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A DEMAND ELASTICITY REPRESENTATION: METHODOLOGY AND CALIBRATION

ANDY S. KYDES

July 1979

ENERGY DATA AND MODELS GROUP
NATIONAL CENTER FOR THE ANALYSIS OF ENERGY SYSTEMS
DEPARTMENT OF ENERGY AND ENVIRONMENT

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ABSTRACT

This paper describes the methodology incorporated within a version of the Brookhaven Energy System Optimization Model (BESOM) to yield first-order approximations of the response of a national energy system, in economic equilibrium, to changes in prices, quantity of fuel available, and changes in the technological structure of the energy conversion devices - including efficiency changes. The methodology is not intended as a substitute for the comprehensive energy-economic analysis derivable from the BNL-DJA suite of models. It is intended as an inexpensive tool for performing first-order sensitivity analysis of energy systems around an established energy-economic "equilibrium" point.

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I. INTRODUCTION

Linear programming optimization models have been and continue to be used extensively as tools for energy policy analysis. In these models total energy demand, resource constraints, technological availability, and prices are generally specified deterministically, and the models calculate the optimal supply-demand configuration that optimizes a quantifiable (linear) objective function. Since price changes in reality affect the quantity of energy used, the supplies that may be forthcoming, and the level of energy service demands, some mechanism for price responsiveness of supply and demand is required. One such mechanism that has successfully closed the loop between the economy and the energy sectors is the BNL-DJA^{1,13} approach. In this integrated modeling approach, the outputs of the Dale Jorgenson Associates (DJA) macro-economic growth model and interindustry model iteratively exchange information with the BNL combined model until the energy quantities supplied in the supply models equal the energy quantities demanded in the DJA inter-industry model. Figure 1 illustrates the interactions between the BNL and DJA combined models. Figure 2 shows the iterative nature of the integrated DJA combined growth and interindustry energy model, and Figure 3 illustrates the Brookhaven combined IO-LP model solution procedure.

Such an iterative procedure, although invaluable and comprehensive in its approach, can prove cumbersome and expensive for performing large-scale sensitivity analysis "near" an established economic-energy equilibrium point. Furthermore, first-order responses of the energy system to changes in costs, resource availability, and/or technological characterization of energy conversion devices (both supply and demand) are often sufficient to satisfy the needs of some studies. The "flip-flop" behavior of linear programs (LP) usually does not permit a consistent examination of the relationship between market behavior and fuel prices. These considerations motivated the development of the elasticity representation of the Brookhaven Energy System Optimization Model.^{3,4}

Figure 1. Relation Between DJA Combined Model and BNL Combined Model

Figure 2. DJA Combined Growth and Interindustry Energy Model

Figure 3. Combined 10-LP Model Solution Procedure

The methodology is an adaptation of a procedure suggested by Cherniavsky⁵ and is similar to work by Bergendahl⁶ and Manne.⁷ Cherniavsky explored several formulations for introducing price responsiveness of the system to supply and demands within the energy system optimization framework (both linear and quadratic) and suggested an iterative scheme for finding the equilibrium demand points at which the demand curve prices equal the marginal costs.

We discuss here an adaptation for which the demand for each end use and fuel combination (e.g., oil to space heat) is approximated by stepwise linear functions, and the approximation to the new equilibrium point is determined through a simple modification to the linear programming algorithm. Two useful variants of this algorithm have been implemented and tested, and the preliminary results are summarized in Section III.

Variant 1

The first deals only with "first-order" effects of changing prices and/or technological characterizations of the energy system. The algorithm uses only "own-price" elasticities of specific fuels to demands and does not directly account for the second-order effects involved in using the cross-price demand elasticities, since constraints on energy service demands are non-binding. The basic idea behind the algorithm is that, if the marginal costs of meeting certain demands with certain fuels rise, the price mechanisms will pressure consumers to moderate their fuel specific demands. As the level of demand drops, the marginal costs decrease and the price pressures acting to depress demands are relaxed.

When calibrated, this formulation can be expected to simulate approximately the short-term response of an energy system to shocks such as a sudden change in price and/or availability of a resource during the period when interfuel substitution is inappropriate, insignificant, or impossible, and the only consumer response is a net decrease (possibly temporary) of energy service demands. The basic capital structure of the economy is assumed to remain unchanged.

Data for the period around the oil embargo of 1973 are currently being used to calibrate the model.

Finally, consistency between the underlying assumptions of two different energy consumption patterns can be approached with variant 1 by searching for elasticities that transform one energy system state to another. Existence of such elasticities is a necessary condition for consistency. One such application of AET-8⁸ with another reference scenario (FORECAST II)⁹ for the year 2000 is described in Section III.

Variant 2

The second variant deals with the case in which the new energy service demands are found through exogenous econometric estimation. The long-term response of the energy system to changes in fuel pricing patterns, resource availability, and technological and market factors (e.g., efficiencies of conversion and first date of availability) can be examined. In variant 2, own-price effects are explicitly accounted for while cross-price effects are implicit in the interfuel substitutions.

Briefly, each algorithm entails solving the original energy-economy equilibrium problem to derive a price-quantity equilibrium point for the energy component. Elasticities are then assumed for each fuel-demand combination, and marginal demand curves are generated and approximated by stepwise linear curves. The new problem (with potentially new prices and/or technological characterizations) is then modified by incorporating the marginal demand curves and resolved for the new approximated energy equilibrium. Energy service demands are either free or set as equalities, depending on whether a short-term or long-term response is sought.

One additional comment is appropriate. The demand elasticity representation, unlike the original linear programming formulation, implicitly values nonprice factors (e.g., institutional factors, regional variability, etc.) as implied by the shadow prices on the activities to the original constrained energy equilibrium solution. Since bounds on activity variables are replaced by stepwise linear

curves in the new perturbed problem, the elasticity representation yields a much more realistic methodology for characterizing market behavior of consumers. The elasticities indicated are not, strictly speaking, own-price elasticities but rather, aggregated gross elasticities of response that incorporate both price and nonprice factors (e.g., solar-heated homes are cost ineffective but continue to be bought and used for nonprice reasons).

II. ALGORITHMIC DETAILS

The details of the algorithm used are given here without the mathematical derivations or justifications for its use, which appear in Appendix A. The derivations are separated from the methodology for clarity of presentation.

The general outline of the methodology is reviewed briefly before the details are given. The terms "methodology" and "algorithm" are used interchangeably. Since the two variants differ only in that energy service demands can be determined by the model in variant 1 whereas they require exogenous econometric estimates in variant 2, both variants are referred to below as "the methodology."

The "equilibrium problem" is defined as that linear programming formulation of the national energy system whose optimal solution is consistent with the energy-economic equilibrium considerations. The "perturbed problem" is that linear programming formulation of the national energy system which deviates from the "equilibrium problem" in cost assumptions, efficiency assumptions, fuel availability assumptions, and/or market penetration assumptions. The algorithm begins by solving the original equilibrium problem to obtain reference flows of energy (by fuel form) to demands and reference prices. The reference quantities, prices, and own-price elasticities are used to "modify the perturbed problem," which is then resolved for approximate equilibrium flows, by fuel forms.

Variant 1 gives the short-term response since demands are endogenously determined, whereas variant 2 gives the long-term response since demands are exogenously determined.

The usual accounting information normally available with BESOM runs continues to be available for the new perturbed problem.

The "equilibrium" energy problem can be cast into the familiar linear optimization framework in Eq. (1) below. Note that the variables X_i are in quads (10^{15} Btu) of intermediate energy form (IEF): IEF can be thought of as "delivered fuel" prior to utilization, e.g., electricity at the outlet, gasoline before combustion in the automobile engine, or No. 2 oil before burning in the home burner. The C_i are the costs, usually including the annualized cost of end-use devices, per 10^6 Btu of IEF.

$$\begin{array}{ll} \text{Minimize} & \sum_{i=1}^n C_i X_i \\ \text{Subject to} & \sum_{j=1}^n a_{ij} X_j + S_i = b_i, \quad i = 1, 2, \dots, m \end{array} \quad (1)$$

where the S_i are the "slack" variables. The constraints in Eq. (1) are divided into four functional and logically related sets.

$$(a) \text{ Demand equations: } \sum_{j=1}^n n_{dj} X_j = b_d$$

where index d ranges only over the set of demand equations and n_{dj} are the nonzero end-use conversion efficiencies for each specific end-use and device combination possible, e.g., oil to space heat for conventional burners has $\eta = 0.59$ for some potential future scenarios. These equations dictate the level of energy service demands which must be satisfied, and hence $S_d = 0$.

$$(b) \text{ Limiting equations: } \sum_{\lambda} x_{\lambda} \leq b_{\ell} \quad \text{and/or} \quad \sum_{\lambda} x_{\lambda} \geq b_{\ell}$$

where λ is an index over the relevant indicies in the limiting equations ℓ . For example, the minimum and maximum levels of refined oil used for satisfying space heat demand may be limited to reflect

more accurately the range of feasible interfuel substitutions for space heat in the future.

(c) Resource constraints:
$$\sum_{j=1}^N a_{rj} x_j \leq b_r$$

where r is an index over the set of resource equations; the nonzero a_{rj} are the inverse of the combined efficiencies that transform crude fuel to delivered energy (IEF); and b_r represents the total limit to crude fuel use of type r . Stepwise linear supply functions are included in this set.

(d) Other constraints:
$$\sum_{j=1}^N a_{oj} x_j + S_o = b_o.$$

These are constraints that are important for characterizing the energy system properly but not for defining the methodology.

The "dual" to the equilibrium problem, Eq. (1), will be required in describing the algorithm and is given in Eq. (2) below.

$$\begin{array}{ll} \text{Maximize} & \sum_{i=1}^m b_i Y_i \\ \text{Subject to} & \sum_{i=1}^m a_{ij} Y_i - u_j = c_j, \quad j = 1, 2, \dots, n \end{array} \quad (2)$$

or, in matrix notation,

$$\text{Subject to} \quad \begin{bmatrix} A_d^T & A_\ell^T & A_r^T & A_o^T \end{bmatrix} \begin{bmatrix} Y_D \\ Y_L \\ Y_R \\ Y_O \end{bmatrix} - U = C.$$

Note that every constraint in Eq. (2) can be rewritten as

$$\sum_{\substack{i=1 \\ i \neq d, \ell}}^m a_{ij} Y_i + n_{dj} Y_d + Y_\ell - u_j = c_j, \quad j = 1, 2, \dots, n$$

and, since the complementary slackness conditions imply that $u_j * X_j = 0$ at optimality, then $X_j > 0$ implies $u_j = 0$. It will turn out that the quantity $\eta_{dj} Y_d + Y_\lambda$, the marginal price of the fuel to demand, will play a critical role in the new perturbed formulation.

The following terms are defined for ease of exposition:

$Q(IEF)(DM)$ = the quantity of IEF, say, oil, to demand (DM), say space heat, in the optimal solution. For example, $Q(OIL)(14)$ is the total oil allocated for space heat in the optimal solution of the equilibrium problem.

$P(IEF)(DM)$ = the reference marginal value of IEF to demand DM in the optimal solution of the equilibrium problem. This is defined by $P(IEF)(DM) = \text{eff}(IEF)(DM) * Y(DM) + Y(IEF)(LM)$ where $\text{eff}(IEF)(DM)$ is λ_{dj} ; $Y(DM)$ is the marginal value on the demand equation; and $Y(IEF)(LML)$ is the marginal value on the limiting equation of IEF to this demand.

The conventions associated with $Q(IEF)(DM)$ will be used in what follows.

Algorithm I (Discrete formulation)

1. Solve the "equilibrium problem", Eq. (1).
2. From the "equilibrium problem" compute $Q(IEF)(DM)$ and $P(IEF)(DM)$ as defined above. These are the reference quantities and prices.
3. Input
 - (a) NPOINT: The number of points to be used in approximating the demand-price curves. NPOINT must be odd and ≥ 3 . Empirically $27 \leq \text{NPOINT} \leq 47$ seems to be a useful range.
 - (b) NLEFT: The number of points to the left of the reference point.
 - (c) NRIGHT: The number of points to the right of the reference point.
 - (d) $\text{ETA}(IEF)(DM)$: The "own-price" elasticity of fuel form IEF for demand DM. See Table 1 for the base-case elasticities used.

TABLE 1				
Short-Run Own-Price Elasticities for Use in the BNL Linear Programming Optimization Model for ERDA 1976 Forecasts				
	Electricity	Gas	Oil	Coal
Residential & Commercial				
Space heat	-0.30	-0.12	-0.14	-0.25
Air conditioning	-0.10	NA	NA	NA
Misc. thermal (water heat cooling)	-0.33	-0.25	-0.24	-0.25
Misc. electric (lights, motors)	-0.30	NA	NA	NA
Industrial				
Process heat & misc.	-0.10	-0.05	0.28	-0.25
Petrochemicals	-0.10	-0.25	0.28	-0.25
Misc. electric drive	-0.28	NA	NA	NA
Primary aluminum	-0.09	NA	NA	NA
Iron & steel production	-0.09	0.25	0.25	0.25
Transportation				
Electrified mass transit	-0.25	-0.25	-0.25	NA
Automobile	-0.25	-0.25	-0.12	NA
Air transport	NA	-0.25	-0.25	NA
Bus, truck, rail	-0.25	-0.25	-0.40	-0.25
Ship	-0.25	-0.25	-0.25	-0.25

(e) PERL: The percentage of reference quantity $Q(IEF)$ (DM) that is to serve as the leftmost point on the piecewise linear marginal demand-price curve ($0 < PERL < 1$).

(f) PERL: The percentage of the reference quantity that is to serve as the rightmost point ($PERR > 1$).

4. Generate stepwise linear points on the price-quantity curves as follows (the reference to indices IEF and DM are omitted in the variables for notational convenience):

For each demand (DM) and (IEF)

(a) Compute:

$$R = NLEFT + 1$$

$$Q(R) = Q_0 \text{ (reference point)}$$

$$P(R) = P_0$$

$$Q(1) = PERL * Q_0 \text{ (leftmost point)}$$

$$Q(NPOINT) = PERR * Q_0 \text{ (rightmost point)}$$

$$DELTA = (1 - PERL) * Q_0 / NLEFT$$

$$DELTAR = (PERR - 1) * Q_0 / NRIGHT$$

(b) For IP = 2 to NLEFT, compute

$$Q(IP) = DELTA + Q(IP - 1)$$

(c) For IP = R + 1 to NPOINT - 1

$$Q(IP) = DELTAR + Q(IP - 1)$$

(d) For IP = 1 to NPOINT, compute

$$P(IP) = P_0 * (Q(IP) / Q_0) ** (1/ETA)$$

(e) $C(1) = K(P_0) * P_0 * Q_0 * (PERL) ** (1 + 1/ETA)$

For IP = 2 to NPOINT, compute

$$C(IP) = C(IP - 1) + K(P_0) * (Q(IP) - Q(IP - 1)) * P(IP)$$

$$\text{where } K(P_0) = 0.99 \text{ if } P_0 > 0$$

$$= 1.01 \text{ if } P_0 < 0.$$

In the perturbed formulation new variables, $W(IEF)(DM)(IP)$, and new constraints, $EL(IEF)(DM)$, are generated, and existing constraints are modified to solve the augmented problem, Eq. (3).

$$5. \text{ Minimize } \sum_{j=1}^n C_j * X_j + \sum_{IP, IEF, DM} C(IEF)(DM)(IP) * W(IEF)(DM)(IP)$$

$$6. \text{ Subject to } \sum_{j=1}^n \eta_{dj} * X_j$$

made nonbinding for the set of demand equations - variant 1 only.

$$7. \sum_{\lambda} X_{\lambda} - \sum_{IP=1}^{NPOINT} Q(IEF)(DM)(IP) * W(IEF)(DM)(IP) = 0$$

8. $W(\text{IEF})(\text{DM})(\text{IP}) = 1$ (constraints $\text{EL}(\text{IEF})(\text{DM})$).

IP

$A_r^* X \leq b_r$: unchanged in form

$A_o^* X + S_o = b_o$: unchanged in structure

Note that η_{dj}^* does not necessarily equal η_{dj} ; similarly $A_r^* \neq A_r$, $A_o^* \neq A_o$, $C_j^* \neq C_j$. It is shown in Appendix A that when $\eta_{dj}^* = \eta_{dj}$, $A_r^* = A_r$, $A_o^* = A_o$, and $C_j^* = C_j$, then the solution to the modified perturbed problem, Eq. (3), is the same as the solution to the equilibrium problem, Eq. (1).

The solution to

$$\frac{\partial Q/Q}{\partial P/P} = \sigma \quad \text{with} \quad P(Q_o) = P_o \quad \text{is}$$

$$\begin{aligned} P(Q) &= P_o * (Q/Q_o)^{(1/\sigma)} \quad \text{for } \sigma \neq 0 \\ &= P_o \quad \text{for } \sigma = 0. \end{aligned} \quad (4)$$

The continuous version takes note of the fact that step 4(e) above is a discrete formulation of the differential equation (5) below. Again the indices are omitted, and $Q_o = Q(\text{IEF})(\text{DM})$, $P_o = P(\text{IEF})(\text{DM})$, and $\sigma = \text{ETA}(\text{IEF})(\text{DM})$.

$$dC/dQ = P(Q) \quad \text{with} \quad C(Q_o) = P_o Q_o. \quad (5)$$

Use of relations (4) and (5) yields the continuous solution to Eq. (5),

$$\begin{aligned} C(Q) &= P_o Q \quad \text{for } \sigma = 0 \\ &= P_o Q_o \left\{ 1 + \ln Q - \ln Q_o \right\} \quad \text{for } \sigma = -1 \\ &= \frac{P_o Q_o}{1 + \sigma} \left\{ \sigma \left(Q/Q_o \right)^{1+(1/\sigma)} + 1 \right\} \quad \text{for } \sigma \neq 0, -1 \end{aligned} \quad (6)$$

and has the computational advantage of bypassing the explicit computation of $P(Q)$ and avoiding roundoff or discretization errors.

Algorithm II (Continuous Version)

Steps 1 to 3 and 5 to 8 are the same as in the discrete version. Step 4 should be modified as follows: (a) to (c) unchanged; (d) and (e) deleted; (f) and (g) added.

4(f) Compute, using Eq. (6) above, the nontrivial case,
$$\text{CONS} = P(R) * Q(R) / (\text{ETA} + 1)$$
$$\text{EXP} = 1 + 1/\text{ETA}$$

4(g) For $IP = 1$ to $NPOINTS$, compute, for the nontrivial case,
$$C(IP) = \text{CONS} * \{ \text{ETA} * (Q(IP) / Q(R)) ** \text{EXP} + 1 \}.$$

Figures 4 and 5 illustrate the process.

III. CASE STUDIES

This section presents some of the computational experience derived from using the elasticity representation of demands described in Section II. Three basic classes of tests were performed. One comparison was made between a Reference Case (FORECAST II) and AET-8 for 2000. See Figures 6 and 7 for the corresponding RESSs, and Tables 2 to 5 for the corresponding cost assumptions.* The second case involved an analysis of a preliminary National Energy Policy (NEP) scenario versus another BASE Case for the year. Figures 8 and 9 show the RESSs, and Tables 6 and 7 give the corresponding price structure.

The third class was used as a calibration and verification set for variant 2, the exogenously determined demands case. For the energy-economy equilibrium, a BASE Case year 2000 energy scenario (Figure 10) was used as the starting point, and a significantly different point (Figure 11) was used to determine appropriate elasticities sufficient to transform one equilibrium to the other. The transformation was considered acceptable if the derived energy (e.g., oil to space heat) capturing at least 10% of the targeted

*Figures 6 to 11 appear in Appendix B. Tables 2 to 16 follow the references.

Figure 4. $P(Q)$ vs Q

Figure 5. Cost vs Quantity

system was within 5% of the target solution for each technology energy service market. The derived elasticities were used to estimate the energy component of a third known consistent energy-economy equilibrium solution. The results are acceptable; the details of the input assumptions defining all three scenarios in the verification runs and conclusions are described more fully later in this section.

Some interesting observations are summarized below.

1. When the "perturbed problem" is the original problem, the solution to the modified "perturbed problem" is identifiable to the solution to the original problem. This lends empirical support for the validity of the formulation, underscores the consistency of the methodology with the more comprehensive full equilibrium approach, and reiterates the implicit incorporation of price and nonprice effects in the reformulation.

2. The qualitative response of the system to changes in price and/or technological characterization (e.g., efficiency changes) is entirely consistent with economic intuition. That is, price increases of fuel to demands tend to decrease the overall demand for energy services (and the demand for the specific fuel type) as well as to cause fuel switching (for variant 2). On the other hand, improvements in efficiency for energy conversion devices tend to reduce the pressure moderating consumer demand and has the net effect of increasing the demand for that fuel type.

3. With variant 1, own-price elasticities of fuels to demands were sought which force a transition in the energy system from the AET-8 case to the corresponding Reference Case. Elasticities required for making this transition were successfully found in every case except oil to process heat and jet fuel for air transport, for which that assumptions of consistency between the two structures of the economy implied by the energy components could not be verified. The two scenarios do not represent a short-term response of the energy system to price-efficiency changes, but some longer-term adjustment reflecting more basic structural changes, especially in the process heat and air transport sectors.

4. With variant 1 used on the second class (NEP to BASE) and third class (verification set) of tests indicated, it was demonstrated empirically that it is impossible to transform these energy-economic equilibria from one state to another without major economic adjustments; i.e., the transformation cannot be short-term or represent minor structural adjustments to the economy.

The results summarizing the first two classes of tests with variant 1 are given in Tables 8 and 9 respectively.

Below are some comments on the consistency between two sets of scenarios of at least passing interest, when variant 1 is used.

1. AET-8 versus Reference Case year 2000 scenario: The behavior of the demand elasticity model is consistent with the pricing and technical assumptions implicit in the two scenarios. For example, the delivered price of oil to process heat decreases from \$4.18/10⁶Btu in AET-8 to \$3.43/10⁶Btu in the Reference Case; the relative conversion efficiencies do not compensate for this difference, and, since the elasticity formulation assumes a downward sloping (inverse) relationship between the quantity demanded and cost, it is not surprising that the flow of oil to process heat cannot be decreased in variant 1 by using any reasonable elasticity value.

The two energy scenarios are closely related but not entirely consistent, with greatest deviation in air transport and oil to process heat. See Table 8 for a summary review.

2. NEP 1 versus BASE scenarios year 2000: The NEP and BASE are reasonably consistent in the transportation and petrochemical area but show severe distortion for oil to space heat and process heat (see Table 9). Evidently these two scenarios do not represent energy components for similar structures of the economy.

Case High Oil Price III, calibration and verification of Variant 2: Two extreme but known energy-economy equilibrium cases⁹ were selected to calibrate the model and test its predictive value. The initial equilibrium is designated as BASE (see Figure 10 for

the RES); Tables 10 and 11 define the Base Case assumptions. The target equilibrium solution is High Oil Price III (see Figure 11 for the RES); Tables 12 and 13 define the underlying assumptions with a comparison to the BASE Case assumptions.

Application of variant 1 clearly indicated that the energy components of the two energy-economy equilibriums represent different economic structures.

The calibration involved the simultaneous variation of the elasticities for the sectors for which interfuel substitution was possible. The procedure was terminated when actual fuel flows were within 5% of the target flows. Calibration was not rigorously attempted for flows not capturing at least 10% of the market for that energy sector.

With use of the derived elasticities from the above and the price/technology assumptions of a Mid-Range Oil Price energy-economy scenario (Tables 14 and 15), the demand representation formulation was used to forecast the behavior of the Mid-Range Oil Price/technology case. Table 16 summarizes the results.

Evidently the demand elasticity representation forecasts the resulting flows due to price and efficiency changes to within 5% of the target solution. Since this level of uncertainty is clearly "noise" in the system, the results of this verification provide substantial cause for optimism.

Additional tests of the model which systematically examine model sensitivity to variations in the elasticity parameters will be required.

Finally, to apply the demand elasticity representation for short-term impact analysis, variant 1 will have to be calibrated with the period encompassing the oil embargo of 1973 used for derivation of elasticities related to oil shortages. Coal strikes will be used to determine the coal-related elasticities.

IV. CONCLUSIONS

Two variants of a demand elasticity representation have been formulated and implemented within BESOM.

Variant 1 is being calibrated and will be verified. Variant 2 has been calibrated and its predictive value affirmed. Implicit in variant 2, however, is that behavioral decision patterns and institutional/societal environments affecting technological choice remain unchanged.

Variant 2 is a useful tool for policy analysis in that, once a reference equilibrium case set is developed with use of the complete set of linkages between energy and the economy, the energy sector component can easily be studied for sensitivity to price-technology-resource availability variations in a realistic manner. The "all-or-none" or "flip-flop" behavior characteristic of LPs is eliminated by this formulation. Stepwise linear fuel demand curves are substituted for simple upper and lower bound constraints, and this adds a consistent relationship between market behavior and fuel prices.

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Table 2
Fuel Costs - Reference Case Year 2000 (1975 Dollars)
and Supplementary Fuel Costs (\$/MMBtu)

	Residential/ comm. and transportation	Utility	Industrial
Gasoline	2.54	-	-
Distillate	1.29	0.65	0.28
Residual	-	0.65	0.65
Kerosene	1.60	-	-
Coal	-	0.13	0.13
Nat. gas	0.85	0.22	0.22

	Crude fuel costs (\$/MMBtu)	Refining efficiency
Domestic oil	2.87	0.91
Imported oil	2.87	0.91
Oil shale	2.62	0.91
Nat. gas	2.19	0.91
Underground coal	0.71	0.97
Stripmined coal	0.66	0.97
Nuclear fuel	0.71	1.00

Table 3
Central Station Characteristics -
Reference Case year 2000 (1975 Dollars)

	Capital Cost (\$/kW)	Conversion efficiency, 2000	O&M (mills/kWh)
Coal steam	445	0.34	4.40
Coal combined cycle	450	0.45	5.80
Oil steam electric	300	0.35	4.00
Gas turbines	140	0.30	4.80
Gas steam electric	270	0.35	3.80
Total energy systems	330	0.30	6.35
LWR	585	0.34	3.00
LMFBR	690	0.40	2.80
HTGR	585	0.40	3.00
Hydroelectric	310	0.34	3.10
Geothermal electric	840	0.34	4.80
Solar electric	1200	0.34	4.80
Pumped storage	200	0.75	3.10

Table 4
Fuel Costs - AET-8 Case year 2000 (1975 Dollars)
and Supplementary Fuel Costs (\$/MMBtu)

	Residential/ comm. and transportation	Utility	Industrial
Gasoline	0.94	-	-
Distillate	1.29	0.65	0.64
Residual	-	0.65	0.64
Kerosene	1.44	-	-
Coal	-	0.32	0.46
Nat. gas	0.89	0.49	0.49

	Crude Fuel Costs (\$/MMBtu), REF	Refining efficiency
Domestic oil	3.54	1.00
Imported oil	3.54	1.00
Oil shale	5.70	1.00
Nat. gas	0.69	0.92
Underground coal	0.69	1.00
Stripmined coal	0.49	1.00
Nuclear fuel	0.28	1.00

Table 5
Central Station Characteristics -
AET-8 Case Year 2000 (1975 Dollars)

	Capital Cost* (\$/kW)	Conversion efficiency, Year 2000	O&M (mills/kW hr)
Coal steam	438	0.35	4.40
Coal combined cycle	450	0.45	4.10
Oil steam electric	330	0.35	4.00
Gas turbines	147	0.30	4.80
Gas steam electric	290	0.35	3.80
Total energy systems	235	0.30	6.35
LWR	425	0.31	3.00
LMFBR	554	0.40	2.80
HTGR	585	0.40	3.00
Hydroelectric	272	0.35	3.10
Geothermal electric	191	0.35	4.80
Solar electric	1200	0.35	4.80
Pumped storage	220	0.75	3.10

Table 6
Fuel Costs - BASE Case and NEP Year 2000 (1975 Dollars)
and Supplementary Fuel Costs (\$/MMBtu)

	Residential/comm. and transportation		Utility		Industrial	
	BASE	NEP	BASE	NEP	BASE	NEP
Gasoline	3.02	3.02	-	-	-	-
Distillate	1.29	1.29	0.65	0.90	0.65	0.90
Residual	-	-	0.65	0.95	0.65	0.95
Kerosene	1.44	1.44	-	-	-	-
Coal	-	-	0.32	0.32	0.46	0.46
Nat. gas	0.88	0.88	0.49	1.45	0.49	1.45

Crude fuel costs (\$/MMBtu)		
	BASE	NEP
Domestic Oil	2.88	3.21
Imported Oil	3.21	3.21
Oil shale	3.21	3.21
Nat. gas	2.82	2.82
Coal	1.18	0.65
Nuclear fuel	0.65	1.18

Table 7
Central Station Characteristic -
BASE Case and NEP Year 2000 (1975 Dollars)

	Capital cost (\$/kW)	Conversion	O&M
		efficiency, 2000	(mills/kW hr)
Coal steam	445	0.35	4.40
Coal combined cycle	450	0.45	4.10
Oil steam electric	300	0.33	4.00
Gas turbines	140	0.30	4.80
Gas steam electric	270	0.33	3.80
Total energy systems	330	0.50	8.15
LWR	585	0.34	4.00
Hydroelectric	310	0.35	3.1
Geothermal electric	840	0.35	4.8
Solar electric (central)	1200	0.35	4.8
Photovoltaics	500	0.35	1.8
Pumped storage	200	0.75	3.1
LMFBR	690	0.40	2.8
HTGR	585	0.40	3.0

Table 8
Summary Calibration Using Variant 1 -
Transitions From AET-8 to Reference Case, Year 2000

Demands	Old equil. flow	New equilibrium flow		Elasticity used
		Target	Actual	
Process heat:				
Oil	6.04	4.46	6.04	-0.06
Nat. gas	16.81	11.73	11.77	-0.52
Coal	5.99	5.70	5.77	-0.06
Electricity	4.68	3.96	3.88	-0.60
Solar	NA			
Geothermal	NA			
Low-Btu	NA			
Direct thermal	NA			
Air transport:				
Oil	14.5	8.63	10.25	-8.20
Truck/bus & Diesel				
rail: Oil	9.26	10.53	10.50	-0.95
Rail: Electric	0.29	0.23	0.27	-0.90
Auto: Gasoline	14.7	12.84	12.50	-0.33
Electric	-	-	-	-
Space Heat:				
Oil	6.75	6.46	6.91	-0.06
Nat. gas	5.14	4.97	5.04	-0.30
Electricity res.	2.89	2.38	2.46	-0.20
Heat pump	NA			
Solar	NA			
Thermal	NA			
Water heat:				
Oil	-	0.31	0.10	-0.06
Nat. gas	2.58	2.63	2.74	-0.30
Electric	1.35	1.37	1.30	-0.20
Solar	-	-	-	-
Air conditioning:				
Electricity conv.	1.24	1.59	1.61	-0.20
Gas	0.30	0.30	0.30	-0.70
Solar	-	-	-	-
Misc. res./comm.				
elec.	11.59	9.86	9.85	-0.45
Misc. industrial				
elec.	2.41	2.05	2.14	-0.65
Iron production:				
Coke	3.54	3.54	3.41	-0.06
Petrochemicals:				
Oil	8.73	8.61	8.73	-0.10
Nat. gas	1.57	1.70	1.62	-0.05
Coal	0.48	0.32	0.32	-0.39
Total resources				
(Quads)	179	163	167	
(1 Quad=10 ¹⁵ Btu)				

Table 9
Summary Calibration Using Variant 1 -
Transitions From NEP to BASE Case, Year 2000

Demands	Old equil. flow	New equilibrium flow		Elasticity used
		Target	Actual	
Process heat:				
Oil	0.90	7.02	4.17	-9.90
Nat. gas	5.15	7.25	7.02	-1.40
Coal	10.88	6.25	6.28	-6.50
Electricity	1.75	1.70	1.75	-0.60
Solar	0.55	0.67	0.55	-0.60
Geothermal	0.47	0.33	0.64	-0.40
Low-Btu syngas	0.90	0.75	0.75	-0.25
Direct thermal	1.0	0.75	0.88	-0.25
Air transport: Oil	5.47	5.74	5.47	-0.25
Truck/bus and Diesel Rail: Oil	7.25	7.70	7.25	-0.40
Rail: Electric	0.10	0.10	0.10	-0.25
Auto: Gasoline	9.42	10.00	9.42	-0.40
Electric	0.11	0.09	0.10	-1.50
Space heat:				
Oil	2.90	6.10	5.00	-9.90
Nat. gas	6.01	4.74	4.62	-7.00
Electricity res.	0.71	0.93	0.66	-0.10
Heat pump	1.00	1.00	0.96	-0.06
Solar	0.25	0.20	0.24	-0.25
Thermal	0.12	0.10	0.09	-0.25
Water heat:				
Oil	0.10	0.10	0.10	-0.24
Nat. gas	1.14	1.44	1.35	-2.80
Electric	1.45	1.45	1.39	-0.40
Solar	0.05	0.10	0.13	-0.40
Air conditioning:				
Electricity conv.	1.18	1.49	1.18	-0.30
Gas	-	-	-	-
Solar	0.05	0.03	0.05	-0.40
Misc. res./comm. elec.	8.55	9.72	10.10	-3.80
Misc. industrial elec.	1.78	2.02	2.10	-1.40
Iron production:				
Coke	2.70	3.21	3.19	-4.50
Petrochemicals:				
Oil	5.69	6.41	6.72	-0.87
Nat. gas	4.10	4.44	4.85	-0.33
Coal	1.10	1.36	1.30	-0.15
Total resources (Quads)	123	139	133	
(1 quad=10 ¹⁵ Btu)				

Table 10
Energy Usage Summary Verification -
BASE Case Results for 2000 and for 1975-2000 Growth Rates
(From Ref. 10)

	1977	1985	1990	2000	%/yr Growth 1977-2000
Energy-Aggregate					
Consumption (quads)					
Domestic crude					
Oil	20.9	22.40	20.10	18.80	-0.5
Imported crude					
Oil	17.0	21.95	23.83	26.89	2.0
Shale oil	0.0	0.30	1.00	3.00	-
Domestic Nat.					
Gas	17.0	18.73	18.14	17.67	0.2
Imported Nat.					
Gas	0.9	1.41	1.36	1.33	1.7
Coal	14.90	19.77	25.79	32.66	3.5
Nuclear	2.20	7.98	14.30	28.49	11.8
Other, via					
central elec.	3.0	3.99	4.65	7.65	4.2
Other via non-					
central	0.0	0.21	0.85	2.00	-
Total	75.9	96.74	110.01	138.49	2.7
Energy-Prices (1975 \$/10 ⁶ Btu)					
Primary energy					
Domestic oil	1.37	1.85	2.14	2.88	3.0
Imported oil	2.24	2.50	2.63	3.21	1.4
All crude oil	1.65	2.17	2.41	3.08	2.5
Domestic nat. gas	0.41	2.20	2.31	2.82	8.0
Imported nat. gas	2.40	2.60	2.65	3.10	1.0
Coal	0.60	1.02	1.07	1.18	2.7
Uranium	0.43	0.50	0.55	0.65	1.7
Av. supply price					
of energy	1.05	1.79	1.81	2.02	2.
Delivered energy					
Del. oil					
products	3.58	3.83	4.05	4.74	1.1
Del. nat. gas					
products	1.29	3.08	3.21	3.78	4.4
Del. coal	0.81	1.49	1.54	1.65	2.9
Del. electricity	7.92	10.85	11.08	11.37	1.5
Av. delivered					
price of					
energy	3.12	4.52	4.85	5.55	2.3
Energy-Imported					
Oil and Gas					
Quads of					
imports	17.90	23.36	25.19	28.22	2.0
As % of total					
primary					
consumption	23.6	24.1	22.9	20.4	-0.6

Table 11
Aggregate Energy-Economy Assumption Verification -
BASE Case Results for 2000 and for 1975-2000 Growth Rates
(From Ref. 10)

	1977	1985	1990	2000	%/yr Growth 1977-2000
<hr/>					
Economy - GNP and Its Components (10 ⁹ 1972 \$)					
GNP	1329.7	1773.3	2060	2790.6	3.3
Consumption	859.3	1144.3	1340	1808.1	3.3
Investment	189.0	264.2	303	411.3	3.4
Government	266.0	344.6	408	567.0	3.3
Net exports	15.4	20.2	9	4.2	-5.8
<hr/>					
Economy - Prices (Index: 1.00 in 1972)					
Agriculture	1.527	2.372	5.025	8.198	6.9
Manufacturing	1.327	1.782	3.778	5.181	5.6
Transportation	1.231	1.659	2.589	3.232	4.0
Services	1.442	2.214	2.525	3.743	3.9
Consumption	1.405	1.937	2.284	3.205	3.7
GNP	1.414	2.017	2.425	3.535	3.7
<hr/>					

Table 12
High Oil Price III - Energy Usage Summary
(From Ref. 10) (% Ch. = % Change from BASE)

	1985		1990		2000	
	Amt.	% Ch.	Amt.	% Ch.	Amt.	% Ch.
Energy - Aggregate Consumption (quads)						
Domestic						
crude oil	23.32	4.1	21.51	7.0	20.42	8.6
Imported						
crude oil	8.18	-62.7	0.00	-100.0	0.00	-100.0
Shale oil	0.32	6.7	1.05	5.0	3.10	3.3
Domestic						
nat. gas	20.29	8.3	19.90	9.7	20.37	15.3
Imported						
nat. gas	2.00	41.8	1.31	-3.7	0.00	-100.0
Coal	23.58	19.3	28.95	12.3	34.49	5.6
Nuclear	9.36	17.3	14.06	-1.7	24.69	-13.3
Other, via						
central						
elec.	4.23	6.0	5.25	12.9	8.75	14.4
Other, via						
noncentral	0.35	66.7	1.41	65.9	3.50	75.0
Total	91.63	-5.3	93.43	-15.1	115.82	-16.4
Energy - Prices (1975 \$/10 ⁶ Btu)						
Primary energy						
Crude oil	3.59	65.4	7.16	197.1	8.59	178.9
Crude nat.						
gas	2.39	7.2	2.92	25.3	3.38	19.0
Coal	1.15	12.7	1.45	35.5	1.50	27.1
Uranium	0.50	0.0	0.55	0.0	0.65	0.0
Av. supply						
price	2.29	27.9	3.14	73.4	3.24	60.4
Delivered Energy						
Del. oil						
products	5.65	47.5	9.62	137.5	10.81	128.1
Del. gas						
products	3.33	8.1	3.90	21.5	4.36	15.3
Del. coal	1.62	8.7	1.92	24.7	1.97	19.4
Del. elec-						
tricity	12.87	18.6	11.07	0.01	11.09	-2.5
Av. del.						
price	5.89	30.3	6.81	40.4	7.63	37.5
Energy - Imported						
Oil and Gas						
Quads of						
imports	10.18	-56.4	1.31	-94.8	0.00	-100.0
As % of total						
primary						
consumption	11.1	-53.9	1.4	-93.9	0.00	-100.0

Table 13
High Oil Price III - Energy Usage Summary
(From Ref. 10) (% Ch. = % Change from BASE)

	1985		1990		2000	
	Amt.	% Ch.	Amt.	% Ch.	Amt.	% Ch.
Energy - Quantities (10 ¹⁵ Btu)						
Aggregate primary energy consumption	91.63	-5.3	93.43	-15.1	115.82	-16.4
Oil and gas imports	10.18	-56.4	1.31	-94.8	0.00	-100.0
Delivered oil products	27.62	-28.0	19.95	-47.9	22.44	-46.5
Energy - Prices (1975 \$/10 ⁶ Btu)						
Aggregate supply price	2.29	27.9	3.14	73.4	3.24	60.4
Refined oil price	5.65	47.5	9.62	137.5	10.81	128.1
Economy (10⁹ 1975 \$)						
GNP	1747.1	-1.5	1972.0	-4.2	2674.1	-4.2
Consumption	1124.2	-1.8	1267.9	-5.4	1723.8	-4.7
Investment	259.9	-1.6	288.1	-4.8	391.2	-4.9
GNP price (Index: 100 in 1972)	2.055	1.9	2.491	2.7	3.628	2.6

Table 14
Mid-Range Oil Price - Energy Usage Summary
(From Ref. 10) (% Ch. = % Change from BASE)

	1985		1990		2000	
	Amt.	% Ch.	Amt.	% Ch.	Amt.	% Ch.
Energy - Aggregate						
Consumption (quads)						
Domestic crude oil	23.08	3.0	20.60	2.5	19.36	3.0
Imported crude oil	13.31	-39.4	8.10	-66.0	12.34	-54.1
Shale oil	0.32	6.7	1.05	5.0	3.10	3.3
Domestic nat. gas	20.26	8.2	19.80	9.2	20.31	14.9
Imported nat. gas	2.00	41.8	2.00	47.1	2.00	50.4
Coal	21.88	10.7	28.11	9.0	33.14	1.5
Nuclear	8.30	4.0	13.79	-3.6	26.67	-6.4
Other, via central elec.	4.07	2.0	4.93	6.0	8.10	5.9
Other, via noncentral	0.31	47.6	1.18	38.8	2.77	38.5
Total	93.62	-3.2	99.96	-9.5	127.80	-7.7
Energy - Prices (1975 \$/10 ⁶ Btu)						
Primary energy						
Crude oil	2.71	24.9	4.16	72.6	4.71	52.9
Crude nat. gas	2.27	1.8	2.53	8.6	2.98	4.9
Coal	1.07	4.9	1.19	11.2	1.27	7.6
Uranium	0.50	0.0	0.55	0.0	0.65	0.0
Av. supply price	1.99	11.2	2.34	29.4	2.45	21.3
Delivered energy						
Del. oil products	4.45	16.2	6.10	50.6	6.71	41.6
Del. gas products	3.28	6.5	3.79	18.1	3.89	2.9
Del. coal	1.54	3.4	1.66	7.8	1.74	5.5
Del. electricity	12.15	12.0	11.23	1.4	11.46	0.8
Av. del. price	5.06	11.9	5.73	18.1	6.29	13.3
Energy - Imported Oil and Gas						
Quads of imports	15.31	-34.5	10.10	-59.9	14.34	-49.2
As of total primary consumption	16.4	-32.0	10.1	-55.9	11.2	-45.1

Table 15
Mid-Range Oil Price - Aggregate Energy-Economy Summary
(From Ref. 10) (% Ch. = % Change from BASE)

	1985		1990		2000	
	Amt.	% Ch.	Amt.	% Ch.	Amt.	% Ch.
Energy - Quantities (10 ¹⁵ Btu)						
Aggregate primary energy consumption	93.62	-3.2	99.96	-9.5	127.80	-7.7
Oil and gas imports	15.31	-32.0	10.10	-59.9	14.34	-49.2
Delivered oil products	33.60	-12.4	25.22	-34.1	30.7	-26.8
Energy - Prices (1975 \$/10 ⁶ Btu)						
Aggregate supply price	1.99	11.2	2.34	29.4	2.45	21.3
Refined oil price	4.45	16.2	6.10	50.6	6.71	41.6
Economy (10⁹ 1975)						
GNP	1756.4	-1.0	2006.9	-2.6	2729.3	-2.2
Consumption	1133.0	-1.0	1296.8	-3.2	1765.8	-2.3
Investment	261.7	-0.9	293.4	-3.1	395.9	-3.7
GNP Price (Index: 100 in 1972)	2.047	1.5	2.479	2.2	3.625	2.5

Table 16
Summary of Calibration and Verification Using Variant 2

Demands	Base	Medium		High		Elasticity
		Target	Actual	Target	Actual	
Space heat:						
Oil	4.05	3.26	3.43	1.75	1.75	-1.45
Nat. gas	5.99	6.35	6.38	5.79	6.01	-0.13
Electricity	2.20	1.39	1.33	1.38	1.39	-0.90
Heat pump	0.60	0.65	0.67	0.75	0.70	-1.50
Solar	0.70	0.73	0.70	0.76	0.70	-0.30
Thermal	0.10	0.10	0.08	0.10	0.08	-0.40
Water heat:						
Oil	0.34	0.10	0.20	0.01	0.08	-1.95
Nat. gas	1.19	1.18	1.14	1.02	1.01	-1.20
Electricity	1.35	1.04	1.03	1.06	1.07	-0.67
Solar	0.18	0.20	0.17	0.22	0.17	-0.20
Air conditioning:						
Electricity conv.	1.20	1.32	1.30	1.14	1.16	-0.60
Solar	0.09	0.08	0.09	0.12	0.09	-0.30
Misc. res./comm.						
elec.*	3.86	3.58	3.58	3.27	3.27	-
Misc. industrial						
elec.*	6.12	5.67	5.67	5.19	5.19	-
Iron production:						
Coal	3.83	3.62	3.54	3.42	3.37	-0.60
Elec.	0.26	0.24	0.26	0.22	0.23	-0.95
Petrochemicals:						
Oil	6.16	4.36	4.18	1.48	1.50	-1.62
Nat. gas	3.40	4.15	4.36	5.00	4.96	-0.70
Coal	1.10	1.20	1.16	2.50	2.52	-0.12
Aluminum*	0.58	0.69	0.69	0.64	0.64	
Process heat:						
Thermal	0.00	0.10	0.10	0.25	0.25	-0.25
Oil	6.46	2.03	2.31	0.69	0.84	-2.50
Nat. gas	5.95	6.89	6.47	6.46	6.47	-0.40
Coal	6.70	6.77	6.64	6.89	6.80	-0.15
Electricity	2.50	2.34	2.50	2.50	2.50	-0.15
Solar	0.53	0.59	0.62	0.65	0.62	-0.90
Low-Btu gas	1.00	1.00	0.96	1.00	1.0	-0.30
Geothermal	0.50	1.27	1.28	1.35	1.31	-3.00
Air transport: Oil*	6.80	5.91	5.91	5.09	5.09	-
Truck/bus: Oil*	8.79	8.46	8.46	6.75	6.75	-
Rail: Electric*	0.22	0.21	0.21	0.20	0.20	-
Auto: Oil	7.80	6.71	6.73	5.77	5.82	-0.75
Elec.	0.08	0.10	0.08	0.13	0.08	-0.20
Ship: Oil*	1.54	1.67	1.67	1.42	1.42	-
Total resources						
(quads)	138	128	126	116	115	

*No fuel substitution is possible in these demands.

1 Quad = 10^{15} Btu

APPENDIX A. MATHEMATICAL DERIVATION

We shall show that an optimal solution to the original energy equilibrium problem is identical to the solution to the "perturbed problem" of variant 1 using the price-quantity information from the original problem. The basic sequence will be to define three related LP formulations and their dual counterparts; to show that the first two have the same optimal primal solution with the dual optima simply related; and to show that a similar relationship holds for the second and third formulations. Some preliminary definitions and a lemma will be required first, however.

Problem 1:

$$\min \sum_{i,j=1} C_{ij} x_{ij}$$

$$\text{s.t. } A_O X \leq b$$

$$A_D X = b_D$$

$$X_L \leq b_L$$

where

D = set of demand equations
 $d \in D$

L = set of limiting equations
 $l \in L$

O = remaining constraint set

$x_{l,d}$ = total flow of a particular intermediate energy from (l) which services demand (d) e.g., oil to space heat

$(A_D X)_d = \sum_j e_{j,d} x_{j,d}$; $e_{j,d}$ is efficiency of end-use conversion for demand d for fuel type j

Problem 2:

$$\min \sum_{i,j=1} C_{ij} X_{ij}$$

$$\text{s.t. } A_O X \leq b_O$$

$$X_{\ell,d} = b'_{\ell} \quad , \quad \ell \in L' \quad \text{for all } d \in D$$

where $b'_{\ell} = X_{\ell,d}^*$ (optimal, flow in problem 1). Note that the constraints $A_D X = b_D$ are superfluous.

Problem 3:

$$\min \sum_{i,j=1} C_{ij} X_{ij} + \sum_{d,\ell,p} \zeta_{\ell,d}^p \cdot w_{\ell,d}^p$$

$$\text{s.t. } A_O X \leq b_O$$

$$X_{\ell,d} - \sum_{p=1}^{NPC} Q_{\ell,d}^p \cdot w_{\ell,d}^p = 0; \quad \ell \in L'$$

$$\sum_{p=1}^{NPC} w_{\ell,d}^p = 1; \quad \text{for all } \ell, d$$

where $Q_{\ell,d}^p = X_{\ell,d}^*$ for $p=0$

$$Q_{\ell,d}^1 < Q_{\ell,d}^2 < \dots < Q_{\ell,d}^{NPC}$$

$$P_{\ell,d}^0 \equiv e_{\ell,d} Y_d^* + Y_{\ell(d)}^*; \quad Y_j^* \text{ defined in Dual Problem 1.}$$

$$Q_{\ell,d}^0 \equiv X_{\ell,d}^*$$

$$\zeta_{\ell,d}^p = \frac{P_{\ell,d}^0 Q_{\ell,d}^0}{1 + \sigma_{\ell,d}} \left\{ \sigma_{\ell,d} \left(\frac{Q_{\ell,d}^p}{Q_{\ell,d}^0} \right)^{1 + \frac{1}{\sigma_{\ell,d}}} + 1 \right\}$$

$\sigma_{l,d}$ = own price elasticity; $\sigma_{l,d} \neq 0, -1$

$e_{l,d}$ = end-use conversion efficiency of fuel type l
to demand d

Dual Problem 1:

$$\text{maximize} \quad \sum_{i \neq d, l(d)} b_i Y_i + \sum_{d \in D} b_d \cdot Y_d + \sum_{l(d)} b_{l(d)} \cdot Y_{l(d)}$$

$$\text{s.t.} \quad \sum_{i=1} a_{ij} Y_i \geq C_j \quad j \neq d, l(d)$$

$$\sum_{\substack{i=1 \\ i \neq d, l(d)}} a_{ij} Y_i + e_{l,d} \cdot Y_d + Y_{l(d)} \geq C_{l,d}; j = l(d), d$$

Y_d = dual variable for demand equation d

$Y_{l(d)}$ = dual variable for limiting equation of variable
 $X_{l,d}$

Dual Problem 2:

$$\text{maximize} \quad \sum_{i \notin L} b_j \pi_j + \sum_{j \in L} b'_j \pi_j$$

$$\text{s.t.} \quad \sum_{i=1} a_{ij} \pi_i \geq C_j \quad ; j \notin L'$$

$$\sum_{\substack{i=1 \\ i \notin L'}} a_{ij} \pi_i + \pi_l \geq C_{l,d}; j = l \in L'$$

Dual Problem 3:

$$\text{maximize} \quad \sum_{j \neq l, S} b_j \lambda_j + \sum_{i \in L'} \lambda_i$$

$$\text{s.t.} \quad \sum_{i=1} a_{ij} \lambda_i \geq c_j \quad ; \quad j \neq d, l$$

$$\sum_{\substack{i=1 \\ i \neq l, d}} a_{ij} \lambda_i + \lambda_{l, d} \geq c_{l, d}$$

$$\text{where} \quad \lambda_{l, d}^s - Q_{l, d}^p \lambda_{l, d} \geq \hat{c}_{l, d}^p \quad ; \quad s \in S$$

$Q_{l, d}^p$ = quantity defined in Problem 3

$\lambda_{l, d}$ = dual value for rows

$$x_{l, d} - \sum_{p=1}^{NPC} Q_{l, d}^p \cdot w_{l, d} = 0$$

$\lambda_{l, d}^s$ = dual value for rows

$$\sum_{p=1}^{NPC} w_{l, d}^p = 1$$

S = set of equations for the new rows

$$\sum_{p=1}^{NPC} w_{l, d}^p = 1$$

NPC = number of points to divide interval around
original equilibrium flow

Notice that dual formulations to Problems 1-3 are defined in
"Dual" Problems 1-3.

Lemma: If X^* is an optimal solution to Problem 1 and Y^* a corresponding dual solution to Dual Problem 1 then X^* is optimal for Problem 2. Moreover, the optimal dual solution to Problem 1, Y^* , is related to the optimal dual solution of Problem 2, π^* , by (1) below

$$\pi_i^* = \begin{cases} Y_i^* & \text{for } i \notin D, L \\ e_{\ell, d} \cdot Y_d^* + Y_{\ell(d)}^* & \text{otherwise} \end{cases} \quad (1)$$

Proof: (a) That X^* is optimal for Problem 2 is obvious since the conditions

$$X_{\ell, d} = X_{\ell, d}^* \equiv b_{\ell}' \text{ in Problem 2}$$

satisfy

$$A_D X = b_D \text{ and } X_L \leq b_L \text{ of Problem 1 by construction.}$$

(b) We show that the optimal solution π^* is related to Y^* by the relation:

$$\pi_i = \begin{cases} Y_i^* & \text{for } i \notin D, L \\ e_{\ell, d} Y_d^* + Y_{\ell(d)}^* & \text{otherwise} \end{cases} \quad (1')$$

Note that the relation

$$\sum_{i=1} a_{ij} Y_i^* \geq C_j \text{ for } j \neq d, \ell$$

is satisfied for Dual Problems 1 and 2.

Since

$$\sum_{i \neq d, \ell(d)} a_{ij} Y_i^* + e_{\ell, d} Y_d^* + Y_{\ell(d)}^* \geq C_{\ell, d}$$

holds for Dual Problem 1 then $\pi_{\ell} = e_{\ell, d} Y_d^* + Y_{\ell(d)}^*$ and $\pi_i = Y_i^*$ for $i \neq \ell, d$ is a feasible solution to Dual Problem 2.

Now since X^* and Y^* are solutions to the primal and Dual Problem 1 then

$$\sum_{i, j} C_{ij} X_{ij}^* = \sum_{i \neq d, \ell(d)} b_i Y_i^* + \sum_d b_d \cdot Y_d^* + \sum_{\ell(d)} b_{\ell(d)} \cdot Y_{\ell(d)}^*.$$

Substituting π in the objective function of the Dual Problem 2 we obtain

$$\sum_{i \notin L', D} b_i \pi_i + \sum_{i \in L'} b_i' \pi_i = \sum_{i \notin L'} b_i Y_i^* + \sum_{\ell \in L'} b_{\ell}' (e_{\ell, d} Y_d^* + Y_{\ell(d)}^*) .$$

But X^* is the optimal solution to Problem 2 implies that the Dual Problem 2 optimum, π^* , has objective function value

$$\sum_{i \in L'} b_i \pi_i^* + \sum_{i \in L'} b'_i \pi_i^* = \sum_{i \notin L, D} b_i Y_i^* + \sum_{d \in D} b_d Y_d^* + \sum_{i \in L} b_{\ell(d)} Y_{\ell(d)}^*$$

We will show next that π is an optimal solution to Dual Problem 2 corresponding to X^* in the primal Problem 2. In general, the optimal optimal solution to Dual Problem 2 is related to any other solution π by the relation

$$\begin{aligned} \sum_i b_i \pi_i^* &\geq \sum_i b_i \pi_i, \text{ that is,} \\ \sum_{i \notin L, D} b_i Y_i^* + \sum_{d \in D} b_d Y_d^* + \sum_{\ell \in L} b_{\ell(d)} \cdot Y_{\ell(d)}^* &\geq \sum_{i \notin L'} b_i Y_i^* \\ &+ \sum_{\ell \in L'} b'_\ell (e_{\ell, d} Y_d^* + Y_{\ell(d)}^*). \end{aligned}$$

This implies

$$\sum_{d \in D} b_d Y_d^* + \sum_{\ell \in L} b_{\ell(d)} Y_{\ell(d)}^* \geq \sum_{\ell \in L'} b'_\ell e_{\ell, d} Y_d^* + Y_{\ell(d)}^*$$

Notice that $b_d \equiv \sum_{\ell} e_{\ell, d} x_{\ell, d}^*$ and $b'_\ell \equiv x_{\ell, d}^*$ by definition.

Substituting these above yields

$$\begin{aligned} \sum_{d \in D} \sum_{\ell \in L} b'_\ell \cdot e_{\ell, d} Y_d^* + \sum_{\ell \in L} b_{\ell(d)} \cdot Y_{\ell(d)}^* \\ \geq \sum_{\ell \in L} b'_\ell \cdot e_{\ell, d} \cdot Y_d^* + \sum_{\ell \in L'} Y_{\ell(d)}^* \cdot b'_\ell \text{ and} \\ \sum_{\ell \in L} b_{\ell} \cdot Y_{\ell(d)}^* \geq \sum_{\ell \in L'} b'_\ell \cdot Y_{\ell(d)}^* \end{aligned}$$

Two cases are to be considered for the solution to Problem 1: the complementary slackness conditions imply

$$(i) \quad 0 \leq x_{\ell,d}^* < b_{\ell} \rightarrow y_{\ell}(d)^* = 0$$

$$(ii) \quad x_{\ell,d}^* = b_{\ell} = b_{\ell}' \rightarrow y_{\ell}(d)^* \geq 0$$

Since set L is a subset of set L' then case (i) implies

$$b_{\ell}(d) y_{\ell}(d)^* = b_{\ell} y_{\ell}(d)^* = 0 \text{ while case (ii) implies}$$

$$b_{\ell}(d) y_{\ell}(d)^* = b_{\ell}' y_{\ell}(d)^*.$$

Hence π_i as defined by (1) is the optimal solution to Dual Problem 2.

Finally, we show that the optimal solution to Problem 1, x^* , is a component of the optimal solution to Problem 3.

Theorem: The optimal solution to Problem 1, x^* , is a component of the optimal solution to Problem 3 (x^* , $\{w_{\ell,d}^0 = 1\}$). Moreover the optimal solution to Dual Problem 3 is related to the optimal solutions to Dual Problems 1 and 2 by relation (2).

Proof: The optimal solution to Problem 1, x^* , is clearly a feasible component for Problem 3 with $w_{\ell,d}^0 = 1$. It remains to show that the dual solution to Problem 3, λ , corresponding to the primal feasible solution (x^* , $w_{\ell,d}^0$), is also feasible.

Consider the Dual Problem 3 constraints. Define

$$\lambda_i = \begin{cases} y_i^* & \text{for } i \notin L' \\ \pi_i^* & \text{for } i \in L' \\ c_{\ell,d}^0 + b_{\ell,d} \pi_{\ell}^* & \text{for } \ell \in S \end{cases} \quad (2)$$

then λ , as defined in (2), is a feasible solution to Dual Problem 3 with an objective function value given by

$$v_D = \sum_{j \notin L', S} b_j y_j^* + \sum_{\ell, d} c_{\ell,d}^0 + \sum_{\ell, d} b_{\ell,d}' \pi_{\ell,d}^* \quad (3)$$

The value of the corresponding objective for primal Problem 3 for feasible solution (x^* , $\{w_{\ell,d}^0 = 1\}$), in light of the Lemma, is

$$\begin{aligned}
v_p &= c^T x^* + \sum \bar{c}_{\ell,d}^o \\
&= \sum_{j \in L', S} b_j y_j^* + \sum_{\ell,d} b_{\ell,d} \pi_{\ell(d)}^* + \sum_{\ell,d} \bar{c}_{\ell,d}^o \quad (4)
\end{aligned}$$

Now for any feasible solutions to the primal and dual, x, v respectively, the relation $c^T x \geq b^T v$ holds with equality permitted only at optimality (see Wagner (11) or Hillier and Lieberman (12)). Relations (3) and (4) imply the objective functions are equal and hence $(x^*, w_{\ell,d}^o)$ and λ_i are optimal solutions to the primal and dual problems respectively. Hence $(x^*, w_{\ell,d}^o)$ is an optimal solution to Problem 3 and λ_i as defined by (2) is the related optimal solution to Dual Problem 3.

APPENDIX B. REFERENCE ENERGY SYSTEMS

Figure 6. Reference Energy System, Year 2000:AET-8

Figure 7. Reference Energy System, Year 2000: Reference Case

Figure 3. Reference Energy System, Year 2000 NEP

Figure 10. Reference Energy System, Year 2000 Base Case:
Calibration and Verification

Figure 11: Reference Energy System, Year 2000 High Oil Price III Case: Target Scenario