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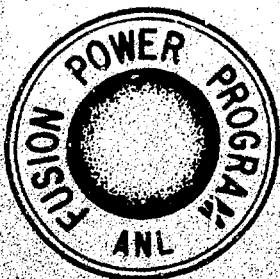
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**KINETIC SOLUTION OF THE SHEATH REGION
IN A FUSION REACTOR**

by

Jeffrey N. Brooks

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FUSION POWER PROGRAM

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KINETIC SOLUTION OF THE SHEATH REGION IN A FUSION REACTOR

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ABSTRACT

The sheath region in a fusion reactor is studied with a one-dimensional kinetic code. The sheath potential, heat transmission, and sputtering of the boundary are all quite sensitive to electron re-emission. The expected heat and particle fluxes in future fusion reactors leads to a prediction of keV-edge temperatures.

INTRODUCTION

An important concern for future tokamak fusion reactors is the behavior of the plasma near the boundary. The presence of a sheath potential and phenomena such as electron reemission can affect the flux of particles and energy to the boundary and the sputtering of impurities from the boundary. This subject has been studied, for the case of flow to the boundary along magnetic field lines, by numerically solving a system of time-independent, Vlasov kinetic equations, for the hydrogen and impurity ions, and the electrons, in which the distribution functions depend on the velocity vector and a single spatial coordinate (the distance from the boundary). Figure 1 shows the problem geometry. The boundary is referred to as a limiter but could also be a divertor plate. The point $x = a$ represents the start of the sheath region.

Boundary conditions are postulated at the start of the sheath region and at the limiter. Particles are assumed to flow towards the limiter along magnetic field lines and so the magnetic field can be neglected. The electric field, which lies in the x -direction, is given self-consistently by Poisson's equation. The equations are then solved in the sheath region. In contrast to fluid model solutions, a kinetic solution shows the fine details of the sheath region and permits a better description of the boundary conditions. However, an important aspect needing further analysis is to connect the boundary conditions with conditions in the plasma proper — this is beyond the scope of the present analysis.

MODEL

The equations solved are as follows:

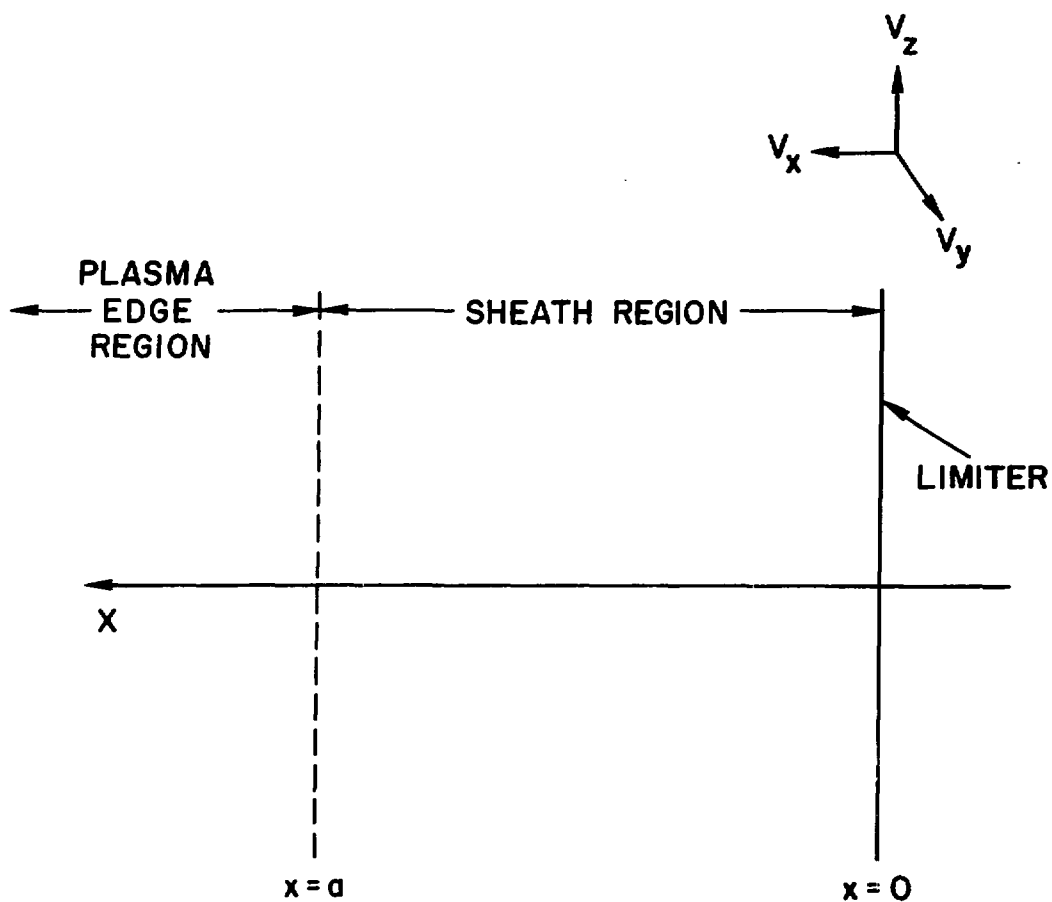


Fig. 1. Problem geometry.

$$V_x \frac{\partial f_{DT}}{\partial x} + \frac{e E(x)}{M_{DT}} \frac{\partial f_{DT}}{\partial V_x} = 0, \quad (1)$$

$$V_x \frac{\partial f_z}{\partial x} + \frac{Z_e E(x)}{M_z} \frac{\partial f_z}{\partial V_x} = 0, \quad (2)$$

$$V_x \frac{\partial f_e}{\partial x} - \frac{e E(x)}{M_e} \frac{\partial f_e}{\partial V_x} = 0, \quad (3)$$

$$\frac{dE(x)}{dx} = \frac{e}{\epsilon_0} (N_i + Z N_z - N_e), \quad (4)$$

where $f_k = f_k(x, \vec{V})$ is the distribution function of the k -th species, M_k is the mass, e is the proton charge, Z_e is the impurity ion charge where Z is the atomic number of the impurity, E is the electric field, V is the (three-dimensional) velocity vector; and where the subscripts denote DT = deuterium/tritium fuel ions, z = sputtered impurity ions and e = electrons. Beryllium has been used as the impurity species because of its proposed use as a coating material⁽¹⁾ for future fusion devices. (The effect of helium and iron impurities have also been examined.) The densities in Eq. (4) are given by the zeroth moment of the distribution functions:

$$N_k(x) = \int_{-\infty}^{\infty} f_k d^3V. \quad (5)$$

Other variables of interest are the potential ϕ , particle flow Γ_k , heat flow q_k , and the average sputtering coefficient of the ions impinging on the limiter \bar{S}_k . These are given by:

$$\phi(x) = \int_0^x E(x) dx, \quad (6)$$

$$\Gamma_k(x) = \int_{-\infty}^{\infty} V_x f_k d^3V, \quad (7)$$

$$q_k(x) = \int_{-\infty}^{\infty} V_k f_k d^3V, \quad (8)$$

$$S_k = \frac{\int_{-\infty}^{\infty} V_k S_k(U_k) f(0, \vec{V}) d^3V}{\Gamma_k(0)}. \quad (9)$$

where $S_k(U_k)$ is the monoenergetic sputtering coefficient for a particle of species k having an energy $U_k = 1/2 M_k (V_x^2 + V_y^2 + V_z^2)$. The form of S_k is taken as Eq. (7) of Ref. 2. An additional computed quantity is a factor γ which describes the heat transmission capability of the sheath and is defined by:

$$\sum q_k(a) = \gamma K T_e^a \sum \Gamma_k(a). \quad (10)$$

The boundary conditions used are as follows: In the plane perpendicular to the X -direction, the distribution functions are Maxwellian everywhere. The form of the distribution functions in the X -direction are specified at $X = a$, and for $V_x \leq 0$ by half-Maxwellian distributions characterized by a temperature T_k^a , a drift velocity V_k^D , and a density N_k^a as follows:

$$f_k(a, \vec{V}) = N_k^a f_k^{yz}(V_y, V_z) f_k^x(V_x), \quad (11)$$

where

$$f_k^{yz}(v_y, v_z) = \frac{M_k}{2\pi T_k} e^{-[M_k(v_y^2 + v_z^2)/2T_k^a]}, \quad -\infty < v_y, v_z < \infty, \quad (12)$$

and

$$f_k^x(v_x) = \left(\frac{M_k}{2\pi T_k^a} \right)^{\frac{1}{2}} e^{-[M_k(v_x - v_D^k)^2/2T_k^a]}, \quad v_x \leq 0. \quad (13)$$

The value of a , i.e. the length of the sheath region, is solved for by the code as part of the overall solution. The impurity density at $x = a$ has been set equal to 5% of the DT density, i.e. $N_z^a = 0.05 N_{DT}^a$. Note that the distribution functions in the x -direction are only specified for particles moving towards the limiter. No ions come back from the limiter (they come back, if at all, as neutrals and there is no appreciable ionization in the sheath region) and so $f_k^x(x, v_x) = 0$ for $v_x > 0$ and for all x , for $k = DT, Z$. This is not the case for electrons for two reasons: (1) electrons are decelerated by the sheath potential and some, therefore, turn around and come back; and (2) because of electron re-emission at the limiter. For electrons the form of $f_e^x(a, v_x)$ for $v_x \geq 0$ is determined by the code. Likewise, the full electron temperature T_e and density N_e at $x = a$ are determined by integrating over the forward and backward-going particles. Additional boundary conditions at $x = a$ are that the plasma is electrically neutral so that

$$N_{DT}(a) + 4N_z(a) = N_e(a), \quad (14)$$

and that the net flow of charge to the limiter is zero:

$$\Gamma_{DT}(a) + 4\Gamma_z(a) = \Gamma_e(a). \quad (15)$$

At the limiter, electrons can be re-emitted with a probability " R_e " such that the flow of electrons from the limiter is equal to R_e times the flow to the limiter:

$$\Gamma_e^+(0) = R_e \Gamma_e^-(0) ,$$

where

$$\Gamma_e^{(\pm)}(0) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dV_y dV_z \int_{0(-\infty)}^{\infty(0)} dV_x f_e(0, \vec{V}) d^3V , \quad (16)$$

It has been assumed for this work that the re-emitted electrons are all at near-zero velocity (a few eV).

Several normalized parameters are used in the analysis; distance is normalized to an electron debye length at $x = a$:

$$\lambda_0 = \sqrt{\frac{\epsilon_0 K T_e^a}{N_e(a) e^2}} . \quad (17)$$

Two parameters are used in specifying the boundary conditions:

$$\alpha = \frac{\frac{1}{2} M_i (V_D^i)^2 + \frac{3}{2} K T_i^a}{\frac{3}{2} K T_e^a} , \quad \beta = \frac{T_{DT}^a}{T_e^a} , \quad (18)$$

where α is approximately the ratio of the average energy of an ion to the average energy of an electron at the start of the sheath region, β is the ratio of ion to electron temperature at this point, and a value of $V_D^e = 0$ has been employed. Other conditions used are that $T_z = T_{DT}$, and $V_D^z = V_D^i$.

This system of equations has been solved numerically using a midpoint-valued finite difference scheme, and using an iterative process to solve for the self-consistent electric field and the value of a .

RESULTS

Figure 2 shows the numerical solution for a case with $R_e = 0$, i.e. with no electron reflux, and for typical parameters $\alpha = 1$, $\beta = 0.25$. The quantities shown are all normalized to their maximum values. The positive charge density is defined as $N^+ = N_{DT} + 4 N_Z$. The plasma is positively charged with respect to the wall, with the potential reaching a maximum of $e\phi \simeq 3 K T_e^a$ at the limiter. This is similar to classical results. The width of the sheath region is about 8 debye lengths but most of the charge separation occurs over about the first 4 debye lengths from the limiter. All densities fall off monotonically towards the limiter. The electron temperature (not shown) falls off from about T_e^a at $x = a$ to about 80% of this value at the limiter.

Figure 3 shows a solution for the same case except that $R_e = 0.535$, a value equal to the so-called space-charge limit. Because of the high re-emission of electrons, N_e near the limiter is actually higher than at $x = a$. As a result, the electric field reverses itself in the sheath region and falls to zero at the limiter. If R_e were greater than this value, the electric field could go negative which would, in theory, serve to repel any additional electrons; the condition of $E(0) = 0$ defines the space-charge limit. The potential for this case is only about $1/2 K T_e^a$.

The electron distribution function for the space-charge limited case is shown in Fig. 4 for the points $x = 0$ and $x = a$. Electrons traveling to the right ($V_x < 0$) and at $x = a$ are specified as per Eq. (11). At $x = 0$ they are displaced in velocity due to the sheath potential. The spike in Fig. 4

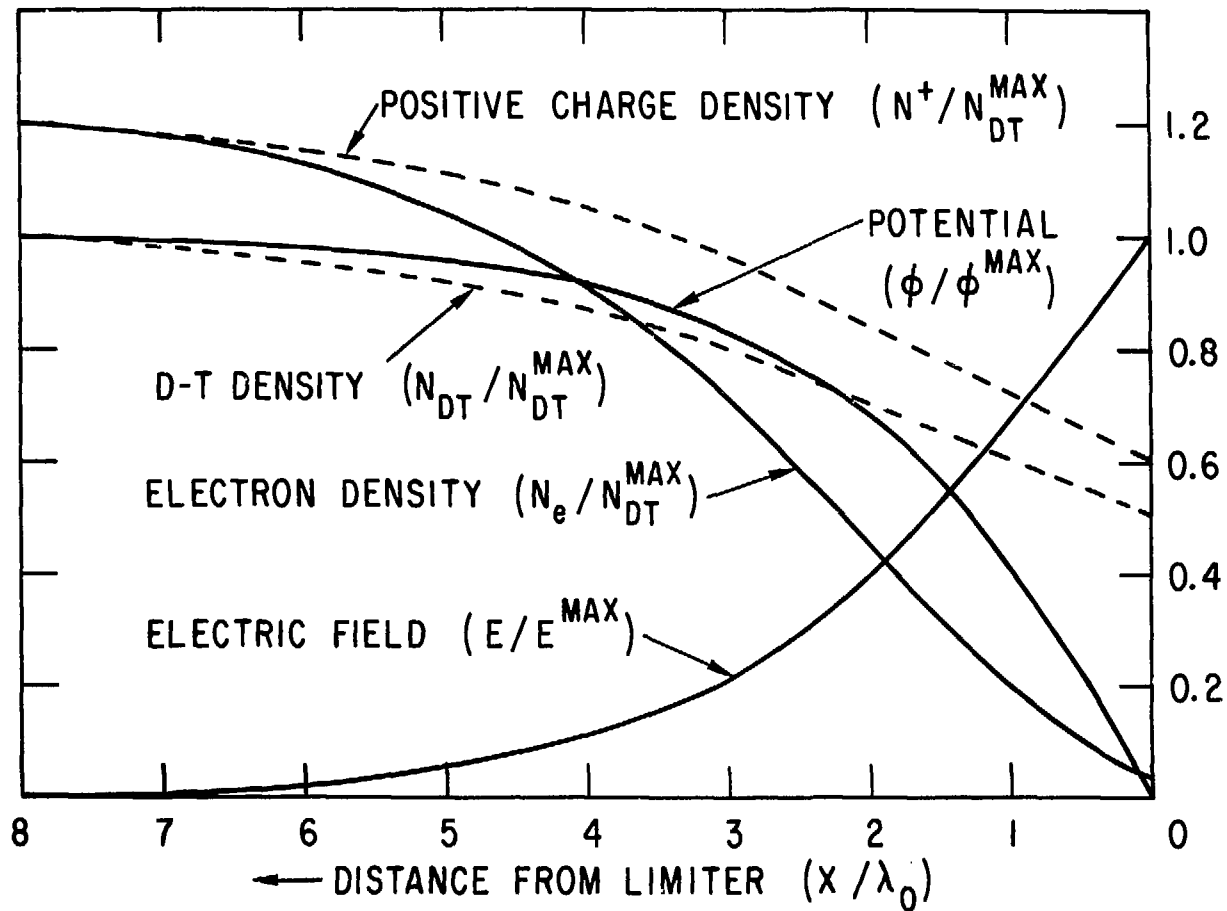


Fig. 2. Densities, electric field, and potential in the sheath region for no electron re-emission.

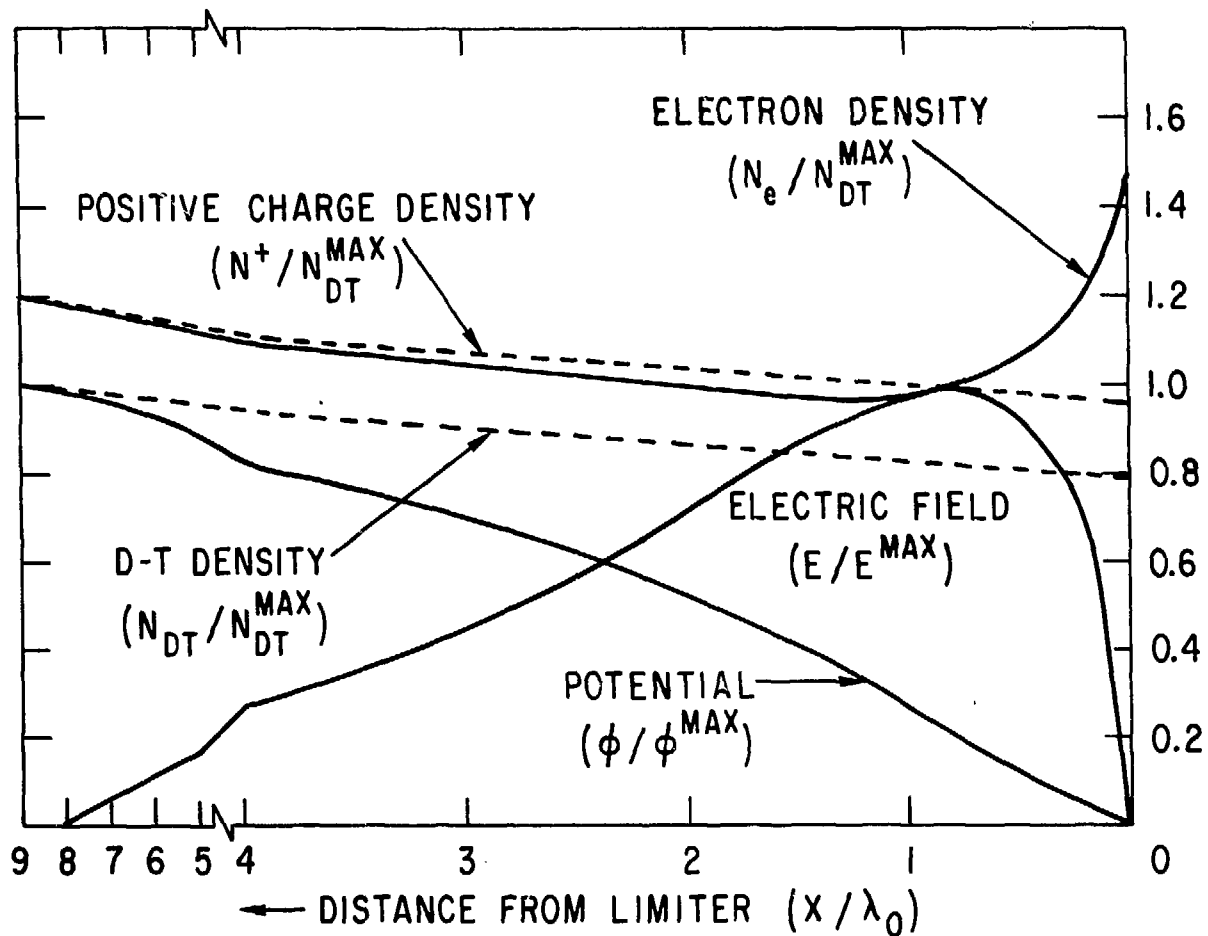


Fig. 3. Densities, electric field, and potential in the sheath region for space-charge limited re-emission.

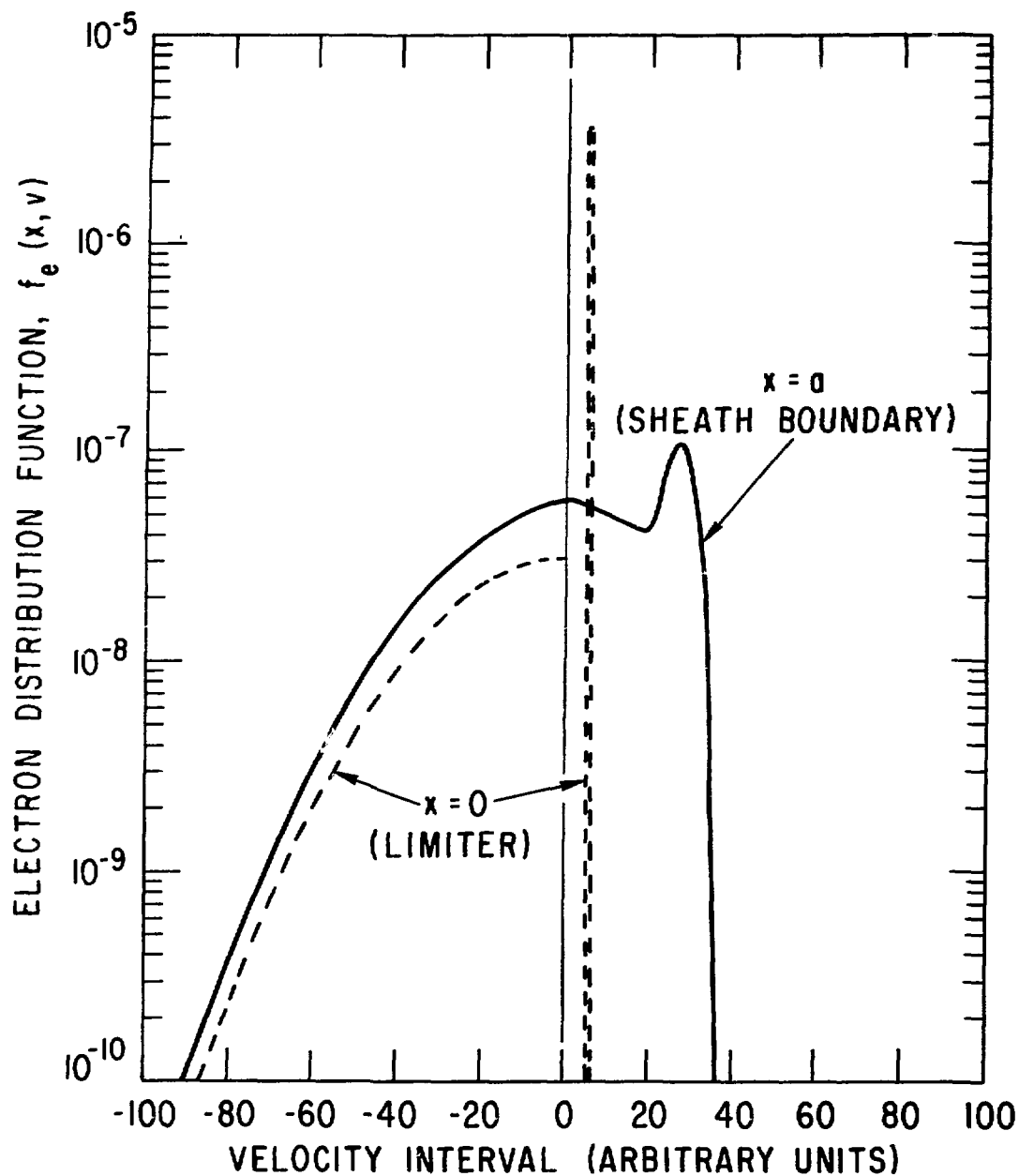


Fig. 4. Electron distribution function at $x = 0$ and $x = a$ for space-charge limited re-emission.

represents the low-energy re-emitted electrons at the limiter. These are accelerated through the sheath and hence have the form shown at $x = 0$.

Figure 5 shows the variation of sheath potential, heat transmission factor, and the average sputtering coefficients as a function of R_e and for $\alpha = 1$ and $\beta = 0.25$. A specific value of $T_e^a = 400$ eV was used for the computation of the sputtering coefficients, the other variables are normalized to any T_e^a as shown. Over the range of R_e from zero to the space-charge limit, ϕ^{\max} varies by about 6 to 1 and γ varies by about 4 to 1. The behavior of γ with R_e follows a simple, nearly straight-line dependence. The explanation for this is that for large R_e , the heat flow is dominated by electron conduction whereby the flux of cold electrons from the limiter permit more hot electrons from the plasma to hit the limiter while still maintaining charge flow balance from the plasma. This also explains the reduction in ϕ^{\max} with increasing R_e .

The form of the sputtering curves is explained as follows: As R_e increases and ϕ decreases, ions from the plasma are accelerated less as they go through the sheath and therefore hit the limiter with less energy. (The DT ions gain an energy of $e\phi^{\max}$ and the impurities gain $4 e\phi^{\max}$ in the sheath.) Because the sputtering of B_e peaks at lower energies, this has the effect of increasing the sputtering coefficients. However, for other choices of T_e^a and/or other coating materials, the trends could be different.

Figure 6 shows the effect of varying α on the sheath potential and heat flow parameter (a value of $\beta = 0.25$ has been used; there is little variation found with β). The minimum value of α for which a time-independent solution exists was found to be $\alpha = 0.65$ and only for $R_e \lesssim 0.2$. Below this so-called Bohm limit, the ions travel too slowly as they enter the sheath and the ion density falls off rapidly and the electric field oscillates.

The value of α at the edge of a tokamak reactor is uncertain; however, it will probably be greater than the Bohm limit and lower than the upper limit

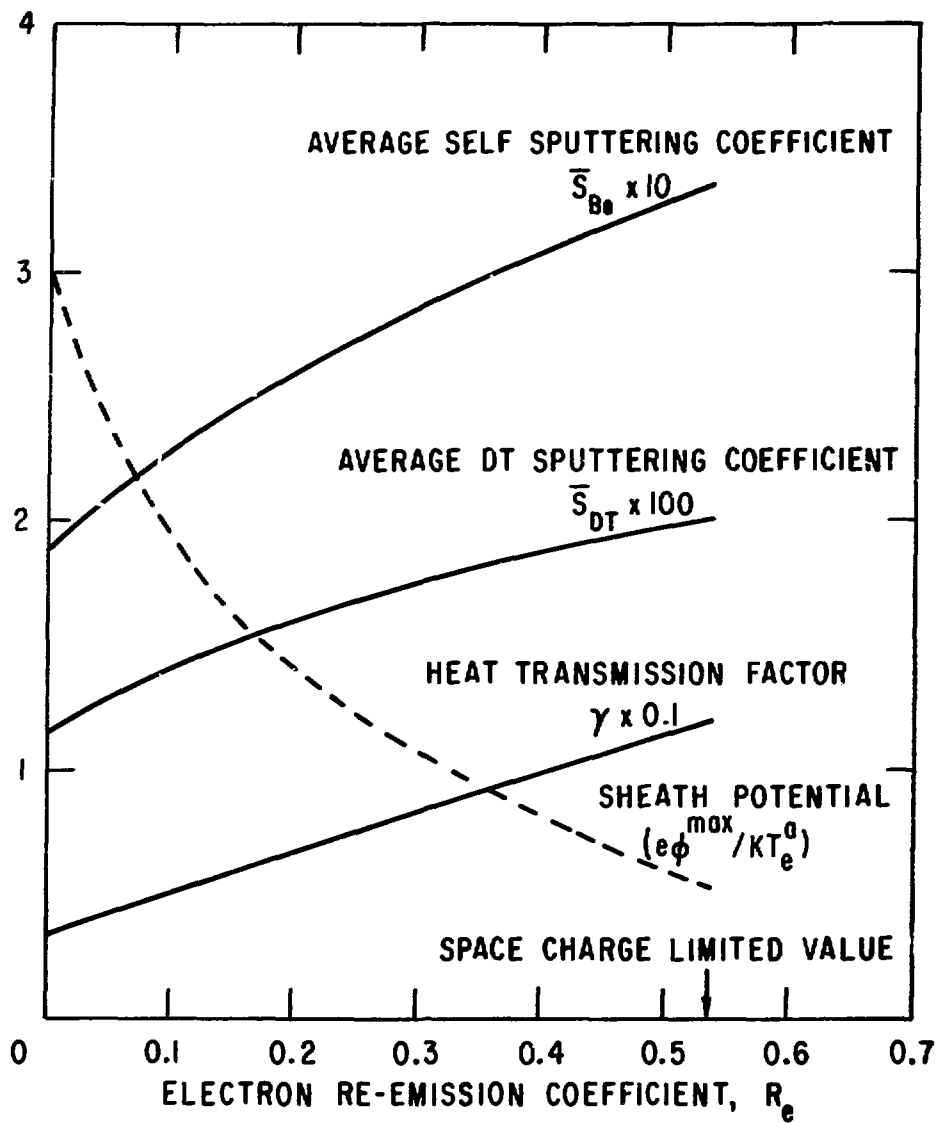


Fig. 5. Sensitivity of sheath parameters and sputtering coefficients to R_e , for $\alpha = 1$.

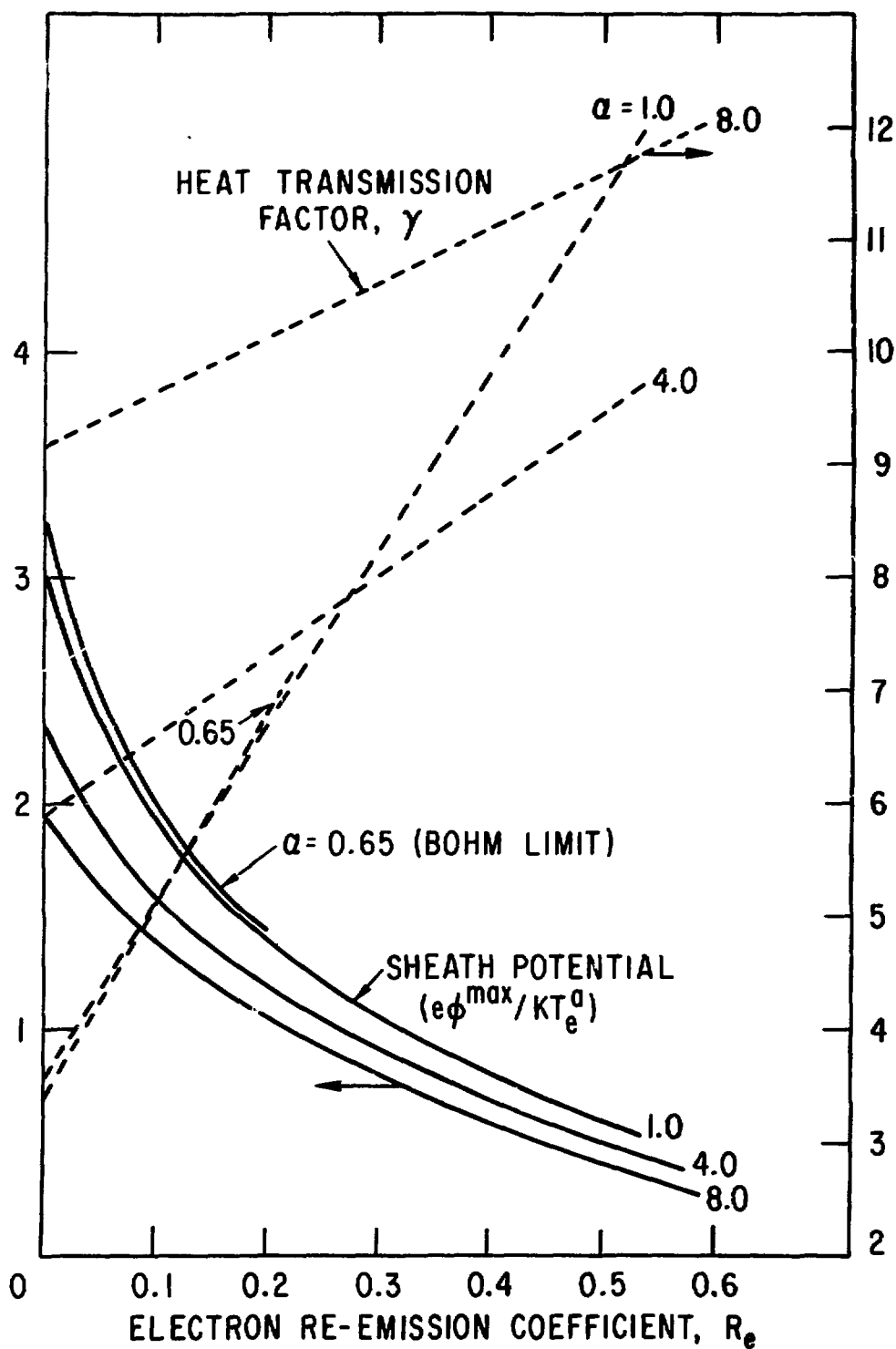


Fig. 6. Sensitivity of sheath parameters to R_e and α .

of $\alpha = 8$, shown in Fig. 5. For $\alpha \gtrsim 1$ there is only a small variation in the space-charge limit (where the curves terminate), and the potential. While the slope of γ versus R_e differs with α , the value of γ has a spread of only about 20%, at the space charge limit. Alpha is a measure of a type of flow in the boundary region, e.g. supersonic versus subsonic. Therefore, the basic conclusions from these results should not be particularly sensitive to the flow type as long as the Bohm limit is satisfied. It was also found that using helium and iron in the computer code, instead of beryllium, did not appreciably alter the results as long as the ratio of electron-to-ion density was kept fixed.

REACTOR IMPLICATIONS

In a fusion plasma at ignition, the alpha-heating power to the plasma will be balanced by radiation and transport losses. Transport throughout the bulk plasmas can be by conduction and convection but heat can only flow to the limiter (or divertor plate) by convection. If the transport power, particle flux, and the heat flow parameter γ , is specified, the edge temperature T_e^a can be determined as per Eq. (10).

As an example, for a 3000-MWth fusion reactor, the alpha heating power would be 600 MW, of which 500 MW could typically be radiated (possibly by injecting high-Z impurities) leaving 100 MW to be transported. The heat flow of Eq. (10) is therefore

$$\sum q_k(a) = 100 \text{ MW} .$$

The DT particle flow to the limiter is given by:

$$\Gamma_{DT} = \frac{\bar{N}_{DT} v_p}{\tau_p} ,$$

where \bar{N}_{DT} is the average DT density in the bulk plasma, V_p is the plasma volume, and τ_p is the average particle containment time. Using the following typical parameters: $\bar{N}_{DT} = 1.0 \times 10^{20} \text{ m}^{-3}$, $V_p = 750 \text{ m}^3$, and $\tau_p = 5 \text{ s}$; and using the assumption that $r_e = 1.2 r_{DT}$ and $r_z = 0.05 r_{DT}$, the total particle flow is:

$$\sum r_k(a) = 3.4 \times 10^{22} \text{ s}^{-1}.$$

Assuming that R_e is at the space-charge limit, then a value of $\gamma = 11$ would be typical. The edge temperature using Eq. (10) is therefore:

$$T_e = \frac{100 \times 10^6 \text{ W}}{1.6021 \times 10^{-19} \text{ J/eV} \times 11 \times 3.4 \times 10^{22} \text{ s}^{-1}} \approx 1500 \text{ eV}.$$

Thus the edge temperature would be about 20% of the average temperature (typically ~8 keV) and about 10% of the central temperature. A DT ion would gain an energy of about 750 eV in the sheath. DT ions would therefore hit the limiter with an average energy of about 3000 eV. For $\alpha = 1$, beryllium ions would gain 3000 eV in the sheath and could hit the limiter at 5000 eV or greater average

energy, depending on their initial energy. Helium ions would likewise be expected to hit at somewhere in this range of 3000-5000 eV. For low-Z coatings (be, B_4O , B_4C , etc.) this range of energies would result in significantly less sputtering than "worst case" calculations⁽¹⁾ which used the peak values to be conservative. On the other hand, high-sheath potentials of the keV magnitude would seem to make operation with bare-wall structural materials, e.g., iron, exceptionally difficult, because of the huge self-sputtering coefficient, ≥ 10 sputtered atoms per incident ion,⁽²⁾ predicted for keV energies. The sheath potential, and hence the bare-wall self-sputtering, could be even worse if (1) not as much of the alpha energy could be radiated; and (2) there were less electron re-emission.

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REFERENCES

- (1) W. M. STACEY, JR., et al., "Tokamak Experimental Power Reactor Conceptual Design," Argonne National Laboratory Report ANL/CTR-76-3 (1976).
- (2) D. L. SMITH, "Physical Sputtering Model for Fusion Reactor First Wall Materials," *J. Nucl. Mater.* 75, 20 (1978).