CONVECTIVE HEAT TRANSFER IN MHD CHANNELS
AND
ITS INFLUENCE ON CHANNEL PERFORMANCE

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CONVECTIVE HEAT TRANSFER IN MHD CHANNELS AND ITS INFLUENCE ON CHANNEL PERFORMANCE

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ABSTRACT

The limitations of the integral boundary layer methods and the potential of the differential boundary layer method in analyzing MHD channel flows are assessed. The sensitivity of results from the integral method to the parametrization of boundary layer profiles and calculation of wall heat transfer is established. A mixing-length type turbulence model for flow on rough walls is developed and validated by comparison with experimental data. The turbulence model is used in a quasi-three-dimensional boundary layer model to evaluate the influence of wall roughness and pressure gradients on the flow characteristics and performance of MHD channels. The behaviors of skin friction and Stanton number calculated from the analytical model are found to differ considerably from the empirical correlations valid for non-MHD flows without pressure gradients.

Nomenclature

A, B	parameters in mixing length model
B	magnetic field
E	electric field
H	total enthalpy
I_L	current load
J	current density
k	thermal conductivity
k_s	wall roughness
λ_m, λ_h	mixing lengths for momentum and enthalpy
Pr	Prandtl number
q	heat flux
Re_x	Reynolds number based on x
St	Stanton number
T	static temperature
u, v	velocity components
V_d	voltage drop
W	channel width
x, y, z	Cartesian coordinates
β	Hall parameter
δ	boundary layer thickness
δ_1	displacement thickness
δ_2, δ_3	momentum and enthalpy thicknesses as defined in Eq. (27)
δ_H	enthalpy thickness
ϵ_m, ϵ_h	turbulent momentum and enthalpy diffusivities
ρ	mass density
θ	momentum thickness
τ	shear stress
μ	molecular viscosity
ν	kinematic viscosity
σ	electrical conductivity
Δy	coordinate displacement, defined in Eq (19)

Subscripts

c	centerline or free stream
eff	effective, laminar plus turbulent
s	sidewall
w	wall

1. Introduction

In the past, both integral and differential methods have been used to study the boundary layer characteristics of flow in MHD channels. Although the integral methods are computationally fast and, are, therefore, ideally suited for parametric analyses¹ and overall system study, they are inherently less accurate than the differential method. Yet, the integral method has sometimes been employed to make performance predictions,² to compare experimental results with theory,³ and to estimate the contribution of finite segmentation effects to the electrical characteristics of the channel.⁴ An objective of this work is to establish carefully the sensitivity of results to the assumptions on which the integral methods are founded.

This paper specifically investigates the sensitivity of results from the integral method to two founding assumptions: 1) parametrization of boundary layer profiles, and 2) specification of wall flux quantities. In Sections II and III, it is shown to be inappropriate to compare the electrical behavior of a channel at two locations, one upstream and the other downstream, using a fixed "enthalpy-velocity" type relationship, such as that of Crocco, without regard to the changing enthalpy profiles at the two locations. By carrying out a numerical experiment with the integral boundary layer method in which the wall heat flux is finitely perturbed, it is also disclosed that the prediction of channel performance is sensitive to the wall heat flux. These results emphasize the importance of prescribing accurate boundary layer profiles and wall heat flux in the integral analyses.

Instead of directing effort toward refining the boundary layer profiles to account for the MHD effect (for use in the integral method), attention is focused in Section IV on developing a mixing-length type turbulence model for flow and heat transfer to rough walls. This turbulence model is used in the differential boundary layer method to relate the turbulent eddy viscosity and conductivity to mean flow quantities. The chief advantage of the differential method is that it does not require any unnec-

essary assumptions relating to boundary layer profiles, skin friction, or wall heat transfer. The developed turbulence model is validated by comparing model predictions with experimental data for skin friction, and heat transfer to rough surfaces. It is observed that the maximum disparity between the analytical and experimental results is no larger than 10 percent for skin friction and 15 percent for Stanton number. The agreement is even better at higher Reynolds number, which, fortunately, is the region of interest in practical MHD flows.

The newly developed rough-wall turbulence model is used in conjunction with a quasi-three-dimensional MHD channel flow model to study the effects of wall roughness and pressure gradients on the flow behavior and performance of a representative channel. It is found that, despite the greater heat flux associated with wall roughness, the influence on channel power output is not very significant. The reasons for this result are explained. Wall roughness and pressure gradients are observed to exert profound influence on other channel characteristics, such as heat transfer, skin friction, blockage at channel exit, which determines the diffuser performance, and the tendency toward flow separation.

II. Local Analysis

By "local analysis" is meant a procedure that computes the local or even overall generator performance on the basis of a temperature profile expressed in terms of a single parameter, which is selected either from experience or from some other external means of deduction. In order to highlight the inadequacy of local analysis to serve as bases for such tasks as making performance predictions and inferring the importance of finite segmentation effects, we show the extreme sensitivity of results from local analysis to the assumed temperature profile. The electrode boundary layer voltage drop, an important electrical parameter, for infinitely fine electrode segmentation is given by

$$V_d = \left(\frac{1 + \beta^2}{\sigma_c} \right) j_y \int_0^\delta \left[1 - \frac{1 + \beta^2}{1 + \beta^2} \left(\frac{\sigma}{\sigma_c} \right) \right] dy \\ - \beta_c E_x \left(1 - \frac{\langle \beta \rangle}{\beta_c} \right) \delta + u_c B \left(1 - \frac{\langle u \rangle}{u_c} \right) \delta, \quad (1)$$

where $\langle \rangle$ denotes an averaged boundary layer quantity. The first two terms in Eq. (1) arise from temperature nonuniformity, the third from velocity nonuniformity in the boundary layer. For turbulent flows, the last two terms may be neglected in comparison to the first. For illustration purposes, let us adopt the following boundary layer variations for temperature and electrical conductivity⁵:

$$\frac{T - T_w}{T_c - T_w} = \left(\frac{y}{\delta} \right)^{1/N} \quad (2)$$

$$\frac{\sigma}{\sigma_c} = \left(\frac{T}{T_c} \right)^{-7/4} \exp(7.25 \left(\frac{T_c}{T} - 1 \right)).$$

Fig. 1 Sensitivity of Local Analysis to N

Figure 1 shows a four-fold variation in voltage drop for a change of exponent N from 5 to 12--a characteristic range of turbulent profiles. A similar observation was made in the parametric study of Ref. 4. For the application considered, Ref. 4, found that changing the power law exponent to 5 from 10 resulted in halving of the open circuit voltage and the short circuit currents.

III. Integral Boundary Layer Analysis

In integral analysis, the flow is subdivided into a uniform core and boundary layer regions near the walls. Generally, the electrode boundary layers are analyzed by solving the integrated momentum and enthalpy equations, with similar boundary layers assumed on the side-walls. The electrode boundary layer equations are:

$$\frac{d\theta}{dx} = \frac{c_f}{2} - \frac{\theta}{u_c} \frac{du_c}{dx} (H_c + 2) - \frac{\theta}{\rho_c} \frac{d\rho_c}{dx} \quad (3)$$

$$\frac{d\delta_H}{dx} + \delta_H \left(\frac{1}{\rho_c} \frac{d\rho_c}{dx} + \frac{1}{u_c} \frac{du_c}{dx} \right) \\ = \frac{\dot{q}_w - (\delta_1 + \delta_H) (E \cdot J)_c + \delta (E \cdot J)_c - \langle E \cdot J \rangle}{\rho_c u_c (H_c - H_w)} \\ + \frac{\delta_H}{H_c - H_w} \frac{dH_w}{dx} \quad (4)$$

Fig. 2 Integral boundary layer analysis

Equations (2) and (3) can be solved in conjunction with the core equations once a form for velocity profile is assumed and the enthalpy profile parametrized in terms of a free parameter. With this approach, the skin friction and Stanton number need to be functionally specified. Alternatively, boundary layer profiles can be assumed for both velocity and enthalpy, and skin friction and Stanton number computed from Eqs. (2) and (3).⁶ We will pursue the former approach with the intention of (a) demonstrating how the parameter N in Eq. (2) changes from point to point in the channel, and (b) investigating the sensitivity of results to wall heat flux.

1. Variation of N : A simple numerical experiment can be carried out by assuming the following profiles:

$$\frac{u}{u_c} = \left(\frac{y}{\delta}\right)^{1/5} \quad \text{and} \quad \frac{H - H_w}{H_c - H_w} = \left(\frac{y}{\delta}\right)^{1/N}$$

The one-fifth power law for velocity approximates the rough-wall flow behavior. The deviation of N from 5 represents departure from Crocco's

profile and is attributed to the combined effects of nonunity Prandtl number, rough wall character, i.e., $St \neq C_f/2$, presence of free stream pressure gradients, and the MHD interaction terms in Eqs. (3) and (4). For the purpose of illustration, let us employ the Prandtl-Schlichting relationship⁷ for skin friction and Nunner's correlation (see Eq. (22)) to relate the Stanton number to skin friction for rough walls. The results of the numerical experiment are recorded in Fig. 2 for flow in a typical U.S. MHD channel² with wall roughness of 3 mm. Figure 2 reveals a rather large variation in N , from 5 to 15, along the channel. This indicates that great caution should be exercised in interpreting results from the local analysis approach in which a constant value of N is pre-assigned everywhere in the channel⁴.

2. Sensitivity to Heat Flux: An inherent source of inaccuracy in the integral method is that the flow field and the temperature field in MHD channels are modified by the Lorentz force in the momentum equation and by the Ohmic heating term in the energy equation. This makes the task of obtaining universal representation for velocity and enthalpy profiles very diffi-

Fig. 3 Sensitivity of results from integral analysis to heat flux

cult. It is equally difficult to prescribe a priori the modification of wall heat transfer by the MHD effects. It is instructive to note that, for a single parameter enthalpy profile, Eq. (3) determines that value of the free parameter as would globally satisfy the energy equation. Because it is difficult to prescribe accurately the wall heat flux in the presence of the pressure gradient and MHD effects, an important question devolves: just how sensitive is the parameter N and, hence, voltage drop to wall heat flux?

To investigate the question posed above, we have obtained, and plotted in Fig. 3, the behavior of an MHD channel as a function of arbitrary perturbations in wall heat flux. In this example, the wall heat flux was arbitrarily increased by 50% and 100% of a reference case. Figure 3 depicts a significant influence of heat flux on the parameter N and the boundary layer voltage drop; both increase with heat flux. Results presented in Fig. 3 need two further qualifications. First, the finite perturbation has been exaggerated beyond the limit to which MHD effects are likely to enhance the wall heat flux. Secondly, as the channel is scaled up, because the ratio of power extracted to heat loss varies in the volume to surface area ratio, the sensitivity of N and, hence, voltage drop to heat flux will decrease.

3. Boundary Layer Profiles and Local Descriptors:

The foregoing section explained the importance of prescribing precisely the boundary layer profiles and wall heat flux. An accurate velocity profile can be obtained by integrating the following shear stress equation:

$$\tau_w = \mu \frac{du}{dy} + 0.16 \rho (y + \Delta y)^2 \left(\frac{du}{dy} \right)^2 \quad (5)$$

where y is the coordinate displacement effect due to wall roughness and is explained in the next section. In the outer turbulent region, where the turbulence effects prevail over the viscous effects and $y \gg \Delta y$, Eq. 5 leads to the following profile:

$$\frac{u - u_c}{u_\tau} = 2.5 \ln \left(\frac{y}{\delta} \right) \quad (6)$$

In the near wall region, the following approximate profile can be obtained:

$$\frac{u}{u_\tau} = 2.5 \ln \left(1 + \frac{0.4 y^+}{1 + 0.4 \Delta y^+} \right) \quad (7)$$

Next, we parametrize the enthalpy profile by a Crocco-like relationship

$$\frac{H - H_w}{H_c - H_w} = \left(\frac{u}{u_c} \right)^\alpha \quad (8)$$

The solution of the integral equations, Eqs. (3) and (4), subject to the profiles of Eqs. (6)-(8) has been quite satisfactory for the purpose of performing parametric studies. It is worth mentioning here that Gertz, et al⁸ have proposed profiles in which the MHD effects are

manifest. We choose, however, not to expend any effort toward refining the boundary layer profiles, but will concentrate on developing an accurate turbulence model for use in the differential boundary layer analysis.

Finally, a manner in which the effects of the pressure gradient and MHD terms on skin friction and heat transfer can be incorporated is to express c_f and St in terms of the local descriptors of the boundary layer behavior, e.g., momentum thickness and enthalpy thickness. For low-speed incompressible flow over a rough surface, Moffat, et al,⁹ have presented the following correlations of their experimental data:

$$c_f = 0.0068 \left(\frac{\delta}{k_s} \right)^{-0.25}, \quad 0.8 \leq \frac{\delta}{k_s} \leq 8 \quad (9)$$

$$St = 0.0032 \left(\frac{\delta_H}{k_s} \right)^{-0.25}, \quad 0.8 \leq \frac{\delta_H}{k_s} \leq 8 \quad (10)$$

It is suggested that further work be pursued in generalizing the above correlations for compressible high-speed flow--the flow situation in practical MHD generators.

IV. Differential Boundary Layer Analysis

There are two major limitations in the use of integral boundary layers model for analyzing the performance of MHD channels. First, flow turbulence necessitates the use of empirical relationships to input the heat flux and shear stress at the walls to the model. Second, whereas the electrical solution depends intimately on the detailed, not the gross behavior of the boundary layer, the integral method is intrinsically incapable of calculating the details of flow variables in the boundary layer, especially with volumetric MHD effects present. Not only is it difficult to prescribe heat flux empirically (particularly for rough walls) as a function of all the relevant variables that influence heat transfer but, also, heat transfer is modified by the MHD effects.

The differential boundary layer model¹⁰ purports to eliminate both of these limitations. Rather than requiring a priori the knowledge of surface losses, it predicts them. For smooth walls, this model can provide the boundary layer details to as much accuracy as desired. The restriction here has been that a satisfactory mixing length model for heat transfer to rough surfaces is not available in the literature. In view of the extreme sensitivity of channel performance prediction to boundary layer profiles and wall heat flux, as exhibited in Fig. 3, the attention in the next section is focused on developing a suitable turbulence model for flow over rough surfaces. This turbulence model is later used to elucidate many aspects of flow in MHD generator.

1. Turbulence Model

A first-order closure of turbulence equations is effected by expressing the turbulent

shear stress and turbulent heat flux quantities as

$$-\overline{\rho u'v'} = \rho \epsilon_m \frac{\partial u}{\partial y} \quad (11)$$

$$-\overline{\rho u'h'} = \rho \epsilon_h \frac{\partial h}{\partial y}$$

where ϵ_m and ϵ_h are given in terms of mixing lengths for transfer of momentum and enthalpy,

$$\epsilon_m = l_m^2 \left| \frac{\partial u}{\partial y} \right| \quad (12)$$

$$\epsilon_h = l_m l_h \left| \frac{\partial u}{\partial y} \right| \quad (13)$$

Following the eddy-viscosity formulation of Cebeci and Smith,¹¹ the above representation of ϵ_m and ϵ_h is used in the inner turbulent region. In the outer region,

$$\epsilon_m = 0.0168 \int_0^\delta (u_c - u) dy \quad (14)$$

and

$$\epsilon_h = 0.0185 \int_0^\delta (u_c - u) dy$$

For flow over a smooth wall, the two mixing lengths are:

$$l_m = 0.4 y (1 - \exp(-y/A)) \quad (15)$$

and

$$l_h = 0.44 y (1 - \exp(-y/B)), \quad (16)$$

where A and B depend on the pressure gradient,

$$A = 26 \frac{v}{u_\tau} \left(\frac{\rho}{\rho_w} \right)^{1/2} / \left(1 - 11.8 \frac{v_w u_c}{u_\tau^3} \frac{\rho_c}{\rho_w} \frac{du_c}{dx} \right)^{1/2}$$

and

$$B = \left(\frac{35}{26} \right) A. \quad (17)$$

Cebeci and Chang¹² proposed that, because of the similarity of velocity profiles for smooth and rough walls, to represent the influence of surface roughness, Eq. (15) can be modified by displacing the coordinate as,

$$l_m = 0.4 (y + \Delta y) (1 - \exp[-(y + \Delta y)/A]), \quad (18)$$

where Δy is a function of the roughness Reynolds number k_s^+ ($\equiv \frac{k_s u_\tau}{\nu}$),

$$\Delta y = 0.9 (v_w/u_\tau) [\sqrt{k_s^+} - k_s^+ \exp(-k_s^+/6)]. \quad (19)$$

Nothing was reported, however, in regards to

the eddy conductivity model for rough walls. It would appear at first that, as a corollary, the method of coordinate displacement should be applied to the calculation of enthalpy mixing length as well, to reflect the influence of wall roughness on heat transfer. Such a procedure, however, predicts a heat flux that is much too high.

It is well known that, for turbulent flow, the increase in heat transfer caused by roughening a surface is less than the corresponding increase in skin friction. The explanation for this is straightforward. In flow over a rough surface, extra shear is imparted to roughness elements as form drag. Thus, even very close to the wall, shear stress is not transmitted by the molecular transport alone, but is augmented by the form drag as well. In fact, in the fully rough regime, the molecular viscosity ceases to have much influence on the velocity profile. Equation (18) simulates the roughness-induced form drag by displacing the coordinate such that eddy viscosity has a non-zero value at the wall. There are two physical features of Eq. (18) that are noteworthy. First, as roughness elements increase in height, the term $(1 - \exp[-(y + \Delta y)/A])$ approaches one even close to the wall. This is consistent with the physical picture that the effect of viscous damping should vanish once the protuberances project outside the viscous sublayer. Second, well outside the inner region where $y \gg \Delta y$, the flow remains unaffected by wall roughness--an experimentally observed fact

At the wall, there is no mechanism for transfer of heat analogous to form drag for shear stress. Therefore, heat transfer to a rough wall is limited by the rate at which heat can be conducted to the surface. The increase in heat flux associated with wall roughness is caused by greater turbulent mixing away from the wall. This implies that, unlike eddy viscosity, eddy conductivity should be zero at the wall. This suggests that a simple rough-wall eddy conductivity model is one in which the modified momentum mixing length expression of Eq. (18) and enthalpy mixing length of Eq. (16) are adopted, i.e., in the inner region

$$\epsilon_m = 0.16 (y + \Delta y) (1 - \exp[-(y + \Delta y)/A])^2 \left| \frac{\partial u}{\partial y} \right| \quad (20)$$

$$\epsilon_h = 0.176 y (y + \Delta y) (1 - \exp[-(y + \Delta y)/A])$$

$$(1 - \exp(-\frac{y}{B})) \left| \frac{\partial u}{\partial y} \right| \quad (21)$$

Equations (20) and (21) constitute our turbulence model for flow over rough surfaces.

2. Validation of Turbulence Model

Equation (20) of our roughness model has been previously tested by Cebeci and Chang,¹² for incompressible, isothermal rough-wall boundary layer flows. Their calculated values of skin friction, displacement thickness, and momentum thickness agree reasonably well with

the experimental measurements for flows with adverse, zero, and favorable pressure gradients. Our calculations indicate that the skin friction calculated from Eq. (20) is generally lower than Schlichting's⁷ by about 10 to 20 percent. This is, perhaps, not a serious shortcoming, because the experimental measurements of Moffat, Healzer, and Kays⁹ also exhibit the same trend. Figure 4 shows a comparison between the values of skin friction calculated from use of Eq. (20) and the correlations of experimental data by Moffat, et al.,⁹ and by Pimenta [as reported in Ref. (13)]. The agreement is excellent and the corresponding values never differ by more than 10 percent, the error bound of the correlations. Furthermore, although the experiments were performed under identical conditions, the two correlations in themselves differ by up to 5 percent.

Fig. 4 Validation of turbulence model--skin friction

Figure 5 is our predicted Moody-like diagram in turbulent regime for flow over flat plate. The broken lines demarcate the limit of complete roughness, in which the skin friction becomes a function of the relative roughness parameter (x/k_s) only. In particular, it is noted that, in the transitionally rough regime where the skin friction depends both on relative roughness parameter and the local Reynolds number, the skin friction behavior resembles that of pipe flow.

Figures 6 and 7 show Stanton number predictions of the presently developed turbulence model for two different free stream velocities and surface roughnesses of 1 mm, 3 mm, and 5 mm. The analytical results are compared with the correlations of Nunner (see Ref. 14), Owen and Thomson,¹⁵ and Moffat, Healzer, and Kays.⁹ Strictly speaking, Nunner's correlation is valid for pipe flow; but, because of its generality, the correlation of Owen and Thomson can be applied to flow over a flat plate. Much less work has been done on boundary layer heat transfer with roughness than on pipe flow. Therefore, the experimental data of Moffat, et al., assume special significance, because they

Fig. 5 Local skin friction for flow over rough walls

Fig. 6 Validation of Turbulence Model--Heat Transfer-I

performed direct measurements on boundary layer heat transfer on rough surfaces. Their important finding was that the Stanton number could be correlated as a function of enthalpy thickness (more exactly, δ_H/k_s) even in the transitionally rough regime.

Figure 6 shows excellent agreement of analytical results for 25 m/s free stream-velocity with the correlation of Moffat, et al.⁹ for the case of 3 mm surface roughness. For the 5 mm roughness case, the disparity is as high as 15 percent, with a progressive improvement in agreement observed as local Reynolds number is increased. No comparison is shown with Moffat's correlation for the 1 mm roughness case, because δ_H/k_s was much higher than 8, the upper limit of Moffat's correlation. Analytical results are almost identical with the Stanton number values calculated from Nunner's correlation for 1 mm roughness and from Owen and Thomson's correlation for 3 mm roughness. In general, the agreement among the three correlations and between the correlations and analytical results improves at higher Reynolds numbers.

For 100 m/s free stream velocity, Fig. 7 shows the same trend as did Fig. 6 for 25 m/s free stream velocity. Once again, the maximum disparity is 15 percent, and the turbulence model does better at higher Reynolds number, which is the region of interest in MHD flows. The Stanton number plotted in Fig. 7 is defined on the basis of adiabatic wall temperature, so that Moffat's correlation required a slight adjustment. In all calculations, the wall

Fig. 7 Validation of Turbulence Model--Heat Transfer-11

temperature was held at 700 K and the free stream temperature at 720 K ($Pr = 0.7$). The density variation for this small temperature change is about 3 percent.

Figure 8 is a generalized chart of Stanton number as a function of the roughness parameter (x/k_s) and Reynolds number. This graph can be useful in performing many of the routine calculations of boundary layer heat transfer to rough walls. Note that the Stanton number does not have the asymptotic behavior of skin friction seen earlier in Fig. 5. This is best seen from the Nunner's correlation:

$$St = St_s \left(\frac{c_f}{c_{f_s}} \right)^n \quad (22)$$

where the subscript s denotes smooth wall and the exponent n is a function of the laminar Prandtl number. At high Reynolds number, c_{f_s} becomes independent of Re_x , but $St_s/(c_{f_s})^n$ does not (for smooth wall, $St_s/c_{f_s}^n$ is nearly a constant from Reynolds analogy). Therefore, even in the fully rough regime, the Stanton number remains a function of both the roughness parameter (x/k_s) and the Reynolds number.

The good agreement between the model predictions and the experimental data is encouraging and provides ample confidence to use the model for analysis of compressible MHD flows where appreciable pressure gradients are present. At this juncture, we reaffirm that the turbulence model being discussed has been extensively validated in the literature for compressible flow over smooth walls as well as, for incompressible flow over rough walls in the presence of pressure gradients¹² (only the hydrodynamic part). Therefore, by analogy, it is expected to behave correctly for compressible flows (including heat transfer) over rough

Fig. 8 Local Stanton number for flow over rough walls

walls in the presence of pressure gradients. The reason for the expectation is that the functional form of the smooth wall turbulence model has been retained. Before elaborating on the flow behavior of rough-wall MHD channels, we will introduce a quasi-three-dimensional flow model which forms the basis for studying the MHD flow behavior.

3. Quasi-Three-Dimensional Model

The MHD channel flow equations being considered are, basically, the two-dimensional compressible, turbulent boundary layer equations along the electrode walls, modified, however, to account for the three-dimensional effects of the sidewalls. The modifications are incorporated by partially integrating the three-dimensional boundary layer equations in the sidewall direction; the equations are as follows:¹⁶

Continuity

$$\frac{\partial}{\partial x} [\rho u(W-2\delta_{1s})] + \frac{\partial}{\partial y} [\rho v(W-2\delta_{1s})] = 0. \quad (23)$$

x-Momentum

$$\begin{aligned} \frac{\partial}{\partial x} [\rho u(W-2\delta_{2s})] + \frac{\partial}{\partial y} [\rho uv(W-2\delta_{2s})] \\ = -W \frac{\partial p}{\partial x} + W \frac{\partial}{\partial y} (\mu_{eff} \frac{\partial u}{\partial y}) - 2\tau_s + \int_0^W J_y B_z dz \end{aligned} \quad (24)$$

y - Momentum

$$W \frac{\partial p}{\partial y} = - \int_0^W J_x B_z dz \quad (25)$$

Total Energy

$$\begin{aligned} \frac{\partial}{\partial x} [\rho u(h + \frac{1}{2} u^2)(W-2\delta_{3s})] + \frac{\partial}{\partial y} [\rho v(h + \frac{1}{2} u^2)(W-2\delta_{3s})] \\ = W \frac{\partial}{\partial y} (k_{eff} \frac{\partial h}{\partial y}) + W \frac{\partial}{\partial y} (u \tau_{xy}) - 2 \dot{q}_s \\ + \int_0^W (J_x E_x + J_y E_y) dz \end{aligned} \quad (26)$$

where δ_1 , δ_2 and δ_3 are, in that order, the displacement, momentum, and enthalpy thicknesses, defined for convenience as,

$$\begin{aligned} \delta_{1s} &= \int_0^s (1 - \frac{\rho u}{\rho_c u_c}) dz \\ \delta_{2s} &= \int_0^s (1 - \frac{\rho u^2}{\rho_c u_c^2}) dz \\ \delta_{3s} &= \int_0^s [1 - \frac{\rho u(h + \frac{1}{2} u^2)}{\rho_c u_c(h_c + \frac{1}{2} u_c^2)}] dz. \end{aligned} \quad (27)$$

Note that δ_2 and δ_3 defined as above can be easily related to their conventionally defined counterparts.

In the foregoing equations, the corrections for heat transfer and friction on the sidewalls appear explicitly. Also, the flux terms have "built-in" allowance for displacements resulting from sidewall boundary layers. The allowances enter in terms of the displacement thickness in the continuity equation, momentum thickness in the momentum equation, and enthalpy thickness in the energy equation. The turbulence model, developed in the previous section for flow over rough surfaces is used in Eqs. (24) and (26) to express μ_{eff} and k_{eff} . Equations (23)-(26) are solved from channel wall to wall by an implicit finite difference scheme, without the introduction of "wall functions" in the near-wall Couette flow region. The details of the solution method are discussed in Ref. 16. Also described in Ref. (16) is a scheme for calculating the axial pressure gradient, that does not require any iterations over the coefficients of the difference equations. In this quasi-three-dimensional model, the total mass, momentum, and energy are conserved in the three-dimensional sense along the channel. For the application considered in Ref. (16), it was found that the plasma free stream temperature could be over-predicted by as much as 120 K if the three-dimension effects were not included in the analysis.

4. Effect of Surface Roughness

The quasi-three-dimension formulation described above, coupled with the newly-developed turbulence model for flow over rough surfaces has been used to predict the performance of a contemporary U.S. channel design. The channel has an active length of 7 m with a cross-sectional area of 0.36 X 0.73 m² at the entrance and of 0.63 X 1.18 m² at the exit. The magnetic field is about 2 Tesla. The plasma consists of the products of combustion of natural gas seeded with potassium. The thermal input to the channel is approximately 250 MW. The details of the input data and specifications of this channel are provided in Ref. (2). In the present work a 40 kg/s is assumed as the mass flow rate for subsonic mode, and the channel performance is analyzed for smooth walls and for different wall roughness values of up to 3 mm.

Figure 9 presents the distribution of skin friction coefficient over the electrode walls for smooth walls as well as for 3-mm rough walls. The solid curves are the results of the subject model, the dashed curves are obtained from the following empirical relationships:

$$\text{smooth walls: } c_f = 0.0592 \text{ Re}_x^{-0.20}$$

$$\text{rough walls: } c_f = (2.87 + 1.58 \log \frac{x}{k_s})^{-2.5}$$

In using the empirical relationships, a half-meter is assumed as the approach distance.

Both of these formulas are for flow over flat plates with zero pressure gradient. The results from the quasi-three-dimensional model, which includes both the pressure gradient and the surface roughness effects, indicate that the skin friction drops in the first half-meter of the channel, followed by an increase in the next half-meter, and then decreases along the channel, with a substantial drop in the last 1.5 m. The behavior exactly follows the axial velocity variation, showing that the adopted turbulence model responds, as it should, to the flow velocity gradient. Such behavior is not peculiar, rather it has been theoretically predicted and experimentally confirmed for non-MHD flows with adverse and favorable pressure gradients.¹² The results of the subject model show that there can be a substantial difference between the predicted values of the skin friction coefficient and the values calculated from the flat plate empirical relationships, especially for rough surfaces.

Figure 10 presents the variation of the heat flux over the electrode walls for smooth and rough walls. The results exhibit the expected enhancement in heat flux with surface roughness. However, as explained earlier, the increase with roughness is not proportional to the enhancement in skin friction coefficient. The wall heat flux response to the axial velocity gradients is slower than the response of the wall shear stress. Therefore, any attempt to estimate the channel heat flux by using the empirical skin friction correlations and the modified Reynolds analogy for rough surfaces can lead to gross over-estimation of the wall heat flux.

Figure 11 depicts the boundary layer growth along the channel with smooth walls and with 3-mm rough walls. As expected, the surface roughness thickens the boundary layer. The displacement thickness over the electrode walls, not shown here, follows the same trend; however, it increases more rapidly near the channel exit because of the flow deceleration there, as evidenced by the sharp drop of skin friction in that region (Fig. 9). As a consequence of the thickening of boundary layer with surface roughness, the boundary layer voltage drop increases, as shown in Fig. 12. However, the rate of increase of voltage drop diminishes as the roughness is increased. Figure 13 shows the effect of surface roughness on the overall channel performance in terms of power output and the total heat rejected. The increase of heat transfer with wall roughness is as expected. The decrease of power output with wall roughness may be less than expected. The two reasons for this are: 1) the reduced level of skin friction because of flow deceleration, as compared to the values obtained from the empirical relationships, and 2) the enhanced displacement thickness, which, in turn, increases the core flow velocity and, hence, the $\underline{U} \times \underline{B}$ emf.

This application has indicated the sensitivity of channel characteristics, particularly, the wall skin friction and heat flux, to the size of roughness elements and to the channel flow operating conditions; therefore, an accurate turbulence model that accounts for wall roughness and pressure gradients should be included in all

Fig. 9 Axial variation of skin friction

Fig. 10 Axial variation of heat flux

Fig. 11 Boundary layer growth

Fig. 12 Effect of roughness on heat transfer and power output

MHD channel flow analyses.

Conclusions

The present study was initiated to investigate the limitations of the integral boundary layer methods and the potentials of the differential boundary layer method in analyzing MHD channel flows. Based on the findings of the study, the following general conclusions can be drawn.

1. The results from the local analysis are sensitive to the assumed boundary layer temperature profile. A numerical experiment with the integral boundary layer method revealed how much the temperature profile changes along the channel in response to the local conditions. This observation indicates that great caution should be exercised in interpreting results from the local analysis. It is especially inappropriate to compare the electrical behavior at two locations, using a fixed "enthalpy-velocity" type relationship such as that of Crocco, without regard to the changing enthalpy profiles at the two locations.

2. For a single parameter enthalpy profile, the integral method determines that value of the parameter which would globally satisfy the energy equations. For non-unity Prandtl number MHD flows over rough wall in the presence of pressure gradients, the resulting enthalpy profile deviates considerably from the Crocco relationship (see Fig. 2).

3. The sensitivity of the results from integral boundary layer methods to wall heat flux has been established. The reason for studying this sensitivity is that the integral methods generally require the specification of functional forms for wall heat flux and wall friction. However, not only is it difficult to empirically prescribe wall heat flux and skin friction, particularly for rough surfaces and in presence of pressure gradients, as a function of all the relevant variables that influence them, but,

Fig. 13 Effect of roughness on voltage drop

also, heat transfer and skin friction are modified by the MHD effects. The sensitivity was established by conducting a numerical experiment in which finite, though arbitrary perturbations in wall heat flux were introduced. The results of the experiment, recorded in Fig. 3, indicate strong influence of the heat flux on the enthalpy profile and, hence, the boundary layer voltage drop.

4. As opposed to the integral methods, the differential boundary layer method does not require any unnecessary assumptions concerning the boundary layer profiles of velocity and enthalpy or the wall heat transfer and skin friction. The influence of MHD effects on the boundary layer profiles and wall flux quantities is naturally and exactly built into the differential analysis. The restriction here has been the nonavailability of a satisfactory turbulence model for heat transfer to rough surfaces. In view of the sensitivity of channel performance prediction to boundary layer profiles and wall heat flux, and the potential of the differential method in analyzing MHD flows, efforts were next directed toward developing a suitable turbulence flow for heat transfer to rough surfaces.

5. The developed turbulence model is the extension of an extant hydrodynamic model for incompressible flow over rough surfaces. For smooth walls, the developed model reduces to the well tested two-layer eddy viscosity model of Cebeci and Smith.¹¹ The hydrodynamic part of the developed model has been previously tested by Cebeci and Chang¹² for incompressible flow over rough walls with adverse, zero, and favorable pressure gradients. The full turbulent model has been validated in Section IV.2, by comparing its predictions of skin friction and Stanton number with the correlations of experimental data for flat plate flow by Moffat, Healzer and Kays,⁹ by Pimenta (as reported in Ref. 13), by Nunner (as reported in Ref. 14), and by Owen and Thomson.¹⁵ The maximum disparity between the analytical pre-

dictions and the experimental data is 10% for skin friction and 15% for Stanton number. A progressive improvement in agreement between the analytical and experimental results is observed as the local Reynolds number is increased. The analytical model does best in the range of roughness parameter and Reynolds number to be encountered in practical MHD flows.

6. The good agreement between the analytical predictions and experimental data provides ample confidence in the use of the developed turbulence model for analyzing compressible MHD channel flows. In order to study the influence of wall roughness and flow pressure gradient on the flow characteristics and performance of channel, the developed turbulence model was coupled to a quasi-three-dimensional channel flow model. The quasi-three-dimensional formulation contains certain correction terms to account for the sidewall boundary layer displacement effects and for the friction and heat losses on the sidewalls. Thus, the flow model ensures absolute conservation of mass, momentum, and energy.

The representative calculations, documented in Figs. 9-13, clearly demonstrate the influence of the pressure gradient on flow behavior. For example, despite the boundary layer growth, the favorable pressure is large enough to raise the local skin friction coefficient in Fig. 9. This trend is exhibited by both smooth and rough wall flows. Similarly, the adverse pressure gradient in the last meter and a half of the channel causes a substantial drop in skin friction in this region. It is also demonstrated in Fig. 9 that the flat plate correlations--generally used in the integral analyses--will miss these important pieces of information.

The surface roughness, as expected, is found to enhance the heat transfer and the boundary layer voltage drop. The degree of decrease in electrical channel power output with roughening of channel walls was less than expected. The reason is traced to the decrease of skin friction coefficient because of flow deceleration and to the thickening of boundary layer and consequent increase in displacement thickness. This caused the core flow velocity and, hence, the induced $U \times B$ emf to increase and partly compensate for the higher boundary layer voltage drop.

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