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AUTHOR(S) D. P. Rusthoi, Paul Allison, and Ken R. Crandall

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Los Alamos Los Alamos National Laboratory
Los Alamos, New Mexico 87545

ACCELERATOR COLUMN MODELS FOR LOW-CURRENT BEAMS*

D. P. Rusthol, Paul Allison, K. R. Crandall, AT-2, MS H818
Los Alamos National Laboratory, Los Alamos, NM 87545 USA

Summary

This paper describes three analytic approaches used to model electrostatic accelerator columns in beam-transport codes for low-current beams and compares the results of each approach with the results obtained by numerically calculating the electric field based on charge distribution on equipotential surfaces. The three analytic approaches described are (1) a cubic energy-gain approximation, (2) a cubic longitudinal electric-field approximation, and (3) the aperture equation. The first two approaches calculate impulse approximations at the apertures, whereas the third is an integration of particle trajectories through the column field. The conditions under which the solutions tend to break down are discussed.

Introduction

We inject a 20-kV H^- beam into an 80-kV accelerating column on the Los Alamos accelerator test stand (ATS). The column has a length/aperture ratio (L/R_a) of approximately 2 for the accelerating portion and an $L/R_a \sim 1$ for the electron suppressor. Our purpose for examining the analytical models discussed here was to determine which one best represented the ATS accelerator column. We calculated the beam dynamics as a function of L/R_a for energy gain factors of 5 and 25 to compare results for these models. We assumed the beam did not significantly alter the charge distribution on the column electrodes.

Two of the models described use a thin-lens (TL) impulse modeling technique (Fig. 1). The column entrance and exit apertures apply focusing and defocusing impulses, respectively, and between these apertures is a linear energy gain. The difference between the two impulse models is the derived value of the electric field potential at each lens that is used to calculate the impulse approximation. The third model described uses the well-known aperture formula,¹ representing the column fields by superimposing the potentials calculated by the aperture formula at the column entrance and exit (Fig. 2).

Fig. 1. Thin lens impulse column model. (equipotential lines shown)

We compared the transformation matrices for each of the models at the two energy-gain factors with a

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Fig. 2. Superposition of apertures column model. (equipotential lines shown)

range of L/R_a ratios. To check the validity of these models, we used the program CHARGE-2D², which can calculate electric fields of an arbitrary set of cylindrical electrodes by first computing the charge distribution on the surfaces. By calculating the motion of two particles through the fields near the column, an equivalent transfer matrix was constructed in both CHARGE-2D and the aperture model.

Cubic- ϕ Model

The cubic- ϕ model approximates the energy gain near the aperture with a cubic between $z = 0$ and $z = 3kR_a$ (R_a = aperture radius, k = a multiplicative constant) with the plane of the aperture at $z = 2kR_a$ (Fig. 3). This model is used in computer codes SPEAM³ and TRACE⁴, with the constant $k = 1$. The cubic energy-gain function gives us the following expression:

$$\phi(z) = \phi_0 + \left(\frac{z}{3kR_a}\right)^3 kR_a \Delta\phi/L, \quad (1)$$

Fig. 3. Cubic energy-gain function through an accelerator column.

where ϕ_0 is the potential at the column entrance and $\Delta\phi$ is the energy change through the column. By calculating the radial force, the change in transverse momenta and the average velocity near the aperture, and by treating this result as the effect of a thin lens, the entrance aperture focal length becomes

$$\frac{1}{f} = \frac{\Delta\phi/L}{4(\phi_0)^{1/2} (\phi_0 + 0.25kR_a \Delta\phi/L)^{1/2}}, \quad (2)$$

with a similar expression for the focal length at the column exit. Because the longitudinal electric field

E_z is given by $\Delta\phi/\Delta z$ and the radial electric field E_r is proportional to $\partial E_z/\partial z$, $E_r(z)$ becomes linear in the region of the aperture, which does not appear to be very realistic.

Cubic- E_z Model

In this model, the longitudinal field on-axis near each aperture is approximated by a cubic function that makes a smooth transition between the constant fields E_1 and E_2 on the two sides of the aperture (Fig. 4). If the cubic extends a distance kR_a on each side of the aperture, then

$$E_z(z) = \frac{E_1 + E_2}{2} + \frac{E_2 - E_1}{4} \left[3 \left(\frac{z}{kR_a} \right) - \left(\frac{z}{kR_a} \right)^3 \right], \quad (3)$$

$-kR_a \leq z \leq kR_a$.

Fig. 4. Cubic longitudinal electric field function through an accelerator column.

If we let $E_1 = 0$ and $E_2 = \Delta\phi/L$, we obtain a similar expression for the entrance focal length:

$$\frac{1}{f} = \frac{\Delta\phi/L}{4 (\phi_0)^{1/2} (\phi_0 + 0.3kR_a \Delta\phi/L)^{1/2}}. \quad (4)$$

With this model, $E_r(z)$ near the aperture is parabolic with z and perhaps more realistic than the previous model.

SAF Model

The potential distribution produced by an accelerating-column gap may also be approximated by using a Superposition of Apertures Formula (SAF). The potential distribution produced by a conducting aperture plate separating two regions, which have asymptotic uniform fields $E_1(-\infty)$ and $E_2(+\infty)$, is well known.¹ For a plate at zero potential located at $z = 0$, the potential distribution on axis is given by

$$\phi(z) = -(E_1 + E_2) \frac{z}{2} + (E_1 - E_2) \frac{z}{\pi} \tan^{-1} z/R_a. \quad (5)$$

For two or more apertures, superposition would be expected to be valid for $L/R_a \gg 1$, where L is the separation of the apertures. For two apertures at potential $\pm V/2$ located at $\pm L/2$ and with $E(\pm\infty) = 0$, the on-axis potential distribution is then given by the SAF:

$$\phi(z) = (R_a V/L\pi) (U_+ \tan^{-1} U_+ - U_- \tan^{-1} U_-), \quad (6)$$

where $U_{\pm} = (z \pm L/2)/R_a$.

Models Compared

A comparison of these models was made by calculating the transfer matrices through each column model at L/R_a ratios ranging from 0.1 to 1000. Because the transfer-matrix calculations for the different models involved different physical lengths (very large for the SAF model), the matrices were multiplied by fore and aft negative drifts (as required) to make the equivalent length equal to L in all cases. The main differences in the transfer matrices were found in element R_{21} , which represents the focusing strength of the column. The values were normalized to the column length and plotted against the L/R_a ratio, as can be seen in Figs. 5 and 6, corresponding to the two energy-gain factors G (output/input energies) of 5 and 25.

Fig. 5. Column models compared ($G = 5$, $k = 1$)

Fig. 6. Column models compared ($G = 25$, $k = 1$)

The SAF model appears to be the most accurate of the three, following the CHARGE-2D numerical calculation fairly closely. (Note that the SAF cannot take into account the electrode thickness, which is essential in the CHARGE-2D calculation.) The thin-lens impulse models both show significant departures from the CHARGE-2D curve, notably in the area of interest to us where $1 \leq L/R_a \leq 2$, and where the difference is as much as a factor of 2. However, if the multiplicative factor k in both impulse models is set equal to 1.8 instead of 1.0, we find reasonable concurrence between all column models in the L/R_a range >1 , as can be seen in Figs. 7 and 8.

Fig. 7. Column models compared ($G = 5$, $k = 1.8$)

Fig. 8. Column models compared ($G = 25$, $k = 1.8$)

Limiting Cases

For the two impulse models, the formulation breaks down in the limit as $L/R_a \rightarrow 0$, and, thus, this limiting case cannot be calculated. For the SAF model, as $L/R_a \rightarrow 0$, we obtain

$$\phi(z) \rightarrow V/\pi[(\tan^{-1}U + U/(1+U^2))], \text{ where } U = z/R. \quad (7)$$

Meanwhile, the exact solution for $L = 0$ can be obtained analytically,* and the result is

$$\phi(r,z) = V \left[1 - \int_0^\infty e^{-x|U|} J_1(x) J_0(xr/R) dx \right]. \quad (8)$$

with the on-axis potential then given by

*We are indebted to Richard K. Cooper for assistance in solving this problem. This solution can also be derived from Ref. 1, p. 489, 490.

$$\phi(z) = (V/2) U/\sqrt{U^2 + 1}.$$

The transfer matrices for these two $L = 0$ cases were calculated and compared [Eqs. (7) and (8)], resulting in an asymptotic limit for R_{21} in the SAF model that was larger by 38 and 47% for the energy-gain factors of 5 and 25, respectively, when compared with the exact solution. Thus, it appears that significant error in the SAF model exists in the limit $L \rightarrow 0$, although it gives very reasonable results, even to very small L/R_a ratios (~ 0.1).

In the limit as $L/R_a \rightarrow \infty$, the transfer-matrix elements R_{21} of each of the three models were in excellent agreement with R_{21} of the analytical formula, based on a uniform accelerating field between two apertures of vanishingly small radius:

$$R_{21} = \frac{3}{8} \left(\frac{G-1}{GL} \right) (1-\sqrt{G})$$

where G represents the energy gain factor. For each model at an L/R_a ratio of 1000, agreement was within 1% of the value calculated using the above formula.

Conclusion

We found that the analytical impulse models as they are used in computer codes TRACE and SPEAM are both limited and inaccurate over a wide range. When a multiplicative factor k of 1.8 was introduced into these models, reasonable agreement with the numerical calculations of CHARGE-2D was demonstrated for L/R_a ratios > 1 . For L/R_a ratios < 1 , these models cannot be used.

The SAF model works for the entire range of $0.1 \leq L/R_a \leq 1000$. The aperture formula modeled the fields near the apertures very well, even when two apertures were used and superposed to represent a column as is the case in the SAF model. We have, therefore, found an analytical model that is both acceptably accurate over the wide range of L/R_a ratios and capable of modeling multielectrode columns and einzel lenses as well.

Again, these results are valid only for low-current beams. We plan to modify TRACE to include the aperture formula and to update our version of SPEAM to include the multiplicative factor of $k = 1.8$.

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