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ENERGY FROM HUMID AIR

Final Report

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SOUTH DAKOTA SCHOOL of MINES & TECHNOLOGY

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ABSTRACT

This is a report of results to date of a research project which is in progress at the South Dakota School of Mines and Technology. The goal of the research is to find a cost-effective process to convert the energy in humid air into mechanical work, which will be used to drive an electrical generator. The research is being carried out by computer modeling.

Results for a natural draft tower show that it is not a cost-effective way to get energy from humid air. Parametric studies are presented for expansion-compression cycles. With suitable conditions, including large amounts of cooling during compression, this cycle has an attractive net work output. To avoid using all the output power to overcome machine losses, it appears necessary to use a one-machine mechanization. The most promising uses vortex flow to achieve the necessary expansion and subsequent compression with cooling. Power output and costs have been estimated for a vortex plant located in Puerto Rico.

EXECUTIVE SUMMARY

This is a report of results of a research project at the South Dakota School of Mines and Technology. The goal of the research is to find a cost-effective process to convert the energy in humid air into mechanical work, which will be used to drive an electrical generator.

A vast amount of energy is contained in the latent heat of vaporization of the water vapor in humid air. If the water vapor is made to condense, the latent heat is released into the surrounding air. The mechanizations for converting the energy in humid air which are being considered in this project can be thought of as heat engines in which the air itself serves as the working fluid. The research has been carried out primarily by computer modeling.

The attractiveness of the concept of extracting energy from humid air has been published¹ and will not be developed in this report.

Natural Draft Tower

The simplest mechanization to extract energy from humid air is a vertical natural-draft, condensation tower (see Figure below).

Natural Draft Tower.

The efficiency of such a tower is dependent upon (1) the degree of saturation of the ambient air at the inlet; (2) tower height; and (3) the prevailing atmospheric temperature lapse rate. As the air rises in the tower, its temperature and pressure follow the moist adiabat. The air inside the tower is then at a higher temperature than outside ambient air, and it rises because of natural buoyancy. The process is similar to that which nature uses in a thunderstorm.

Computer models have been developed for the natural draft tower, and extensive studies have been made using the computer. The power output naturally increases proportional to mass flow rate of air through the tower, expressed in kilograms of air per second. Hence various configurations are analyzed in terms of power output per kg./sec. of air processed.

Several shapes for a natural draft tower were studied. Power output was found to be essentially independent of tower shape. Power output as a function of tower height has been examined. It was discovered that power output is approximately proportional to the 1.4 power of tower height or:

$$\text{Power out} = K (\text{Tower Height})^{1.4}$$

Hence, to get a large power output there is a strong incentive to build a very high tower.

The natural draft tower was tested for power output at each of 5 geographical locations; (San Juan, P.R.; Tampa, Fla.; Boothville, La.; Lake Charles, La.; and Hilo, Ha.) for each day of the year 1975 and at two times each day. These correspond to the times when upper air meteorological observations were recorded. At the most favorable location, Hilo, Ha., the overall average power output from a tower averaging 586 meters in height was found to be 52 watts per kg./sec. of air processed. This falls far short of a goal we have set of 1620 watts per kg./sec. (see page 15)

Our conclusion based on all of the computer results is that the natural draft tower does not appear to be a cost-effective way to get useful energy from humid air. However, the natural draft tower was a logical first step in our computer modelling process.

Expansion-Compression Mechanization

An expansion-compression mechanization was then modelled on the computer. With this cycle, humid air first expands adiabatically into a low pressure region and work is extracted from this process. During the expansion moisture condenses and falls out at its dewpoint and is removed from the system. If the air plus the remaining water vapor is compressed with enough cooling, the compression work put in is less than the expansion work extracted, and positive work output is theoretically possible from the cycle.

Expansion-Compression Tower.

Parametric studies have been made of the expansion-compression cycle looking at:

- (1) Power output vs. cooling temperature
- (2) Power output vs. degree of expansion
- (3) Power output vs. degree of saturation
- (4) Power output vs. tower height

As might be expected, power output increases as cooling temperature becomes lower. Power output increases with degree of expansion; however, it is not profitable to expand to a temperature that is more than about 10°C lower than the available cooling temperature. Power output increases with degree of saturation. The output decreases slightly with tower heights; hence, a structure with negligible vertical height appears best for the expansion-compression cycle. This has the effect of tending to hold down structural costs.

Related Investigations

Investigations which relate to the mechanizations have been made into (1) a solar saturator, which would supply saturated humid air at the inlet to a natural draft tower (2) means of obtaining cooling for an expansion-compression mechanization and (3) prevailing atmospheric lapse rates at various locations.

Absorption Cycle

During the course of the research, a new possible way to extract the energy from humid air has occurred to us. This is to use an absorbing solution, such as calcium chloride or lithium bromide to remove the moisture from the air. A preliminary investigation has been made, and a very attractive-looking cycle has been developed. This process merits further detailed investigation.

Practical Mechanization of Expansion-Compression Cycle

An expansion-compression system has several advantages as compared to a natural draft tower:

- (1) The power output in terms of watts per kilogram /second of air processed is much greater.
- (2) A solar saturator is not required.
- (3) As compared to a natural draft tower of reasonable height, a much larger fraction of the moisture in the air can be condensed.

- (4) The expansion-compression mechanization is not dependent for its effectiveness on tower height. A structure with negligible height appears best. This holds down structural costs. It also makes performance independent of prevailing atmospheric lapse rates.

An obvious disadvantage of the expansion-compression mechanization relates to machine efficiencies. If two independent machines are required for the turbine which extracts power and the compressor which absorbs power, then each machine must have a very high efficiency in order to have any net power output.

However, it is possible to devise an expansion-compression system which involves only one machine. The one machine, in effect, processes only the net power out of the cycle; hence, machine efficiency is less critical. A system involving vortex flow appears promising. Expansion is carried out by letting humid air spiral inward to the center of a vortex, where there is a low pressure area. The air then passes into a separate vortex chamber which is used for recompression with cooling. The air spirals outward to the periphery of the vortex and is recompressed to atmospheric pressure in the process. Cooling is introduced in the compression chamber by a fine spray of cooling water; hence, no large and expensive heat exchanger surface is necessary.

There is an important question which has not yet been answered, which is "How large are the drag losses within such a vortex?" Additional research to answer this question has been proposed.

Performance and Cost Estimates

Ignoring drag losses within the vortex itself, performance and cost estimates were made for a plant located at Punta Tuna, P.R. At this location for cooling purposes it is possible to get ocean water at a temperature of 6°C about 1.5 miles offshore. Net power output was obtained over the year 1975 using surface weather data to provide temperature and relative humidity of the air at the inlet to the plant. Costs for the plant were estimated using standard references and estimation procedures. Plant cost is found to be about \$1500 per rated kilowatt. The cost of electrical energy produced is about 4 cents per kw. hr. The humid air plant will produce maximum output when the air is hot and humid, which is exactly when maximum electrical demand will occur for purposes of air conditioning. Hence, the humid air plant should get full capacity credit from an electric utility.

It must be noted that the preceding estimates are very preliminary ones, made for a process which has not yet been proved feasible, but which, on the other hand, has not been optimized. The estimates are made at this time only to justify further research.

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1. Natural Draft Tower

This mechanization assumes a vertical tower, preferably circular in cross section, similar to those used for cooling in large power plants. Ambient air entering the tower will first be saturated by a solar saturator.* As the air rises in the tower, its temperature and pressure drop, but at a slower rate than the outside atmosphere. This is so because of the latent heat evolved from the moisture condensed in the tower. The air inside the tower is, therefore, at a higher temperature than the outside ambient air, and it will rise due to natural buoyancy. This buoyancy force can be utilized to generate work either in the form of excess kinetic energy or as flow work. In the first case the released latent heat is transferred into kinetic energy resulting in higher velocity as the air moves up the tower. This kinetic energy can be extracted at the top of the tower by a wind turbine. In the second case the released latent heat causes lower density and results in higher pressure for the air inside the tower as compared to the air outside. This pressure differential at the top of the tower can be used to drive a low pressure turbine. In either case the result is a net power output.

The efficiency of such a tower will depend upon (1) the degree of saturation of the inlet air; (2) the tower height which controls the amount of condensation; (3) the drag or friction loss inside the tower; and (4) the prevailing atmospheric temperature lapse rate.

* See Section 3.3 on solar saturator.

1.1 Assumptions and Equations

Symbols

A	cross sectional area of tower
A_s	surface area of control volume
C_p	specific heat
D	hydraulic diameter
δF_f	wall friction force per unit mass
f	Fanning friction coefficient
g	acceleration of gravity
h	specific enthalpy
m	mass
$\dot{m}$	mass flow rate of moist air
$\dot{m}_a$	mass flow rate of dry air component
P	pressure of air mixture
r	mixing ratio (m_{wv}/m_a)
Re_z	local Reynolds number ($\frac{WZ}{\nu}$)
S	specific entropy
T	temperature of air mixture
W	velocity in flow direction
WP	wetted perimeter
Z	coordinate in the flow direction
ϵ	equivalent sand grain diameter
ρ	density of air mixture
ν	kinetic viscosity

Subscripts

a	properties of dry air component of air mixture
e	exit conditions
i	inlet conditions
lw	properties of liquid water component of air mixture
wv	properties of water vapor component of air mixture
2	conditions after expansion process

In order to obtain results for such a tower, a one-dimensional steady flow treatment of the problem is used. The basic assumptions used in the analysis are:

- 1) The flow is steady and one-dimensional and in the vertical direction.
- 2) All fluid properties are uniform over any cross section along the tower.
- 3) Moist air is considered to be a mixture of dry air and water vapor. The dry air and the water vapor each occupies the total volume of the mixture and acts as if the other were not present.
- 4) Properties for the dry air component are calculated from the ideal gas equation, while the international steam table is used to determine the properties for the water vapor.
- 5) The air mixture is assumed to be inviscid in the main stream. In the boundary layer the sole friction force acting on the fluid element is due to wall friction.
- 6) There is no external work or heat transfer across the tower walls ($\delta W = 0$, $\Delta Q = 0$).

Figure 1 illustrates the control surface surrounding an elementary control volume ($A dz$) of the flowing fluid. The corresponding continuity, energy, and momentum equations take the following forms.

Continuity equation

$$\dot{m} = \rho A W \quad (1a)$$

or for the dry air

$$\dot{m}_a = \rho_a A W \quad (1b)$$

Pseudo-adiabatic energy equation

$$m_a C_{p_a} \Delta T + m_a (1+r) \left[\Delta \left(\frac{W^2}{2} \right) + \Delta Z \right] + m_{wv} \Delta h_{wv} + m_{lw} \Delta h_{lw} = 0 \quad (2)$$

Fig. 1. Forces acting on a differential element for a steady one-dimensional adiabatic flow with wall friction.

Momentum equation

$$W \frac{dW}{dz} + \frac{1}{\rho} \frac{dP}{dz} + g + \delta F_f = 0 \quad (3)$$

The wall force δF_f may be expressed in terms of the hydraulic characteristic of the flow passage and the Fanning friction coefficient f , which is defined as

$$f = \frac{\text{shear stress at the wall}}{\frac{1}{2} \rho W^2} = \frac{\tau_w}{\frac{1}{2} \rho W^2} \quad (4)$$

The determination of this friction coefficient for the natural draft tower is explained in detail in Appendix A and will not be repeated here.

The friction force can be written in terms of this friction coefficient as

$$\delta F_f = \frac{2fW^2}{D} \quad (5)$$

where D denotes the corresponding hydraulic diameter, which is defined as

$$D = \frac{(4)(\text{flow area})}{\text{wetted perimeter}} = \frac{4A}{WP} \quad (6)$$

The definition of D is chosen so that it is equal to the diameter for a flow passage having a circular cross section. It should be noted that for a truly isentropic flow f should be equal to zero. For this case Eq. 3 when combined with Eq. 2 will reduce to the second law of thermodynamics identically.

In addition to the above conservation equations, the equations of state in general functional form can be written as

$$\rho = \rho(P, T) \quad (7)$$

$$h = h(\rho, T) \quad (8)$$

$$S = S(\rho, T) \quad (9)$$

In the present study the properties of water vapor are evaluated based on the International Steam Tables using ASME specified equations.²

Equations 1, 2, 3, 7, 8, and 9 comprise a system of six equations involving the seven variables T , P , ρ , h , S , W , A . For given upstream properties and specified value of the friction coefficient, these six equations may be solved for six of the seven variables. This implies that one of these seven variables must be known throughout the tower if solutions of this set of equations are to be found. Numerical solutions for three different flow conditions were developed. They include cases of A, fixed pressure distribution or free acceleration, B, constant velocity, and C, fixed tower shape. These conditions are chosen to represent the broadest spectrum of tower geometries and serve to define limiting flow conditions for the tower design.

Typical computer generated tower shapes for cases A and B are shown in Fig. 2 with the Trojan tower as case C.

Fig. 2. Tower shapes.

1.2 Tower Shapes

1.21 Free Acceleration Tower

In this case pressure inside the tower is matched to the ambient outside pressure at each point along the vertical tower. In this tower all of the buoyancy force generated by moisture condensation is used in accelerating the air upward. Work can be extracted in the form of kinetic energy by a wind turbine. For this case the pressure distribution $P(z)$ is derived from the meteorological data. The unknowns to be solved are T , ρ , h , S , W , and A . So for given inlet conditions, the exit flow conditions at any point along the flow path can be found. This mechanization is easy to model and represents a natural convective updraft in the atmosphere.

The momentum Eq. (3) for this case can be simplified to a simple ordinary differential equation in W^2 as

$$\frac{d}{dz}\left(\frac{W^2}{2}\right) = -\frac{1}{\rho} \frac{dP}{dz} - g - \frac{2fW^2}{D} \quad (10)$$

Equations 1, 2, 7, 8, 9, and 10 now form a closed set of equations. This set of six equations in six unknowns can be solved by a predictor-corrector iteration scheme. The sequence of steps for solutions is as follows:

- 1) Calculate the properties at the inlet using appropriate equations of state or tables and specify the final condition of $P(z)$ for a given z .
- 2) Assume a value for the temperature T . For the first predicted value of T , employ the result of a perfect gas analysis. The second predicted value of T is obtained from the results of the first calculation. Subsequent values of T are obtained by applying the Secant method of iteration.
- 3) Determine the values of ρ , h , S from the equations of state or tables.
- 4) Determine the values of W from Eq. 10 by the modified Euler's predictor-corrector equations.
- 5) Substitute these values into the energy Eq. 2 and check to see if the result is zero or within the desired tolerance.
- 6) If not, steps 2, 3, 4, and 5 must be repeated to achieve convergence.
- 7) When convergence is achieved, the solution is complete.
- 8) Move to the next point along the flow path and repeat steps 1 to 7.

The numerical algorithm discussed above is simple to apply and converges rapidly. If wall friction is negligible, then the solution becomes much simpler. Temperature can be computed without iteration by keeping entropy constant. If flow velocities are kept low, wall friction is indeed negligible (see Section 1.5). Hence, wall friction was assumed to be zero in all the results presented both for natural draft towers and for expansion-compression cycles.

1.22 Constant Velocity Tower

For this case the flow velocity inside the tower is held fixed. Pressure force will develop due to energy released by the latent heat of condensation. Consequently, a pressure differential

exists at the top of the tower provided there is a favorable atmospheric pressure lapse rate. This pressure differential can then be utilized to drive a low pressure turbine.

Since W is known and equal to a constant, the unknown variables are T , P , ρ , h , S , and A . The momentum equation can be rearranged as

$$\frac{dP}{dz} = -\rho \left(g + \frac{2fW^2}{D} \right) \quad (11)$$

This again is a simple ordinary differential equation which can be solved by the predictor-corrector method. The numerical algorithm discussed in the previous section is also applicable here. The only change necessary is to replace Eq. 10 in Sect. 1.21 by Eq. 11, and calculate the variable P instead of W from this equation. The rest of the procedures can be followed without change.

1.23 Arbitrary Shape Tower

In this case the tower shape is given along with the ambient inlet flow conditions. The unknown variables are T , P , ρ , h , S , and W . The latent heat of condensation appears both in the form of pressure differential and increased kinetic energy. For this study the principal shape tested is the hyperbolic cooling tower shape, typified by the Trojan installation in Oregon.

The mass conservation equation can be rewritten as

$$W = \left(\frac{R_a T}{P_a} \right) \frac{\dot{m}_a}{A} \quad (12)$$

Substituting this equation into Eq. 3, the momentum equation can be expressed as

$$\frac{dP_a}{dz} = \left[\frac{1}{A^2 P_a} \frac{dA}{dz} - \frac{g A P_a}{(\dot{m}_a R_a T)^2} - \frac{2f}{D A P_a} \right] / \left[\frac{A}{(1+r) \dot{m}_a^2 R_a T} - \frac{1}{A P_a^2} \right] \quad (13)$$

This equation is again an ordinary differential equation. The solution algorithm, used in Sect. 1.21, is also applicable here if Eq. 11 is replaced by the above Eq. 13.

1.3 Tower Shape Results

Results of analyses of cases A, B, and C show that, for equal tower heights and equal mass flow rates, power outputs are essentially equal. Another assumption involved is that drag losses are small when comparing one tower shape to another.

A couple of factors seem to favor the constant velocity tower shape. First, since the air velocity is kept constant, it is easiest with the constant velocity tower to keep drag losses small. Second, the machine which converts the power output to mechanical form probably will work most efficiently with the constant velocity tower. With the free acceleration tower, the output power takes the form of increased kinetic energy of the air at the top of the tower. A machine is faced with a problem somewhat like that of the windmill of converting this kinetic energy efficiently. With the constant velocity tower, the output work per unit of air processed is $\int V dp$. The situation is ideal for a turbomachine, across which the pressure difference can appear.

If one includes structural considerations, it would probably be desirable to build the constant velocity tower as a double-walled structure, somewhat as in the sketch below (Fig. 3).

Fig. 3.

The constant velocity tower was chosen as the one to be used for studies of power output as a function of tower height and power output as a function of geographic location, season of the year, and time of day, which follow in Sect. 1.6.

1.4 Significance of Saturating Inlet Air

The significance of saturating the inlet air is shown in Fig. 4. Here the available power from a constant velocity tower is plotted versus the entrance relative humidity. It is immediately apparent that the success of the natural draft tower is heavily dependent on the degree of saturation provided at the entrance. In fact, the power output is seen to be zero or negative when the inlet relative humidity is less than 92⁰/o for Lake Charles' July ambient conditions. Therefore, in order to allow for standard comparison of results from different tower geometries, the inlet relative humidity was assumed to be 100⁰/o in all the tower calculations except as otherwise stated. This condition can be achieved by installing a solar saturator unit at the entrance as mentioned earlier.

1.5 Effect of Wall Friction

The effect of wall friction on the available power is shown in Fig. 5. In this calculation the constant velocity tower was used. Different flow velocities were tried while the other inlet conditions (T, P, ϕ) were held constant. The friction coefficient was determined from the Schultz-Grunow relation when the boundary layer is turbulent (see Appendix A for details). The pressure differential and the available power are plotted versus flow velocities. It is seen that the wall friction effects are unimportant in the prevailing wind velocities (1.5–15 m/s). At higher velocities, the friction effect becomes significant and finally, at about 30.5 m/s, the frictional drag dissipated all the available power. Hence the natural draft tower will not operate above a certain critical velocity. The magnitude of this critical speed depends on the inlet flow conditions and the tower shape. For the purpose of the present study it is sufficient to state that the wall frictional effect on the fluid flow is insignificant for the existing wind velocity range.

FIGURE 4. POWER OUTPUT VERSUS INLET HUMIDITY

FIGURE 5. FRICTIONAL DRAG EFFECT

1.6 Power vs Height

1.61 Results

In the investigation of power output as a function of height, atmospheric conditions were taken from U.S. Standard Atmosphere Supplements, 1966.³ Figure 6 contains plots of power vs height for three different atmospheric conditions: (1) Standard atmosphere for 30° N Latitude in July, (2) Standard atmosphere for 30° N Latitude in January, and (3) Standard atmosphere for 15° N Latitude, annual. Power is plotted as watts per kilogram/sec of air mass flow through the tower. It will be noted that for 30° N Latitude in January, negative power is indicated. This standard atmosphere has an unfavorable lapse rate of pressure versus altitude; hence, on a "standard" day, the natural draft tower would not function. It should be borne in mind that in the actual atmosphere at 30° N Latitude in January, the natural draft tower will probably produce usable power less than half of the time; but day-to-day and hour-to-hour variations will result in some times when it will function and produce power.

For 30° N Latitude in July and 15° N Latitude, annual, it was found that power output varies approximately with the 1.4 power of height, or: $P_{out} = K(H_T)^{1.4}$.

1.62 Analysis of Power vs Height Curve

1.621 Goal

To put this power output into perspective with respect to costs, let us look at a preliminary cost estimate.

Preliminary Cost Estimate

Estimate of power which might be obtained from a large humid-air energy-conversion plant:

Assumptions:

Inlet air has Key West summer average ambient conditions.

$T = 28.3^{\circ}\text{C}$, Dew Point = 23.9°C

Fig. 5. Power vs height.

50,000 kg/sec of humid air flow rate through the tower (typical of large cooling towers).

50% of the moisture in the air is removed.

Energy conversion efficiency is 7%.

Under the conditions stated above, the plant would produce an output of 81 megawatts of power plus, as a by-product, 475 kilograms per second of fresh water.

A crude cost estimate for such a plant can be obtained from cost estimates for large cooling towers.⁴ Estimated costs are:

Tower	\$49,000,000
Machinery (turbine and generator)	<u>30,000,000</u>
Total Cost	\$79,000,000

The capital investment then becomes about \$976 per kilowatt of generating capacity, a very attractive figure for a plant whose fuel is free.

Using this estimate, the power output per kg/sec of air processed would be:

$$\frac{81,000,000 \text{ watts}}{50,000 \text{ kg/sec}} = 1620 \frac{\text{watts}}{\text{kg/sec}}$$

1.622 Comparison

Now if we look at the power vs height curve, even for 30° N Latitude in July, to get 1620 watts per kg/sec of air processed would require a tower height of about 3900 meters. If such a tower were feasible and could be built for the same cost as a 300 meter tower, then the natural draft process would look attractive! We are forced to the conclusion that a better mechanization than the natural draft process is required.

1.7 Power Output as a Function of Location and Time

Upper air meteorological observations were obtained on magnetic tape from the National Climatic Center at Asheville, North Carolina for the calendar year 1975 at San Juan, Puerto Rico; Tampa, Florida; Boothville, Louisiana; Lake Charles, Louisiana; and Hilo, Hawaii. These observations are taken each day of the year and at two times each day (0000 and 1200 Universal Standard Time.)

Observations are taken at standard pressure levels: ground level, 1000 mb, 950 mb, 900 mb, etc. For each level records are made of pressure, height, temperature, relative humidity, etc.

In order to investigate the influence of geographical location on the output of a natural draft tower, and also the influences of time of day and season of the year, the constant velocity natural draft tower was run on the computer for each of the five locations and for every observation (i.e., twice each day). So as to avoid having to interpolate between the levels at which meteorological observations are available, the height of the tower always was made to correspond to the 950 mb pressure level.* This leads, of course, to a non-constant tower height, which averages about 583 meters. Results are summarized in Tables 1 through 5, which follow. The column headed "⁰/o of times operational" lists the percentage of days when some positive power would be obtained. The column headed "Avg. power when operating" gives the average of the positive numbers; or in other words, zeros corresponding to the days the tower would not operate are left out of the averaging process. The final column headed "Average power for month" is an overall average and does take the zero-power days into account.

A marked difference in pattern will be seen between the island locations (San Juan and Hilo) and the Gulf Coast locations (Tampa, Boothville, and Lake Charles). The coastal locations tend to have a temperature inversion near the surface in the morning. This leads to a pressure lapse rate which is unfavorable for natural draft tower operation.

In all of these natural draft tower runs, the inlet air was assumed to be at prevailing ambient temperature and at 100⁰/o relative humidity; or, in other words, in a saturated condition through the use of a solar saturator. For a natural draft tower of reasonable height, even 580 meters, for example, to operate at all, it is essential for the inlet air to be saturated. The differences in power output from the tower are caused by differences in the prevailing atmospheric pressure lapse rate. This, in turn, is largely, but not entirely, a function of the temperature lapse rate.

* Height corresponding to the 950 mb pressure level is included in the data obtained from the National Climatic Center.

TABLE 1

SAN JUAN, PUERTO RICO

Observations at 8:00 a.m. and 8:00 p.m. local time.

Average tower height = 583 meters.

Std. deviation = 16 meters

Power is in watts per Kg/sec of air processed.

	% of times operational		Avg. power when operating		Std. deviation in power		Maximum power	Average power for month
	AM	PM	AM	PM	AM	PM		
January	81	100	35.01	54.84	18.90	23.64	109.66	41.54
February	65	100	32.43	46.90	25.34	18.81	89.49	34.03
March	32	90	21.63	30.17	16.75	18.29	87.70	34.23
April	73	93	25.14	29.52	13.96	16.06	53.84	22.93
May	90	97	28.80	28.41	16.17	19.75	67.42	26.60
June	97	97	34.01	29.99	19.89	17.20	87.37	30.93
July	87	77	32.74	34.90	17.23	17.26	66.39	27.77
August	81	90	22.01	23.45	14.45	12.38	62.89	19.46
September	86	70	23.59	26.85	19.46	18.91	78.76	19.57
October	58	87	16.02	23.24	9.29	13.30	58.13	14.77
November	27	77	15.92	21.64	11.90	12.33	50.64	10.42
December	48	90	25.41	26.56	21.92	14.85	84.19	18.14
Average for Year	69	89						25.03

TABLE 2

TAMPA, FLORIDA

Observations at 7:00 a.m. and 7:00 p.m. local time.

Average tower height = 586 meters.

Std. deviation = 26 meters

Power is in watts per Kg/sec of air processed.

	% of times operational		Avg. power when operating		Std. deviation in power		Maximum power	Average power for month
	AM	PM	AM	PM	AM	PM		
January	10	71	10.39	20.63	8.38	12.36	48.67	7.82
February	14	89	8.91	26.51	6.92	16.85	54.96	12.47
March	19	100	11.15	38.83	10.65	20.04	81.27	20.49
April	17	97	16.35	59.93	12.73	18.65	93.81	30.33
May	19	97	6.71	76.99	5.73	20.87	118.41	37.90
June	17	93	23.70	72.00	20.24	30.43	123.26	35.40
July	32	77	25.70	61.15	12.76	35.32	119.11	27.82
August	0	61	---	55.06	---	25.62	92.62	16.88
September	10	67	11.80	29.12	8.43	20.83	70.02	10.29
October	3	74	10.80	23.96	---	10.19	45.72	9.06
November	13	60	11.61	15.48	9.73	9.77	31.57	5.42
December	10	45	4.09	15.90	6.27	9.83	29.13	3.79
Average for Year	14	78						18.14

TABLE 3

BOOTHVILLE, LOUISIANA

Observations at 6:00 a.m. and 6:00 p.m. local time.

Average tower height = 586 meters.

Std. deviation = 31 meters

Power is in watts per Kg/sec of air processed.

	% of times operational		Avg. power when operating		Std. deviation in power		Maximum power	Average power for month
	AM	PM	AM	PM	AM	PM		
January	19	63	20.16	16.99	9.74	13.24	44.64	7.33
February	21	78	21.47	30.70	11.66	16.38	63.39	14.24
March	26	81	13.73	36.36	9.02	20.29	74.16	16.43
April	17	72	18.19	38.91	8.25	18.85	73.60	15.61
May	32	93	22.09	55.28	15.30	26.97	104.65	29.36
June	63	100	24.56	61.70	16.91	17.53	97.81	38.63
July	58	100	15.75	58.83	12.78	25.98	117.35	33.99
August	77	94	18.41	60.66	13.02	30.32	112.07	35.43
September	62	100	24.95	51.28	17.98	20.72	88.33	33.38
October	45	90	21.38	41.21	13.40	15.81	79.34	23.44
November	33	87	15.61	23.04	14.17	16.04	60.62	12.59
December	39	58	25.38	19.60	10.89	11.69	43.61	10.60
Average for Year	41	85						22.59

TABLE 4

LAKE CHARLES, LOUISIANA

Observations at 6:00 a.m. and 6:00 p.m. local time.

Average tower height = 575 meters.

Std. deviation = 37 meters

Power is in watts per Kg/sec of air processed.

	% of times operational		Avg. power when operating		Std. deviation in power		Maximum power	Average power for month
	AM	PM	AM	PM	AM	PM		
January	19	87	16.09	25.75	13.13	13.36	53.67	12.77
February	14	86	16.10	39.29	11.40	18.19	69.07	17.99
March	32	90	12.72	40.78	10.96	20.84	85.27	20.47
April	27	87	13.85	49.36	6.83	22.41	101.91	23.24
May	16	94	9.06	50.53	6.42	17.71	83.30	24.37
June	17	90	10.88	51.82	5.14	25.96	106.32	24.23
July	0	81	---	53.03	---	23.01	102.12	21.38
August	0	84	---	41.63	---	22.28	78.34	17.46
September	10	97	5.91	54.81	6.25	20.90	112.99	26.79
October	10	90	6.44	45.62	8.49	23.76	99.05	20.91
November	7	77	2.37	22.99	2.24	14.49	51.07	17.79
December	6	55	26.46	19.06	8.79	9.40	38.02	12.16
Average for Year	13	85						19.96

TABLE 5

HILO, HAWAII

Observations at 2:00 a.m. and 2:00 p.m. local time.

Average tower height = 586 meters.

Std. deviation = 19 meters

Power is in watts per Kg/sec of air processed.

	% of times operational		Avg. power when operating		Std. deviation in power		Maximum power	Average power for month
	AM	PM	AM	PM	AM	PM		
January	39	100	16.55	74.29	12.37	26.58	116.82	40.35
February	50	100	18.85	78.52	12.84	29.13	139.17	43.97
March	48	100	12.25	79.40	10.85	26.42	125.12	42.66
April	57	100	17.84	103.32	14.87	23.54	143.60	56.72
May	39	100	17.73	114.32	8.64	30.87	181.11	60.59
June	67	100	14.09	109.78	10.24	23.63	162.56	59.59
July	61	100	17.37	104.68	15.59	27.91	154.53	57.66
August	58	100	15.67	106.17	11.64	19.95	138.30	57.63
September	40	100	10.69	111.21	7.63	20.84	146.21	57.74
October	42	100	17.45	98.26	10.62	22.71	147.14	52.79
November	37	100	12.72	84.49	11.49	33.04	133.23	44.58
December	32	100	16.75	96.25	12.91	24.47	131.64	50.84
Average for Year	48	100						52.09

It will be noted that for natural draft tower operation, Hilo, Hawaii, is by far the most favorable location among those investigated. Even at Hilo the overall average power of 52 watts per kg/sec falls woefully short of the target of 1620 watts per kg/sec.

An extremely crude cost estimate for a natural draft tower follows. A tower 150 meters high capable of processing 50,000 kg/sec of air is estimated to cost \$27 million.⁴ A tower 450 meters high, assuming that cost increases with the square of the height, would cost 9 times as much. Total plant costs might be:

\$243 M	- tower, 450 meters high
30 M	- machinery
185 M	- solar saturator
<u>\$458 M</u>	- Total Plant Cost

From $P = K (Ht)^{1.4}$ at Hilo, Ha., the plant might be expected to put out:

$$52 \left(\frac{450}{150} \right)^{1.4} = 35.9 \text{ watts per kg/sec}$$

$$\text{Total power out} = (35.9)(50,000) = 1800 \text{ Kw.}$$

Plant cost = \$250,000 per average kilowatt, a very unattractive figure.

Our conclusion based on all of the computer results is that the natural draft tower is not a cost-effective way to get useful energy from humid air.

1.71 Related

Climatological Data

Monthly average temperatures and dewpoints for San Juan, Tampa, Lake Charles, and Hilo were obtained and plotted. (Figs. 7 through 10). Surface data for Boothville were not available. However, one can surmise that Boothville would be similar to Lake Charles and Tampa. In these monthly averages we see quite dramatically one pattern for the Gulf Coast locations, Tampa and Lake Charles. The island locations have very little seasonal variation in temperatures and dewpoints as compared to the Gulf Coast locations.

2. Expansion-Compression Cycle Analyses

It appears that to enhance power output it is desirable to expand to lower temperatures in order to obtain more condensation than can be obtained with the natural draft tower. A possible mechanization for such a process is through an expansion-compression cycle. In this cycle moist ambient air is first expanded adiabatically

Figure 7.

Figure 8.

Figure 9.

Figure 10.

to a low pressure and work is extracted from this process. During the expansion moisture condenses and falls out at its dewpoint and is removed from the system. Now if the air, plus the remaining water vapor, is compressed adiabatically back to atmospheric pressure, work has to be supplied. From the first and second law of thermodynamics, it can be shown that for this case the compression work expended will have to be greater than the expansion work. However, if we can accomplish the compression with cooling through a polytropic process, the work expended will be less than the corresponding adiabatic compression process without cooling. Then by supplying enough cooling, positive work output is possible from an expansion-compression cycle. In addition, the other parameters like the inlet conditions and the degree of expansion can also be adjusted to provide maximum power output. Another possible modification is to let the expanded air rise through the natural draft tower before compressing it back to the prevailing ambient pressure at the top of the tower. Since the ambient pressure at higher altitude is smaller than the surface pressure, it appears that the compression work required might be less.

In general, then, the performance of an expansion-compression cycle machine should depend on:

- 1) Amount of cooling and cooling water temperature available.
- 2) The degree of expansion the inlet air will undergo.
- 3) The degree of saturation of the inlet air.
- 4) Height of the tower.
- 5) Efficiencies of the expansion, compression machines, and the heat exchanger.

The following sections will analyze the effect of Items 1 to 4.

2.1 Assumptions used in Parametric Studies

The following assumptions are used in the analyses throughout this section.

1. The expansion process is assumed to be isentropic. Moisture condensed is removed at the end of the process.
2. The compression process is assumed to consist of two stages. The first stage is an isentropic compression from the existing low pressure back to a fixed temperature, corresponding to the available cooling water temperature. The second stage of the compression process is assumed to be isothermal at the given water temperature.

This two-stage compression is necessary because the available cooling water temperature T_c is fixed, which means that no cooling is possible when the air temperature is below T_c .

3. Work done in the isentropic process is determined from the energy equation. The changes in kinetic and potential energy ($\Delta KE, \Delta PE$) are neglected and the work expression is given by

$$W_{IEN}^* = -m_a C_{p_a} \Delta T - m_{lw} \Delta h_{lw} - m_{wv} \Delta h_{wv} \quad (14)$$

4. The isothermal work expended is obtained by assuming the air mixture is ideal and follows a polytropic process curve ($Pv^n = \text{constant}$). When $n \neq 1$ the work is expressed as

$$W_c = -\frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right] \quad (15)$$

For the special case where $n=1$ representing isothermal process, the work is given by

$$W_c = -P_1 V_1 \ln \frac{P_2}{P_1} \quad (16)$$

The cooling required in this process can be calculated from the energy equation as

$$Q_c = m_a C_{p_a} \Delta T + m_{wv} \Delta h_{wv} + W_c \quad (17)$$

5. The constant velocity tower option is used in the study of tower height effect.

6. No machine losses are included. In this expansion-compression cycle analysis, the work output of a theoretical cycle is being calculated.

2.2 Parametric Studies

2.21 Power versus Cooling Temperature

In these cases the entrance flow conditions and the final pressure of the expansion process were fixed. The temperature of the cooling water used varies from (4.4°C) to (23.9°C). This range covered all possible water temperature from ground to deep sea water. Three different cases were studied and their flow conditions are given in the following.

* W_{IEN} is the work output from the isentropic expansion.

Case	$T_i (^{\circ}\text{C})$	$P_i (\text{mb})$	ϕ_i	$P_2 (\text{mb})$
A	28.3	1013	.77	540
B	28.3	1013	1.00	540
C	28.3	1013	1.00	700

Where T_i , P_i , and ϕ_i are the inlet temperature pressure and relative humidity respectively, P_2 is the final pressure of the expansion process. The results from these three cases are tabulated in Tables 6 to 8. The net power output and the required cooling

TABLE 6

A. Power vs Degree of Cooling

Inlet Conditions: $T_i = 28.3^{\circ}\text{C}$; $P_i = 1013 \text{ mb}$; $\phi_i = .77$ o/o; $P_2 = 540 \text{ mb}$

Cooling Temp ($^{\circ}\text{C}$)	Expansion Work (J/Kg)	Compression Work (J/Kg)	Net Work (J/Kg)	Cooling Required (J/Kg)
23.9	53017.3	-53504.1	-486.9	-31420.6
21.1	53017.3	-53197.0	-179.8	-32946.8
18.3	53017.3	-52863.1	154.1	-36446.3
15.6	53017.3	-52502.3	515.0	-38918.7
12.8	53017.3	-52114.2	903.1	-41363.9
10.0	53017.3	-51698.5	1318.7	-45632.7
7.2	53017.3	-51255.1	1762.1	-51212.1
4.4	53017.3	-50783.6	2233.7	-56460.7
1.7	53017.3	-50171.9	2845.3	-61036.2

TABLE 7

B. Power vs Degree of Cooling

Inlet Conditions: $T_i = 28.3^\circ\text{C}$; $P_i = 1013 \text{ mb}$; $\phi_i = 100\%$; $P_2 = 540 \text{ mb}$

Cooling Temp ($^\circ\text{C}$)	Expansion Work (J/Kg)	Compression Work (J/Kg)	Net Work (J/Kg)	Cooling Required (J/Kg)
23.9	54063.5	-54141.8	-78.2	-36907.7
21.1	54063.5	-53783.3	280.3	-39400.5
18.3	54063.5	-53397.9	665.7	-4866.4
15.6	54063.5	-52985.3	1078.3	-46252.2
12.8	54063.5	-52545.3	1518.3	-53232.5
10.0	54063.5	-52077.5	1986.0	-59510.8
7.2	54063.5	-51581.9	2481.7	-65178.2

TABLE 8

C. Power vs Degree of Cooling

Inlet Conditions: $T_i = 28.3^\circ\text{C}$; $P_i = 1013 \text{ mb}$; $\phi_i = 100\%$; $P_2 = 700 \text{ mb}$

Cooling Temp ($^\circ\text{C}$)	Expansion Work (J/Kg)	Compression Work (J/Kg)	Net Work (J/Kg)	Cooling Required (J/Kg)
23.9	32369.0	-32243.4	125.5	-24181.0
21.1	32369.0	-32003.8	365.1	-29084.6
18.3	32369.0	-31737.1	631.9	-37971.6
15.6	32369.0	-31333.8	1035.2	-45823.6
12.8	32369.0	-31137.7	1231.2	-53104.4
10.0	32369.0	-30948.6	1420.3	-59717.9
7.2	32369.0	-30765.4	1603.6	-65754.4
4.4	32369.0	-30587.1	1781.9	-71293.1

for each case are summarized in Fig. 11. It is easily seen that the power output is highly dependent on the cooling temperature. The lower the temperature of the cooling water, the better the cycle will perform. It should be pointed out, however, the actual cooling temperature available depends on the location and the source of the water.

2.22 Power versus Degree of Expansion

For this case the entrance flow conditions and the cooling temperature were fixed, while the expansion process is allowed to proceed to different pressures. Four different flow conditions were studied and the given conditions are as follows.

Case	$T_i(^{\circ}\text{C})$	$P_i(\text{mb})$	ϕ_i	$T_c(^{\circ}\text{C})$
D	28.3	1013	.77	21.1
E	28.3	1013	1.00	21.1
F	28.3	1013	.77	15.6
G	28.3	1013	1.00	15.6

T_c is the cooling water temperature.

The results from this comparison are tabulated in Tables 9 to 12. The net power output and required cooling are summarized in Fig. 12. These results indicate that for a given inlet flow condition there is an optimum pressure which the air should be expanded to. If the air is expanded beyond this pressure, the available power will actually decline. The location of this maximum varies with different flow conditions. It appears that this maximum shifts to lower values as the cooling water temperature is lowered. The existence of this maximum is due to the fact that the pressure-volume relationships for the adiabatic and isothermal processes are not linear. When the air mixture is overexpanded, the additional latent heat released by condensation is not converted to available work, but instead to the internal energy of the molecules. This finding suggests that for given inlet conditions and cooling temperature, the expansion ratio has to be carefully controlled for optimum power production from the cycle.

FIGURE 11. POWER OUTPUT, COOLING VERSUS COOLING TEMPERATURE

TABLE 9

D. Power vs Degree of Expansion

Inlet Conditions: $T_i = 28.3^\circ\text{C}$; $P_i = 1013 \text{ mb}$; $\phi_i = 77\%$; $T_c = 21.1^\circ\text{C}$

Degree of Expansion	Expansion Work (J/Kg)	Compression Work (J/Kg)	Net Work (J/Kg)	Cooling Required (J/Kg)
950 mb	5670.9	-5589.4	81.4	-14747.0
900 mb	10382.9	-10271.3	111.6	-14717.5
800 mb	20515.9	-20391.7	124.2	-15977.9
750 mb	25991.6	-25880.5	111.1	-19105.4
700 mb	31785.1	-31704.0	81.1	-22379.8
650 mb	37938.8	-37908.4	30.4	-25810.7
600 mb	44504.3	-44550.1	-45.8	-29406.7
540 mb	53017.3	-53197.0	-179.8	-33946.8
500 mb	59141.8	-59442.8	-301.0	-37108.0

TABLE 10

E. Power vs Degree of Expansion

Inlet Conditions: $T_i = 28.3^\circ\text{C}$; $P_i = 1013 \text{ mb}$; $\phi_i = 100\%$; $T_c = 21.1^\circ\text{C}$

Degree of Expansion	Expansion Work (J/Kg)	Compression Work (J/Kg)	Net Work (J/Kg)	Cooling Required (J/Kg)
950 mb	5748.5	-5640.5	108.0	-29144.3
900 mb	10548.1	-10349.5	198.5	-29047.7
800 mb	20874.5	-20556.6	318.0	-28977.2
750 mb	26458.2	-26109.9	348.3	-29008.7
700 mb	32369.0	-32003.8	365.1	-29084.6
650 mb	38651.0	-38286.0	365.0	-30506.8
600 mb	45358.2	-45014.9	343.3	-34408.5
540 mb	54063.5	-53783.3	280.3	-39400.5

TABLE 11

F. Power vs Degree of Expansion

Inlet Conditions: $T_i = 28.3^\circ\text{C}$; $P_i = 1013 \text{ mb}$; $\phi_i = 77^\circ/\text{o}$; $T_c = 15.6^\circ\text{C}$

Degree of Expansion	Expansion Work (J/Kg)	Compression Work (J/Kg)	Net Work (J/Kg)	Cooling Required (J/Kg)
950 mb	5670.9	-5516.2	154.7	-32131.0
900 mb	10382.9	-10119.2	263.7	-31996.8
800 mb	20515.9	-20016.2	499.7	-31767.2
750 mb	25991.6	-25465.5	526.1	-31739.8
700 mb	31785.1	-31227.3	557.8	-31816.3
650 mb	37938.8	-37367.0	571.8	-31905.6
600 mb	44504.3	-43941.0	563.3	-34481.6
540 mb	53017.3	-52502.3	514.9	-38918.7
500 mb	59141.8	-58688.5	452.3	-42008.5

TABLE 12

G. Power vs Degree of Expansion

Inlet Conditions: $T_i = 28.3^\circ\text{C}$; $P_i = 1013 \text{ mb}$; $\phi_i = 100^\circ/\text{o}$; $T_c = 15.6^\circ\text{C}$

Degree of Expansion	Expansion Work (J/Kg)	Compression Work (J/Kg)	Net Work (J/Kg)	Cooling Required (J/Kg)
950 mb	5748.5	-5566.8	181.7	-46630.4
900 mb	10548.1	-10214.1	333.9	-46444.1
800 mb	20874.5	-20211.1	663.5	-46104.4
750 mb	26458.2	-25614.9	843.3	-45955.3
700 mb	32369.0	-31333.8	1035.2	-45823.6
600 mb	45358.2	-44311.0	1047.3	-46046.3
540 mb	54063.5	-52985.3	1078.3	-46252.2
500 mb	60333.3	-59259.6	1073.7	-47757.1

FIGURE 12. POWER OUTPUT, COOLING VERSUS DEGREE OF EXPANSION

2.23 Power versus Degree of Saturation

In these cases the inlet air temperature and pressure, the final temperature after expansion, and the cooling temperature are held constant. The inlet relative humidity is allowed to vary between 0 and 100⁰/. Four cases were studied and the given conditions are as follows.

Case	$T_i(^{\circ}\text{C})$	$P_i(\text{mb})$	$T_2(^{\circ}\text{C})$	$T_c(^{\circ}\text{C})$
H	28.3	1013	10.0	21.1
I	37.8	1013	10.0	21.1
J	54.4	1013	10.0	21.1
K	71.1	1013	10.0	21.1

The results for these cases are tabulated in Tables 13 to 16. The power output and required cooling are summarized in Fig. 13. These results clearly indicate that the available power increases with increasing relative humidity at the inlet. This means that a solar saturator will also help to increase the power output for this mechanization. However, it should be noted that in this case saturation is not as critical as in the natural draft tower. At low inlet relative humidity the expansion process does not produce significant condensation and the air mixture acts essentially like dry air. This explains why the indicated power is almost constant at low relative humidities.

2.24 Power versus Tower Height

For these cases air at a given flow condition was allowed to expand to a fixed pressure. The expanded air was then allowed to flow up a constant velocity tower. At the top of the tower the air mixture was then recompressed back to the prevailing outside ambient pressure at that height. The variable in these cases is the height of the tower. The flow conditions studied are as follows.

Case	$T_i(^{\circ}\text{C})$	$P_i(\text{mb})$	ϕ_i	$P_2(\text{mb})$	$T_c(^{\circ}\text{C})$
L	28.3	1013	.77	540	21.1
M	28.3	1013	1.00	540	21.1

TABLE 13

35.

II. Power vs Degree of Saturation

Inlet Conditions: $T_i = 28.3^\circ\text{C}$; $P_i = 1013 \text{ mb}$; $T_2 = 10.0^\circ\text{C}$; $T_c = 21.1^\circ\text{C}$

Relative Humidity	Expansion Work (J/Kg)	Compression Work (J/Kg)	Net Work (J/Kg)	Cooling Required (J/Kg)
1.0	47285.3	-46952.5	332.8	-35521.2
0.9	42587.4	-42402.6	184.8	-30985.0
0.8	37872.8	-37800.8	72.0	-26396.3
0.7	33143.3	-33144.4	-1.1	-21752.6
0.65	30773.5	-30794.5	-21.0	-19408.8
0.6	28400.6	-28429.4	-28.754	-17049.7
0.5	23646.9	-23650.4	-3.5	-12282.5
0.4	18884.7	-18799.4	85.3	-7442.7
0.2	18576.3	-18488.0	88.4	-7229.7
0.1	18494.8	-18406.9	88.0	-7197.9
0	18413.9	-18326.4	87.6	-7166.8

TABLE 14

I. Power vs Degree of Saturation

Inlet Conditions: $T_i = 37.8^\circ\text{C}$; $P_i = 1013 \text{ mb}$; $T_2 = 10.0^\circ\text{C}$; $T_c = 21.1^\circ\text{C}$

Relative Humidity	Expansion Work (J/Kg)	Compression Work (J/Kg)	Net Work (J/Kg)	Cooling Required (J/Kg)
1.0	86228.5	-83755.8	2472.7	-83510.9
0.9	78615.2	-76662.2	1952.9	-72531.3
0.8	70931.6	-69441.4	1490.2	-61651.7
0.7	63181.2	-62088.7	1092.5	-50861.0
0.65	59282.2	-58361.1	921.1	-46892.4
0.6	55368.3	-54598.2	770.1	-43142.3
0.55	51440.0	-50799.0	641.0	-39355.5
0.5	47498.2	-46962.1	536.1	-35530.7
0.45	43543.8	-43085.9	457.8	-31666.2
0.4	39577.6	-39168.5	409.2	-27760.1
0.2	28320.6	-27856.8	463.8	-16529.5
0.1	28108.9	-27648.5	460.3	-16405.3
0	27900.0	-27443.1	456.9	-16283.0

TABLE 15

J. Power vs Degree of Saturation

Inlet Conditions: $T_i = 54.4^\circ\text{C}$; $P_i = 1013 \text{ mb}$; $T_2 = 10.0^\circ\text{C}$; $T_c = 21.1^\circ\text{C}$

Relative Humidity	Expansion Work (J/Kg)	Compression Work (J/Kg)	Net Work (J/Kg)	Cooling Required (J/Kg)
1.0	190316.0	-176689.1	13626.9	-263380.2
0.8	159384.5	-149672.5	9712.0	-202993.0
0.6	127103.7	-120864.5	6239.1	-146445.7
0.4	93231.1	-89775.3	3455.8	-93069.4
0.2	57772.3	-55901.0	1871.3	-44440.6
0.1	45431.1	-43635.5	1795.6	-32279.8
0	44640.0	-42875.8	1764.2	-31716.0

TABLE 16

K. Power vs Degree of Saturation

Inlet Conditions: $T_i = 71.1^\circ\text{C}$; $P_i = 1013 \text{ mb}$; $T_2 = 10^\circ\text{C}$; $T_c = 21.1^\circ\text{C}$

Relative Humidity	Expansion Work (J/Kg)	Compression Work (J/Kg)	Net Work (J/Kg)	Cooling Required (J/Kg)
1.0	366422.2	-324786.5	41635.8	-733218.7
0.8	302112.5	-271274.4	30838.1	-537986.1
0.6	240427.3	-219272.5	21154.8	-374747.6
0.4	177471.6	-164897.4	12574.2	-236088.0
0.2	109414.3	-103548.9	5865.4	-115842.6
0.1	72721.3	-68651.8	4069.5	-60479.3
0	61380.1	-57542.6	3837.6	-46382.6

FIGURE 13. POWER OUTPUT, COOLING VERSUS DEGREE OF SATURATION

TABLE 17

L. Power vs Height

Inlet Conditions: $T_i = 28.3^\circ\text{C}$; $P_i = 1013 \text{ mb}$; $\phi_i = 77\%$; $P_2 = 540 \text{ mb}$

Tower Height (m)	Expansion Work (J/Kg)	Compression Work (J/Kg)	Net Work (J/Kg)	Cooling Required (J/Kg)
0	53017.3	-53197.0	-179.8	-33946.8
76.9	53017.3	-53313.1	-295.9	-33682.7
153.5	53017.3	-53349.0	-331.7	-33333.0
229.7	53017.3	-53383.6	-366.4	-32987.9
305.7	53017.3	-53418.1	-400.8	-32637.1
381.3	53017.3	-53452.3	-435.0	-32280.7
457.8	53017.3	-53480.5	-463.3	-31979.8

TABLE 18

M. Power vs Height

Inlet Conditions: $T_i = 28.3^\circ\text{C}$; $P_i = 1013 \text{ mb}$; $\phi_i = 100\%$; $P_2 = 540 \text{ mb}$

Tower Height (m)	Expansion Work (J/Kg)	Compression Work (J/Kg)	Net Work (J/Kg)	Cooling Required (J/Kg)
0	54063.5	-53783.3	280.3	-39400.5
76.9	54063.5	-53892.3	171.2	-39165.9
153.5	54063.5	-53920.3	143.2	-38850.6
229.7	54063.5	-53947.7	115.8	-38534.8
305.7	54063.5	-53974.4	89.1	-38218.7
381.3	54063.5	-54000.6	62.9	-37902.1
457.8	54063.5	-54026.6	36.9	-37579.8

Their corresponding power output and required cooling are summarized in Fig. 14. These results indicate that letting the expanded air rise through a natural draft tower actually produces a negative effect. The power output decreases as the tower height is increased. For the expansion-compression mechanization, addition of a natural draft tower has no advantage at all.

3. Related Investigations

3.1 Lapse Rate Study

As explained in the previous section, the work of a natural draft tower is heavily dependent on the ambient conditions. For maximum work, the tower must be located in an area with a favorable atmospheric lapse rate. The outside atmosphere must have a lapse rate greater than the corresponding saturated adiabatic lapse rate. If a column of ambient air has the same average density as the air inside the tower, no air movement will result, because there is no buoyancy force. Conversely, if the ambient air is drier, and therefore heavier, than the air inside the tower, the outside air will force the moist and lighter air inside to rise. Consequently, a natural convection stream will flow through the tower. Therefore, an understanding of the atmospheric lapse rate distribution for different locations is essential for the analysis of natural draft towers.

The lapse rates for thirteen representative locations in the U.S. were determined from the radiosonde data. The data studied were those of January and July 1965. This lapse rate determination involved the calculation of:

FIGURE 14. POWER OUTPUT, COOLING VERSUS HEIGHT

- a) Saturation vapor pressure
- b) Saturation mixing ratio
- c) Actual mixing ratio
- d) Virtual temperature

Saturation Vapor Pressure (e_s, mb):

$$e_s = 6.11(10^{\frac{AT}{T+B}}) \quad (18)$$

$$A = 7.5$$

$$B = 237.3^\circ\text{C}$$

and T is the atmospheric temperature

Saturation Mixing Ratio (r_s):

$$r_s = (0.62197)\left(\frac{e_s}{P - e_s}\right) = \frac{\text{g}}{\text{kg}} \quad (19)$$

where P is the atmospheric pressure

Actual Mixing Ratio (r):

$$r = (r_s)\left(\frac{\text{RH}}{100}\right) = \frac{\text{g}}{\text{kg}} \quad (20)$$

where RH is the atmospheric relative humidity

Virtual Temperature (T_v $^\circ\text{K}$):

$$T_v = (0.6r)(T+273) + (T+273) \quad (21)$$

$$\text{LAPSE RATE} = \Gamma = \frac{-\Delta T_v}{\Delta Z}$$

$$= \frac{T_{v1} - T_{v2}}{Z_2 - Z_1} = ^\circ\text{C/Km} \quad (22)$$

where Z is the geopotential height.

The lapse rate results for the locations surveyed are summarized in Tables 19 and 20.

Figures 15 and 16 show the results of virtual temperature versus height for the different locations. Lapse rates are represented by the slope of the lines in these graphs. The standard atmospheric lapse rate, the saturation adiabat, and the dry adiabat for 15°C at sea level are also shown in the figures. Since

TABLE 19

The average lapse rates for July at 6:00 p.m.

	<u>Lapse Rate</u> <u>(°C/Km)</u>	<u>Applicable</u> <u>Height (m)</u>
El Paso	16.7	324
Tucson	11.5	720
Amarillo	12.8	432
Albuquerque	11.2	429
Winslow	13.5	550
Las Vegas	15.6	339
Santa Monica	-11.2	488
San Diego	-0.6	435
Salt Lake City	13.1	723
Boothville	12.0	597
Del Rio	11.8	740
Portland ME	4.2	520
Lake Charles	8.8	596

TABLE 20

The average lapse rates for January at 6:00 p.m.

	<u>Lapse Rate</u> <u>(°C/Km)</u>	<u>Applicable</u> <u>Height (m)</u>
El Paso	8.5	317
Tucson	8.9	641
Albuquerque	12.0	375
Las Vegas	8.5	520
Salt Lake City	7.9	689
Boothville	5.0	624
Del Rio	8.6	738
Portland	6.1	507
Lake Charles	7.4	615

JANUARY 1965, pm-hour

LAPSE RATES
 $\Gamma = ^\circ\text{C}/\text{Km}$

1-ALBUQUERQUE
 2-LAS VEGAS
 3-DEL RIO
 4-SALT LAKE CITY
 5-LAKE CHARLES

6-BOOTHVILLE
 7-PORTLAND
 SATURATION ADIABAT
 ——— STANDARD ATMOSPHERE
 --- DRY ADIABAT

Fig. 15.

JULY 1965, pm-hour

LAPSE RATES
 $\Gamma = ^\circ\text{C}/\text{Km}$

1-ALBUQUERQUE
 2-LAS VEGAS
 3-DEL RIO
 4-SALT LAKE CITY
 5-LAKE CHARLES
 6-BOOTHVILLE

7-FORTLAND
 8-SAN DIEGO
 9-SANTA MONICA
 SATURATION ADIABAT
 ——— STANDARD ATMOSPHERE
 --- DRY ADIABAT

Fig. 16.

each location has a different ground level and ground temperature, all lapse rates were plotted with ground level and 15°C as their origin. This modification is allowable because the main interest is in the slopes of the curves and they are not changed by shifting the origin.

It should be pointed out that lapse rates greater than the dry adiabat (Line 1 in Fig. 15 and Lines 1, 2, 3, 4, and 6 in Fig. 16) are called super adiabats. Such adiabats are thought to exist only temporarily in an unstable atmosphere. However, the survey of the radiosonde data indicates that these super adiabats exist and are maintained frequently in desert locations. Negative lapse rates were observed for Portland, San Diego, and Santa Monica. Their data indicate an increase in temperature with height. This occurrence is due to temperature inversions at altitudes above ground level at these locations. The Southern Pacific inversions take place because of the existence of a subsidence air cell in which the periphery brings cold water in from the Northern Pacific. The air moves over the cold water and brings the cold air inland. A low level temperature inversion then takes place. Around the North Atlantic coast, temperature inversions also take place because of a sea breeze. The land is heated more than the water by the sun. The warm air that rises over the land is cooled by the expansion, then sinks over the water and is drawn inland. This air is cooled even more as it moves over the water, thus producing a low level temperature inversion on the coast.

In general, the most favorable locations are in desert regions where super adiabatic lapse rates are found. The Pacific and the Atlantic Coasts are unfavorable due to the low level temperature inversions. Gulf Coasts are marginally favorable because the flow around the periphery of the Atlantic subtropical high pressure cell passes over the warm waters of the Caribbean, Gulf Stream, or Gulf of Mexico, providing humid air along the South Atlantic and Gulf coastal areas. Strictly on the basis of ambient lapse rates, the best year-round location is found to be El Paso, Texas.

However, it should be recalled that the natural draft tower operates on the premise that air flowing into the tower must be saturated before entering the tower. This will require an abundant supply of water for the chosen location. This adds another constraint to the selection of a favorable tower site. Unfortunately, the favorable locations are usually in the desert regions where water is rare. The only location found to be near a useful source of water and still have an atmospheric super adiabatic lapse rate is Del Rio, Texas. Del Rio is approximately 10 miles from the Amistad Reservoir. However, it is highly unlikely that this water will be available for use in a humid air tower. Therefore, from a practical standpoint, the Gulf Coast locations are good compromise between lapse rates and availability of water supply.

3.2 Cooling

An investigation has been made into the coolant temperatures that might be obtained by using ground water. Such water might be obtained by drilling wells. The temperature of ground water at a depth of 30 to 60 ft. is almost constant. At a depth of 30 ft. the annual range in temperature may be expected to be less than 1°F . At a depth of 30 to 60 ft., ground water temperature will generally exceed the mean annual air temperature by 1 to 1.5°C . Below these depths the temperature increases approximately 1°C for each 100 ft. of depth in accordance with the geothermal gradient. A ground water temperature map of the United States is shown in Fig. 17.⁵

Cooling temperatures that might be obtained using fresh ground water are not low enough to permit satisfactory operation of an expansion-compression cycle. Hence, it is proposed instead to use ocean water for cooling. Plant performance estimates and cost estimates based on use of ocean water for cooling appear in Section 6 of this report.

Fig. 17. Approximate temperature of ground water in the United States at depths of 30 to 60 feet (after Collins).

3.3 Solar Saturator

The humid air power cycles computed to date have found that best results are obtained when the inlet air is saturated. By starting with saturated air we allow the expansion process of the cycle to condense the maximum possible amount of water vapor. Should we take the ambient, unsaturated air at the inlet of the humid air machine, then part of the expansion process would only serve to lower the air temperature to the dew point where condensation could begin. Since the ambient air is usually at a relative humidity of less than 100⁰%, even in high humidity areas, some means of saturating the air is desirable. Therefore, consideration has been given to a collector of solar energy which would produce water vapor for mixing with the ambient air in order to saturate the inlet air. We named this the solar saturator.

A preliminary study of the design requirements for a solar saturator revealed that there are four important considerations. These are: solar insolation, ambient air conditions, thermal storage capacity, and energy losses to the surroundings. More

information on each of these considerations is presented in the following paragraphs.

Solar insolation varies with latitude, time of year, time of day, transmittance of the atmosphere, and the angle of inclination of the energy absorbing surface. For a horizontal surface, the solar insolation may be expressed in the form:

Solar insolation = (solar constant)($\cos \delta_H$)(transmittance) where δ_H is the angle of incidence at any daylight hour angle H and may be predicted by well known equations relating latitude, time of year, and time of day. The transmittance of the atmosphere depends primarily on the opacity of the cloud cover. It is difficult to predict transmittance if the only meteorological data available is an estimate of cloud cover. A general observation is that on those days having low relative humidity there is a high probability of few clouds, and vice versa. Thus, on days where more energy is required to reach saturation conditions, more solar energy reaches the collector. For a design limit, the transmittance can be taken as the value for clear sky, and the lowest values of relative humidity may be used to compute energy requirements.

Ambient air conditions of temperature and relative humidity have influence on the amount of water vapor required to saturate the air. Also high wind conditions will cause greater heat loss at the collector. These conditions must be evaluated for each proposed site.

Thermal storage capacity has a great influence on the operating characteristics of the saturator. If the thermal storage capacity is very small, then the saturator will warm up quickly and provide maximum flow rates of water vapor during the daylight hours. On the other hand, if the thermal storage capacity is very large, then the saturator will have enough energy stored to provide water vapor on a continuous basis. The thermal storage capacity of the saturator can easily be adjusted by varying the amount of water contained in the saturator.

Energy losses to the surroundings may be described as:

Loss from the cover glass via reflection, convection, and reradiation

Loss from the saturator walls

Loss due to heating the water added as replacement for water vaporized and blow-down.

All of these losses are strongly dependent on the temperature difference between saturator and surroundings, i.e., as the temperature difference approaches zero, the losses also diminish.

The inter-relationship between the parameters is best expressed by an energy rate balance equation:

$$\left(\begin{array}{c} \text{solar} \\ \text{insolation} \\ \text{rate} \end{array} \right) = \left(\begin{array}{c} \text{energy rate} \\ \text{for produc-} \\ \text{tion of water} \\ \text{vapor} \end{array} \right) + \left(\begin{array}{c} \text{rate of} \\ \text{energy} \\ \text{stored} \\ \text{in water} \end{array} \right) + \left(\begin{array}{c} \text{rate of} \\ \text{energy} \\ \text{lost to} \\ \text{surroundings} \end{array} \right)$$

One possible configuration for a solar saturator is shown in Fig. 18. This design absorbs solar energy directly in the water

- a. ambient air inlet
- b. single pane glass cover and supporting structure
- c. water pool
- d. black butyl rubber pool liner
- e. water pump
- f. water spray for saturation of air
- g. saturated air rises up tower
- h. water blowdown

Fig. 18. Solar saturator.

pool and also on the black butyl rubber pool bottom. The cover glass serves to prevent loss of water vapor to the surrounding air and thus will greatly decrease the energy losses if the saturator operates at a temperature higher than ambient temperature. Water vapor escaping from the pool surface mixes with the air stream that is en route to the inlet of the humid air machine. At the exit of the solar saturator, a spray of water from the pool assures that the air has become saturated. The water pool also provides thermal storage capacity. This configuration strongly resembles that of solar heated water distillation units, for which there is considerable data on costs.⁶ Taking cost figures for solar stills and allowing for inflation, the range in initial cost is \$50-100 per square meter of glass-covered collector surface. Largest cost item is the cover glass and supporting framework. Glass has a long life and requires little maintenance other than an occasional cleaning.

A calculation has been made of the collector area required in order to arrive at a cost estimate for a solar saturator. Assume Key West, Florida average conditions for June or December. Using published data for average solar insolation, the collector area required was computed as shown in Table 21. For a cost

TABLE 21

SOLAR SATURATOR CALCULATIONS
LOCATION: KEY WEST, FLORIDA

Month	June	December
avg temp, °C	28.3	21.3
avg dew point, °C	22.8	15.6
enthalpy increase to saturate air $\frac{\text{joules}}{\text{kg}}$	18140	12560
possible hours of sunshine per day	13.63	10.65
avg solar insolation, on horiz. surface joules/M ² day	2.3×10^7	1.256×10^7
air flow, kg/s	1.0	1.0
collector efficiency	100 ⁰ /o	100 ⁰ /o
collector area, M ² /kg/s (daylight operations)	38.7	38.3
(24 hr. operation)	68.1	86.3
collector cost, \$/M ²	\$55	\$55
collector cost, \$/kg/s daylight operation	2128	2109
24 hour operation	3747	4753

estimate of $\$55/M^2$, the capital cost of the solar saturator is $\$2100$ per kg/s air flow during the daylight hours, or $\$3700/kg/s$ for continuous operation. If the solar saturator is used with a natural draft tower having a height of 586 m, the capital cost amounts to approximately $\$100,000/kw$ in June when the power output is at a maximum. (Power output is taken for Boothville or Tampa.)

The high cost of $\$100,000/kw$ may be reduced by increasing the operating temperature of the solar saturator. When temperature of the saturated air is increased above the ambient temperature, the power output of the humid air machine increases dramatically, whereas the collector efficiency drops off rather slowly. This suggests that optimization of the total system, i.e., solar saturator plus humid air machine, will improve the cost figures greatly. Energy conversion (output/solar input) amounts to only .2% for the natural draft tower discussed here. This efficiency may be raised to above 5% by operation of the system at higher temperatures. Such an improvement would reduce capital cost of the solar saturator by at least an order of magnitude.

4. Absorption Cycle

A search for a means of extracting more humidity from the air than is possible with the adiabatic expansion process has produced an idea for employment of an absorbing solution. When an absorbing solution, such as $CaCl_2$ or $LiBr$, contacts humid air, the humidity is taken into the solution. The latent heat of vaporization is released, warming the solution and the air. It is possible to raise the air temperature to $40^\circ C$ above the dried air dewpoint, i.e., the dewpoint may be lowered by $20^\circ C$ and the air temperature may be raised by $20^\circ C$. About 30% to 50% of the moisture in the air can be extracted, and the absorption works with unsaturated air. Of course, the diluted absorbing solution must be reconcentrated, and this must be done at a temperature about $40^\circ C$ higher than the final air temperature. This reconcentration could be feasible with a solar collector/evaporator. The absorbing solution may be stored so that the system can operate continuously.

Since the absorption process is a form of air heating, a thermodynamic cycle different from the expansion/cooling/compression cycles would be required. The Brayton cycle (the same as used in gas turbines) appears well suited for use with the absorption process. Processes in the cycle would be:

- 1 to 2 Compression of the humid air, preferably with cooling, but avoiding condensation of moisture.
- 2 to 3 Heating/dehumidifying of the compressed air with absorption.
- 3 to 4 Adiabatic expansion with production of shaft work via a turbine.
- 4 to 1 Exhaust of dried air to atmosphere.
Intake of humid air from atmosphere.

A diagram of the cycle is shown in Fig. 19. In this cycle,

Fig. 19. Absorption Brayton cycle.

the water vapor that is driven off from the absorbing solution during the reconcentration process should not be wasted, since the vapor has a large energy content. Mixing the water vapor with the intake air would raise its dewpoint, thereby enabling the absorption process to achieve higher temperatures. This would improve the cycle efficiency.

As with all low temperature compression/expansion cycles, losses due to low efficiency in the compression and expansion processes can use up all of the ideal cycle work. Probably, turbomachines cannot achieve the high efficiency ($> 98\%$) needed to make the cycle operable. An alternative means of compression-expansion, avoiding the use of two separate machines, might be the use of a deep hole like a mine shaft, as shown in Fig. 20.

Fig. 20. Deep hole mechanization of absorption Brayton cycle.

In this arrangement, the compression is accomplished by the hydrostatic pressure change as the air flows down the shaft. The air is then heated at the bottom of the shaft by absorption and then allowed to expand as the air rises up the return shaft. Work is extracted via a turbine at the hot air exit. Flow losses in the

shaft can be minimized by keeping air velocities low, so that the necessary high efficiencies can be attained. The required depth of hole appears to be in the range of 1000 to 2000 m, which is feasible with current mining techniques. Note that this configuration is a form of a natural draft or wind augmentation process. The natural draft occurs because the column of air on the inlet side (1 to 2, Fig. 20) has a higher density than the column on the exit side (3 to 4, Fig. 20).

5. Single-machine Mechanization of Expansion-Compression Cycle Using Vortex Flow

The greatest problem in obtaining useful power from an expansion-compression cycle is one involving machine efficiencies. A large amount of power is extracted during the expansion. A large but slightly less amount of power is put back into the air during compression. The difference is the useful output we are trying to obtain. With the two-machine mechanization, unless machine efficiencies are extremely high (in excess of 98⁰/o), the output hoped for is all consumed in machine losses, and no useful output is obtained.

An attractive approach is a mechanization of an expansion-compression cycle which requires only one machine. The one machine in effect sees only the difference between the expansion power out and the compression power going back into the system; hence, machine efficiency is no longer critical.

To illustrate the concept, one such mechanization would use a very high tower built as an inverted U-tube as in Fig. 21.

The humid air rises on the left side of the tower and expands, with condensation of moisture. Just past the top of the tower, cooling is introduced. The result is that air on the right side is cooler and more dense than air on the left side. A flow of air through the inverted U-tube tower results. Compression takes place as the air descends on the right. An expansion-compression

cycle has been realized via hydrostatics. Flow losses can be minimized by use of low velocities. One machine placed anywhere along the flow path can be used to extract power.

The above scheme is not economically practical because a very high tower would be required. It is presented only to illustrate the concept of a one-machine mechanization.

A more promising approach from the practical standpoint is to use vortex flow in a manner so as to get a one-machine realization of the expansion-compression cycle.

In Fig. 22 humid air enters through guide vanes on the upwind side of the upper or expansion chamber. Air spirals in toward the center of the chamber, where a low pressure prevails. Hence, the air is expanding as it approaches the center. While maintaining its vorticity and kinetic energy* the air passes to the lower or compression chamber. Here it spirals outward and is compressed in the process. Cooling is introduced during the compression. The air exits through vanes on the downwind side. Since this cycle is a net power producer, the vortex will sustain itself, provided the losses are not excessive, and one machine, placed anywhere along the flow path, can be used to extract power.

The above is intended only to serve as an outline of the essentials of the process. The diagram is intended to be only schematic in nature. Exact shapes and dimensions must be determined after modeling of the desired flow pattern. Further treatment of this concept appears in Sections 6 and 7 of this report.

*There has been some question as to whether the air would indeed maintain vorticity and kinetic energy in passing from one chamber to another. Provided losses are not excessive, the law of conservation of energy calls for kinetic energy, and, hence, vorticity to be maintained. A small model of the vortex chamber has been built which demonstrated maintenance of vorticity and kinetic energy as the air passed from the expansion chamber to the compression chamber.

Fig. 21. Inverted U-tube mechanization of an expansion-compression cycle.

Fig. 22. Vortex mechanization of an expansion-compression cycle.

6. Power Generation Potential and Cost Estimates for a Large Humid Air Processing Plant

6.1 Power Generation Potential

The computer analysis described in Section 2 was employed to predict the hourly and monthly power generation for the year 1975 at a site on the island of Puerto Rico near Punta Tuna. At this location cooling water at a temperature of 6°C can be obtained about a mile and a half off-shore. The plant is assumed to be on land, with a large concrete pipe placed on the ocean bottom going out to the location of the 6°C cooling water. The plant size was chosen somewhat arbitrarily, so that it can process 10,000 kg/sec of air. To provide the cooling water required, the concrete pipe has a length of 2773 meters and a diameter of 2.917 meters.

To calculate the power output from the plant surface weather data for San Juan, PR were used. (Detailed surface data are not available for Punta Tuna.) There is every reason to believe that Punta Tuna temperatures and relative humidities are very close to those prevailing at San Juan. In calculating power output from the plant, power necessary to pump the cooling water is subtracted from the power output from the turbine. We start with the theoretical power output from the expansion-compression cycle. A turbine is used to extract the net output power and is assumed to be 90% efficient. Flow losses in the vortex itself have been neglected. The research we are proposing addresses this problem, but we have no good way of estimating the losses at this time.

On the suction side of the pumps, fluid friction in the pipe was calculated for a hydraulic smooth, insulated pipe with a water velocity of 1.5 m/sec. A well-rounded bell mouth entrance was assumed. The equivalent pipe length was increased by 32 pipe diameters to allow for pump inlet connections. On the discharge side of the pumps, a constant head of 3 meters was assumed to account for lifting and spraying the water into the vortex chamber. A cooling water pipe length of 2773 meters is used.

For the plant described above, Fig. 23 shows the variation in output power as a function of time of day for an average day in January and for an average day in August. Surface weather data are provided at three hour intervals for these average days. Hence, the computer has calculated 8 different cycles for each curve. The output power reaches its maximum at 2:00 p.m., the hottest time for the average day. Hence, power output is greatest just when it is most needed to supply electrical air conditioning loads.

Fig. 24 shows variation of power versus month of the year for 1975 data. The center curve is average power for each month. The top curve is the maximum for each month, calculated for the most favorable weather condition during the month. The bottom curve is the minimum calculated at the least favorable time. Overall average power output for this plant is 6.73 megawatts. It is noteworthy that even in the winter months a respectable average power output is maintained. Furthermore, the minimum power out always exceeds one megawatt, indicating that the plant could theoretically operate all year on a continuous basis. No unfavorable condition of temperature or relative humidity at the inlet would close it down.

Fig. 23. Net power output vs time of day.

Fig. 24. Net power output vs month of year.

6.2 Cost Estimates

A capital cost has been computed for a one-machine-expansion-compression cycle of the type shown in Fig.22. Since the detailed flow analysis necessary for a complete vortex-design has yet to be done, a number of assumptions were used in lay-out of a rough preliminary design for cost estimation purposes. The assumptions are:

1. The plant has an air flow rate of 10,000 kg/s except when the air flow must be reduced because of cooling water pump limitations. Plant rating is 10.5 megawatts.
2. The plant site and electric power transmission lines are not included in the capital cost estimate.
3. Expansion and compression are ideal, frictionless processes.

4. Cooling of the air during compression is by direct contact between the air and a spray of cold water. No heat exchanger is necessary. A heat transfer effectiveness for the parallel flow is assumed at 0.9.
5. The structure for the vortex chamber is very large in diameter in order to accelerate the air from 5 m/s at inlet to 150 m/s at the center. The construction design and cost are similar to an airplane hangar.
6. The turbine is located at the orifice between the expansion and compression chambers. The turbine drives a synchronous generator.
7. The circulation of water for coolant is by twelve large turbine type pumps of 20,000 gallon/minute capacity each. These pumps are connected to a large diameter (2.917 m) concrete pipe. Water velocity in the pipe is limited to 1.5 m/s in order to minimize flow losses.
8. The plant has an automatic control system capable of controlling the air inlet doors in order to limit maximum power rating to 10.5 mw. Also, it selects the optimum number of pumps to operate as required by the ambient air conditions. If the air conditions are not favorable for operations, the control system will shut the plant down until favorable air conditions occur.

The capital cost relationships developed to fit the above assumptions are described in 6.21 below.

For the plant of 10.5 mw rated net power, the estimated capital cost is 15.91 million dollars, or about \$1500 per rated kilowatt. Figure 25 shows a breakdown of the cost. The rather large structure selected in this design accounts for the major portion of the cost. It must be emphasized that these are preliminary design figures based on a configuration that would be workable, but has in no way been optimized. Figure 26 is a schematic diagram of the same plant.

Total Cost: \$15,900,000 (exclusive of site)
 Design Rating: 10.5 MW at $T = 52.2^{\circ}\text{C}$, RH = 65%, 6°C cooling water.

Fig. 25. Capital cost of Humid Air Power Plant.

Fig. 26 Schematic diagram of a humid air power plant

Since the humid air power plant operates without fuel, the annual fixed cost is a figure which will come close to the total annual cost. Computing the annual fixed cost at 15⁰/o of the capital investment for the plant, and using the total kw-hr generated if the plant operates 12 months per year, the cost per kw-hr is 4.0 cents/kw hr. This appears to be an attractive cost figure for a non-energy consuming power plant.

A schematic diagram of the plant layout is shown in Fig. 26

6.21 Capital Cost Relationships

Structure cost was developed on a dollars per square foot basis for a structure similar to an airplane hangar. Cost data from Reference 7 were used. Data from this same reference were used for estimating cost of the cooling water pipe. Cost of laying the pipe on the ocean floor was assumed to be similar to ditching costs for putting the pipe underground on land. Electrical machinery costs were estimated for a 10.5 mw generator using published data from Reference 8.

Pump costs were based on off-the-shelf prices for large pumps of the type used in waste water treatment plants.⁹

Installed costs of machinery were estimated on the basis of multiplying factors recommended in Reference 10. These factors include overhead and direct labor, transportation, and installation costs.

All costs were updated for price escalation for the time interval from the date of the cost quote up to 1977. Data in Reference 11 were used for 1969 to 1975 and an escalation of 10⁰/o/year was estimated for 1976 and 1977.

6.22 Equations Used in Cost Estimating

Structure (labor and material):

$$\text{Cost} = \$682/(\text{kg air/s})$$

This assumes $R_1/R_2 = 10$, metal exterior building on a 6" thick concrete ground slab. The inner cones are of stressed skin plywood. For an air flow rate of 10,000 kg/s, structure would be 288 m in diameter.

Turbine, gear box, generator (installed module):

$$\text{Cost} = \$4176 \left[3.58 \left(\frac{\text{HP}}{1000} \right)^{1/5} + \left(10 + 14.4 \left(\frac{\text{HP}}{1000} \right) \right) + 25 \left(\frac{\text{Kw}}{1500} \right)^{1/3} + 17.34 \left(\frac{\text{HP}}{1000} \right)^{1/3} \right]$$

The exponents were determined from curve fit of published cost data for generator and gear boxes. The turbine cost was estimated for a 2 blade, variable pitch propellor type of about 10 m in diameter. Turbine rpm to be 100, generator rpm to be 900. Mounting frame for the components was estimated at 20% of the component cost and is included in the equation.

Pumps (installed module):

Cost/pump = \$33,000 including motor, wiring, control panel, piping manifold.

Pump house (labor and material):

$$\text{Cost} = \left(\frac{\text{number of pumps}}{4} \right)^{.75} \times \$154,100$$

This was developed taking into consideration the necessary spacing of the pumps in a multiple pump installation. Construction of the pump house is of poured concrete.

Water pipe (labor and material):

$$\text{Cost} = \$.737 (\text{diameter in inches})^{1.6} \times (\text{length in m})$$

This was derived from curve fitting cost data for cast concrete pipe of large diameters such as used in sewer and drainage systems.

Control system (module installed):

$$\text{Cost} = \$3480 \left[28 + 20 \left(\frac{\text{air kg/s}}{1000} \right)^{1/3} \right]$$

The size of the installation has a moderate effect on the cost of the control system. This is because of the number of actuators required for the control louvers increases as the air flow increases.

Control house (labor and material):

$$\text{Cost} = \$44,200$$

This is typical of a 20' x 30' building such as might be used for a telephone exchange.

Site development (labor and material):

Cost = \$30,800

This includes lighting, fencing, water return flume, etc.

6.3 Comments Regarding Cost Estimates

It must be emphasized that the preceding cost estimates are made for a very rough preliminary design. Further research is necessary to prove that the vortex mechanization can really be made to operate successfully. There will be some power losses in the vortex, and we are not yet in a position to estimate these losses. On the other hand, the estimates are based on a design for a plant which has not in any way been optimized.

It is entirely possible that in our proposed research a way will be found to do the job with a smaller structure. This could provide a major cost reduction. It is also very likely that pressures of economic competition and resulting application of American ingenuity will lead to lower costs for the cold water pipe and another major cost reduction. Our principal conclusion is that the cost estimates look sufficiently attractive that further research in this area is warranted.

Estimated costs for future coal-fired steam plants are now in the range of \$1200 per rated kilowatt (Wall Street Journal, Jan. 12, 1978, page 10). Compared to this our estimate of \$1500 per rated kilowatt is not unattractive. Capacity credit should be given to a humid air plant, because it produces maximum power under the same weather conditions that lead to maximum electrical loads for air conditioning.

It is true that lower costs per rated kilowatt can be quoted for conventional wind turbines. However, the favorable duty cycle of the humid air plant must be taken into account (see Figs. 12 and 13). It must also be emphasized that in our cost estimates we have not employed any learning curve to reduce costs; we have not assumed any special tax credits; we have not assumed that the money is borrowed by a municipality issuing tax-free bonds at a low interest rate.

7. VORTEX, PRELIMINARY CALCULATIONS

7.1 Preliminary Calculations

Preliminary calculations have been made with the objective of obtaining at least a crude estimate of maximum tangential velocity required in a vortex in order to produce a given pressure drop in the center of the vortex. For this purpose pressure, air density, and tangential velocity are assumed to be functions only of r , the radius measured from the axis of the vortex.

Symbols: v_t - tangential velocity of air
 r - radius measured from axis of the vortex
 ρ - air density
 P - pressure

A functional relationship between V_t and r was assumed. Then pressure as a function of r was calculated using the familiar equation:

$$\frac{dP}{dr} = \rho \frac{v^2}{r} \quad (23)$$

This relationship is, of course, based on the requirement that the pressure gradient must supply the necessary centripetal force to hold a parcel of air in a circular orbit.

7.11 "Ideal" Vortex

For an "ideal" vortex as defined by Loth¹² and also used by Yen,¹³ we assume a central core to the vortex having radius a . Within this central core tangential velocity is proportional to r :

$$V_t \sim r \quad r < a$$

When r exceeds a , V_t is assumed to be inversely proportional to r or

$$V_t \sim \frac{1}{r} \quad r > a$$

This leads to a tangential velocity profile as sketched in Fig. 27.

Fig. 27. Tangential velocity, "ideal" vortex.

7.12 "Real" Vortex

For a "real" vortex, as defined by Loth, we again assume $V_t \sim r$ within a central core of radius a . When r exceeds $4.6a$, $V_t \sim \frac{1}{r}$. When r is between a and $4.6a$, V_t is inversely proportional to the square root of r .

In summary:

$$\begin{aligned} V_t &\sim r & r < a \\ V_t &\sim \frac{1}{\sqrt{r}} & a \leq r \leq 4.6a \\ V_t &\sim \frac{1}{r} & r > 4.6a \end{aligned}$$

This leads to the tangential velocity profile sketched in Fig. 28.

Fig. 28. Tangential velocity, "real" vortex.

7.13 Maximum Tangential Velocity Required

Let us assume that it is desired to reduce pressure from an ambient value of 14.7298 lbf/in^2 at the outer periphery of the vortex to a value of 9.5150 lbf/in^2 at the center of the vortex. These pressures correspond to ambient pressure and pressure after isentropic expansion used in the Expansion-Compression Cycle calculations of Section 2.3. Pressure drop for a given tangential velocity profile can be found by integrating Eq. 23:

$$\frac{dP}{dr} = \rho \frac{v^2}{r}$$

ρ was assumed to vary with P as in the isentropic expansion. To obtain the desired pressure drop, it was found:

	$V_{t_{\max}}$ ft/sec
"Ideal" Vortex	630
"Real" Vortex	543

7.14 Energy Balance

From the first law of thermodynamics, for an adiabatic expansion process with external work to the turbine:

$$\cancel{Q} + W = \Delta h + \Delta KE$$

Using the results from page B-6, $\Delta h = -15.9546 \text{ BTU}$ for 1.01 lb of air + water vapor. The turbine work, for frictionless flow, would be .4543 BTU. Assuming an inlet velocity of 5 m/s (16.4 ft/s), the maximum velocity is:

$$0.4543 = -15.9546 + \frac{V_2^2 - 16.4^2}{2 \times g_c \times 778}$$

$$V_2 = 881 \text{ ft/s}$$

Note that V_2 exceeds the maximum value of tangential velocity, $V_{t_{\max}}$. The difference can be made up of radial velocity and axial velocity. V_2 is the vector sum of tangential, radial, and axial velocity components.

7.15 Conclusions from Vortex Calculations

It is recognized that the preceding calculations are of a very preliminary nature. However, the possibility of the vortex sustaining itself when humid air is expanded appears attractive. It should also be kept in mind that a great many variations in the process are possible and that it is not necessarily essential to obtain as large a pressure drop as was assumed above. It may be that velocities will have to be kept lower to minimize drag losses and that a lesser amount of output energy per unit of air processed will have to be accepted. A great many parameters can be adjusted in any complete optimization of the process.

7.2 Future Research Needed

Using a vortex mechanization, will it indeed be possible to obtain a net power output at a reasonable cost? The first and foremost question to be answered is, "How great are the power losses within the vortex itself?" These power losses will be caused by wall friction, by viscous friction within the air itself, and by turbulence. Wall friction can be minimized in various ways. The first obvious way is to build the whole structure very large. The larger the structure, the greater the ratio of mass flow rate of air processed to wall area. The power output from the cycle is proportional to mass flow rate; wall friction is proportional to wall area. Another way of reducing wall friction is to make the walls very smooth. Still another possibility concerns the walls of the central hole between the expansion and compression chambers. It is adjacent to this part of the walls that maximum air velocities occur. The walls of the central hole might themselves be allowed to rotate about the axis of the vortex. In fact, this rotating wall might be the energy extracting turbine; and, in this process, wall friction power losses would be minimized.

Losses due to viscous friction and turbulence will be more difficult to minimize. It appears probable that a full-scale plant would involve flow at such a high Reynolds number that viscous effects would be negligible. It may be possible to keep losses due to turbulence

under control by careful adjustment of the flow path and by minimizing the path length traveled by a parcel of air as it goes through the vortex chambers.

Additional research appears desirable with the objective of calculating or measuring these losses in the vortex itself.

8. Conclusions

8.1 Natural Draft Tower

By orders of magnitude, the natural draft tower is not a cost-effective way to get useful energy from humid air. Factors contributing to its lack of cost effectiveness are:

- (a) The power versus height function (see Fig. 6) favors an extremely high tower. Such a tower would be extremely costly and perhaps structurally beyond the capability of present technology.
- (b) Power output is dependent on prevailing atmospheric lapse rate. At locations which are favorable from the standpoint of being hot and humid, the lapse rate tends to be unfavorable a considerable percentage of the time (see Tables 1 through 5).
- (c) An expensive solar saturator is required.

8.2 Expansion-Compression Cycles

Parametric studies of expansion-compression cycles lead to the following conclusions:

- (a) As would be expected from the second law of thermodynamics, the power output increases as cooling temperature decreases.
- (b) Power output increases with degree of expansion up to a point and then decreases. The optimum point is dependent on the temperature of available coolant.
- (c) Power output increases with degree of saturation of humid air and is maximum with 100% relative humidity.

- (d) Power output decreases with tower height. A structure with relatively small vertical dimension is best for an expansion-compression cycle mechanization.

8.3 Related Investigations

8.31 Cooling

Fresh ground water, obtainable from springs or wells, is not cold enough to be useful for cooling with an expansion-compression mechanization. However, at suitable locations, sea water can be obtained which is at a low enough temperature to be attractive for cooling.

8.32 Solar Saturator

A solar saturator might be used to increase the humidity of ambient air at the input to a humid air energy conversion plant. Because the overall efficiency of such a plant is low, such a solar saturator would not be a cost-effective way to use solar energy.

8.4 Mechanization of Expansion-Compression Cycle Using Vortex Flow

A system using vortex flow to mechanize an expansion-compression cycle has possibilities of being cost effective provided that drag losses within the vortex can be kept sufficiently small. In this area, further research appears to be warranted.

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APPENDIX A

WALL FRICTION

In the analysis for the natural draft tower simple frictional flows are assumed; that is, flows that are steady, one-dimensional, and involve only wall friction. To analyze this problem the flow field can be considered as two regions: a central region where the fluid flow is considered inviscid, and a thin boundary layer region next to the wall. It is assumed that all of the frictional effects are confined to the boundary layers where the velocity gradient is large. This frictional force may be expressed in terms of the hydraulic characteristics of the flow passage and the Fanning friction coefficient f , which is defined as

$$f = \frac{\text{shear stress at the wall}}{\frac{1}{2} \rho W^2} = \frac{\tau_w}{\frac{1}{2} \rho W^2} \quad (\text{A-1})$$

The frictional force can then be written as

$$F_f = \frac{\rho W^2}{2} \left(\frac{4f dZ}{D} \right) A \quad (\text{A-2})$$

where F_f is the frictional force and D is the hydraulic diameter ($4A/WP$).

So in order to calculate this frictional force, the Fanning coefficient has to be determined first, either experimentally or analytically. Due to the small length to diameter ratio of the tower, the boundary layers formed on the walls are very thin and will not merge to form a fully developed velocity profile. Hence these boundary layers are acted upon by the external free stream velocity, similar to a flat plate boundary layer. Therefore, in the present study, this coefficient is determined from the flat plate boundary layer analysis.

When the Reynolds number ($Re = \frac{WZ}{\nu}$) is less than 3.0×10^5 the boundary layer is entirely laminar. The friction coefficient

is strictly a function of Reynolds number and is independent of the roughness of the surface. Schlichting^{A1} showed that the following relation is applicable.

$$f = .664/\sqrt{Re_z} \quad (A-3)$$

For Reynolds number greater than 3.0×10^5 and ϵ/z (ϵ is the equivalent sand grain diameter) small, the surface can be considered hydraulic smooth, and the friction coefficient can be determined by the Schultz-Grunow relationship.

$$f = .370(\log_{10} Re_z)^{-2.584} \quad (A-4)$$

If the surface is very rough, that is, large value for ϵ/z , and if Re_z also has large value, experiments show that f is independent of the Reynolds number. Such a surface is said to be wholly rough. For flow in this regime, the friction coefficient is given by

$$f = (2.87 + 1.58 \log_{10} \frac{z}{\epsilon})^{-2.5} \quad (A-5)$$

For detailed derivation of these equations and values of ϵ for different surfaces, the readers are referred to References A1 and A2.

For the present study the hydraulic smooth Eq. A-4 will be used when

$$\epsilon \leq 100 \frac{\nu}{W} \quad (A-6)$$

and Eq. A-5 will be used when

$$\epsilon > 100 \frac{\nu}{W} \quad (A-7)$$

The step by step determination of the frictional force can be summarized as follows.

1. Determine the local Reynolds number and the relative roughness of the surface at the given location.
2. If $Re_z \leq 3.0 \times 10^5$, use Eq. A-3 to evaluate f .
3. If $Re_z > 3.0 \times 10^5$ and $\epsilon \leq 100 \nu/W$, use Eq. A-4 to find f . If $\epsilon > 100 \nu/W$, use Eq. A-5 instead.

4. Substitute the value of f into Eq. A-2 to calculate the frictional force. This can then be substituted back into the momentum equation.

Finally it should be pointed out that, strictly speaking, Eqs. A-3, A-4, and A-5 are applicable only to flat plate boundary layer at zero pressure gradient. This assumption cannot, certainly, be exact, because the velocity distribution in the tube is formed under the influence of a pressure gradient, whereas on a plate the pressure gradient is zero. However, small differences in the velocity distribution are unimportant, because the drag is calculated from the integral of momentum. Furthermore, Schlichting^{A1} indicated that in instances where the pressure gradient differs from zero, the skin friction is not materially different from that on a flat plate, provided that there is no separation. Therefore, results from these calculations should be accurate enough for the one-dimensional analysis used in the natural draft tower option.

References

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Expansion-Compression Cycle - Details of Computation

The expansion-compression process has been modeled on the CDC 6400 computer and extensive parametric studies have been made. Since the cycle calculations are basic to this report, the equations used and method followed will be set forth in detail. For the air component, the ideal gas law and constant specific heats are assumed. For the water or water vapor component, thermodynamic properties are obtained from a subroutine on our computer which calculates the equivalent of the International Steam Tables using ASME equations. As we go through the cycle calculations, we will demonstrate how the results can be verified, to at least four significant figures, using Keenan and Keyes Steam Tables. Because the steam subroutine on our computer is in terms of English units, these will be used in going through a representative sample cycle. We have been converting final results to MKS units.

Table of symbols and constants usedSymbols

P	pressure
RH	relative humidity
P _a	partial pressure of air
V	volume
ρ	density
M	mass
h	specific enthalpy
s	specific entropy
H	total enthalpy
S	total entropy
W _e	work output from expansion
W _c	work input in compression
C _p	specific heat at constant pressure
R	gas constant
Q	cooling required, BTU
^o F	degrees, Fahrenheit
^o R	degrees, Rankine
^o C	degrees, Celsius

Subscripts

sat saturated

a air

wv water vapor

lw liquid water

1.....state at inlet

2.....state after isentropic expansion

3.....state after expansion and after removal of
 liquid water

4.....state after polytropic compression with cooling

Constants

$$^{\circ}\text{R} = ^{\circ}\text{F} + 459.67$$

$$C_{pa} = 0.2399 \frac{\text{BTU}}{\text{lbm}^{\circ}\text{R}} \quad \left(\text{This value was derived from the enthalpy change of air in the Keenan and Kaye Gas Tables over the temperature range in which we are operating.} \right)$$

$$R_a = 53.34 \frac{\text{ft lbf}}{\text{lbm}^{\circ}\text{R}}$$

or

$$R_a = \frac{53.34}{778.16} \frac{\text{BTU}}{\text{lbm}^{\circ}\text{R}}$$

Initial conditions

Ambient conditions are obtained from local climatological data published by the National Climatic Center, Asheville, N.C. We start with air temperature in $^{\circ}\text{F}$ and relative humidity. For the sample calculation, start with

$$T_1 = 77^{\circ}\text{F} = 536.7^{\circ}\text{R}$$

$$\text{RH}_1 = 91\%$$

$$P_1 = 14.7298 \text{ lbf/in}^2$$

This is not an unusual condition, even in winter, for the Gulf Coast of the U.S.

From the steam table, for $T = 77^{\circ}\text{F}$ find the saturation pressure of water vapor.

$$P_{\text{sat wv1}} = 0.4596 \text{ lbf/in}^2 \quad (\text{B-1})$$

Then

$$P_{\text{wv1}} = (RH_1)(P_{\text{sat wv1}}) = 0.41824 \text{ lbf/in}^2 \quad (\text{B-2})$$

$$P_{\text{a1}} = P_1 - P_{\text{wv1}} = 14.3116 \text{ lbf/in}^2 \quad (\text{B-3})$$

$$\text{We take } M_{\text{a1}} = 1 \text{ lbm}$$

Throughout the cycle calculation we will carry 1 lbm of air plus associated water or water vapor.

$$\rho_{\text{a1}} = \frac{P}{RT} = \frac{(14.3116)(144)}{(53.34)(536.67)} = 0.071993 \text{ lbm/ft}^3 \quad (\text{B-4})$$

ρ_{wv1} is obtained indirectly from the steam table.

Specific volume = $694.6 \text{ ft}^3/\text{lbm}$ for saturated vapor.

$$\rho_{\text{wv1}} = \frac{0.91}{694.6} = 0.0013101 \text{ lbm/ft}^3 \quad (\text{B-5})$$

$$M_{\text{wv1}} = \frac{M_{\text{a}} \rho_{\text{wv1}}}{\rho_{\text{a1}}} = 0.018198 \text{ lbm} \quad (\text{B-6})$$

$$h_{\text{a1}} = C_{\text{pa}} T_1 = (0.2399)(536.67) = 128.7471 \text{ BTU/lbm} \quad (\text{B-7})$$

h_{wv1} is obtained from the steam table for saturated vapor. The degree of superheat is so small that no correction is necessary.

$$h_{\text{wv1}} = 1095.1 \text{ BTU/lbm}$$

$$M_{\text{wv1}} h_{\text{wv1}} = (1095.1)(0.018198) = 19.9286 \text{ BTU}$$

Entropy of the air is obtained from

$$s_{\text{a}} = (C_{\text{pa}} - R) \ln T - R \ln \rho_{\text{a}} \quad (\text{B-8})$$

$$s_{\text{a1}} = (0.2399 - \frac{53.34}{778.16}) \ln 536.67 - \frac{53.34}{778.16} \ln 0.071993$$

$$s_{\text{a1}} = 1.2574 \text{ BTU/lbm}^{\circ}\text{R}$$

Entropy of water vapor is obtained from the steam table (Table 3 in Keenan and Keyes) via a double interpolation process. We interpolate between temperatures of 70°F and 80°F to get 77°F and between pressures of 0.4 and 0.6 lbf/in² to get pressure = 0.41824.

We get $s_{wv1} = 2.0553 \text{ BTU/lbm}^{\circ}\text{R}$

$$M_{wv1}s_{wv1} = (0.018198)(2.0553) = 0.0374 \text{ BTU}/^{\circ}\text{R}$$

Total enthalpy:

$$H_1 = M_{a1}h_{a1} + M_{wv1}h_{wv1} = 148.6757 \text{ BTU} \quad (\text{B-9})$$

Total entropy:

$$S_1 = M_{a1}s_{a1} + M_{wv1}s_{wv1} = 1.2948 \text{ BTU}/^{\circ}\text{R} \quad (\text{B-10})$$

Now let us compare our result using the Keenan and Keyes steam tables with what the computer has printed out for state 1:

	<u>From Steam Table</u>	<u>Computer Printout</u>
P_{wv1}	0.41824	0.41786
P_{a1}	14.3116	14.3120
ρ_{a1}	0.071993	0.071995
ρ_{wv1}	0.001310	0.001309
M_{wv1}	0.018198	0.18178
h_{a1}	128.7471	128.7471
$M_{wv1}h_{wv1}$	19.9286	19.9085
s_{a1}	1.2574	1.2574
$M_{wv1}s_{wv1}$	0.0374	0.0374
H_1	148.6757	148.6557
S_1	1.2948	1.2947

It must be recognized that the computer calculates the steam tables to a greater number of significant figures than the four which appear in the Keenan and Keyes steam tables; hence, the slight discrepancies.

State after isentropic expansion

We decide to expand to a temperature of 8°C , which is 46.4°F or 506.07°R . (This decision is based on availability of cooling water at $T = 6^{\circ}\text{C}$).

$$T_2 = 46.4^{\circ}\text{F}$$

Finding P_2 now is an iterative or trial-and-error process, the objective of which is to force the entropy change to be zero, or:

$$\Delta S = M_{a1}s_{a1} + M_{wv1}s_{wv1} - M_{a2}s_{a2} - M_{wv2}s_{wv2} - M_{lw2}s_{lw2} = 0 \quad (B-11)$$

Here the value of letting the CDC 6400 do the job becomes apparent. Let us just verify the state which the computer has found:

$$P_2 = 9.5150 \text{ lbf/in}^2$$

For calculations similar to ones already done in 2.32 above, we will present results generated by the computer and cease comparing with what we would get using the published steam table.

$$P_{wv2} = 0.15548 \text{ lbf/in}^2$$

$$P_{a2} = 9.3596 \text{ lbf/in}^2$$

$$\rho_{a2} = 0.049929 \frac{\text{lbm}}{\text{ft}^3}$$

$$\rho_{wv2} = 0.000516 \frac{\text{lbm}}{\text{ft}^3}$$

$$M_{wv2} = 0.010336 \text{ lbm}$$

$$M_{lw2} = M_{wv1} - M_{wv2} = 0.007842 \text{ lbm}$$

$$M_{a2}s_{a2} = 1.2724 \frac{\text{BTU}}{\text{°R}}$$

$$M_{wv2}s_{wv2} = 0.0221 \frac{\text{BTU}}{\text{°R}}$$

$$M_{lw2}s_{lw2} = 0.0002 \frac{\text{BTU}}{\text{°R}}$$

Total entropy:

$$S_2 = M_{a2}s_{a2} + M_{wv2}s_{wv2} + M_{lw2}s_{lw2} = 1.2947 \quad (B-12)$$

Hence

$$\Delta S_{\text{total}} = S_1 - S_2 = 0 \quad (B-13)$$

It can be noted that the entropy of the air component has increased in this process as the entropy of the water/water vapor component has decreased.

Enthalpy:

$$M_{a2} h_{a2} = 121.4062 \quad \text{BTU}$$

$$M_{wv2} h_{wv2} = 11.1816 \quad \text{BTU}$$

$$M_{lw2} h_{lw2} = 0.1132 \quad \text{BTU}$$

$$H_2 = 132.7010 \quad \text{BTU}$$

Expansion work:

$$\begin{aligned} W_e &= H_1 - H_2 \\ &= 15.9546 \quad \text{BTU} \end{aligned} \quad (\text{B-14})$$

State after removal of liquid water:

In going from state 2 to state 3, the liquid water is removed. The state of the air and water vapor remains the same. Hence:

$$P_3 = P_2 = 9.5150 \quad \text{lb f/in}^2$$

$$T_3 = T_2 = 506.07^\circ \text{R}$$

$$\rho_{a3} = \rho_{a2} = 0.049929 \quad \text{lbm/ft}^3$$

$$M_{wv3} = M_{wv2} = 0.010336 \quad \text{lbm}$$

$$M_{lw3} = 0$$

$$\begin{aligned} H_3 &= H_2 - M_{lw2} h_{lw2} \\ &= 132.5878 \quad \text{BTU} \end{aligned} \quad (\text{B-15})$$

$$\begin{aligned} S_3 &= S_2 - M_{lw2} s_{lw2} \\ &= 1.2945 \quad \text{BTU/}^\circ \text{R} \end{aligned} \quad (\text{B-16})$$

There are at least two possibilities for removing the liquid water. We have used an adiabatic process in which the liquid water is removed after completion of the expansion. Another possibility is a pseudo-adiabatic process (often assumed by atmospheric scientists) in which the liquid water is removed bit by bit as soon as it condenses. For our purposes, computation has verified that the difference between these two processes is negligible.

Compression with cooling

In order to be able to exhaust to the atmosphere, we must compress back to atmospheric pressure. Hence:

$$P_4 = P_1 = 14.7298 \text{ lbs/in}^2$$

We decide to compress with cooling to a final temperature of 12°C . This figure is a compromise. With cooling water available at 6°C , if our final temperature were less than 12°C more cooling water would be required; and, hence, more power would be used to pump the water. On the other hand, if T_4 were lower than 12°C , more work would be available, per pound of air processed, from the cycle.

Let us use:

$$T_4 = 12^\circ\text{C} = 53.6^\circ\text{F} = 513.27^\circ\text{R}$$

Both pressure and temperature for state 4 have been fixed and the final state is thus determined:

$$P_{wv4} = P_{\text{sat}} = 0.2032 \text{ lbf/in}^2$$

$$P_{a4} = P_4 - P_{wv4} = 14.5266 \text{ lbf/in}^2$$

$$\rho_{wv4} = 0.000665 \text{ lbm/ft}^3$$

$$\rho_{a4} = 0.076406 \text{ lbm/ft}^3$$

$$M_{wv4} = 0.008707 \text{ lbm}$$

$$M_{lw4} = M_{wv3} - M_{wv4} = 0.001629 \text{ lbm}$$

(A small amount of water vapor has condensed during the compression.)

Let us assume that during the compression, cooling is supplied at a rate such that:

$$\frac{P}{P_3} = \left(\frac{V_3}{V}\right)^n \quad (\text{B-17})$$

(V is defined as the volume occupied by one pound of air and the associated water vapor. P is static pressure of the moist air.)

If this is true, then:

$$\begin{aligned}
 W_c &= -\int_3^4 V \, dp \\
 &= -\frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]
 \end{aligned}
 \tag{B-18}$$

Notice that during the compression the water/water vapor component does not behave as an ideal gas, in that part of the water vapor condenses. However, our expression for compression work, W_c , is not dependent on the ideal gas law. It is only dependent on the pressure vs volume relationship assumed in Eq. (17).

Next we use:

$$\frac{P_4}{P_3} = \left(\frac{V_3}{V_4} \right)^n
 \tag{B-19}$$

to calculate a value for n .

$$V_3 = \frac{1}{\rho_{a3}} = 20.0284 \text{ ft}^3$$

$$V_4 = \frac{1}{\rho_{a4}} = 13.0880$$

Solving for n ,

$$n = 1.02714$$

We cannot use T_3 and T_4 in computing n , because of the latent heat released by the small amount of water which condensed.

Now use Eq. (18) to calculate the compression work required:

$$W_c = -15.5004 \text{ BTU}$$

Now let us calculate the amount of cooling required:

$$Q = -H_3 + W_c + H_4$$

$$M_{a4} h_{a4} = 123.133 \text{ BTU}$$

$$M_{wv4} h_{wv4} = 9.4469 \text{ BTU}$$

$$M_{lw4} h_{lw4} = 0.035264 \text{ BTU}$$

$$H_4 = M_{a4} h_{a4} + M_{wv4} h_{wv4} + M_{lw4} h_{lw4} = 132.6157 \text{ BTU}$$

Using Eq. (19)

$$Q = -132.5878 - 15.5004 + 132.6157 = -15.4725 \text{ BTU}$$

The net work out of the cycle,

$$W = W_c + W_c = 0.4543 \text{ BTU per lbm of air processed}$$

This can be converted to:

$$W = 1057 \text{ joules/kg of air}$$

or to 1057 watts per kg/sec of air processed.

To examine the change in entropy between state 3 and state 4 we need to include entropy gained by the cooling water. The temperature change of the cooling water is from 6°C to 12°C or $\Delta T = 10.8^\circ\text{F}$. For one pound of air processed, the cooling water required is:

$$\frac{15.4725 \text{ BTU}}{10.8^\circ\text{F}} = 1.4326 \text{ lbm}$$

The increase in entropy of this cooling water is $0.03058 \frac{\text{BTU}}{^\circ\text{R}}$.

We found $S_3 = 1.2945 \text{ BTU}/^\circ\text{R}$.

$$S_4 = M_{a4}S_{a4} + M_{wv4}S_{wv4} + M_{lw4}S_{lw4} = 1.2641 \text{ BTU}/^\circ\text{R}$$

Total entropy change in the universe in going from state 3 to state 4:

$$\Delta S = S_4 - S_3 + \Delta S_{\text{cooling water}} = 0.0002 \text{ BTU}/^\circ\text{R}$$

Total entropy has increased. The second law has not been violated.

Temperature vs entropy diagrams are included for the air, Fig. 4, for the water/water vapor component, Fig. 5, and total entropy, Fig. 6.

Efficiency vs Carnot efficiency

η - efficiency

η_c - Carnot efficiency

$$T_1 = 536.67^\circ\text{R}$$

$$T_4 = 513.27^\circ\text{R}$$

$$\eta_c = \frac{536.67 - 513.27}{536.67} = 4.36\%$$

Let us suppose the work out of the cycle ($W = 0.4543$ BTU) all came from the dry air.

$$H_{a1} = 128.7471$$

$$H_{a't} = 123.1335$$

$$\Delta H_a = 5.6137 \text{ BTU}$$

$$\eta = \frac{W}{\Delta H_a} = 8.09\%$$

This exceeds the Carnot efficiency. The work out did not all come from the dry air. Now use total enthalpies.

$$H_1 = 148.6557$$

$$H_h = 132.6157$$

$$H_{lw2} = 0.1132$$

$$\Delta H = H_1 - H_h - H_{lw2} = 15.9268$$

$$\eta = \frac{W}{\Delta H} = 2.852\%$$

This is less than the Carnot efficiency, as would be expected.

Fig. 4. Temperature vs entropy for air component.

Fig. 5. Temperature vs entropy for water/water vapor component.

Fig. 6. Temperature vs total entropy.

Fig. 7. Mass balance for expansion-compression cycle.

Fig. 8. Energy balance of expansion-compression cycle

Work output as a function of relative humidity

For the cycle just calculated, the input ambient air was at $T = 77^{\circ}\text{F}$, $\text{RH} = 91\%$. Net work out of the cycle is 1056.6 joules per kg of air. The same calculation has been performed for $\text{RH} = 100\%$, $\text{RH} = 50\%$, and $\text{RH} = 0$. Results are summarized in the table below. For all cases, $T_1 = 77^{\circ}\text{F}$; the air is expanded to $T_2 = 46.4^{\circ}\text{F}$ and is compressed with cooling to $T_4 = 53.6^{\circ}\text{F}$.

<u>RH($^{\circ}/\%$)</u>	<u>Work Out (joules/kg of air)</u>
100	1276
91	1057
50	418
0	379

The relationship is plotted in Fig. 9.

Fig. 9. Work output vs relative humidity.

Note that with 100% relative humidity the work out per kg of air processed is about 3.4 times as much as for 0% relative humidity, or dry air.