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EBT Reactor Analysis

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EBT REACTOR ANALYSIS

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ABSTRACT

This report summarizes the results of a recent ELMO Bumpy Torus (EBT) reactor study that includes ring and core plasma properties with consistent treatment of coupled ring-core stability criteria and power balance requirements. The principal finding is that constraints imposed by these coupling and other physics and technology considerations permit a broad operating window for reactor design optimization. Within this operating window, physics and engineering systems analysis and cost sensitivity studies indicate that reactors with $\langle \beta_{\text{core}} \rangle \sim 6\text{--}10\%$, $P \sim 1200\text{--}1700 \text{ MW(e)}$, wall loading $\sim 1.0\text{--}2.5 \text{ MW/m}^2$, and recirculating power fraction (including ring-sustaining power and all other reactor auxiliaries) $\sim 10\text{--}15\%$ are possible. A number of concept improvements are also proposed that are found to offer the potential for further improvement of the reactor size and parameters. These include, but are not limited to, the use of (1) supplementary coils or noncircular mirror coils to improve magnetic geometry and reduce size, (2) energetic ion rings to improve ring power requirements, (3) positive potential to enhance confinement and reduce size, and (4) profile control to improve stability and overall fusion power density.

1. INTRODUCTION

The ELMO Bumpy Torus (EBT) reactor studies summarized in this report differ from earlier ones primarily by including theoretical stability criteria developed in the past several years [particularly the Lee-Van Dam-Nelson (LVN) limit^{1,2}] and their coupling to the ring power requirements.

In EBT, the stability requirements of core and energetic ring plasmas are closely coupled. The ring-core interactions define the stable operating regimes of both plasmas and determine the limit on the achievable stable core beta β_c , where the beta value of the plasma is defined as $\beta = p/(B^2/2\mu_0)$ with p the pressure, B the magnetic field strength, and $\mu_0 = 4\pi \times 10^{-7}$ H/m. In this study, ring and core plasma properties are considered with consistent treatment of coupled ring-core stability criteria and power balance requirements.

The principal finding is that these criteria and other physics and technology constraints permit a broad operating window for reactor optimization. Within this operating window, physics and engineering systems analysis and cost sensitivity studies indicate that reactors with $\langle \beta_{\text{core}} \rangle \sim 6\text{--}10\%$, $P \sim 1200\text{--}1700$ MW(e), wall loading $\sim 1.5\text{--}2.5$ MW/m², and recirculating power fraction (including ring sustaining power and all other reactor auxiliaries) $\sim 10\text{--}15\%$ are possible.

The physics rationale for this study includes (1) equilibrium, particle orbits, and magnetics optimization; (2) confinement/transport scaling; (3) stability (beta limits); (4) electron and ion ring power requirements; (5) optimal pressure profiles; and (6) coupling of ring and core plasma properties. In addition, the EBT systems code has been

developed to integrate physics, engineering, and cost constraints to establish a macrostable, steady-state operating window for a family of EBT reactors that are competitive with other fusion reactors.

Whereas most of the discussions in this report are centered on stability, ring power requirements, optimal pressure profiles, and their impact on reactor operating space, we consider briefly some of the other highlights from the study. Also, a number of concept improvements are introduced that could offer the potential for further improvement of the reactor parameters (such as size, recirculating power fraction, etc.). Detailed descriptions of the studies are given in Appendixes A-G.

2. COUPLED CORE AND RING STABILITY LIMITS

The basic operation of EBT requires the use of high beta, hot electron rings for the stability of the overall plasma. These rings form near contours of constant mod-B in the midplane of each mirror section and produce a local magnetic well near the edge of a toroidally confined plasma. Typical profiles of the magnetic field, toroidal core and ring pressure profiles, and $U = \oint d\ell/B$ are shown in Fig. 1.

In EBT, as mentioned earlier, the stability requirements of core and energetic ring plasmas are closely coupled,^{3,4} and their interaction defines the stable operating regimes of both plasmas and determines core beta (β_c) limits, as well as the allowable upper and lower limits of the ratio of the core plasma density to the hot plasma density (n_c/n_h).

Fig. 1. Typical profiles of $U \equiv \oint d\ell / B$, the magnetic field B , and pressures (core and ring) as a function of midplane radius.

The role of high beta, hot electron rings in maintaining the stability of the toroidal core plasma against flute and interchange modes may be seen from Fig. 1 and the simple stability criterion⁵ given by

$$(p'U + \gamma p U') [U' - p' \oint (d\ell/B^3)] > 0, \quad (1)$$

where $U = \oint d\ell/B$, p is the core plasma pressure, γ is the ratio of specific heats, and the derivative is with respect to the flux ψ (the prime indicates $d/d\psi$). In the central region and at the inside edge of the ring (regions I and II in Fig. 1), stability is attained if $p' = 0$ or

$$p'/p > -\gamma U'/U. \quad (2)$$

At the outer edge (region III), stability is possible if

$$|p'| \leq |U'| / \oint d\ell/B^3. \quad (3)$$

Thus, the electron ring modifies B and $\oint d\ell/B$ in such a way as to stably support core plasma pressure gradients in the regions in which they are largest. The stability threshold is then related to the critical value of ring beta $\beta_{h,crit}$ necessary to reverse the gradients at the outer edge of the plasma, which is given by

$$\beta_{h,crit} \approx 2\Delta \langle R_c \rangle, \quad (4)$$

where Δ is the ring half-width and $\langle R_c \rangle$ is the average magnetic field radius of curvature.

Although the hot electron rings provide a local minimum in B and stabilize low beta interchange modes of the toroidal core plasma, a number of instabilities (curvature-driven modes) may still persist because of electromagnetic coupling between ring and core plasmas. Among them is the core pressure-driven interchange mode, which is stable if

$$\beta_c < \frac{2\Delta}{R_c} \left(1 + \frac{p_{\parallel h}}{p_{\perp h}} \right), \quad (5)$$

where R_c is the local radius of curvature in the midplane and $p_{\parallel h}$ and $p_{\perp h}$ are the hot electron ring pressures parallel and perpendicular, respectively, to the B field. This is the core beta limit^{1,2} (so-called LVN limit) near the outside edge of the core plasma (see Fig. 1, region III), which is typically on the order of 10-15% for a reactor. The volume-averaged core beta depends on the core pressure profiles in the core plasma (regions I and II in Fig. 1).

Because the hot electron rings are localized in the region near the outside edge of the core plasma (see Fig. 1), the radial structure of the instability eigenmodes is assumed to be more significant near the rings on the outside edge, and almost all of the theoretical calculations carried out to date for region III assume a core pressure gradient $p' = 0$ in regions I and II. As discussed in Sect. 3, this assumption (made for convenience in calculation) is neither realistic nor favorable for reactor applications.

Summaries of the various modes that may be relevant to a reactor and approximate analytic expressions^{1,5} for their respective stability

criteria are listed in Table 1. These are the core interchange, hot electron interchange, and interacting ring-core interchange (LVN) modes. The compressional Alfvén mode does not pose any problem for reactorlike devices. Typically, these modes impose restrictions on upper and lower limits of n_c/n_h , an upper limit on β_c , a lower limit on β_h , etc.; these restrictions are determined by the ring scale length Δ , radius of curvature R_c (local and/or field line averaged), $q_0 = v_{cv1}/\omega_{ci}\Delta$ (ratio of curvature drift frequency to ion cyclotron frequency), anisotropy, radial and poloidal wave numbers, etc.

Although core beta values on the order of 10–15% have been estimated, the coupling of beta limits with ring dimensions (through Δ) and power balance (through n_c/n_h) is a concern for a reactor. Boundaries of a reactor plasma operating window as a function of n_c/n_h and q_0 are shown in Fig. 2. Within the constraints (1) that reactor parameters must lie within the possible operating window shown in

Table 1. Summary of stability conditions

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MODE	STABILITY CONDITION
CORE PLASMA INTERCHANGE	$\beta_{hot} \gtrsim 4\Delta/(R_c)$
HOT ELECTRON INTERCHANGE	$\frac{n_{core}}{n_{hot}} > \frac{4}{(k_{\perp}\Delta)^2} \frac{1}{q_0} \frac{1}{(1 - \beta_c R_c/2\Delta)^2}$
COMPRESSIONAL ALFVÉN	$\frac{n_{core}}{n_{hot}} < \frac{[q_0(1 - \tilde{\beta}_c)(k_{\theta}/k_{\perp})^2 - 1]^2}{(k_{\theta}/k_{\perp})^2 q_0 (2\Delta/R_c)^2 \tilde{\beta}_h (\tilde{\beta} - 1)(1 - \beta_c)}$
INTERACTING RING-CORE INTERCHANGE	$\beta_{core} < \frac{2\Delta}{R_c} \left(1 + \frac{p_{ h}}{p_{\perp h}} \right)$
DEFINITIONS: $q_0 = \frac{v_{cv1}}{\Delta\omega_{ci}}$, $\tilde{\beta} = \beta \frac{R_c}{2\Delta}$, $k_{\theta} = \frac{m}{a}$, $k_{\perp}^2 = k_r^2 + k_{\theta}^2$, $k_r \sim \frac{2}{\Delta}$

Fig. 2. Boundaries of stable operating window in $n_{\text{core}}/n_{\text{hot}}$ ($=n_c/n_h$) vs q_0 ($\sim T_{\text{hot}}/B^2 R_c \Delta$) space in a reactor. Various boundaries are indicated.

Fig. 2 for stability and (2) that the core pressure profile inside the ring (regions I and II in Fig. 1) must satisfy core interchange criteria (see Sect. 3) for profile considerations, we find that a favorable reactor design can be formulated in regard to both the recirculated power fraction and the total fusion power produced.

Detailed discussions of coupled core-ring stability limits and their implications for a reactor are given in Appendix A.

3. OPTIMAL PRESSURE PROFILES

The economic viability of an EBT reactor improves noticeably if the volume-averaged core beta $\langle \beta_c \rangle$ significantly exceeds ~5%. The radial profiles of core pressure (which, for simplicity, are assumed to be constant in most of the stability calculations discussed in Sect. 1) are found to play an important role in both fusion and ring power balance. Since the magnetic fields in EBT vary both radially and axially (along the field lines), for constant pressure profiles the volume-averaged core beta $\langle \beta_c \rangle$ is much smaller than the theoretically calculated β_c near the outside edge of the ring (region III in Fig. 1):

$$\langle \beta_c \rangle \lesssim 0.25 \beta_c , \quad (6)$$

where the limit on β_c is given by Eq. (5). This imposes a serious economic penalty for a reactor. However, if pressure gradients in the core plasma (in regions I and II of Fig. 1) are allowed, $\langle \beta_c \rangle$ is found to be comparable to the local core beta at the outer edge of the ring:

$$\langle \beta_c \rangle \approx (0.5-1.0) \beta_c . \quad (7)$$

This permits the fusion power density to be high enough for a sufficiently small ring width and, in turn, for an allowable recirculating power fraction for ring-sustaining power.

In the absence of detailed kinetic derivations, stable pressure profiles are derived from the MHD criterion⁵ given by Eq. (1) for the core plasma (regions I and II in Fig. 1), where ∇B is not reversed, and from kinetically derived relations⁴ for the outer edge (region III).

In regions I and II the stability condition is given by Eq. (2), and marginal stability is achieved if

$$p'/p = -\gamma U'/U = -\gamma/\langle R_B \rangle, \quad (8)$$

where $U = \oint d\ell/B$, $1/\langle R_B \rangle = U'/U$, and

$$U' = \partial U / \partial \psi = - \oint (\partial B / \partial \psi + \kappa/r) d\ell/B^2, \quad (9)$$

with $\kappa = -1/R_c$ the curvature. Equilibrium relations are used in Eq. (9) to calculate $\langle R_B \rangle$.

In region III, the corresponding expression for Eq. (8) is

$$p'/p = -1/\Delta. \quad (10)$$

Combining Eqs. (8) and (10), the stable core pressure profile can be derived from

$$p'(r) + \alpha(r)p(r) = 0 \quad (11)$$

to give

$$p(r) = C \exp[-\int \alpha(r) dr], \quad (12)$$

where C is constant and expressions for $\alpha(r)$ are given by (see Appendix B)

$$\alpha(r) = \begin{cases} \frac{2\gamma r}{\langle r_0 R_c(r_0) \rangle}, & 0 < r < r_0 - \Delta \\ \frac{2\gamma r}{\langle r_0 R_c(r_0) \rangle} \left(1 + \frac{\beta_n \langle R_c \rangle}{4\Delta} \right), & r_0 - \Delta < r < r_0 \\ \frac{1}{\Delta}, & r_0 < r < r_0 + \Delta \end{cases}$$

where the angle brackets represent field line averaging, r_0 is the ring radius, β_h is the ring beta, and $\gamma = 5/3$ is the ratio of specific heats (see Fig. 1 for geometry).

Considering, for example, the central core plasma region in Fig. 1, where $0 < r < r_0 - \Delta$,

$$p(r) = p(0) \exp[-\gamma r^2 / r_0 R_c(r_0)] . \quad (13)$$

For a representative set of reactor parameters [$\Delta/r_0 \approx 0.1$, $r_0/R_c(r_0) \approx 1/2-1/3$] one finds $p(r_0 - \Delta)/p(0) \approx 0.6 \pm 0.1$ and $p(r_0)/p(0) \approx 0.3 \pm 0.1$. This implies that $\beta_c(0) \approx (1-2)\beta_c(r_0)$ and $\langle \beta_c \rangle \sim 0[\beta_c(r_0)]$, where $\beta_c(r_0)$ is the core beta value that enters in theoretical stability calculations. It should be noted that the radial pressure profiles obtained from one-and-one-half-dimensional (1-1/2-D) transport calculations (discussed in Sect. 7 and Appendix F) are found to be similar to those required for stability.

Detailed derivations of stable pressure profiles and descriptions of their role in reactor power balance are given in Appendix B.

4. STEADY-STATE RING POWER REQUIREMENTS

In present EBT experiments hot electron rings [formed by electron cyclotron heating (ECH)] are used to achieve gross plasma stability. Power losses from these hot electron rings are observed to scale classically,^{3, 6, 7} primarily determined by drag cooling, pitch angle scattering, and synchrotron radiation. Ring temperature is observed to be limited by $\rho/L \approx 5 \times 10^{-2}$ scaling,⁸ corresponding to the

nonadiabatic limit (where ρ = average hot electron gyroradius and L = magnetic field radius of curvature).

The use of ion rings⁹ that could be formed by ion cyclotron heating (ICH) is also proposed. The properties of ion rings are compared with those of electron rings to determine the potential physics and/or technology advantages. It is found that (1) ion rings would require devices with reactorlike size and magnetic field strengths for stable, adiabatic confinement if the criterion $\rho/L \approx 5 \times 10^{-2}$ limits the ion ring temperature (as it appears to do for hot electron rings); (2) stability conditions for curvature-driven modes (discussed in Sect. 2) are the same for ion and electron rings, although additional diamagnetic drift and anisotropy-driven modes may be present; and (3) power requirements for ion rings can be qualitatively lower than those for electron rings, provided the ion ring volume does not greatly exceed the electron ring volume.

4.1 ELECTRON RING POWER LOSSES

The power absorbed in the ring $P_{\text{ring},e}$ is balanced by losses due to drag cooling of hot electrons on core electrons, pitch angle scattering of the ring electrons on core ions and electrons, and synchrotron radiation. The power loss is given (in mks units with temperature in kiloelectron volts and power in megawatts) by⁸ (see Appendix C)

$$P_{\text{ring},e} \text{ (MW)} \approx 0.16 \beta_h B_h^4 V_{\text{ring},e} G(\gamma, \beta_h, n_c/n_h), \quad (14)$$

where $V_{\text{ring},e} = 2\pi a \cdot \Delta_e \cdot (NL_{\text{ring}})$ is the ring volume, with $a = r_0$ = plasma radius, Δ_e = electron ring half-width, L_{ring} = ring

length ($\approx a$) and N = number of rings (sectors); β_h is the ring beta ($\beta_h \propto n_h T_h / B_R^2$, with n_h and T_h the density and temperature of the ring); and B_R is the magnetic field at the ring location. Here,

$$G(\gamma, \beta_h, n_c/n_h) \approx 1 + 2.5(\gamma - 1) + 7.4 \beta_h \left(\frac{n_c}{n_h} \right) \frac{\gamma/(\gamma - 1)^2}{(\gamma^2 - 1)^{1/2}} \left(1 + \frac{4}{\gamma + 1} \right), \quad (15)$$

where $\gamma = 1 + T_h/511$ is the relativistic factor and n_c/n_h is the ratio of the core to ring plasma density at the ring location.

4.2 ION RING POWER LOSSES

Power loss from the ion rings is found to be dominated by Coulomb drag cooling of energetic ions on background core electrons, although drag and scattering by background core ions are not entirely negligible.¹⁰ At the ion ring energies [on the order of megaelectron volts (MeV)] required for stable reactor operation, charge-exchange losses have a small effect on the power balance. The total power required for ion rings can then be approximated as⁹ (see Appendix D)

$$P_{\text{ring},i} \text{ (MW)} \approx 10^3 \frac{\beta_c \beta_h B_R^4}{u_h T_{ec}^{5/2}} V_{\text{ring},i} [1 + f(T_{ec}/T_h)], \quad (16)$$

where $V_{\text{ring},i} = 2\pi a \cdot \Delta_i \cdot (NL_{\text{ring}})$ is the ion ring volume of half-width Δ_i , T_{ec} is the core plasma electron temperature at the ring location, $u_h = m_h/m_p$ is the ratio of the hot ion mass to the proton mass, β_c is the core beta at the ring location, and the function f is small and on the order of $\sim 10^{-1}$.

4.3 COMPARISON OF ELECTRON AND ION RINGS

The power required to sustain an ion ring ($P_{\text{ring},i} \equiv P_i$) relative to that required for an electron ring ($P_{\text{ring},e} \equiv P_e$) can be calculated from Eqs. (14) and (16) to be

$$P_i/P_e = 2 \bar{n}_h^{-1} \beta_c (10/T_{ec})^{5/2} V_{\text{ring},i}/V_{\text{ring},e} , \quad (17)$$

where it is assumed that ion and electron ring betas are the same ($\beta_{h,i} = \beta_{h,e}$) with equal ring temperatures of $T_h \sim 1-2$ MeV. The temperature scaling of Eq. (17) favors a large core electron temperature in the ring region. Considering comparable ion and electron ring volumes with $\beta_c \sim 0.1$ and $T_{ec} \sim 5-10$ keV, one has

$$\frac{P_i}{P_e} \sim \begin{cases} 0.1-0.5 & \text{for } D^+ \text{ ion ring ,} \\ 0.2-1.0 & \text{for } H^+ \text{ ion ring .} \end{cases} \quad (18)$$

Thus, one sees that ion ring power requirements can be quantitatively favorable, provided that (1) the ion ring volume is not much larger than the electron ring volume and (2) the core electron temperature is not much smaller than about 5 keV near the vicinity of the ring. One would expect $\Delta_i = \rho_i$ to be a minimum ring thickness. For ion rings with $T_h \sim 1-2$ MeV and $B_R \sim 2$ T, $\rho_i \sim 10$ cm [the corresponding gyro-radius for the electron ring is $\rho_e < 0.5$ cm with $\Delta_e \sim (10-30)\rho_e$]. Thus, the ion rings must be localized enough that Δ_i is not much larger than ρ_i .

Detailed discussions of the electron ring scaling and power balance and the assessment of ion rings are given in Appendixes C and D, respectively.

5. COUPLING OF STABILITY AND RING POWER BALANCE

Limitations imposed on core beta and pressure profiles, ring scale lengths, and power balance by stability constraints in EBT are critical in determining the operating windows of future experiments and in assessing the desirability of a reactor.

Coupling of core beta with ring volume (through the ring thickness Δ) and with ring power losses (through n_c/n_h) affects reactor power balance. Minimizing ring power requires thin (small Δ) rings, whereas high beta (high fusion power density) requires thick (large Δ) rings; this implies opposing constraints. Taking into account the various stability constraints (LVN limit, etc.), profile effects, and other physics and technology constraints, the results of system studies (discussed in Sect. 8 and Appendix G) indicate the existence of a comfortable operating window with acceptable ring power losses (recirculating power fraction) in a reactor. One such operating window is shown in Fig. 2. Boundaries of this window are determined from the stability constraints, microwave cutoff, beta limit, and net power production requirements.

For a typical reactor of ~ 1500 MW(e), required sustaining powers are $P_{\text{ring},e} \approx 50\text{--}60$ MW and $P_{\text{ring},i} \approx 5\text{--}6$ MW. Here, $T_h \sim 2$ MeV, $\beta_h \sim 0.15$, $\beta_c \sim 0.1$, $B_R \sim 2$ T, average plasma radius $\langle a \rangle \sim 1.5$ m, and average core plasma temperature $\langle T_{ec} \rangle = \langle T_{ic} \rangle = 25$ keV. (Note that the core plasma temperature near the ring location depends on profiles; it

is roughly one-half to one-third the central value.) For this reactor case, the difference in cost of electricity is about 8-12% (in favor of ion rings) depending on the absorption efficiencies of ECH (for electron rings) and ICH (for ion rings).

6. MAGNETICS

The spatial position of the hot electron rings (formed near contours of constant $|B|$ in the midplane) is critical to efficient utilization of the magnetic volume within the vacuum chamber because it determines the boundary of the toroidally confined core plasma. The results of this study indicate that centering mod-B (centering the ring) through symmetrizing (SYM) or inverse-D (ID) magnetic configurations can be more efficient than using the coil configurations assumed in previous studies [i.e., axis-encircling aspect ratio enhancement (ARE) coils¹⁰].

One of the critical parameters in the assessment of any EBT reactor magnetics configuration is the radial distance $\Delta r = \delta$ between the first wall and the mirror coil winding with $\delta =$ blanket and shield thickness plus various gaps, dewar, etc., under the coil. Typically, $\delta \approx 1.0\text{--}1.5\text{ m}$ for adequate shielding and tritium breeding and $\delta/r_{\text{coil}} \approx 1/3$ to $1/2$ for a reactor, in comparison with $\delta/r_{\text{coil}} \approx 1/8$ in the present experiments (where r_{coil} is the mean coil radius). A primary consideration is to optimize δ/r_{coil} in a reasonably sized device while modifying the vacuum magnetic fields to improve single-particle confinement and centering of mod-B either with supplementary (i.e., SYM) coils or with noncircular (i.e., ID) coils.

In an earlier study¹⁰ we showed that the use of SYM coils, with circular mirror coils and with $\delta = 1.5$ m, resulted in a magnetic field geometry within the reactor chamber in which the midplane mod-B was symmetric across the minor axis at the ring location. The corresponding magnetic ARE factors were approximately 2 (1.5) for trapped (passing) particles. Similar results were obtained with axis-encircling ARE coils. Both ARE and SYM coils improve volume utilization and single-particle confinement by lengthening the magnetic field lines inside the torus relative to those outside, with SYM coils being more efficient in terms of magnetic field utilization in a reactor.

Reactor applications of noncircular ID coils have also been examined. The parameters describing the noncircularity are similar to those of typical tokamak D-shaped toroidal field (TF) coils. In a bumpy torus, however, the vertical "straight" leg is optimally located on the outside of the torus rather than on the inside; that is, the coil is reversed from the tokamak configuration. In a toroidal array of these coils, this orientation results in a partial cancellation of the vertical drift that arises from the toroidicity. A finite-aspect-ratio torus with ID coils has a mod-B in the midplane that is almost perfectly symmetric about the magnetic axis.

The ratio of the area of the largest confined passing particle ($V_{\parallel} = V$) drift orbit to that for trapped particles ($V_{\parallel} = 0$) is compared in Fig. 3 for ID coils and for circular mirror coils (with and without SYM coils) as a function of δ for various reactor sizes. In all cases, ID coils are superior to simple circular coils. However, for $\delta \geq 1.0$ –1.2 m in the example shown in Fig. 3, the improvement obtained

Fig. 3. The ratio of the area of the largest confined passing particle ($V_{||} = V$) drift orbit to that for trapped particles ($V_{||} = 0$) is shown for circular coils (a) without and (b) with SYM coils and (c) for ID coils as a function of blanket-shield thickness ($\Delta_{B-S} = \delta$) under the coils for various reactor sizes.

with ID coils is not as large as that obtained with SYM coils. With nonuniform distribution of blanket and shield thickness under and between the coils, if future detailed engineering and neutronics calculations indicate that $\delta \leq 1.0\text{--}1.2\text{ m}$ gives adequate neutron shielding and a minimal breeding ratio under the coil, then ID coils will certainly compare favorably with systems utilizing SYM coils. The ID coil magnetics system is significantly less complicated; hence, assembly, maintenance, and access should not be adversely affected, as they are with configurations in which there are two supplementary coils per sector. Configurations with noncircular tori (square, racetrack, etc.) are also under consideration.

Discussions of reactor magnetics and particle confinement for various magnetic configurations (wedged mirror coils, ARE and SYM coils, and noncircular mirror coils) are given in Appendix E.

7. CONFINEMENT SCALING

The operating parameter space of a reactor is investigated by using both time-dependent zero-dimensional (0-D) and 1-1/2-D transport calculations. The 0-D model is used for parametric burn studies to examine the global plasma performance and the sensitivity of these performance parameters to various transport assumptions and operational scenarios. The 1-1/2-D radial transport code, on the other hand, is used for determination of density, temperature, and ambipolar potential profiles and the effect of these profiles in energy confinement scaling.

In EBT, a simple neoclassical electron transport scaling is given by

$$n\tau \sim A^2 T^{3/2} \left(1 + \frac{R_B}{R_E} \frac{e\phi}{T_e} \right)^2 f(\text{profiles}) , \quad (19)$$

where $A = R_T/R_B$ is the magnetic aspect ratio; R_T , R_B , and R_E are the major radius of the torus, magnetic field scale length, and electric field scale length, respectively; and $e\phi$ is the ambipolar potential. Because the neoclassical electron losses dominate the experimental behavior, it is difficult to compare the theoretical ion losses and the experimental observations. Scaling laws governing ion behavior are important in a reactor. At present, ion transport principally determines the radial electric field ($E = \nabla\phi$) needed to maintain ambipolarity. In the experiments $E_r < 0$; however, $E_r > 0$ is also possible, which could lead to enhanced confinement.

In a global analysis with $E_r < 0$ [which leads to the simple scaling given by Eq. (19)], it is observed that large devices ($A > 20$) are required for ignition. Analysis with the 1-1/2-D transport simulation code, however, provides an important new insight into confinement scaling of EBT in going from present experiments [EBT-Scale (EBT-S)] to an EBT reactor (EBT-R). It is observed that in the EBT-S to EBT-R sequence profile effects near the outer edge of the hot electron ring lead to enhanced confinement beyond the simple scaling obtained from the global analysis (which omits these profile effects). In Fig. 4 the dashed curves represent the values of $n\tau$ calculated from Eq. (19) for the sequence of EBT devices: EBT-S, -P, -Q, and -R. The

Fig. 4. Confinement parameter $n\tau_E$ vs collisionality $\bar{\nu}_e = \nu_e / \Omega_{oe}$ for a sequence of EBT devices. Dashed curves represent values of $n\tau$ calculated from Eq. (19). Solid curves (solid circles) are results from 1-1/2-D transport calculations.

solid curves show the results of 1-1/2-D transport simulations (for $E_r < 0$) of these increasingly larger devices. Note that the confinement is better than predicted by the simple $A^2 T_e^{3/2}$ scaling of Eq. (19). Profile effects suggest that confinement exceeds the simple scaling by at least a factor of 3-5. A similar enhancement factor is also observed to be possible with $E_r > 0$.

Detailed discussions of the 0-D and 1-1/2-D models, the appropriate transport coefficients for nonresonant and resonant particles, and the role of profile effects in energy confinement time scaling are given in Appendix F.

8. SYSTEMS INTEGRATION

The EBT systems code is used to carry out parametric studies. This code includes consideration of the coupling of the ring and core plasma properties, a consistent treatment of the stability and power balance requirements, and preliminary cost calculations. A reactor operating space obtained from the systems code is similar to one shown in Fig. 2. Within this operating window, a representative (though unoptimized) set of reactor parameters has been derived and is given in Table 2.

The essential elements of the systems code and the calculational sequences leading to the specification of the reactor options and their associated costs are described in Appendix G.

Table 2. Representative EBT reactor parameters

Thermal power	5000 MW(t)
Net electric power	
With electron ring	1550 MW(e)
With ion ring	1650 MW(e)
Major radius	38 m
Average plasma radius	1.5 m
Number of sectors	34
Average magnetic field	4 T
Neutron wall loading	1.4 MW/m ²
Average core density	$8 \times 10^{19} \text{ m}^{-3}$
Average core temperature	25 keV
Core/ring beta	10%/15%
Ring temperature	2 MeV
Ring power losses	
Electron ring	60 MW
Ion ring	5 MW

9. CONCLUSIONS

A macrostable, steady-state operating window for a family of EBT reactors is calculated considering both ring and core plasma properties with consistent treatment of coupled ring-core stability criteria and power balance requirements. The EBT systems code has been developed to integrate the constraints imposed by these couplings, as well as by other physics, engineering, and cost considerations. The principal conclusion of this study is that a broad operating window exists for reactor optimization. Within this operating window, physics and engineering systems analysis and cost sensitivity studies indicate that steady-state, ignited reactors with $\langle \beta_{\text{core}} \rangle \sim 6\text{--}10\%$, $P \sim 1200\text{--}1700$ MW(e), and recirculating power fraction (including ring sustaining power and all other reactor auxiliaries) $\sim 10\text{--}15\%$ are economically possible.

Consideration of profile effects is found to play an important role in determination of an acceptable operating window for a reactor. The radial pressure profiles are derived from stability requirements and from transport calculations. The optimal profiles obtained from both considerations are found to be similar.

Recently, several ideas have emerged for improving the performance of the basic EBT concept. These include (but are not limited to) configurations with noncircular mirror coils, square or racetrack geometries, and bumpy torus-stellarator hybrids, all of which show potential for developing EBT into an even more attractive approach to a fusion reactor. In addition to these advanced configurations, concept improvements such as the use of ion rings for reduced recirculating power fraction, positive ambipolar potential for enhanced confinement,

and various radiofrequency and microwave heating scenarios for profile control offer possibility of enhanced performance. Preliminary results from some of these configurational approaches and concept improvement schemes are discussed in this report. Detailed assessments of advanced bumpy torus configurations and concept improvement ideas are presently under way and results will be reported in future reports.

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Appendix A

COUPLED CORE AND RING STABILITY LIMITS
AND THEIR IMPLICATIONS FOR A REACTOR

In EBT, the stability requirements of core and energetic ring plasmas are closely coupled, and their interaction defines the stable operating regimes of both plasmas and determines limits on core beta β_c , as well as the allowable upper and lower limits of the ratio of the hot plasma density to the core plasma density [Refs. A.1-A.11], where the beta value of the plasma is defined as $\beta = p/(B^2/2\mu_0)$, with p the pressure, B the magnetic field strength, and $\mu_0 = 4\pi \times 10^{-7}$ H/m. From both the kinetic theory models and the experiments, it appears that the relative density of the toroidal core plasma is a critical factor in ring stability. However, the stability of the toroidal core plasma requires a diamagnetic well (provided by the rings) near the edge of the plasma to support a pressure gradient at the edge. The magnetic well and resulting maximum in $U \equiv \oint d\ell/B$ ($U' < 0$ at the outer edge) can only stabilize flutes and interchanges up to some maximum core beta at which the ring and core begin to interact.

In all EBT experiments to date, the hot electron rings [formed by electron cyclotron heating (ECH) in each of the mirror sectors] are used to achieve gross plasma stability. In these experiments, transport losses and heating capability limit the core plasma beta to $\beta_c < 0.5\%$, whereas a ring beta $\beta_h \sim 5-30\%$ is routinely attained. Theoretical studies indicate the existence of several modes, as well as the existence of a stable, finite beta operating window with

$\beta_c \sim 2\Delta/R_c \sim 10\%$ (where Δ is the ring half-width and R_c is the radius of curvature). Although the present experiments cannot investigate this core beta range, the proposed EBT Proof-of-Principle (EBT-P) experiment is expected to have β_c values on the order of a few percent to enable the testing of some of the theoretical predictions.

Section A.1 discusses the role of energetic rings in core plasma stability. The results of coupled core-ring stability limits are summarized in Sect. A.2. Discussion of results is in Sect. A.3.

A.1. INFLUENCE OF HOT ELECTRON RINGS ON TOROIDAL CORE PLASMA STABILITY

The role of high beta, hot electron rings in maintaining the stability of the toroidal core plasma against flute and interchange modes may be seen from Fig. A.1 and the simple stability criterion given by Nelson and Hedrick [Ref. A.8], derived from a modified energy principle,

$$(p'U + \gamma pU')[U' - p' \oint (d\ell/B^3)] > 0, \quad (A.1)$$

where $U = \oint d\ell/B$, p is the core plasma pressure, γ is the ratio of specific heats, and the derivative is with respect to the flux ψ (the prime denotes $d/d\psi$). For stability, Eq. (A.1) must be satisfied on each flux surface ψ . In the central region and at the inside edge of the ring (regions I and II in Fig. A.1), where $p' \leq 0$ and $U' > 0$, stability is attained if

$$p' = 0 \quad (A.2)$$

Fig. A.1. Typical profiles of $U \equiv \oint dl/B$, the magnetic field B , and the pressures (core and ring) are given as a function of radius at the midplane of an EBT sector. Rings modify $\oint dl/B$ and B to support pressure gradients in regions where they are largest.

or if

$$\frac{p'}{p} > - \frac{\gamma U'}{U} . \quad (\text{A.3})$$

Because $U'(\text{II}) > U'(\text{I})$, somewhat larger pressure gradients can be tolerated at the inside edge of the ring plasma (region II) than in the central region (region I). At the outer edge (region III), where $p \rightarrow 0$, $p' < 0$, and $U' < 0$, stability is possible if

$$|p'| \lesssim \frac{|U'|}{\oint d\ell/B^3} . \quad (\text{A.4})$$

Thus, the electron ring modifies B and $\oint d\ell/B$ in such a way as to stably support core plasma pressure gradients in the regions in which they are largest. The stability threshold is then related to the critical value of ring beta necessary to reverse the gradients at the outer edge of the plasma. The value for U' can be written as

$$U' = - \oint \frac{d\ell}{B^2} \left(\frac{\partial B}{\partial \psi} + \frac{\kappa}{r} \right) < 0 \quad (\text{A.5})$$

for stability. The field line curvature $\kappa [= \vec{\kappa} \cdot \hat{e}_\psi]$, where $\vec{\kappa} = (\hat{b} \cdot \nabla) \hat{b}$ with $\hat{b} = \vec{B}/B$ and $\hat{e}_\psi = \nabla\psi/|\nabla\psi|$ is negative in the midplane and positive in the coil throat. Because the factor $1/rB^2$ tends to weight the midplane contribution more heavily than that from the throat, $\oint d\ell (\kappa/rB^2)$ will be negative. To counteract this, $\partial B/\partial \psi$ must be positive. The critical value of ring beta $\beta_{h,\text{crit}}$ required to make $\partial B/\partial \psi$ just positive can be estimated from the equilibrium condition

given in Eq. (B.9). Replacing gradients by scale lengths reveals that $\partial B / \partial \psi = 0$ when

$$\beta_{h,crit} \approx 2 \Delta / R_c, \quad (A.6)$$

where Δ = ring scale length and R_c = magnetic field radius of curvature. This ring beta (~ 5 -10%) is in reasonable agreement with the experimental observation of the C-T transition to the quiescent stable mode of operation.

A.2. COUPLED CORE-RING STABILITY LIMITS

Although the hot electron rings provide a local minimum in B and stabilize low beta interchange modes of the toroidal core plasma, a number of instabilities may still persist because of electromagnetic coupling between ring and core plasmas that, in turn, limits plasma parameters and/or enhances losses. Because the hot electron rings are localized in the region near the outside edge of the core plasma (see Fig. A.1), the radial structure of the instability eigenmodes is assumed to be more significant near the rings on the outside edge. Recently, curvature-driven instabilities for a plasma interacting with energetic electron rings were analyzed by means of radial eigenmode [Refs. A.2, A.3, A.11], variational principle [Refs. A.12 and A.13], and gyrokinetics [Ref. A.14] approaches. Radial analysis is based on a Z-pinch model for modes with frequency ω comparable to the ion cyclotron frequency ω_{ci} and less than the hot electron grad-B drift frequency in the zero Larmor radius flute limit. In this and other

local models, several modes are identified that can limit the core beta β_c , core density n_c , and ring density n_h . These are short wavelength WKB-type modes and include (1) low ($\omega < \omega_{ci}$) and high ($\omega > \omega_{ci}$) frequency, hot electron interchange modes; (2) the compressional Alfvén wave; and (3) the core interchange mode.

A radial normal mode analysis was also carried out by El-Nadi [Ref. A.15] to study the stability of fluidlike flute modes. In that study, consideration was given only to the case when $m/r_0 \gg 2\pi/L$, where m is the poloidal mode number, r_0 is the radius of the hot electron ring (see Fig. A.1), and L is the characteristic length for the axial variation of the equilibrium quantities. Instead of a Z-pinch model, a straight cylindrical model is used in which the curvature of the field lines at the ring location is represented by an artificial, radial, velocity-dependent gravitational force.

Results from stability studies continue to show that if the dynamics of the hot electrons can decouple from those of the core plasma and if the hot electron beta (β_h) is sufficiently large that

$$\beta_h > \frac{4\Delta}{\langle R_c \rangle} - \beta_c, \quad (\text{A.7})$$

then the hot electrons create a diamagnetic well that stably confines the core plasma against the core interchanges. (Here, Δ is the ring half-width and $\langle R_c \rangle$ is the average radius of curvature.) The core beta limit in that well is given by Nelson [Ref. A.9] and Van Dam and Lee [Ref. A.10] in that the core-pressure-driven interchange is stable if

$$\beta_c < \frac{2\Delta}{R_c} (1 + p_{\parallel h}/p_{\perp h}) , \quad (\text{A.8})$$

where R_c is the local radius of curvature in the midplane and $p_{\parallel h}$ and $p_{\perp h}$ are the hot electron ring pressures parallel and perpendicular, respectively, to the B field. This is the core beta limit [so-called Lee-Van Dam-Nelson (LVN) limit] near the outside edge of the core plasma (see Fig. A.1, region III), which is typically on the order of 10-15% for a reactor. Volume-averaged core beta depends on the core pressure profiles in the core plasma (regions I and II in Fig. A.1). A study of stability on the inside edge of the ring (region II) is now under way [Ref. A.16].

A low frequency ($\omega < \omega_{ci}$), hot electron interchange mode is driven by the hot electron pressure. Stability requires a sufficient ratio of core ion to hot electron density [Ref. A.11]:

$$\frac{n_c}{n_h} > \frac{4}{(k_{\perp} \Delta)^2} \frac{1}{q_0} [1 - \beta_c R_c / 2\Delta + O(\Delta/R_c)]^{-2} , \quad (\text{A.9})$$

where $q_0 = v_{cv\perp} / \Delta \omega_{ci}$ is the ratio of the hot electron curvature drift frequency ($v_{cv\perp} / \Delta$) to the ion cyclotron frequency and $k_{\perp}^2 = k_r^2 + k_{\theta}^2$, with the poloidal wave number $k_{\theta} = m/r_0$ (m is poloidal mode number and r_0 is ring radius) and radial wave number $k_r \gg 1/\Delta$. The hot electron interchange imposes restrictions on n_c/n_h (setting a lower limit on core density or an upper limit on ring density). Because $q_0 > 1$ and $\beta_c \ll 2\Delta/R_c$ in present experiments, this criterion is easily satisfied except for low m mode numbers. However, deterioration of stability was

not observed experimentally. For reactorlike devices this mode is restrictive [Ref. A.17] because $q_0 \lesssim 0.1$ and because of the economic necessity for core beta values approaching the $2\Delta/R_c$ limit.

A high frequency, hot electron instability is driven by the hot electron pressure if $\omega > \omega_{ci}$ [Ref. A.7]. To excite this mode one needs [Ref. A.11]

$$q_0 > \frac{1}{4} \frac{k_{\perp}^2}{k_{\theta}^2}, \quad (\text{A.10})$$

and stability requires

$$\frac{n_c}{n_h} > \left[1 - \left(\frac{k_{\perp}^2}{q_0 k_{\theta}^2} + \frac{\beta_c R_c}{2\Delta} \right)^{1/2} \right]^{-2}. \quad (\text{A.11})$$

This criterion is easily satisfied for future reactorlike devices because $q_0 < 1$ and $k_{\perp}/k_{\theta} \sim r_0/\lambda \gg 1$ (for low m numbers). However, in present experiments $q_0 > 1$, and it may be possible to excite this mode. It is believed that the high frequency fluctuations (~ 10 – 20 MHz) observed in the experiments near the T-M transition may be related to insufficient cold plasma density [Ref. A.18].

When the core plasma ion density is too high, a compressional Alfvén mode is excited. This mode is stable if [Ref. A.11]

$$\frac{n_c}{n_h} < \left(\frac{k_{\perp}}{k_{\theta}} \right)^2 \frac{R_c}{2\Delta\beta_h q_0} \frac{[1 - q_0(k_{\theta}/k_{\perp})^2(1 - \beta_c R_c/2\Delta)]^2}{(\beta_t R_c/2\Delta - 1)(1 - \beta_c R_c/2\Delta)}, \quad (\text{A.12})$$

where $\beta_t = \beta_h + \beta_c$. This limit is either a density or a beta condition, depending on the plasma parameters. The compressional Alfvén mode does not appear to represent a problem for reactorlike devices.

Recent calculations [Ref. A.19] included the effects of both finite ion and hot electron Larmor radii. For flute modes in the purely radial Z-pinch model, a variational treatment shows that finite Larmor radius (FLR) effects can simultaneously stabilize both core plasma interchange and compressional modes, with

$$(k_{\perp} \rho)^2 > 2 \Delta / R_c \beta_h \quad (\text{A.13})$$

being sufficient to ensure the existence of a stable window (where ρ = hot electron or ion gyroradius). Also, an approximate analysis of the coupling of the shear Alfvén (finite $k_{\parallel}$) mode in a bumpy cylindrical system with axial variation shows that ballooning effects stabilize low m modes.

A.3. DISCUSSION OF RESULTS

Although core beta values on the order of 10% have been estimated, the coupling of beta limits with ring dimensions (through Δ) and power balance (through n_c/n_h) is a concern for a reactor. Note, however, that some of the stability constraints discussed here are only approximate, and assumptions made in their derivation break down for reactorlike devices. For example, it is shown [Ref. A.17] that several key dimensionless parameters ($\omega_{\text{dhot}}/\omega_{\text{ci}}$, β_c/β_h , Δ/R_c , etc.) and

frequency ordering in a reactor are outside the limits of applicability of present theories. In addition, the correct geometry of the bumpy torus configuration is yet to be included in the stability analyses.

Typical parameters of various EBTs (present and future) are shown in Table A.1, and characteristic frequencies (evaluated near the hot electron ring) corresponding to each are given in Table A.2. Characteristic frequencies in present EBT-I/S experiments are such that the frequency spread is small:

$$\omega_{\text{dhot}} \sim \omega_{\text{ci}} \sim kV_A \sim \omega_{\text{be}} \gg \omega_{\text{bi}} . \quad (\text{A.14})$$

Existing theories omit ω_{be} and its effects. In the near-future EBT-P experiment, the frequency spread is modest, whereas in a reactor (see Table A.2),

$$\omega_{\text{dhot}} \sim \omega_{\text{bi}} \ll kV_A \ll \omega_{\text{be}} < \omega_{\text{ci}} . \quad (\text{A.15})$$

Here, ω_{dhot} is the hot electron curvature drift frequency, V_A is the Alfvén speed, and ω_{be} and ω_{bi} are the bounce frequencies of core electrons and ions, respectively. The ordering of $\omega_{\text{dhot}} \sim \omega_{\text{bi}}$ could be important for ballooning modes on the ring.

Summaries of the short wavelength modes (discussed in Sect. A.2) that may be relevant to a reactor and approximate analytic expressions for their respective stability criteria are listed in Table A.3. Typically, these modes impose restrictions on upper and lower limits on n_c/n_h , an upper limit on β_c , a lower limit on β_h , etc.; these restrictions are determined by the ring scale length Δ , radius of

Table A.1. Typical parameters

	EBT-S	EBT-P	EBT-R
Magnetic field, T			
$B_{\text{ring}} \equiv B_R$	~ 0.5	~ 1.0	~ 2.0
$\langle B \rangle \sim B_{\text{res}}$	~ 1.0	2.0	4.0
Temperature, keV			
$T_{\text{ring}} \equiv T_h$	~ 500	$\sim 1-2 \times 10^3$	$\geq 2 \times 10^3$
$T_e(\text{core})$	$\sim 0.4-1.0$	$\sim 2-5$	$\sim 15-25$
$T_i(\text{bulk/tail})$	$\sim 0.04/0.25$	$\sim 0.5-2.5/2.5$	$\sim 15-25$
Density of core, m^{-3}	$\sim 10^{18}$	10^{19}	$\sim 10^{20}$
Plasma radius $\langle a \rangle$, m	~ 0.1	~ 0.18	~ 1.0
Sector length L, m	~ 0.4	~ 0.8	$\sim 6-9$
Radius of curvature R_c , m	$\sim 0.16 \pm$	$\sim 0.3 \pm$	$\sim 3.0 \pm$
Ring scale length Δ , m	$\sim 0.01 \pm$	$\sim 0.02 \pm$	$\sim 0.1 \pm$

Table A.2. Characteristic frequencies (evaluated near the hot electron ring in mks units with temperatures in kiloelectron volts)

	EBT-S	EBT-P	EBT-R
Ion cyclotron			
$\omega_{ci}/2\pi = 15.2 \mu^{-1} B$, MHz	~7.5	~15	~15
Hot electron drift			
$\omega_{dhot}/2\pi = 10^{-3} T_h/2\pi B R_c a$, MHz	~10	~3-6	≥50
Alfvén			
$kV_A/2\pi \approx 3.5 \left(\frac{10^{18}}{n}\right)^{1/2} \left(\frac{B}{a}\right) \mu^{-1/2}$, MHz	~15	~5	~0.5
Bounce			
$\omega_{be}/2\pi \approx 6.63 T_e^{1/2}/L$, MHz	~10-15	~10-20	~3-5
$\omega_{bi}/2\pi \approx 155 T_i^{1/2} \mu^{-1/2}/L$, kHz	~75-200	~125-300	~50-75
$q_0 = \frac{v_{cv1}}{\Delta\omega_{ci}} = \left(\frac{\omega_{dhot}}{\omega_{ci}}\right) \left(\frac{a}{\Delta}\right)$	~10	~1	≥5 × 10 ⁻²

Table A.3. Summary of stability conditions

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MODE	STABILITY CONDITION
CORE PLASMA INTERCHANGE	$\beta_{\text{hot}} \geq 4\Delta/\langle R_c \rangle$
HOT ELECTRON INTERCHANGE	$\frac{n_{\text{core}}}{n_{\text{hot}}} > \frac{4}{(k_{\perp} \Delta)^2} \frac{1}{q_0} \frac{1}{(1 - \beta_c R_c / 2\Delta)^2}$
COMPRESSIONAL ALFVÉN	$\frac{n_{\text{core}}}{n_{\text{hot}}} < \frac{[q_0(1 - \tilde{\beta}_c)(k_{\theta}/k_{\perp})^2 - 1]^2}{(k_{\theta}/k_{\perp})^2 q_0 (2\Delta/R_c)^2 \tilde{\beta}_h (\tilde{\beta} - 1)(1 - \beta_c)}$
INTERACTING RING-CORE INTERCHANGE	$\beta_{\text{core}} < \frac{2\Delta}{R_c} \left(1 + \frac{p_{\parallel h}}{p_{\perp h}} \right)$
DEFINITIONS: $q_0 = \frac{v_{c\perp l}}{\Delta \omega_{ci}}$, $\tilde{\beta} = \beta \frac{R_c}{2\Delta}$, $k_{\theta} = \frac{m}{a}$, $k_{\perp}^2 = k_r^2 + k_{\theta}^2$, $k_r \sim \frac{2}{\Delta}$	

curvature R_c (local and/or field line averaged), $q_0 = v_{c\perp l}/\omega_{ci}\Delta$, anisotropy, radial and poloidal wave numbers, etc. For example, the required n_c/n_h is limited from below by the low frequency ($\omega < \omega_{ci}$), hot electron interchange mode and from above by either the compressional Alfvén wave or the interacting ring-core interchange mode. For low plasma temperatures (as in present experiments), the compressional Alfvén wave limits the core plasma density; conversely, for higher core plasma temperatures (and for higher magnetic fields), the interacting ring-core interchange mode (or the microwave cutoff) determines the upper density limit.

Figure A.2 shows the stable operating window for the ratio n_c/n_h as a function of q_0 . This figure indicates that the stable window is very sensitive to the assumed value of radial wave number k_r . As pointed out earlier, these modes are short wavelength modes, and, by definition ($k_r \Delta \gg 1$), the radial wavelength fits across the outer edge (region III of Fig. A.1) of the ring of half-width Δ . A simple analytical model [Ref. A.11] indicates $k_r = 2/\Delta$ in that only a fraction of a wavelength fits to the outer edge of the ring. This k_r value is commonly used in various analytical stability studies; however, note that $k_r \Delta \gg 1$ is not really satisfied for these wave numbers. The comparison of the cases for $k_r = 2/\Delta$, π/Δ , and $2\pi/\Delta$ is evident from Fig. A.2.

The modes most pertinent to the reactor regime in Table A.3 are the low frequency, hot electron interchange; the core plasma interchange; and the interacting ring-core interchange. Boundaries of a reactor plasma operating window as a function of n_c/n_h and q_0 are shown in Fig. A.3. Within the constraints (1) that reactor parameters must lie within the possible operating window shown in Fig. A.3 for stability and (2) that the core pressure profile inside the ring (regions I and II in Fig. A.1) must satisfy core interchange criteria (see Appendix B for profile considerations), a favorable reactor design can be formulated in regard to both the recirculated power fraction and the total fusion power produced.

Fig. A.2. Stable operating window for the ratio $n_{\text{core}}/n_{\text{hot}}$ ($=n_c/n_h$) as a function of $q_0 = v_{cV1}/\Delta\omega_{ci}$ ($\sim T_h/B^2 R_c \Delta$) for various values of radial wave number k_r . The size of the stable window is very sensitive to the assumed value of k_r (as well as the poloidal mode number m).

Fig. A.3. Boundaries of stable operating window in n_c/n_h vs q_0 ($\sim T_h/B^2 R_c \Delta$) space in a reactor. Various boundaries are indicated which are determined by the hot electron interchange mode, ring beta limits, microwave cutoff, and net power production requirements. Here $k_r = 2\pi/\Delta$ is assumed and the compressional Alfvén mode (shown in Fig. A.2) does not appear to represent a problem for a reactor.

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Appendix B

STABLE PRESSURE PROFILES

The economic viability of an EBT reactor improves noticeably if the volume-averaged core beta $\langle \beta_c \rangle$ significantly exceeds ~5%. The radial profiles of core pressure (which, for simplicity, are assumed to be constant in most of the stability calculations discussed in Appendix A) are found to play an important role in both fusion and ring power balance. Since the magnetic fields in EBT vary both radially and axially (along the field lines), for constant pressure profiles, the volume-averaged core beta $\langle \beta_c \rangle$ is much smaller than the theoretically calculated core beta near the outside edge of the ring (region III in Fig. A.1):

$$\langle \beta_c \rangle \lesssim 0.25 \beta_c, \quad (B.1)$$

where β_c is given by Eq. (A.8). This imposes a serious economic penalty for a reactor. However, if pressure gradients in the core plasma (in regions I and II of Fig. A.1) that can be derived either from the modified energy principle (see Sect. B.2) or from kinetic theory (see Sect. B.3) are allowed, $\langle \beta_c \rangle$ can be found to be comparable to local core beta at the outer edge of the ring:

$$\langle \beta_c \rangle \approx (0.5-1.0) \beta_c. \quad (B.2)$$

This permits fusion power density to be high enough for a sufficiently small ring width and, in turn, an allowable recirculating power fraction for ring-sustaining power.

Preliminary calculations indicate the existence of equilibria with the pressure profiles discussed in Sects. B.2 and B.3 with the three-dimensional (3-D) equilibrium codes. Also, the radial profiles obtained from one-and-one-half-dimensional (1-1/2-D) transport calculations are found to be similar to those required for stability (see Appendix F).

B.1. EQUILIBRIUM RELATIONS

The basic equations for a finite beta equilibrium are

$$\vec{\nabla} \cdot \vec{P} = \vec{J} \times \vec{B}, \quad (\text{B.3})$$

$$\vec{\nabla} \times \vec{B} = \vec{J}, \quad (\text{B.4})$$

$$\vec{\nabla} \cdot \vec{B} = 0, \quad (\text{B.5})$$

where $\vec{J}$ is the total current (ring plus core), $\vec{P} = p_{\perp} \vec{I} + (p_{\parallel} - p_{\perp}) \hat{b} \hat{b}$ is the pressure tensor (anisotropic ring plus isotropic core), and

$$\vec{\nabla} \cdot \vec{P} = \vec{\nabla} p_{\perp} + \hat{b} \frac{\partial}{\partial \ell} (p_{\parallel} - p_{\perp}) + (p_{\parallel} - p_{\perp}) \vec{\kappa} - \frac{1}{B} \frac{\partial B}{\partial \ell} \hat{b}. \quad (\text{B.6})$$

Here,

$$\vec{\kappa} = (\hat{b} \cdot \vec{\nabla}) \hat{b} = \frac{1}{B} \vec{\nabla}_{\perp} B + \frac{1}{B^2} (\vec{\nabla} \times \vec{B}) \times \vec{B} \quad (\text{B.7})$$

is the curvature; $\hat{b} = \vec{B}/B$; $p_{\parallel}$ and $p_{\perp}$ are the total pressures parallel and perpendicular to the $\vec{B}$ field, respectively; and l = arc length along the field line.

The parallel component of Eq. (B.3) gives constraint conditions on the B-field dependence of $p_{\parallel}$ and $p_{\perp}$:

$$\frac{\partial p_{\parallel}}{\partial B} = \frac{p_{\parallel} - p_{\perp}}{B}. \quad (\text{B.8})$$

The perpendicular component of Eq. (B.3),

$$\vec{\nabla}_{\perp}(p_{\perp} + B^2/2) = \kappa B^2 \left(1 + \frac{p_{\perp} - p_{\parallel}}{B^2} \right), \quad (\text{B.9})$$

is solved to obtain equilibrium fields and pressure distributions.

B.2. PRESSURE PROFILES USED IN COUPLED STABILITY AND RING POWER BALANCE CALCULATIONS — MHD DERIVATION

Because the hot electron rings are localized to the region near the outside edge of the core plasma, stability calculations (at present) are mostly carried out near the rings on the outside edge, where ∇B is reversed (region III in Fig. A.1). In the absence of detailed kinetic derivations, stable pressure profiles are derived from the MHD criterion given by Eq. (A.1) for the core plasma (regions I and II in Fig. A.1), where ∇B is not reversed, and from kinetically derived relations that are given in Appendix A (Sect. A.2) for the outer edge

(region III in Fig. A.1). In Sect. B.3, preliminary kinetic derivations are given.

In regions I and II the stability condition is given by Eq. (A.3), and marginal stability is achieved if

$$\frac{p'}{p} = - \gamma \frac{U'}{U}, \quad (\text{B.10})$$

where $p' = \partial p / \partial \psi$ and $U' = \partial U / \partial \psi$ are given by Eq. (A.5). Concentrating at the midplane for simplicity (without loss of generality), derivatives with respect to the flux ψ can be transformed to derivatives with respect to r ; $\partial / \partial \psi \rightarrow (1/rB) \partial / \partial r$. Equation (B.10) then reads as

$$\frac{\partial \ln p}{\partial r} = - \frac{\gamma}{\langle R_B \rangle}, \quad (\text{B.11})$$

where

$$\frac{1}{\langle R_B \rangle} = \frac{\partial \ln U}{\partial r} = - \left\langle \frac{1}{B} \frac{\partial B}{\partial r} + \kappa \right\rangle. \quad (\text{B.12})$$

The curvature $\kappa = -1/R_c$ is not affected much by the hot electron rings, but the magnetic gradients change near the vicinity of the rings such that they can be calculated from Eq. (B.9):

$$\frac{1}{B} \frac{\partial B}{\partial r} = - \frac{1}{B^2} \frac{\partial p_{\perp}}{\partial r} - \frac{1}{R_c} \left(1 + \frac{p_{\perp} - p_{\parallel}}{B^2} \right). \quad (\text{B.13})$$

Here, $p_{\perp} = p_{\perp \text{ring}} + p_{\perp \text{core}}$, with $p_{\perp \text{core}} = p_{\parallel \text{core}}$. Expressions for $\langle R_B \rangle^{-1}$ are then given as follows:

In region I, the central region ($0 < r < r_0 - \Delta$; $p_{\perp \text{ring}} = 0$),

$$\frac{1}{\langle R_B \rangle_I} = \frac{2}{\langle R_c \rangle} \frac{1}{(1 + \gamma \beta_c/2)} = \frac{2}{\langle R_c \rangle}. \quad (\text{B.14})$$

In region II, the inside edge ($r_0 - \Delta < r < r_0$; $\partial p_{\text{ring}}/\partial r > 0$),

$$\begin{aligned} \frac{1}{\langle R_B \rangle_{II}} &= \left\langle \frac{\beta_h}{2\Delta} + \frac{2}{R_c} (1 + \beta_h/2) \right\rangle \frac{1}{(1 + \gamma \beta_c/2)} \\ &\approx \frac{2}{\langle R_c \rangle} \left(1 + \beta_h \frac{\langle R_c \rangle}{4\Delta} \right). \end{aligned} \quad (\text{B.15})$$

Here, β_c and β_h are core and ring plasma betas defined as $2p_{\perp}/B^2$, and Δ is the ring pressure scale length.

In region III, the outer edge ($r_0 < r < r_0 + \Delta$; $\partial p_{\text{ring}}/\partial r < 0$; $\partial \ln B/\partial r > 0$), the corresponding expression for Eq. (B.11) is

$$\frac{\partial \ln p}{\partial r} \approx - \frac{1}{\Delta}, \quad (\text{D.16})$$

which is used in kinetic treatment.

B.3. STABLE PRESSURE PROFILES — PRELIMINARY KINETIC CALCULATIONS

On the inside edge of the hot electron ring (region II in Fig. A.1), a finite pressure gradient may be supported due to the stabilizing effect of compressibility. Determination of the maximum stable value of this pressure gradient is an important factor in relating the core beta limit on the outside edge of the ring to the volume-averaged core beta $\langle \beta_c \rangle$.

Since kinetic effects enter into the stability of region II (see Fig. A.1), a calculation that takes into account ring-core coupling (see Ref. A.11) has been employed. The following core pressure profile and the typical reactor parameters given in Table A.1 are used in the radially dependent stability code:

$$p_c = n_{co} T_{co} \begin{cases} (1 - s) [(x + 1)^2 - 1]^4 + x & x < 0 \\ (s - s_0)(x^2 - 1)^4 + s_0 & 0 < x < 1, \\ s_0 & x > 1 \end{cases} \quad (B.17)$$

where n_{co} and T_{co} are the core plasma density and temperature, respectively, at $r = r_0 - \Delta$ ($x = -1$), $s_0 = 0.1$, $s = 0.1-1.0$, and $x = (r - r_0)/\Delta$.

Here, s is a parameter that is varied from 0.1 (all the pressure gradient on the inside edge of the ring) to 1.0 (all the pressure gradient on the outside edge of the ring). Typical core density profiles for various values of s are shown in Fig. B.1. These profiles

Fig. B.1. Typical core density profiles in regions II (inside edge of ring) and III (outside edge of ring) of Fig. A.1 for various values of profile parameter s (left). These profiles, if sharp enough, result in unstable roots with eigenfunctions peaked on the inside edge of the rings, as shown (right) for $s = 0.65$.

result in unstable roots with eigenfunctions peaked on the inside edge of the hot electron ring, as shown in Fig. B.1 for $s \approx 0.65$.

In numerical calculations these unstable roots are followed as a function of s and are observed to stabilize with the pressure gradient still finite on the inside edge of the ring (as shown in Fig. B.2). Effectively, the mode is stabilized when the growth rate is less than $(|\omega_{*c} \omega_{dc}|)^{1/2}$, since the theory is not valid below this point ($\omega \gg \omega_{*c}$, ω_{dc} is assumed), where ω_{*c} is the diamagnetic drift frequency and ω_{dc} is the drift frequency of the core plasma in region II. The result indicates stability around $s \approx 0.7$. Note that this result is not optimized and that further consideration of parametric and profile effects could give stability at somewhat lower s values ($s \lesssim 0.6$). For example, for higher values of ring beta (or ring temperature), the stability limit moves to lower s values.

B.4. TYPICAL RESULTS

Typical pressure profiles used in many stability calculations and derived here (in Sects. B.2 and B.3) are shown in Fig. B.3 for a comparison. For a representative set of parameters,

$$\Delta/r_0 \approx 0.1 ,$$

$$r_0/R_c(r_0) \approx 1/2 - 1/3 , \quad (B.18)$$

$$\gamma = 5/3 ,$$

one finds from Eqs. (B.11)–(B.15) that

Fig. B.2. Growth rate as a function of profile parameter s . Effectively, the mode is stabilized when the growth rate is less than $|\omega_{*c} \omega_{dc}|^{1/2}$, where ω_{*c} is the diamagnetic drift frequency and ω_{dc} is the drift frequency of the core plasma in region II.

$$p_c(r_o - \Delta)/p_c(0) \approx 0.6 \pm 0.1 \quad (\text{B.19a})$$

and

$$p_c(r_o)/p_c(0) \approx 0.3 \pm 0.1 \quad (\text{B.19b})$$

This implies that

$$\beta_c(0) \approx (1.0-2.0) \beta_c(r_o) \quad (\text{B.20})$$

Fig. B.3. Comparison of typical pressure profiles. Many stability calculations use profiles shown on the top. Stable profiles derived in this report are shown on the bottom. For a reactor, pressure profiles shown on the top are not favorable.

and

$$\langle \beta_c \rangle \sim O[\beta_c(r_0)] , \quad (B.21)$$

where $\beta_c(r_0)$ is the core beta value that enters in theoretical stability calculations [see Eq. (A.8)].

Appendix C

ELECTRON RING SCALING AND POWER BALANCE

There is a wealth of information on hot electron ring physics, both from present EBT experiments and from past mirror experiments. The properties of hot electron plasmas created by electron cyclotron heating (ECH), particularly the high beta, hot electron rings, have been described elsewhere in detail [Refs. C.1-C.5]. Here, a brief summary of the results, especially those pertinent to the full power balance expressions, is presented for completeness.

The temperature of the hot electron rings (where temperature is referred to the slope of the log of the hot electron distribution function) is observed to be determined by a simple scaling law [Ref. C.4] given by

$$\rho/L \approx \text{const} \approx 5-6 \times 10^{-2}. \quad (\text{C.1})$$

Here, ρ is the hot electron gyroradius (relativistic) and L is the magnetic field scale length (radius of curvature). Most of the ECH hot electron plasma experiments to date are governed by this scaling law where single-frequency heating is used; however, results from several experiments were no different for multiple ECH frequencies. [One possible exception is, probably, the use of ion cyclotron heating (ICH) on EBT-S, where an increase in ring temperature is observed with corresponding ρ/L values about 10-20% higher than those given by

Eq. (C.1); however, the mechanism for this process is not known at present.]

The value of $\rho/L \approx 0.05$ is roughly the value at which conservation of the adiabatic invariant μ begins to break down, which suggests that the hot electron temperatures are probably limited by nonadiabatic particle behavior. It is also possible that the ρ/L limit might also be related to the heating [Ref. C.6].

C.1. STEADY-STATE RING POWER REQUIREMENTS

The ring power losses in EBT experiments are observed to scale classically [Refs. C.1-C.3 and C.7]. The power absorbed in the ring per unit volume, $P_{\text{ring}}/V_{\text{ring}}$ (where V_{ring} = ring volume), is balanced by losses due to drag cooling of hot electrons on core electrons, pitch angle scattering of the ring electrons on core ions and electrons, and synchrotron radiation. The power loss is given (in mks units with temperature in kiloelectron volts and power in megawatts) by

$$P_{\text{ring}} = P_{\text{drag}} + P_{\text{scatt}} + P_{\text{synch}} , \quad (\text{C.2})$$

with

$$\frac{P_{\text{drag}}}{V_{\text{ring}}} \text{ (MW/m}^3\text{)} \approx 5 \times 10^{-38} n_h n_c \frac{\gamma}{(\gamma^2 - 1)^{1/2}} , \quad (\text{C.2a})$$

$$\frac{P_{\text{scatt}}}{V_{\text{ring}}} \text{ (MW/m}^3\text{)} \approx 2 \times 10^{-37} n_h n_c \frac{\gamma(\gamma - 1)}{(\gamma^2 - 1)^{3/2}} , \quad (\text{C.2b})$$

and

$$\frac{P_{\text{synch}}}{V_{\text{ring}}} \text{ (MW/m}^3\text{)} \approx 3.2 \times 10^{-20} n_h B_R^2 (\gamma - 1) \times \left[1 + 2.5(\gamma - 1) + \frac{1.5(\gamma - 1)^2}{\gamma - 0.3} \right], \quad (\text{C.2c})$$

where $\gamma = 1 + T_h/511$; n_h and T_h are the density and temperature of the ring, respectively; B_R is the magnetic field at the ring location; and n_c is the toroidal core plasma density near the vicinity of the ring.

Equation (C.2) can be rewritten as

$$P_{\text{ring}} \text{ (MW)} \approx 0.16 \beta_h B_R^4 V_{\text{ring}} G(\gamma, \beta_h, n_c/n_h), \quad (\text{C.3})$$

where $V_{\text{ring}} = 2\pi a \cdot \Delta \cdot (NL_{\text{ring}})$ is the ring volume, with N the number of rings (sectors); L_{ring} is the ring length per sector; and β_h is the ring beta. The ring occupies a fraction of a sector length, and in the experiments $L_{\text{ring}} \sim a$ is measured. Here,

$$G(\gamma, \beta_h, n_c/n_h) \approx 1 + 2.5(\gamma - 1) + 7.4 \beta_h \left(\frac{n_c}{n_h} \right) \frac{\gamma(\gamma - 1)^2}{(\gamma^2 - 1)^{1/2}} \left(1 + \frac{4}{\gamma + 1} \right). \quad (\text{C.4})$$

C.2. COUPLING OF STABILITY AND RING POWER BALANCE

Limitations imposed on core beta and pressure profiles, ring scale lengths, and power balance by stability constraints are critical in EBT in determining the operating windows of future experiments and in assessing the desirability of a reactor. In their present forms, among the constraints of various modes discussed in Appendix A (see Table A.3), the low frequency ($\omega < \omega_{ci}$), hot electron interchange (which sets lower limits on n_c/n_h) and the core-pressure-driven interchange (which limits the core beta to $\beta_c \leq 2\Delta/R_c$) are found to be most restrictive for reactor applications. In addition, radial profiles of the core pressure (see Appendix B) are important in the power balance of a reactor.

For an ignited reactor, the core plasma heating would be turned off after ignition. However, the ring power would have to be sustained throughout the steady-state operation. The amount of power required to sustain the rings is important, since it determines the recirculating power fraction and affects the economics. Defining a Q value that is the ratio of the fusion power produced (P_{th}) to the ring sustaining power (P_{ring}), one has

$$Q = \frac{P_{th}}{P_{ring}} \sim \frac{1}{\epsilon}, \quad (C.5)$$

where ϵ is the recirculating power fraction.

The total thermal fusion power (corresponding to 21 MeV per fusion event in a D-T plasma) in mks units is given by

$$P_{th} \text{ (MW)} \approx 8.5 \times 10^{-19} n^2 \langle \sigma v \rangle_{DT} \cdot V_p, \quad (C.6)$$

where n is the average core plasma (ion) density and V_p = core plasma volume = $2\pi^2 a^2 R_T$ with average plasma radius a and major radius R_T . In terms of core plasma $\langle \beta_c \rangle$ and average magnetic field strength B ,

$$P_{th} \text{ (MW)} = 1.3 \times 10^{24} \frac{\langle \sigma v \rangle_{DT}}{T^2} \langle \beta_c^2 B^4 \rangle V_p. \quad (C.7)$$

Here, $T_e \approx T_i \approx T$ is assumed.

Using Eqs. (C.3) and (C.7) in Eq. (C.5), one can see several features:

$$Q \sim \frac{1}{\epsilon} \sim F\left(\frac{V_p}{V_{ring}}, \langle \beta_c \rangle, \beta_h, \frac{n_c}{n_h}, \gamma\right); \quad (C.8)$$

that is, Q (or the recirculating power fraction ϵ) is a function of several dimensionless parameters, most of which are either determined or influenced by the stability constraints. For example, minimizing ring power (maximizing Q or minimizing ϵ) requires thin (small Δ) rings, whereas high beta (fusion power density) requires thick (large Δ) rings; this imposes opposing constraints on the ring dimensions and core beta. Taking into account the various stability constraints (LVN limit, etc.) discussed in Appendix A, the profile effects discussed in Appendix B, and other physics and technology

constraints, the results of systems studies (see Appendix G) indicate the existence of a comfortable operating window with acceptable ring power losses (recirculating power fraction). One such operating window is shown in Fig. A.2 (see Appendix A).

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Appendix D

ASSESSMENT OF ION RINGS

A preliminary assessment of energetic ion rings for use in an EBT reactor (EBT-R) has been carried out [Ref. D.1]. The properties of ion rings are compared with those of electron rings to determine if potential physics advantages are sufficient to consider replacing hot electron rings formed by electron cyclotron heating (ECH) with ion rings to stabilize the EBT configuration and to improve its reactor performance. Included in the analysis of the physical properties of ion rings are considerations of (1) device size and magnetic field strength required for their adiabatic confinement, (2) ring energy loss processes, from which a model for the ion ring power balance has been developed, and (3) stability requirements of the rings and the ring-core plasmas.

From considerations of device size, it is concluded that stable, adiabatic confinement of ion rings, assuming that the criterion [Ref. D.2] $\rho/L \approx 5 \times 10^{-2}$ (ρ = the average energetic ion gyroradius and L = the magnetic scale length) limits ion ring temperature, would require experiments with dimensions and magnetic field strengths comparable to those envisaged for EBT reactors designed to operate with electron rings.

Power loss from the ion rings is found to be dominated by Coulomb drag cooling of energetic ions on background core electrons, although drag and scattering by background core ions are not entirely negligible. At the ion ring energies [on the order of megaelectron volts (MeV)] required for stable reactor operation, charge-exchange

losses are shown to have a small effect on the power balance [Ref. D.3]. Results of power balance studies show that macrostable, steady-state, ion ring operation having acceptable power requirements is theoretically possible. In comparing ion ring power requirements with electron ring requirements (see Appendix C), there can be quantitative advantages in using ion rings instead of electron rings if the ion ring volume does not greatly exceed the volume of acceptable electron rings.

The stability conditions for curvature-driven modes (discussed in Appendix A) are the same for ion rings as they are for electron rings, with appropriate redefinitions. In addition, small-scale modes driven by either diamagnetic drifts (which could affect ring size) or temperature anisotropy (which could affect ring energy loss), if present, are considered. Drift waves such as the lower hybrid drift mode are found to be very little affected by the energetic ring ions and are probably of no consequence. However, an electromagnetic, hot ion mode driven by ring temperature anisotropy appears to be possible for ion rings whose formation results in $T_{\perp}/T_{\parallel} > 3-4$. This mode, which is referred to as an Alfvén ion cyclotron instability [Ref. D.4] or ion whistler [Ref. D.5], has been observed in neutral beam injection on the Tandem Mirror Experiment (TMX) [Ref. D.6]. Unlike the whistler in an electron ring, where relativistic effects are stabilizing, no relativistic stabilization is possible for relevant ion ring energies.

A list of the major issues that must eventually be considered in a comprehensive study of energetic ion rings in EBT is given in Sect. D.1. Section D.2 contains a discussion of device dimensions

required for experimental tests of ion rings. Section D.3 describes power balance calculations and compares steady-state ion ring power requirements with those for hot electron rings. Stability properties of ion rings are discussed in Sect. D.4, and Sect. D.5 summarizes the main conclusions.

D.1. MAJOR ISSUES FOR ION RINGS

A full set of issues that must be considered in a comprehensive assessment of ion rings is given in Ref. [D.1]. The present assessment has been limited to consideration of only some of these issues. Table D.1 summarizes the spectrum of problem areas that must be addressed to fully assess ion rings and compare them in detail with electron rings.

Because the gyroradius of an energetic ion is much larger than that of an electron of similar energy, an important issue for ion rings is the minimum machine size required to stably confine ion rings.

Power requirement considerations for ion rings, as for electron rings, must be assessed in terms of both start-up requirements and steady-state operating requirements. For the former, the concern is that there is sufficient heating power to overcome collisional losses at low energies and thus begin to extract a high energy population from the cold ion distribution. In the latter case, where an energetic population of ions has been formed, the power balance considerations are different. Here, the concern is to maintain the energetic ions against loss on the background core plasma. The energy dependence of the various loss processes will, in general, impose different power

Table D.1. Major issues for ion rings in EBT

Device size and magnetic field requirements

Heating techniques

- Mechanisms (neutral beam, ICH, etc.)
- Theoretical models

Ring start-up

- Power loss processes at low energy
- Heating greater than loss to extract an energetic ion population

Steady state

- Power loss processes at high energy
- Heating equals loss
- Ring size and power requirements

Engineering, technology, and cost trade-offs

- ECH vs ICH and/or neutral beams
 - Dollars per watt of ECH power vs ion heating power
-

requirements in these two different regimes. To produce a self-consistent model of ring start-up, size, and power balance, it will be necessary to develop adequate theories of ion heating for the EBT geometry [for example, ion cyclotron heating (ICH) or neutral beam heating].

Stability properties of ion rings must also be considered on both the macroscopic scale for beta limits and the microscopic scale for possible effects on power balance.

In addition to these physics issues, which are issues for either ion or electron rings, there are the important issues of engineering and technology requirements and economics. The latter considerations could be dominant factors in assessing advantages and disadvantages of ion vs electron rings for EBT reactors in cases where a clear-cut choice is not dictated by the physics.

D.2. DEVICE REQUIREMENTS FOR ION RINGS

To adiabatically confine ion rings at the energies required for stable operation in EBT devices, minimum size requirements and magnetic field strengths must be met. The condition

$$\omega_d / \omega_{ci} = \frac{\rho^2}{aR_c} \ll 1 \quad (D.1)$$

clearly must be satisfied for the ions to sample the magnetic geometry, where ω_d and ω_{ci} are the magnetic precessional drift and cyclotron frequencies of the energetic ions, ρ is the average gyroradius of the ring ions, a is the device minor radius, and R_c is the radius of curvature of the device. In electron ring experiments, it has been observed [Ref. D.2] that the hot electron gyroradius scales as $\rho/R_c \approx 5-6 \times 10^{-2}$ for a variety of cases. Table D.2 displays the ratios of ω_d / ω_{ci} and ρ/R_c for EBT-S, EBT-P, and EBT-R, using the

Table D.2. Values of ω_d/ω_{ci} , ρ/R_c , and $T_{\max}$ for proton (deuteron for reactor) rings in EBT-S, EBT-P, and EBT-R using parameters in Table A.1

	EDT-S	EBT-P	EBT-R
ω_d/ω_{ci}	~ 1.3	$\sim 0.2-0.4$	$\sim 3.4 \times 10^{-3}$
ρ/R_c	~ 0.9	$\sim 0.3-0.5$	≥ 0.03
$T_{\max}^a$	~ 2.0 keV	~ 22.0 keV	~ 4.0 MeV

^aCalculated assuming that $\rho/R_c = 5 \times 10^{-2}$ limits the ion ring temperature. The ring energies used to calculate ω_d/ω_{ci} and ρ/R_c are those assumed previously for electron ring operation.

parameters in Table A.1 for these devices and assuming the use of proton rings (deuteron ring for a reactor) with energies corresponding to those of the electron rings they replace. The maximum ion ring temperature is also tabulated assuming that, as with hot electrons, the condition $\rho/R_c = 5 \times 10^{-2}$ limits the temperature due to nonadiabatic losses. One sees that only in EBT-R is adiabatic confinement assured at the energies required for stable ring operation. In both EBT-S and EBT-P, because of their smaller size and lower magnetic field strengths (see Table A.1), the confinement criteria are violated at the relevant

ring temperatures (for stability) of 500 keV and 1-2 MeV, respectively. The maximum ring temperatures allowed by $\rho/R_c \approx 5 \times 10^{-2}$ are uninterestingly low for these devices. The conclusion is that device sizes comparable to a reactor will be required to adequately confine ion rings at suitable energies (those necessary to stabilize the core plasma). This assumes that nonadiabatic effects will limit the ion ring temperatures through the condition $\rho/R_c \approx 5 \times 10^{-2}$, as they appear to do for hot electron rings.

D.3. STEADY-STATE ION RING POWER REQUIREMENTS

To maintain the energetic rings in steady state, power sufficient to overcome that lost by a variety of processes must be continuously supplied. Table D.3 lists the classical energy loss processes that can affect ring power balance for both ion and electron rings. Here, instability-induced losses, which may be present for both electron and ion rings in certain circumstances, are not considered.

The case for electron rings is discussed in Appendix C. Basically, the dominant loss processes for present experiments are Coulomb drag on core plasma electrons and scattering off both core electrons and ions. These effects are of comparable importance. In future experiments, where the ring energies are expected to be >1 MeV, synchrotron radiation becomes important. As mentioned previously, the ring electron temperature presently appears to be limited by nonadiabatic effects. In future (higher energy) experiments, however, the ring temperature is expected to be determined by the onset of the

Table D.3. Classical power loss processes affecting electron and ion rings

Electron rings	Ion rings
Drag on core plasma electrons and ions	Drag on core plasma electrons and ions
Scattering off core plasma electrons and ions	Scattering off core plasma electrons and ions
Diffusion/convection	Diffusion/convection
Synchrotron radiation	Charge exchange
Bremsstrahlung	Nuclear and atomic processes
Atomic processes	Nonadiabatic effects at $\rho/L \sim 0.05$
Nonadiabatic effects at $\rho/L \sim 0.05$	

synchrotron-radiation-dominated regime at temperatures below the adiabatic limit [Ref. D.2].

In the case of ion rings, Coulomb drag of the energetic ions on the core plasma electrons is expected to be the dominant loss process for ring energies of reactor interest (>1 MeV). Coulomb drag and scattering by the core plasma ions are not entirely negligible, however, and can account for ~ 10 - 20% of the overall power losses. Charge exchange is a small effect for ions at megaelectron volt

energies, since the cross section falls off sharply with energies above 100 keV [Ref. D.3].

The power loss formulae for all the important classical loss processes for ion rings are given in Ref. [D.1]. The total steady-state power required for ion rings (due to classical Coulomb processes) is given by

$$P_{\text{ring}} = \sum_{j,k} \nu_{hj}^k n_h T_h V_{\text{ring}} = P_{\text{scatt}} + P_{\text{drag}}, \quad (\text{D.2})$$

where the summations are over all the species $j = e, i$ ($e =$ core electron, $i =$ core ion) and over all the loss processes k ($=$ scattering, drag); ν is the collision frequency; n_h and T_h are the density and temperature, respectively, of the ion ring; and V_{ring} is the ring volume. The components of Eq. (D.2) are (in mks units with temperature in kiloelectronvolts and power in megawatts):

$$\begin{aligned} \frac{P_{\text{scatt}}}{V_{\text{ring}}} \text{ (MW/m}^3\text{)} &= (P_{\text{scatt},e} + P_{\text{scatt},i})/V_{\text{ring}} \\ &\approx 3.24 \times 10^{-40} \frac{n_h n_e}{\mu_h T_{ec}^{1/2}} \left[1 + 56.25 \mu_h^{1/2} \left(\frac{T_{ec}}{T_h} \right)^{1/2} Z_c Z_h^2 \right]; \end{aligned} \quad (\text{D.3a})$$

$$\begin{aligned} \frac{P_{\text{drag}}}{V_{\text{ring}}} \text{ (MW/m}^3\text{)} &= (P_{\text{drag},e} + P_{\text{drag},i})/V_{\text{ring}} \\ &\approx 3.24 \times 10^{-40} \frac{n_h n_e T_h}{\mu_h T_{ec}^{3/2}} \left[1 + 56.25 \mu_h^{1/2} \frac{\mu_h}{\mu_e} \left(\frac{T_{ec}}{T_h} \right)^{3/2} Z_c Z_h^2 \right], \end{aligned} \quad (\text{D.3b})$$

where n_c and T_{ec} are the core plasma electron density and temperature at the ring location, respectively; $\mu_c = m_{ic}/m_p$ with m_{ic} the core ion mass and m_p the proton mass; $\mu_h = m_h/m_p$ with m_h the ring ion (hot) mass; and Z_c and Z_h are the charge states of the core and ring ions, respectively.

Comparing the various loss terms in Eq. (D.3) for ion rings, one sees that drag on the core electrons ($P_{drag,e}$) is the dominant energy loss mechanism for

$$T_h/T_{ec} \gtrsim 20. \quad (D.4)$$

For typical reactor parameters, $T_h/T_{ec} \sim O(10^2)$ and drag losses on the core electrons are 5-10 times those of drag and scattering losses on the core ions. Thus, combining both parts of Eq. (D.3) in Eq. (D.2) and rewriting it, one has

$$P_{ring} \text{ (MW)} \approx 10^3 \frac{\beta_c \beta_h B_R^4}{\mu_h T_{ec}^{5/2}} V_{ring} [1 + f(T_{ec}/T_h)], \quad (D.5)$$

where β_c and β_h are the core and ring plasma betas, respectively; B_R is the magnetic field at the ring location; $V_{ring} = 2\pi a \cdot \Delta \cdot (NL_{ring})$ is the ion ring volume of half-width Δ with N the number of rings (sectors); L_{ring} is the ring length per sector; and the function f is small and is on the order of $\sim 10^{-1}$,

$$f(T_{ec}/T_h) \approx \frac{T_{ec}}{T_h} \left[1 + 56.25 \mu_h^{1/2} \left(1 + \frac{\mu_h}{\mu_c} \right) \left(\frac{T_{ec}}{T_h} \right)^{1/2} Z_c Z_h^2 \right]. \quad (D.6)$$

The ion ring power loss can be minimized for a given core beta by operating at the minimum value of ring beta required for stability. Also, note that ion ring losses vary as $T_{ec}^{-5/2}$; thus, a considerable reduction in power loss can be realized for relatively large core plasma temperatures in the vicinity of the ring. This implies that nearly flat core plasma temperature profiles are favored from the standpoint of ion ring power requirements.

For typical reactor cases, the power required to sustain an ion ring relative to that required to sustain an electron ring can be calculated from Eqs. (D.5) and (C.3) to be

$$P_{ring,i}/P_{ring,e} \approx 2 \mu_h^{-1} \beta_c (10/T_{ec})^{5/2} (V_{ring,i}/V_{ring,e}) , \quad (D.7)$$

where the assumption is that $\beta_{h,i} = \beta_{h,e}$ and $T_h \sim 1-2$ MeV; for an electron ring this implies that $P_{synch}/(P_{drag} + P_{scatt}) \sim O(1)$. The temperature scaling of Eq. (D.7) favors a large core electron temperature in the ring region. Considering comparable ion and electron ring volumes with $\beta_c \sim 0.1$ and $T_{ec} \sim 5-10$ keV, one has

$$P_{ring,i}/P_{ring,e} \sim \begin{cases} 0.1-0.5 \text{ for } D^+ \text{ ion ring ,} & (D.8a) \\ 0.2-1.0 \text{ for } H^+ \text{ ion ring .} & (D.8b) \end{cases}$$

Thus, one sees that ion ring power requirements can be quantitatively favorable, provided that (1) the ion ring volume is not much larger than the electron ring volume and (2) the core electron temperature is not much smaller than about 5 keV near the vicinity of the ring. One

would expect $\Delta = \rho_i$ to be a minimum ring thickness. For ion rings with $T_h \sim 1-2$ MeV and $B_R \sim 2$ T, $\rho_i \sim 10$ cm (corresponding gyroradius for electron ring is $\rho_e < 0.5$ cm). Ion rings must be sufficiently localized so that Δ is not much larger than ρ_i .

D.4. STABILITY PROPERTIES OF ION RINGS

For a classical description of power balance, as discussed in Sect. D.3, to adequately predict ring power requirements, the rings must operate stably. In a preliminary stability survey of ion rings, three classes of modes have been examined:

1. Curvature-driven modes, familiar from electron ring studies, which determine ring and core beta limits, as well as required ring-core density ratios;
2. Diamagnetic drift-driven modes, which, if present, could affect ring size through anomalous diffusion and resistivity; and
3. Anisotropy-driven modes, which could lead to enhanced power loss, for example, through enhanced particle scattering.

D.4.1. Curvature-Driven Modes

Curvature-driven modes have been studied for electron rings (see Appendix A and references therein), and refinements to the theory [e.g., finite Larmor radius (FLR) effects] are under continuing investigation. Table A.3 summarizes the stability properties of curvature-driven interchange modes for electron rings, namely, the core

interchange, hot electron interchange, and interacting ring-core interchange. An examination of the equations describing these low frequency interchange modes shows that the stability conditions are identical for energetic ion rings with energetic ion quantities replacing the hot electron quantities in the stability formulae. Note that, since an energetic ion gyroradius is much larger than an electron gyroradius at the same energy, the stabilizing influence of FLR effects will be important. A sufficient condition for FLR stabilization of low frequency modes [Ref. D.7] is given by

$$(k_{\perp} \rho)^2 > 2 \Delta / R_c \beta_h, \quad (D.9)$$

where $k_{\perp}$ is the perpendicular wave number ($k_{\perp}^2 = k_r^2 + k_{\theta}^2$ with poloidal wave number $k_{\theta} = m/r_0$, poloidal mode number m , ring radius r_0 , and radial wave number $k_r \gg 1/\Delta$).

From Table A.3, to estimate the core-to-ring density ratio required for stability of the hot electron (ion) interchange mode, one can take $\Delta k_{\perp} = 2$ [Ref. D.8] and write the stability condition approximately as

$$\frac{n_c}{n_h} > \frac{R_c}{\rho} \frac{\Delta}{\rho}. \quad (D.10)$$

For typical reactor parameters (see Table A.1), this leads to $n_c/n_h > 30$ for ion rings with $\Delta \sim \rho$. Rewriting Eq. (D.10) as a condition on beta, one finds

$$\frac{\beta_c}{\beta_h} > \frac{R_c}{\rho} \frac{\Delta 2T_c}{\rho T_h}, \quad (D.11)$$

where $T_{ec} = T_{ic} = T_c$ is assumed.

Thus, to satisfy both the ring-core and hot electron interchange criteria, β_c is restricted to

$$\frac{2\Delta}{R_0} \geq \beta_c > \beta_h \frac{R_c}{\rho} \frac{\Delta 2T_c}{\rho T_h}. \quad (D.12)$$

This provides a somewhat restricted but reasonable window of stability, consistent with acceptable power losses, for EBT reactors using deuterium ion rings at ~ 2 MeV operating near minimum β_h .

D.4.2. Drift Modes

In addition to understanding the stability of ion rings for macroscopic modes, their microstability properties must also be assessed, since microinstabilities, if present, could influence ring size and/or loss rates and, thus, ring power balance. Here, the stability of small-scale modes driven by the diamagnetic drifts due to the pressure gradients in the ring region is considered. Since the minimum ion ring thickness will be the energetic ion gyroradius, drift modes that could occur for $L_p \geq \rho$, where L_p is the ring pressure gradient scale length, are of interest. If present, such modes could result in thicker rings and thus larger ion ring power requirements due to the additional transport they might induce. For values of L_p

comparable to or somewhat larger than ρ , the relevant modes are lower hybrid drift (LHD) modes [Ref. D.9]. The linear stability of these modes has been analyzed for a three-component plasma made up of an energetic ion ring component in a cold background of core plasma electrons and ions. The LHD modes are considered in the electrostatic limit in which ∇B and curvature effects are ignored in order to assess the relevance of the basic LHD modes. Only density gradient terms are retained for the background plasma; both density and temperature gradients are retained for the ring. The scale lengths are different, in general, for the ring and core plasmas. Figure D.1 contains a schematic of the modes.

The frequency and growth rate of LHD modes for an energetic ion ring in a colder background plasma in a slab approximation are

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Fig. D.1. Schematic of diamagnetic drift-driven modes.

$$\omega = -kV_e / \left[1 - \Gamma_0 + k^2 \lambda_e^2 + \frac{T_e}{T_i} (1 + \lambda_i^2 / \lambda_h^2) \right] \quad (D.13a)$$

and

$$\frac{\gamma}{\omega} = \frac{T_e}{T_i} \frac{\pi}{2}^{1/2} \frac{1}{kv_i} \frac{\omega}{kV_e} \left[\omega - kV_i + \left(\frac{T_i}{T_h} \right)^{3/2} \frac{n_h}{n_i} (\omega - kV_h) \right], \quad (D.13b)$$

with

$$V_e = \frac{-v_e^2}{\omega_{ce}} \Gamma_0 \frac{\partial}{\partial x} \ln n_e; \quad \Gamma_0 = e^{-b} I_0(b); \quad b = k^2 \rho_e^2;$$

$$V_i = \frac{v_i^2}{\omega_{ci}} \frac{\partial}{\partial x} \ln n_i; \quad V_h = \frac{v_h^2}{\omega_{ci}} \left(1 - \frac{n_{Th}}{2} \right) \frac{\partial}{\partial x} \ln n_h;$$

$$n_{Th} = (\partial \ln T_h / \partial x) / (\partial \ln n_h / \partial x);$$

where e, i, and h refer to background electrons, background ions, and ring ions, respectively. Thus, the contribution to the growth rate from the energetic ion component compared with that from the background plasma is in the approximate ratio

$$\left(\frac{T_i}{T_h} \right)^{3/2} \frac{n_h}{n_i} \frac{V_h}{V_i} \sim \frac{L_i}{L_h} \frac{n_h}{n_i} \left(\frac{T_i}{T_h} \right)^{1/2} \ll 1$$

for typical EBT reactor parameters. It can be concluded that an ion ring will not significantly influence the drift wave stability of an EBT reactor; drift wave stability is determined mainly by the background plasma structure. Unless the background plasma gradients are quite large ($L_i \sim \rho$), significant drift wave activity is not expected.

D.4.3. Anisotropy-Driven Modes

For ring formation techniques that result in preferentially energizing ring particles perpendicular to the magnetic field, resulting in $T_{\perp} > T_{\parallel}$, unstable, small-scale, anisotropy-driven modes become theoretically possible. Such modes could lead to enhanced scattering of ring particles and thus contribute to ring power loss.

In the case of electron rings, conventional whistler modes must be considered. These modes are subject to relativistic stabilizing effects at the high energies relevant for EBT electron rings. At these energies, relativistic broadening of the whistler wave resonance by the hot electrons substantially reduces the growth rate of these modes [Ref. D.10]. Thus, in practice these modes may not significantly affect electron ring power balance.

For ion rings, however, there is an analogous electromagnetic mode, referred to as an Alfvén ion cyclotron mode [Ref. D.4] or ion whistler mode [Ref. D.5], driven by temperature anisotropy. Unlike the whistler mode for hot electron rings, where the relativistic electron energy is a stabilizing influence, the ion whistler modes will not be subject to relativistic stabilizing effects at the nonrelativistic ion energies (on the order of megaelectron volts) expected in EBT ion ring operation. These modes have been observed in recent neutral beam injection experiments [Ref. D.6] on the end cell plasma of the TMX experiment. The instability resulted in degraded plasma confinement. Calculations show that the threshold value of $T_{\perp}/T_{\parallel}$, at which ion Landau damping is overcome by anisotropy driving terms, is dependent on plasma dimensions. A numerical example given in Ref. [D.6], which may

be relevant for EBT reactors, indicates a threshold of $T_{\perp}/T_{\parallel} \sim 3-4$ in the range $\beta_h \sim 0.2-0.5$ for $a/\rho = 15$ and $L_{\parallel}/\rho = 50$ ($L_{\parallel}$ is the axial length of energetic ions). These calculations indicate the possible importance of the modes for ion ring operation in EBT when the ions are formed with an anisotropic temperature, for example, with neutral beams or ICH. More detailed linear stability calculations for EBT geometry are called for, as is a theoretical assessment of the effects on ion ring power balance from these modes.

D.5. DISCUSSION OF RESULTS

The main conclusions of this preliminary assessment of ion rings for EBT include the following.

1. Stable energetic ion rings in EBT geometry probably require reactorlike device size and magnetic field strengths, if the condition $\rho/L = 5 \times 10^{-2}$ limits ion ring temperature.
2. Ion ring losses are dominated by Coulomb drag on the background electrons for ring energies of reactor interest.
3. Stable windows for steady-state ion ring operation with acceptable power requirements appear to exist for EBT reactors.
4. Power loss from ion rings is minimized (a) for operation at the minimum β_h required for ring stability and (b) for high core plasma electron temperature at the ring location in that flat temperature profiles are favored.
5. High β_c (and thus large fusion power density) is favored by large ring thickness Δ , while low ring power loss is favored by small Δ ; the optimum appears to be $\Delta \sim \rho$, $\beta_c \sim 10\text{--}15\%$ for $T_h \sim 2$ MeV.
6. For typical reactor parameters, the ratio of power per unit volume required for an ion ring to that required for an electron ring is $P_i/P_e \sim 0.1\text{--}0.5$ (for a D^+ ion ring); thus, power requirements to sustain ion rings in steady state can be quantitatively lower than those for electron rings, provided the ion ring volume does not greatly exceed the electron ring volume [$V_{\text{ring},i} < (2\text{--}10)V_{\text{ring},e}$].

7. Even in cases where ion ring power requirements are comparable to or somewhat larger than those for electron rings, there may be engineering, technology, and/or cost advantages that could be determining factors.
8. Small-scale Alfvén ion cyclotron modes may occur in ion ring operation for sufficient ring temperature anisotropy $T_{\perp}/T_{\parallel} \gtrsim 4$; thus, the theory of these modes and their influence on power balance should be assessed for ion rings in EBT geometry.
9. More detailed studies of ion rings will be required (and appear warranted) to determine if there are advantages sufficient to replace electron rings with ion rings in EBT reactors. Studies should include formation methods and heating models, start-up, power balance and stability, engineering, technology, and cost trade-off studies.

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Appendix E

REACTOR MAGNETICS AND PARTICLE CONFINEMENT

The spatial position of the hot electron rings (formed near contours of constant $|B|$ in the midplane) is critical to efficient utilization of the magnetic volume within the vacuum chamber because it determines the boundary of the toroidally confined core plasma. Various methods have been proposed for centering the mod-B in the midplane and for improving single-particle confinement in the vacuum magnetic field of a bumpy torus [Ref. E.1].

Methods of improving single-particle confinement in the vacuum magnetic field of an EBT reactor have focused on enhancement of the effective magnetic aspect ratio through the addition of relatively low current supplementary coils to the basic EBT configuration of toroidally linked circular mirror coils [Ref. E.1]. Recently, the Nagoya Bumpy Torus (NBT) group has investigated confinement with noncircular mirror coils in the planned NBT-II experiment [Ref. E.2]. The parameters describing the noncircularity (ellipticity and triangularity) are similar to those of typical tokamak D-shaped toroidal field (TF) coils. In a bumpy torus, however, the vertical "straight" leg is optimally located on the outside of the torus rather than on the inside; that is, the coil is reversed from the tokamak configuration. In a toroidal array of these coils, this orientation results in a partial cancellation of the vertical drift that arises from the toroidicity.

A critical parameter in the assessment of any EBT reactor magnetics configuration is the radial distance δ between the first wall of the reactor chamber and the mirror coil windings. This space is occupied by the first wall, blanket/shield assembly, coil dewar expansion voids, thermal insulation, etc. In general, $\delta \approx 1.0\text{--}1.5$ m is typically required for adequate neutron shielding and acceptable tritium breeding ratio. It is instructive to compare the ratio of δ to the mean coil radius R_c for present (EBT-I/S) and near-term (EBT-P) experiments with that for a reactor. In EBT-I/S, δ is just the coil bobbin thickness and $\delta/R_c \approx 1/8$; for the proposed EBT-P experiment with superconducting magnets, $\delta/R_c \approx 1/4$. Assuming $R_c \approx 3$ m for the mirror coils gives $\delta/R_c \approx 1/3$ to $1/2$ for a reactor, where blanket and shield are uniformly distributed under and between the coils around the torus. Alpha particle first-orbit losses and efficient plasma volume utilization are primary considerations in an assessment of the impact of large δ/R_c on reactor economics and performance. Without supplementary coils to modify the vacuum magnetic field in the reactor chamber, a large value of δ leads to a large reactor volume (either a large mechanical aspect ratio or a large major radius so that the orbits are better centered, or a large minor radius and correspondingly smaller δ/R_c). By taking advantage of the orbit topology and device geometry, a reduction in δ/R_c can also be realized with nonuniform distribution of blanket and shield.

Reference [E.1] showed that the use of symmetrizing (SYM) coils, with circular mirror coils and $\delta = 1.5$ m, resulted in a magnetic field geometry within the reactor chamber in which the midplane mod-B was symmetric across the minor axis at the ring location. The

corresponding magnetic aspect ratio enhancement (ARE) factors were approximately 2 (1.5) for trapped (passing) particles. Similar results were obtained with axis-encircling ARE coils. Both ARE and SYM coils improve volume utilization and single-particle confinement by lengthening the magnetic field lines inside the torus relative to those outside; SYM coils are more efficient in terms of magnetic field utilization in a reactor.

Recent studies [Ref. E.3] show that noncircular inverse-D (ID) coils are extremely effective in symmetrizing (centering) mod-B (thus centering the ring and toroidal plasmas) in the midplane, thereby giving good trapped particle confinement and reactor volume utilization. While the magnetic system with ID coils is significantly less complicated than those with SYM coils, the question of whether the use of ID coils alone in the basic EBT configuration permits a significant reduction in the size of an EBT reactor is still open. Configurations with noncircular tori (square torus, racetrack, etc.) are also under consideration [Ref. E.4]. Brief summaries of configurations with wedged mirror coils, ARE and SYM coils, and noncircular mirror coils are given in the following sections.

E.1. WEDGED MIRROR COILS

Wedging the mirror coils could provide a method for improving trapped particle confinement (centering of mod-B) and gaining access to the high field region for microwave heating. For a reactor, two types of wedged coils are used in Refs. [E.1] and [E.3]. In the split-wedged coil, the two coil halves are separated by a distance of 50 cm and

wedged about this position. This separation permits separate dewars for each coil half and support structure to maintain the coil positions. To isolate the effects of wedging, a configuration in which the coil halves are simply wedged without any additional separation is also examined. In general, wedging improves trapped particle confinement by better centering mod-B in the midplane about the minor axis [Refs. E.1 and E.3]. If standard mirror coils in a torus are wedged or split and wedged, the mirror ratio is reduced. The mirror ratio can be restored to its value in the standard configuration by increasing the major radius with the number of sectors N fixed, by decreasing N while keeping the major radius fixed, or by decreasing the mean coil radius. The third option (which leads to an increase in δ/R_c) tends to nullify any beneficial effects of wedging, and the first two options lead to larger reactors. Table E.1 shows that the reduced mirror ratio in a wedged coil system does not significantly affect the trapped-particle ARE factor but does tend to degrade the confinement of transitional and, to a lesser extent, passing particles [Ref. E.3]. This is reflected in lower values of $f_A^{(2)}$ (the enhancement factor based on $\oint d\ell/B$) in group A compared with the corresponding cases with $M = 2.24$ in group B.

The conclusion is that, whereas trapped particle confinement and the symmetry of mod-B across the minor axis in the midplane are improved by wedging the mirror coils, the confinement of transitional and passing particles is not improved. Since it is necessary to increase the size of the reactor to retain the same mirror ratio (and the same transitional and passing particle confinement), it is not clear that wedging offers a significant advantage over simply

Table E.1. Effects of wedging on ARE factor f_A for unsplit wedged coils with R_T adjusted so that $B_{00} = 3$ T (Case A) and for split wedged coils with a 50-cm gap and R_T adjusted so that $M = 2.24$ (Case B)

Wedge angle (deg)	Major radius (m)	Mirror ratio	$r(B_{\max}^{\text{mid}})$ (m)	$f_A^{(1)a}$	$r(\oint d\ell/B)_{\min}$ (m)	$f_A^{(2)b}$
Case A						
0	34	2.24	0.524	1.0	1.258	1.0
5	34.3	2.178	0.476	1.09	1.229	1.02
10	34.7	2.105	0.408	1.26	1.185	1.05 ^c
15	35.3	2.032	0.317	1.59	1.124	1.10 ^c
Case B						
0	35.4	2.24	0.517	0.97	1.275	0.95
5	36.3	2.24	0.451	1.09	1.194	1.00
10	37.5	2.24	0.369	1.29	1.089	1.07
15	38.8	2.24	0.280	1.64	0.974	1.18

$f_A^{(1)}$ is the ARE factor based on the radial position of the maximum of mod-B in the midplane -- trapped particle ARE factor.

$f_A^{(2)}$ is the ARE factor based on the radial position of the minimum of $\oint d\ell/B$, and represents a pitch-angle-averaged ARE factor.

^cThe lower mirror ratio for large wedge angles degrades transitional particle confinement. This is reflected in $f_A^{(2)}$ but not in $f_A^{(1)}$.

increasing the number of sectors and/or the mirror ratio in the standard (unwedged) configuration. While the cost-effectiveness of wedged coils as a means of improving particle confinement is questionable, the increased complexity and cost of the magnetics system could be justified if, for example, access to the high field region through the center of the coil were needed for microwave heating.

E.2. ARE AND SYM COILS

The supplementary coil technique of aspect ratio enhancement consists of modifying the vacuum magnetic field within the reactor chamber by placing two or more (as required by symmetry) relatively low current supplementary coils in each mirror sector. These coils are in addition to the mirror coils that provide the EBT toroidal field. Two types of supplementary coil configurations have been studied [Ref. E.1].

Axis-encircling ARE coils (two per sector) typically require $\leq 10\%$ of the ampere-turns in the mirror coils (I_{ARE}), with the polarity of the current opposite to that in the mirror coils. Enhancement of the effective aspect ratio with ARE coils results from two effects: (1) increased mirror ratio and (2) lengthening of the magnetic field lines inside the torus relative to those outside. These effects come about largely through partial cancellation of the mirror coil field. Therefore, for operation at a given field strength with ARE coils, the mirror coils must be capable of generating a field that is approximately 3% stronger for each 1% of ARE current than the required field in the absence of ARE coils. To lowest order, the ARE factor for a reactor with ARE coils can be approximated by $f_A \approx (1 + 2I_{ARE})^2 / (1 - 2I_{ARE})$. For example, single-particle confinement in a 24-sector reactor with $I_{ARE} \approx 8-10\%$ of the mirror coil ampere-turns compares favorably with that in a reactor having 42-48 sectors without ARE coils. Although ARE coils are quite effective in improving both single-particle confinement and magnetic field geometry

within the reactor chamber, increased size and field requirements of mirror coils result in a costly magnetics system.

The SYM coils serve the same purpose but offer several distinct advantages over ARE coils. SYM coils do not encircle the minor axis of the torus and therefore permit easier assembly and maintenance and better accessibility. In addition, there is no global field cancellation, so it is not necessary to operate the mirror coils at higher fields than would be required without SYM coils. They improve single-particle confinement by symmetrizing mod-B across the minor axis in the midplane and lengthening magnetic field lines inside the torus (see Figs. E.1 and E.2). The required ampere-turns per SYM coil (2 per sector) are roughly twice the value for ARE coils, but the total ampere-turns (SYM + mirror coils) for a given operational field are less for a reactor using SYM coils than for one using ARE coils.

Thus, of the two supplementary coil configurations examined, SYM coils offer the most promise for improving magnetic field geometry and single-particle confinement. Even though the magnetics system is more complicated than that for simple mirror coils, the attendant reduction, by approximately a factor of 2, in the required size of an EBT reactor makes the SYM coil configuration economically attractive. This is particularly true if more detailed neutronics and engineering calculations indicate that the required thickness of the first wall, blanket/shield assembly, dewar, etc., under the coil is $\delta \geq 1.3-1.5$ m.

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Fig. E.1. Three-dimensional, finite-beta tensor pressure equilibria in one sector of a reactor without and with SYM coils ($I_{SYM} = 20\%$). The region occupied by the ring (dark dots) and the toroidal core plasma (light dots) is shown. With SYM coils, mod-B across the minor axis is nearly symmetric and the maximum radius of the hot electron rings as well as the core plasma is nearly doubled.

Fig. E.2. Variation of aspect ratio enhancement factor f_A with SYM coil current I_{SYM} for trapped (T) and passing (P) particles.

E.3. NONCIRCULAR MIRROR COILS

The use of noncircular (specifically, semicircular) mirror coils was recently suggested by the NBT group as a means of improving particle confinement in the planned NBT-II experiment [Ref. E.2]. Here possible reactor applications of noncircular mirror coils have been examined. Rather than using semicircular coils, the coil shape has been parameterized in the same way as is done for a tokamak's D-shaped TF coils. In a bumpy torus the vertical "straight" leg of the coil is optimally located outside the torus rather than inside; that is, the coil is reversed from that in tokamak configuration. A typical inverse-D (ID) coil configuration for a bumpy torus is shown in Fig. E.3. In a toroidal array of these coils, this orientation results in a partial cancellation of the vertical drift that arises from the toroidicity.

To study the effects of mirror coil noncircularity, we consider a system with a given major radius R_T , number of sectors N , mirror ratio M , etc., in which the circular mirror coils are replaced with ID coils whose diameter, ellipticity, and triangularity are chosen to give the required mirror ratio on axis. The resulting mean diameter of the ID coils across the equatorial plane is typically about 80% of that for the circular coils for ellipticity and triangularity values of 1.75 and 0.7, respectively. If the mechanical aspect ratio is defined as the major radius of the torus divided by the mean coil radius in the equatorial plane, then the aspect ratio of the ID coil system is about 25% larger than that with circular coils. This is evident in the results presented here for passing particles. Defining a quantity to

Fig. E.3. Typical ID coil configuration. Mean diameter of the ID coil across the equatorial plane is about 80% of that for the circular coil for a given major radius, number of sectors, and mirror ratio.

represent the magnetic aspect ratio enhancement of ID coils compared with circular coils is not without ambiguity. For example, the magnetic axis in an infinite-aspect-ratio ID coil system is outside the geometric center of the coil (defined in the equatorial plane). This means that a finite-aspect-ratio ID coil torus can have mod-B in the midplane almost perfectly symmetric about the geometric axis, as shown in Fig. E.4 for $N = 36$, $M = 2.36$. If the inward shift of the minimum of the longitudinal invariant $r(J_{\min})$ is measured relative to the magnetic axis and scaled by the mean coil radius in the equatorial plane, then the ARE factors for trapped and passing particles are seen to be ~ 1.7 and 1.35 , respectively, for $M = 2.36$.

As pointed out, the radial distance δ between the first wall and the mirror coil windings is an important parameter that influences the selection of a magnetics system. The ratio of the area of the largest confined passing particle ($V_{\parallel} = V$) drift orbit to that for trapped particles ($V_{\parallel} = 0$) is compared in Fig. E.5 for ID coils and for circular mirror coils (with and without SYM coils) as a function of δ for various reactor sizes with $N = 18, 24, 30$, and 36 coils. In all cases, ID coils are superior to simple circular coils. However, for $\delta \gtrsim 1.0$ – 1.2 m in the example shown in Fig. E.5, the improvement obtained with ID coils is not as large as that obtained with SYM coils.

A commonly used method [Ref. E.1] of characterizing the confinement of single particles in an EBT vacuum magnetic field consists of measuring the inward shift, from the minor axis, of the minimum of the longitudinal adiabatic invariant $J = \int V_{\parallel} d\ell$, $[r(J_{\min})]$. In the limit $V_{\parallel} \rightarrow 0$, this shift is given by the radial position of the peak of mod-B in the midplane, $[r_{\max}(\text{midplane mod-B})]$. The magnitude

Fig. E.4. Mod-B contours (solid) and magnetic field lines (dashed) in the equatorial plane for devices with 24 (left column) and 36 (right column) sectors. Inverse-D coils are shown on the top, and circular coils are shown on the bottom. Note the excellent centering of the plasma and the ring (centering of mod-B about the minor axis) in the midplane with ID coils, as well as the smaller clear bore in the equatorial plane.

Fig. E.5. The ratio of the area of the largest confined passing particle ($V_{||} = V$) drift orbit to that for trapped ($V_{||} = 0$) particles is shown for circular coils (a) without and (b) with SYM coils and (c) for ID coils as a function of blanket-shield thickness ($\Delta_{B-S} = \delta$) under the coils for various reactor sizes.

of the inward shift for deeply trapped particles ($V_{\parallel} = 0$) and for extreme passing particles ($V_{\parallel} = V$) is shown in Fig. E.6 for ID coils as a function of N . The curve in Fig. E.6 shows the N^{-1} dependence of the shift for $V_{\parallel}/V \approx 0$. On the other hand, the shift for passing particles ($V_{\parallel}/V \approx 1$) exhibits roughly an $N^{-\alpha}$ dependence with $\alpha \approx 0.74$ for fixed $M (\approx 2.36)$.

It should be noted that while the dependence of the trapped particle shift upon α ($N^{-\alpha}$) is independent of mirror ratio M and the coil shape ($\alpha = 1$), the exponent α for passing particles depends on both of these quantities. This dependence is shown in Fig. E.7. Note that α is larger for ID coils than for simple circular coils by approximately the ratio of the mean coil diameters in the equatorial plane. A convenient parameterization is

$$\alpha = \beta \tan^{-1}(M/2.7) ,$$

where $\beta = 1$ for ID coils and $\beta \approx 0.78$ for circular coils.

In conclusion, noncircular coils or ID coils represent an attractive alternative to the supplementary coil technique of aspect ratio enhancement. If future detailed engineering and neutronics calculations indicate that $\delta \leq 1.0\text{--}1.2$ m gives adequate neutron shielding and an acceptable minimal breeding ratio under the coil, then ID coils will certainly compare favorably with systems utilizing SYM coils. If $\delta \gtrsim 1.3\text{--}1.5$ m is found to be necessary, then SYM coils would present the most attractive alternative in terms of minimizing reactor size. The ID coil magnetics system is significantly less complicated; hence, assembly, maintenance, and access should not be adversely

affected, as they are with configurations in which there are two supplementary coils per sector.

Fig. E.6. Magnitude of inward shift for deeply trapped particles ($V_{\parallel} = 0$) and for extreme passing particles ($V_{\parallel} = V$) for ID coils as a function of number of sectors and major radius.

Fig. E.7. The centering of passing particle drift orbits scales with number of sectors N as $N^{-\alpha}$, where α depends on mirror ratio M and coil shape. Note that $\alpha(M)$ for ID coils is larger than that for circular coils.

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Appendix F

CONFINEMENT SCALING FOR A REACTOR

The particle and energy containment times and the variations of these with plasma and device parameters are critical in determining the detailed energy balance of a reactor. In present EBT devices, the core plasma losses are largely governed by the neoclassical electron transport, leading to a simple scaling [Refs. F.1-F.4],

$$n\tau \sim A^2 T_e^{3/2} \left(1 + \frac{R_B}{R_E} \frac{e\phi}{T_e} \right)^2 f(\text{profiles}) , \quad (\text{F.1})$$

where $A = R_T/R_B$ is the magnetic aspect ratio; R_T , R_B , and R_E are the major radius of the torus, magnetic field scale length, and electric field scale length, respectively; and $e\phi$ is the ambipolar potential. Because the neoclassical electron losses dominate the experimental behavior, it is difficult to make comparisons between the theoretical ion losses and experimental observations. Scaling laws governing ion behavior are important in a reactor. At present, ion transport principally determines the radial electric field ($E = -\nabla\phi$) needed to maintain ambipolarity.

The operating parameter space of a reactor is investigated by using both time-dependent zero-dimensional (0-D) and one-and-one-half-dimensional (1-1/2-D) transport calculations [Ref. F.5]. The 0-D model is used for parametric burn studies to examine the global plasma performance and the sensitivity of these performance parameters to

various transport assumptions and operational scenarios. The 1-1/2-D radial transport code, on the other hand, is used for determination of density, temperature, and ambipolar potential profiles and the effect of these profiles in energy confinement scaling [Refs. F.6 and F.7].

In the experiments, the observed radial electric field points in a direction opposite to the radial magnetic field gradient inside the ring (e.g., $E_r < 0$). As a result, radial electron transport is dominated by nonresonant processes, whereas ion transport (at low collisionalities, $\omega/\Omega < 1$) is dominated by resonant processes. Although in the present experiments (in which $T_e > T_i$) $E_r < 0$ in the central plasma column, $E_r > 0$ is also possible if the roles of ions and electrons are interchanged ($T_i > T_e$), which is favored in a reactor plasma. In this case ions are nonresonant and electrons are resonant (for small ω/Ω). Recently [Ref. F.8] analytic transport coefficients have been derived for finite electric fields in EBT (which reduce to Kovrizhnykh [Ref. F.1] coefficients in the large electric field limit) for both nonresonant and resonant particles. With these coefficients, the results from 1-1/2-D transport calculations are in good agreement with the experiments [Ref. F.8]. The same diffusion coefficients are used for reactor calculations.

In a global analysis with $E_r < 0$ [which leads to the simple scaling given by Eq. (F.1)], it is observed that large device sizes ($A > 20$) are required for ignition. Analysis with the 1-1/2-D transport simulation code, on the other hand, has provided much better results, giving an important new insight on confinement scaling of EBT in going from present experiments (EBT-S) to EBT-R. It is observed from the 1-1/2-D code that, in the sequence from EBT-S to EBT-R,

profile effects near the outer edge of the hot electron ring lead to enhanced confinement beyond the simple scaling that is obtained from the global analysis (which omits these profile effects). Profile effects suggest that confinement exceeds the simple scaling by at least a factor of 2-5. A similar enhancement factor is also observed to be possible with $E_r > 0$ [Ref. F.5].

The 0-D and 1-1/2-D models of transport equations are given in Sect. F.1. Section F.2 gives the transport coefficients for nonresonant and resonant particles. The profile effects in energy confinement time scaling are discussed in Sect. F.3 and profile form factors used in the 0-D model are given in Sect. F.4.

F.1. TRANSPORT EQUATIONS

The particle and energy balance equations solved are of the form (for the j th species, $j = e, D, T, \alpha, \dots$)

$$\frac{\partial n_j}{\partial t} + \nabla \cdot \Gamma_j = S_{pj} , \quad (F.2)$$

$$\frac{3}{2} \frac{\partial}{\partial t} (n_j T_j) + \nabla \cdot Q_j - q_j \Gamma_j E_r = P_{Ej} , \quad (F.3)$$

where S_p is the net particle source (sink) and P_E is the net energy source (sink). Here Γ and Q are the particle and energy (heat) fluxes, respectively, and in one-dimensional (1-D) cylindrical geometry their radial components can be written (for each species j with charge q) as

$$\Gamma_r = - D_n (\nabla n - q n E_r / T) - D_T \nabla T , \quad (F.4)$$

$$Q_r = -K_T \nabla T - K_n (\nabla n - qnE_r/T) , \quad (F.5)$$

where the D's and K's are transport coefficients. Charge neutrality for electrons and ions ($i = D, T, \alpha, \dots$) is imposed with the equation

$$n_e = \sum_i n_i Z_i \quad i = D, T, \alpha, \dots , \quad (F.6)$$

and the ambipolar potential ϕ ($E = -\nabla\phi$) is determined from

$$\Gamma_e = \sum_i Z_i \Gamma_i . \quad (F.7)$$

F.1.1. 1-1/2-D Model

The effect of realistic magnetic geometry is included in 1-1/2-D transport calculations [Ref. F.9], in that the transport equations are averaged along magnetic field lines computed from a two-dimensional (2-D) solution to the magnetic equilibrium equation in mirror geometry. Defining the geometric form factors [Ref. F.9],

$$U_1 = \left(\frac{B_m}{L} \oint \frac{d\ell}{B} \right)^{-1} , \quad (F.8)$$

$$U_2 = \oint \frac{r^2}{\rho^2} \frac{B}{B_m} \frac{d\ell}{L} ,$$

where L is the length of a mirror sector, $B_m \equiv B_{mid}$ is the midplane field, ℓ is length measured along the field line, and ρ is the spatial variable that measures the radial position of a constant flux (ψ)

surface as it intersects the mirror midplane, one has the following expressions for the transport equations [Eqs. (F.2) and (F.3)]:

$$\frac{\partial n_j}{\partial t} + \frac{U_1}{\rho} \frac{\partial}{\partial \rho} (\rho U_2 \Gamma_{\rho j}) = \langle S_{pj} \rangle_V, \quad (F.9)$$

$$\frac{3}{2} \frac{\partial}{\partial t} (n_j T_j) + \frac{U_1}{\rho} \frac{\partial}{\partial \rho} (\rho U_2 Q_{\rho j}) + q_j \frac{\partial \phi}{\partial \rho} \Gamma_{\rho j} U_1 U_2 = \langle P_{Ej} \rangle_V. \quad (F.10)$$

Here, Γ_ρ and Q_ρ are given by Eqs. (F.4) and (F.5), respectively, with ρ replacing r ; $\langle \rangle_V$ defines a flux surface average over a volume contained within the surface; and

$$V = 2\pi \int \rho B_m d\rho \oint d\ell / B \quad (F.11)$$

is the volume enclosed by a surface of constant ψ . In the 1-D cylindrical limit $\rho = r$ and $V = \pi^2 \oint d\ell = 2\pi^2 R_T r^2$.

F.1.2. 0-D Model

In the 0-D, time-dependent model, equations are averaged over a volume and the particle and energy confinement times (τ_p and τ_E , respectively) are calculated from

$$\tau_{pj} = \frac{\langle n_j \rangle}{\langle \nabla \cdot \Gamma_j \rangle} = \frac{a}{2} \frac{\langle n_j \rangle}{\Gamma_{rj}(a)} ,$$

(F.12)

$$\tau_{Ej} = \frac{\left\langle \frac{3}{2} n_j T_j \right\rangle}{\langle \nabla \cdot Q_j \rangle} = \frac{a}{2} \frac{\left\langle \frac{3}{2} n_j T_j \right\rangle}{Q_{rj}(a)} ,$$

where a is the average plasma radius and the angle brackets $\langle \rangle$ represent a volume average.

Note that the charge neutrality [Eq. (F.6)] and ambipolarity [Eq. (F.7)] conditions impose

$$\tau_{pe} = \tau_{pi} , \quad (F.13a)$$

where

$$\frac{1}{\tau_{pi}} = \sum_{j \neq e} \frac{Z_j \langle n_j \rangle}{\langle n_e \rangle} \frac{1}{\tau_{pj}} \quad j = D, T, \alpha, \dots . \quad (F.13b)$$

Replacing gradients by scale lengths, Eqs. (F.4) and (F.5) can be written (for each j th species) as

$$\Gamma_r = n \cdot D_n \cdot \left[\ell_n^{-1} + \left(\frac{T}{n} \frac{D_T}{D_n} \right) \ell_T^{-1} + \frac{qE_r}{T} \right] \\ \equiv n \cdot D_n \cdot L_p , \quad (F.14)$$

$$\begin{aligned}
Q_r &= n \cdot K_n \cdot \left[\ell_n^{-1} + \left(\frac{T}{n} \frac{K_T}{K_n} \right) \ell_T^{-1} + \frac{qE_r}{T} \right] \\
&= n \cdot K_n \cdot L_E ,
\end{aligned} \tag{F.15}$$

where $\ell_n \equiv -(\partial \ln n / \partial r)^{-1}$, $\ell_T \equiv -(\partial \ln T / \partial r)^{-1}$, and L_p and L_E represent the terms (which are functions of scale lengths, electric field, and ratios of transport coefficients) in square brackets. Using Eqs. (F.14) and (F.15) in Eq. (F.12) one has (subscript j omitted for brevity)

$$\tau_p = \frac{a}{2} \frac{\langle n \rangle}{n(a)} \frac{1}{D_n(a) L_p(a)} = \frac{a}{2} \frac{f_n(a)}{D_n(a) L_p(a)} , \tag{F.16}$$

$$\tau_E = \frac{a}{2} \frac{\left\langle \frac{3}{2} nT \right\rangle}{n(a) T(a)} \frac{T(a)}{K_n(a) L_E(a)} = \frac{3a}{4} \frac{f_{nT}(a) T(a)}{K_n(a) L_E(a)} ,$$

where $f_n(a) \equiv \langle n \rangle / n(a)$ and $f_{nT}(a) \equiv \langle nT \rangle / [n(a) T(a)]$ are the profile form factors, which can be determined either from assumed density and temperature profiles or from the requirements of stable pressure profiles discussed in Appendix B. The results for the latter case are given in Sect. F.4.

F.2. TRANSPORT COEFFICIENTS

F.2.1. Nonresonant Particles ($q_j E_r > 0$)

Neoclassical transport coefficients have been calculated by Kovrizhnykh [Ref. F.1] in the limit of large electric fields ($\Omega_{E \times B} > \Omega_{VB}$, where Ω is the poloidal precessional drift frequency). For finite electric fields, numerical [Ref. F.10] and analytical [Refs. F.8 and F.11] expressions have been developed that are in good agreement with each other. The analytic expressions [Ref. F.11] (which reduce to Kovrizhnykh coefficients in the large electric field limit) for nonresonant particles are summarized below:

$$D_n = a_1 \bar{D} [1 + a_2/W_0 - a_3 W_0 / (\bar{v}^2 + W_0^2)] , \quad (F.17)$$

$$D_T = b_1 \left(\frac{n}{T} \right) \bar{D} [1 + b_2/W_0 - b_3 W_0 / (\bar{v}^2 + W_0^2)] , \quad (F.18)$$

$$K_n = c_1 T \bar{D} [1 + c_2/W_0 - c_3 W_0 / (\bar{v}^2 + W_0^2)] , \quad (F.19)$$

$$K_T = d_1 n \bar{D} [1 + d_2/W_0 - d_3 W_0 / (\bar{v}^2 + W_0^2)] \quad (F.20a)$$

$$= \frac{7}{2} n \bar{D} , \quad (F.20b)$$

where

$$\begin{aligned} \bar{D} &= \frac{1}{2} (v_T / \Omega)^2 [w / (\bar{v}^2 + W_0^2)] \\ &= 2(r R_B / R_T)^2 [w / (\bar{v}^2 + W_0^2)] , \end{aligned} \quad (F.21)$$

$\bar{\nu} = \nu/\Omega_{VB}$, and $W_0 = \Omega_E \times B/\Omega_{VB} = |qR_B E_r/T|$. The numerical coefficients in Eqs. (F.17)-(F.20) are:

$a_1 = 0.0684$	$b_1 = 0.137$	$c_1 = 0.2395$	$d_1 = 3.5167$
$a_2 = 8.4795$	$b_2 = 12.2115$	$c_2 = 12,1832$	$d_2 = -0.9094$
$a_3 = 1.8128$	$b_3 = 5.5945$	$c_3 = 6.1557$	$d_3 = 2.6313$

In the present experiments, $E_r < 0$ and electron transport is governed by nonresonant processes. For $E_r > 0$, ions would be nonresonant.

F.2.2. Resonant Particles ($q_j E_r < 0$)

Recently, neoclassical transport coefficients for resonant particles have been developed for the entire range of resonant particle collisionality (ν/Ω) from banana to plateau, which permits analytic approximations for the entire transport matrix [Refs. F.8 and F.12].

For the plateau regime ($\epsilon^{3/2} < \nu/\Omega_{VB}$, where $\epsilon = a/R_T$ is the inverse plasma aspect ratio) the density diffusion coefficient [Refs. F.12 and F.13] is (in mks units with T in kiloelectron volts)

$$D_n = 1.25 \times 10^3 \left(\frac{T}{B} \frac{rR_c}{R_T^2} \right) e^{-W_0} . \quad (F.22)$$

The remaining coefficients are

$$D_T = \left(\frac{n}{T} \right) D_n (W_0 + 1) , \quad (F.23)$$

$$K_n = T D_n (W_0 + 5/2) , \quad (F.24)$$

$$K_T = nD_n \left(W_0^2 + \frac{7}{2} W_0 + 5 \right) . \quad (\text{F.25})$$

In contrast to the plateau limit, the more collisionless expressions (banana limit, $v/\Omega < \epsilon^{3/2}$) depend explicitly on the banana width, and the ambipolar potential scales with magnetic field as

$$\frac{\phi'''}{\phi'} \approx \frac{B'''}{B'} , \quad (\text{F.26})$$

where differentiation is with respect to radius.

In present experiments, $E_r < 0$ and ions are the resonant particles. (Note that for resonant particles, cancellation occurs between the $E \times B$ and ∇B precessional drifts.)

F.3. PROFILE EFFECTS IN ENERGY CONFINEMENT SCALING — 1-1/2-D MODEL

As indicated before, profile effects near the outer edge of the hot electron ring (see Fig. A.1) lead to confinement estimates for reactor-size devices that are significantly better than the estimates obtained from the simple scaling given by Eq. (F.1). To see this effect and to explain the results of 1-1/2-D calculations, we give the following simple arguments.

From conservation of energy, the total power absorbed by the toroidal core plasma P_{total} must equal the integral of the heat flux $\vec{Q}$ over the plasma surface S ,

$$P_{\text{total}} = \oint \vec{Q}_{pe} \cdot d\vec{S} \approx 4\pi^2 R_T a U_2 Q_{pe}(a) , \quad (\text{F.27})$$

where all the quantities (a , R_T , U_2 , ρ) have been defined earlier. Defining the energy containment time as energy density times volume (V) divided by total power absorbed [see Eq. (F.12)],

$$\tau_E \equiv \frac{\langle (3/2)nT_e \rangle V}{P_{\text{total}}} , \quad (\text{F.28})$$

we have

$$n\tau_E = \frac{3a}{4U_1U_2} \frac{n^2T_e}{Q_{pe}(a)} . \quad (\text{F.29})$$

Here Eqs. (F.11) and (F.27) are used and the $\langle \rangle$ notation (representing averages) has been dropped for brevity. The electron heat flux [see Eq. (F.5)] is on the order of (ignoring density gradients for simplicity)

$$Q_{pe} \sim K_{Te} \frac{\partial T_e}{\partial \rho} \left(1 + \frac{e\phi}{T_e} \right) \quad (\text{F.30})$$

where $E_r < 0$ [$E_r = -\nabla\phi$ is assumed (i.e., electrons are nonresonant)] and the thermal conductivity K_{Te} is given by Eq. (F.20b). Substituting Eqs. (F.20b) and (F.30) into Eq. (F.29) in the collisionless limit ($\bar{v}_e \ll W_{oe}$) and using

$$\nu_e = \nu_{ee} + \nu_{ei} \approx 3.79 \times 10^{-15} nT_e^{-3/2} \quad (\text{F.31})$$

(in mks units with T in kiloelectron volts) and

$$\partial T_e / \partial \rho \sim T_e / \Delta \rho \sim T_e / \ell_T, \quad (\text{F.32})$$

we have

$$n_T \approx 1.13 \times 10^{14} \left(\frac{\Delta \rho}{a} \right) \left(\frac{R_T}{a} \right)^2 \left(\frac{a^2 W_{oe}}{2 R_B \rho} \right)^2 T_e^{3/2} \quad (\text{m}^{-3} \cdot \text{s}). \quad (\text{F.33})$$

Conventional scaling assumes parabolic profiles valid near $\rho \ll a$. In this case $\Delta \rho / a \sim 1/2$ and $(A^2 W_{oe} / 2 R_B \rho) \sim e \phi / T_e \sim 0(1)$ so that Eq. (F.33) reduces to Eq. (F.1). [Note that the functional form of Eq. (F.33) is the same if density gradients were not ignored.]

In Fig. F.1, the dashed curves represent the values of n_T calculated from Eq. (F.1) for the sequence of EBT devices: EBT-S, -P, -Q, and -R. The solid curves show the results of 1-1/2-D transport simulations of these increasingly larger devices. Note that the confinement is better than that predicted by the simple $A^2 T_e^{3/2}$ scaling of Eq. (F.1). To understand this, we must first consider the detailed geometry for these cases, which is shown in Fig. F.2.

Figure F.2 shows the 2-D mod-B contours along with normalized magnetic field profiles in the midplane for the devices in the sequence from EBT-S to EBT-R. Note that the depression in B due to the hot electron ring occurs progressively closer to the magnetic axis for the larger machines. This results in relatively larger gradients in B on the outer edge of the magnetic well for these cases. The ambipolar electric field is calculated from the ordinary differential equation [see Eq. (F.26)]

Fig. F.1. Confinement parameter $n\tau_E$ as a function of collisionality, $\bar{\nu}_e = \nu_e / \Omega_{0e}$, for the sequence of EBT devices. Dashed curves represent values of $n\tau$ calculated from Eq. (F.1) (i.e., $A^2 T_e^{3/2}$ scaling). Solid curves (for a reactor, solid circles) are the results of 1-1/2-D transport calculations.

Fig. F.2. 2-D mod-B contours (a) and normalized magnetic field profiles (b) for the sequence of EBT devices. Note that the depression in B due to hot electron ring occurs progressively closer to the magnetic axis for larger machines.

$$\frac{E^{\sim}(\rho)}{E} = \frac{B^{\sim}(\rho)}{B^{\sim}(\rho)}, \quad (\text{F.34})$$

which results from the assumption of a higher energy ion tail whose transport is dominated by resonant processes. The resulting ambipolar

potential profiles are shown in Fig. F.3. Note that relatively larger gradients in B on the outer edge of the magnetic well lead to larger electric fields in this same region. This in turn improves confinement for the larger machines and allows steeper pressure gradients near the outer edge. A typical pressure profile is shown in Fig. F.4. Note

Fig. F.3. Ambipolar potential profiles obtained from 1-1/2-D transport calculations in the midplane for the devices in the EBT-S to EBT-R sequence. (Normalized magnetic field profiles are shown in the insert.) Note the larger gradients in B and larger electric fields.

Fig. F.4. Typical pressure profile obtained from 1-1/2-D transport simulations. (Note that the somewhat hollow profile is due to present fueling deficiency in the code.)

from Figs. F.4 and B.3 [or Fig. A.1] that the radial profile of core pressure obtained from 1-1/2-D transport simulations is similar to those theoretically required for stability. A somewhat hollow profile of pressure near the plasma center in Fig. F.4 is due to the hollow density profile, which has to do with the fueling of the plasma. Because the 1-1/2-D code is not fully developed for reactor-size devices, incorporation of pellet fueling should eliminate this problem.

These profile effects (shown in Figs. F.3 and F.4) enter the simple point model of Eq. (F.33) through the electric field term,

$W_0 a^2 / 2R_B \rho$. Global confinement time is determined not by the magnitude of this term near the magnetic axis, but rather by its magnitude near the outer edge of the magnetic well, where diffusion is minimum. For example, near the magnetic axis, $W_0 a^2 / 2R_B \rho$ is on the order of unity for all cases in Fig. F.1. However, near the outer edge it reaches progressively larger peak values of ~ 2.2 to 4.5 for the EBT-S to EBT-R sequence. Using these values in Eq. (F.33) with $\Delta \rho / a \sim 0.5-1$ gives a reasonably close approximation to the 1-1/2-D results of Fig. F.1.

F.4. PROFILE FORM FACTORS USED IN 0-D MODEL

As discussed in Appendix B, the radial profile of core pressure p_c can be derived from

$$p_c'(r) + \alpha(r)p_c(r) = 0 \quad (F.35)$$

to give

$$p_c(r) = C \exp[-\int \alpha(r) dr] , \quad (F.36)$$

where C is constant and

$$\alpha(r) = \begin{cases} \frac{\gamma}{\langle R_B \rangle_I} \approx \frac{2\gamma r}{\langle r_0 R_c(r_0) \rangle} , & 0 < r < r_0 - \Delta \\ \frac{\gamma}{\langle R_B \rangle_{II}} \approx \frac{2\gamma r}{\langle r_0 R_c(r_0) \rangle} \left(1 + \beta_h \frac{\langle R_c \rangle}{4\Delta} \right) , & r_0 - \Delta < r < r_0 \\ \frac{1}{\Delta} , & r_0 < r < r_0 + \Delta \end{cases} \quad (F.37)$$

Here Δ is the ring half-width, R_c is the radius of curvature, r_0 is the ring radius, β_h is the ring beta, and γ is the ratio of specific heats (see Appendix B and Fig. A.1).

Considering, for example, the central core plasma region (Region I in Fig. A.1, $0 < r < r_0 - \Delta$),

$$p_c(r) = p_c(0) \exp\left[-\frac{\gamma}{r_0 R_c(r_0)} r^2\right], \quad (\text{F.38})$$

where to a first-order approximation $r R_c = r_0 R_c(r_0) \cong \text{constant}$. Because

$$\frac{p'}{p} = \frac{n'}{n} + \frac{T'}{T},$$

several combinations of density and temperature profiles can satisfy Eq. (F.38). For example, if $T'/T \ll n'/n$,

$$n(r) \cong n(0) \exp\left[-\frac{\gamma}{r_0 R_c(r_0)} r^2\right] = n(0) \exp(-\epsilon r^2), \quad (\text{F.39})$$

where $\epsilon = \gamma/r_0 R_c(r_0)$. The density scale length is

$$l_n = -\left(\frac{1}{n} \frac{dn}{dr}\right)^{-1} \cong \frac{R_c(r_0)}{2\gamma}, \quad (\text{F.40})$$

and the profile form factor is

$$\begin{aligned}
 f_n(r_o) &= \frac{\langle n \rangle}{n(r_o)} \cong (1/er_o^2) [\exp(er_o^2) - 1] \\
 &= 2(\ell_n/r_o) [\exp(er_o^2) - 1]. \quad (F.41)
 \end{aligned}$$

Confinement times are determined from Eq. (F.16) by using the scale length and profile form factor given by Eqs. (F.40) and (F.41), respectively. Here the ambipolar electric field is calculated from Eq. (F.7); that is, $\Gamma_e = \sum_i Z_i \Gamma_i$. The resulting ambipolarity condition (ignoring the alpha particle contribution, since it is small, and taking $\partial T / \partial r \approx 0$ for simplicity) is given by

$$e^{W_o} \cong \frac{15}{7\bar{\nu}} (\bar{\nu}^2 + W_o^2) \frac{(R_B/\ell_n) - W_o}{(R_B/\ell_n) + W_o}. \quad (F.42)$$

Here $\bar{\nu}$ is the collisionality parameter of the nonresonant species and $W_o = R_B |eE_r| / T$. In obtaining Eq. (F.42) it is assumed that the diffusion coefficient for the nonresonant particle (e or i) is given by Eq. (F.17) and that the resonant particle (i or e) is in the plateau regime and the associated diffusion coefficient is in the form of Eq. (F.22). The solution for Eq. (F.42) is displayed in Fig. F.5.

Fig. F.5. Ambipolarity condition [solution of Eq. (F.42)]: W_0 vs collisionality parameter $\bar{\nu} = \nu / \Omega_{\nabla B}$ for nonresonant particles.

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Appendix G

REACTOR SYSTEMS ANALYSIS

An essential task in any reactor design study is the integration of physics, engineering, and cost constraints to determine the range of acceptable reactor design options and to demonstrate the most attractive technical and economic characteristics of these reactors. For this study, such an integration was carried out using a systems code that incorporates recent advances in EBT physics. A unique feature of this code, which differs from those used previously, is the coupling of ring and core plasma properties and the self-consistent treatment of the coupled ring-core stability and power balance requirements. The code establishes a reactor operating space on the basis of this coupling and power balance along with additional limits imposed by other physics and engineering constraints.

The basic EBT operation requires a toroidal array of simple mirrors with energetic electron (or ion) rings contained in each mirror for stability of the overall plasma configuration [Ref. G.1]. The stability of both core and ring plasmas (Appendix A), the role of profile effects (Appendix B), the steady-state power requirements of electron (Appendix C) and ion (Appendix D) rings, and their coupling to one another are discussed in Appendixes A-D.

The stability of the core plasma and that of the rings are closely coupled [Refs. G.2-G.4], and consistent analysis of the EBT operation requires consideration of these processes. Basically, the core plasma requires energetic rings (to produce a diamagnetic well) for its stability, and the maximum allowable core beta is governed by the core

pressure-driven interchange [Lee-Van Dam-Nelson (LVN) limit] near the ring and is given by $\beta_c \lesssim 2\Delta/R_c$, where 2Δ is the electron ring width and R_c is the magnetic radius of curvature near the ring. The ring stability, on the other hand, is established by the hot electron interchange mode and requires a sufficient amount of core plasma density [e.g., large $n_{\text{core}}/n_{\text{hot}} = n_c/n_h$].

In this appendix, the essential elements of the systems code are reviewed and the calculational sequences leading to the specification of the reactor options and their associated costs are described.

G.1 ELEMENTS OF THE SYSTEMS CODE

The sequence of calculations carried out in the systems code is shown schematically in Fig. G.1. To properly consider the coupling of the stability and power balance and to determine the operating space in which the plasma parameters lie within the bounds of the constraints imposed by the physics, the input parameters in this code are physics data. In other systems codes [see, for example, Ref. G.5] the input parameters were selected from engineering considerations and size (e.g., plasma major radius, number of coils, net electric power, etc.), which may or may not be acceptable from physics considerations.

To separate stability and ring power balance requirements from those of transport and size requirements (for an ignited and/or driven system), the code is structured, as far as possible, so that all of the calculations are performed for a single reactor sector. Structuring the code in disjointed modules ensures that uncertainties in each area (stability, power balance, transport scaling, etc.) are somewhat

Fig. G.1. Sequence of systems code calculations.

decoupled. The dimensions of a single reactor sector are calculated from the specification of a plasma radius (r_p) under the coil, the mirror ratio (M), and the magnitude (maximum or minimum) of the magnetic field (B). The total number of mirror sectors (number of coils) and, correspondingly, the major radius are determined from neoclassical confinement scaling, $n\tau \sim A^2 T^{3/2}$, where T is the plasma temperature and A is the magnetic aspect ratio (see Appendix F).

G.1.1 Magnetics Calculations

The average magnetic field curvature is related to the plasma major radius (R_T), the mirror ratio (M), the number of sectors (N), and the coil radius (r_c). Approximating the coils with a current sheet of finite length and assuming a large aspect ratio, the magnetic field in cylindrical coordinates (r, θ, z), as shown in Fig. G.2, is given by [Ref. G.6]

$$B_z(r, z) = \mu_0 \frac{I}{L} \left[1 + \sum_n (-1)^n \alpha_n(D) I_0(k_n r) \cos(k_n z) \right] \quad (G.1)$$

and

$$B_r(r, z) = \mu_0 \frac{I}{L} \sum_n (-1)^n \alpha_n(D) I_1(k_n r) \sin(k_n z) , \quad (G.2)$$

where

$$k_n = \frac{2\pi n}{L} = n(N/R_T) , \quad (G.3)$$

$$\alpha_n(D) = 2 \frac{\sin(n\pi D/L)}{n\pi D/L} (k_n r_c) K_1(k_n r_c) . \quad (G.4)$$

Fig. G.2. EBT sector geometry and ring dimensions.

In these equations,

L = sector length = $2\pi R_T/N$,

D = length of a current sheet,

r_c = coil current center (radius of the current sheet),

I = total coil current,

I_0, I_1, K_1 = modified Bessel functions.

The total magnetic field is $B = (B_z^2 + B_r^2)^{1/2} \approx B_z$. Then, the mirror ratio, $M = B_{\max}/B_{\min} = B_z(r=0, z=\pm L/2)/B_z(r=0, z=0)$, is given by

$$M = \frac{1 + \sum_n \alpha_n(D)}{1 + \sum_n (-1)^n \alpha_n(D)} = f(Nr_c/R_T, D/L), \quad (G.5)$$

In the limit as $D \rightarrow 0$, the results given above reduce to the current loop approximation. The relationship between the mirror ratio (M) and $y = Nr_c/R_T$ is plotted in Fig. G.3 for the ratio $D/L = 1/4$.

The magnetics module determines the magnet coil and sector dimensions for the specified values of the mirror ratio and D/L . The coil radius is obtained by solving Eqs. (G.1) and (G.2) for the coil half-width Δ_c , which is related to r_c by

Fig. G.3. Relationship between mirror ratio M , number of coils N , coil radius r_c , and major radius R_T for $D/L = 1/4$, where D is the length of a coil and L is the sector length.

$$r_c = r_p + \Delta_{cz} + \Delta_{BS} + \Delta_m + \Delta_c , \quad (G.6)$$

where

r_p = plasma radius under the coil,

Δ_{cz} = thickness of the scrapeoff zone (cold zone),

Δ_{BS} = blanket-shield thickness,

Δ_m = distance between the shield and magnet,

Δ_c = coil half-thickness.

In Eqs. (G.1) and (G.2) the total current I is given by

$$I = J_0 D = J_{\max} A(\Delta_c) , \quad (G.7)$$

where J_0 is the current density in the sheet (in the limit that the coil width $\rightarrow 0$), $J_{\max}$ is the maximum current density, and $A(\Delta_c)$ is the cross-sectional area of the real magnet calculated from Δ_c . (In the calculations, a superconducting coil having a current density of 16 MA/m² is assumed.) Equations (G.6) and (G.7) are substituted into Eqs. (G.1) and (G.2), the equations are solved for Δ_c with $B(r = 0, z = \pm L/2)$, and the ratio

$$R_T/N = r_c/y \quad (G.8)$$

is determined. Then, the mirror sector dimensions are obtained from

$$\text{sector length} = L_s = 2\pi(R_T/N) , \quad (G.9)$$

$$\text{sector volume} = V_s = \pi L_s (a_p)^2 , \quad (G.10)$$

$$\text{sector surface area} = A_s = 2\pi L_s(\bar{a}_p + \Delta_{cz}) , \quad (G.11)$$

where $\bar{a}_p \approx \sqrt{Mr_p}$ is the average plasma radius.

G.1.2 Stability Calculations

The core-ring stability calculation defines a set of EBT reactors that lie within a stable operating region and have core and ring properties specified by β_c , β_h , and T_h (where β_c and β_h are core and ring betas, respectively, and T_h is the ring temperature). A typical reactor operating window defined from the core-ring stability calculations is shown in Fig. G.4. The window is calculated for a reactor having a mirror ratio of 2.2; a minimum magnetic field on axis, $B(r=0, z=0)$, of 2.5 T; and stabilization of the core accomplished by hot electron rings with 15% beta (β_h) and an electron temperature of 2 MeV (T_h). The expressions summarized in Table A.3 (see Appendix A) are used in obtaining the boundaries of the operating window.

For a given core beta (β_c), the ring width (2Δ) is determined from the LVN limit, that is,

$$\beta_c \text{ (at the ring location)} < \frac{2\Delta}{R_0} \left(1 + \frac{p_{\parallel}}{p_{\perp}} \right) , \quad (G.12)$$

where $(p_{\parallel}/p_{\perp})$ is the pressure anisotropy and is assumed to be 0.25. It is assumed in all cases that the core beta at which the reactor operates is 80% of the LVN limit given by Eq. (G.12),

Fig. G.4. A typical reactor operating window obtained from the core-ring stability calculations.

$$\beta_c (\text{operation}) = 0.8 \beta_c (\text{LVN limit}) = 0.8 \frac{2\Delta}{R_c} \left(1 + \frac{p_{\parallel}}{p_{\perp}} \right). \quad (\text{G.13})$$

For this beta value, the limit on the core density is determined from the low frequency, hot electron interchange mode [Eq. A.9, Appendix A],

$$(f_R)_{\min} = \frac{n_c}{n_h} > \frac{4}{(k_{\perp} \Delta)^2} \cdot \frac{1}{q_0} \frac{1}{\left[1 - \beta_c \frac{R_c}{2\Delta} + O(\Delta/R_c)\right]^2}, \quad (G.14)$$

where q_0 is the ratio of hot electron curvature drift frequency to the ion cyclotron frequency, given by

$$q_0 = v_{cv\perp} / \Delta \omega_{ci} \cong 10^{-5} \mu \frac{T}{B_R^2 \Delta} \quad (G.15)$$

with

$\mu = m_i/m_p$ ratio of the ion mass to proton mass

($\mu \cong 2.5$ for a 50-50 D-T plasma),

B_R = magnetic field near the ring, and

$k_{\perp}$ = wave number,

all of which are discussed in Appendix A.

The lower curve in Fig. G.4 is Eq. (G.14) with $\beta_c = 10\%$. This sets the lower limit on the density ratio (n_c/n_h).

For a given ring beta (β_h), ring temperature (T_h), and $B_R \cong 0.7 B_{\min}$ [$B_{\min} = B(r=0, z=0)$; see Fig. G.2], the ring density is given by

$$n_h \cong 2.48 \times 10^{21} \frac{\beta_h \beta_R^2}{T_h} = 4.85 \times 10^{18} \frac{\beta_h B_R^2}{\gamma - 1}, \quad (G.16)$$

where $\gamma = 1 + T_h/m_o c^2$ is the relativistic factor. Thus, the minimum core plasma density near the ring that satisfies the stability criterion [Eq. (G.14)] is

$$(n_c)_{\min} = n_h(f_R)_{\min} . \quad (G.17)$$

Introducing an average core density

$$\langle n \rangle = k n_c , \quad (G.18)$$

then

$$\langle n \rangle_{\min} = k n_h(f_R)_{\min} . \quad (G.19)$$

where k is a profile factor that can be determined from the arguments presented in Appendix B.

The maximum core plasma density, on the other hand, is calculated from the microwave cutoff or the compressional Alfvén mode [Eq. (A.12), Appendix A], depending on which one is more stringent. In realistic reactor plasmas, however, the compressional Alfvén mode limit exceeds the microwave cutoff. Taking the cutoff density to be the maximum allowable density,

$$\langle n \rangle_{\max} = n_{\text{cutoff}} = 9.73 \times 10^{18} B_{\text{resonance}}^2 , \quad (G.20)$$

where $B_{\text{resonance}} \approx 2B_R$ is the resonance field (in tesla). From Eqs. (G.18) and (G.20),

$$(f_R)_{\max} = (n_c/n_h)_{\max} \approx 1.6 \times 10^{-2} \frac{T_h}{k\beta_h}, \quad (\text{G.21})$$

where k is the same profile factor defined earlier. The upper curve in Fig. G.4 is obtained from Eq. (G.21).

At the present, the systems code treats the magnetic field, core pressure, and ring pressure profiles (discussed in Appendix B) in an approximate manner. More detailed treatment of profile effects will be added to the code as improvements are made to include equilibrium calculations along with more realistic magnetics and stability analyses. The approximate treatment of the radial dependence of the field does not, however, significantly impact the final results of this analysis.

Equations (G.14) and (G.21) establish the extremes of the upper and lower boundaries of the operating space in which stable plasmas are achieved for the specified ring parameters. Depending on the desired core beta value, ring temperature, magnetic field, and ring beta, the stable reactor operating window can be opened or closed. The directions in which the window boundaries move with changes in these parameters are indicated in Fig. G.4.

For q_0 calculated from the input parameters, there exists a finite range of n_c/n_h that falls within the stable plasma region. The set of specified input parameters, $q_0 = q_0(\beta_h = 15\%, T_h = 2 \text{ MeV}, \langle B \rangle = 4 \text{ T}) = 0.14$, is also plotted in Fig. G.4. At this point, one additional check is made in the code to ensure that the range of plasma temperatures of reactors operating within the allowed n_c/n_h range always exceeds 5 keV (bremsstrahlung limit). Now, for the set of

allowed reactors we have defined essentially all of the plasma properties (β_c , B, n, T) that can be maintained in a stable operating state by electron rings with the specified β_h and T_h .

In the remainder of this appendix, the reactor performance and the engineering and cost constraints are compared for the set of reactors having core beta values $5\% < \beta_c < 15\%$.

G.1.3 Ring Power Calculations

The set of EBT reactors stabilized by hot electron rings is specifically calculated in the stability subroutine (see Fig. G.1). It is assumed that the reactors defined here in terms of electron ring stability criteria can also be equivalently stabilized by ion rings within the constraint that ion ring parameters can be scaled from the electron ring parameters (see Appendixes C and D). The scaling is accomplished by assuming that the ion ring half-width Δ_i is

$$\Delta_i = \max(\Delta_e, \rho_i) , \quad (G.22)$$

where $\rho_i \sim T_h^{1/2}/\langle B_R \rangle$ is the ion gyroradius. The electron and ion ring volumes are calculated assuming that both ring species have the same overall length ℓ_R (see Fig. G.2) but with the ring volume being a function of Δ_e for electron rings and Δ_i for ion rings. The ring temperature and beta specified in the input are taken to be the same for both electron and ion rings (however, different values can be assumed). Steady-state ring power requirements are calculated based on expressions developed in Appendix C for electron rings and Appendix D

for ion rings. These data are required to determine the recirculating power requirements and the net electric power produced by the reactors. For the case of electron rings (see Appendix C), the power lost in the rings is due to synchrotron radiation, drag losses, and scattering. The required ring sustaining power is given by [see Eq. (C.3), Appendix C]

$$P_{\text{ring},e} \text{ (MW)} = 0.16 \beta_h B_R^4 V_{\text{ring},e} G(\gamma, \beta_h, n_c/n_h), \quad (\text{G.23})$$

where

$$G(\gamma, \beta_h, n_c/n_h) \cong 1 + 2.5(\gamma - 1) + 7.4 \beta_h \left(\frac{n_c}{n_h} \right) \frac{\gamma/(\gamma - 1)^2}{(\gamma^2 - 1)^{1/2}} \left(1 + \frac{4}{\gamma + 1} \right). \quad (\text{G.24})$$

For the case of ion rings (see Appendix D), the power lost in the rings is due mainly to drag and scattering losses (charge-exchange losses are negligible for reactors with energetic ion temperatures exceeding several hundred kiloelectron volts) and can be approximated by [see Eq. (D.5), Appendix D]

$$P_{\text{ring},i} \text{ (MW)} \cong 10^3 \frac{\beta_c \beta_h B_R^4}{\mu_h T_{ec}^{5/2}} V_{\text{ring},i}, \quad (\text{G.25})$$

where T_{ec} is the core electron temperature near the ring and $\mu_h = m_h/m_p$ is the ratio of the hot ion mass to proton mass. The ratio of the

power required to sustain an electron ring to that required to sustain an electron ring [see Eq. (D.7), Appendix D] is given by

$$P_{\text{ring},i}/P_{\text{ring},e} \cong 2\mu^{-1}\beta_c(10/T_{ec})^{5/2}(V_{\text{ring},i}/V_{\text{ring},e}) \quad (\text{G.26})$$

The plasma Q value, that is, the ratio of the fusion power produced to the ring sustaining power, is obtained from

$$Q_j = P_{\text{th}}/P_{\text{ring},j} = \eta_j/\epsilon_j, \quad j = e, i, \quad (\text{G.27})$$

where ϵ is the required recirculating power fraction to sustain the rings and η is the efficiency (which includes thermal-to-electric conversion efficiency and microwave coupling efficiency).

G.1.4 Confinement Time and Reactor Size Scaling

The reactor dimensions and the number of sectors are determined from neoclassical confinement scaling $n\tau \sim A^2T^{3/2}$ where T is the plasma temperature and A is the magnetic aspect ratio. Detailed discussions of confinement scaling and a full expression for $n\tau$ as a function of collisionality (ν/Ω) are given in Appendix F. Calculations are carried out for the set of ignited reactors having a range of plasma densities determined for the operating space shown in Fig. G.4. In determining the required confinement times for these ignited reactors, the only (core plasma) radiation loss terms that are considered are those due to bremsstrahlung and synchrotron radiation; that is,

$$(n\tau)_{\text{ignition}} = \frac{3n^2T}{P_{\alpha} - P_{\text{rad}}} , \quad (\text{G.28})$$

where

$$P_{\text{rad}} = P_{\text{brem}} + P_{\text{synch}} . \quad (\text{G.29})$$

Then the required aspect ratio for ignition is estimated from Eq. (G.28), that is, $A^2 \propto n\tau/T^{3/2}$, where T is evaluated for the range of core plasma densities (n_c) obtained from Fig. G.4. The number of sectors (N) is obtained by evaluating

$$N = A \cdot r_c / (R_T/N)$$

where r_c is obtained from Eq. (G.6) and (R_T/N) is obtained from Eq. (G.8).

G.1.5 Power Balance Calculations

The final set of calculations in the sequence of physics calculations determines the thermal power produced in a reactor sector along with the neutron wall loading and fusion power density. The thermal power production includes the blanket multiplication factor M_b as an input parameter so the power production capabilities of the reactor options are assessed without considering specific blanket designs at this time.

All of the physics data are now available to carry out a systematic analysis of the reactor options that lie within the operating window shown in Fig. G.4. The physical dimensions of the reactors are specified along with the ring parameters required to ensure stability. In addition, the blanket-shield and magnet coil dimensions are specified and a sufficient database exists to complete a cost analysis of each reactor option. These data form the input to the cost analysis module.

G.1.6 Cost Calculations

The algorithms and sequences of calculations to determine the cost of the reactors used here are the same as those used in the McDonnell-Douglas EBT Reactor Systems Code that was developed for the Los Alamos National Laboratory EBT Reactor Study [Ref. G.5]. However, most of the subroutines were modified by McDonnell-Douglas Astronautics Company for use with this systems code, but they do not include the design optimization loops that were used to converge the reactor design envelope in Ref. [G.5].

G.2 TYPICAL RESULTS OF REACTOR SYSTEMS ANALYSIS

The results of a typical reactor systems analysis are reviewed in the remainder of this appendix for the set of reactors that fall within the operating window shown in Fig. G.4. The operating space plotted in Fig. G.4 was calculated using the following input parameters:

Mirror ratio $M = 2.2$

Minimum plasma radius under the coil $r_p = 1.0 \text{ m}$

Core beta $\beta_c = 0.10$

Ring beta $\beta_h = 0.15$

Ring temperature $T_h = 2.0$ MeV

Minimum magnetic field on axis $B_{\min} = 2.5$ T

Blanket energy multiplication = 0.25

However, the allowed operating space, indicated in Fig. G.4, is also valid for values of β_c between 5% and 15% (all other values of the input being constant). The comparisons of reactor performance and costs discussed in the remainder of this appendix are presented as a function of β_c for a fixed average plasma density of $8 \times 10^{19} \text{ m}^{-3}$. (For displaying data, this value for the plasma density is chosen as a representative value, consistent with that anticipated in a typical EBT power reactor. However, it is not a magic number.) A complete data set is, however, available for all possible densities within the stable operating window shown in Fig. G.4.

Figure G.5 shows the variation of the reactor major radius (R_T) and the number of sectors (N) with core beta. The reactor size and the number of sectors decrease with increasing core beta for a fixed density because of the temperature increase. The major radius is decreased by ~30% where the number of sectors goes down by ~38%. The constant values for R_T and N for values of core beta between 6% and 8% arise from roundoff in the calculation of the integer number of sectors. Also, near these beta values, the collisionality of electrons (ν/Ω) becomes nearly unity (i.e., transition from collisional to collisionless transport, see Appendix F).

Fig. G.5. Variation of the major radius R_T and number of sectors N with core plasma beta β_c .

Figure G.6 compares the power density (MW/m^3), the neutron wall loading (MW/m^2), and the thermal power produced per sector as a function of core beta for plasma density $n = 8 \times 10^{19} \text{ m}^{-3}$. These data were calculated assuming a blanket multiplication factor M_b of 0.25. All of these power parameters increase with increasing core beta as expected.

Fig. G.6. Variation of fusion power density, neutron wall loading, and thermal power with core plasma beta for core plasma density $n = 8 \times 10^{19} \text{ m}^{-3}$.

The reactor performance as a function of core beta is plotted in Fig. G.7. Shown in the figure is the comparison between the total thermal power production and total power losses in both electron and ion rings. These data provide a basis for estimating the amount of

Fig. G.7. Reactor performance parameters (fusion power, ring sustaining power, plasma Q value, cost of electricity, etc.) as a function of core plasma beta for $n = 8 \times 10^{19} \text{ m}^{-3}$.

power that must be recirculated to maintain the rings. This is manifested in the $Q_p(\text{electron})$ and $Q_p(\text{ion})$ curves, which are the ratios of $P_{\text{thermal}}/P_{\text{ring}}$ for the appropriate rings. The reciprocal of Q_p is the recirculating power fraction. The cost of producing electricity is plotted in the lower portion of the figure. The cost of electricity is ~10% larger when electron ring stabilization is used.

Table G.1 shows a representative set of performance parameters for an EBT reactor having an average plasma density of $8 \times 10^{19}/\text{m}^3$ and operating with a core beta of 10%. These data were taken from the curves plotted in Fig. G.7 and from the systems code output.

Table G.1. Results of the systems code analysis for an EBT reactor
with core beta = 10%

Net electric power	
With electron ring	1540 MW
with ion ring	1650 MW
Thermal power	5000 MW
Average plasma radius	1.5 m
Number of sectors	34
Neutron wall loading	1.4 MW/m ²
Average magnetic field	4 T
Inner coil radius	2.75 m
Aspect ratio enhancement	2
Average core plasma density	$8 \times 10^{19}/\text{m}^3$
Average core ion temperature	25 keV
Ring temperature	2 MeV
Core/ring beta values	10%/15%
Ring power losses	
Electron rings	60 MW
Ion rings	5 MW
Cost of electricity	
Electron rings	51 mills/kWh
Ion rings	45 mills/kWh

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