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A DESIGN METHOD TO DETERMINE THE OPTIMAL DISTRIBUTION AND AMOUNT OF INSULATION FOR IN-GROUND HEAT STORAGE TANKS

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ABSTRACT

The seasonal sensible heat storage model developed by F.C. Hooper and C.R. Attwater (1) is modified to describe the thermal behaviour of the soil regime surrounding cylindrical, in-ground, heat storage tanks with optimally distributed insulation. The model assumes steady-state heat transfer, and the surrounding soil is considered to be homogeneous and isotropic. Changes in soil thermal properties due to moisture migration, whether driven by thermal or hydrostatic gradients, are assumed negligible (2). The optimal distribution is determined using the method of Lagrange multipliers. It is shown that the marginal cost per unit of energy lost and per unit of tank surface area must be the same at all points on the surface of the tank as the condition for minimum total heat loss with a given total investment in insulation. This condition appears to apply for all axi-symmetric in-ground tank geometries. For a given volume of insulation, the incremental increase in storage efficiency with an optimal redistribution of the insulation is a function of tank geometry. The problem of determining the optimal total investment in insulation for a given marginal cost of fuel is described and a method of solution is outlined.

1. INTRODUCTION

All solar heating systems incorporate heat storages, usually adequate to carry the heating load for one to three days. However, at higher latitudes, seasonal storage of heat from summer to winter has major advantages and it can be argued that north of 40° latitude it often provides an economically superior choice.

Such systems are principally characterized by collector type, orientation, efficiency and area, by the building heating load, the storage capacity, the soil conditions and storage insulation. Overall system optimization is to a great extent site specific, however site characteristics appear to be of minor significance in the optimization of insulation distribution.

The insulation and waterproofing components contributed approximately one quarter of the total cost of an installed 0.9 million litre below-ground water thermal storage tank constructed to serve a 30 unit apartment building. This proportion will be slightly higher in smaller tanks and lower in larger tanks due to the square-cube relationship between heat loss and capacity.

2. MODEL

A cut-away view showing the geometry of the heat flow field under study is given in Fig.1. Steady-state heat transfer occurs by conduction from the tank to the air at the ground surface, to the water table

Fig.1 Storage Geometry

below, and to an assumed vertical isothermal boundary located at a position where more than 90% of the heat flow has reached the horizontal sinks. The soil surrounding the tank is considered homogeneous and isotropic. The latent heat of fusion of soil moisture under freezing and thawing conditions, and the latent heat of vaporization during surface drying are known to be factors (2) but are not considered here. The ground surface heat transfer coefficient and the air temperature are taken to be constant. The tank wall and floor, having thermal properties assumed similar to those of the soil, are thus assimilated into the soil regime. The tank contents are considered to be well-mixed.

Temperatures throughout the heat flow field are determined numerically by subdividing it into smaller regions and solving the finite-difference equations for the nodal network.

3. THE DISTRIBUTION PROBLEM

The resistance to heat flow from an area dA_1 on the tank surface can be considered to be the sum of the resistance of the insulation, RV_1 , and that of the soil, RS_1 , along the corresponding heat flow path from the tank to the appropriate heat sink. These terms

may be factored into insulation and soil unit resistances, RI and RS' , and effective path lengths, t_1 and ts_1 , respectively.

The total resistance at a node i on the tank is then:

$$RT_i = RV_i + RS_i = (RI \cdot t_i) + (RS' \cdot ts_i) \quad (1)$$

The heat flux through an area dA_i on the tank surface is:

$$dQ_i = (\Delta T_i dA_i) / (RV_i + RS_i) \quad (2)$$

Typical heat flow lines are shown in Fig.1.

The method of Lagrange multipliers (3) is used to obtain the necessary relationship between the heat loss (performance) and cost (constraint) functions. This relationship can be shown to be:

$$\nabla Q_T - \lambda \nabla C_T = 0 \quad (3)$$

where:

$$Q_T = Q_w + Q_f + Q_t \quad \text{and} \quad C_T = C_w + C_f + C_t$$

with w , f , and t representing the coordinates of a thickness space.

In this representation, the gradient vector is:

$$\nabla = \frac{\partial}{\partial t_w} \vec{i}_w + \frac{\partial}{\partial t_f} \vec{i}_f + \frac{\partial}{\partial t_t} \vec{i}_t$$

As $\partial C_f / \partial t_w = \partial C_t / \partial t_w = 0$, and similarly for the other cross-products, then:

$$\nabla C_T = \frac{dC_w}{dt_w} \vec{i}_w + \frac{dC_f}{dt_f} \vec{i}_f + \frac{dC_t}{dt_t} \vec{i}_t \quad (4)$$

This is also true for Q_T . Substituting into (3), we obtain:

$$\nabla Q_T - \lambda \nabla C_T = \left\{ \frac{dQ_w}{dt_w} \vec{i}_w + \frac{dQ_f}{dt_f} \vec{i}_f + \frac{dQ_t}{dt_t} \vec{i}_t \right\} - \lambda \left\{ \frac{dC_w}{dt_w} \vec{i}_w + \frac{dC_f}{dt_f} \vec{i}_f + \frac{dC_t}{dt_t} \vec{i}_t \right\} \quad (5)$$

Since the vector components must be equivalent:

$$dQ_w/dC_w = dQ_f/dC_f = dQ_t/dC_t = \lambda \quad (6)$$

This requires that the marginal cost per unit of energy lost must be the same everywhere about the tank for the minimum total heat loss for a given total investment in insulation.

Assuming that the insulation cost is a linear function of the thickness $dC_w = \chi_w t_w dA$;

$dC_f = \chi_f t_f dA$; $dC_t = \chi_t t_t dA$ where χ is the unit cost. Therefore:

$$dQ_w/dC_w = -RI_w \cdot \Delta T / \chi_w \cdot (RI_w \cdot t_w + RS_w)^2$$

$$dQ_f/dC_f = -RI_f \cdot \Delta T / \chi_f \cdot (RI_f \cdot t_f + RS_f)^2$$

$$dQ_t/dC_t = -RI_t \cdot \Delta T / \chi_t \cdot (RI_t \cdot t_t)^2$$

From equation (6):

$$RI_w / \chi_w \cdot (RI_w \cdot t_w + RS_w)^2 = RI_f / \chi_f \cdot (RI_f \cdot t_f + RS_f)^2 = RI_t / \chi_t \cdot (RI_t \cdot t_t)^2 \quad (7)$$

A solution may be obtained by defining:

$$\psi = RI_w \cdot t_w + RS_w; \quad \phi = RI_f \cdot t_f + RS_f; \quad \Gamma = RI_t \cdot t_t$$

$$\text{Therefore, } \psi = (\chi_f \cdot RI_w / \chi_w \cdot RI_f)^{1/2} \quad \phi = (\chi_t \cdot RI_t)^{1/2} \Gamma \quad (8)$$

If insulation with the same thermal and cost properties is used on all tank surfaces, $\chi_f = \chi_w = \chi_t$ and $RI_f = RI_w = RI_t$, and hence $\psi = \phi = \Gamma$. In this case, the total resistance at all nodal positions on the tank surface must be the same for an optimal distribution. ψ , ϕ , and Γ define the distribution of insulation, not the optimal amount, as an arbitrary value for the total investment C_T was assumed. An optimal C_T can only be determined by considering the relation between the storage system and the other components of the solar system. Equation (6) indicates that there exists a constant λ which is related to this amount. λ is a function of the total insulation volume applied, VI , or alternatively, C_T . This aspect of the problem will be discussed later in the context of determining this optimum.

This optimal insulation distribution has been studied for various storage geometries. A parameter in the analysis is SINKV, the dimensionless distance to the water table. SINKV is the ratio of the depth of the water table to the depth of the tank.

Applying insulation which has uniform physical, thermal, and cost properties to the tank, the distribution was determined for an R/D value of unity as shown in Fig.2. For the three values of SINKV considered in this figure, TI, the sum of the dimensionless equivalent soil and insulation thickness is 0.05.

Fig.2 Optimized Insulation Distribution Along Tank Wall and Floor for R/D = 1.0.

As would be expected the maximum insulation thickness is required at the upper corner, and a relative thickness is required to offset the fin effect at the lower corner. As SINKV increases, the amount of insulation attributable to the soil increases. This is particularly evident at the corners. For larger R/D values (more disk-like tanks), the degree of curvature in the insulation distribution can be shown to be reduced.

The effective thermal resistance or ETR (1), a lumped parameter, is a direct measure of the performance of in-ground heat storage tanks. For a given ΔT between the stored water and ambient air temperatures, increasing the ETR decreases the net tank heat loss. The effect on the ETR of distributing the insulation was studied for systems with equal insulation volumes (equal investments in insulation). The distributed case was compared with the uniform distribution case. The corresponding increase in storage performance is shown in Fig.3 for various values of SINKV. This shows clearly that the effect of distribution becomes negligible at large TI's, i.e. when the insulation component of TI increasingly dominates the soil component. The percentage change in the ETR is defined as: $\% \Delta ETR = 100 \cdot (ETR_{dist} - ETR_{unif}) / ETR_{dist}$.

Fig.3 Variation in Storage Performance With TI and SINKV.

At small TI's, when the insulation resistance at same nodal points is roughly equal to the soil resistance, the effect is much greater. For $R/D = 1$, the $\% \Delta ETR$ at a given TI is greatest for $SINKV = 2.0$. In this case, as the water table is relatively far away, the required insulation along the floor is small and that elsewhere on the tank is relatively greater. However, when the water table is higher the floor insulation must be thicker. The $\% \Delta ETR$ is higher for $SINKV = 1.2$ than for $SINKV = 1.5$ at small thicknesses, as the insulation in the mid-tank wall region is in effect transferred to the tank floor.

If we include seasonal variations in ground thermal conductivity in our heat transfer analysis, distributing the insulation in a non-uniform manner may not show as an advantage. During the spring thaw, the soil about the tank could be flushed by cold water. This could quickly cool the storage, especially if the insulation has been thinned-out in the mid-wall region in the process of distribution. In

practice, this may be minimized by paying careful attention to installation procedures. A protective waterproof plastic diaphragm around the tank just below the surface would act as a shield from percolation of the heavy rains of spring and fall. Work is continuing in the study of seasonal variability in soil conditions and their effect on storage performance (2).

The ETR for the uniformly distributed tank does not increase linearly with TI at small insulation thicknesses, Fig.4.

Fig.4 Comparison of Uniform and Distributed ETR's.

4. THE AMOUNT PROBLEM

A unique determination of the optimal amount of insulation for a storage system can only be arrived at when the system behaviour is known in detail. If we assume that the system can be described by three primary variables: collector area A_c ; storage volume, V_s ; and insulation volume V_i , then there exists an infinite set of cost and performance spaces for specified tank temperature variations at various collector areas, insulation thicknesses and storage volumes. Such a space is shown in Fig.5 for a given insulation volume. The slight waviness in the 100% line is an artifact of a slight instability in the computer generated solution at the balance point.

Fig.5 System Performance Space For a Given Insulation Volume.

The optimal system configuration may be determined in a manner similar to that previously applied to the distribution problem. Again we employ the method of Lagrange multipliers:

$$\nabla P_T - \eta \nabla C_T = 0 \quad (9)$$

where the objective function, P_T , is the heat delivered or saved, by each of the independent parameters ($P_T = P_A + P_S + P_I$) and the constraint function, C_T , is the total system cost,

($C_T = C_A + C_S + C_I$). In this representation, we employ the unit vectors: $\vec{i}_A$, $\vec{i}_S$ and $\vec{i}_I$ and obtain:

$$\nabla C_T = dC_A/dA \vec{i}_A + dC_S/dS \vec{i}_S + dC_I/dI \vec{i}_I \quad (10)$$

where the cross-products are zero. Similarly,

$$\nabla P_T = \left\{ \frac{\partial P_T}{\partial A} \vec{i}_A + \frac{\partial P_T}{\partial S} \vec{i}_S + \frac{\partial P_T}{\partial I} \vec{i}_I \right\} \cdot \{ P_A + P_S + P_I \} \quad (11)$$

the cross-product terms in (11) are generally not zero. Substituting (10) and (11) into (9) and equating vector terms gives:

$$\eta = \frac{dA}{dC_A} \cdot \frac{\partial P_T}{\partial A} = \frac{dS}{dC_S} \cdot \frac{\partial P_T}{\partial S} = \frac{dI}{dC_I} \cdot \frac{\partial P_T}{\partial I} \quad (12)$$

This equation defines the optimal statepoint for the solar system. It requires a constraint η . This is similar to the constant λ which was required to determine ψ , ϕ , and Γ . Clearly, $\lambda = \lambda(\eta)$. The overall constraint on the system is the marginal cost of energy, η , with which the system can be optimized, given the necessary performance and cost functions. Knowledge of η will in turn specify the optimal distribution constraint λ , through equation (12). It is not within the scope of this paper either to produce the relation for P_A , P_I , and P_S , nor to determine η . Both these tasks are the subject of further study.

5. CONCLUSIONS

A method for the determination of the optimum economic distribution of thermal insulation on in-ground annual heat storage tanks is described. Further, a method for determining the optimum insulation amount in a three variable collector area, storage volume, insulation quantity space is outlined assuming a knowledge of the cost factors such as the marginal cost of energy.

Generally the reductions available in the annual heat loss from a tank due to redistribution of insulation from a uniform to an optimum pattern appear to be on the order of 2 to 5 percent. Other geometries will probably show somewhat similar saving potentials. While not large such savings appear to be cost free in most applications, and are worthy of consideration by thermal storage designers.

6. ACKNOWLEDGEMENT

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7. NOMENCLATURE

A	collector area
C	cost function
dA	elemental area on tank surface
dQ	heat flux
D	tank depth
ETR	effective thermal resistance
ETR _{dist}	ETR for distributed insulation
ETR _{unif}	ETR for uniform insulation
$\vec{i}$	unit vector
P	performance function (heat delivered)
Q	heat flux
R	tank radius
RI	insulation unit resistance
RS'	soil unit resistance
RS	soil resistance
RT	total resistance
RV	insulative resistance
S	storage volume
SINKV	dimensionless distance to water table
TI	dimensionless total equivalent thickness
t	insulation thickness
ts	soil thickness
VI	total volume of insulation applied
ΔT	temperature difference
η	Lagrange multiplier
∇	gradient operator
λ	Lagrange multiplier
χ	unit insulation cost
ψ	total resistivity at wall node
ϕ	total resistivity at floor node
Γ	total resistivity at top node

Subscripts

A	collector
I	insulation
S	storage
T	total function value
f	position on tank floor
t	position on tank top
w	position on tank wall
i	the ith nodal position

8. REFERENCES

1. Hooper, F.C. and C.R. Attwater, "A Design Method for Heat Loss Calculation for In-Ground Heat Storage Tanks", ASME Annual Meeting, Atlanta, Georgia, November 1977.
2. Hooper, F.C. et. al., Solar Space Heating Systems Using Annual Heat Storage, Progress Reports for the U.S. Department of Energy, Contract EY-76-C-02-2939000.
3. Kaplan, Wilfred, Advanced Calculus, Second Edition, (Addison - Wesley Publishing Company: 1973).