributions, suggestions, and assistance are being acknowledged with sentiments of sincere gratitude.

In addition, the principal author (KDM) wishes to express his appreciation to his two co-authors and advisors (HJD and TFY) for their help, encouragement, and interest in this study.

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Printed in the United States of America
Available From
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road
Springfield VA 22161

NTIS Price Codes
Printed Copy: A04
Microfiche Copy: A01

WFPS:TME-79-032
DECEMBER 1979
UC-20d

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ABSTRACT

A conceptual design of a fuel injection system for CTHR (Commercial Tokamak Hybrid Reactor) is discussed. Initially, relative merits of the cold-fueling concept are compared with those of the hot-fueling concept; that is, fueling where the electron temperature is below 1 eV is compared with fueling where the electron temperature exceeds 100 eV. It is concluded that cold fueling seems to be somewhat more free of drawbacks than hot fueling. Possible implementation of the cold-fueling concept is exploited via frozen-pellet injection. Several methods of achieving frozen-pellet injection are discussed and the light-gas-gun approach is chosen from these possibilities. A modified version of the ORNL Neutral Gas Shielding Model is used to simulate the pellet injection process. From this simulation, the penetration-depth dependent velocity requirement is determined. Finally, with the velocity requirement known, a gas-pressure requirement for the proposed conceptual design is established. The cryogenic fuel-injection and fuel-handling systems are discussed. A possible way to implement the conceptual device is examined along with the attendant effects on the total system.

SUMMARY

Possible fueling concepts were examined to determine which would provide the highest probability of success for application in the CTHR (Commercial Tokamak Hybrid Reactor). Cold fueling was chosen with this idea in mind. Thereafter, the particular scheme chosen was frozen-pellet fueling, since it provides the advantage of maximum fuel-particle density. Several methods of frozen-pellet injection were studied to determine their capabilities, their experimental verification, and projections for the future. Any characteristics which might adversely affect the normal operation of the tokamak were taken into account. To this effect, the light-gas gun was tentatively chosen to provide pellet acceleration.

The ORNL Neutral Gas Shielding Model is the basic theory used to determine the required pellet velocity. It has been modified, however, to account for operation in the commercial temperature regime (as opposed to experimental devices with temperatures around 1 keV). The required pellet velocity is a function of the depth at which the pellet has disappeared as a solid entity.

From this, the pressure level and other essential estimates to be made on the fuel injector design have been made. This leaves only the design of a fuel handling system to implement operation. The fuel handling system has been designed so that a sufficient fuel-pellet supply is produced and quality control systems may be integrated into the system at a later date.

Various ways were studied to locate the fuel injection system so as to minimize the total-system perturbation. The suggested design incorporating the fuel injector into CTHR accounts for both this aspect and that of providing the shortest possible path to the plasma center.

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NOMENCLATURE AND NOTATION

- $u \equiv$ Required pellet velocity to achieve penetration as a solid entity to (l/a) , (in m/sec).
- $(l/a) \equiv$ Fraction of pressure center of tokamak to which pellet is to penetrate; i. e., if $(l/a) = 1$, pellet penetrates to the pressure center of the reactor.
- $a \equiv$ Distance from the outer edge of the plasma to the pressure center (in m). Note that this quantity is taken as 0.9 m for CTHR.
- $J \equiv$ Electron current, further defined in Eqn. 1d.
- $E_{\infty} \equiv$ Thermal energy (in eV) of electrons in fusion plasma. Note: This quantity is position dependent as illustrated in Eqn. 1e.
- $E_0 \equiv$ Thermal energy (in eV) of electrons at pellet surface. Note: This quantity has been taken as 10 eV in Eqn. 1g.
- $L(E) \equiv$ Electron energy loss function, approximated by Milora and Foster¹ as shown in Eqn. 1b, (in eV m^2), where E is the electron energy (in eV).
- $m \equiv$ Mass of fuel molecule (in kg).
- $m_e \equiv$ Mass of electron (in kg) $\doteq 9.109537 \times 10^{-31}$.
- $m_H \equiv$ Mass of proton (in kg) $\doteq 1.673560 \times 10^{-27}$.
- $m_D \equiv$ Mass of deuteron (in kg) $\doteq 3.344549 \times 10^{-27}$.
- $\rho_D \equiv$ Mass density of solid deuterium (in kg/m^3) $\doteq 2.059 \times 10^2$.
- $\gamma \equiv$ Ratio of specific heats.
- $e \equiv$ Charge of the electron (in C) $\doteq 1.6021892 \times 10^{-19}$.
- $r_p \equiv$ Initial pellet radius (in m) $= 3.073 \times 10^{-3}$.
- $\mu \equiv$ Ratio of mass of fuel molecule to the mass of a proton.
- $n_{\infty} \equiv$ Number density of electrons in the tokamak (in m^{-3}). Note: This quantity is position dependent, as in Eqn. 1f.
- $\langle T_{\infty} \rangle \equiv$ Average electron temperature of fusion plasma (in eV) $= 1.3 \times 10^4$.
- $\langle n_{\infty} \rangle \equiv$ Average electron number density in fusion plasma (in m^{-3}) $= 9.8 \times 10^{19}$.
- $r \equiv$ Position variable in fusion plasma, i. e., (r/a) is dimensionless.

NOMENCLATURE AND NOTATION (continued)

- $M \equiv$ Mach number of the ablatant.
- $\hat{\rho} \equiv$ Ratio of the mass density of the ablatant to the mass density of the ablatant at the pellet surface.
- $\hat{r} \equiv$ Ratio of distance from pellet center to pellet radius.
- $m_{s+p} \equiv$ Sum of the mass of a pellet injector shell and the mass of the fuel pellet (in kg). In the present design, this is $\approx 1.947 \times 10^{-3}$ kg.
- $A_s \equiv$ Surface area of the pellet injector shell exposed to the propellant gas (in m^2), (numerically, $\pi/4 (0.03)^2 m^2$).
- $L \equiv$ Distance over which the fuel injector shell and pellet are accelerated (in m), taken here to be 0.30 m.
- $u_{max} \equiv$ Maximum attainable velocity of light-gas-gun-projectile (in m/sec).
- $r_o \equiv$ Radius of pellet as a function of time (in m).
- $\rho_o \equiv$ Mass density of ablatant as a function of time (in kg/m^3).
- $\rho_s \equiv$ Mass density of fuel pellet (in kg/m^3).
- $v_o \equiv$ Speed of the ablatant (in m/sec). For the purposes of the determination of the value of ξ (see Eqn. 2), this quantity is taken to be 400 m/sec.

1.0 Introduction

Westinghouse Fusion Power Systems Department is developing a design for a Commercial Tokamak Hybrid Reactor, referred to henceforth as CTHR. In the course of the design of the reactor, it became apparent that fueling system design must be given considerable attention. This is due to the fact that the fuel must cross magnetic-field lines in order to effect penetration of the reacting plasma body. In spite of this, the subject of refueling mechanisms is new to the tokamak-reactor conceptual design. The idea of crossing magnetic field lines leads, basically to two concepts of fueling. These are hot fueling, wherein the fuel consists of either ionized particles or neutral particles with a high thermal energy, and cold fueling, wherein the fuel consists of particles with very small thermal energy.

In the hot fueling concept, the particle temperatures are generally above 100 eV. Some examples of this concept are cluster beams, neutral beams, and plasma guns. The primary disadvantage to using hot ionized-particle fueling lies in the requirement that the refueling "mixture" must have at least as high a thermal energy as the reacting body; clearly, in order to allow electrically charged fuel particles to enter and penetrate the confining magnetic field, their average energy must exceed the energy which the magnetic field was designed to contain. The primary disadvantage to employing hot neutral-particle fueling is due to the rather small probability for interaction with the plasma; obviously, a wastefully excessive amount of fuel would have to be injected to maintain the necessary plasma density. These requirements also cause a secondary disadvantage. The inherent inefficiency of heating the fuel by an electromechanical device includes both the inefficiency of the device and the

Carnot/Rankine inefficiencies accompanying the electrical-power generation upon which the device is operated. Thus, it is probable that cold fueling, if feasible, will be a more efficient and a safer process.

There are, basically, three different schemes for implementation of a cold-fueling concept. These are fueling by frozen D+T pellets, squirting a jet of liquid D+T, and puffing gaseous D+T from the surface of the reactor. All three have in common the fact that the fuel particles are unaffected by the magnetic field since they are electrically neutral. The requirement for refueling here is that the fuel must penetrate the pressure "wave" of the reacting body. Obviously, the greater the density of the fuel, the greater is the probability of penetration, provided that the fuel-injection velocity is held constant. (Remember that, upon injection into the reacting body, the fuel will tend to become very quickly ionized by the surrounding plasma.) This, effectively, justifies the concentration of the study on frozen-pellet fueling while, at least temporarily, eliminating further consideration of the liquid-jet and surface-gas schemes.

There have been many methods suggested by which pellet injection may take place and, presently, some of these will be discussed briefly. However, a far more pressing question arises to the surface. How fast, i.e., with what velocity, must a pellet be injected in order to achieve penetration of the reacting body and thereby refuel the tokamak? The model used in this paper to describe the behavior of the pellet is the ORNL Neutral Gas Shielding Model of Foster and Milora.¹ The model has been modified, here, to fit more closely the realities of the CTHR temperature regime of operation. Explanation of the modification will follow in a later section.

The first method for consideration is that of linear-resonance acceleration. In this method,² a pellet of frozen D+T receives a metallic coating

and is, subsequently, electronically charged. It is then accelerated through an electric field to achieve terminal velocity. There are several disadvantages to this system, pertaining to use in a fusion-power facility, as the following illustrates:

1. The metallic coating, received by the pellet initially, enhances bremsstrahlung losses, which, in turn, increases the value of the critical n_T required for fusion breakeven quite dramatically; for example, a 10% increase in bremsstrahlung would be realized from a pellet containing only 0.01% of iron.
2. Very high accelerating potentials are projected for use in such a reactor (~ 100 MV). To assure that there is no electric-field breakdown, such a device is expected to require a length of around 100 m.² The drift tube could be expected to be enormous.

In spite of these disadvantages, however, velocity achievable by this type of system is predicted to be as high as 10^4 m/sec, although no experimental data are available to confirm the theoretical projection.

Another method for consideration is that of freezing a jet of liquid into a rod which is subsequently shattered into pellets.³ The pellets are then allowed to drift towards the target plasma without further acceleration. This method has been tested experimentally to speeds of 100 m/sec and is expected to deliver an ultimate pellet velocity of 2000 m/sec.³

A third possibility for accelerating the pellet lies in striking it with a laser beam on one side.⁴ The resulting ablatant mass causes the pellet to recoil and to be accelerated into the tokamak with an estimated pellet velocity of 10^4 m/sec. Unfortunately, no experiments have been made to test this idea.⁴

A fourth possibility is to accelerate the pellet mechanically through the use of rotating arbor.^{5,6} The primary disadvantage of this method lies in the hazard to the integrity of the vacuum caused by the rapidly moving

arbor. This would necessitate a protective shield for the vacuum vessel. Terminal velocity is anticipated to exceed 1000 m/sec.⁵

The final method to be discussed here is that of using a light-gas gun to accelerate the pellet.^{6,7,8} In this method, a high-pressure propellant gas strikes a disk and accelerates disk and pellet towards the reacting body. The experimental velocity achieved thus far by this method is ~330 m/sec,⁷ which is higher than that for any other method studied. This method has been projected to achieve velocities of 6000 m/sec.⁸

As this is a conceptual-design paper for a possible fueling system of CTHR, it is requisite that a choice be made of methods by which the scheme of pellet-injection may be achieved. On the basis of inherent advantages and disadvantages (including lack of experimental evidence) and projected capabilities, a choice has been made for study in this paper. The system selected and recommended for use in the CTHR design is that of the light-gas gun for pellet injection.

2.0 Fuel Scheme Modeling (Ablation Modeling)

The model chosen to simulate the behavior of the pellet in the fusion plasma is the ORNL Neutral Gas Shield Model. It is chosen for two reasons. First, the physical concepts upon which it is based seem logical; and the concomitant mathematical formulation is reasonably simple to allow numerical solution without excessive effort. Second, results of the theory in the lower-temperature regime appear to be verified by experiment.⁷ Thus, it is logical to apply this theory to the commercial-reactor temperature regime. All equations to be presented here are taken from reference #1, in one form or another.

First, let us examine the equation which actually predicts the pellet velocity, u .

$$u = K \int_{\left(\frac{r}{a}\right)}^1 \left[\left(\frac{J}{E_{\infty} - E_0} \right) \int_{E_0}^{E_{\infty}} L(E) dE \right]^{1/3} \int_{E_0}^{E_{\infty}} \frac{dE}{L(E)} d\left(\frac{r}{a}\right); \quad (1a)$$

where

$$L(E) = (2.35 \times 10^{18} + 4 \times 10^{15} E + 2 \times 10^{21} / E^2)^{-1}; \quad (1b)$$

$$K = \left(\frac{10}{3} \right) \left(\frac{m_D}{\rho_D} \right) \left[f(\xi) \xi^{1/3} \right]^{-1} \left[\left(\frac{\gamma - 1}{2} \right) \left(\frac{e}{m_H} \right) \right]^{1/3} r_p^{-5/3} \frac{a}{\mu^{1/3}}; \quad (1c)$$

$$J = n_{\infty} \left[\left(\frac{e}{m_e} \right) \left(\frac{E_{\infty}}{3\pi} \right) \right]^{1/2}; \quad (1d)$$

$$E_{\infty} = \frac{45}{16} \langle T_{\infty} \rangle \left[1 - \left(\frac{r}{a} \right)^2 \right]^2; \quad (1e)$$

$$n_{\infty} = \frac{3}{2} \langle n_{\infty} \rangle \left[1 - \left(\frac{r}{a} \right)^2 \right]; \text{ and} \quad (1f)$$

$$E_0 \doteq 10 \text{ eV}. \quad (1g)$$

There are two terms in the above equations which are not direct physical constants or CTHR parameters. The first is ξ and the second is $f(\xi)$.

The factor ξ is a quantity referred to by Foster and Milora as a heating parameter and is defined thus:

$$\xi \equiv \frac{\gamma-1}{2\mu} e^{\frac{L(E)}{m_H}} \frac{r_o}{v_o^3} J \quad (2)$$

where E is in eV and $L(E)$ is in eV m².

The second factor $f(\xi)$ is defined by the following expression:

$$f(\xi) = \int_1^\infty \delta \, d\hat{r}$$

Solving this equation is obviously dependent upon the function of $\hat{r}$ that δ happens to assume. From conservation of flux, momentum flux, and energy flux principles Foster and Milora derive the following coupled differential equations for the reduced density, δ and the Mach number, M as a function of the reduced position $\hat{r}$:

$$\frac{dM}{d\hat{r}} = \frac{M}{1-M^2} \left[\xi (\delta \hat{r}^2)^3 M^2 (1 + \gamma M^2) - \frac{2 + (\gamma-1)M^2}{\hat{r}} \right] \quad (3a)$$

$$\frac{d\delta}{d\hat{r}} = \frac{2M^2\delta}{1-M^2} \left[\frac{1}{\hat{r}} - \xi (\delta \hat{r}^2)^3 \right] \quad (3b)$$

Note: This last equation may be derived from equation 15 of Reference # 1, but does not actually appear in this form in Foster and Milora's report. See Appendix I for derivation and details of solution.

To these equations, we have the following boundary conditions:

$$(i) \quad \delta(\hat{r}=1) = 1$$

$$(ii) \quad \left. \frac{dM}{d\hat{r}} \right|_{\hat{r}=\infty} = 0 \text{ or } M(\hat{r}=\infty) = \sqrt{5/\gamma}$$

It is this system of equations (1, 2, 3a, and 3b) that must be solved in order to model the pellet ablation process. In order to solve equations (3a) and (3b) numerically, it is necessary to have $\rho(\hat{r}=1)$ and $M(\hat{r}=1) = M_0$. These equations may be solved with the available information in one of two ways. The first is to guess an initial value of the Mach number and, thereupon, use a numerical integration and shoot for the appropriate solution. As little or nothing is known about the initial Mach number, this can become a great waste of computer time. To this end, another method was developed to determine the appropriate value of the initial Mach number. Due to the complex nature of the mathematics used in this method, the details are relegated to Appendix II.

Initially, equations (3a) and (3b) were solved for two cases. The first case was a reference case solved previously for $\xi = 1000$.¹ The second case is for the value of ξ given by CTHR parameters. Comparison of the two solutions may be made by examining Figures 1 and 2. What we shall actually compare is the value of $f(\xi)^{-1} \xi^{-1/3}$, for it is this constant upon which the velocity distribution will vary. In the reference case, Foster and Milora obtained a value of 1.23 for $f(\xi)^{-1} \xi^{-1/3}$; whereas the value obtained here is 1.156. For details of solution, Appendix I may be consulted.

These equations were next solved simultaneously for the parameters of CTHR. A value for $f(\xi)^{-1} \xi^{-1/3}$ of about 1.0978 was obtained. The pellet velocity requirement as a function of the quantity (ℓ/a) (of equation (1)) is plotted in Figure 3.

Now we are ready to consider design of the fuel injection system.

3.0 Fuel Injector Design

3.1 Gas Gun Design

The light gas gun is the type of design chosen for the fuel-injection system of CTHR, whose pertinent plasma dimensions are given by a minor radius of 1.40 m, an aspect ratio of 4.33, and an elongation of 1.60. The conceptual design is shown in Figs. 4 and 5. It is a system which is easily used in rapid-fire mode, say, one to three pellets per second. The anticipated average firing rate is one pellet per second. The size of the pellet is such that for a pellet fueling rate of three per second, the reactor will be refueled every 1.64 seconds (the plasma confinement time for CTHR). Calculations show that this is about 6.146 mm in diameter (assuming spherical geometry).

The final parameter required for this design is the pressure used by the light gas gun. Since the required pellet velocity is a function of the penetration depth and since the required gas pressure depends on the velocity needed, the gas pressure may be expressed as a function of penetration depth.

The gas pressure as a function of injection velocity is given by the following expression:⁹

$$P = \frac{m_{s+p}}{A_s} \frac{u \cdot u_{\max}}{2L} \left(\frac{\gamma-1}{\gamma+1} \right) \left[\left(1 - \frac{u}{u_{\max}} \right)^{\frac{\gamma+1}{\gamma-1}} - 1 \right]$$

(assuming a frictionless injection system). Numerically, with assumed parameters where P is the gas pressure (in Pa) and u is the required pellet velocity (in m/sec), one has for hydrogen gas:

$$P = 5033.7 u \left[\left(1 - \frac{u}{6578.4} \right)^{-6} - 1 \right];$$

and for D+T gas:

$$P = 3187.7 u \left[\left(1 - \frac{u}{4166.0} \right)^{-6} - 1 \right]$$

The required gas pressure is plotted as a function of the penetration fraction in Figures 6 and 7 for elemental hydrogen and a deuterium-tritium mixture, respectively.

This brings up one final question. What type of performance can we expect from a light-gas-gun such as that which is conceptualized here? If the performance of the gun were limited to that already achieved by others,⁷ i.e., a pellet-injection speed of 330 m/sec, the pellet would disappear as a solid entity after penetrating about 25% of the distance to the pressure center (see Fig. 3). The maximum performance to be expected of a light-gas gun employed in the refueling of CTHR is determined by assuming that the maximum applicable propellant pressure is 100 GPa. Thus, the performance limit for the D+T propellant is found from Figs. 7 to 3 to result in a penetration of about 45% of the pressure-center distance, achieved with an injection velocity of approximately 3600 m/sec. Based on Figs. 6 and 3, the limit for the H₂ propellant turns out to be somewhat higher, say a maximum penetration of about 50%, corresponding to an injection velocity of approximately 5500 m/sec. The extremely high pressure assumed above is justified when one notes that this value may be decreased by a factor of 5 to 10 were this design modified such that each shell could be used only for a single shot.

3.2 Fuel Handling System

The fuel handling system prepares the D+T pellets and subsequently, makes them available to the fuel injection system. To make fuel handling simpler, the pellets have been given a cylindrical shape in place

of the spherical one assumed earlier in this report. The net result of this is a cylinder diameter $\sqrt[3]{16/3}$ times the sphere's radius and a height equal to the diameter. The effects of this geometry shift on ablation are unknown but, obviously, expected to be small. [Note that the ratio of surface area to volume is increased only by about 15 percent.]

The fuel handling system works on the following principles (see Fig. 8). Initially, the D+T mixture is cooled by a liquid nitrogen stage. The next stage, which liquifies the fuel, is a liquid helium stage. These discrete steps are suggested since the pellets will be mass produced and liquid helium is more expensive than liquid nitrogen. The very cold D+T mixture (5 to 10 K) is injected into a mold whose temperature is maintained at 4.2 K by the liquid helium until the pellets are frozen. The newly formed pellets are subsequently ejected from the mold by the liquid D+T for the next cycle of pellets. Several of these pellet makers could be in operation simultaneously and quality control may be instituted if and where necessary.

Now that the fuel pellets have been formed, they may be loaded into the fuel injector mechanically. The remaining problem is to implement the system into CTHR.

3.3 Implementation into CTHR

For implementation into CTHR, several criteria must be taken into account:

1. The pellet must take the shortest path possible to the fusion plasma.

2. The amount of radiation shielding used to isolate the system from the environment should be minimized within good safety practices.
3. The fuel injection system must be located at a position in the system to make maintenance feasible.
4. Surrounding systems (and subsystems) must be affected as little as possible by the introduction of the fuel injection system.

These goals are attained as shown in Figures 9 and 10. Figure 9 illustrates the overall system and how each of 3 injector systems may be put into CTHR. In Figure 10, a closeup view is provided, to illustrate maintenance possibilities and to illustrate better radiation-shield requirement and system implementation. As can be seen from Fig. 9, interaction of the fuel injection systems with CTHR is very limited, indeed.

4.0 Conclusions

After rejection of several hot-fueling schemes because of inherent difficulties anticipated in practical implementation, the three possibilities of tokamak refueling with cold fuel were considered: liquid-fuel-jet injection, gas puffing, and frozen-pellet injection. It was concluded that pellet fueling appeared to be the most likely of these three alternatives to succeed as a practical scheme. Several methods by which pellet fueling could be implemented into CTHR were then investigated. This led to the further conclusion that the light-gas-gun approach entailed probably the fewest number of real drawbacks. The ORNL Neutral-Gas-Shielding-Model was used to model pellet ablation in CTHR and a required pellet injection velocity was determined as a function of required penetration-depth in the CTHR plasma. On this basis, a conceptual design for a light-gas-gun fuel-injection system for CTHR was developed and the gas-pressure requirement for the device was determined. Finally, suggested means for implementation into the overall design of CTHR have been discussed.

It must be recalled, however, that the penetration-depth requirement for CTHR has not been firmly established. This penetration-depth requirement will ultimately determine the potential feasibility of the design presented in this report. To this end, such a study is highly recommended.

In conclusion, the findings of this study are as follows:

1. If it is determined that the required penetration depth must be 50% of the pressure center distance or more, the light-gas gun will most probably not succeed in refueling CTHR.
2. If the required penetration is between 45% and 50% of the pressure center distance, the D+T propellant mixture will most likely not

succeed in achieving the required injection velocity, but the light-gas-gun design might be practically feasible with H_2 as the propellant.

3. If the required penetration depth is between 25% and 45% of the pressure center depth, the proposed light-gas-gun injection system should be able to refuel the CTHR, with either propellant.
4. If the required penetration depth is less than 25% of the pressure center distance, the proposed scheme of light-gas gun injection will be able to achieve the necessary penetration velocity based on currently tested and proven state-of-the-art technology.

5.0 References

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Figure 1. Solution to system of coupled differential equations for case of $\xi = 1000$ (reference case - Milora and Foster¹).

Figure 2. Solution to system of coupled differential equations for CTHR case ($\xi = 15,301$).

Figure 3. Required pellet velocity is plotted as a function of the point at which the pellet will be totally ablated, which is known as the penetration fraction or depth.

Figure 4. Fuel injection system for CTHR

Figure 5. The shell for the fuel injector is a 1mm by 30mm dia disk intersected by a cylindrical shell whose thickness is 0.0254 mm and whose inner diameter is 6.1648 mm.

Figure 6. The required gas pressure, utilizing H₂ as propellant, is plotted as a function of the penetration fraction.

Figure 7. The required gas pressure, utilizing DT as propellant, is plotted as a function of the penetration fraction.

Figure 8. The fuel handling system is as diagrammed above. The liquid DT mixture enters the cylindrical chamber (above), is frozen by the liquid He, and on the next cycle is forced out as a pellet by the entering DT(l).

Figure 9. Incorporation of fueling systems into CTHR.

1. TF COIL
2. FUEL SYSTEM SHIELDING
3. FUEL DRIFT TUBE
4. FUELING MECHANISM
5. NEUTRAL BEAM SHIELDING
6. NEUTRAL BEAM DRIFT TUBE

Figure 10. Closeup view of incorporation of pellet injector into CTHR.

Appendix I. Solution of the System of Coupled Differential Equations

The set of coupled differential equations (CDE's) given by Foster and Milora¹ are as follows:

$$(1-M^2) \frac{dM}{d\hat{r}} = \frac{1}{2} \left(\frac{\rho_o}{\rho_s} \right)^3 \left| \frac{dr_o}{dt} \right|^{-3} \frac{e L(E)}{m} J r_o (\gamma-1) \hat{\rho}^3 \hat{r}^6 M^3 (\gamma M^2 + 1) - \frac{2M}{\hat{r}} \left(1 + \frac{\gamma-1}{2} M^2 \right) \quad (1)*$$

$$\frac{d}{d\hat{r}} (\hat{\rho} \hat{r}^2)^{-1} + \frac{\hat{r}^2}{\gamma} \frac{d}{d\hat{r}} (M^2 \hat{\rho} \hat{r}^4)^{-1} = 0 \quad (2)*$$

$$\text{where } \xi \equiv \frac{(\gamma-1)}{2} \left(\frac{\rho_o}{\rho_s} \right)^3 \frac{e L(E)}{m} J r_o \left| \frac{dr_o}{dt} \right|^{-3} \quad (3)*$$

$$\text{and where } r_o^2 \rho_o v_o = -r_o^2 \rho_s \frac{dr_o}{dt} \quad (4)*$$

Equations (1), (2), and (4) represent the laws of conservation of energy, momentum, and mass, respectively, and equation (3) is merely a simplifying definition.

* Equations (1) through (4) are taken from Milora and Foster.¹

First, find an approximate value for ξ valid for the commercial plasma density and temperature regime. Equation (3) may be expressed in the following form (through the use of equation (4), if ξ is given by the average over the reacting body:

$$\xi = \frac{(\gamma-1)}{2} \frac{e \langle L(E) \rangle}{m} \frac{r_o}{v_o} J \quad (5a)*$$

where $J = n \sqrt{ekT/2\pi m_e} \quad (5b)*$

Calculations for ξ show that, for CTHR, this value is approximately 1.53×10^4 .

Now, derive a set of coupled differential equations solvable by computational methods wherein the solution for $\rho(r)$ may again be integrated to provide a numerical value for $\xi^{-1/3} f(\xi)^{-1}$.

Substitution of equation (3) into equation (1) yields the following, upon rearrangement of terms:

$$\frac{dM}{dr} = \frac{M}{1-M^2} [\xi(\delta r^2)^3 M^2 (\gamma M^2 + 1) - \frac{(2 + (\gamma-1) M^2)}{r}] \quad (6)$$

Equation (2) is somewhat more complex to solve for $\rho(r)$ but is done in the following manner. Use the chain rule to obtain the following:

$$\frac{d}{dr} \left(\frac{1}{\delta r^2} \right) + \frac{r^2}{\gamma M^2 r^2} \frac{d}{dr} \left(\frac{1}{\delta r^2} \right) + \frac{r^2}{\gamma \delta r^2} \frac{d}{dr} \left(\frac{1}{M^2 r^2} \right) = 0$$

Rearranging terms:

$$\frac{d}{dr} \left(\frac{1}{\delta r^2} \right) \left[1 + \frac{1}{\gamma M^2} \right] + \frac{1}{\gamma \delta} \frac{d}{dr} \left[\frac{1}{M^2 r^2} \right] = 0$$

*Equations (5a) and (5b) are also taken from Milora and Foster.¹

Again using the chain rule yields:

$$\frac{d}{d\hat{r}} \left(\frac{1}{\beta \hat{r}^2} \right) \left[\frac{\gamma M^2 + 1}{\gamma M^2} \right] + \frac{1}{\gamma \beta M^2} \frac{d}{d\hat{r}} \left(\frac{1}{\hat{r}^2} \right) + \frac{1}{\gamma \beta \hat{r}^2} \frac{d}{d\hat{r}} \left(\frac{1}{M^2} \right) = 0$$

This may be reexpressed as follows by differentiating and then multiplying the equation by $-\gamma M^2 \beta \hat{r}^2$:

$$\frac{1}{(\beta \hat{r}^2)} \frac{d}{d\hat{r}} (\beta \hat{r}^2) [1 + \gamma M^2] + \frac{2}{\hat{r}} + \frac{2}{M} \frac{dM}{d\hat{r}} = 0$$

The following results from using the chain rule of differentiation and combining terms:

$$\frac{d\beta}{d\hat{r}} \left[\frac{1 + \gamma M^2}{\beta} \right] + \frac{2(2 + \gamma M^2)}{\hat{r}} + \frac{2}{M} \frac{dM}{d\hat{r}} = 0$$

If eqn (6) is substituted into the above relationship:

$$\frac{d\beta}{d\hat{r}} \left[\frac{1 + \gamma M^2}{\beta} \right] = - \frac{2(2 + \gamma M^2)}{\hat{r}} - \frac{2}{M} \left[\frac{M}{1 - M^2} \left[\xi(\beta \hat{r}^2)^3 M^2 (\gamma M^2 + 1) - \frac{2 + (\gamma - 1)M^2}{\hat{r}} \right] \right]$$

Upon rearranging terms further, the following is found:

$$\left(\frac{1 + \gamma M^2}{\beta} \right) \frac{d\beta}{d\hat{r}} = - \frac{2}{1 - M^2} \left[\xi(\beta \hat{r}^2)^3 M^2 (1 + \gamma M^2) - \frac{2}{\hat{r}} + \frac{2}{\hat{r}} (1 - M^2) - \frac{(\gamma - 1)M^2}{\hat{r}} + \frac{\gamma M^2}{\hat{r}} (1 - M^2) \right]$$

This may be reexpressed in the following form by combining terms:

$$\left(\frac{1 + \gamma M^2}{\beta} \right) \frac{d\beta}{d\hat{r}} = - \frac{2M^2}{1 - M^2} \left[\xi(\beta \hat{r}^2)^3 (1 + \gamma M^2) - \frac{2}{\hat{r}} + \frac{1}{\hat{r}} - \frac{\gamma M^2}{\hat{r}} \right]$$

Combining terms again in the above equation yields:

$$\left(\frac{1 + \gamma M^2}{\beta} \right) \frac{d\beta}{d\hat{r}} = - \frac{2M^2}{1 - M^2} \left[\xi(\beta \hat{r}^2)^3 (1 + \gamma M^2) - \frac{(\gamma M^2 + 1)}{\hat{r}} \right]$$

Finally, the relationship describing $\rho(r)$ is obtained by solving the above equation for $\frac{d\delta}{df}$.

$$\frac{d\delta}{df} = - \frac{2M^2\delta}{1-M^2} \left[\xi(\delta f^2)^3 - \frac{1}{f} \right] \quad (7)$$

and again:

$$\frac{dM}{df} = \frac{M}{1-M^2} \left[\xi(\delta f^2)^3 M^2(1+\gamma M^2) - \frac{2+(\gamma-1)M^2}{f} \right] \quad (6)$$

The boundary condition on the above system of equations (cf. Foster and Milora)* are as follows:

$$\delta(f=1) = 1 \quad (8a)$$

$$M(f=\infty) = \sqrt{5/\gamma} \quad (8b)$$

This system of equations may be solved in the following manner. Since the second boundary condition, shown above, is somewhat inappropriate, two possibilities exist for the solution of eqns (6) and (7). These are:

- (i) Shoot a solution by assuming a value for $M(r=1)$; or
- (ii) Solve the above system of equations for $M(r=1)$ but as this is a somewhat complicated procedure, it is described more or less fully in Appendix II.

In any case, equations (6) and (7) may be solved simultaneously to find $\rho(r)$ (and $M(r)$) and finally, $\rho(r)$ may be integrated to yield a value for $f(\xi)$ and thus, $\xi^{-1/3} f(\xi)^{-1}$. This must be done numerically, since equations (6) and (7) are sufficiently complicated that, were they uncoupled, it is doubtful that they could be solved in closed form, but they can be solved.

In the computer program that follows, Hamming's fifth order general predictor-corrector has been used with a fourth order Runge-Kutta-Gill starter to solve the CDEs. The program then solves for $\xi^{-1/3} f(\xi)^{-1}$ and uses this quantity to determine pellet velocity requirements. It was this program that

generated results for figures 1 and 2 (solutions of equations (6) and (7) for $\xi = 1000$ and $\xi = 1.53 \times 10^4$) and figure 3, velocity requirement as a function of pellet penetration. Pressure requirement figures were generated by a separate computer program, not supplied here.


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C*****A0001
C***A0002
C***A0003
C*** PROGRAM BY KENNETH D. MATNEY. A0004
C***A0005
C***A0006
C*** THE ULTIMATE PURPOSE OF THIS PROGRAM IS TO DETERMINE FUEL PELLET A0007
C*** VELOCITY REQUIREMENTS FOR TOKAMAK TYPE REACTORS USING AN EQUAL A0008
C*** PARTS MIXTURE OF DEUTERIUM AND TRITIUM. A0009
C***A0010
C***A0011
C*****A0012
ISN 0002      IMPLICIT REAL*8(A-H,O-Z) A0013
ISN 0003      REAL EW(8),D,ER,MACH(2005),DENS(2005),RPCS(2005) A0014
ISN 0004      COMMON/FOATA/PSI,GAMMA,T,A,B,C,DARG A0015
ISN 0005      COMMON/HAMDAT/CRIT,IFRKG,LG A0016
ISN 0006      COMMON R(1005),Y(1005,2),O(2),XINIT A0017
ISN 0007      DIMENSION W1(2),W2(2),W3(2),W4(2),W(82),X(82),Y(2),Y1(2),Y2(8) A0018
ISN 0008      2 FORMAT(2(034.27,6X)) A0019
ISN 0009      3 FORMAT(1P022.15,8X,022.15,13X,'MACH DATA ',15) A0020
ISN 0010      4 FORMAT(1P022.15,8X,022.15,13X,'DENS DATA ',15) A0021
ISN 0011      5 FORMAT(4(016.9,4X)) A0022
ISN 0012      6 FORMAT(16,4X,021.14) A0023
ISN 0013      10 FORMAT(///) A0024
ISN 0014      11 FORMAT(1H1,115X,'PAGE ',15) A0025
ISN 0015      12 FORMAT(1H0,5X,'THE VALUE OF THE LOSS FUNCTION IS ',1P025.18) A0026
ISN 0016      13 FORMAT(1H0,5X,'THE VALUE OF THE PELLET RADIUS IS ',1P025.18/6X, A0027
        $,'THE VALUE OF THE ABLATANT VELOCITY AT THE BOUNDARY IS ',025.18) A0028
ISN 0017      14 FORMAT(1H0,5X,'THE VALUE OF THE HEATING PARAMETER IS ',1P025.18) A0029
ISN 0018      15 FORMAT(1H0,5X,'THE STABILITY POINTS ARE ',1P025.18,' AND ',025.18) A0030
ISN 0019      17 FORMAT(1H0,5X,'THE VALUE OF THE DESIRED CONSTANT IS ',1P025.18) A0031
ISN 0020      18 FORMAT(1H0,5X,'THE BOUNDARY MACH NUMBER IS ',1P025.18/ A0032
        $6X,'THE CONVERGENCE EFFICIENCY IS ',025.18) A0033
ISN 0021      19 FORMAT(1H0,5X,14,7X,1P025.18,2(10X,025.18)) A0034
ISN 0022      20 FORMAT( 6X,'TABLE',14,'. INTEGRATION DATA.'/6X,29(1H-)) A0035
ISN 0023      21 FORMAT( 6X,'TABLE',14,'. INTEGRATION DATA -- CONTINUED.'/6X,42(1H A0036
        $-)) A0037
ISN 0024      22 FORMAT(1H0,5X,'R(',16,') = ',1P025.18,', WHEN H = ',025.18,', AND' A0038
        $,'/6X,'RHO = ',025.18,', WHERE OM/OM' = ',025.18) A0039
ISN 0025      23 FORMAT(1H0/1H0,5X,'*F(PSI) = ',1P025.18) A0040
ISN 0026      24 FORMAT(1H0,5X,'STEP',5X,'VALUE OF THE RELATIVE POSITION',5X,'VALU A0041
        $E OF THE RELATIVE DENSITY ',5X,'VALUE OF ABLATANT MACH NUMBER '/ A0042
        $6X,4(' '),3(5X,30(' '))) A0043
ISN 0027      25 FORMAT(16X,'THE NUMBER OF INTEGRATIONS PERFORMED THIS STEP = ',16) A0044
ISN 0028      26 FORMAT(6X,'FOR THE PARAMETER L/A = ',1P022.15,':') A0045
ISN 0029      27 FORMAT(1H0,5X,'THE VALUE OF THE PELLET VELOCITY MUST BE ',1P022.15 A0046
        $,' TIMES THE PRESSURE CENTER DISTANCE.') A0047
ISN 0030      28 FORMAT(1P022.15,3X,022.15,11X,'PELLET SPEED',15) A0048
ISN 0031      FLIE)=1.0+00/(2.35D+18+4.0+15+E+2.0+21/E/E) A0049
ISN 0032      NO=1005 A0050
ISN 0033      LG=0 A0051
ISN 0034      IPAGE=1 A0052
ISN 0035      ITABLE=1 A0053
ISN 0036      K=0 A0054
ISN 0037      NSTART=1 A0055
ISN 0038      WRITE(6,11)IPAGE A0056

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ISN 0039	IPAGE=IPAGE+1	A0057
ISN 0040	READ(5,6)N	A0058
ISN 0041	DO 100 J=1,N,1	A0059
ISN 0042	READ(5,2)X(J),W(J)	A0060
ISN 0043	X(J+N)=(1.0+00+X(J))/2.0+00	A0061
ISN 0044	X(J)=(1.0+00-X(J))/2.0+00	A0062
ISN 0045	W(J)=W(J)/2.0+00	A0063
ISN 0046	W(J+N)=W(J)	A0064
ISN 0047	100 CONTINUE	A0065
ISN 0048	NSAVE=N+N	A0066
ISN 0049	N=NSAVE	A0067
ISN 0050	READ(5,5)EMAX,DN,VO,RP	A0068
ISN 0051	E=1.60218920-19	A0069
ISN 0052	XM=(2.014102220+00+3.016049720+00)*1.6605710-27	A0070
ISN 0053	PI=3.1415926535897930+00	A0071
ISN 0054	GAMMA=1.40+00	A0072
ISN 0055	PSI=0.0+00	A0073
ISN 0056	DO 105 J=1,N,1	A0074
ISN 0057	PSI=PSI+W(J)*FL(EMAX*X(J))	A0075
ISN 0058	105 CONTINUE	A0076
ISN 0059	WRITE(6,12)PSI	A0077
ISN 0060	WRITE(6,13)RP,VO	A0078
ISN 0061	PL=PSI*(GAMMA-1.0+00)/2.0+00+E/XM*RP/VO**3*OSQRT(E*EMAX/3.0+00/PI	A0079
	S/1.6605710-27/548.5790-06)*ON	A0080
ISN 0062	T=MAX*2.0+00/3.0+00	A0081
ISN 0063	READ(5,2)ALSAVE,A2SAVE	A0082
ISN 0064	S=-1.0+00/3.0+00	A0083
ISN 0065	CRIT=OSQRT(5.0+00/GAMMA)	A0084
ISN 0066	READ(5,6)NSAV,HSAY	A0085
ISN 0067	READ(5,6)NTABLE	A0086
ISN 0068	110 CONTINUE	A0087
ISN 0069	READ(5,2,END=999)XMACH,DELTA	A0088
C****		****A0N01
C****		****A0N02
C****	THE PURPOSE OF THIS, AN OBVIOUS ADDITION TO THE MAIN PROGRAM, IS	****A0N03
C****	TO REDUCE THE CPU BUT MAINTAIN THE INTEGRITY IN COMPUTATION.	****A0N04
C****	SINCE THE FOLLOWING SECTION ONLY COMPUTES THE VALUE OF THE	****A0N05
C****	VARIABLE, CONST; IT IS SUBMITTED HERE AS COMPUTED IN EARLIER RUNS	****A0N06
C****	IN THE FORM OF A DATA STATEMENT. THE SECTION OF PROGRAM WHICH	****A0N07
C****	FOLLOWS IS THEREFORE BYPASSED, YIELDING A CPU SAVINGS OF ABOUT	****A0N08
C****	90%.	****A0N09
C****		****A0N10
C****		****A0N11
ISN 0070	DATA CONST/1.09776345588615600/	A0N12
ISN 0071	IF(IPAGE.GT.0)GO TO 337	A0N13
ISN 0073	ROSAV=1.0+00	A0089
ISN 0074	RHOSAV=1.0+00	A0090
ISN 0075	NINC=5.0-3/HSAY+5.0-3	A0091
ISN 0076	IF(NINC.LT.1)NINC=1	A0092
ISN 0078	NEWTAB=0	A0093
ISN 0079	INITIL=1	A0094
ISN 0080	H=HSAY	A0095
ISN 0081	RO=ROSAV	A0096
ISN 0082	YO(1)=XMACH	A0097
ISN 0083	YO(2)=RHOSAV	A0098
ISN 0084	FPSI=0.0+00	A0099
ISN 0085	NC=2	A0100
ISN 0086	A1=ALSAVE	A0101

ISN 0087	A2=A2SAVE	A0102
ISN 0088	ER=1.E-14	A0103
ISN 0089	IFRKG=-1	A0104
ISN 0090	WRITE(6,14)PSI	A0105
ISN 0091	WRITE(6,15)A1SAVE,A2SAVE	A0106
ISN 0092	WRITE(6,19)INITIL,ROSAV,RHCSAV,XMACH	A0107
ISN 0093	WRITE(6,23)FPSI	A0108
ISN 0094	WRITE(6,11)IPAGE	A0109
ISN 0095	IPAGE=IPAGE+1	A0110
ISN 0096	NSTORE=0	A0111
ISN 0097	IF(NEWTAB.EQ.0)WRITE(6,20)ITABLE	A0112
ISN 0099	IF(NEWTAB.EQ.1)WRITE(6,21)ITABLE	A0113
ISN 0101	WRITE(6,24)	A0114
ISN 0102	NEWTAB=1	A0115
ISN 0103	LINES=0	A0116
ISN 0104	120 CONTINUE	A0117
ISN 0105	N=1003	A0118
ISN 0106	CALL HAMMIN(RO,YO,NC,N,H,A1,A2,ER,R,Y,I,Y2,Y1,W1,W2,W3,W4,O,EW,NC)	A0119
ISN 0107	NSTORE=NSTORE+1000	A0120
ISN 0108	N=1001	A0121
ISN 0109	DO 200 J=3,N,2	A0122
ISN 0110	FPSI=FPSI+(Y(J-2,2)+4.0+00*Y(J-1,2)+Y(J,2))*CABS(HI)/3.0+00	A0123
ISN 0111	200 CONTINUE	A0124
ISN 0112	DO 300 J=NSTART,N,NINC	A0125
ISN 0113	WRITE(6,19)J,R(J),Y(J,2),Y(J,1)	A0126
ISN 0114	K=K+1	A0127
ISN 0115	RPOS(K)=R(J)	A0128
ISN 0116	MACH(K)=Y(J,1)	A0129
ISN 0117	GENS(K)=Y(J,2)	A0130
ISN 0118	LINES=LINES+1	A0131
ISN 0119	IF(LINES-20)300,250,250	A0132
ISN 0120	250 CONTINUE	A0133
ISN 0121	IF(J+125 -N)275,300,300	A0134
ISN 0122	275 CONTINUE	A0135
ISN 0123	WRITE(6,11)IPAGE	A0136
ISN 0124	IPAGE=IPAGE+1	A0137
ISN 0125	WRITE(6,21)ITABLE	A0138
ISN 0126	WRITE(6,24)	A0139
ISN 0127	LINES=0	A0140
ISN 0128	300 CONTINUE	A0141
ISN 0129	N=N-1	A0142
ISN 0130	IFRKG=IFRKG+1	A0143
ISN 0131	DO 320 J=1,4,1	A0144
ISN 0132	DO 310 L=1,NC,1	A0145
ISN 0133	Y(J,L)=Y(J+N,L)	A0146
ISN 0134	310 CONTINUE	A0147
ISN 0135	R(J)=R(J+N)	A0148
ISN 0136	320 CONTINUE	A0149
ISN 0137	NSTART=NINC+1	A0150
ISN 0138	IF(NSTORE.GE.NSAV)GO TO 330	A0151
ISN 0140	GO TO 120	A0152
ISN 0141	330 CONTINUE	A0153
ISN 0142	WRITE(6,11)IPAGE	A0154
ISN 0143	IPAGE=IPAGE+1	A0155
ISN 0144	YNEFF=Y(N,1)/CRIT*1.02	A0156
ISN 0145	WRITE(6,18)XMACH,YNEFF	A0157
ISN 0146	WRITE(6,25)NSTORE	A0158
ISN 0147	DMOR=F(1,R,Y,N)	A0159

ISN 0148	WRITE(6,22)NSTORE,R(N1,Y(N,1),Y(N,2),DMOR	A0160
ISN 0149	VALUE=3.00/4.00/(1.00+(GAMMA-1.00)*Y(N,1)**2)/PSI/Y(N,1)**2/(1.00	A0161
	+GAMMA*Y(N,1)**2)**S	A0162
ISN 0150	FPSI=FPSI+R(N1)**(4.00*S)*VALUE	A0163
ISN 0151	CONST=PSI**S/FPSI	A0164
ISN 0152	WRITE(6,23)FPSI	A0165
ISN 0153	WRITE(6,17)CONST	A0166
ISN 0154	IF(YNEFF.LT.90.)GO TO 345	A0167
ISN 0156	DO 333 IP=1,K,1	A0168
ISN 0157	PUNCH 3,RPGS(IP),MACH(IP),IP	A0169
ISN 0158	333 CONTINUE	A0170
ISN 0159	DO 336 IP=1,K,1	A0171
ISN 0160	PUNCH 4,RPOS(IP),OENS(IP),IP	A0172
ISN 0161	336 CONTINUE	A0173
ISN 0162	337 CONTINUE	A0174
ISN 0163	WRITE(6,11)IPAGE	A0175
ISN 0164	IPAGE=IPAGE+1	A0176
ISN 0165	XINIT=0.000	A0177
ISN 0166	ICOUNT=0	A0178
ISN 0167	XMD=2.0141022200*1.6605710-27	A0179
ISN 0168	XME=548.5790-6*1.6605710-27	A0180
ISN 0169	XMH=1.0086652200*1.6605710-27	A0181
ISN 0170	A=CONST/(PI*XME)**(1.00/6.00)*(XMD/2.05902)**(4.00/9.00)*OSQRT(E)*	A0182
	S(2.35019)**(2.00/3.00)*(4.00*PI/3.00)**(5.00/9.00)*(1.500/1.600)**	A0183
	S(7.00/6.00)*(3.00/XMH)**(1.00/3.00)/T**((1.00/6.00)	A0184
	B=9.00/3.7603	A0185
ISN 0171	C=-4.09602/3.80700	A0186
ISN 0172	DARG=2.820801/1.057500	A0187
ISN 0173	FVOL=4.00*PI/3.00*RP*RP*RP*2.05902/DN/XMD	A0188
ISN 0174	338 CONTINUE	A0189
ISN 0175	WRITE(6,26)XINIT	A0190
ISN 0176	H=1.0-2	A0191
ISN 0177	V1=G(T,H)/(XM/XMH)**(1.00/3.00)/FVOL**((5.00/9.00)/DN**((2.00/9.00)	A0192
ISN 0178	WRITE(6,27)V1	A0193
ISN 0179	H=1.0-3	A0194
ISN 0180	V2=G(T,H)/(XM/XMH)**(1.00/3.00)/FVOL**((5.00/9.00)/DN**((2.00/9.00)	A0195
ISN 0181	WRITE(6,27)V2	A0196
ISN 0182	XPENET=1.00-XINIT	A0197
ISN 0183	ICOUNT=ICOUNT+1	A0198
ISN 0184	PUNCH 28,XPENET,V2,ICOUNT	A0199
ISN 0185	XINIT=1.00-XPENET*2.0-1*1.0-2	A0200
ISN 0186	IF(MCD(ICOUNT,6).EQ.0)GO TO 339	A0201
ISN 0187	WRITE(6,10)	A0202
ISN 0188	GO TO 340	A0203
ISN 0189	339 CONTINUE	A0204
ISN 0190	WRITE(6,11)IPAGE	A0205
ISN 0191	IPAGE=IPAGE+1	A0206
ISN 0192	340 CONTINUE	A0207
ISN 0193	IF(V2.GT.1.02)GO TO 338	A0208
ISN 0194	345 CONTINUE	A0209
ISN 0195	WRITE(6,11)IPAGE	A0210
ISN 0196	IPAGE=IPAGE+1	A0211
ISN 0197	ITABLE=ITABLE+1	A0212
ISN 0198	GO TO 110	A0213
ISN 0199	999 CONTINUE	A0214
ISN 0200	STOP	A0215
ISN 0201	END	A0216
ISN 0202		
ISN 0203		
ISN 0204		

```

C*****FUN01
C***FUN02
C***FUN03
C*** THE PURPOSE OF F IS TO PROVIDE THE COUPLED DIFFERENTIAL ***FUN04
C*** EQUATIONS REQUIRED BY RKGSQ AND HAMMIN. COMMON BLOCK FOATA IS ***FUN05
C*** REQUIRED. S IS THE VALUE OF PSI AND G IS THE VALUE OF GAMMA. ***FUN06
C*** R IS THE DISTANCE FROM PELLET CENTER, RELATIVE TO PELLET RADIUS. ***FUN07
C*** ***FUN08
C*** STATEMENT NO. PURPOSE OF STATEMENT ***FUN09
C*** ----- ***FUN10
C*** ***FUN11
C*** 10 DIFFERENTIAL EQUATION FOR MACH NUMBER, M ***FUN12
C*** 20 DIFFERENTIAL EQUATION FOR RELATIVE DENSITY, P ***FUN13
C*** 30 TEST FOR CONTINUITY AT M = 1.0+00 ***FUN14
C*** 40 EQUATION FOR G(T,H) ***FUN15
C*** ***FUN16
C*** ***FUN17
C*****FUN18
ISN 0002 FUNCTION F(L,X,Y,J) FUN19
ISN 0003 IMPLICIT REAL*8(A-H,O-Z) FUN20
ISN 0004 REAL*8 M FUN21
ISN 0005 COMMON/FOATA/S,G,T,A,B,C,D FUN22
ISN 0006 COMMON/HAMCAT/CRIT,IFRKG,LG FUN23
ISN 0007 DIMENSION X(1),Y(1005,1) FUN24
ISN 0008 CAPX(T,RA)=(1.00-(RA)**2)**2*T FUN25
ISN 0009 IF(LG.EQ.4)GO TO 40 FUN26
ISN 0010 A=1.0+00 FUN27
ISN 0011 B=2.0+00 FUN28
ISN 0012 M=Y(J,1) FUN29
ISN 0013 P=Y(J,2) FUN30
ISN 0014 R=X(J) FUN31
ISN 0015 IF(L-1)1,10,20 FUN32
ISN 0016 1 WRITE(6,6)J,L FUN33
ISN 0017 6 FORMAT(1H0,2X,'...ERROR...AT STEP = ',I3,', DIFF EQ CALLED = ',I3) FUN34
ISN 0018 F=0.C+00 FUN35
ISN 0019 RETURN FUN36
ISN 0020 10 F=(S*(P*M*R**3*(A+G*M*M)-M/R*(B+(G-A)*M*M))/(A-M*M) FUN37
ISN 0021 RETURN FUN38
ISN 0022 20 CONTINUE FUN39
ISN 0023 IF(L-3)25,30,1 FUN40
ISN 0024 25 F=B*P*M*M/(A-M*M)*(A/R-S*(P*R**3) FUN41
ISN 0025 RETURN FUN42
ISN 0026 30 CONTINUE FUN43
ISN 0027 F=(S*(P*M*R**3*(A+G*M*M)-M/R*(B+(G-A)*M*M) FUN44
ISN 0028 RETURN FUN45
ISN 0029 40 CONTINUE FUN46
ISN 0030 RA=X(J) FUN47
ISN 0031 IF(RA.GT.9.999999990-01)GO TO 45 FUN48
ISN 0032 ONUM=CAPX(T,RA)+B*CAPX(T,RA)**2+C/CAPX(T,RA)+D FUN49
ISN 0033 DENO=(1.00/CAPX(T,RA)+2-4.00*C/CAPX(T,RA)**3)**(1.00/3.00) FUN50
ISN 0034 F=ONUM/DENO*A FUN51
ISN 0035 RETURN FUN52
ISN 0036 45 CONTINUE FUN53
ISN 0037 F=0.C0 FUN54
ISN 0038 RETURN FUN55
ISN 0039 END FUN56
ISN 0040
ISN 0041

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QS/360 FORTRAN H

	C*****	***GTH01
	C***	***GTH02
	C***	***GTH03
	C*** THE PURPOSE OF THIS FUNCTION IS TO AID IN THE CALCULATION OF THE	***GTH04
	C*** REQUIRED PELLET VELOCITY BY CORRECTING FOR TEMPERATURE AND	***GTH05
	C*** DENSITY DISTRIBUTION EFFECTS.	***GTH06
	C***	***GTH07
	C***	***GTH08
	C*****	***GTH09
ISN 0002	FUNCTION G(T,H)	GTH10
ISN 0003	IMPLICIT REAL*8(A-O,F-H,C-Z)	GTH11
ISN 0004	REAL*8 K1(1),K2(1),K3(1),K4(1)	GTH12
ISN 0005	COMMON/HAMOAT/CRIT,IFRKG,L	GTH13
ISN 0006	COMMON X(1005),Y(1005,2),E(2),XINIT	GTH14
ISN 0007	M=1005	GTH15
ISN 0008	NC=1	GTH16
ISN 0009	L=4	GTH17
ISN 0010	X(1)=XINIT	GTH18
ISN 0011	Y(1,1)=0.00	GTH19
ISN 0012	NSAV=(1.00-X(1))/H+1.100	GTH20
ISN 0013	50 CONTINUE	GTH21
ISN 0014	N=NSAV	GTH22
ISN 0015	IF(NSAV.GT.1000)N=1000	GTH23
ISN 0017	DO 100 J=2,N,1	GTH24
ISN 0018	CALL RKGSQ(J,X,Y,NC,H,E,K1,K2,K3,K4,M)	GTH25
ISN 0019	100 CONTINUE	GTH26
ISN 0020	IF(NSAV.LE.1000)GO TO 150	GTH27
ISN 0022	NSAV=NSAV-999	GTH28
ISN 0023	X(1)=X(N)	GTH29
ISN 0024	Y(1,1)=Y(N,1)	GTH30
ISN 0025	GO TO 50	GTH31
ISN 0026	150 CONTINUE	GTH32
ISN 0027	HNEW=1.00-X(N)	GTH33
ISN 0028	N=N+1	GTH34
ISN 0029	CALL RKGSQ(N,X,Y,NC,HNEW,E,K1,K2,K3,K4,M)	GTH35
ISN 0030	G=Y(N,1)	GTH36
ISN 0031	RETURN	GTH37
ISN 0032	END	GTH38

OS/360 FORTRAN H

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C*****PC005
C***PC010
C***PC015
C*** THE PURPOSE OF HAMMIN IS TO SOLVE A SET OF SIMULTANECUS FIRST PC020
C*** ORDER FUNCTIONALS OF Y WITH RESPECT TO X USING HAMMING'S FIFTH PC025
C*** ORDER PREDICTOR-CORRECTOR. FOURTH ORDER RUNGA-KUITA-GILL PC030
C*** PROVIDES STARTING VALUES WHICH ARE ITERATED UPON BY THREE OTHER PC035
C*** INTEGRATION FORMULAEES UNTIL A CONSISTENT STARTER IS OBTAINED. PC040
C***PC045
C***PC050
C***PC055
C***PC060
C***PC065
C***PC070
C***PC075
C***PC080
C***PC085
C***PC090
C***PC095
C***PC100
C***PC105
C***PC110
C***PC115
C***PC120
C***PC125
C***PC130
C***PC135
C***PC140
C***PC145
C***PC150
C***PC155
C***PC160
C***PC165
C***PC170
C***PC175
C***PC180
C***PC185
C***PC190
C***PC195
C***PC200
C***PC205
C*****PC210
C***SUBROUTINE HAMMIN(XO,YO,NC,N,H,A1,A2,ERR,X,Y,IER,YP,YC,W1,W2,W3,W4 PC215
C***S,E,ER,NO) PC220
C***REAL*8 XO,YO,H,A1,A2,X,Y,YP,YC,W1,W2,W3,W4,C1,C2,C3,C4,C5,C6,C7,EP PC225
C***S,EC,A0,B0,B1,B2,B3,BCN1,BC0,BC1,BC2,CRIT,F PC230
C***COMMON/HAMCAT/CRIT,IFRKG,LC333 PC235
C***DIMENSION YO(1),X(1),Y(ND,1),YP( 4,1),YC(1),E(1),W1(1),W2(1),W3(1) PC240
C***S,W4(1),ER( 4,1) PC245
C***PC250
C***PC255
C***PC260
C***PC265
C***PC270
C***PC275
C***PC280

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ISN 0002

ISN 0003

ISN 0004

ISN 0005

ISN 0006

ISN 0007

ISN 0008

ISN 0009

ISN 0010

ISN 0011

ISN 0012

ISN 0013	C7=8.0+00	PC285
ISN 0014	EP=(2.510+02-C2*A1-C7*A2)/6.0+00	PC290
ISN 0015	EC=(-C2+11.0+00*A1-C7*A2)/6.0+00	PC295
ISN 0016	A0=1.0+00-A1-A2	PC300
ISN 0017	B0=(55.0+00+C1*A1+C7*A2)/C4	PC305
ISN 0018	B1=(-59.0+00+C2*A1+32.0+00*A2)/C4	PC310
ISN 0019	B2=(37.0+00-C3*A1+C7*A2)/C4	PC315
ISN 0020	B3=(A1-C1)/C4	PC320
ISN 0021	BCN1=(C1-A1)/C4	PC325
ISN 0022	BC0=(C2+13.0+00*A1+C7*A2)/C4	PC330
ISN 0023	BC1=(-C3+13.0+00*A1+32.0+00*A2)/C4	PC335
ISN 0024	BC2=(1.0+00-A1+C7*A2)/C4	PC340
ISN 0025	IF(IFRKG.GE.0)GO TO 155	PC345
ISN 0027	X(1)=X0	PC350
ISN 0028	DO 100 L=1,NC,1	PC355
ISN 0029	Y(1,L)=Y0(L)	PC360
ISN 0030	100 CONTINUE	PC365
ISN 0031	DO 115 J=2,4,1	PC370
ISN 0032	CALL RKGS0Q(J,X,Y,NC,H,E,W1,W2,W3,W4,N0)	PC375
ISN 0033	115 CONTINUE	PC380
ISN 0034	JCHECK=0	PC385
ISN 0035	120 CONTINUE	PC390
ISN 0036	JCHECK=JCHECK+1	PC395
ISN 0037	DO 130 L=1,NC,1	PC400
ISN 0038	YP(2,L)=Y(1,L)+H*(C1*F(L,X,Y,1)+C2*F(L,X,Y,2)-C3*F(L,X,Y,3)+F(L,X,Y,4))/C4	PC405
ISN 0039	YP(3,L)=Y(1,L)+H*(F(L,X,Y,1)+C5*F(L,X,Y,2)+F(L,X,Y,3))/C6	PC410
ISN 0040	YP(4,L)=Y(1,L)+H*(F(L,X,Y,1)+C6*(F(L,X,Y,2)+F(L,X,Y,3))+F(L,X,Y,4))/C7*C6	PC415
ISN 0041	DO 125 J=2,4,1	PC420
ISN 0042	ER(J,L)=(Y(J,L)-YP(J,L))/Y(J,L)	PC425
ISN 0043	ER(J,L)=ABS(ER(J,L))	PC430
ISN 0044	125 CONTINUE	PC435
ISN 0045	130 CONTINUE	PC440
ISN 0046	ITEST=0	PC445
ISN 0047	DO 140 L=1,NC,1	PC450
ISN 0048	DO 135 J=2,4,1	PC455
ISN 0049	IF(ER(J,L).GT.ERR)ITEST=1	PC460
ISN 0051	Y(J,L)=YP(J,L)	PC465
ISN 0052	135 CONTINUE	PC470
ISN 0053	140 CONTINUE	PC475
ISN 0054	IF(JCHECK.GE.500)GO TO 145	PC480
ISN 0056	IF(ITEST.EQ.1)GO TO 120	PC485
ISN 0058	145 DO 150 L=1,NC,1	PC490
ISN 0059	YP(1,L)=0.0+00	PC495
ISN 0060	YC(L)=0.0+00	PC500
ISN 0061	150 CONTINUE	PC505
ISN 0062	155 CONTINUE	PC510
ISN 0063	DO 300 J=4,N,1	PC515
ISN 0064	X(J+1)=X(J)+H	PC520
ISN 0065	DO 175 L=1,NC,1	PC525
ISN 0066	YP(2,L)=YP(1,L)	PC530
ISN 0067	175 CONTINUE	PC535
ISN 0068	DO 200 L=1,NC,1	PC540
ISN 0069	YP(1,L)=A0*Y(J,L)+A1*Y(J-1,L)+A2*Y(J-2,L)+H*(B0*F(L,X,Y,J)+B1*F(L,X,Y,J-1)+B2*F(L,X,Y,J-2)+B3*F(L,X,Y,J-3))	PC545
ISN 0070	200 CONTINUE	PC550
ISN 0071	DO 225 L=1,NC,1	PC555
		PC560
		PC565
		PC570

ISN 0072	Y(J+1,L)=YP(1,L)-(EP/(EP-EC))*(YP(2,L)-YC(L))	PC575
ISN 0073	225 CONTINUE	PC580
ISN 0074	DO 250 L=1,NC,1	PC585
ISN 0075	YC(L)=AO*Y(J,L)+A1*Y(J-1,L)+A2*Y(J-2,L)+H*(BCN1*F(L,X,Y,J+1)+BCO*F	PC590
	S(L,X,Y,J)+BC1*F(L,X,Y,J-1)+BC2*F(L,X,Y,J-2))	PC595
ISN 0076	250 CONTINUE	PC600
ISN 0077	DO 275 L=1,NC,1	PC605
ISN 0078	Y(J+1,L)=YC(L)-(EC/(EP-EC))*(YP(1,L)-YC(L))	PC610
ISN 0079	275 CONTINUE	PC615
ISN 0080	IF(J-N)285,300,400	PC620
ISN 0081	285 CONTINUE	PC625
ISN 0082	300 CONTINUE	PC635
ISN 0083	IER=ITEST	PC640
ISN 0084	400 N=N+1	PC645
ISN 0085	RETURN	PC650
ISN 0086	END	PC670

OS/360 FORT RAN H

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C***                                ***RK 01
C***                                ***RK 02
C***                                ***RK 03
C***  THE PURPOSE OF RKGSDQ IS TO SOLVE, USING RUNGA-KUTTA-GILL, A SET ***RK 04
C***  OF SIMULTANECUS DIFFERENTIAL EQUATIONS, WHICH ARE FIRST ORDER IN ***RK 05
C***  A COMMON (DUMMY) VARIABLE.  THE QUADRATURE IS FOURTH ORDER. ***RK 06
C***                                ***RK 07
C***                                ***RK 08
C***  J IS THE ITERATE OF Y BEING SOLVED. ***RK 09
C***                                ***RK 10
C***  X(I,J) IS THE DUMMY VARIABLE. ***RK 11
C***                                ***RK 12
C***  Y(I,J) IS THE SOLUTION TO THE L'ITH DIFFERENTIAL EQUATION. ***RK 13
C***                                ***RK 14
C***  N IS THE NUMBER OF SIMULTANEOUS DIFFERENTIAL EQUATIONS. ***RK 15
C***                                ***RK 16
C***  H IS THE INCREMENT TO BE ADDED TO X(J-1) TO GET X(J). ***RK 17
C***                                ***RK 18
C***  E IS A PARAMETER USED IN THE RULE OF COLLATZ, OF LENGTH,N. ***RK 19
C***                                ***RK 20
C***  K1 IS A REAL WORK ARRAY OF LENGTH, N. ***RK 21
C***                                ***RK 22
C***  K2 IS A REAL WORK ARRAY OF LENGTH, N. ***RK 23
C***                                ***RK 24
C***  K3 IS A REAL WORK ARRAY OF LENGTH, N. ***RK 25
C***                                ***RK 26
C***  K4 IS A REAL WORK ARRAY OF LENGTH, N. ***RK 27
C***                                ***RK 28
C***                                ***RK 29
C***                                ***RK 30
C*****
ISN 0002  SUBROUTINE RKGSDQ(J,X,Y,N,H,E,K1,K2,K3,K4,M) RK 31
ISN 0003  REAL*8 X,Y,H,K1,K2,K3,K4,AMC,A00,AMS,AM,A0,AX,F RK 32
C  REAL K1,K2,K3,K4 RK 33
ISN 0004  DIMENSION X(1),Y(M,1),E(1),K1(1),K2(1),K3(1),K4(1) RK 34
ISN 0005  A00=2.0+00 RK 35
ISN 0006  A00=1.0+00+DSQRT(5.0-01) RK 36
ISN 0007  AMS=5.0-01-DSQRT(5.0-01) RK 37
ISN 0008  AM=2.0+00-DSQRT(2.0+00) RK 38
ISN 0009  A0=2.0+00+DSQRT(2.0+00) RK 39
ISN 0010  AX=6.0+00 RK 40
ISN 0011  X(J)=X(J-1) RK 41
ISN 0012  DO 100 L=1,N,1 RK 42
ISN 0013  Y(J,L)=Y(J-1,L) RK 43
ISN 0014  100 CONTINUE RK 44
ISN 0015  DO 110 L=1,N,1 RK 45
ISN 0016  X1(L)=F(L,X,Y,J)*H RK 46
ISN 0017  110 CONTINUE RK 47
ISN 0018  DO 120 L=1,N,1 RK 48
ISN 0019  Y(J,L)=Y(J,L)+(K1(L)/A00) RK 49
ISN 0020  120 CONTINUE RK 50
ISN 0021  X(J)=X(J)+(H/A00) RK 51
ISN 0022  DO 130 L=1,N,1 RK 52
ISN 0023  K2(L)=F(L,X,Y,J)*H RK 53
ISN 0024  130 CONTINUE RK 54
ISN 0025  DO 140 L=1,N,1 RK 55
ISN 0026  Y(J,L)=Y(J,L)+(K2(L)-K1(L))*(A00-A00) RK 56

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ISN 0027	140. CONTINUE	RK 57
ISN 0028	DO 150 L=1,N,1	RK 58
ISN 0029	K3(L)=F(L,X,Y,J)*H	RK 59
ISN 0030	150 CONTINUE	RK 60
ISN 0031	DO 160 L=1,N,1	RK 61
ISN 0032	Y(J,L)=Y(J,L)+AMS*K1(L)-K2(L)+ADD*K3(L)	RK 62
ISN 0033	160 CONTINUE	RK 63
ISN 0034	X(J)=X(J-1)+H	RK 64
ISN 0035	DO 170 L=1,N,1	RK 65
ISN 0036	K4(L)=F(L,X,Y,J)*H	RK 66
ISN 0037	170 CONTINUE	RK 67
ISN 0038	DO 180 L=1,N,1	RK 68
ISN 0039	Y(J,L)=Y(J-1,L)+(K1(L)+AM*K2(L)+AD*K3(L)+K4(L))/AX	RK 69
	E(L)=(K2(L)-K3(L))/(K2(L)-K1(L))	RK 70
	E(L)=ABS(E(L))	RK 71
ISN 0040	180. CONTINUE	RK 72
ISN 0041	RETURN	RK 73
ISN 0042	END	RK 74

Appendix II. Determination of Pellet-Surface Mach Number.

Although the appropriate value for the pellet-surface Mach number, $M(\hat{r})|_{\hat{r}=1} = M_0$, can be determined, in principle, by a trial-and-error approach, the computational effort involved may turn out to be very substantial and perhaps prohibitively expensive. The degree and severity of this concomitant practical difficulty tends to increase with increasing values of the heating parameter, ξ . While such a trial-and-error approach is perhaps viable for the heating-parameter values considered by Milora and Foster ($\xi = 10, 100, 1000$), the corresponding heating parameter for CTHR ($\xi=15300$) is much larger and a more sophisticated semi-analytical method merits serious consideration.

The given system of differential equations to be solved is comprised of equations (6) and (7), i. e.,

$$\frac{dM}{d\hat{r}} = \frac{M}{1-M^2} \left(\xi M^2 (\delta \hat{r}^2)^3 (1+\gamma M^2) - \frac{2+(\gamma-1)M^2}{\hat{r}} \right) \quad (6)$$

$$\frac{d\delta}{d\hat{r}} = \frac{2M^2\delta}{1-M^2} \left(\frac{1}{\hat{r}} - \xi (\delta \hat{r}^2)^3 \right) \quad (7).$$

The relevant boundary conditions, expressed by equations (8a) and (8b), are

$$\delta(\hat{r})|_{\hat{r}=1} = 1 \quad (8a)$$

and

$$M_\infty = \lim_{\hat{r} \rightarrow \infty} M(\hat{r}) = \sqrt{5/\gamma} \quad (8b)$$

However, the companion condition

$$M(\hat{r})|_{\hat{r}=1} = M_0 \quad (9)$$

is not given in terms of a known boundary value. Thus, immediate numerical integration is precluded, save for a trial-and-error approach in which one tries to guess the correct value of M_0 .

Several other conditions must be satisfied on physical grounds. First of all,

$$\frac{dM}{df} > 0 \quad (10)$$

which implies, as noted by Milora and Foster, that the bracket expression on the right-hand side of equation (6) must be positive at subsonic velocities, where $M < 1$, and negative at supersonic velocities, for which $M > 1$; at the sound barrier, characterized by the Mach-number value $M=M_1=1$, the expression must vanish to preclude an unacceptable singularity. Similarly, one must expect that

$$\frac{d\delta}{df} < 0 \quad (11)$$

to avoid a condition which would trigger recalescence; correspondingly, the bracket term on the right-hand side of equation (7) must be negative when $M < 1$, zero when $M=M_1=1$, and positive when $M > 1$.

The last sound-barrier condition, which requires that

$$\left(\frac{1}{f} - \xi(\delta f^2)^3 \right)_{M=1} = 0 \quad (12)$$

to prevent a physically unacceptable singularity, provides the motive for introducing the substitution

$$\delta = \eta \xi^{-1/3} f^{-7/3} \quad (13),$$

where $\eta = \eta(f)$ represents a new dependent variable. From equation (13) it follows by rearrangement that

$$\xi(\delta f^2)^3 = \frac{\eta^3}{f} \quad (14).$$

Consequently, the sound-barrier condition reflected in $M=M_1=1$, equation (12), can be reformulated as

$$\eta_1 = \eta(f(M)) \Big|_{M=1} = 1 \quad (15)$$

where $\hat{r} = \hat{r}(M)$ is the inverse function of $M = M(\hat{r})$; thus the sound-barrier value of the new variable η is known to be unity.

Pursuant to the substitution given in equation (13), the differential equations (6) and (7) can be rewritten in terms of the variables $\hat{r}$, M , and η ; one finds easily:

$$\frac{d\eta}{d\hat{r}} = \left[\frac{7 - M^2(1+6\eta^3)}{3(1-M^2)} \right] \frac{\eta}{\hat{r}} \equiv \frac{V}{(1-M^2)\hat{r}} \quad (16)$$

$$\frac{dM}{d\hat{r}} = - \left[\frac{2 - M^2[(1+\eta^3) - \gamma(1-M^2\eta^3)]}{1-M^2} \right] \frac{M}{\hat{r}} \equiv - \frac{U}{(1-M^2)\hat{r}} \quad (17)$$

The two polynomials $U = U(M, \eta)$ and $V = V(M, \eta)$, introduced for convenience in notation, are defined uniquely by virtue of the identities in equations (17) and (16), respectively. By introducing the further simple transformation

$$s = \ln \hat{r} \quad (18)$$

one obtains an autonomous system of differential equations, namely:

$$\frac{d\eta}{ds} = \frac{[7 - M^2(1+6\eta^3)]\eta}{3(1-M^2)} \equiv \frac{V}{1-M^2} \quad (19)$$

$$\frac{dM}{ds} = - \frac{\{2 - M^2[(1+\eta^3) - \gamma(1-M^2\eta^3)]\}M}{1-M^2} \equiv - \frac{U}{1-M^2} \quad (20)$$

Then, elimination of the parametric variable s from these equations yields

$$\frac{dM}{d\eta} = - \frac{3M\{2 - M^2[(1+\eta^3) - \gamma(1-M^2\eta^3)]\}}{\eta[7 - M^2(1+6\eta^3)]} \equiv - \frac{U}{V} \quad (21)$$

By way of their definition, the polynomials $U = U(M, \eta)$ and $V = V(M, \eta)$ represent the numerator and denominator of the given fraction, respectively.

This formulation of the problem at hand allows the application of standard methods germane to the theory of nonlinear differential equations, particularly geometric considerations developed originally by Poincaré and

Liapounov (see, for example, Chapter 9 in "Elementary Differential Equations and Boundary Value Problems" by W. E. Boyce and R. C. DiPrima, John Wiley & Sons, 3rd edition, 1977).

The regime of physical interest is contained in the positive real domain of the variables, that is, where $M > 0$ and $\eta > 0$; this domain is bounded by the two obvious solutions $M=0$ when $\eta > 0$ and $\eta=0$ when $M > 0$, with the origin $M=0$ and $\eta=0$ being a critical point (readily recognized on geometric grounds to be a saddle point). Within the positive real domain, there are two critical points at which the numerator and denominator on the right-hand side of equation (21) vanish simultaneously. Solution of the appropriate simultaneous algebraic equations yields readily the critical-point coordinate values.

One of these critical points is given by

$$M_{\infty} = \sqrt{5/\gamma} \quad \text{and} \quad \eta_{\infty} = \sqrt[3]{(7\gamma-5)/30} \quad (22a)$$

Comparison with equation (8b) reveals that this critical point matches the asymptotic limit as $\lim \hat{r} \rightarrow \infty$. It is easy to show that the condition

$$\eta_{\infty} = \lim_{\hat{r} \rightarrow \infty} \eta(\hat{r}) = \sqrt[3]{(7\gamma-5)/30} \quad (23)$$

is, indeed, the appropriate condition for the asymptotically stable solution as suggested by Milora and Foster.* Thus, this critical point must be the asymptotic limit of the desired solution; the asymptotic expression

$$\delta(\hat{r}) - \eta_{\infty} \xi^{-1/3} \hat{r}^{-7/3} \quad (24)$$

follows by substitution into equation (13). This guarantees the necessary convergence of the improper integral defining $f(\xi) = \int_1^{\infty} \delta(\hat{r}) d\hat{r}$. For the specific case at hand in this report, $\gamma=7/5$, then $M_{\infty} \approx 1.88982$ and $\eta_{\infty} = 0.54288$.

* Assuming $M \sim M_{\infty}$, one obtains readily $\delta \sim C \hat{r}^{-7/3}$ by separation of variables in equation (2). Subsequent substitution into equation (7) yields $C \sim \eta_{\infty} \xi^{-1/3}$ in agreement with equation (24).

The other critical point is found to be at

$$M_1 = 1 \quad \text{and} \quad \eta_1 = 1 \quad (22b).$$

Comparison with equation (15) shows that this critical point represents the sound-barrier condition. Therefore, it must be contained in the solution being sought.

To exploit this most essential feature, the specific character of this critical point must be explored in detail. In a sufficiently small vicinity of the critical point, with $M \approx 1 + \mu$ and $\eta \approx 1 + \epsilon$, the linear autonomous system of differential equations

$$\frac{d\epsilon}{d\tau} = 18\epsilon + 14\mu \quad (25a)$$

$$\frac{d\mu}{d\tau} = -9(\gamma+1)\epsilon + 6(\gamma+2)\mu \quad (25b)$$

where τ is a suitable parameter, is mathematically equivalent with the differential equation (21) being investigated. The concomitant characteristic equation

$$\lambda^2 - 6(\gamma-1) + 18(\gamma-5) = 0 \quad (26)$$

has the two distinct characteristic roots

$$\lambda_1 = 3[(\gamma-1) + \sqrt{(\gamma-2)^2 + 7}] \quad (27a)$$

and

$$\lambda_2 = 3[(\gamma-1) - \sqrt{(\gamma-2)^2 + 7}] \quad (27b)$$

Since the specific-heat ratio $\gamma = (\nu+2)/\nu$, where the integer $\nu \geq 3$ is the number of degrees of freedom for the gas-molecule motion, is thus bounded on physical grounds by $1 < \gamma \leq 5/3$, it follows that their product is negative, i.e.,

$$\lambda_1 \lambda_2 = 18(\gamma-5) < 0 \quad (28),$$

so that the two characteristic roots are of the opposite sign. Consequently, the critical point of interest is diagnosed to be a saddle point.

To determine the slopes $M' = dM/d\eta$ for the two solutions traversing the saddle point, let the differential equation (21) be rewritten as

$$V\left(\frac{dM}{d\eta}\right) + U = 0 \quad (29),$$

where U and V represent the numerator and denominator of the fraction on the right-hand side, respectively. When evaluated at the critical point, both coefficients U and V vanish, of course, and equation (29) degenerates into a trivial identity. Differentiation of equation (29) while regarding η as the independent variable and $M = M(\eta)$ as the dependent variable yields

$$V \frac{d^2 M}{d\eta^2} + \frac{\partial V}{\partial M} \left(\frac{dM}{d\eta}\right)^2 + \left(\frac{\partial V}{\partial \eta} + \frac{\partial U}{\partial M}\right) \frac{dM}{d\eta} + \frac{\partial U}{\partial \eta} = 0 \quad (30),$$

where the coefficients can be determined easily by partial differentiation of $U = U(M, \eta)$ and $V = V(M, \eta)$. At the critical point, the coefficient V is known to vanish and equation (30) becomes a quadratic equation for the saddle point slope M'_s ; after straightforward but slightly tedious mathematical manipulations to evaluate the relevant coefficients at the saddle point one finds

$$14 M'^2_s + 6(\gamma+5)M'_s + 9(\gamma+1) = 0 \quad (31);$$

the concomitant solutions are then

$$M'_{s1} = -3[(\gamma+5) - \sqrt{(\gamma-2)^2 + 7}]/14 \quad (32a)$$

and

$$M'_{s2} = -3[(\gamma+5) + \sqrt{(\gamma-2)^2 + 7}]/14 \quad (32b)$$

The product of these two roots is certainly positive, because of

$$M'_{s1} M'_{s2} = 9(\gamma+1) > 0 \quad (33).$$

Since it is obvious that $M'_{s2} > 0$ based on simple inspection of the expression (32b), it follows pursuant to equation (33) that $M'_{s1} < 0$; it is furthermore evident readily that $M'_{s1} > M'_{s2}$. Consequently,

$$0 > M'_{s1} > M'_{s2} \quad (34)$$

For the specific case at hand in this report, $\gamma=7/5$, thus $M'_{s1}=-0.79009$ and $M'_{s2}=-1.95277$; their relative magnitude is obviously in accordance with inequality (34).

With the two saddle-point slopes known, there are but two different solutions of the differential equation (21) which satisfy the already established condition that the saddle point must be part of the solution. One of these two solutions is then, of course, the physically realistic solution being sought here; which of these two possible solutions remains to be determined. To this end, a quite detailed analysis, straightforward in principle but rather tedious and cumbersome in practice, must be carried out, involving exploration of the isoclines $dM/d\eta = 0$, for which $U = U(M, \eta) = 0$, and $d\eta/dM = 0$, for which $V = V(M, \eta) = 0$, subsequent determination of regions in terms of the sign the values of the polynomials $U = U(M, \eta)$ and $V = V(M, \eta)$ assume in such regions within the positive real domain for the variables M and η , in combination with a distinction between subsonic and supersonic regimes, in which $(1-M^2) > 0$ or $(1-M^2) < 0$, respectively. By virtue of equations (21), (17), and (16) the results of such an analysis permit a solution to be characterized in terms of the sign for the derivatives $dM/d\eta$, $dM/d\hat{r}$, and $d\eta/d\hat{r}$. Furthermore, one can infer by differentiation of equation (13) that $d\delta/d\hat{r} = [d\eta/d\hat{r} - 7\eta/3\hat{r}]\xi^{-1/3}\hat{r}^{-7/3} < 0$ whenever $d\delta/d\hat{r} < 0$; however no conclusion may be drawn regarding the sign of the derivative $d\delta/d\hat{r}$ when $d\eta/d\hat{r} > 0$.

When the procedure just outlined is applied for the solution with the steeper negative slope, namely $M'_{s2} = -3[(\gamma+5) + \sqrt{(\gamma-2)^2 + 7}]/14$, at the saddle point, it reveals as the essential feature that $dM/d\hat{r} < 0$. This is exactly contrary to the constraining condition set forth in inequality (10). Consequently, this particular choice among the two mathematically possible saddle-point solutions must be rejected as physically unrealistic in its consequences.

By process of elimination, the alternate possibility left for practical consideration is represented by the solution with the shallower negative slope, namely $M'_{s1} = -3[(\gamma+5) - \sqrt{(\gamma-2)^2 + 7}]/14$, at the saddle point. In this case, the results of the analysis employing the procedure outlined above shows that $dM'/d\tau > 0$, $d\eta/d\tau < 0$, and $d\beta/d\tau < 0$ in agreement with the physical-constraint conditions stated in inequalities (10) and (11). Thus, the desired solution compatible with physical reality exists indeed and has been duly identified. The analysis indicates also that $dM/d\eta < 0$, which means that the solution of interest is represented by a monotone decreasing function $M=M(\eta)$.

With the acquired knowledge about the saddle-point characteristics, the differential equation (21) can be integrated numerically commencing at the saddle point at $M=M_1=1$ and $\eta=\eta_1=1$ with slope $M'_{s1} = -3[(\gamma+5) - \sqrt{(\gamma-2)^2 + 7}]/14$. For the present work, two fourth-order schemes of numerical integration were chosen: the standard fourth-order Runge-Kutta method and Gill's modified method (see Chapter 25, entries 25.5.10 and 25.5.12 in "Handbook of Mathematical Functions," edited by M. Abramowitz and I. A. Stegun, 9th printing, Dover Publications, Inc., 1972). Naturally, the numerical solutions for this report were carried out for this specific case at hand, where $\gamma=7/5$ as appropriate for gas with diatomic molecules. The results of the numerical solution thus obtained are graphically illustrated in Figure A; within the limits of numerical precision attainable in the computations, the results from both numerical-integration schemes were found to be in agreement.

The value of the acquired solution $M=M(\eta)$ of differential equation (21) corresponding to the pellet-surface condition where $\tau=\tau_0=1$ and $\beta=\beta_0=1$, provides the desired pellet-surface condition given in equation (8a) into equation (13) yields the pellet-surface value

$$\eta(\tau) \Big|_{\tau=1} = \eta_0 = \sqrt[3]{\xi} \quad (35);$$

using the solution $M=M(\eta)$ one obtains then for the pellet-surface Mach number the value

$$M_0 = M(\eta) \Big|_{\eta=\eta_0} = M(\eta_0) \quad (36).$$

Obviously, this process for determining the pellet-surface Mach number M_0 required that differential equation (21) be integrated numerically over the interval $1=\eta_1 \leq \eta \leq \eta_0 = \sqrt[3]{\xi}$, starting at the saddle point, where $M=M_1=1$ and $\eta=\eta_1=1$, with slope $M'_{s1} = -3[(\gamma+5) - \sqrt{(\gamma-2)^2 + 7}]/14$.

As a corollary note, the veracity of the original conjecture, that the trial-and-error approach for determining the pellet-surface Mach number M_0 becomes less attractive and viable with increasing heating-parameter values ξ , can now be elucidated as follows. After choosing, by way of a guess, at tentative value of M_0 , the trial-and-error method is tantamount mathematically to the numerical integration of differential equation (21) in the reverse direction, commencing at the pellet surface, where $\eta=\eta_0 = \sqrt[3]{\xi}$, and proceeding towards the sound barrier value, where $\eta=\eta_1=1$, in the hope of meeting the desired target given as the saddle point. Clearly, the computational effort will increase and the chance of a lucky bullseye hit will decrease as the distance from the starting point to the target grows with increasing values of the heating parameter ξ .

One final remark is in order. Determination of the pellet-surface Mach number M_0 is not an end in itself, but the starting point for the numerical evaluation of various physical parameters, such as $M(\tau)$, $\beta(\tau)$, and $f(\xi) = \int_1^\infty \beta(\tau) d\tau$. For the results presented in this report, these parameters were obtained by numerical integrations starting with a predictor-corrector scheme applied to the system of simultaneous differential equa-

tions (6) and (7). This approach was motivated in terms of convenience and expedience by the ready availability of a directly applicable computer code, which had been established to support an ill-fated attempt to implement the trial-and-error approach of finding M_0 . The authors realize that other avenues, perhaps conceptually and computationally simpler, could have been taken. For example, use of the suggested technique for determining M_0 entails numerical knowledge of the solution $M=M(\eta)$ of the differential equation (21) in the interval $1=\eta_1 \geq \eta \geq \eta_0$ as the result of a numerical-integration process; one could use the same scheme to find $M=M(\eta)$ also in the interval $1=\eta_1 \leq \eta \leq \eta_\infty$, commencing the numerical-integration scheme once more at the saddle point, where $M=M_1=1$ and $\eta=\eta_1=1$, with the known slope $M'_{s1} = -3[(\gamma+5) - \sqrt{(\gamma-2)^2 + 7}]/14$. Upon substitution of $M=M(\eta)$ into differential equation (16), the function $\eta=\eta(\hat{r})$ can be found by separation of variables and simple quadrature; $\beta(\hat{r})$ is then obtained using equation (13). Finally, determination of $f(\xi) = \int_1^\infty \beta(\hat{r}) d\hat{r}$ requires another simple quadrature, and $M(\hat{r})$ is constructed by substitution of $\eta=\eta(\hat{r})$ into $M=M(\eta)$, i.e., $M(\hat{r})=M(\eta(\hat{r}))$.

Figure A. Solution of differential equation (21) for the physically realistic sound-barrier conditions.

WFPS:TME-79-032
DECEMBER 1979
UC-20b

CONCEPTUAL DESIGN OF A
COMMERCIAL TOKAMAK HYBRID REACTOR FUELING SYSTEM

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BY THE WESTINGHOUSE ELECTRIC CORPORATION
CONTRACT EG-77-C-02-4231-A000

ACKNOWLEDGEMENT

This work was performed for the U.S. Department of Energy, under Contract EG-77-C-02-4231-A000, Cooperative Graduate Education Program in Fusion Technology. Reproduction, translation, publication, use and disposal, in whole or in part, by or for the United States Government is permitted.

Special personal thanks are being extended to Dr. Edwin E. Kintner of the U.S. Department of Energy and to Drs. Zalman Shapiro and Daniel Klein of Westinghouse Electric Corporation for their contributions in making this superb program a viable and continuing reality.

During the course of this study, the authors have become deeply indebted to Dr. Z. Shapiro, Dr. D. Klein, Dr. G. Gibson, Dr. E. W. Sucov, Mr. G. Collier, Mrs. E. Iwinski, Mr. J. Easoz, Mr. J. Selembo, and Mrs. Yvonne Harlow of Westinghouse, as well as Ms. T. Gleue, Mr. C. O'Brian, and Mrs. J. Findley of Kansas State University; their contributions, suggestions, and assistance are being acknowledged with sentiments of sincere gratitude.

In addition, the principal author (KDM) wishes to express his appreciation to his two co-authors and advisors (HJD and TFY) for their help, encouragement, and interest in this study.

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