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K. Kolehmainen

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PION INTERFEROMETRY OF ULTRA-RELATIVISTIC HADRONIC COLLISIONS

Karen KOLEHMAINE

Nuclear Science Division, Lawrence Berkeley Laboratory
University of California, Berkeley, California 94720 USA

and

Department of Physics, University of Arizona, Tucson, Arizona 85721 USA

Pion interferometry of ultra-relativistic hadronic collisions is described in the context of the inside-outside cascade model using a current ensemble method capable of describing an arbitrary distribution of pion sources with an arbitrary velocity distribution. The results are quite distinct from the usual Gaussian and Kopylov parameterizations. Extraction of the temperature parameter, effective source lifetime, and transverse size requires a full three-dimensional analysis of the correlation function in terms of the momentum difference.

1. INTRODUCTION

Correlations in momentum space between pairs of identical particles have long been used to provide information about the space-time structure of the source of the particles. The correlation function is defined as

$$C(k_1,k_2) = \frac{P_2(k_1,k_2)}{P_1(k_1)P_1(k_2)} \quad (1)$$

where $P_1(k)$ and $P_2(k_1,k_2)$ are the single particle and two-particle inclusive distributions, respectively. If one takes the two particle wave function to be a Bose symmetrized (or Fermi antisymmetrized) plane wave state, one finds that $C(k_1,k_2)$ is related to the Fourier transform of the space-time density distribution of particle sources. Experimental results for two pion correlations in hadronic collisions are usually fit by expressions which assume that this density of sources is given by a gaussian in space and time or a uniformly radiating disc with exponential time dependence. Neither of these parameterizations, however, contains the correlations between the dynamics and geometry appropriate for high energy hadronic collisions.

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Such collisions are expected to be described by the inside-outside cascade. Due to Lorentz time dilation, the production of a particle with longitudinal rapidity $y$ cannot be localized to within $\Delta z \sim \tau_0 \sinh(y)$, where $\tau_0 \sim 1$ fm/c. This means that the momentum and the production point are strongly correlated, in contrast to the situation at lower energy. It also results in Lorentz invariance under boosts along the beam (z) axis and a central plateau in rapidity distributions. In ref. 6, it was shown that such correlations could significantly alter the interference patterns. However, the Wigner function approximations used in that study lead to Lorentz noncovariant expressions. Therefore, we employ the general current ensemble formalism developed in ref. 4 to investigate this question. We extend this formalism to allow each source to be moving with an arbitrary velocity in an arbitrary direction. Details of this extension will be published elsewhere. In particular our goal is to find out exactly how $\pi^- \pi^-$ correlation measurements can be used to verify the basic phenomenon of space-momentum correlations.

2. METHOD

We start by considering an ensemble of currents with Fourier transform

$$j(k) = \sum_{i=1}^{N} e^{i \phi_i} e^{ikx_i} j_0(y - y_i, k_T),$$

where $y_i$ is the longitudinal rapidity of pion source $i$ in a frame where $k^H = (m_T \cosh y, \vec{k}_T, m_T \sinh y)$, i.e., the laboratory frame. The longitudinal rapidity of the pion in this frame is $y$, and its transverse mass $m_T^2 = m^2 + k_T^2$ is invariant under boosts along the z axis. Source $i$, as characterized by a current $j_0(x)$, is centered initially at space-time point $x_i$. The Fourier transformed current $j_0(k)$ is interpreted as the amplitude for the source to emit a pion with momentum $k$ in the lab frame. We further assume that the phases $\phi_i$ of the different sources are random. The ensemble is specified by the probability $D(x, y_0)$ for finding a source with rapidity $y_0$ at $x_0$. The single pion inclusive distribution is then

$$P_1(k) = \langle |j(k)|^2 \rangle = \langle N \rangle \int dy_0 D(q = 0, y_0) |j_0(y - y_0, k_T)|^2,$$

where $D(q, y_0)$ denotes the Fourier transform of $D(x, y_0)$ in $x$. Similarly, the two pion inclusive distribution is

$$P_2(k_1, k_2) = \langle |j(k_1)|^2 |j(k_2)|^2 \rangle.$$
The correlation function is given by

\[ C(k_1, k_2) = 1 + \frac{|G(k_1, k_2)|^2}{G(k_1, k_1)G(k_2, k_2)} \]  

where

\[ G(k_1, k_2) = \int dy_0 D(k_2 - k_1, y_0)J_0(y_1 - y_0, k_1T)J_0(y_2 - y_0, k_2T) \]  

We characterize the inside-outside cascade model by

\[ D(x, y_0) = D(\tau, \eta, y_0) = \frac{1}{\tau_0} \delta(\tau - \tau_0) \delta(\eta - y_0) \exp \left( -\frac{r^2}{r_T^2} \right) \]  

where \( \tau = (t^2 - z^2)^{1/2} \) is the proper time, \( \eta = \frac{1}{2} \ln \left( (t+z)/(t-z) \right) \) is the rapidity-like variable, \( y_0 \) is the source rapidity, and \( r_T \) is the radius of the production region in the transverse direction. The parameter \( \tau_0 \) characterizes the proper time at which interactions between the particles cease. From Eq. (6) it is clear that \( G \) depends not only on \( D \) but also on the dynamical details described by \( J_0 \).

We consider a simple pseudothermal source function given by

\[ J_0(y - y_0, k_T) = \exp \left( -\frac{m_T}{2T} \cosh(y - y_0) \right) = \exp \left( -\frac{E}{2T} \right) \]  

where \( E \) is the pion energy in the rest frame of the pion emitter. This parameterizes the sources in terms of an effective temperature \( T \). We also consider another model

\[ J_0(y - y_0, k_T) = \epsilon^{1/2} \exp \left( -\frac{E}{2T^1} \right) \]  

which one expects to be characteristic of a source in thermal equilibrium. This thermal model results in nonanalytic expressions for \( C \), and hence we present results here only for the pseudothermal model. We find, however, that the results for the two models agree closely if \( T' \) and \( T \) are related by a complex expression which we empirically can approximate by

\[ T = 1.42T' - 12.7 \text{ MeV} \]  

Results for the thermal source are presented in more detail in ref. 7.
3. RESULTS AND CONCLUSIONS

The invariant single inclusive pion distribution with the pseudothermal source is

\[ P_1(k) = \frac{d^3n_\pi}{dyd^2k_T} = aK_0(m_T/T), \]

where \( a \) is a normalization constant. Thus \( T \) characterizes the average transverse momenta of the pions, which are uniformly distributed in rapidity. The function \( G \) is found to be

\[ \hat{G}(k_1,k_2) = aK_0(u) \exp \left( -\frac{q^2r^2}{4} \right) \]

where

\[
u = \left\{ \begin{array}{l}
\frac{2m_1m_2}{4} \left( \frac{1}{4T^2} + \frac{1}{\tau_0^2} \right) \cosh \Delta y \\
+ \left( m_{1T}^2 + m_{2T}^2 \right) \left( \frac{1}{4T^2} - \frac{1}{\tau_0^2} \right) + \left( \frac{1}{T} \right) \left( m_{1T}^2 - m_{2T}^2 \right) \right\}^{1/2}
\]

and \( q_T = \hat{r}_{2T} - \hat{r}_{1T} \). The rapidity difference is \( \Delta y = y_2 - y_1 \), and \( K_0(z) = \int dt \exp(-z\cosh t) \) is the conventional modified Bessel function of a complex argument.

In this model the correlation function depends on three parameters, \( \tau_0, T, \) and \( r_T \), which we would like to extract. It is a function of four variables, the two transverse masses \( m_{1T}, m_{2T} \), the rapidity difference \( \Delta y \), and the angle \( \phi \) between \( \hat{r}_{1T} \) and \( \hat{r}_{2T} \) (which enters through \( q_T \)). The structure of \( G \) is obviously rather different than conventional Gaussian or Kopylov parameterizations\(^3,4\). This means that a full three dimensional analysis of pion correlations is necessary to disentangle \( T, \tau_0, \) and \( r_T \).

This procedure involves three steps. First, \( T \) is determined by fitting the single inclusive distribution \( P_1 \) as a function of \( m_T \) by Eq. (11), as shown in Figure 1. Next, we examine the correlation function for the case where \( q_T = 0 \), thus eliminating the dependence on \( r_T \). Since \( T \) is known from the first step, \( \tau_0 \) can now be determined. In practice, this is done by examining either \( C \) as a function of \( \Delta y \) for fixed \( m_T \) (\( m_{1T} = m_{2T} \) for this case) or \( C \) as a function of \( m_T \) for fixed \( \Delta y \). These two cases are shown in Figures 2 and 3, respectively. Finally, \( r_T \) is determined by fitting the correlation function by Eqs. (12) and (13) for the general case where \( q_T \) is not zero. \( C \) can be examined as a function of any of the four variables, \( m_{1T}, m_{2T}, \Delta y, \) or \( \phi \), while the other three are held fixed. An example of \( C \) vs. \( \phi \) for fixed \( m_{1T}, \)
Invaraint single particle distribution $P_1$ vs. $m_T$ for several values of the temperature parameter $T$ (in MeV) in the pseudothermal model.

Correlation function $C$ vs. $\Delta y$ for $q_T = 0$ and fixed $m_T$. Labels are values of the lifetime $\tau_0$ in fm. Pseudothermal and thermal results are identical to within the accuracy of the figure.

$C$ vs. $m_T$ for $q_T = 0$ and fixed $\Delta y$. Several values of $\tau_0$ (in fm) are shown. For $\tau_0 = 2$ fm, pseudothermal and thermal results are shown separately. Otherwise, pseudothermal and thermal results are identical to within the accuracy of the figure.

$C$ vs. $\phi$ for fixed $m_{1T}$, $m_{2T}$, and $\Delta y$. Labels indicate values of the transverse radius $r_T$ in fm. Pseudothermal and thermal results are identical to within the accuracy of the figure.
$m_2$, and $d\gamma$ is shown in Figure 4.

Since in the general case, the effects of $T$, $\tau_0$, and $r_T$ are all mixed together, this three step approach is necessary to extract meaningful parameters from correlation data. The usual procedure is to plot the correlation function as a function of one variable, for example $|q|$, integrating over all other variables. We have verified that the source radius thus obtained is a nonlinear function of the three parameters in our model. The more complex analysis outlined above is required to see the inside-outside cascade nature of the dynamics.

REFERENCES