2212

PPPL-2212

DR 1104-5

0020-A, D, G

I-21618

196
6-28-85 w8.

(1)

PLASMA TRANSPORT NEAR MATERIAL BOUNDARIES

By

C.E. Singer

JUNE 1985

PLASMA
PHYSICS
LABORATORY

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,
UNDER CONTRACT DE-AC02-76-CH0-3873.

RESTRICTION OF THIS DOCUMENT IS UNLIMITED

PLASMA TRANSPORT NEAR MATERIAL BOUNDARIES*

C. E. Singer
Plasma Physics Laboratory, Princeton University
Princeton, New Jersey 08544

ABSTRACT

The fluid theory of two-dimensional (2-d) plasma transport in axisymmetric devices is reviewed. The forces which produce flow across the magnetic field in a collisional plasma are described. These flows may lead to up-down asymmetries in the poloidal rotation and radial fluxes. Emphasis is placed on understanding the conditions under which the known 2-d plasma fluid equations provide a valid description of these processes. Attempts to extend the fluid treatment to less collisional, turbulent plasmas are discussed. A reduction to the 1-d fluid equations used in many computer simulations is possible when sources or boundary conditions provide a large enough radial scale length. The complete 1-d fluid equations are given in the text, and 2-d fluid equations are given in the Appendix.

INTRODUCTION

Understanding the interaction of plasma flows with material boundaries is of fundamental importance in experimental plasma physics. Several of the lectures in this series refer to the development and application of fluid models of these plasma flows. These models are beginning to play an important role in the design and interpretation of tokamak experiments. The physicist interested in this area therefore needs at least a qualitative understanding of the physics used in these models. Where do the model equations come from? When is the general approach used adequate, and when does it have to be abandoned or supplemented with a more sophisticated treatment? The purpose of this review is to provide some answers to these questions. The discussion will be kept intuitive and mostly qualitative. For those interested in the complete transport equations, however, an Appendix is provided.

CONSERVATION LAWS

The only equations which give a complete description of boundary plasma flows are the collisional Vlasov equations.

*This paper is one of a series of lectures¹³ given at a NATO Advanced Studies Institute International course on the "Physics of Plasma-Wall Interactions in Controlled Fusion" at Val-Morin, Québec, Canada (July 30-August 10, 1984), to be published by Plenum Press.

These consist of the Fokker-Planck equation for each type of particle

$$\frac{\partial f}{\partial t} + v_a \frac{\partial f}{\partial x_a} + a_a \frac{\partial f}{\partial v_a} = C + \Sigma, \quad (1)$$

and Maxwell's equations. f represents the number of particles in a unit cube centered on physical location x_a with velocities in a unit cube centered on v_a . In one dimension, $\partial f / \partial t$ depends on the difference in the number of particles advecting towards point x (and, hence, on $\partial f / \partial x$). $\partial f / \partial t$ similarly depends on the acceleration, a , of particles up to velocity, v , and the acceleration away from v (and, hence, on $\partial f / \partial v$). In three cartesian coordinates, we sum over the index a and note that $\vec{a} = (e/m)(\vec{E} + \vec{v} \times \vec{B}/c)$. Coulomb collisions between particles separated by small distances can also change the distribution function f . The effect of $C_a = \Sigma_b C_{ab}(f_a, f_b)$ of all collisions on species a results in a contribution roughly proportional to the Coulomb collision frequencies

$$\nu_{aa} = \frac{4\pi^{1/2} n_a e^4 \lambda_{aa}}{3m_a^{1/2} T_a^{3/2}}, \quad (2)$$

for like-particle collisions and, e.g., $\nu_{e1} = 4(2\pi m_e)^{1/2} n_e e^4 \lambda_{e1} / (3m T_e^{3/2})$ for collisions between hydrogen ions of mass m , electrons of mass m_e , and temperature T_e , where λ_{ab} is the Coulomb logarithm.² The rate, Σ , of creation and scattering of particles by neutral and inelastic atomic collisions depends on the cross sections described elsewhere by M. Harrison¹³. Collisions with neutrals are generally computed using the Monte Carlo methods described elsewhere by D. Reifetz¹³.

Now we make three enormous simplifications. First, we assume that the magnetic field in the region of interest is externally determined. For 2-d calculations this is generally the case, since currents in the poloidal field coils and core plasma are much larger than in the boundary plasma. (With extreme gradients as in so-called edge relaxation phenomena (ERPS) and in high confinement (H-mode) divertor plasmas this condition may be violated.

Second, we assume $\partial / \partial t = 0$, which eliminates all forms of microturbulence. We therefore cannot generally treat low-density plasmas, which are typically microturbulent. The trend towards higher density scrape-off plasmas in future experiments gives hope that higher collisionality will rapidly damp out any microturbulence driven by core plasma fluctuations. Later, we will discuss attempts to reintroduce turbulence effects phenomenologically into the fluid equations.

Third, we assume collisions and gyromotion are sufficient to maintain nearly Maxwellian ion distribution functions

3

$$f = f_0 (1 + \Phi), \quad (3)$$

where $\Phi \ll 1$ and $f_0 = n\pi^{-3/2} v_{th}^{-3} \exp[(\vec{v} - \vec{u})^2/v_{th}^2]$ with $v_{th} = (2T/m)^{1/2}$. (From now on we suppress the species index when referring to ions.) A similar equation is used for the electron distribution function f_e . This places three restrictions on the validity of our treatment. First, inspection of the Fokker-Planck equation shows that advection parallel to the magnetic field will overly distort the nearly Maxwellian distribution unless $v_{th} \epsilon/L_{\parallel} \ll v_f$, where $L_{\parallel} = Bf/B \cdot \nabla f$ is the parallel scale height. This requires a small "inverse collisionality," $\Delta = \lambda_{mfp}/L_{\parallel} = (v_{th}/v)/L_{\parallel}$. Second, local gyromotion will also overly distort the ion Maxwellian distribution unless $\delta = \rho/L_{\perp} \ll 1$ (where $\rho = v_{th}/\Omega = v_{th}/[eB/(mc)]$ is the ion gyroradius and $L_{\perp} = |\nabla \psi| f / (\nabla \psi \cdot \nabla f)$, where ψ is the magnetic flux surface label in orthogonal coordinates.) The contributions to f of order Δ and δ have been computed in works on classical transport theory^{3,4} and pose no problem.

However, there is also a distortion of the distribution function due to the radial drifts experienced by particles in a curved magnetic field. In the absence of collisions, there is a contribution of order δ_p to the correction function Φ , where $\delta_p = \delta B/B_p$ and B_p is the poloidal magnetic field. But particles can only execute a fraction Δ of their poloidal drift before being scattered by collisions, so a collisional plasma experiences a resulting distortion of order $\Delta \delta_p$. Suppose we rely on existing work for computation of the perturbation, Φ . Then the unknown correction of order $\Delta \delta_p$ should be small compared to the known corrections of order Δ and δ . This requires $\Delta \delta_p \ll \Delta$ and $\Delta \delta_p \ll \delta$, i.e.,

$$\delta_p \ll 1, \quad (4)$$

$$\Delta \ll B_p/B, \quad (5)$$

where B_p is understood to be averaged over a region significantly smaller than $L_{\psi} \times L_{\parallel}$. The first of these requirements is, in general, more severe. For typical large tokamak parameters, $(n/10^{14} \text{ cm}^{-3})/(T_e/10 \text{ eV})^{3/2} \gtrsim 1$ is required.⁵ This is achieved only in high-density divertors.

When the above conditions are satisfied, we can usefully integrate the Fokker-Planck equations over velocity space⁴ after multiplying by various powers of v_{α} . This gives the conservation equations

$$\nabla \cdot (n \vec{u}) = S, \quad (6)$$

$$\nabla \cdot (m n \vec{u}) + \nabla \cdot (n T) + \nabla \cdot \Pi_{vis} + en(\vec{\nabla} \phi - \vec{u} \times \vec{B}/c) = \vec{F}_{Coul} + \vec{F}_{Non-Coul}, \quad (7)$$

$$\begin{aligned} \nabla \cdot \left(\frac{1}{2} m u^2 \vec{u} + \frac{5}{2} n T \vec{u} + \vec{u} \cdot \Pi_{vis} + \vec{q}_{cond} \right) + en \vec{u} \cdot \nabla \phi, \\ = Q_{Coul} + \vec{u} \cdot \vec{F}_{Coul} + \vec{F}_{Non-Coul}, \end{aligned} \quad (8)$$

for each plasma species, with similar equations for higher velocity moments. The Coulomb and non-Coulomb friction and energy sources are obtained by integrating over the collision terms C (using polynomial expansion techniques) and Σ (by summing over Monte Carlo calculations). Here we have used $\partial/\partial t = 0$ to write $\vec{E} = -\nabla \phi$; this requires the usual situation of a small externally applied electric field, $e\phi_{ext} \ll kT_e$.

In general, these equations should be supplemented by Poisson's equations, $\nabla^2 \phi = n_i - n_e$, but an excellent approximation in practical pure hydrogen collisional plasma regions is $n_e = n_i \equiv n$ as described elsewhere by P. Stangeby.¹³ We can therefore replace Poisson's equation by this approximation. We are left with ten equations for the ten unknowns, n , $\vec{u}$, T , $\vec{u}_e$, T_e , ϕ .

CROSS-FIELD TRANSPORT

In toroidally symmetric situations, the above equations become tractable if we retain only the dominant terms. Since the mathematical details are outlined elsewhere for the case of tokamak boundary plasmas,⁵ we present here only the most interesting physics.

Ion continuity provides a useful starting point. The continuity equation is

$$g^{-1/2} \left[\frac{\partial}{\partial \phi} (n u_\phi |\vec{\nabla} \psi|^2)^{1/2} + \frac{\partial}{\partial \theta} (n u_\theta |\vec{\nabla} \theta|^2)^{1/2} \right] = S, \quad (9)$$

where $g^{1/2} = 1/|\vec{\nabla} \psi \times \vec{\nabla} \theta \cdot \vec{\nabla} \xi|$ is the Jacobian, and ξ is the toroidal symmetry angle. The orthogonal coordinates (ϕ, θ, ξ) have the advantage of geometric simplicity. However, we have to resolve the poloidal flow,

$$u_\theta = (B_\phi/B) u_i + (B_\xi/B) u_d, \quad (10)$$

into components in the "parallel" direction $\hat{B} = \vec{B}/B$, and the electron "diamagnetic" direction $B \times \nabla\psi/|\nabla\psi|$ perpendicular to the magnetic field within the magnetic flux surfaces. We start our discussion of cross-field transport by analyzing the diamagnetic ion flow velocity, u_d .

Diamagnetic Ion Flow Velocity

Due to the large $\vec{u} \times \vec{B}$ (Lorentz) term in the momentum balance, cross-field transport in one direction is determined by the momentum balance in the orthogonal direction across the magnetic field. This is a reflection of the fact that a cross-field force tends primarily to make particles drift in the picture direction perpendicular to that force. Thus, the diamagnetic flow is determined from the radial momentum balance. The only significant radial forces are due to the radial gradients of the pressure and potential, so

$$nu_d = \frac{|\nabla\psi|}{m\Omega} \left(\frac{\partial(nT)}{\partial\psi} + en \frac{\partial\phi}{\partial\psi} \right). \quad (11)$$

Normally, nT and ϕ decrease radially outward, so the poloidal projection $(B_\theta/B)u_d$ corresponds to a unidirectional poloidal rotation. This rotation tends to compress the plasma against one side of the limiter or divertor(s). Even in an up-down symmetric geometry, the flow will not be up-down symmetric. The magnitude of the rotation is comparable to that needed to explain observed up-down asymmetries in the ASDEX tokamak.⁶ The sign of the rotation depends on the sign of the toroidal magnetic field,⁷ as do experimentally observed up-down asymmetries.⁸

Radial Ion Flow

The cross-field transport perpendicular to magnetic flux surfaces is more complicated. This "radial" flux is driven by forces in the diamagnetic direction. For very subsonic flows, only the gradients of nT and ϕ are significant. Since $\partial/\partial\xi = 0$, only $\partial(nT)/\partial\theta$ and $en\partial\phi/\partial\theta$ contribute. The radial flux can be expressed directly in terms of these gradients, as illustrated in Fig. 1. Alternatively, the electron momentum balance parallel to the magnetic field

$$\frac{B_\theta}{B} \left[\frac{\partial(nT)}{\partial\theta} + en \frac{\partial\phi}{\partial\theta} |\nabla\theta| \right] = F_{||} = -F_{||}^e = -\frac{\alpha_0 m v_{ei} J_{||}}{e} + \beta_0 n \frac{B_\theta}{B} \frac{\partial T_e}{\partial\theta} |\nabla\theta|,$$

(12)

can be used to eliminate the poloidal gradients. Choosing this alternative, we find two contributions to the radial flux

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

Fig. 1. Force balance which describes neoclassical radial particle flows. (a) Overall geometry; looking down through the indicated port, one views from above the tangent plane shown in Fig. 1b. (b) Force balance in a plane tangent to the magnetic flux surface (solid vectors). The Lorentz force $enu_{\perp}B/c$ is balanced by the projection (long dashed vector) of the poloidal force, $V_{\theta}(nT) + enV_{\theta}\phi$. For subsonic flows, the poloidal force can in turn be related to the dominant Coulomb frictional force, $F_{||} = (B_{\theta}/B)[V_{\theta}(nT) + enV_{\theta}\phi]$. (For near-sonic flows, additional inertial and other contributions to $F_{||}$ make the radial flux different for ions and electrons, cf. Sec. 3.4.)

$$\Gamma_{PS} = \frac{1}{mQ_p} \frac{B_c}{B} \frac{\alpha_0 m v_{ei} J_{||}}{e}, \quad (13)$$

and

$$\Gamma_F = \frac{1}{mQ_p} \frac{B_c}{B} n \nabla_{||} T_e, \quad (14)$$

where $\alpha_0 = 0.5129$, $\beta_0 = 0.711$, $\nabla_{||} = (B_{\theta}/B)\partial/\partial\theta$, and $J_{||}$ is the current parallel to the magnetic field.

The Scrape-off Thickness as a Function of Coulomb Collisionality

The usual Pfirsch-Schlüter (PS) term is significant only at very high collisionality, $\{\Delta^{-1} \gg (m/m_e)^{1/2}\}$. However, the

V_{Te} term is independent of collisionality as pointed out by Feneberg.⁹ This so-called Meinst is intimately related to the poloidal rotation, as is evident from Eqs. (9-12). The net effect of these two terms is to map particles onto the material boundary not from a magnetic flux surface, but rather from a surface shifted downwards (for the configuration shown in Fig. 2). There may therefore be a "superplateau regime" where the scrape-off flows are dominated by forces independent of the collisionality. This superplateau regime would lie in the collisionality regime $1 \ll \Delta^{-1} \lesssim (m/m_e)^{1/2}$. At lower collisionality, the thermal friction is no longer independent of collisionality. At very high collisionality, the usual Pfirsch-Schluter flux produces a thicker scrapeoff. In the putative superplateau regime, the poloidal rotation adjusts to be competitive with the radial flux due to thermal friction. The actual scrape-off thickness then depends on the other poloidal and radial flows. Flow along the magnetic field will tend to cancel part of the poloidal rotation. In addition, there are radial Pfirsch-Schluter flows, which always go towards increasing major radius. At very high collisionality, the Pfirsch-Schluter flux overwhelms the other radial fluxes, leading to a different flow pattern. In the superplateau regime, the poloidal gyroradius is a consistent, but not unique possibility for the scrape-off thickness. Only in the subsonic supercollisional regime is the scrape-off thickness known for certain to be larger than the poloidal gyroradius.

Fig. 2. Poloidal streamlines which would result from poloidal rotation, $\Gamma_{rot} = (B_\theta/B)nu_d$, and from thermal friction force, Γ_F if these fluxes were not perturbed by parallel flows and by other radial fluxes (dashed line). These unperturbed streamlines are shifted downwards from the magnetic flux surfaces by about a poloidal gyroradius. The axis of symmetry is to the left and the toroidal magnetic field comes out of the page. (84P0215)

Breakdown of Local Ambipolarity

In the subsonic flow regime upstream from an intense recycling region, the radial ion and electron flows are equal. This local ambipolarity results from the fact that the friction force is equal and opposite for electrons and ions, as is the Lorentz force. (Thus the same velocity $\vec{U}$ balances the Lorentz force against the poloidal forces held off by parallel friction.) For sonic flows, however, the larger ion mass leads to Coriolis and viscous forces which act almost exclusively on the ions. This complicates the flow pattern and destroys the ambipolarity of the radial particle flows. The poloidal flow of the resulting radial electric current can be returned through the plasma and/or an electrically conducting limiter.

Terminology for Radial Flows

According to the nomenclature used in the authoritative review by Hinton and Hazeltine,¹⁰ all of the above-mentioned radial particle fluxes are neoclassical, in the sense that they are absent in a rectilinear magnetic field geometry. There are also classical radial particle fluxes resulting from resistance to the diamagnetic current, $J_{\perp} = (c/B) |\nabla\psi| \partial[n(T_e + T_i)]/\partial\psi$. However, the classical radial particle flux is small compared to the neoclassical fluxes.

PARALLEL FLOWS

When transport across the magnetic field is negligible, the plasma fluid equations become much simpler. One way this can happen is that reionization of charge-exchange neutrals may dominate cross-field energy transport. This may occur in some high-recycling divertor geometries. An example of such broad scrape-off thickness is given in the D-III probe data given elsewhere by D. Post.¹³ Alternatively, anomalous transport upstream from a collision-dominated region can produce a broad scrape-off plasma. This may occur near the tokamak midplane in divertor experiments. If the midplane turbulence damps away as the plasma approaches a collision-dominated divertor throat, then 1-D fluid equations can be used to integrate the flows through the divertor throat. Under these circumstances, it is traditional to use the distance x along the magnetic field as a coordinate. The fluid equations then become

$$B \frac{d(nu_{\parallel}/B)}{dx} = S, \quad (15)$$

$$mnu_{\parallel} \frac{du_{\parallel}}{dx} = - \frac{dp_{\text{tot}}}{dx} + (X_{\parallel} - mu_{\parallel} S), \quad (16)$$

$$\frac{3}{2} nT_j + nT_j B \frac{d(u_{\parallel}/B)}{dx} = B \frac{d}{dx} \left(\frac{\kappa_j^j}{B} \frac{dT_j}{dx} \right) + W_j, \quad (17)$$

for j = ions or electrons, where the source terms S , X_j , and W_j can be gleaned from the Appendix. In numerical simulations, the small variation of field strength B is sometimes neglected. Only the sum of the ion and electron momentum balances is needed, so only the total pressure $P_{\text{tot}} = n(T_e + T_i)$ enters in the momentum balance. This is adequate if no significant current flows through the region, an assumption which is valid for 1-D flows unless a potential difference $e\Delta\phi \gg T_e$ is deliberately applied across the region of interest. The ion thermal conductivity $\kappa_{\parallel} \ll \kappa_{\parallel}^e$ is formally small in the fluid regime, as is the bulk viscosity η_0 , but these effects are sometimes nevertheless included in numerical simulations.

Since the solution of these equations is discussed elsewhere,¹³ comments here are restricted to collecting the above-mentioned constraints on their applicability. These are summarized in Table 1.

Table 1. Restrictions on 1-D Fluid Equations.

REQUIREMENT	FORMULA	WHERE VIOLATED
Collision-dominated	$\Delta \ll \lambda$	Near plate; usually near midplane
Broad scrapeoff	$\delta_p \ll 1$	Usually marginal
$\vec{B}$ determined externally	$e d \ln \beta_p / d \ln \phi < 1$	Steep gradients
Steady State	$\partial \ln f / \partial t \ll v_{th} / L_{\parallel}$	When fluctuations are observable
Axisymmetry	$\partial / \partial \xi = 0$	Nonaxisymmetric limiters
No excess biasing	$e\phi_{\text{applied}} \lesssim T_e$	RF heating?
Pure hydrogen	$Z_{\text{eff}}^{-1} \ll 1$	Many experiments

Given the many limitations listed in Table 1, one might wonder why 1-D plasma transport calculations have been so successful in helping to explain the difference between low- and high-recycling tokamak divertors. The basic reason is that a qualitative treatment requires primarily a careful analysis of neutral gas transport and is relatively insensitive to the details of plasma transport. A quantitative comparison with detailed experimental observations will require more careful

analysis of several problems, however. The most difficult problems are a phenomenological treatment of turbulence and inclusion of impurities with appropriate boundary conditions.

TURBULENCE, IMPURITIES, AND BOUNDARY CONDITIONS

Turbulence

There are a number of microinstabilities which occur when the collisionality is insufficient to suppress them. In some cases, it may be possible to obtain stationary conservation equations for the turbulence-averaged properties of the plasma. It is therefore worth examining possible types of turbulence for hints as to how the fluid equations could be modified.

The simplest instability occurs when radial gradients are relatively flat but the parallel collisionality approaches unity, $\Delta \sim 1$. Then the electron distribution is highly distorted along field lines. Energetic electrons can then escape from hot regions unimpeded by collisions. This sets up a positive potential which draws back a current of cold electrons. Ion cyclotron waves resonant with the cold electron current can be driven unstable. These waves propagate obliquely across the magnetic field and damp on the ions.¹⁰ The energy they extract from the parallel electron flow should therefore result in anomalous ion heating.

Even at somewhat higher collisionality (up to $\Delta^{-1} \gtrsim (m/m_e)^{1/2}$), the known neoclassical effects may lead to a scrape-off thickness comparable to the poloidal gyroradius, $\rho_p = \rho B/B_0$ (where $\rho = v_{th}/Q$ is the ion Larmor radius). Loss of poloidally drifting ions may then generate loss-cone instabilities,¹¹ which mix ions from different magnetic flux surfaces.

However, both of these instabilities may be overshadowed by resonance of waves with poloidal phase velocity near the diamagnetic drift velocity, u_d . While such instabilities have been studied extensively for core-plasma conditions, strong poloidal and radial scrape-off gradients make most of these studies inapplicable. The only method available so far for studying the possible 2-d effects of such instabilities has been to extrapolate empirical enhancements of some cross-field transport coefficients from core-plasma transport studies. Since the 2-d fluid equations are much more complicated than their flux-surface-averaged counterparts, this can be dangerous. For example, enhancing the cross-field particle transport alone can force the ions in a two fluid model to do more work than is available in the stored ion energy. The resulting negative ion temperatures may be an indication that the ions have been told to flow into a region where the high electrostatic potential ought to exclude them. Unless Ohm's law is modified consistently with the cross-field diffusion, such

paradoxes will occasionally arise. One way to avoid such problems is to modify the underlying expressions for the electron-ion, ion-ion, and/or electron-electron collisionality. This will avoid such inconsistency problems, and it may give some insight into the action of different kinds of turbulence. However, the actual instabilities are localized and anisotropic in velocity space, so enhancement of the collision frequencies is only a first crude step towards a useful qualitative picture of turbulence effects. In the equations in the Appendix, the various collisionalities are nevertheless kept separate to facilitate this approach.

Impurities

For a multispecies plasma, the neoclassical radial transport of the various ions is driven by the same type of forces which determine hydrogen transport. However, the Coulomb friction between various ions is much stronger than between ions and electrons. Small differences between the parallel flow velocities of the various ions can therefore contribute strongly to radial transport.

For a broad scrapeoff, the 1-d transport of impurities along the magnetic field is trivial in a collisional plasma: collisions force the impurities to flow with the bulk plasma flow. In the sonic flow regime, other effects such as thermal friction only become significant when the collisionality is of order one and the fluid approximation breaks down. Numerical fluid studies including such terms, discussed elsewhere by K. Lackner,¹³ give quantitative insight to the extent the results are independent of the detailed particle kinetics, or the computation is confined to an intense recycling regime with subsonic flow throughout.

The lowest order approximation of equal parallel ion flow velocities is unfortunately inadequate for determining the cross-field transport. In principle, there is no difficulty in numerically solving directly for the difference between the various flow velocities in the fluid regime. The major difficulty lies in determining the boundary condition on the relative velocities.

Boundary Conditions

A scrape-off fluid is always bounded by a region near the material boundary, where fluid theory breaks down. We must therefore turn to kinetic solutions of the Fokker-Planck equation for boundary conditions on the fluid equations. There is at present no general solution relevant to scrape-off problems which includes a correct quantitative treatment of the transition from a realistic sheath plasma to a collision-dominated fluid. However, there are a number of treatments of this interface which contain qualitatively reasonable

assumptions about the sheath and fluid regions.* Our understanding of the qualitative behavior of hydrogen plasma flows is therefore not necessarily compromised by uncertainties about the boundary conditions.

The same cannot be said about impurity flows. At present, there is to our knowledge not even a qualitative theory which adequately predicts the relative velocities of impurities in the kinetic regime. Since this is essential for studying radial impurity transport, a careful study of the multispecies kinetic theory is a high priority.

CONCLUSIONS

Considerable progress has been made in the past few years in understanding the physics of axisymmetric plasma flows near material boundaries. Despite the above-mentioned limitations of the present fluid theory of these flows, at least four significant results have emerged. First, 1-d analytic and computational work has pointed out the importance of localized intense recycling near the material boundary as described elsewhere by D. Post.¹³ Second, up-down asymmetry in the plasma flow can be related to poloidal rotation and up-down asymmetry in the radial transport. Third, the poloidal gyroradius is identified as a possible minimum radial thickness of a collisional scrape-off plasma. Fourth, the very high collisionalities envisioned for advanced divertor plasmas such as ASDEX-Upgrade and INTOR should have a larger neoclassical scrape-off thickness due to the radial Pfirsch-Schluter fluxes. For such cases, a complete fluid treatment is relatively straightforward. Since the very high collisionality in such plasmas should help prevent kinetic microturbulence, detailed numerical solution of the 2-d neoclassical transport equations for such devices may provide considerable insight. It should be possible to discover which transport equilibria lead to acceptable rates of plasma pumping and material boundary erosion. It should also be possible to perform accurate fluid and kinetic stability analyses on the computational equilibria to investigate limits on the achievable boundary gradients of pressure, current, and resistivity.

*Truly quantitative treatments of the sheath boundary conditions may soon be forthcoming. To be computationally feasible, this will require matching a kinetic computation near the material boundary to a fluid computation away from the boundary. The fluid calculation will have to have transport coefficients accurate to at least second order in the small parameter $\Delta^{-1} = L/L_{mf}$ (e.g., parallel electron heat conduction $q^e = -\kappa_1 \nabla T_e + \kappa_2 \nabla(V T_e) + \dots$). (From the point of view of numerical efficiency alone, third or fourth order accuracy in Δ^{-1} is optimal, but this is not likely to be practical.)

ACKNOWLEDGMENTS

13

The author is grateful to A. Boozer, W. Feneberg, K. Lackner, J. Neuhauser, and D. Post for useful discussions and to B. Braams for pointing out the problem of negative ion temperatures in 2-d transport simulations.

This work was supported by the U.S. Department of Energy Contract No. DE-AC02-76-CHO-3073.

APPENDIX

Here we outline a complete set of axisymmetric steady-state transport equations for a quasineutral, pure hydrogen plasma. The plasma is assumed to be collision dominated. The applied electrostatic potential is assumed to be of no greater order of magnitude than that arising from gradients in the plasma.

The transport equations are derived from the lowest four velocity moments of the Fokker-Planck equation. The zeroth velocity moments yield the electric current and ion conservation laws

$$\nabla \cdot \vec{J} = 0, \quad (A1)$$

$$\nabla \cdot \vec{\Gamma} = S, \quad (A2)$$

where $\nabla \cdot \vec{A} = g^{-1/2} \{ \partial(A_\psi |\nabla \psi| g^{1/2}) + \partial(A_\theta |\nabla \theta| g^{1/2}) / \partial \theta \}$, $g^{1/2} = 1/|\nabla \psi \times \nabla \theta \cdot \nabla \xi|$, and (ψ, θ, ξ) are right-handed orthogonal toroidal coordinates with toroidal symmetry angle ξ . The poloidal current and the poloidal flux $\Gamma_\theta = n u_\theta$ are decomposed into components perpendicular and parallel to the magnetic field, $\vec{B}$, to obtain $J_\theta = (B_\xi/B) J_d + (B_\theta/B) J_\parallel$ and $\Gamma_\theta = (B_\xi/B) \Gamma_d + (B_\theta/B) \Gamma_\parallel$.

The dominant terms in the radial total and ion momentum balances give the diamagnetic flows:

$$J_d = (c/B) \nabla_\psi [n(T + T_e)], \quad (A3)$$

$$\Gamma_d = \frac{1}{m\Omega} [\nabla_\psi (nT) + en \nabla_\psi \phi], \quad (A4)$$

where $n = n_e$ is the ion density, T the ion temperature, T_e the electron temperature, m the ion mass, $\Omega = eB/(mc)$ the ion gyroradius, e the ion charge, ϕ the electrostatic potential, and $\nabla_\psi = |\nabla \psi| \partial/\partial \psi$. The total and ion toroidal momentum balances give the radial current and radial ion flow

$$J_{\phi} = \frac{e}{mQ_p} F_{\text{Mach}} \quad (\text{A5})$$

$$\text{where } \Gamma_{\phi} = \frac{J_{\phi}}{e} - \frac{1}{mQ_p} \frac{B_z}{B} F_{\parallel} \quad (\text{A6})$$

$$F_{\parallel} = -\alpha_0 m v_{ei} J_{\parallel} / e \quad (\text{A7})$$

is the Coulomb friction on the ions parallel to the magnetic field, $\alpha_0 = 0.5129$, $\beta_0 = 0.711$, m_e is the electron mass, $v_{ei} = 4(2\pi)^{1/2} m_e^{1/2} e^4 \lambda_{ei} / (3m T_e^{3/2})$ is the electron-ion collision frequency, λ_{ei} is the Coulomb logarithm, $\nabla_{\parallel} = (B_{\theta}/B) \partial/\partial\theta$ and $Q_p = (B_{\theta}/B)Q$ is the ion poloidal gyrofrequency. We have neglected the small classical contribution to Γ_{ϕ} resulting from resistance to the diamagnetic current. For nearly sonic flows, there can be several forces producing radial transport which act primarily on the ions

$$F_{\text{Mach}} = F_{\text{Cor}} + F_{\text{Vis}} + F_{\text{CX}} \quad (\text{A9})$$

(We call the sum of these forces " F_{Mach} " since they are only significant when the parallel Mach number is of order 1.) The Coriolis force is

$$F_{\text{Cor}} = -(\mathbf{m} \Gamma_{\phi} \nabla_{\phi} u_{\xi} + \mathbf{m} \Gamma_{\theta} \nabla_{\theta} u_{\xi}), \quad (\text{A10})$$

where u_{ξ} can be replaced by $u_{\parallel}$, and an accurate solution is retained. The classical viscous force can be adequately approximated by

$$F_{\text{Vis}} = \frac{B_z}{B} [\nabla_{\phi} (-\eta_2 \nabla_{\phi} u_{\parallel} + \eta_4 \nabla_{\theta} u_{\parallel}) - \nabla_{\theta} (\eta_4 \nabla_{\phi} u_{\parallel})], \quad (\text{A11})$$

where $\eta_4 = nT/Q$ and $\eta_2 = 6v_{ii}\eta_4/(5Q)$ are viscosity coefficients, $v_{ii} = (4\pi)^{1/2} ne^4 \lambda_{ii} / (3m^{1/2} T^{3/2})$ is the ion-ion collision time, and $\nabla_{\theta} = |\nabla\theta| \partial/\partial\theta$. The toroidal charge-exchange friction

$$F_{\text{CX}} = x_{\xi} - m u_{\xi}^2 \quad (\text{A12})$$

is obtained by integrating the non-Coulomb collision operator over a Maxwellian approximation to the distribution functions

$$X_E = \int d^3v \, m v_E \Sigma(f_O, f_O^e), \quad (A13)$$

and subtracting out the momentum dilution term, $mu_E S$.

The total and electron momentum balances parallel to the magnetic field help determine the parallel ion flux and the electrostatic potential. An adequate approximation of the total parallel momentum balance is

$$\frac{B_E}{B} \nabla_{\parallel} [n(T + T_e)] = F_{Mach}.$$

The parallel electron momentum balance (Ohm's law) can be approximated by

$$en \nabla_{\parallel} \phi = -F_{\parallel} - \nabla_{\parallel} (n T_e). \quad (A15)$$

Finally, the ion and electron heat balances are

$$\begin{aligned} \frac{3}{2} (\Gamma_{\psi} \nabla_{\psi} T + \Gamma_{\theta} \nabla_{\theta} T) + n T \nabla \cdot \vec{u} &= -Q_{\Delta} + W_{vis} \\ + (Y \cdot \vec{u} \times \vec{x} - \frac{3}{2} T S + \frac{m}{2} u^2 S) - \nabla \cdot \vec{q}, \end{aligned} \quad (A16)$$

$$\begin{aligned} \frac{3}{2} (\Gamma_{\psi}^e \nabla_{\psi} T_e + \Gamma_{\theta}^e \nabla_{\theta} T_e) + n T_e \nabla \cdot \vec{u}_e &= Q_{\Delta} - F_{\parallel} J_{\parallel} / (en) \\ + (Y^e \cdot \vec{u}_e - \frac{3}{2} T_e S) - \nabla \cdot \vec{q}^e. \end{aligned} \quad (A17)$$

Here $Q_{\Delta} = 3n v_{ei} (T - T_e)$ is the Coulomb interchange ion heating, $Y = \int d^3v \, (1/2) m v^2 \Sigma(f_O, f_O^e)$ is the non-Coulomb ion energy source, $\vec{u}^e = n \vec{u}^e = \vec{I} - \vec{J}/e$ is the electron flux. The poloidal components of the heat conduction fluxes, $\vec{q}$ and $\vec{q}^e$ are decomposed into parallel and diamagnetic components, e.g.,

$$q_{\theta}^e = (B_{\psi}/B) q_d^e + (B_{\theta}/B) q_{\parallel}^e. \quad (A18)$$

The classical diamagnetic heat fluxes

$$q_d^e = -\frac{5}{2} \frac{n T_e}{m \Omega} \nabla_{\psi} T_e, \quad (A19)$$

$$q_d = \frac{5}{2} \frac{nT}{m\Omega} \nabla_{\psi} T, \quad (A20)$$

cancel each other for equal ion and electron temperatures. The classical parallel electron heat flux is

$$q_{\parallel}^e = -(\kappa_{\parallel}^e \nabla_{\parallel} T_e + \beta_0 T J_{\parallel} / e), \quad (A21)$$

where $\kappa_{\parallel}^e = 3.1616 nT / (2^{1/2} m_e v_{ee})$, $v_{ee} = 4\pi^{1/2} n e^4 / (3m_e^{1/2} n_e^{3/2})$, and $\beta_0 = 0.711$. The parallel ion heat conduction is formally small in the fluid regime. The radial electron heat flux is dominated by the neoclassical contribution

$$q_{\psi}^e = -\frac{B_{\theta}}{B} \frac{5nT_e}{2m\Omega_p} \nabla_{\parallel} T_e. \quad (A22)$$

Due to the larger ion mass, the radial ion heat flux, q_{ψ} , is considerably more complicated. An exact expression for q_{ψ} is obtained from the diamagnetic component of the $(1/2, mv^2)$ moment of the Fokker-Planck equation

$$\begin{aligned} q_{\psi} = & -\frac{1}{\Omega_p} \left\{ \frac{B_{\theta}}{B} \left[(\nabla_{\parallel} \tilde{R} - \frac{5T}{2m} \nabla_{\parallel} \tilde{P})_d + \frac{e}{m} (\nabla_{\psi} \tilde{\Pi})_d - enu_d \tilde{u} \cdot \nabla \phi \right] \right. \\ & + \frac{B_{\theta}}{B} \frac{enu^2}{2} \nabla_{\parallel} \phi + \frac{B_{\theta}}{B} [(G_d + Z_d) - \frac{5T}{2m} (F_d + X_d)] \} \\ & - (u \cdot \tilde{\Pi})_{\psi} - \frac{m}{2} nu^2 u_{\psi}. \end{aligned} \quad (A23)$$

For very subsonic flows, this reduces to $q_{\psi} = (B_{\theta}/B) [(5nT)/(2m\Omega_p)] \nabla_{\parallel} T + q_{\psi}^{\text{classical}}$ where

$$q_{\psi}^{\text{classical}} = -\frac{G_d}{\Omega} = \frac{2nT}{mv_{ii}} \left(\frac{v_{ii}}{\Omega} \right)^2 \nabla_{\psi} T. \quad (A24)$$

For near sonic flows, one can straightforwardly obtain an expression valid for q_{ψ} at very high collisionality [$\Delta^{-1} \gg (m/m_e)^{1/2}$] by using the method of Braginskii (cf. Wong¹²) to obtain the correction Φ to the distribution function $f = f_0(1 + \Phi)$. One then determines the heat flux using the definitions $\tilde{R} = \int d^3v (mv^2/2) \tilde{v} \tilde{f}$, $\tilde{P} = \int d^3v m \tilde{v} \tilde{f}$. (Usually, the diamagnetic contribution from $\tilde{Z} = \int d^3v (mv^2/2) \tilde{v} \tilde{\Sigma}$ should be negligible.) The viscous heating $Q_{vis} = (\nabla \tilde{u} \cdot \tilde{\Pi}_{vis})$ is also potentially important for near-sonic flows and can be similarly computed. While this computation of q_{ψ} is straightforward, we have not carried it

through because the results are valid only at very high collisionality. Note that in this "super-Pfirsch-Schluter" regime, the electron and ion temperatures are very nearly equal, and the diamagnetic and neoclassical electron heat fluxes are cancelled by similar ion heat flux terms. It is therefore highly desirable to add ion and electron heat balance equations to obtain a simpler total heat balance.

For lower collisionality, where one cannot generally exclude the condition $\delta_p \sim 1$, drift effects must be included in computing the transport coefficients. After this is done, one could include one of the ion or electron heat balances along with the total heat balance to obtain a complete set of ten differential plasma transport equations. It might then be computationally convenient to add the total mechanical energy to the total heat balance to obtain the total energy transport equation

$$\begin{aligned} \nabla \cdot \left(\frac{1}{2} \mu u^2 \vec{u} + \frac{5}{2} n T_i \vec{u} + \frac{5}{2} n T_e \vec{u}_e + \vec{u} : \vec{\Pi}_{vis} + \vec{q}_g + \vec{q}_g^* + \vec{q}_\psi + \vec{q}_\psi^* \right) \\ = -F_{\parallel} J_{\parallel} / (en) + \vec{u} \cdot \vec{F}_{Coul} + Y + Y^* \end{aligned} \quad (A25)$$

Use of this "conservative" form has some computational advantages for determining the total energy density. Due to the large mechanical and other energy interchanges between the ions and electrons, however, such conservative forms are unlikely to be useful for separately determining the electron and ion temperatures. Rather, one should combine Eq. (A25) with either Eq. (A16) or Eq. (A17) to determine the difference between the ion and electron temperatures.

REFERENCES

- ¹D. Sharkovsky, T. W. Johnston, and M. P. Bachyuski, The Particle Kinetics of Plasmas (Addison-Wesley, London, 1966).
- ²D. L. Book, NRL Plasma Formulary, Naval Research Laboratory (Washington, DC, 1983).
- ³S. I. Braginskii, ZHETF **33**, 479 (1957); Sov. Phys. JETP **6**, 358 (1958).
- ⁴S. I. Braginskii, in Reviews of Plasma Physics, edited by M. A. Leontovich (Consultants Bureau, NY, 1965).
- ⁵C. E. Singer and W. Langer, Phys. Rev. **A28**, 994 (1983).
- ⁶J. Neuhauser, Max Planck Institut für Plasmaphysik, private communication (1984).
- ⁷U. Dayhelge, Nucl. Fusion **21**, 1589 (1981).
- ⁸G. Fussmann et al., "Impurity Retainment in the Divertor of ASDEX," in Proceedings of the Eleventh European Conference on Controlled Fusion and Plasma Physics (Aachen, Germany, 1983) European Physical Society (1983), Vol. 7D, Part II, p.

- ⁹W. Feneberg, JET Joint Undertaking, Abingdon, Oxfordshire, England, private communication (Feb. 1984).
- ¹⁰C. E. Singer, J. Geophys. Res. 82, 2686 (1977).
- ¹¹F. H. Hinton and R. D. Hazeltine, Rev. Mod. Phys. 48, 239 (1976).
- ¹²S. K. Wong, Matrix Elements of the linearized Fokker-Planck Operator, GA Technologies, Inc. Report No. GA-A1749 (1984), submitted to Phys. Fluids.
- ¹³NATO Advanced Study Institute on the Physics of Plasma-Wall Interactions in Controlled Fusion, Val-Morin, Quebec, Canada, August 1984, eds. D. Post and R. Behrisch (Plenum Press, NY).

EXTERNAL DISTRIBUTION IN ADDITION TO UC-20

Plasma Res Lab, Austro Nat'l Univ, AUSTRALIA
 Dr. Frank J. Paoloni, Univ of Wollongong, AUSTRALIA
 Prof. I.R. Jones, Flinders Univ., AUSTRALIA
 Prof. M.H. Brennan, Univ Sydney, AUSTRALIA
 Prof. F. Cap, Inst Theo Phys, AUSTRIA
 Prof. Frank Verheest, Inst theoretische, BELGIUM
 Dr. D. Palumbo, Dg XII Fusion Prog, BELGIUM
 Ecole Royale Militaire, Lab de Phys Plasmas, BELGIUM
 Dr. P.H. Sakanaka, Univ Estadual, BRAZIL
 Dr. C.R. James, Univ of Alberta, CANADA
 Prof. J. Teichmann, Univ of Montreal, CANADA
 Dr. H.M. Skarsgard, Univ of Saskatchewan, CANADA
 Prof. S.R. Sreenivasan, University of Calgary, CANADA
 Prof. Tudor W. Johnston, INRS-Energie, CANADA
 Dr. Hannes Barnard, Univ British Columbia, CANADA
 Dr. M.P. Sachynski, MEB Technologies, Inc., CANADA
 Chalk River, Nucl Lab, CANADA
 Zhengwu Li, SW Inst Physics, CHINA
 Library, Tsing Hua University, CHINA
 Librarian, Institute of Physics, CHINA
 Inst Plasma Phys, Academia Sinica, CHINA
 Dr. Peter Lukac, Komenského Univ, CZECHOSLOVAKIA
 The Librarian, Culham Laboratory, ENGLAND
 Prof. Schatzman, Observatoire de Nice, FRANCE
 J. Radet, CEN-EP6, FRANCE
 AM Dupas Library, AM Dupas Library, FRANCE
 Dr. Tom Mual, Academy Bibliographic, HONG KONG
 Preprint Library, Cent Res Inst Phys, HUNGARY
 Dr. S.K. Trehan, Panjab University, INDIA
 Dr. Indra Mohan Lal Das, Banaras Hindu Univ, INDIA
 Dr. L.K. Chauda, South Gujarat Univ, INDIA
 Dr. R.K. Chhajlani, Vikram Univ, INDIA
 Dr. B. Dasgupta, Saha Inst, INDIA
 Dr. P. Kaw, Physical Research Lab, INDIA
 Dr. Phillip Rosenau, Israel Inst Tech, ISRAEL
 Prof. S. Cupeiman, Tel Aviv University, ISRAEL
 Prof. G. Rostagni, Univ Di Padova, ITALY
 Librarian, Int'l Ctr Theo Phys, ITALY
 Miss Clelia De Palo, Assoc EURATOM-ENEA, ITALY
 Biblioteca, del CNR EURATOM, ITALY
 Dr. H. Yamato, Toshiba Res & Dev, JAPAN
 Ginec. Dept. Lg, Tokamak Dev. JAERI, JAPAN
 Prof. Nobuyuki Inoue, University of Tokyo, JAPAN
 Research Info Center, Nagoya University, JAPAN
 Prof. Kyoji Nishikawa, Univ of Hiroshima, JAPAN
 Prof. Sigeru Mori, JAERI, JAPAN
 Library, Kyoto University, JAPAN
 Prof. Ichiro Kawakami, Nihon Univ, JAPAN
 Prof. Satoshi Itoh, Kyushu University, JAPAN
 Dr. D.I. Choi, Adv. Inst Sci & Tech, KOREA
 Tech Info Division, KAERI, KOREA
 Bibliothek, Rijn-Inst Voor Plasma, NETHERLANDS
 Prof. B.S. Liley, University of Waikato, NEW ZEALAND
 Prof. J.A.C. Cabral, Inst Superior Tecn, PORTUGAL
 Dr. Octavian Petrus, ALI OIZA University, ROMANIA
 Prof. M.A. Hellberg, University of Natal, SO AFRICA
 Dr. Johan de Villiers, Plasma Physics, Nucor, SO AFRICA
 Fusion Div. Library, JEN, SPAIN
 Prof. Hans Wilhelmson, Chalmers Univ Tech, SWEDEN
 Dr. Lennart Stenflo, University of UMEA, SWEDEN
 Library, Royal Inst Tech, SWEDEN
 Centre de Recherches, Ecole Polytech Fed, SWITZERLAND
 Dr. V.T. Toldk, Kharkov Phys Tech Ins, USSR
 Dr. D.D. Ryutov, Siberian Acad Sci, USSR
 Dr. G.A. Eliseev, Kurchatov Institute, USSR
 Dr. V.A. Glukhikh, Inst Electro-Physical, USSR
 Institute Gen. Physics, USSR
 Prof. T.J.M. Boyd, Univ College N Wales, WALES
 Dr. K. Schindler, Ruhr Universitat, W. GERMANY
 Nuclear Res Estab, Julich Ltd, W. GERMANY
 Librarian, Max-Planck Institut, W. GERMANY
 Bibliothek, Inst Plasmaforschung, W. GERMANY
 Prof. R.K. Janev, Inst Phys, YUGOSLAVIA