EFFECTS OF IMPURITY RADIATION ON REVERSED-FIELD PINCH EVOLUTION

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ABSTRACT

We study the effects of impurity radiation on the evolution of a reversed-field pinch plasma by means of a one-dimensional MHD simulation code that includes both plasma transport and impurity effects, and follows the plasma through a series of equilibrium states. The equations are split into two sets, one that contains plasma transport and another that contains atomic physics effects. Two codes were developed and linked together to solve the full problem. Results are presented for the ZT-S, ZT-40, and RFX experiments with a fixed set of parameters, typical or envisioned, and various concentrations of impurities. Radiation barriers are encountered, and limits are found on the acceptable level of low Z impurities that may be present in these devices. It is shown that the temperatures currently observed in the ZT-S experiment are radiation limited. Next, a criterion for radiation barrier burn-through is derived in terms of appropriate nondimensional parameters and calibrated by comparison to the numerical simulations for the case of oxygen. In addition, the subject of thermal instability in the reversed-field pinch is explored. Namely, a current carrying plasma can become thermally unstable due to current diffusion into regions of low resistivity, which through ohmic heating, leads to a lower resistivity feeding an instability. This instability is studied for the case of a reversed-field pinch in a nearly force free equilibrium, after the formation phase, as the plasma attempts to ohmically heat to a significant beta. Two cases, stationary electron temperature (where impurity radiation balances ohmic heating), and non-stationary electron temperature are considered. It is found that fine-scale current perturbations remaining from the formation phase can be greatly enhanced by the presence of impurity cooling. The theory of this phenomena is developed analytically and compared to numerical simulation.

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CHAPTER 1

INTRODUCTION

The reversed-field pinch (RFP) is a promising approach to controlled fusion that has received considerable attention in recent years. It is an axisymmetric toroidal device that has demonstrated theoretically and experimentally MHD stable plasma confinement at the relatively high beta of 30-40%.^(1,2) It has roughly equal poloidal and toroidal magnetic fields which provide the shear that permits MHD stable operations at current densities substantially above the Kruskal-Shafranov limit. Hence, in comparison to the tokamak the RFP has a large toroidal current and the potential to reach fusion temperatures with ohmic heating alone. Therefore the RFP is attractive as a possible fusion reactor.

Unfortunately, recent RFP experiments such as 2T-S⁽³⁾ HBTX-1,⁽⁴⁾ and ETA-BETA⁽⁵⁾ have not been able to fully realize this potential. These experiments are relatively small (~7cm minor radius), use field programming to set up the reversed toroidal field and operate at relatively high densities ($1 \times 10^{15} \text{ cm}^{-3}$). Their main difficulty is a very low electron temperature (30 eV in 2T-S), seemingly due to energy loss caused by radiation from low Z plasma impurities, principally oxygen.

The high particle density in these experiments offsets the advantage of high current density by lowering the Ohmic heating rate per particle and by increasing the radiation loss rate, which for a fixed impurity concentration is proportional

to the square of the electron density. In addition, measurements of oxygen impurities in ZT-S show a concentration of .3% - 1% relative to deuterium ($2 \times 10^{15} \text{ cm}^{-3}$).⁽⁶⁾ This is a higher density of oxygen than is observed in tokamaks and in conjunction with the high electron density may explain why RFP experiments encounter a low temperature radiation barrier and tokamaks do not.

At the low temperatures currently observed in these devices, the transport of particles and magnetic fields in the post-implosion phase, where magnetic forces and pressure gradients are in magnetohydrostatic equilibrium, can be explained by classical coefficients. The discharge terminates because the initially stable magnetic field profiles diffuse to unstable ones on the classical diffusion time.⁽⁷⁾ Whether the transport remains classical at temperatures of a few hundred electron volts is unknown. Hence, it is imperative that future RFP experiments, such as ZT-40 and RFX, overcome low temperature radiation barriers so that the potential of this concept can be fully explored, and anomalous transport processes, if any, identified.

It is the purpose of this paper to study theoretically some of the effects of impurities in the RFP with numerical and analytical methods. We have developed a computer code based on the equations of MHD, which includes both transport and radiation effects in the post-implosion phase, to simulate existing RFP experiments and to predict results for

future ones. This code enables us to normalize analytical treatments of radiation barrier burn-through and thermal instability.

Section 2 presents the physics model used in the numerical computations; section 3 presents results for typical operating parameters of ZT-S, ZT-40, and RFX with various concentrations of impurities. Section 4 develops a general criterion, in terms of nondimensional parameters, for whether a RFP will burn through a radiation barrier. This criterion is normalized by comparison with numerical results for the case of oxygen. Section 5 discusses the subject of thermal instabilities in the RFP. We find an instability due to the coupling of current diffusion and Ohmic heating. This instability was first derived by Kadomtsev, who calls it the "Superheating Instability."⁽⁸⁾ He assumed a uniform magnetic field, small current and a stationary zeroth order temperature that is the same for electrons and ions. We remove these restrictions and develop the theory of this instability in the post-implosion phase of the RFP, in cylindrical geometry, for both stationary and evolving zeroth order electron and ion temperatures.

The importance of impurities in creating fine-scale current density and temperature gradients in RFP's is demonstrated.

CHAPTER 2

EQUATIONS

Our basic approach to investigating the combined effects of impurity radiation and classical resistive diffusion on the evolution of plasma and magnetic field profiles in the RFP is to split the problem into two parts. The resistive diffusion is governed by the two fluid MHD equations given by Braginskii.⁽⁹⁾ Our computations of impurity radiation rests on a multifluid MHD model which provides heat sink terms in the electron temperature equation but is otherwise ideal. Electrons, deuterium ions, and each charge state of an impurity species are modelled as separate fluids. These two sets of equations taken together define our complete physical model.

Both sets of equations are transformed to a Lagrangian coordinate system based on a normalized poloidal flux coordinate. The dependent variables are changed to quantities that remain invariant during adiabatic motion of the plasma. This representation of the equations is useful for both numerical and analytical work.

2.1. Transport Model Equations

Starting with the full two fluid MHD equations in Ref (9), we assume that the plasma evolves in time through a series of equilibrium states. This is equivalent to ordering the resistivity, velocity, and the time derivative of any quantity as a small number ϵ and then neglecting terms of order ϵ^2 . Thus the inertia and all terms arising from the plasma

viscosity are neglected. This yields a set of equations valid on the resistive diffusion time scale and is equivalent to ordering the velocity small in comparison to the Alfvén speed. Cylindrical geometry and symmetry is assumed so that all quantities vary only in the radial direction. With these approximations the equations become:

$$\frac{\partial n}{\partial t} + \vec{v} \cdot (\vec{v}n) = 0 \quad , \quad (1)$$

$$\frac{\partial B_\theta}{\partial t} + \frac{\partial}{\partial r} (v_r B_\theta - \frac{cR_z}{en}) = 0 \quad , \quad (2)$$

$$\frac{\partial B_z}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} r(v_r B_z + \frac{cR_\theta}{en}) = 0 \quad , \quad (3)$$

$$\frac{3}{2} \frac{\partial}{\partial t} nT_e + \frac{3}{2} \vec{v} \cdot \vec{v} nT_e + nT_e \vec{v} \cdot \vec{v} = -\vec{v} \cdot \vec{q}_e + \frac{\vec{R} \cdot \vec{j}}{en} - Q_i \quad , \quad (4)$$

$$\frac{3}{2} \frac{\partial}{\partial t} nT_i + \frac{3}{2} \vec{v} \cdot \vec{v} nT_i + nT_i \vec{v} \cdot \vec{v} = -\vec{v} \cdot \vec{q}_i + Q_i \quad , \quad (5)$$

$$\frac{\partial p}{\partial r} = \frac{-1}{4\pi} [B_z \frac{\partial}{\partial r} B_z + \frac{B_\theta}{r} \frac{\partial}{\partial r} rB_\theta] \quad , \quad (6)$$

where $p = n(T_e + T_i)$, $\vec{R} = \vec{R}_u + \vec{R}_T$ is the total force between electrons and ions due to friction $\vec{R}_u$ and temperature gradient $\vec{R}_T$, $\vec{q}_e$ and $\vec{q}_i$ are the electron and ion heat fluxes respectively, Q_i is the electron-ion equilibration term, and $\vec{j}$ is the current.

$$\vec{R}_u = en\eta_n \vec{j}_n + en\eta_i \vec{j}_i$$

$$\vec{R}_T = s_\lambda^{uT} \vec{v}_T \times \frac{\vec{B}}{|B|}$$

$$\vec{Q}_e = -\chi_e^e \frac{\partial T_e}{\partial r} \hat{r} + \beta_\lambda^{uT} \frac{cT_e}{en|B|} \frac{\partial p}{\partial r} \hat{r}$$

$$\vec{Q}_i = -\chi_i^i \frac{\partial T_i}{\partial r} \hat{r}$$

$$Q_i = \frac{3m_e n}{M \tau_e} (T_e - T_i)$$

$$\vec{j} = \frac{c}{4\pi} \vec{v} \times \vec{B}$$

For the resistivity η , the thermal conductivity χ , and the thermoelectric coefficient β_λ^{uT} , the classical values in the strong field limit are assumed,

$$\eta_n = \frac{.513 m_e}{e^2 n \tau_e} ,$$

$$\eta_i = \frac{m_e}{e^2 n \tau_e} ,$$

$$\chi_e^e = \frac{4.66 n^2 T_e c^2}{B^2} \eta_i ,$$

$$\chi_i^i = \frac{2nT_i}{M\omega_{ci}\tau_i}$$

$$\beta_\lambda^{uT} = \frac{3}{2} \frac{en^2 c}{B} \eta_i ,$$

where $\omega_{ci} = eB/MC$. The electron and ion collision times, τ_e and τ_i , are,

$$\tau_e = \frac{3m_e^{1/2} T_e^{3/2}}{4(2\pi)^{1/2} \lambda e^4 n}$$

$$\tau_i = \frac{3m_i^{1/2} T_i^{3/2}}{4(2\pi)^{1/2} \lambda e^4 Z^2 n}$$

and λ is the Coulomb logarithm.

In our model we assume that the plasma extends to the vacuum chamber wall at radius a , and that there may be a conducting wall at a larger radius b . The region between these two radii is a magnetic flux reservoir.

The boundary conditions at the plasma edge on Eqs.

(1) - (6) are the electric fields E_z and E_θ calculated from values given at the conducting wall and specified "pedestal" values for n , T_e , and T_i . The electric field is not only the natural quantity to use as a boundary condition when magnetic flux is a variable, but also the quantity controlled by external circuits. Magnetic flux variables occur in the Lagrangian representation of these equations.

The origin is an artificial boundary due to the use of cylindrical coordinates. Here we require there be no flux of any quantity.

2.2. Radiation Model Equations

An ideal MHD model, which includes all radiation effects and treats every charge state of a given impurity species as a separate fluid, is now presented. There is adiabatic expansion and contraction of the plasma but no transport. The equations describing this model are given for one impurity species of charge Z . The extension to more than one impurity species is obvious.

$$n_e = n_i + \sum_{j=1}^{Z+1} (j-1)n_j \quad , \quad (7)$$

$$\frac{\partial n_i}{\partial t} + \vec{v} \cdot \vec{\nabla} n_i = 0, \quad (8)$$

$$\frac{\partial n_j}{\partial t} + \vec{v} \cdot \vec{\nabla} n_j = n_e [n_{j-1} \beta_{j-1} - n_j (\beta_j + \alpha_j) + n_{j+1} \alpha_{j+1}], \quad (9)$$

$$\frac{\partial \vec{B}}{\partial t} - \vec{v} \times (\vec{v} \times \vec{B}) = 0, \quad (10)$$

$$\frac{3}{2} \frac{\partial n_e T_e}{\partial t} + \frac{3}{2} \vec{v} \cdot \vec{\nabla} n_e T_e + n_e T_e \vec{v} \cdot \vec{\nabla} = n_e \sum_j n_j A_j \quad , \quad (11)$$

$$\frac{3}{2} \frac{\partial}{\partial t} n_i T_i + \frac{3}{2} \vec{v} \cdot \vec{\nabla} n_i T_i + n_i T_i \vec{v} \cdot \vec{\nabla} = 0 \quad , \quad (12)$$

$$\vec{v}_p = \frac{\vec{j}}{c} \times \vec{B} \quad (p = n_e T_e + n_i T_i + T_i \sum_j n_j) \quad , \quad (13)$$

where,

α_j = coefficient for recombination from state
j to j-1 ($\alpha_1 = 0$),

β_j = coefficient for ionization from state j - 1 to
j ($\beta_{Z+1} = 0$).

A_j is the energy loss rate per electron per impurity ion of charge j+1 due to excitation and recombination radiation and bremsstrahlung. It does not currently include the energy loss from electrons due to the ionization potential of the impurity. This energy is generally small compared to the energy lost by radiation during the ionization process, but it will be included in future work. All ion species have been assumed to have the same temperature. (10)

The boundary condition imposed on this set of equations is that the electric field, and thus the radial velocity, vanish at the vacuum chamber wall. Consequently, there is no flux of any quantity at $r = a$.

2.2.1. Atomic Physics

For the range of operating densities ($n < 2 \times 10^{15} \text{ cm}^{-3}$) and time scales ($t > 10 \mu\text{s}$) relevant to RFP experiments, the time-dependent coronal atomic physics model is valid. (11,12,13) In this model, ionization and excitation occur by the collision of an electron with an atom or ion in its ground state; recombination is to the ground state by means of radiative and dielectric recombination; de-excitation is due to spontaneous emission and occurs on a time scale short compared

to all other processes. This model ignores collisional recombination, collisional de-excitation, photo-ionization, and photo-excitation.

It is assumed that all radiation is lost from the plasma. This is true even if the plasma is not optically thin (though RFP's are generally optically thin), provided that the collisional de-excitation time is sufficiently long. (13)

The ionization and recombination coefficients and radiation loss rates are obtained, for an arbitrary element, from an atomic physics code developed by Dr. Russell Hulse. This code utilizes generalized formulae given in Ref. (14). For each ionic species of the element under consideration, it calculates energy levels, oscillator strengths, and transition energies. The generalized formulae for the rates of electron collisional ionization, radiative recombination, and dielectronic recombination are evaluated for each species using these oscillator strengths and energies together with the given plasma density and electron temperature. Similarly, radiation rates are computed for each ionic species, including excitation of $\Delta n = 0$ and $\Delta n \neq 0$ line transitions, radiation associated with the recombination processes, and bremsstrahlung.

2.3. Poloidal Flux Coordinate System

The two sets of equations (1) - (6) and (7) - (13) must be solved numerically, but are not in a convenient form for numerical solution because the velocity in these equations is determined indirectly by the magnetohydrostatic equilibrium constraint. By transforming to a Lagrangian coordinate system and by changing the dependent variables to adiabatic invariant quantities, this velocity can be eliminated and the equations so obtained can be directly advanced in time.

Our Lagrangian coordinate system is defined by:

$$x(r,t) = \int_0^r x(r',t) dr', \quad \frac{\partial x}{\partial t} + \frac{\partial}{\partial r} v_x = 0 \quad (14a,b)$$

$$\frac{\partial}{\partial t_L} = \frac{\partial}{\partial t} + v \frac{\partial}{\partial r}, \quad \frac{\partial r}{\partial t_L} = v \quad (14,c,d)$$

where $\frac{\partial}{\partial t_L}$ is the Lagrangian time derivative and

$$x = \frac{B_\theta}{\phi_{PT}}, \quad \phi_{PT}(t) = \int_0^a B_\theta(r,t) dr,$$

$$v = v_r - \frac{cR_z}{enB_\theta} + \frac{F(t)}{B_\theta} + \frac{x}{B_\theta} \frac{\partial \phi_{PT}}{\partial t},$$

$$F(t) = \frac{cR_z}{en} (r = 0, t),$$

$$\frac{\partial \phi_{PT}}{\partial t} = cE_z(r = a, t) - F.$$

$\chi(r, t)$ is the Lagrangian coordinate defined as the poloidal flux at a given radius divided by the total poloidal flux ϕ_{PT} in the system at any given time. V is the velocity followed in the Lagrangian frame. It contains a term F/B_θ that is a consequence of poloidal flux destruction at the origin, and another term $\frac{\chi}{B_\theta} \frac{\partial \phi_{PT}}{\partial t}$, that is due to normalizing R_θ by ϕ_{PT} in the definition of χ . Physically, this latter term relabels the dependent variables so that the domain of the independent variable χ remains $0 \leq \chi \leq 1$.

For this transformation to be valid, $V(r = 0, t)$ must equal zero. This requires that n , T_e , and j_z be parabolic near the origin, a result that follows from the diffusive nature of the equations.

2.4. Transport Model Equations in Lagrangian Form

Equations (1) - (6) are transformed in nondimensional form to the Lagrangian coordinate system defined by Eqs. (14). Let us define a reference poloidal flux ϕ_0 , temperature T_0 , and length a (vacuum wall radius) and construct a reference resistivity η_0 , time τ_0 , and density n_0 from the relations

$$\eta_0 = \frac{4(2\pi)^{1/2}}{3} \frac{\pi e^{1/2} a^2}{T_0^{3/2}} \lambda(T_0), \quad \tau_0 = \frac{\pi a^2}{c^2 \eta_0},$$

$$\beta_0 = \frac{8\pi n_0 T_0 a^2}{\phi_0^2} = 1.$$

Then the nondimensional time τ and resistivity $\bar{\eta}$ are $\tau = \frac{t}{\tau_0}$ and $\bar{\eta} = \frac{\eta}{\eta_0}$.

We choose $Y(X, t) = r^2/a^2$ as the radial coordinate. Then from Eqs. (14) $Y' \equiv \frac{\partial Y}{\partial X} = \frac{2r\phi_0}{a^2 B_\theta}$ (where $\phi = \phi_{PT}/\phi_0$) is the nondimensional volume per normalized poloidal flux. This is used to construct new dependent variables. In place of the Eulerian variables B_z, n, T_e , and T_i we define:

$$P = \frac{aB_z}{2\phi_0} Y', \quad (15a)$$

$$N = \frac{n_e}{2n_0} Y', \quad (15b)$$

$$\theta_e = \frac{T_e}{T_0} Y'^{2/3}, \quad (15c)$$

$$\theta_i = \frac{T_i}{T_0} Y'^{2/3}, \quad (15d)$$

P is the toroidal flux per normalized poloidal flux or the pitch of a field line, an important quantity in considering the MHD stability of the RFP. (15) N is the number density per normalized poloidal flux; θ_e and θ_i are a measure of the electron and ion entropy per particle respectively. Using these variables, one can recast equations (1) - (6) into a pitch equation

$$\frac{\partial P}{\partial \tau} = \frac{\partial}{\partial X} \left[\left(\frac{X\phi_0}{\phi} + \frac{f}{\phi} - \frac{\tilde{n}_n}{Y'} \right) P + \frac{\tilde{n}_n Y}{Y'^2} \frac{\partial P}{\partial X} \right], \quad (16)$$

and a density equation

$$\frac{\partial N}{\partial \tau} = \frac{\partial}{\partial X} UN, \quad (17)$$

where

$$U = \frac{\dot{X}\phi}{\phi} + \frac{f}{\phi} + \frac{\tilde{n}_e y^2 P}{y^2 (P^2 + Y\phi^2)} \frac{\partial}{\partial X} \left(\frac{P}{Y} \right) +$$

$$\frac{\tilde{n}_e Y}{4(P^2 + Y\phi^2)} \left[\frac{\partial}{\partial X} \frac{N(\theta_e + \theta_i)}{Y^{5/3}} - \frac{3N}{2Y'} \frac{\partial}{\partial X} \left(\frac{\theta_e}{Y^{2/3}} \right) \right].$$

The electron temperature equation becomes

$$\frac{\partial \theta_e}{\partial \tau} = \frac{\partial}{\partial X} U \theta_e - \frac{1}{3} \theta_e \frac{\partial U}{\partial X} + \frac{Y^{2/3}}{6N} \frac{\partial}{\partial X} \left[\frac{\tilde{n}_e Y \phi_e}{Y' (P^2 + Y\phi^2)} \right.$$

$$\times \left[4.66 N^2 \frac{\partial}{\partial X} \frac{\theta_e}{Y^{2/3}} - \frac{3}{2} Y' N \frac{\partial}{\partial X} \frac{N(\theta_e + \theta_i)}{Y^{5/3}} \right] \quad (18)$$

$$- \Lambda \tilde{n}_e \frac{N(\theta_e - \theta_i)}{2Y'} + \frac{8\phi^2 \tilde{n}_e Y^{5/3}}{(P^2 + Y\phi^2) N} \left(\frac{Y}{Y'} \right)^4 \left[\frac{\partial}{\partial X} \left(\frac{P}{Y} \right) \right]^2$$

$$+ \frac{\tilde{n}_e Y Y^{5/3}}{6N (P^2 + Y\phi^2)} \left[\frac{\partial}{\partial X} \frac{N(\theta_e + \theta_i)}{Y^{5/3}} \right]^2$$

$$- \frac{\tilde{n}_e Y Y^{2/3}}{4(P^2 + Y\phi^2)} \left[\frac{\partial}{\partial X} \frac{\theta_e}{Y^{2/3}} \right] \left[\frac{\partial}{\partial X} \frac{N(\theta_e + \theta_i)}{Y^{5/3}} \right],$$

the ion temperature equation is

$$\frac{\partial \theta_i}{\partial \tau} = \frac{\partial}{\partial X} U \theta_i - \frac{1}{3} \theta_i \frac{\partial U}{\partial X} + \frac{\Lambda \tilde{n}_e N(\theta_e - \theta_i)}{2Y'} \quad (19)$$

$$+ \frac{2}{3} \left(\frac{M}{m} \right)^{1/2} \frac{Y^{2/3}}{N} \left[\frac{\partial}{\partial X} \left(\frac{Y N^2}{\theta_i^{1/2} Y^{2/3} (P^2 + Y\phi^2)} \frac{\partial}{\partial X} \frac{\theta_i}{Y^{2/3}} \right) \right],$$

and the magnetohydrostatic equilibrium equation takes the form

$$\frac{\partial^2 \psi}{\partial X^2} = Y' \left[\frac{\frac{\partial}{\partial X} (P^2 + 2Y\phi^2) + \frac{1}{2} Y'^{1/3} \frac{\partial}{\partial X} N(\theta_e + \theta_i)}{2(P^2 + Y\phi^2) + 5/6 Y'^{1/3} N(\theta_e + \theta_i)} \right]. \quad (20)$$

The net rate of change of the normalized poloidal flux $\dot{\phi}$ is determined by $\dot{\phi} = e_z - f$, where e_z and f are the nondimensional z electric fields at the plasma edge and

$$f = (\phi \tilde{n}_n / Y') \quad x = 0$$

In the electron-ion equilibration term,

$$\Lambda = \phi_0^2 e^2 / T_0 M c^2$$

is the square of the plasma radius divided by the reference ion gyroradius.

Unlike Eqs. (1) - (6), Eqs. (16) - (20) can be directly advanced in time. First P, N, θ_e, θ_i and ϕ are advanced; then equation (20) can be solved for the new equilibrium $\psi(X, t)$. In addition, equations (16) - (20) display the physics more clearly than did their Eulerian counterparts. For instance, the pitch P depends only on the parallel electric field $\eta_n j_n$, and the current $\vec{j}$ has been resolved into its parallel ($j_n = \partial/\partial x B_z / r B_\theta = \partial/\partial x P/Y$) and perpendicular ($j_\perp = \frac{\partial P}{\partial r} = \frac{\partial}{\partial x} N(\theta_e + \theta_i)$) components rather than along the θ and z directions, which have no physical significance.

2.4.1. Boundary Conditions

The boundary condition on Eqs. (16) - (19) at $x = 0$ is that the flux of P, N, θ_e and θ_i vanish, which is automatically satisfied by the definition of f .

The pedestal values of density and temperature, specified at the wall for Eqs. (1) - (6), become for Eqs. (16) - (20),

$$\left(\frac{N}{Y^T}\right)_{x=1} = \text{const.}, \quad (21a)$$

$$\left(\frac{\theta_e}{Y^{2/3}}\right)_{x=1} = \text{const.}, \quad (21b)$$

$$\left(\frac{\theta_i}{Y^{2/3}}\right)_{x=1} = \text{const.} \quad (21c)$$

The z electric field is specified at the plasma edge $\epsilon_z(x=1)$, and along with f determines $\dot{\phi}$, which enters as a convective term in Eqs. (16-19). The boundary condition on the pitch equation (16) at $x=1$ is the poloidal electric field ϵ_θ . Given the poloidal electric field at the conducting wall V_θ , ϵ_θ at the plasma edge can be expressed solely in terms of plasma quantities. This yields a time dependent boundary condition for the pitch equation at $x=1$

$$V_\theta + \left(\frac{b^2}{a^2} - 1\right) \frac{\partial P}{\partial \tau} = - \epsilon_z P - \frac{\tilde{n}_n Y^2}{Y^{1/2}} \frac{\partial}{\partial X} \left(\frac{P}{Y}\right). \quad (22)$$

The boundary condition on the magnetohydrostatic equilibrium equation (20) at $x=0$ and at $x=1$ is

$$Y(x=0) = 0 \quad , \quad Y(x=1) = 1 \quad . \quad (23a,b)$$

2.5. Radiation Model Equations in Lagrangian Form

In the Lagrangian coordinate system defined by Eq. (14) the radiation model equations (7)-(13) become in nondimensional form,

$$N_e = N_i + \sum_{j=1}^{Z+1} (j-1) N_j \quad (24)$$

$$\frac{\partial N_i}{\partial \tau} = 0 \quad (25)$$

$$\frac{\partial N_j}{\partial \tau} = \frac{N_e}{Y'} \{ N_{j-1} \tilde{\beta}_{j-1} - N_j \{ \tilde{\beta}_j + \tilde{\alpha}_j \} + N_{j+1} \tilde{\alpha}_j \} \quad (26)$$

$$\frac{\partial P}{\partial \tau} = 0 \quad (27)$$

$$\frac{\partial S_e}{\partial \tau} = \frac{N_e}{Y' 1/3} \sum_j N_j \tilde{A}_j \quad (28)$$

$$\frac{\partial S_I}{\partial \tau} = 0 \quad (29)$$

$$\frac{\partial^2 Y}{\partial X^2} = Y' \left[\frac{\frac{\partial}{\partial X} (P + 2Y\phi^2) + \frac{1}{2} Y' 1/3 \frac{\partial}{\partial X} (S_e + S_I)}{2(P^2 + Y\phi^2) + \frac{5}{6} Y' 1/3 (S_e + S_I)} \right] \quad (30)$$

where $N_j = \frac{2n_j}{n_0} Y'$, and $S_e = N_e \theta_e$, $S_I = N_i \theta_i + \theta_i \sum_j N_j$ are a measure of the electron and ion entropies respectively. α_j , β_j , and $\bar{A}_j$ are the nondimensional form of α_j , β_j , and A_j defined previously.

Note that in the Lagrangian representation all of the radiation model equations have become ordinary differential equations. A convective velocity, common to all the plasma particle species and the magnetic fields, caused by radiation energy loss, has been explicitly eliminated!

2.6. Summary of the Physics Model

The multi-fluid MHD equations, where each charge state of an impurity is treated as a separate fluid and the plasma evolves through a series of equilibria, has been split into two independent and self-consistent sets. This decomposition was motivated by noticing that the velocity, determined by all dissipative processes, separates into two components, one which arises from plasma resistivity and another from atomic processes. The original set of equations is approximated by numerically solving the two simpler sets in succession.

In the transport model equations the impurity ions have been neglected; therefore, the number of ions is not the same in the two models. In successively solving these two sets of equations the electron and ion entropy and the electron density are kept constant to insure that the same

equilibrium is passed between the two models. When the transport model equations are advanced in time, the impurity density is held fixed in physical space. Impurity diffusion can be important and will be treated in future work.

Standard numerical techniques are used to solve both sets of equations, the details of which will be presented elsewhere. (16)

CHAPTER 3

RESULTS FOR POINT CASES

Results of the numerical solution of our physics model for various concentrations of impurities are presented for ZT-S, ZT-40, and RFX with a fixed set of parameters, typical or envisioned, for each experiment. The effect of oxygen, and also aluminum for ZT-40, on the electron and ion temperatures and the magnetic energy is studied.

The parameters chosen to simulate these experiments are given in Table 1. In Figure 1 the normalized pitch $P = \frac{rB_z}{aB_\theta}$ and the parallel current density $j_{||}$ versus r/a are shown at the initial time. The pitch is the same for all three cases. The parallel current density, given by the dashed line for ZT-S and ZT-40 and by the dash-dot line for RFX, has a 60% peak off axis corresponding to a current sheath in the early post-implosion phase. The hydrogen density is specified by a parabolic distribution which decreases from a maximum on axis to a value at the vacuum chamber wall of 5% of the on axis density (see Table 1). The initial electron and ion temperature is assumed to be uniform at 10 eV for all cases; the electric fields at the conducting wall are set to zero; the pedestal values of density and temperature at the vacuum chamber wall are given by their initial value or the value at the point next to the wall, whichever is less.

3.1. Results for ZT-S

In Fig. 2 is shown the B_z and B_θ fields for the ZT-S simulation at the initial time and at 35 μ s (the lifetime of a typical ZT-S discharge) for 0% and .4% oxygen.

For the impurity free case the total Z current ($-B_\theta(a)$) in the plasma at 35 μ s is greater (by about 5%) than the initial, even though the energy contained in the poloidal field has decreased. This is because the diamagnetic current created by ohmic heating has a substantial component along the Z direction in the central and outer regions of the plasma. The numerical simulations show this to be a characteristic feature of diffuse RFP profiles.

The degradation of the magnetic field profiles from enhanced ohmic heating due to low electron temperatures caused by radiation from oxygen impurities can also be seen in Fig. 2. At 35 μ s one third more magnetic energy has gone into ohmic heating in the .4% oxygen case than in the zero impurity case.

In Fig. 3(a), (b), and (c) results of ZT-S simulations are given for 0%, .2% and .4% oxygen impurity, distributed as a constant fraction of the hydrogen density ($1 \times 10^{15} \text{ cm}^{-3}$ on axis). In Fig. 3(a) the average electron temperature $\bar{T}_e$, defined as

$$\bar{T}_e \equiv \frac{1}{n_L} \int_0^a 2\pi n T_e r dr$$

(where $n_L = \int_0^a 2\pi n r dr$ is the line density), is plotted as a function of time for these three cases. On the right-hand-

side of Fig. 3(a) is labeled the nondimensional quantity β_E , defined as

$$\beta_E \equiv \frac{1}{W_p(0)} \int_0^a 2\pi n T_e r dr$$

where $W_p(0) = \int_0^a \frac{B_\theta^2}{4} (r,0) r dr$ is the poloidal magnetic energy at the initial time. β_E is a measure of the electron poloidal beta (also a nondimensional temperature) referenced to the initial poloidal magnetic energy. This energy is roughly the amount of magnetic energy available to ohmic heat the plasma. Analogous quantities $\bar{T}_i$ and β_i , defined with respect to the ion temperature T_i , are given as a function of time in Fig. 3(b). In Fig. 3(d) the internal energy $W_I = \int_0^a 3\pi n(T_e + T_i) r dr$, the magnetic energy $W_M = \int_0^a (B_z^2 + B_\theta^2)/4r dr$, and the energy radiated until the current time $W_R = \int_0^t \int_0^a 2\pi n \sum_{j=1}^{z+1} n_j A_j r dr$ are plotted as a function of time for all three cases.

From Fig. 3(a) one sees that for the impurity free case T_e rises steadily in time but at a decreasing rate due to the $T_e^{-3/2}$ dependence of the resistivity. For .2% oxygen impurity $\bar{T}_e$ also increases steadily but at a rate that is substantially less than the zero impurity case after the first 5-10 μ s. The 4% oxygen impurity case shows a radiation limited electron temperature for 2T-S. Here the average electron temperature rises rapidly from 10 eV to 24 eV in the first 10 μ s and then remains flat for the next 50 μ s. What we see is that after the first few microseconds the plasma encounters the oxygen radiation barrier which is sharply peaked at about 25 eV (See Fig. 12).

In Fig. 3(c) the electron temperature as a function of radius is shown at 35 μ s for the zero impurity and for the .4% oxygen impurity case. It is seen that the electron temperature remains between 20 eV and 30 eV nearly everywhere for the latter case. There is a temperature peak off axis due to the combined effect of the off axis current peak and the falling density.

Fig. 3(d) shows for the .4% oxygen case that although the internal plasma energy is constant after 10 μ s, the magnetic energy continues to decrease rapidly, appearing as radiation energy loss (the flux of energy at the plasma edge is small). At 35 μ s nearly 60% of the magnetic energy that has been converted into plasma energy has been lost to radiation. For the .2% oxygen case this number is reduced to 35%.

From Fig. 3(b) one sees that the average ion temperature is more nearly equilibrated with the average electron temperature for a higher concentration of impurity since the resistivity and thus the electron-ion equilibration rate is increased. The density profile for all cases remains relatively unchanged from its initial value, although the electron density increases somewhat due to impurity ionization.

All of this demonstrates that for a .4% oxygen concentration at $1 \times 10^{15} \text{ cm}^{-3}$ density the temperatures in the ZT-S experiment are very radiation limited. Since this amount of impurity is at the low range of the measured amount of oxygen in this device⁽⁶⁾ (.3%- 1% oxygen at $2 \times 10^{15} \text{ cm}^{-3}$ density), it is seen that ZT-S has not been able to operate in a regime where impurity radiation is not the principal physics effect.

The major uncertainty in the results leading to the above conclusion is the manner in which the oxygen impurity is spatially distributed. For instance, if all of the oxygen were concentrated very near the edge, the effect on the plasma could be slight. However, this is not expected since it is well known that for classical transport the impurity ions always diffuse preferentially toward the hydrogen maximum. (17)

To see what happens for a different spatial distribution of impurity, we show in Fig. 4 the average electron temperature as a function of time for oxygen distributed uniformly in space at a concentration of 0%, .2%, and .4% that of the line density. Fig. 4 shows the same qualitative behavior for $\bar{T}_e$ as did Fig. 3(a). At 35 μ s there has been 30% less energy radiated by the .4% uniform oxygen density case. This is not significant enough to modify our above conclusion.

3.2.1. Results for ZT-40

ZT-40 simulations were performed for the operating parameters listed in Table 1 for both oxygen and aluminum as an impurity since this device has an aluminum oxide ceramic vacuum vessel. In Fig. 5 results are shown for 0%, .4%, .8%, and 1.6% oxygen impurity distributed as a constant fraction of the hydrogen density ($5 \times 10^{14} \text{ cm}^{-3}$ on axis). The first three cases are run to 1.5 ms and the last to 150 μ s.

In Figs. 5(a) and 5(b) the average electron and ion temperatures $\bar{T}_e$ and $\bar{T}_i$ are given as a function of time for these four cases. For the 1.6% oxygen case $\bar{T}_e$ undergoes a

sharp change in slope at about 30 eV. The temperature in the inner 7 cm of the discharge becomes radiation limited, but a middle region of the discharge where the current density is maximum does burn through the radiation barrier. Only this middle region contributes to the increase in $\bar{T}_e$. Thus one sees in Fig.5(a) a distinct difference between a case where a significant portion of the plasma is radiation limited and one where the radiation barrier is overcome at all spatial points.

Because of the low value of β (less than 2.5%) the electron thermal conductivity is not effective in transporting energy from the hot middle region to the cold inner region. In fact, the principal classical transport process is magnetic field diffusion (see section 5). The current in the radiation limited region diffuses into the hot, lower resistivity middle region of the discharge, further exacerbating the situation in the cold region. This leads to very large electron temperature gradients as is illustrated in Fig.5(c), where the electron temperature is plotted as a function of radius at 150 μ s for the zero impurity and 1.6% oxygen impurity case. This is markedly different from ZT-S (see Fig.3(c)) where the electron temperature profile was radiatively depressed by approximately the same amount throughout the middle and inner regions, so that the size of the temperature gradient was not substantially different from the impurity free case. This qualitative

difference between these radiation-limited cases (note that these two cases follow the expected density squared dependence of the radiation loss rate for a fixed impurity concentration) is due to the large difference in beta for these devices ($\beta \sim 2\%$ for ZT-40; $\beta \sim 25\%$ for ZT-S). This follows from the fact that $\beta \sim n/a^2$ for a fixed current density.

In Fig.5(d) the internal, magnetic, and radiated energy is given as a function of time. Unlike in ZT-S, the radiated energy is only a small fraction of the magnetic energy. This is again a consequence of the low value of beta in ZT-40. Thus ZT-40 should be able to radiate a substantial portion of its ohmic heating power for a long time, slowly surmounting a radiation barrier, without depleting a large amount of its magnetic energy reservoir.

3.2.2. Oxygen Plus Aluminum

The radiation cooling curve for aluminum⁽¹⁴⁾ is much the same as that for oxygen (Fig. 12) except that the peak is shifted to about 100 eV and broadened. Thus it is to be expected that a given amount of aluminum will present a much more difficult radiation barrier to burn through than the same amount of oxygen since the ohmic heating power at the peak has decreased by a factor of 8.

To study the effect of aluminum as well as oxygen on ZT-40, the .4% oxygen case shown in Fig. 5 was repeated with .2% and .4% aluminum added to it. These results are shown in Figure 6.

In Figs. 6(a) and 6(b) $\bar{T}_e$ and $\bar{T}_i$ are given as a function of time for four cases: zero impurity, .4% O, .4% O plus .2% Al, and .4% O plus .4% Al. We see that radiation from aluminum is severe after $\bar{T}_e$ rises above 75 eV and completely dominates the energy loss from oxygen. The .2% Al case burns through the aluminum radiation barrier everywhere. In the inner portion of the plasma this burn through is slow, as it takes nearly a millisecond for the temperature in the central region to rise above 100 eV. For the .4% Al case the temperature in the inner 6 cm of the plasma is radiation limited at a temperature of about 60 eV while temperatures in the middle portion of the plasma continue to rise. The current in this inner region diffuses away. Thus we see on a longer timescale the same qualitative phenomena as occurred for the oxygen radiation barrier.

Figure 6(c) shows that the energy radiated by aluminum can be a significant fraction (greater than 10%) of the magnetic energy. At one millisecond, for the .4% Al case, 10% of the initial magnetic energy has been lost to radiation (80% of this due to aluminum), which is greater than the internal energy contained in the plasma at this time.

3.3. Results for RFX

RFX has a much lower current density than ZT-40 and therefore less ohmic heating power. However it also has a radius three times as large and is expected to operate at a low density ($5 \times 10^{13} \text{ cm}^{-3}$ on axis) and on a long time-scale. (18) From Fig. 1 we see that for the parameters chosen here it has a beta of about half that of ZT-40 for the same temperature and one order of magnitude lower density. Because of this lower density the fractional concentration of impurities is greater.

In Figs. 7(a) and 7(b) $\bar{T}_e$ and $\bar{T}_i$ are given as a function of time for 0%, 2%, and 4% oxygen impurity distributed as a constant fraction of the hydrogen density. The impurity free and 2% oxygen cases are run to 5 ms and the 4% oxygen case to 1 ms. In the 2% oxygen case at 1 ms 45% of the ohmic heating power has gone into radiation. At 5 ms the amount of ohmic heating power lost is 33%. The temperatures for the 4% oxygen case are radiation limited in the inner 20 cm of the discharge. $\bar{T}_e$ shows the same characteristic behavior as the radiation limited case of ZT-40 in Fig. 5(a). At 1 ms 75% of the ohmic heating power has been dissipated as radiation. However, this is still only 6% of the initial magnetic energy.

In absolute terms RFX becomes radiation limited for a much lower amount of oxygen than does ZT-40. We therefore conclude that for this experiment to be successful a plasma that is cleaner than ZT-40 by more than a factor of four must be maintained.

With respect to the limits on impurities found for RFX and ZT-40 assuming classical transport, one point must be kept in mind. For a low beta plasma there is enough magnetic energy available to burn through low Z radiation barriers with higher impurity limits than those given here if an anomalous mechanism is present to convert the magnetic energy to plasma energy fast enough. Since the depletion of the magnetic field is small, this can be done without necessarily causing the plasma to go MHD unstable.

3.4. Stability of Point Cases

The first requirement for stability of the point cases is that the reversed toroidal field not disappear. The reversed field disappears at $62\mu s$, $46\mu s$, and $38\mu s$ for the 0%, .2%, and .4% oxygen impurity ZT-S simulations shown in Fig. 3, otherwise, it remains for all other cases.

It is always seen that the Suydam criterion is violated in the outer regions of the plasma. This can have important consequences for two reasons: first, it can cause enhanced transport in the vicinity of the B_z reversal point leading to a more rapid annihilation of the reversed field; second, it has been shown that violation of the Suydam criterion implies the existence of a nonlocalized rapidly growing $m = 1$ kink mode.⁽¹⁹⁾ To illustrate the extent to which our profiles are Suydam unstable, we present in Figs. 8(a), (b), and Figs. 9(a), (b) three dimensional graphs

of the pitch and electron temperature as a function of radius and time for ZT-40 and RFX respectively, for the impurity free case. In each of these graphs the Suydam unstable region is sketched. From Fig. 8 we see that the ZT-40 simulation becomes Suydam unstable after 100 μ s and that this unstable region rapidly grows to cover the outer quarter of the discharge. From Fig. 9 the RFX simulation becomes unstable after the first 2ms in the outer 10 cm of the plasma. The field reversal point is not included in the Suydam unstable region until after 1ms for ZT-40 and 20ms for RFX. The violation of the Suydam criterion is less severe for RFX because the broader current profile gives rise to a smaller temperature gradient near the plasma edge than in ZT-40 (c.f. Figs. 8(b) and 9(b)).

Impurities are seen to steepen temperature gradients and in particular to create very hollow temperature profiles where $\vec{\nabla}n$ and $\vec{\nabla}T$ are in opposite directions. This, as is well known, may give rise to the universal instability. In addition, it will be shown in section 5 that impurities may aid the growth of fine-scale gradients in the current and temperature.

CHAPTER 4

SCALING OF RFP DEVICES

A general criterion is now developed in terms of nondimensional parameters for whether a RFP can overcome a specified radiation barrier given only general design parameters. This criterion is then calibrated by comparison with the point cases for ZT-S, ZT-40, and RFX discussed in section 3.

To make a consistent and uniform comparison of the three devices the results of our point cases must be expressed in terms of nondimensional parameters. This can be done by defining a nondimensional representation of the temperature as

$$\beta_T \equiv \beta_E + \beta_I \quad (32a)$$

(where β_E and β_I are the electron and ion poloidal beta defined in section 3 and specified by a second axis parallel to $\bar{T}_e$ and $\bar{T}_i$ on the RHS in Figs. 3(a,b), 5(a,b), and 7(a,b)), and a nondimensional time τ by the decay of the poloidal magnetic energy W_p ,

$$W_p(t) = W_p(0)e^{-2\tau} \quad (32b)$$

The results of Figs. 3(a,b), 5(a,b), and 7(a,b) for ZT-S, ZT-40, and RFX are shown in Figs. 10(a), 10(b), and 10(c), respectively, in terms of β_T versus τ . From Fig. 10 we see that in the impurity free case $\beta_T(\tau)$ is a straight line that has nearly the same slope for all three devices (note that β_T and τ have different scales in Figs. 10(a,b,c)) and that the effect of impurity radiation is to change the slope of this line toward the horizontal. For example, in Fig. 10(b) for ZT-40 the .4% and .8% oxygen cases show a decrease in the slope of $\beta_T(\tau)$, compared to the impurity free case, at small values of τ where the radiation barrier is being surmounted. At large τ where the radiation barrier has been overcome and energy loss from radiation is small, $\beta(\tau)$ is a straight line with almost the same slope for all three cases. For these values of τ the effect on the magnetic field of overcoming the oxygen radiation barrier can be seen by the amount that τ has increased over the impurity free case. The radiation limited case (1.6%O) is quite distinct from the rest. Here $\beta_T(\tau)$ has a small slope that does not increase with τ as do the two cases which burn through the radiation barrier. Notice that a value of τ has been reached by this case that is as large as that attained by the 0%, .4%, and .8% O simulations, even though it has been run only a tenth as far in physical time. Results similar to those noted above are seen in Fig. 10(c) for RFX. Fig 10(a) once again shows the severe radiation problems encountered by ZT-S. Thus it is seen that this nondimensional

representation of our point cases provides a good correlative description of these results.

Next, a radiation barrier burn through criterion is derived. Consider the zero-dimensional temperature equation

$$\frac{3}{2} n \frac{\partial T}{\partial t} = \eta j^2 - n n_I C(T), \quad (33)$$

where $C(T)$ is the energy loss rate per electron per impurity ion, and n_I is the impurity density. By means of the resistive diffusion time $\tau_0 = \pi a^2 / c^2 \eta$, a nondimensional time $\tau = t / \tau_0$ can be defined and Eq. (33) becomes

$$d\tau = \frac{dT}{\epsilon [1 - \sum(T)]} \quad , \quad (33a)$$

where $\epsilon = \pi a^2 j^2 / c^2 n$ is the magnetic energy per particle ($\epsilon = \epsilon_0 e^{-2\tau}$ and $\epsilon_0 = \pi a^2 j^2 / c^2 n|_{t=0}$) and $\sum = n n_I C(T) / \eta j^2$ is the ratio of the radiation rate to the ohmic heating rate. $\sum(T)$ is assumed to have a sharply peaked Lorentzian shape (see Fig. 12) given by

$$\sum(T) = \frac{\sum_0}{1 + \frac{(T - T_0)^2}{(\Delta T)^2}} \quad ,$$

where $\sum_0$ is the peak of $\sum(T)$ at temperature T_0 and ΔT is the half width of the peak at half maximum. With this, Eq. (33a) can be integrated to some temperature $T > T_0$, with the result

$$\tau = \frac{T - T_1}{\epsilon_0} + \frac{\pi \Delta T}{\epsilon_0} \frac{\sum_0}{(1 - \sum_0)^{1/2}} \quad (34)$$

where T_1 is the initial temperature and τ has been assumed small. For a RFP to attain a temperature T above the radiation barrier, τ must be small; otherwise significant decay of the magnetic fields would occur. This implies that both L_0 and $(T-T_1)/\epsilon_0$ (a measure of beta) must be small.

We can rewrite Eq. (34) in a form more suited to comparison with the point cases.

$$\tau = a_1 \beta_D \left(1 + a_2(T) \frac{L_0}{(1-L_0)^{1/2}} \right). \quad (34a)$$

To compare this with the numerical simulations we take β_D to be

$$\beta_D = \beta_T(\tau) - \beta_T(0) \quad . \quad (35a)$$

$$L_0 = \langle n \rangle \langle n_I \rangle C(T) / \eta \langle j \rangle^2 \Big|_{T_0} \quad (35b)$$

is computed for oxygen ($C(T) = 9 \times 10^{-19} \text{ erg cm}^3/\text{sec})^{14}$, where $T_0 = 25\text{ev}$, using the zero dimensional quantities $\langle n \rangle$, $\langle n_I \rangle$, and $\langle j \rangle$ defined from the one dimensional profiles by a density weighted average, i.e.,

$$\langle A \rangle = \frac{\int_0^a 2\pi n A r dr}{\int_0^a 2\pi n r dr} \quad ,$$

since it is quantities measured relative to the plasma density rather than physical space that are of interest. $\bar{\lambda}_O$ is given in Figs. (10a,b,c). It is seen that $\bar{\lambda}_O = .45$ corresponds to the radiation limited cases of ZT-S, ZT-40, and RFX. a_1 and a_2 are parameters determined by fitting Eq. (34a) to the results of the numerical simulations as given in Figs. (10a,b,c). a_1 is determined from the impurity free cases to be .318. a_2 is found, by fitting Eq. (34a) to the radiation limited cases, to have value of about 2π (if a_2 were determined by fitting Eq. (34a) to other cases it would be found to have a value smaller than 2π , and would give a less restrictive limit on the parameters necessary for a RFP to burn through the oxygen radiation barrier).

Using these values of a_1 and a_2 , we plot in Fig. 11 curves of β_D versus $\bar{\lambda}_O$ for constant τ (notice the sharp dependence of β_D on $\bar{\lambda}_O$). In this figure squares, circles, and triangles are drawn that represent ZT-S, ZT-40, and RFX respectively at values of β_D and $\bar{\lambda}_O$ that correspond to the point cases at a temperature of 40ev, which is above the 25ev peak of the oxygen radiation barrier. This figure is then divided into four regions: region 1, where ($\bar{\lambda}_O > .45$) the oxygen impurity content is too great for the plasma to overcome the radiation barrier; region 2, where the energy lost to radiation is substantial and the τ at which burn-through occurs may be large, corresponding to a significant degradation of the magnetic fields; region 3, where the

device size is too small, i.e., the beta corresponding to 40ev already implies a large value of τ ; and region 4, where the radiation barrier can be safely overcome. We see that for ZT-S the device size is too small regardless of the impurity concentration. ZT-40 and RFX are at a low enough beta at 40ev that they can potentially lie in the safe region if the oxygen impurity density is low enough.

Figure 11 may be used to predict the rough behavior of any proposed RFP confronted with the oxygen radiation barrier simply by constructing β_D and $\int_0$ using the design parameters.

CHAPTER 5

THERMAL INSTABILITY IN THE RFP

As pointed out previously, ^(8,20) an ohmically heated plasma can be thermally unstable. Under proper conditions, the current will diffuse into a region of low resistivity, increase the ohmic heating in that region, further lower the resistivity, and thereby feed an instability. Here this phenomena is studied in the context of the RFP.

We consider a low beta plasma model governed by the Lagrangian MHD equations. We construct an equilibrium on the ohmic heating time scale and develop the theory of this instability for a stationary zeroth order electron temperature. The case of nonstationary temperature is then considered, and the theoretical predictions are compared to results of numerical simulations, wherein the full set of equations is solved.

5.1. Physical Considerations

To determine the important physical effects at low beta, we construct order of magnitude estimates of the terms in Eqs. (1)-(6) by scaling all gradients to the system size and by setting all coefficients constant. The following simple relations are obtained among the particle diffusion rate γ_{PD} , the magnetic field diffusion rate γ_{MD} , the ohmic heating rate γ_{OH} , the electron thermal conduction rate γ_{ETC} , the ion thermal conduction rate γ_{ITC} , and the electron-ion equilibration rate γ_{EI} :

$$\gamma_{PD} = \beta^2 \gamma_{OH} \quad , \quad \gamma_{MD} = \beta \gamma_{OH} \quad (36a,b)$$

$$\gamma_{EI} = \lambda \beta^2 \gamma_{OH} \quad , \quad \gamma_{BTC} = \beta^2 \gamma_{OH} \quad (36c,d)$$

$$\gamma_{ITC} = \beta^2 \left(\frac{M}{m}\right)^{1/2} \gamma_{OH} \quad , \quad (36e)$$

where β is the plasma beta, $\frac{M}{m}$ is the ion to electron mass ratio, and λ is the square of the plasma radius divided by the ion gyroradius. It is seen that for low beta the ohmic heating rate dominates all other plasma processes.

In the electron temperature equation (18) a radiation loss term is included that has the functional form $C N_e N_I \theta_e^{\alpha} / Y^{2\alpha+1}$, where C and α are constants and N_I is the impurity density. Then a low beta steady-state of Eqs. (16)-(20) on the ohmic heating time scale is given by

$$\frac{\partial P}{\partial \tau} = \frac{\partial N}{\partial \tau} = 0 \quad , \quad (37a,b)$$

$$\frac{\phi^2 \tilde{n}_n Y^{5/3}}{N(P^2 + Y\phi^2)} \left(\frac{Y}{Y}\right)^4 \left[\frac{\partial}{\partial X} \frac{P}{Y}\right]^2 = \frac{C N_e N_I \theta_e^{\alpha}}{Y^{2\alpha+1}} \quad (38)$$

$$\frac{\partial^2 Y}{\partial X^2} = Y \left[\frac{\frac{\partial}{\partial X} (P^2 + 2Y\phi^2)}{2(P^2 + Y\phi^2)} \right] \quad (39)$$

where the electron-ion equilibration term, and therefore the ion temperature equation, has been neglected

$$(\Lambda\beta^2 = \frac{e^2 \phi_0^2 \beta^2}{T_0 Mc^2} = \beta \frac{8\pi n_0 a^2 e^2}{Mc^2} \text{ and can be large or small}$$

depending on β , n_0 , and a). The case where this term is included ($\Lambda\beta^2$ of order unity) is considered later.

The character of this low beta equilibrium is now investigated by examining the form of a typical radiation cooling curve such as the one shown in Fig. 12 for oxygen, where the radiation power versus electron temperature is plotted logarithmically. It follows from Eq. (37a)

($\frac{\partial P}{\partial T} = \frac{\partial}{\partial X} n_e j_e = 0$) that $n_e j_e = \text{const.}$ Therefore, either both the electron temperature and current density are spatially uniform or $j_e \sim n_e^{-1}$. In Fig. 12 the ohmic heating rate is plotted as a function of electron temperature for both cases. The uniform temperature and current density case ($n_e j_e^2 \sim T_e^{-3/2}$) is shown as the solid line intersecting points A and B of the cooling curve, which represent equilibria described by Eqs. (37) - (39). For this case, point A is stable and point B is unstable with respect to small perturbations in temperature (an equilibrium is stable if the slope of the cooling curve is greater than that of the heating curve). However, if $j_e \sim n_e^{-1}$ the ohmic heating rate ($n_e j_e^2 \sim T_e^{3/2}$) is represented by the dashed curve intersecting point A. In this case perturbations in temperature about point A are unstable.

Consequently, if a low beta plasma reaches a radiation barrier, as illustrated by the solid line intersecting point A, it can burn through this barrier at isolated spatial points by following the $j_e \sim n^{-1}$ ohmic heating curve. The plasma switches from the $j_e = \text{const.}$ to $j_e \sim n^{-1}$ heating curve via current diffusion into regions of lower resistivity, and thus splits up into high and low temperature regions. This may occur with little change in the plasma density since the rate of particle diffusion is a factor of β less than the rate of current diffusion (c.f. Eq. (36)).

5.2. Derivation of the Thermal Instability

The theory of the thermal instability previously discussed in physical terms is now developed through means of a linear stability analysis.

We begin by considering Eqs. (16)-(20) and assume the existence of a stationary equilibrium described by Eqs. (37)-(39). A small perturbation in the parallel current $\epsilon P_1' (j_{\parallel} \sim \frac{\partial}{\partial X}(\frac{P}{Y}) \sim P')$, where the smallness is in ϵ , and in the electron temperature $\epsilon \theta_1$ is assumed that has a scale length $\bar{x}$ small compared to one. Therefore,

$$P' = P_0' + \epsilon P_1'(X, \tau), \quad (40a)$$

$$\theta_e = \theta_0 + \epsilon \theta_1'(X, \tau), \quad (40b)$$

where the zeroth order quantities $(\frac{P_0}{Y})'$ and θ_0 are assumed uniform in space and stationary in time.

The perturbation in the current P_1' causes, through Eq. (39), perturbations in Y , Y' , and Y'' . These perturbations are now estimated and it is shown that:

$$Y'' = Y_0'' + \epsilon Y_1'', \quad (41a)$$

$$Y' = Y_0' + \epsilon \bar{x} Y_1', \quad (41b)$$

$$Y = Y_0 + \epsilon \bar{x}^2 Y_1. \quad (41c)$$

First from Eq. (40a)

$$P = P_0 + \epsilon \bar{X} P_1, \quad (42)$$

since P_1' is a monotone function only over a scale $\bar{X}$. Eq. (41a) follows directly from using Eqs. (40a) and (42) in Eq. (39). To show Eq. (41b) we write Eq. (39) as,

$$Y'(X) = (P^2 + Y\phi^2) \exp - \int_0^X \frac{PP'}{P^2 + Y\phi^2} dX. \quad (43)$$

Then with Eq. (40a), (42) and because $\int_0^X \epsilon P_1' P_0 dX \sim O(\bar{X}\epsilon)$, Eq. (41b) follows. Similarly, Eq. (41c) can be shown by integrating Eq. (43) and proceeding as before. Since $N = nY'$, and because n is stationary for low beta (see Eq. (36)),

$$N = N_0 + \epsilon \bar{X} N_1. \quad (44)$$

The assumption that the perturbed current is due mainly to P_1' can now be verified. Using Eqs. (40) and (41) we have

$$j_n \sim \left(\frac{P}{Y}\right)' \sim \left(\frac{P_0}{Y_0}\right)' + \epsilon \left(\frac{P_1}{Y_0}\right)' - \epsilon \bar{X} \left(\frac{P_0}{Y_0^2} Y_1'\right)',$$

which shows that the perturbed current $\delta j_n \sim \epsilon \left(\frac{P_1}{Y_0}\right)'$.

Next the density and time are scaled by nondimensional factors $\bar{N}$ and $\bar{\tau}$ respectively. Since a reference density n_0 was defined by setting the reference beta equal to one in deriving Eqs. (16)-(20), the low beta assumption implies $\bar{N} = \beta$. The size of $\bar{\tau}$ and $\bar{X}$ is now related to $\bar{N}$ and equations are derived that describe the time evolution of θ_1 and $\left(\frac{P_1}{Y_0}\right)'$.

Including the radiation loss term in Eq. (18) and scaling it by $\bar{\tau}$ and $\bar{N}$, the order of the terms is found to be

$$1 = \bar{\tau}, \bar{N}\bar{\tau}: \bar{\tau}, \bar{N}\bar{\tau}: \bar{N}\bar{\tau}: \bar{N}\bar{\tau}\Lambda: \frac{\bar{\tau}}{\bar{N}}: \bar{\tau}\bar{N}: \bar{\tau}\bar{N}: \bar{\tau}\bar{N}: C\bar{\tau}\bar{N}^2.$$

By setting $\bar{\tau} = N = \beta$, the equilibrium equation (38) is retrieved. Ohmic heating can be balanced against radiation loss since C is an arbitrary parameter. All other terms are small of order β or β^2 ($\Lambda\beta^2$ is still assumed small). Thus, the nondimensional time scale of the equilibrium is of order β .

Taking the derivative with respect to X of the pitch equation (16) we have,

$$\frac{\partial}{\partial \tau} \epsilon P'_1 = \frac{\partial^2}{\partial X^2} \left[\left(\frac{X\phi}{\phi} + \frac{f}{\phi} - \frac{\eta_0}{Y} \right) P + \frac{\tilde{\eta}_0 Y}{Y^2} P' \right]. \quad (45)$$

Since $\left(\frac{P_0}{Y_0} \right)' = 0$, the leading term on the RHS of Eq. (45) is of order ϵ . Scaling the independent variables to $\bar{\tau}$ and $\bar{X}$, the terms are of relative order,

$$1 = \frac{\bar{\tau}}{\bar{X}} : \frac{\bar{\tau}}{\bar{X}^2}.$$

This equation can be balanced by setting $\bar{X} = \bar{\tau}^{1/2} = \beta^{1/2}$, giving the scale length of the perturbation. Using Eqs. (40), (41), (42) and the relation

$$\tilde{\eta} = \tilde{\eta}_0 \left(1 - 3/2 \frac{\theta_1}{\theta_0} \right) \quad (46)$$

in Eq. (45), dropping terms of order $\beta^{1/2}$, and keeping only terms linear in ϵ , we obtain the result

$$\frac{\partial}{\partial \tau} \left(\frac{P_1}{Y_0} \right)' = \frac{\partial^2}{\partial X^2} [C_1 \tilde{\eta}_0 \left(\frac{P_1}{Y_0} \right)' - \frac{3}{2} C_1 \left(\frac{P_0}{Y_0} \right)' \frac{\theta_1}{\theta_0}] , \quad (47)$$

where $C_1 = Y_0^2 / X_0'^2$.

An equation is now derived for θ_1 . With the scale length $\tilde{X} \sim \beta^{1/2}$ for perturbed quantities, terms in Eq. (18) that were of order $\tilde{\tau} \sim \beta$, $\tilde{N} \tilde{\tau} \sim \beta^2$ in the unperturbed quantities become of order $\frac{\tilde{\tau}}{\tilde{X}} \sim \beta^{1/2}$ and $\frac{\tilde{N} \tilde{\tau}}{\tilde{X}^2} \sim \beta$ in an equation for θ_2 and can still be dropped. With this in mind and using Eqs. (38), (40), (41), (42), (44), and (46) in Eq. (18), and keeping terms linear in ϵ we find that

$$\begin{aligned} \frac{\partial \theta_1}{\partial \tau} = & 2C_2 \tilde{\eta}_0 \left(\frac{P_0}{Y_0} \right)' \left(\frac{P_1}{Y_0} \right)' - (3/2) C_2 \frac{\tilde{\eta}_0}{\theta_0} \left(\frac{P_0}{Y_0} \right)'^2 + \\ & + C_3 \alpha \theta_0^{\alpha-1} \theta_1, \end{aligned} \quad (48)$$

where Eq. (38) can be written as $C_2 \tilde{\eta}_0 \left(\frac{P_0}{Y_0} \right)'^2 = C_3 \theta_0^\alpha$, defining C_2 and C_3 .

Assuming $\left(\frac{P_1}{Y_0} \right)' \sim \theta_1' \sim e^{ikx + \omega t}$ and using Eq. (38), the growth rate ω is found from Eqs. (47) and (48) to be

$$\omega = \frac{-(a_1 + a_2) \pm \sqrt{(a_1 + a_2)^2 - 4(a_1 a_2 - b_1 b_2)}}{2} \quad (49)$$

$$\begin{aligned} \text{where, } a_1 = & C_1 k^2 \eta_0, \quad a_2 = \frac{C_2 \eta_0 \left(\frac{P_0}{Y_0} \right)'}{\theta_0} (\alpha + 3/2), \quad b_1 = \\ & - \frac{3}{2} \frac{C_1 k^2 \eta_0}{\theta_0} \left(\frac{P_0}{Y_0} \right)', \quad b_2 = - 2C_2 \eta_0 \left(\frac{P_0}{Y_0} \right)'. \end{aligned}$$

There are two possibilities to be considered:

$a_2 < 0$ and $a_2 > 0$. The case $a_2 < 0$, or $\alpha < \frac{3}{2}$, corresponds to the unstable equilibrium shown in Fig. 12 as point B. This is the zero-dimensional case; the growth rate is the ohmic heating rate.

For the case $a_2 > 0$ there is an instability if $b_1 b_2 > a_1 a_2$, which becomes $\alpha < 3/2$. This is the equilibrium shown in Fig. 12 as point A, where an instability can develop due to current diffusion into regions of low resistivity. By examining Eqs. (47) and (48), it is seen that b_1 and b_2 represent the driving terms for this process. The growth rate ω is found to be a monotone increasing function of k , vanishing at $k = 0$ and reaching a limiting value for large k of

$$\omega = a_2 = \gamma_{OH} (3/2 - \alpha). \quad (50)$$

Notice that Eq. (50) does not apply for $k \gg \beta^{-1}$, since stabilizing terms, such as thermal conductivity, from Eq. (18) must then be included in Eq. (48).

Results of numerical simulations that verify the predicted $\alpha < 3/2$ thermal instability condition are presented. Typical ZT-40 parameters were chosen with initially uniform electron and ion temperatures (100 eV) and density ($2 \times 10^{13} \text{ cm}^{-3}$). The initial conditions contain a 10% perturbation of several wavelengths in an otherwise uniform parallel current. An ad-hoc electron energy loss term of the form $C(X) (T_e/T_o)^\alpha$ is

included in the computer code, where $T_0 = 100$ eV and $C(x)$ is the ohmic heating rate of the unperturbed current at 100 eV, and allows us to simulate the stationary zeroth order electron temperature case.

Figures 13(a) and 13(b) present graphs of the parallel current and the electron temperature as a function of radius at the initial time and at 50 μ s for different values of α . There is good agreement with the instability condition $\alpha < 3/2$. For the unstable case ($\alpha = 0$) the initial current perturbation decays during the time in which a temperature perturbation is being created, but then both grow. In a stable case ($\alpha = 2$) the current perturbation creates a temperature perturbation, but then both decay. For the unstable $\alpha = 1$ case the current perturbation at 50 μ s is only slightly larger than its initial value, therefore, only the temperature perturbation is shown.

Let us now consider the role of electron-ion temperature equilibration in this thermal instability. Assume that the only loss term in the electron temperature equation (18) is the electron-ion equilibration term (now $\Lambda\beta^2$ is considered to be of order unity), that the zeroth order electron temperature θ_0 is still constant (an artificial assumption), and that no perturbations in the ion temperature may arise, i.e., γ_{ITC} is infinite. Then, proceeding as before we find that the growth rate ω in the limit of large k is

$$\omega = \gamma_{OH} \left(3 - \frac{\theta_o}{\theta_o - \theta_{io}} \right), \quad (51)$$

and the instability threshold becomes $\theta_{io} < 2/3 \theta_o$ (note that for $\theta_{io} = 0$ this reduces to Eq. (50) with $\alpha = -1/2$).

For the case where both the electron-ion equilibration term and an arbitrary loss term proportional to θ_e^α is present, the growth rate in the limit of large k is

$$\omega = \gamma_{OH} (3/2 - \alpha) + \gamma_{EI} \left[(3/2 + \alpha) \frac{\theta_o - \theta_{io}}{\theta_o} - 1 \right]. \quad (52)$$

5.3. Nonstationary Electron Temperature

When a low beta plasma burns through a radiation barrier there is no stationary θ_o since the electron temperature increases rapidly everywhere. What happens to our thermal instability when this is the case is now considered.

Suppose that initially there are some small, short scale length perturbations in the current that remain from the setting up phase, and that the radiation loss rate is a small fraction of the ohmic heating power. Then the current distribution will give rise to an electron temperature profile that can be written as

$$\theta_e = \theta_o(\tau) + \epsilon \theta_1(X, \tau), \quad (53)$$

where both the zeroth order temperature and the perturbation are increasing in time.

Including the electron-ion equilibration term, this case is described by Eqs. (47) and (48) for $(\frac{P_1}{Y_0})'$ and θ_1 , along with an equation for $\theta_0(\tau)$,

$$\frac{\partial \theta_0}{\partial \tau} = C_2 \bar{n}_0 \left(\frac{P_0}{Y_0}\right)'^2 - C_3 \theta_0^\alpha - C_4 \bar{n}_0 (\theta_0 - \theta_{10}) \quad (54)$$

where $C_4 = \frac{\Delta N_0}{Y_0}$. By Fourier analyzing Eq. (47) in space, we can write the solution for P_1' as

$$P_1' = \frac{3}{2} \frac{P_0'}{\theta_0} \theta_1 + \text{const.} \exp \frac{-\bar{n}_0 Y_0 k^2 \tau}{Y_0'^2} \quad (55)$$

Taking the short wavelength limit one obtains the steady-state solution,

$$P_1' = \frac{3}{2} P_0' \frac{\theta_1}{\theta_0} \quad (56)$$

This shows that the current perturbation P_1' is driven by the relative temperature perturbation $\frac{\theta_1}{\theta_0}$, and not θ_1 alone. If θ_1 and θ_0 are growing at the same rate, P_1' is stationary and the instability does not exist (if Eq. (56) is used in Eq. (48) along with Eq. (38) the result for ω in short wavelength, stationary θ_0 case, Eq. (50), is obtained). Therefore, the role of the radiation loss term is to decrease the growth rate of θ_0 , thereby increasing the growth of the relative perturbation θ_1/θ_0 , and through Eq. (56) P_1' .

By combining Eqs. (48) and (54), an equation for the relative temperature perturbation $\xi \equiv \theta_1/\theta_0$ can be obtained

$$\begin{aligned} \frac{\partial \xi}{\partial \tau} = & \frac{C_2 \bar{n}_0 \left(\frac{P_0}{Y_0}\right)^2}{2\theta_0} \xi + C_3 (1-\alpha) \theta_0^{\alpha-1} \xi + \\ & + C_4 \bar{n}_0 \left(\frac{3}{2} - \frac{5}{2} \frac{\theta_{10}}{\theta_0}\right) \xi \end{aligned} \quad (57)$$

which gives a growth rate

$$\begin{aligned} \omega_\xi = & \frac{C_2 \bar{n}_0 \left(\frac{P_0}{Y_0}\right)^2}{2\theta_0} + C_3 (1-\alpha) \theta_0^{\alpha-1} + \\ & + C_4 \bar{n}_0 \left(\frac{3}{2} - \frac{5}{2} \frac{\theta_{10}}{\theta_0}\right). \end{aligned} \quad (58)$$

The actual growth of ξ is then given by

$$\xi(\tau) = \xi(0) e^{\int_0^\tau \omega_\xi d\tau}. \quad (59)$$

Whereas in the θ_0 stationary case the instability condition was $\alpha < 3/2$, ω_ξ will nearly always be positive, independent of α , since the radiation loss rate is much less than the ohmic heating rate. In fact, it is positive when there is no energy loss from the electron temperature equation ($C_3=C_4=0$). Using Eqs. (54) and (58) in Eq. (59) ξ can be calculated for this case, with the result

$$\xi(\tau) = \xi(0) (1 + 5/2 \gamma_{OH} \tau)^{1/5}, \quad (60)$$

where γ_{OH} is the ohmic heating rate at the initial temperature. This shows that the growth of ξ , which was exponential for stationary θ_0 , has been greatly reduced, but is not zero.

Although Eq. (60) indicates that current perturbations in a low beta RFP, remaining from the formation phase, have a natural tendency to grow in the absence of any electron energy loss, it does not estimate that growth rate realistically because the electron-ion equilibration term was omitted. As seen from Eq. (58), this term is destabilizing if the electron and ion temperatures are not closely equilibrated (for this term to be net stabilizing $\theta_{i0}/\theta_0 > \frac{3}{5}$). The effect of this term and that of various impurity concentrations on the growth of initial current perturbations in the RFP are studied numerically.

5.3.1. Numerical Simulation

This section presents results of numerical studies of the effect of impurities on the growth of fine-scale current and temperature gradients in the RFP. Plasma parameters typical of ZT-40 are chosen (see table 1) with a uniform plasma density. There is an initial 10% perturbation in the parallel current from a radius of 4 cm to 17 cm. The unperturbed current is uniform until 17 cm after which it drops sharply to zero by 18.5 cm.

In Figs. 14(a) and 14(b) the parallel current and electron temperature is plotted as a function of radius at the initial time and at 100 μ s for three cases: impurity free, 2% oxygen, and 4% oxygen, where there is a uniform plasma density of 10^{14} cm $^{-3}$. The initial current perturbations have a 2.5 cm wavelength. For the impurity free case the current perturbations have grown by 25% at 100 μ s. This is due to the effect of the electron-ion equilibration term, since these perturbations are nearly stationary when the same case is run with this term omitted. For the 2% oxygen case the current perturbations have grown by 120%, and for the 4% oxygen case by a huge 350%. From Fig. 14(b) we see that although the zeroth order temperature (seen from the inner 4 cm) has climbed well above the peak of the oxygen radiation barrier for all cases, the temperature depressions corresponding to the 4% oxygen case are actually hung up on this barrier. This has been brought about by current diffusion out of these regions. The temperature actually rises above the peak of the oxygen radiation barrier but then collapses as the current diffuses away. This figure shows that the role of the impurity radiation is to decrease the growth of the zeroth order temperature and thereby increase the growth of the relative temperature perturbation driving the perturbed current, a point made previously from Eq. (56).

From the expression for ω in the stationary θ_0 case, Eq. (49), it was noted that the growth rate of this instability decreases as the wavelength increases. Figs. 15(a) and 15(b)

show results at 100 μ s for cases identical to those in figure 14 except that the wavelength of the initial current perturbation has been increased to 4 cm. We see that the growth of the initial current perturbation has been greatly reduced from that in the 2.5 cm wavelength case. For the 2% oxygen simulation the current perturbation has grown by 55% in 100 μ s and for 4% oxygen the temperature depressions are not caught on the oxygen radiation barrier.

Note that there is a sharp peak in the current just near the wall in Fig. 15(a). This is due to current in the cold outermost region diffusing inward. The current channel tries to constrict but cannot do so.

since the ohmic heating rate is inversely proportional to the plasma density but the radiation loss rate is directly proportional to the density squared for a fixed impurity concentration, it is interesting to compare simulations at different densities. A series of simulations were performed for initial conditions identical to those cases shown in figure (14) except for a density of $4 \times 10^{15} \text{ cm}^{-3}$ with oxygen impurity concentrations of 0%, .25% and .5%. The results of these simulations are shown in Figs. 16(a,b). Although no simple scaling between different density cases can be given in general (due to the temperature dependence of η), the magnitude of the current and temperature perturbations in the .5% O and .25% O simulations of figure (16) and the 4% O and 2% O simulations of figure (14) are very similar except that

the .5 0 case does not have temperature wells that are caught on the radiation barrier (note the difference in the zeroth order temperatures between Figs. 14(b) and 16(b)). Thus we see that by increasing the plasma density by a factor of four and decreasing the impurity density by a factor of two we have achieved very similar results.

It is often seen in the numerical simulations that the wells in the initial current perturbation grow faster than the peaks. This is due to the dependence of the radiation loss term on position. That is, the perturbation in ξ is caused not only by the current perturbation, with the radiation loss as a uniform background slowing the growth of θ_0 , but also by the spatial variation of the radiation loss term (the assumption that θ_1/θ_0 is small can break down so that the growth rate ω_ξ must be expressed in terms of θ_e instead of θ_0). An extreme case of this is where there is an isolated point at which the radiation barrier is not overcome (c.f. Fig. 14(b)). The current will diffuse out of this region creating a deep well but not a comparable peak.

5.3.2. Saturation of the Instability

If beta becomes large enough the temperature perturbations will be smoothed by the electron thermal conductivity and by the ion thermal conductivity once the electron and ion temperatures are sufficiently equilibrated, and the instability will be quenched. In Figs. 17(a,b,c) the parallel

current and electron and ion temperatures are plotted, respectively, as a function of radius at the initial time and at three later times for the .25% oxygen impurity case of Fig. (16). The instability reaches a peak at about 100 μ s and then begins to decay as beta increases. We see that at 310 μ s the growth of the current perturbation has decreased to about half of its maximum value and that by 910 μ s it is substantially less than its initial value. The decay of the current perturbation and the relative temperature perturbation begins at a beta of about 4% and corresponds to the point where the ratio of the zeroth order ion to electron temperature becomes greater than 3/5, the threshold at which the electron-ion equilibration term becomes stabilizing (c.f., Eq. (58)). Thus it is the ion thermal conductivity which is the damping mechanism. Notice from Eq. 17(c) that perturbations in the ion temperature do not develop.

Because beta must increase to a value such that the electron and ion temperatures become closely equilibrated before these perturbations begin to damp, it is expected that some anomalous process will actually provide the damping mechanism.

5.3.3 Anomalous Transport

It is not known what transport coefficients actually apply in the post-implosion phase of a RFP. The coefficients may be anomalous due to many mechanisms, including unstable drift waves driven by fine-scale current and temperature gradients that are created by the thermal instability just discussed.

How anomalous transport effects our thermal instability picture, based on classical transport, depends on how χ_{le}/η_e scales from the classical to the anomalous case. If χ_{le}/η_e is enhanced sufficiently, then the electron thermal conductivity can smooth perturbations in the electron temperature. If χ_{le}/η_e remains unchanged, as in the case where χ_{le} and η_e are both modified by an effective collision frequency (for example, the ion acoustic instability), then only the time scale changes and the character of the thermal instability is the same as in the classical case.

It is interesting to note that in the tokamak the experimentally observed electron thermal conductivity is approximately one hundred times its classical value. When simulations were performed with an initial 10%, 2.5 cm wavelength current perturbation, a modest level of oxygen impurity, and the electron thermal conductivity enhanced by this amount, the current perturbation simply decayed. Since the electron thermal conductivity must be enhanced by such a large factor to stabilize this perturbation, one would expect that for a classical electron thermal conductivity very fine-scale (wavelengths greater than

1 mm) current perturbations will be unstable.

The anomalous, hundred times classical electron thermal conductivity observed in tokamaks is unexplained theoretically. It is possible that this phenomena could be the consequence of a microinstability that is the result of the need to stabilize these short wavelength current perturbations. However, since a microinstability that could be responsible for this effect is not known, this hypothesis can only be meant as mere speculation.

Chapter 6

CONCLUSION

The effect of energy loss due to impurity radiation on the evolution of RFP plasma and magnetic field profiles in the post-implosion phase has been studied by numerical simulation for the ZT-S, ZT-40, and RFX experiments using oxygen (and also aluminum in ZT-40) as a typical impurity. Assuming classical transport and typical operating parameters, limits are found on the levels of these impurities above which the electron temperature becomes radiation limited. The ability of a reversed-field pinch to burn through a radiation barrier is shown to depend on the electron and impurity densities, current density, and device size.

It is found that the ZT-S experiment has a radiation limited temperature in agreement with the experimental observations and the measured oxygen impurity level. Because of its small size and high density, the value of beta in ZT-S is large at the temperature where the oxygen radiation barrier is a maximum ($\beta \sim 17\%$ at 30 eV with a $1 \times 10^{15} \text{ cm}^{-3}$ density). Therefore, there is only a small magnetic energy reservoir available to heat the plasma above this value or to be lost to radiation without causing a large degradation of the magnetic fields and consequent loss of stability. Both ZT-40 and RFX have the advantage that for projected operating parameters the peak of the oxygen radiation barrier occurs at a low value of beta. However, RFX has a much lower current density than ZT-40 and we find, for similar plasma and magnetic field profiles and

for the projected operating parameters, that the density of oxygen necessary to produce a radiation limited temperature in RFX is approximately four times less than in ZT-40.

The results show that if a low beta plasma burns through the radiation barrier in some region the classical electron thermal conductivity will not be large enough to aid burn-through in the rest of the plasma and steep temperature gradients will be formed leading presumably to anomalous transport. It is seen that the Suydam criterion is violated in the outer regions of the plasma for ZT-40 and RFX even if there is no steepening of temperature gradients due to impurity effects. Thus, classical transport cannot be expected in this region.

The one-dimensional results were suitably reduced to zero-dimensional ones and used to calibrate an analytical formula that describes the ability of a plasma to burn through the oxygen radiation barrier given only the design parameters: size, total current, electron density, and impurity density. This criterion is useful in roughly estimating how a proposed RFP will perform when confronted with the oxygen radiation barrier.

The role of impurities in producing a thermal instability in the RFP that is due to the coupling of current diffusion and ohmic heating was studied for the case where there is a nearly force free, low beta equilibrium, after the formation phase, as the plasma attempts to ohmic heat to a significant beta. Two cases, stationary and nonstationary

electron temperature, were considered.

For the stationary electron temperature case it was shown that in a radiation limited plasma the current (and, therefore, the temperature, but not the density) can break up into filaments. It was found that the growth rate of this phenomena is a monotone increasing function of k , which vanishes for $k = 0$ and for large k is proportional to the ohmic heating rate. The condition that must be satisfied for this instability to occur was derived, and verified through numerical simulation. This indicates how a radiation limited RFP plasma may destroy itself.

The nonstationary electron temperature case, which is of more general interest, yields important new results. It is found that the thermal instability previously considered is still operable even in the case where there is no energy loss from radiating impurities. Thus, fine-scale current perturbations remaining from the formation phase, have a natural tendency to grow, although the growth rate is much reduced from the ohmic heating rate. It was shown analytically and verified numerically that the role of a uniform background energy loss is to decrease the growth of the uniform, zeroth order electron temperature θ_0 , while not affecting the growth of the perturbed, short wavelength electron temperature θ_1 . The growth of the current perturbations depends upon the relative temperature perturbation θ_1/θ_0 , which explains the role of impurity radiation in aiding the growth of the current perturbations. Numerical simulations show that only modest

impurity levels are necessary to produce large current and temperature gradients in relatively short periods of time.

The classical damping mechanism is the ion thermal conductivity, which makes the electron-ion equilibration term net stabilizing after the electron and ion temperatures have equilibrated sufficiently. This requires a modest value of beta. Therefore, it is expected that the effect of a low level of plasma impurities in conjunction with fine-scale perturbations in the initial current distribution will be to create large current and temperature gradients, via thermal instability, that will be saturated by a nonclassical mechanism, which preferentially increases the electron thermal conductivity.

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	ZT - S	ZT - 40	RFX
Vacuum Wall Radius	7.7 cm	20 cm	60 cm
Conducting Wall Radius	7.9 cm	22 cm	63 cm
I _z Current	70 KA	500 KA	700 KA
Toroidal Field On Axis	4000 G.	10,500 G.	5100 G.
Density On Axis	$1 \times 10^{15} \text{ cm}^{-3}$	$5 \times 10^{14} \text{ cm}^{-3}$	$5 \times 10^{13} \text{ cm}^{-3}$
Initial Temperature	10 ev	10 ev	10 ev
Initial Beta	5.1 %	.38 %	.16 %

Table 1

FIGURE CAPTIONS

- Fig. 1. The initial pitch ($P = \frac{rB_z}{aB_\theta}$) and current profiles used in the simulations of ZT-S, ZT-40, and RFX are plotted as a function of r/a , the radius divided by the wall radius.
- Fig. 2. ZT-S magnetic field profiles. Solid curves: B_z and B_θ at the initial time; dashed curves: B_z and B_θ at 35 μ s with no impurity present; chain dash curves: B_z and B_θ at 35 μ s with an oxygen impurity concentration of .4% of the deuterium density ($1 \times 10^{15} \text{ cm}^{-3}$ on axis).
- Fig. 3(a). The average electron temperature $\bar{T}_e$ and β_e in ZT-S as a function of time with oxygen impurity concentrations of 0%, .2%, and .4% that of deuterium ($1 \times 10^{15} \text{ cm}^{-3}$ on axis).
- Fig. 3(b). The average ion temperature $\bar{T}_i$ and β_i as a function of time.
- Fig. 3(c). The electron temperature as a function of radius at 35 μ s for 0% and .4% oxygen impurity. The electron temperature at the initial time is a uniform 10eV.
- Fig. 3(d). The magnetic energy W_m and internal energy W_i , and the total energy radiated W_R as a function of time for 0%, .2%, and .4% oxygen impurity.

- Fig. 4. The average electron temperature $\bar{T}_e$ in ZT-S as a function of time for oxygen concentrations of 0%, .2%, and .4% of the line density distributed uniformly in space.
- Fig. 5(a). The average electron temperature $\bar{T}_e$ and β_E in ZT-40 as a function of time with oxygen impurity concentrations of 0%, .4%, .8%, and 1.6% that of deuterium ($5 \times 10^{14} \text{ cm}^{-3}$ on axis).
- Fig. 5(b). The average ion temperature $\bar{T}_i$ and β_i as a function of time for the above mentioned impurity concentrations.
- Fig. 5(c). The electron temperature as a function of radius at 150 μ s for 0% and 1.6% oxygen impurity. The electron temperature at the initial time is a uniform 10eV.
- Fig. 5(d). The magnetic energy W_M and internal energy W_I , and the total energy radiated W_R as a function of time for 0%, .8%, and 1.6% oxygen impurity.
- Fig. 6(a). The average electron temperature $\bar{T}_e$ in ZT-40 as a function of time with impurity concentrations of 0%^O, .4%O, .4%O + .2%Al, and .4%O + .4%Al that of deuterium ($5 \times 10^{14} \text{ cm}^{-3}$ on axis).
- Fig. 6(b). The average ion temperature $\bar{T}_i$ as a function of time for the above mentioned impurity concentrations.

- Fig. 6(c). The magnetic energy W_m and internal energy W_I , and the total energy radiated W_R as a function of time for the above mentioned impurity concentrations.
- Fig. 7(a). The average electron temperature $\bar{T}_e$ and β_e in RFX as a function of time with oxygen impurity concentrations of 0%, 2%, and 4% that of deuterium ($5 \times 10^{13} \text{ cm}^{-3}$ on axis).
- Fig. 7(b). The average ion temperature $\bar{T}_i$ and β_i as a function of time for the above mentioned impurity concentrations.
- Fig. 8(a). The nondimensional pitch $P = rB_z/aB_\theta$ as a function of radius and time for ZT-40 with no impurities. The Suydam unstable region intersects the pitch reversal point at about 1ms.
- Fig. 8(b). The electron temperature as a function of radius and time.
- Fig. 9(a). The nondimensional pitch $P = rB_z/aB_\theta$ as a function of radius and time for RFX with no impurities. The Suydam unstable region does not intersect the pitch reversal point.
- Fig. 9(b). The electron temperature as a function of radius and time.

Fig. 10(a). The nondimensional temperature $\beta_T = \beta_E + \beta_I$ is given as a function of the nondimensional time τ (see Eqs. (32a,b) for the ZT-S cases (0%, .2%, and .4% oxygen) shown previously in Fig. 3. The curves are labeled by $\sum_0$, the ratio of the impurity radiation rate to the ohmic heating rate at the 25eV peak of the oxygen cooling curve (see Eq. (35b).

Fig. 10(b). β_T versus τ is given for the ZT-40 cases (0%, .4%, .8%, and 1.6% oxygen) shown in Fig. 5.

Fig. 10(c). β_T versus τ is given for the RFX cases (0%, 2%, and 4% oxygen) shown in Fig. 7.

Fig. 11. $\beta_D = \beta_T(\tau) - \beta_T(0)$ (Eq. (35a)) is drawn as a function of $\sum_0$ (Eq. (35b)) for constant values of τ (see Eq. (34a)). The ZT-S, ZT-40, and RFX cases shown in Figs. 10(a), (b), and (c) are represented by squares, circles and triangles respectively, drawn for a β_D corresponding to $\bar{T}_e = 40\text{eV}$. Full, half, and quarter shaded symbols represent respectively the maximum, half and quarter maximum oxygen impurity cases of Figure 10. Four regions of parameter space are delineated: region I, where the plasma temperatures are radiation limited; region II where there is substantial radiation loss; region III, where

the device size is too small for burn through at low beta; region IV, which is safe for burning through the oxygen radiation barrier. The $\tau = .05$ curve corresponds to a 10% degradation in the magnetic energy available for ohmic heating.

Fig. 12. The radiation cooling rate due to oxygen (energy per electron, per oxygen ion, per second) is given as a function of temperature. Also drawn are the ohmic heating rate for a constant current density (solid line), which intersects this curve at points A and B, and the ohmic heating rate for $j_{\parallel} \sim \eta^{-1}$ (dashed line), which intersects the cooling curve at point A.

Fig. 13(a). The parallel current is shown at the initial time with a 10%, 2cm wavelength perturbation from 4cm to 16cm, and at 50 μ s for $\alpha = 0$ and $\alpha = 2$.

Fig. 13(b). The electron temperature is shown at the initial time (a uniform 100eV) and at 50 μ s for $\alpha = 0, 1$, and 2.

Fig. 14(a). The parallel current is shown at the initial time, with a 10%, 2.5cm wavelength perturbation from 4cm to 17cm, and at 100 μ s for 2T-40 parameters with oxygen impurity concentrations of 0%, 2%, and 4% of the deuterium density ($1 \times 10^{14} \text{ cm}^{-3}$).

Fig. 14(b). The electron temperature is shown at the initial time and at 100 μ s for the above mentioned impurity concentrations.

Fig. 15(a). The parallel current is shown at the initial time, with a 10%, 4cm wavelength perturbation from 4cm to 17cm, and at 100 μ s for ZT-40 parameters with oxygen impurity concentrations of 0%, 2% and 4% of the deuterium density ($1 \times 10^{14} \text{ cm}^{-3}$).

Fig. 15(b). The electron temperature is shown at the initial time and at 100 μ s for the above mentioned impurity concentrations.

Fig. 16(a). The parallel current is shown at the initial time, with a 10%, 2.5cm wavelength perturbation from 4cm to 17cm, and at 100 μ s for ZT-40 parameters with oxygen impurity concentrations of 0%, .25%, and .5% of the deuterium density ($4 \times 10^{14} \text{ cm}^{-3}$).

Fig. 16(b). The electron temperature is shown at the initial time and at 100 μ s for the above mentioned impurity concentrations.

Fig. 17(a,b,c). The parallel current, electron temperature, and ion temperature are given in parts (a), (b), and (c) respectively, for the .25% oxygen impurity case of Fig. 16, at the initial time and at 110 μ s, 310 μ s, and 910 μ s.

Fig. 1. 792515

Fig. 3. 802003

Fig. 4. 792517

Fig. 5. 802002

Fig. 6. 792567

Fig. 7. 792510

Fig. 8. 792522

Fig. 9. 792519

Fig. 10. 792566

Fig. 11. 802001

Fig. 12. 792509

Fig. 13. 792516

Fig. 14. 802005

Fig. 15. 802006

Fig. 16. 802004

Fig. 17. 792523