MAXIMUM INFORMATION WITH MINIMUM COMPLEXITY FROM
A COINCIDENCE ASSAY SYSTEM*

BNL--41859

DE89 002909

Martin S. Zucker

Department of Nuclear Energy
Brookhaven National Laboratory
Associated Universities, Inc.
Upton, NY 11973

Summary: Nuclear assay based on coincidence measurements can yield more useful information than is usually derived from them. The additional information can be used to improve assay accuracy and reliability with only a modest increase in the complexity of the electronics. A particular three-channel coincidence system that has had practical application is analyzed as an example.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

*Work performed under the auspices of the U.S. Department of Energy under Contract No. DE-AC02-76CH00016.

Introduction

There are nuclear-based assay situations where measurement of coincident radiation is possible, and also the most appropriate mode of assay, because the coincidence requirement decreases the influence of background and the emission of coincident radiation may be particularly characteristic of the material being assayed.

In these cases, besides the basic coincidence response, the coincidence system can be made to furnish additional useful information that can be used to reduce the sensitivity of the measurement to detector efficiency, provide a continuous test of system performance, and in some instances signal the presence of electronic noise or interferences from nuclear reactions other than the one of assay interest.

The purpose of this paper is to show that the additional information can be attained with only a modest increase in the complexity of the system through care in the logical design of the coincidence electronics, so that the resulting system will still be practical in an industrial setting. This will be done using as an example a three-fold coincidence arrangement that has been successfully employed in two different applications, one assaying nuclear-reactor fuel elements, the other, assaying several kinds of matrices containing low-level nuclear waste.

Overview

The reduction of the dependence of the assay on detection efficiency is based on a principle, known from the early days of nuclear science, that enables absolute calibration of the rate of nuclear reactions which fortuitously involve emission of coincident radiation.¹ It is then possible to define in terms of various observed count-rates assay signatures which are (in the limit of point geometry) independent of detection efficiency. For finite geometries, these signatures are still useful for reducing the effect on the assay of the various factors affecting detection efficiency. These include, for example, amplifier gain, detector bias, discriminator threshold, solid angles subtended, and matrix absorption of emitted radiation.²

The number of such "efficiency independent" signatures available depends in principle on the number or multiplicity of coincident radiations emitted by the reaction used as a signature, assuming

that the detector-electronics coincidence arrangement can take advantage of the multiplicity of radiation emitted.³

For definiteness, a particular system is discussed in the present paper. This system was intended to assay fissionable material by detecting the neutrons and gammas (average numbers ~ 2.5 and ~ 7.5 , respectively) emitted in fissions induced by bombarding the material with neutrons from an external source of uncorrelated neutrons. The emitted radiation, gammas and neutrons, are detected by each of three plastic scintillators each mounted on a photomultiplier tube.

Coincidence counting allows the induced fission radiation to be detected in a background consisting of radiation from the fissionable material, from the neutron source, and from the natural environment, since these will form only "chance" coincidences. Electrical disturbances that affect all channels however will produce apparent "true" coincidences.

As in other coincidence arrangements, it was necessary in the present case to compromise between the order of the coincidence required and the count rate. Moreover, the higher the order of coincidence and the more information in principle that is made available, the greater the complexity of the electronics and the more demands placed on data acquisition and reduction.

Logic Block Diagram

For the present application, the best compromise between the factors mentioned above is probably a three-fold coincidence arrangement. This paper will assume so, and on that basis describe a coincidence system which extracts a maximum amount of information from an assembly of three detectors with a minimum complexity of equipment.

The logical diagram of the apparatus is shown in Fig. 1. The first AND logic unit will produce an output pulse only if there is coincidence between all three detector outputs. This can occur in several ways; there will be an output at U if there is a true triple coincidence T_{ijk} , a chance (accidental) triple coincidence C_{ijk} , or any of the three possible combinations of a two-fold true coincidence T_{ij} with a single chance count in the remaining channel, $T_{ij}C_k$.

By placing delay lines of very different delay times, each delay being much greater than the resolution time τ of the coincidence condition, in two of the lines leading to the second AND unit,

the three pulses entering that AND can be coincident only by chance. Since a triple coincidence is demanded, only $C_{ijk} = C_i C_j C_k$ type of events, i.e., pure chance three-fold coincidences, can produce an output at V.

Unit 3 AND demands a two-fold or three-fold coincidence. Therefore C_{ijk} or any one of the possible two-fold chance coincidences $C_{ij} = C_i C_j$ can produce an output at W.

The AND logic unit 4 is exposed to the input lines without any intervening delays and can be satisfied by any way of producing 2 or 3 coincidences, and so will produce an output X for all events that would produce an output U and additionally, for true doubles and chance doubles, T_{ij} and C_{ij} .

Unit 5 is a logical OR, and so will produce outputs for any event, singles, doubles or triples, true or chance. However, while a triple coincidence involves three pulses, and a double, two pulses, each can only produce a single pulse output at Y. The count of triples of any kind is the output U. The output of doubles of any kind is $(X-U)$. Therefore Y represents the total singles count, diminished by twice the total triples count and by the total doubles count:

$$Y = (S_i + S_k + S_k) - 2U - (X - U) = (S_i + S_j + S_k) - U - X .$$

The equations shown in Fig. 1 may be interpreted as the expected count rate in scalers U, V, W, X, Y, in terms of the count rates of the respective events. These equations may be solved for the quantities of interest in terms of the observables (scaler readings):

$$C_{ijk} = V \tag{1}$$

$$(C_{ij} + C_{jk} + C_{ki}) = W - V \tag{2}$$

$$T_{ijk} + (T_{ij} C_k) + (T_{jk} C_i) + (T_{ki} C_j) = U - V \tag{3}$$

$$(T_{ij} + T_{jk} + T_{ki}) = X - U + V - W \tag{4}$$

$$(S_i + S_j + S_k) = Y + U + X \tag{5}$$

In these equations the symbol $(T_{ij} C_k)$ stands for the count rate of events such that a triple coincidence occurred because of the chance coincidence of a true double T_{ij} in channels i and j with an unrelated single chance pulse in channel k .

Explicit expressions for T_{ijk} , C_{ij} , and C_{ijk} will be given elsewhere in this paper after a discussion of the coincidence resolving time τ .

Block Diagram of the Electronics

Figure 2 shows the actual electronic block diagram used to implement the logic shown in Fig. 1; this set of electronics corresponds to the apparatus "A" in Table I.

Three NIM modules, a discriminator and two logic units, make up the essential core of the electronics. An amplifier unit (1) is used to reduce the required photomultiplier current (in order to increase stability of the detectors), but this is not essential. The discriminator module (2) (a five-fold unit) is used to discriminate against noise and establish pulses of standard height and width τ for each of the three detector channels, and also to reconstitute pulses attenuated and distorted by passage through the delay lines D_2 and D_3 . The last two discriminators also supply pulses of resolution width τ to the following electronics. Logic module (4) is identical to (3), but is used to form two 3/3 coincidences (i.e., give an output only if all three inputs occur within a coincidence resolving time τ of one another). The two remaining sections of this module are used to form two $\geq 2/3$ coincidences, i.e., each gives an output if there are at least two of the three inputs occurring within a time τ . The outputs of the logic units in module (4) are pulses that can be counted by the scalers U, V, W, X, Y, controlled by a scaler acting as a timer (t). These scalers can be interfaced to a PC type computer to effect automatic data reduction, correct for dead time, and solve the equations (1) through (5), and (13), also, e.g., equation (10) for τ (see below).

The Coincidence Resolution Time

The resolution time of the coincidence circuit is the (time) width τ of the pulses delivered to the logic circuit. The value for τ can be established by adjusting the widths of the pulses in the separate channels.

The effective value of τ can be measured using the circuit performance under operating conditions. The count rates of events $(T_{ij}C_k)$, C_{ijk} , and C_{ij} can be expressed in terms of the separate events forming them, by using the classical expressions for the chance rates in coincidence circuits, valid for low enough count rates and detector efficiencies:

$$(T_{ij}C_k) = 2\tau (T_{ij}) \cdot (C_k) \quad (6)$$

$$(C_{ijk}) = 3\tau^2 (C_i) \cdot (C_j) \cdot (C_k) \quad (7)$$

$$(C_{ij}) = 2\tau (C_i) \cdot (C_j) \quad (8)$$

The singles count rate S_i is made up of a true event count rate T_i and the chance rate C_i , $S_i = T_i + C_i$. However typically $C_i \gg T_i$, and then $S_i \approx C_i$ to a good approximation.

If all the channels are made to be close to one another in efficiency (i.e., similar geometry for the detectors, photomultiplier gains, discriminator thresholds, etc.), then $S_i \approx S_j \approx S_k$, and $T_{ij} \approx T_{jk} \approx T_{ki}$, so that

$$\begin{aligned} V = C_{ijk} &\approx 3\tau^2 \langle S_i \rangle^3 \\ &\approx 3\tau^2 \left(\frac{S_i + S_j + S_k}{3} \right)^3 = 3\tau^2 \left(\frac{Y + U + X}{3} \right)^3 \\ \tau &\approx 3(V/(Y + U + X)^3)^{\frac{1}{2}} \end{aligned} \quad (9)$$

From

$$\begin{aligned} W - V &= (C_{ij} + C_{jk} + C_{ki}) = 2\tau (C_i \cdot C_j + C_j \cdot C_k + C_k \cdot C_i) \\ &\approx (2/3)\tau (Y + U + X)^2 \\ \tau &\approx (3/2)(W - V) / (Y + U + X)^2 \end{aligned} \quad (10)$$

These two different expressions for τ imply a relation between the chance triples, doubles, and singles count rates for this coincidence circuit, obtained by equating τ from equations (7) and (8):

$$C_{ijk} = 3C_i \cdot C_j \cdot C_k \left[\frac{(C_{ij} + C_{jk} + C_{ki})}{2(C_i \cdot C_j + C_j \cdot C_k + C_k \cdot C_i)} \right]^2 \quad (11)$$

The equality of τ from the two expressions was tested by computing them from the experimental data from several separate series of measurements involving (a) a ^{252}Cf (spontaneous fission neutron) source, (b) fuel elements containing ^{235}U (several different loadings) bombarded with $^{241}\text{Am-Li}$ neutrons (uncorrelated, with $E_{\text{ave}} \sim 400$ keV) to induce fission, and (c) several types of matrices containing ^{235}U , also bombarded with $^{241}\text{Am-Li}$ neutrons.

The apparatus, detectors as well as electronics in (a) and (b) was a different one than that used in (c), but the logic was the same, though implemented with different commercial modules in certain particulars.

In all cases, the resolution had been set to ~ 20 ns on the basis of that value being the minimum giving no loss in true coincidence counts as τ is reduced from a higher value. This is in turn governed by the transit time of particles and signals in the assembly of three detectors arranged to form a volume for sample insertion of ~ 20 cm D, ~ 45 cm H; the plastic scintillators were ~ 15 cm D by ~ 45 cm H.

The results are listed in Table I. The agreement is good for the assays (a) and (b), apparatus "A," but $\sim 20\%$ off for assays (c), which used apparatus "B." The major difference between measurements made with A and B is that measurements made with A were of concentrated high grade material, whereas in the case of apparatus B, the material was (simulated) diffuse waste. The expressions used to compute τ involve approximations, and perhaps the approximations were poorer in the case of B.

Use of the Resolution Time; Explicit Expression for T_3

Since τ is a constant depending on the electronics and yet can be measured objectively in terms of the actual performance of the electronics as evidenced by scaler readings, the two calculated values can be used as monitors of overall system performance. The value of τ computed from equation (10) is especially useful since it involves each scaler reading, has good statistical precision, and seems to agree better with the set values for τ . It has, for example, proved useful in uncovering

errors in data transcription, and presumably would also indicate failures in scalers or the logic electronics.

The value of τ determined either from the data as above, or from inspection of the pulse width used in the logic circuitry, can then be applied to equation (3), using equation (6), to determine the true three-fold coincidence count rate $T_3 = T_{ijk}$. Replacing T_{ij} and C_k in equation (6) with average values,

$$\begin{aligned}\langle T_{ij} \rangle &= (1/3) (X - U + V - W), \\ \langle C_k \rangle &\approx (1/3) (S_i + S_j + S_k) = (1/3) (Y + U + X) .\end{aligned}$$

Using equation (10) for τ , the following expression results for the count rate of events consisting of a true two-fold coincidence T_{ij} itself in chance coincidence with an uncorrelated event:

$$(T_{ij}C_k) + (T_{jk}C_i) + (T_{ki}C_j) \approx \frac{(W - V)(X - U + V - W)}{(Y + U + X)} \quad (12)$$

Therefore, in terms of the scaler readings

$$T_{ijk} \approx (U - V) - \frac{(W - V)(X - U + V - W)}{(Y + U + X)} \quad (13)$$

With this equation, used together with equations (1) through (5), all the quantities of interest can be determined explicitly in terms of the five scaler readings.

Two-Fold and Three-Fold Coincidence Assay Characteristics

The application of the forgoing to a measurement will be illustrated with data for one of the types of matrix listed in (c) of Table I. The assay is of ^{235}U , four samples with ^{235}U contents from 0 to ~20 g dispersed in a particular inert matrix, each sample filling the same type of container to the same depth.

Figure 3 is of the three-fold true coincidence rate, T_3 , which conventionally would be the only assay signature from a three-fold coincidence assay system that would be utilized. In fact, often

the count U has been used as a signature without any attempt to separate out the accidentals C_{ijk} or $T_{ij}C_k$ (which admittedly can be a small fraction of the total, say less than 20%, and of that, the $T_{ij}C_k$ can be said to have some relevance to the sample being assayed). This figure illustrates typical features of an assay system in which there is appreciably self absorption by the material emitting the radiation being used as the assay signature. There is an initially linear rise followed by an approach to saturation. (The origin is used as a data point as the sample with pure matrix was used for background subtraction.)

Figure 4 shows the two-fold true coincidence rate, T_2 , actually the sum $(T_{ij} + T_{jk} + T_{ki})$, as derived from the logic circuitry. It also exhibits a linear rise which then approaches saturation, but with characteristically less curvature than the T_3 plot.

The reason for this is that self absorption is one of the factors determining the effective detection efficiency ϵ of the sample-detector-logic electronics system. T_2 is a function of ϵ^2 , whereas T_3 depends on ϵ^3 . (For a point geometry each is directly proportional.) Since the self-absorption will to a first approximation vary linearly with the amount of material for a dilute system, the above characteristics then follow.

While T_2 has an order of magnitude higher count rate than T_3 , this does not necessary result in significantly better statistical precision. The less stringent coincidence condition allows a higher proportion of coincidences to be chance and while the logic circuitry allows the correct subtraction on the average, there is still a loss of statistical precision.

There are however reasons for monitoring both T_3 and T_2 . A coincidence system is by its nature susceptible to common mode electrical noise, which therefore affects T_3 . However, T_2 is made up strictly of only two-fold coincidences, and is therefore immune to common mode noise. This phenomenon was actually observed in an industrial application where there was such a noise problem; T_3 became unreproducible on particular occasions, while T_2 remained unaffected, thus acting as an indicator of the noise problem.

Another reason for collecting T_2 data along with T_3 lies in the fact that they are in almost constant ratio, characteristic of the given type of material. This is illustrated in Fig. 5, which shows

that a change in the amount of material by a factor of $(19.4/1.4) = 13.9$ resulted in a corresponding change in (T_2/T_3) of only $(11.1/8.6) = 1.29$. An alternate demonstration of this is the approximate linear fit to a plot of T_2 vs. T_3 (Fig. 6); both these results are typical of the other matrix types listed in Table I.

This feature has applications. As assay of samples proceeds, the two quantities can be compared and significant deviation from the expected ratio used as an indication that the matrix is not what it is supposed to be (hence, e.g., a different calibration curve is needed), or that there is a malfunction in the system (since T_2 and T_3 have a different dependence on the logic electronics).

A more specialized application relates to situations where there is a possibility of the presence in the sample of neutron absorbers which would affect the assay. If the neutron absorption leads to the prompt emission of two gammas in cascade which could result in a two-fold coincidence, the presence of the neutron absorber would be revealed by the altered T_2/T_3 ratio.

It should be noted that, strictly speaking, T_2/T_3 is not constant, and in fact should behave as $\sim 1/\epsilon$ because of the dependence of T_2 and T_3 on efficiency (see following section). Figure 5 can in fact be interpreted as indicating that as the mass of ^{235}U increases, thus increasing self absorption and consequently decreasing the detection efficiency, the ratio increases. Moreover, while Fig. 6 by itself seems to be plausibly fitted by a linear relation, the other matrices seem to show similar divergence from linearity.

However, while for the above applications it would be most convenient for the ratio to be approximately constant, it is still useful as long as the relation is slowly varying or can be represented by an empirical relation characteristic of a given matrix.

An Efficiency Insensitive Assay Signature

In the limit of a point geometry, and a "small" detector efficiency (usually satisfied in typical systems), the various true count rates are simply proportional to the source strength and appropriate powers of the detection efficiency ϵ (assumed for simplicity the same for each channel).

For a three-fold coincidence circuit

$$T_i \propto \epsilon q$$

$$T_{ij} \propto \epsilon^2 q$$

$$T_{ijk} \propto \epsilon^3 q$$

and five quantities can be defined which, subject to the above limitations, are independent of detector efficiency, and yet proportional to the source strength, therefore useful assay signatures:

$$(T_i T_j / T_{ij}) \quad (\text{three quantities}) , \quad (14)$$

$$(T_{ij} T_{jk} T_{ki} / T_{ijk}^2) , \quad (15)$$

$$(T_i T_j T_k / T_{ijk})^{\frac{1}{2}} \quad (16)$$

Of these equations, only (15) does not depend on singles counts. It is therefore useful as a signature in those situations where singles counts from extraneous sources are a large fraction of the total singles rate and are difficult to discriminate against. (This is often the case when "nuclear material" is being assayed. Such material has modes of radioactive decay besides fission.) The singles quantities T_i in equations (14) and (16) must be from the process producing the coincidences.

In a real system with finite (not point) geometry, the various quantities represented in equations (14), (15), and (16) must be replaced with appropriate averages over the sample and detector, and then the cancelling out of the various factors of ϵ and of q which make these expressions proportional only to the first power of source strength will not occur. However, they will often turn out in practical situations to be relatively insensitive to detection efficiency ϵ .^{2,3}

The logic circuit of Fig. 1 gives the sum $T_2 = (T_{ij} + T_{jk} + T_{ki}) = 3(T_2/3)$. Then the expression (15) can be written

$$\begin{aligned} (T_{ij} T_{jk} T_{ki} / T_{ijk}^2) &= \langle T_{ij} \rangle^3 / T_3^2, (T_3 \equiv T_{ijk}) , \\ &= (1/9) (T_2^3 / T_3^2) . \end{aligned}$$

Treating the $(1/9)$ as a scaling factor, then the ratio

$$R \equiv (T_2^3 / T_3^2) \tag{17}$$

is expected to be proportional to source strength and relatively independent of efficiency. From the previous discussion of the curvature of T_3 and T_2 as a function of mass of ^{235}U , this ϵ insensitivity should manifest itself in a more linear relation between R and ^{235}U mass. That this is indeed the case is shown in Fig. 7, typical of plots of R vs. ^{235}U for other matrices. This demonstrates that it is possible to obtain a linear response or calibration curve from a system with self-absorption if coincidence techniques are properly employed.

Table I. Determinations of Resolving Time from Experimental Data

Assay Material	Resolution Time, $\tau \pm \sigma_r$, (10^{-8} s)		Apparatus
	Equation (9)	Equation (10)	
(a) ^{252}Cf fission source	$2.094 \pm .093$	$1.950 \pm .002$	A
(b) ^{235}U fuel element	$2.034 \pm .248$	$1.939 \pm .014$	A
(c) ^{235}U matrix 1	$2.423 \pm .204$	$2.036 \pm .074$	B
(c) ^{235}U matrix 2	$2.473 \pm .145$	$2.015 \pm .059$	B
(c) ^{235}U matrix 3	$2.434 \pm .199$	$2.013 \pm .058$	B
(c) ^{235}U matrix 4	$2.431 \pm .219$	$2.004 \pm .056$	B

References

1. H. Geiger and A. Werner, *Z. Physik* **21**, 187 (1924).
2. M.S. Zucker, T. Gozani, and H. Bernatowicz, *Proceedings, Fifth Annual ESARDA Symposium on Safeguards and Nuclear Material Management*, 1983.
3. T. Gozani, "Active Nondestructive Assay of Nuclear Materials," NUREG/CD-0602 (1981).

Figure Captions

Fig. 1. Logic for a "maximum information, minimum complexity" three-fold coincidence system.

Fig. 2. NIM modular equipment used to carry out the logical operations.

Fig. 3. Triple coincidence rate (T_3) vs. sample mass ^{235}U .

Fig. 4. Double Coincidence Rate (T_2) vs. sample mass ^{235}U .

Fig. 5. (T_2/T_3) vs. sample mass ^{235}U , to show relative constancy over a range of mass values.

Fig. 6. T_2 vs. T_3 plot, to show linear relation between them.

Fig. 7. (T_2^3/T_3^2) vs. sample mass, to demonstrate linearity and usefulness as an assay signature.

$$D_3 > D_2 > (D_3 - D_2) \gg \tau$$

Fig. 1. Logic for a "maximum information, minimum complexity" three-fold coincidence system.

1. AMP. LECROY 612 AM
2. DISC. PHILLIPS 730
3. LOGIC PHILLIPS 755
4. LOGIC PHILLIPS 755
5. SCALER ORTEC 874 (2)

Fig. 2. NIM modular equipment used to carry out the logical operations.