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THE INFLUENCE OF RADIATION TRANSPORT
ON LITHIUM ION MOTION IN AN ICF REACTOR

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THE INFLUENCE OF RADIATION TRANSPORT ON LITHIUM FALL MOTION
IN AN ICF REACTOR

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ABSTRACT

The fluid motion in the cavity of a HYLIFE reactor was calculated for the two extreme cases of optically thin and thick lithium gas in the hot core. The distinction is important because in the optically thin case (emission approximation) a major fraction of the gas energy is re-radiated to the main fall, following the lithium implosion, whereas in the optically thick case (diffusion approximation) virtually none escapes. This, in turn, has a major influence on the impulse ultimately imparted to the reactor walls.

The complete set of finite difference equations used is given in an appendix.

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1.0 INTRODUCTION

In one version of an inertial confinement fusion (ICF) power reactor, the driver-imploded pellet is surrounded by thick annuli¹ or close-packed jets² of liquid lithium. The lithium has three functions: to breed tritium for pellet resupply, to act as an energy sink and heat exchange medium in the primary power loop, and to protect the first wall of the reactor from excessive neutronic and hydrodynamic loading.

Most of the energy release from the microexplosion is in the form of high energy x-rays and 14 MeV neutrons (for which the mean free path in liquid lithium ranges up to 0.3 m^3). The liquid thickness must therefore be 2-3 times this length to provide adequate moderation and to sufficiently reduce the fluence on the first wall⁴. Since the neutrons are distributed over a large mass of lithium, the specific energy is relatively low, generally 1-3 orders of magnitude below the cohesive energy (which for lithium is 23.03 MJ/kg). The liquid is thus isochorically heated, but well below the vaporization point; the subsequent motion has been described elsewhere^{1,5}.

The remaining fusion energy, normally from twenty-five to thirty-five percent of the total, is in the form of x-rays and pellet debris and is unable to penetrate beyond a very thin layer in the innermost exposed jets or at the inner radius of the annular fall. As a result, the specific energy of the lithium in this layer increases by several orders of magnitude and the instantaneous pressure may exceed 100 GPa. A strong shock then moves out of the short-range deposition region, but is quickly caught and attenuated by a rarefaction centered at the free inner surface. The blowoff velocity at the tail of the rarefaction was estimated to be $\sim 40 \text{ km/s}^1$, but this figure is

certainly too low for the high-energy-density systems currently under consideration. Moreover, the velocity will continue to increase as the implosion develops, a consequence of the energy-focusing effect of the convergent geometry. At implosion time, the kinetic energy thus acquired will be reconverted to internal energy, a substantial, if not the major, fraction of which will be in the form of radiation.

The subsequent energy flow will be controlled both by radiation transport and hydrodynamic motion. We have previously described the case where the mean free path for radiation was large compared with the dimension of the reaction chamber¹ so that the hot cavity gas was considered to be optically thin and the emission approximation valid. If the energy and mass density in the cavity are sufficiently high, this approach no longer works. Our main purpose here is to contrast the optically thin results with those obtained for the optically thick case where the diffusion approximation was employed. The limitations and uncertainties associated with both methods are discussed. The complete set of finite difference equations used is given in an appendix along with explanatory notes.

2.0 THE DESIGN BASIS: HY LIFE

Our previous calculations of the lithium implosion and blowback were based on a 700 MJ pulse deposited in a lithium annulus whose inner and outer radii were respectively 2.0 and 2.6 m. The thickness of lithium for which $e \geq e_c^*$ was $\sim 10^{-2}$ mm (this is a measure of the blowoff mass) so that the gas density in the central cavity at implosion time was of the order of 10^{-6} Mg/m³. Figure 1 shows the Rosseland mean opacity for lithium as a function of temperature and density; the data derive from calculations performed at LASL by W. Huebner⁶. For $\rho = 10^{-6}$ Mg/m³, the maximum value for K_R is $\sim 10^3$ m²/kg at a temperature slightly below 1 eV. For higher and lower temperature, the opacity decreases sharply; at 10 eV, $K_R \sim 1$ m²/kg and at 0.125 eV (the lowest temperature for which we have tabulated data), $K_R \sim 10^{-6}$ m²/kg. Thus the radiation mean free path is bounded by: $1 \text{ m} < \lambda_R = (K_R \rho)^{-1} < 10^9 \text{ m}$, so that the emission approximation appears fully justified (calculations based on the Planck mean opacity--cf. Fig. 2--would reduce the minimum bound by about 1 order of magnitude).

More recent reactor designs call for greater energy per pulse and a more compact fall configuration. The High Yield Lithium Injection Fusion Energy converter (HY LIFE) system² employs a 2700 MJ thermonuclear pulse in a close-packed annular jet array (300 jets, each 200 mm in diameter) that surrounds the pellet on an inner radius of ~ 0.5 m. In this case, the thickness of the penetration layer ($e \geq e_c$) is about 300 times higher and the cavity radius 4 times less than before, so that the gas density at implosion is ~ 1200 times higher. The minimum value of λ_R is then

*Refer to Appendix B for the nomenclature.

$\sim 10^{-3}$ m, considerably less than the cavity radius so that, at least at temperatures near 1 eV, the gas cannot reasonably be considered optically thin. This has a very strong influence on the energy transport and the gas motion in the cavity.

3.0 CALCULATIONS WITH THE EMISSION APPROXIMATION

This method was described earlier in connection with our calculations for the smaller (700 MJ) ICF reactor¹. The relevant finite difference equations are given in Appendix A, beginning with eq. (62). In brief, each gas zone ($e \geq e_c$) in the cavity is considered to be a ring source that radiates to the relatively cool ($e < e_c$) residual fall. Radiant energy released by the implosion is absorbed initially by the first interior zone in the fall that is opaque. This zone then heats and expands until it becomes transparent (cf. Note V of Appendix A), at which point no further radiant absorption is allowed and the next adjacent two-phase (liquid-vapor) or liquid zone acts as the radiation sink. This process is coupled with the energy flow by hydrodynamic motion and continues until the radiation flow no longer influences the motion.

Figure 3 shows the Lagrangian motion within the Eulerian reactor cavity as a function of time. The Eulerian cavity radius (ECR) is defined as that surface to which the inner boundary of the innermost lithium annulus (or, approximately, the innermost row of jets) is congruent at $t=0$. Proceeding in counter-clockwise fashion from the initial state in Figure 3, the inner boundary of the annular fall has imploded on the cylindrical axis at $t = 1.5 \mu s$; the collapse velocity is almost 400 mm/ μs , corresponding to an energy of ~ 80 GJ/kg. The fluid initially contained within a 1 μm thick annulus, at a radius of 0.5 m (the first zone), now occupies the cavity core to a radius of 166 mm. The peak temperature in the core is ~ 100 eV and the peak pressure ~ 1 GPa. Approximately 200 Lagrangian zones have entered the Eulerian cavity.

By $t = 5 \mu s$, the peak temperature has dropped to ~ 40 eV. Figure 4, which plots the energy radiated from source to sink as a function of time, shows that 30% of the initial gas energy has been emitted (E_0 is the total energy initially contained in fluid for which the specific internal energy exceeded the cohesive energy).

Most of the radiant emission takes place in the first $50 \mu s$ as gas continues to move towards the axis and compress the core. The work done thereby sustains the radiation transport which, as seen in Figure 4, is essentially completed in this interval. The energy absorbed by the two-phase liquid-vapor region (60% of E_0 at $50 \mu s$) has caused this material to heat, expand and accelerate in towards the axis. In Figure 3, this region is represented by the relatively wide zones that appear near the ECR beginning at $t = 10 \mu s$. The significance of this is that the inward-moving two-phase fluid has a mass that exceeds that of the outward-moving, low density gas by several orders of magnitude. By $60 \mu s$, the boundary between the two regions is approximately half way between the axis and the ECR. In reality, this boundary is unstable and the resulting mixing and molecular transport would be expected to cool and condense the hot gas. This, in turn, would eliminate the requirement for mechanical exhausts--a vexing feature of drywall ICF systems (the ambient pressure between pulses must be as low as 0.1 Pa for some of the driver systems under consideration).

The spacial distribution of pressure, temperature, and particle velocity at $t = 60 \mu s$ are shown in Figures 5-7. Discussion of these results is deferred until after the equivalent results obtained using the diffusion approximation are presented. The time basis of $60 \mu s$ was chosen for the comparison because the diffusion results at this time served as initial conditions for the crossflow of gas through the annular array of jets that constitutes the lithium fall in the HYLIFE design.²

4.0 CALCULATIONS WITH THE DIFFUSION APPROXIMATION

In our earlier calculations with the diffusion approximation, we used an explicit method devised by Trulio⁷. This method allows somewhat larger timesteps than would otherwise be computed when the mean free path for radiation, λ_R , is of the order of or greater than the zone size (in the limit $\lambda_R \rightarrow \infty$, e.g., photon propagation in a near-vacuum, diffusion theory leads to the physically absurd result that for a field of finite energy density, the radiant energy flux is unbounded. Actually, no matter what the value of λ_R , the magnitude of the photon energy flux cannot exceed c even if all the photons moved in one direction. In Trulio's method, the timestep is proportional to the zone size, Δx , rather than to $(\Delta x)^2$, and is independent of λ_R when $\lambda_R \rightarrow \infty$). Still, calculations were far too costly and it was necessary to implement implicit differencing. Our method is based on a scheme derived by Christy⁸. In Christy's method, a simple Newton-Raphson-like iteration procedure is used for direct solution of an implicit form of the non-linear diffusion equation, coupled with explicit Lagrangian hydrodynamics. Our method differs mainly in that we solve the diffusion equation separately, in sequence with the hydrodynamics. This permits an arbitrary, time-dependent coordinate system to be employed, without affecting any conservation requirement; mass, momentum, and energy are exactly conserved. Other minor differences are the boundary conditions, the use of a flux limiter and the employment of tabular data, rather than a simple analytical form for the opacity. Equations (1)-(34) of Appendix A describe the hydrodynamics in a sequence suitable for numerical calculation. The diffusion sequence follows in eq. (35)-(61). Additional details on the outer boundary condition for radiation transport are found in Note V of this appendix.

Figure 8 shows the Lagrangian motion within the Eulerian reactor cavity, calculated with the diffusion approximation, and should be contrasted with Figure 3. Major differences are apparent as early as at $t = 5 \mu s$, the first frame shown after the implosion. The radiant energy generated near the axis diffuses rapidly into the cooler incoming gas, but only until a temperature of ~ 1 eV is reached. At that point, as seen in Figure 1, the gas becomes effectively opaque. This blocks the hot interior core from radiating to the cool liquid or liquid-vapor region, as was the case with the emission approximation. No inward-moving mass of cool fluid is produced and the hot cavity gas moves outward against the main fall unopposed.

Figures 9-11 depict the spacial distribution of pressure, temperature, and particle velocity at $t = 60 \mu s$ and should be compared with the equivalent emission-approximation figures 5-7. The cavity pressure in both cases is approximately the same, ~ 0.4 GPa, but the volume in which this pressure is contained in the emission case is only about 40% as large as for diffusion. This is consistent with Figure 4 which shows that, in the emission case, 60% of the energy initially contained in the cavity has been radiated to the cool zones by $t = 60 \mu s$; effectively, none of the cavity energy has escaped in the diffusion case. Comparison of the temperature profiles tells the same story. With diffusion-limited-radiation flow, Figure 10 shows the peak temperature in the core at $60 \mu s$ to be in excess of 15 eV, gradually dropping off to the ~ 1 eV level at a radius of ~ 0.32 m; a 100 mm wide opaque layer (the dark, densely packed region in Figure 8) insulates the cavity from the rest of the fall. By contrast, Figure 6 shows the peak temperature at $60 \mu s$ with emission-limited-radiation flow to be ~ 2 eV.

Finally, comparison of the particle velocity traces in Figures 7 and 11 shows in more detail the picture sketched in Figures 3 and 8. With diffusion

at 60 μ s, the leading edge of the core gas has blown back almost to the ECR and the peak outward radial velocity exceeds 5 mm/ μ s. With emission, the cooler fluid is rushing inward at speeds up to 4 mm/ μ s and the outward core gas motion is on the verge of being arrested.

5.0 CONCLUDING REMARKS

It is clear that radiation transport strongly affects the cavity motion in the period immediately following the lithium implosion. If the emission approximation was valid, the designer's task would be considerably easier. The core gas would be drowned by cooler elements from the main fall, its momentum countered and its subsequent influence very much reduced. Mixing and molecular transport would likely cool and condense the gas within the Eulerian cavity radius. Unfortunately, this is probably an overly optimistic picture for the HYLIFE system. For this design, radiation diffusion seems to be a better--although still not totally accurate--approximation for the transport. And with diffusion, the hot core gas will blow back against, and interact with, the main fall. This will increase the outward-directed momentum of the fall and thus the impulse imparted to the walls of the reactor will be greater. The heat transfer problem will be more complicated, although not necessarily more worrisome from a practical standpoint.

The main problem with either approximation to the transport equation is the usual one that neither works over the entire region of interest. Generally, emission is better near the axis where the temperature is high and the density low and diffusion is better elsewhere. There exist, of course, several methods which are capable of describing the radiation transport in optically thin and thick regions and which are usefully accurate in intermediate regions as well. The simplest of these is the Variable Eddington method developed by Freeman⁹ and his associates. Even better, albeit more expensive, are the Monte Carlo methods pioneered by Fleck¹⁰. We have not pursued these (as of yet) for several reasons. First, no reliable opacity

data are available for lithium in the near-liquid or two-phase liquid-vapor regions and the employment of more sophisticated transport algorithms is not likely to produce more accurate results without these data. Second, and perhaps even more important, is the assumption that the flow is strictly one-dimensional. This assumption is especially questionable during the implosion phase where, even if the inner fall surface was initially quite smooth, Rayleigh-Taylor instabilities would almost certainly develop. Given that close-packed jets constitute the actual geometry, so that a top view would show a scalloped inner surface, the energy density at implosion time is likely to be much less concentrated near the axis. This would tend to lower and to equalize the core temperature and density and favor a diffusion-like transport of r and n . In any case, it does not seem reasonable to ignore deviations from 1D geometry if a very accurate solution is required.

Finally, the diffusion approximation in one-dimensional cylindrical symmetry seems to represent a worst case scenario and is therefore useful for preliminary designs of the reaction chamber. The initial conditions for the fall motion are based on the energy deposition from the DT pellet in the plane passing through the pellet's equator and normal to the axis of the fall. The energy, of course, falls off with increasing distance from this plane so that axial gradients exist right from the beginning. The radial energy and momentum flow in practice are thus certain to be less than the 1D calculations predict.

APPENDIX A - Difference Equations

$$M_j^n = M_j^{n-1} + \delta^{n-\frac{1}{2}} t [A_j^{n-\frac{1}{2}} (PQ_{j-\frac{1}{2}}^{n-\frac{1}{2}} - PQ_{j+\frac{1}{2}}^{n-\frac{1}{2}}) + (\widetilde{\rho\omega A})_{j-\frac{1}{2}}^{n-\frac{1}{2}} U_{j-\frac{1}{2}}^{n-\frac{1}{2}} - (\widetilde{\rho\omega A})_{j+\frac{1}{2}}^{n-\frac{1}{2}} U_{j+\frac{1}{2}}^{n-\frac{1}{2}}] \quad (1)$$

$$t^n = t^{n-1} + \delta^{n-\frac{1}{2}} t \quad (2)^{I*}$$

$$U_j^n = M_j^n / m_j^n \quad (3)$$

$$U_j^{n+\frac{1}{2}} = 2U_j^n - U_j^{n-\frac{1}{2}} \quad (4)^{II}$$

$$U_{j+\frac{1}{2}}^{n+\frac{1}{2}} = \frac{1}{2} (U_j^{n+\frac{1}{2}} + U_{j+1}^{n+\frac{1}{2}}) \quad (5)$$

$$m_j^n = \frac{1}{2} (m_{j-\frac{1}{2}}^n + m_{j+\frac{1}{2}}^n) \quad (6)$$

$$X_j^{n+1} = X_j^n + S_j^{n+\frac{1}{2}} \delta^{n+\frac{1}{2}} t \quad (7)^{III}$$

$$(X_j^{n+1})_L = X_j^n + U_j^{n+\frac{1}{2}} \delta^{n+\frac{1}{2}} t \quad (8)$$

$$\omega_j^{n+\frac{1}{2}} = U_j^{n+\frac{1}{2}} - S_j^{n+\frac{1}{2}} \quad (9)$$

$$A_j^{n+\frac{1}{2}} = \frac{1}{2} (X_j^n + X_j^{n+1}) \quad (10)$$

$$(A_j^{n+\frac{1}{2}})_L = \frac{1}{2} [X_j^n + (X_j^{n+1})_L] \quad (11)$$

$$\tilde{A}_j^{n+\frac{1}{2}} = \frac{1}{2} [(X_j^{n+1})_L + X_j^{n+1}] \quad (12)$$

*Refer to notes at end of Appendix A.

$$(\delta^{n+\frac{1}{2}}\Delta_{j+\frac{1}{2}})_L = [2(X_{j+1}^n U_{j+1}^{n+\frac{1}{2}} - X_j^n U_j^{n+\frac{1}{2}}) + (U_{j+1}^{n+\frac{1}{2}} - U_j^{n+\frac{1}{2}})(U_{j+1}^{n+\frac{1}{2}} + U_j^{n+\frac{1}{2}})\delta^{n+\frac{1}{2}}t] \delta^{n+\frac{1}{2}}t / 2m_{j+\frac{1}{2}}^n v^0 \quad (13)$$

$$(\Delta_{j+\frac{1}{2}}^{n+1})_L = \Delta_{j+\frac{1}{2}}^n + (\delta^{n+\frac{1}{2}}\Delta_{j+\frac{1}{2}})_L \quad (14)$$

$$(\rho_{j+\frac{1}{2}}^{n+1})_L = 1/v^0 [1 + (\Delta_{j+\frac{1}{2}}^{n+1})_L] \quad (15)$$

$$(P_{j+\frac{1}{2}}^{n+1})_L = P[(\rho_{j+\frac{1}{2}}^{n+1})_L, e_{j+\frac{1}{2}}^n] \quad (16)$$

The equation of state model for lithium was devised by Glenn and Young⁽¹⁾ and includes the liquid, liquid-vapor, and ionized gas regions. Pressure, temperature and sound speed are explicit functions of density and specific internal energy.

$$\text{If } e_{j+\frac{1}{2}}^n > e_c, (P_{j+\frac{1}{2}}^{n+1})_L = (P_{j+\frac{1}{2}}^{n+1})_L + a\theta_{j+\frac{1}{2}}^n/3 \quad (17)$$

$$\text{where } \theta_{j+\frac{1}{2}}^n = (T_{j+\frac{1}{2}}^n)^4 \quad (18)$$

$$(C_{j+\frac{1}{2}}^{n+1}) = C[(\rho_{j+\frac{1}{2}}^{n+1})_L, e_{j+\frac{1}{2}}^n] \quad (19)$$

$$(Q_{j+\frac{1}{2}}^{n+\frac{1}{2}})_L = \left[|D| \{ \max \{-\alpha_1 D, 0\} \} + \alpha_2 D (C_{j+\frac{1}{2}}^{n+1})_L \right] (\rho_{j+\frac{1}{2}}^{n+1})_L \quad (20)$$

$$\text{where } D = (\delta^{n+\frac{1}{2}}\Delta_{j+\frac{1}{2}})_L (X_{j+1}^n - X_j^n) / [1 + (\Delta_{j+\frac{1}{2}}^{n+1})_L] \delta^{n+\frac{1}{2}}t \quad (21)$$

$$(PQ_{j+\frac{1}{2}}^{n+\frac{1}{2}})_L = \frac{1}{2} [(P_{j+\frac{1}{2}}^{n+1})_L + P_{j+\frac{1}{2}}^n] + (Q_{j+\frac{1}{2}}^{n+\frac{1}{2}})_L \quad (22)$$

$$(e_{j+\frac{1}{2}}^{n+1})_L = e_{j+\frac{1}{2}}^n - (\delta^{n+\frac{1}{2}} t / m_{j+\frac{1}{2}}^n) \{ (\rho_{j+\frac{1}{2}}^{n+\frac{1}{2}})_L [(A_{j+1}^{n+\frac{1}{2}})_L U_{j+1}^{n+\frac{1}{2}} - (A_j^{n+\frac{1}{2}})_L U_j^{n+\frac{1}{2}}] \} \quad (23)$$

The Lagrangian calculational sequence is terminated here. For non-Lagrangian grid surfaces, (23) serves merely to define the energy field for advection.

$$(\widetilde{\rho w A})_j^{n+\frac{1}{2}} = \frac{\gamma_j^{n+\frac{1}{2}}}{\rho_j} \omega_j^{n+\frac{1}{2}} \frac{\gamma_j^{n+\frac{1}{2}}}{A_j} \quad (24)$$

$$(\widetilde{\rho w A})_{j+\frac{1}{2}}^{n+\frac{1}{2}} = \frac{1}{2} [(\widetilde{\rho w A})_{j+1}^{n+\frac{1}{2}} + (\widetilde{\rho w A})_j^{n+\frac{1}{2}}] \quad (25)$$

where, depending on the smoothness of the density field in the neighborhood of $(X_j^{n+1})_L$, either

$$\rho_j^{n+\frac{1}{2}} = \frac{1}{2} [(\rho_{j+\frac{1}{2}}^{n+1})_L + (\rho_{j-\frac{1}{2}}^{n+1})_L] - \omega_j^{n+\frac{1}{2}} \delta^{n+\frac{1}{2}} t [(\rho_{j+\frac{1}{2}}^{n+1})_L - (\rho_{j-\frac{1}{2}}^{n+1})_L] / [(X_{j+1}^{j+1})_L - (X_{j-1}^{n+1})_L] \quad (26a)$$

or

$$\rho_j^{n+\frac{1}{2}} = \begin{cases} (\rho_{j+\frac{1}{2}}^{n+1})_L, & \text{if } \omega_j^{n+\frac{1}{2}} < 0 \\ (\rho_{j-\frac{1}{2}}^{n+1})_L, & \text{if } \omega_j^{n+\frac{1}{2}} > 0 \end{cases} \quad (26b)^{IV}$$

$$m_{j+\frac{1}{2}}^{n+1} = m_{j+\frac{1}{2}}^n + \delta^{n+\frac{1}{2}} t [(\widetilde{\rho w A})_j^{n+\frac{1}{2}} - (\widetilde{\rho w A})_{j+1}^{n+\frac{1}{2}}] \quad (27)$$

$$\begin{aligned} \Delta_{j+\frac{1}{2}}^{n+1} = & (\Delta_{j+\frac{1}{2}}^n)_L (m_{j+\frac{1}{2}}^n / m_{j+\frac{1}{2}}^{n+1}) \\ & + (\delta^{n+\frac{1}{2}} t / m_{j+\frac{1}{2}}^{n+1}) [(\omega_j^{n+\frac{1}{2}} \frac{\gamma_j^{n+\frac{1}{2}}}{A_j} - \omega_{j+1}^{n+\frac{1}{2}} \frac{\gamma_{j+1}^{n+\frac{1}{2}}}{A_{j+1}}) / v^0 \\ & - (\widetilde{\rho w A})_j^{n+\frac{1}{2}} + (\widetilde{\rho w A})_{j+1}^{n+\frac{1}{2}}] \end{aligned} \quad (28)$$

$$\tilde{e}_{j+\frac{1}{2}}^{n+1} = e_{j+\frac{1}{2}}^n (m_{j+\frac{1}{2}}^n / m_{j+\frac{1}{2}}^{n+1}) + (\delta^{n+\frac{1}{2}} t / m_{j+\frac{1}{2}}^{n+1}) [(\widetilde{\rho\omega e A})_j^{n+\frac{1}{2}} - (\widetilde{\rho\omega e A})_{j+1}^{n+\frac{1}{2}}] \quad (29)$$

$$(\widetilde{\rho\omega e A})_j^{n+\frac{1}{2}} = (\widetilde{\rho\omega A})_j^{n+\frac{1}{2}} \tilde{e}_j^{n+\frac{1}{2}} \quad (30)$$

and $\tilde{e}_j^{n+\frac{1}{2}}$ is obtained by replacing ρ with e in (26).

$$p_{j+\frac{1}{2}}^{n+1} = P[\rho_{j+\frac{1}{2}}^{n+1}, e_{j+\frac{1}{2}}^{n+1}] \quad (31)$$

$$PQ_{j+\frac{1}{2}}^{n+\frac{1}{2}} = \frac{1}{2} (p_{j+\frac{1}{2}}^{n+1} + p_{j+\frac{1}{2}}^n) + Q_{j+\frac{1}{2}}^{n+\frac{1}{2}} \quad (32)$$

We take $Q_{j+\frac{1}{2}}^{n+\frac{1}{2}} = (Q_{j+\frac{1}{2}}^{n+\frac{1}{2}})_L$, i.e., the artificial viscosity is not normally recomputed after transport. Experience has shown that the added calculations are rarely necessary.

$$\tilde{e}_{j+\frac{1}{2}}^{n+1} = \tilde{e}_{j+\frac{1}{2}}^{n+1} - (\delta^{n+\frac{1}{2}} t / m_{j+\frac{1}{2}}^{n+1}) [PQ_{j+\frac{1}{2}}^{n+\frac{1}{2}} (A_{j+1}^{n+\frac{1}{2}} U_{j+1}^{n+\frac{1}{2}} - A_j^{n+\frac{1}{2}} U_j^{n+\frac{1}{2}})] \quad (33)$$

$$\tilde{T}_{j+\frac{1}{2}}^{n+1} = T[\tilde{e}_{j+\frac{1}{2}}^{n+1}, \rho_{j+\frac{1}{2}}^{n+1}] \quad (34)$$

Radiation Transport by Diffusion

$$e_{j+\frac{1}{2}}^{n+1} = \tilde{e}_{j+\frac{1}{2}}^{n+1} + \frac{1}{2} (\delta^{n+\frac{1}{2}} t / m_{j+\frac{1}{2}}^{n+1}) [(L_j^{n+1} + L_j^n) - (L_{j+1}^{n+1} + L_{j+1}^n)] \quad (35)$$

$$L_j^{n+1} = 2 \frac{ca}{3} \lambda_j^{n+1} x_j^{n+1} (\theta_{j-\frac{1}{2}}^{n+1} - \theta_{j+\frac{1}{2}}^{n+1}) / (x_{j+1}^{n+1} - x_{j-1}^{n+1}) \quad (36)$$

$$\lambda_j^{n+1} = (\lambda_j^{n+1})^* / [1 + 2(\lambda_j^{n+1})^* / (x_{j+1}^{n+1} - x_{j-1}^{n+1})] \quad (37)$$

A flux limiter is necessary because, in certain regions, the "true" mean free path for radiation, λ^* , inevitably exceeds the mesh spacing. But the magnitude of the photon energy flux cannot, on physical grounds, exceed $ca\theta$, even if all the photons moved in one direction. The extent to which the flux limiter is called upon is a measure of the adequacy of the diffusion approximation.

$$(\lambda_j^{n+1})^* = (\theta_{j+\frac{1}{2}}^{n+1} \lambda_{j+\frac{1}{2}}^{n+1} + \theta_{j-\frac{1}{2}}^{n+1} \lambda_{j-\frac{1}{2}}^{n+1}) / (\theta_{j+\frac{1}{2}}^{n+1} + \theta_{j-\frac{1}{2}}^{n+1}) \quad (38)$$

$$\lambda_{j+\frac{1}{2}}^{n+1} = \lambda_R [T_{j+\frac{1}{2}}^{n+1}, \rho_{j+\frac{1}{2}}^{n+1}] \quad (39)$$

where $\lambda_R = 1/\rho K_R$ and K_R is the Rosseland mean opacity.

When evaluating L_j^n in (36), conditions prior to radiation transport are used, i.e., $\tilde{T}_{j+\frac{1}{2}}^{n+1}$ and $\rho_{j+\frac{1}{2}}^{n+1}$.

We follow Christy's method⁽⁸⁾ for solution of the implicit form of the nonlinear diffusion equation (35)--by first making a linear expansion of the various terms therein, and then employing a Newton-Raphson-type iteration procedure.

Defining

$$i_{\delta_{j+\frac{1}{2}}}^{n+1} \theta^{n+1} = i_{\theta_{j+\frac{1}{2}}}^{n+1} - i_{\theta_{j+\frac{1}{2}}}^{n+1} \quad (40)$$

the $(i+1)$ th iterates can be expressed as:

$$i_{e_{j+\frac{1}{2}}}^{n+1} = i_{e_{j+\frac{1}{2}}}^{n+1} + \left. \frac{\partial e}{\partial \theta} \right|_{j+\frac{1}{2}}^{n+1} i_{\delta_{j+\frac{1}{2}}}^{n+1} \theta^{n+1} \quad (41)$$

$$i_{L_j}^{n+1} = \frac{2ca}{3} i_{\lambda_j}^{n+1} x_j^{n+1} [i_{\theta_{j-\frac{1}{2}}}^{n+1} - i_{\theta_{j+\frac{1}{2}}}^{n+1}] / (x_{j+1}^{n+1} - x_{j-1}^{n+1}) \quad (42)$$

$$i_{j+1}^{n+1} = i_{\lambda_j}^{n+1} + \frac{i_{\partial\lambda}^{n+1}}{\partial\theta_{j+\frac{1}{2}}} [i_{\delta_{j+\frac{1}{2}}} \theta^{n+1}] + \frac{i_{\partial\lambda}^{n+1}}{\partial\theta_{j-\frac{1}{2}}} [i_{\delta_{j-\frac{1}{2}}} \theta^{n+1}] \quad (43)$$

and

$$\frac{\partial\lambda_j^{n+1}}{\partial\theta_{j+\frac{1}{2}}} = \left[\frac{\lambda_j^{n+1}}{(\lambda_j^{n+1})^*} \right]^2 [\lambda_{j+\frac{1}{2}}^{n+1} + \theta_{j+\frac{1}{2}}^{n+1} \left(\frac{\partial\lambda}{\partial\theta} \right)_{j+\frac{1}{2}}^{n+1} - (\lambda_j^{n+1})^*] / (\theta_{j+\frac{1}{2}}^{n+1} + \theta_{j-\frac{1}{2}}^{n+1}) \quad (44a)$$

$$\frac{\partial\lambda_j^{n+1}}{\partial\theta_{j-\frac{1}{2}}} = \left[\frac{\lambda_j^{n+1}}{(\lambda_j^{n+1})^*} \right]^2 [\lambda_{j-\frac{1}{2}}^{n+1} + \theta_{j-\frac{1}{2}}^{n+1} \left(\frac{\partial\lambda}{\partial\theta} \right)_{j-\frac{1}{2}}^{n+1} - (\lambda_j^{n+1})^*] / (\theta_{j+\frac{1}{2}}^{n+1} + \theta_{j-\frac{1}{2}}^{n+1}) \quad (44b)$$

When (39) is given in tabular form, as in the present example, $(\frac{\partial\lambda}{\partial\theta})$ in (44) is derived by piecewise differentiation.

Substituting (36)-(44) into (35), and dropping higher order terms, there results:

$$-\alpha_{j+\frac{1}{2}} (i_{\delta_{j+3/2}} \theta^{n+1}) + \beta_{j+\frac{1}{2}} (i_{\delta_{j+\frac{1}{2}}} \theta^{n+1}) - \gamma_{j+\frac{1}{2}} (i_{\delta_{j-\frac{1}{2}}} \theta^{n+1}) = \zeta_{j+\frac{1}{2}} \quad (45)$$

where

$$\alpha_{j+\frac{1}{2}} = (\delta^{n+\frac{1}{2}}_t) \frac{ca}{3} X_{j+1}^{n+1} [i_{\lambda_{j+1}}^{n+1} - \frac{i_{\partial\lambda}^{n+1}}{\partial\theta_{j+3/2}} (i_{\theta_{j+\frac{1}{2}}}^{n+1} - i_{\theta_{j+3/2}}^{n+1})] / (X_{j+2}^{n+1} - X_j^{n+1}) \quad (46)$$

$$\beta_{j+\frac{1}{2}} = \left[\frac{i_{\partial\theta}}{\partial\theta} \right]_{j+\frac{1}{2}}^{n+1} m_{j+\frac{1}{2}}^{n+1} + \alpha_{j-\frac{1}{2}} + \gamma_{j+3/2} \quad (47)$$

$$\gamma_{j+\frac{1}{2}} = (\delta^{n+\frac{1}{2}}_t) \frac{ca}{3} X_j^{n+1} [i_{\lambda_j}^{n+1} + \frac{i_{\partial\lambda}^{n+1}}{\partial\theta_{j-\frac{1}{2}}} (i_{\theta_{j-\frac{1}{2}}}^{n+1} - i_{\theta_{j+\frac{1}{2}}}^{n+1})] / (X_{j+1}^{n+1} - X_{j-1}^{n+1}) \quad (48)$$

$$\zeta_{j+\frac{1}{2}} = - (i_{e_{j+\frac{1}{2}}}^{n+1} - \tilde{e}_{j+\frac{1}{2}}^{n+1}) m_{j+\frac{1}{2}}^{n+1} + [(i_{L_j}^{n+1} - i_{L_{j+1}}^{n+1}) + (L_j^n - L_{j+1}^n)] (\delta^{n+\frac{1}{2}}_t) / 2 \quad (49)$$

Equation (45), together with appropriate boundary conditions, can be solved for $i_{\delta_{j+\frac{1}{2}}} \theta^{n+1}$ by the method of backward substitution. The recursive solution is:

$$i_{\delta_{j+\frac{1}{2}}} \theta^{n+1} = M_{j+\frac{1}{2}} i_{\delta_{j-\frac{1}{2}}} \theta^{n+1} + N_{j+\frac{1}{2}} \quad (50)$$

where

$$M_{j+\frac{1}{2}} = \gamma_{j+\frac{1}{2}} / (\beta_{j+\frac{1}{2}} - \alpha_{j+\frac{1}{2}} M_{j+3/2}) \quad (51)$$

and

$$N_{j+\frac{1}{2}} = (\epsilon_{j+\frac{1}{2}} + \alpha_{j+\frac{1}{2}} M_{j+3/2}) / (\beta_{j+\frac{1}{2}} - \alpha_{j+\frac{1}{2}} M_{j+3/2}) \quad (52)$$

The boundary condition at the axis of symmetry, $j=1$, is:

$$L_1^{n+1} = L_1^n = 0 \quad (53)$$

so that

$$i_{\delta_{\frac{1}{2}}} \theta^{n+1} = 0 \quad (54)$$

For the other boundary, define

$$j^* = \text{Min } j \ni \tilde{e}_{j-\frac{1}{2}}^{n+1} < e_c \quad \text{and} \quad \lambda_{j-\frac{1}{2}}^{n+1} < \lambda_c \quad (55)^V$$

Also, let

$$j^{**} = \text{Max } j \ni \tilde{e}_{j+\frac{1}{2}}^{n+1} \geq e_c; \quad j^{**} < j^* - 1 \quad (56)$$

Then

$$(\lambda_{j^{**}+1}^{n+1})^* = (\theta_{j^{**}+\frac{1}{2}}^{n+1} \lambda_{j^{**}+\frac{1}{2}}^{n+1} + \theta_{j^*-\frac{1}{2}}^{n+1} \lambda_{j^*-\frac{1}{2}}^{n+1}) / (\theta_{j^{**}+\frac{1}{2}}^{n+1} + \theta_{j^*-\frac{1}{2}}^{n+1}) \quad (57)$$

$$L_{j^{**}+1}^{n+1} = 2 \frac{ca}{3} \lambda_{j^{**}+1}^{n+1} X_{j^*-1}^{n+1} (\theta_{j^{**}+\frac{1}{2}}^{n+1} - \theta_{j^*-\frac{1}{2}}^{n+1}) / (X_{j^*}^{n+1} - X_{j^{**}}^{n+1}) \quad (58)$$

and

$$L_j^{n+1} = \begin{cases} L_{j^{**+1}}^{n+1} & \text{for } j^{**+2} \leq j \leq j^{*-1} \\ 0 & \text{for } j \geq j^* \end{cases} \quad (59)$$

from which it follows immediately that

$$i_{\delta_{j+\frac{1}{2}}} \theta^{n+1} = 0 \quad \text{for} \quad \begin{cases} j^{**+1} \leq j \leq j^{*-2} \\ \text{all } j \geq j^* \end{cases} \quad (60)$$

We then proceed as follows:

1. Calculate $\alpha_{j+\frac{1}{2}}$, $\beta_{j+\frac{1}{2}}$, $\gamma_{j+\frac{1}{2}}$ and $\zeta_{j+\frac{1}{2}}$ for $j=1, \dots, j^{*-1}$.
2. Calculate $M_{j+\frac{1}{2}}$ and $N_{j+\frac{1}{2}}$ for $j=j^{*-1}, \dots, 1$. Note that $M_{j^{*-1}} = \gamma_{j^{*-1}}/\beta_{j^{*-1}}$ and $N_{j^{*-1}} = \zeta_{j^{*-1}}/\beta_{j^{*-1}}$.
3. Calculate $i_{\delta_{j+\frac{1}{2}}} \theta^{n+1}$ for $j=1, \dots, j^{*-1}$. Steps 1-3 are repeated until the temperature corrections are reduced to a sufficiently small amount. Our convergence criterion was

$$\max_j |i_{\delta_{j+\frac{1}{2}}} \theta^{n+1}| / \max_j \theta_{j+\frac{1}{2}}^{n+1} < 10^{-6} \quad (61)$$

Convergence was generally quite rapid, usually in 3 or 4 iterations. Occasionally, as at implosion time when the initial "guesses" were far from the mark, the number of iterations required was greater. When the number exceeded 5, the timestep was automatically dropped. The implicit differencing in (35) provides unconditional stability so that no other timestep control is required for the diffusion operator.

After the final sweep through (61), L_j^{n+1} was re-evaluated using (36) and $e_{j+\frac{1}{2}}^{n+1}$ was computed from (35), thus assuring exact conservation of energy. The final step was then to recompute the temperature from (34), substituting $e_{j+\frac{1}{2}}^{n+1}$ for $\tilde{e}_{j+\frac{1}{2}}^{n+1}$.

Radiation Transport by Emission

Retaining the definitions of j^* and j^{**} in (55) and (56),

$$e_{j+\frac{1}{2}}^{n+1} = \tilde{e}_{j+\frac{1}{2}}^{n+1} \begin{cases} -\frac{ca}{4} (\delta^{n-\frac{1}{2}}_t) \epsilon_{j+\frac{1}{2}}^{n+1} x_{j+1}^{n+1} \tilde{\theta}_{j+\frac{1}{2}}^{n+1} / m_{j+\frac{1}{2}}^{n+1}, & j \leq j^{**} \\ + 0, & j^{**} < j < j^*-1, j > j^*-1 \\ + \frac{ca}{4} (\delta^{n-\frac{1}{2}}_t) \sum_{j=1}^{j^{**}} \epsilon_{j+\frac{1}{2}}^{n+1} x_{j+1}^{n+1} \tilde{\theta}_{j+\frac{1}{2}}^{n+1} / m_{j+\frac{1}{2}}^{n+1}, & j = j^*-1 \end{cases} \quad (62)$$

The emissivity coefficient, ϵ , is defined as the ratio of the energy emitted by a gas volume enclosed by a surface of given dimension to the energy that would be emitted by a black surface of this same dimension and at the same temperature. For the cylindrical geometry considered here:

$$\epsilon_{j+\frac{1}{2}}^{n+1} = \text{Min} \{ 2x_{j+1}^{n+1} [1 - (x_j^{n+1}/x_{j+1}^{n+1})^2] / \lambda_{j+\frac{1}{2}}^{n+1}, 1 \} \quad (63)$$

$$\lambda_{j+\frac{1}{2}}^{n+1} = \lambda_p [\tilde{\tau}_{j+\frac{1}{2}}^{n+1}, \rho_{j+\frac{1}{2}}^{n+1}] \quad (64)$$

$\lambda_p = 1/\rho K_p$ and K_p is the Planck mean opacity

$$T_{j+\frac{1}{2}}^{n+1} = T(e_{j+\frac{1}{2}}^{n+1}, \rho_{j+\frac{1}{2}}^{n+1}) \quad (65)$$

Conservation Laws

Equations (1) and (27) are respectively statements of exact conservation of momentum and mass. Taking the scalar product of (1) with $U_j^{n+\frac{1}{2}}$ and adding the result to the sum of (29), (33) and (35), then invoking the continuity equation (27) along with the definitions in (6) and (25), there results:

$$(H_j^{n+1} - H_j^n) / \delta^{n+\frac{1}{2}} t + (HT_{j+\frac{1}{2}}^{n+\frac{1}{2}} - HT_{j-\frac{1}{2}}^{n+\frac{1}{2}}) + (W_{j+\frac{1}{2}}^{n+\frac{1}{2}} - W_{j-\frac{1}{2}}^{n+\frac{1}{2}}) = 0 \quad (66)$$

which is a form-preserving analog⁽¹¹⁾ of the equation for total energy conservation.

Here

$$H_j^n = \frac{1}{2} m_j^n U_j^{n-\frac{1}{2}} U_j^{n+\frac{1}{2}} + \frac{1}{2} (m_{j+\frac{1}{2}}^n e_{j+\frac{1}{2}}^n + m_{j-\frac{1}{2}}^n e_{j-\frac{1}{2}}^n) \quad (67)$$

$$\begin{aligned} HT_{j+\frac{1}{2}}^{n+\frac{1}{2}} = & (\widetilde{\rho u A})_{j+\frac{1}{2}}^{n+\frac{1}{2}} \left(\frac{1}{2} \right) U_j^{n+\frac{1}{2}} U_{j+1}^{n+\frac{1}{2}} + \frac{1}{2} [(\widetilde{\rho u e A})_{j+1}^{n+\frac{1}{2}} + (\widetilde{\rho u e A})_j^{n+\frac{1}{2}}] \\ & + \frac{1}{2} \left[\frac{1}{2} (L_{j+1}^{n+1} + L_{j+1}^n) + \frac{1}{2} (L_j^{n+1} + L_j^n) \right] \end{aligned} \quad (68)$$

$$W_{j+\frac{1}{2}}^{n+\frac{1}{2}} = \frac{1}{2} [PQ_{j+\frac{1}{2}}^{n+\frac{1}{2}} (A_{j+1}^{n+\frac{1}{2}} U_{j+1}^{n+\frac{1}{2}} + A_j^{n+\frac{1}{2}} U_j^{n+\frac{1}{2}})] \quad (69)$$

A similar result is obtained when (62) replaces (35) in the above scheme. The total energy analog, $\sum_j H_j^n$, is exactly conserved, i.e., the interior work and transport terms cancel so that the change in $\sum_j H_j^n$ is determined entirely by boundary conditions. As with the equations for mass and momentum conservation, there is a one-to-one correspondence between the terms of (66) and the fundamental energy conservation equation, with no terms of any order left over.

APPENDIX A--NOTES

Note 1--Timestep Calculation

The hydrodynamic timestep is limited by

$$\delta^{n+\frac{1}{2}}t = \text{Min} [\delta t_1, \delta t_2, \beta_1 \delta^{n-\frac{1}{2}}t] , \quad (\text{I-1})$$

where δt_1 derives from a Courant condition

$$\delta t_1 = \text{Min}_j [(X_{j+1}^n - X_j^n) \beta_2 / C_{j+\frac{1}{2}}^n] , \quad (\text{I-2})$$

and δt_2 is computed from the velocity divergence in each mesh interval:

$$\delta t_2 = \delta^{n-\frac{1}{2}}t \begin{cases} \text{Min}_j [-(1 + \Delta_{j+\frac{1}{2}}^n) \beta_2 / \alpha_1 \delta^{n-\frac{1}{2}}\Delta_{j+\frac{1}{2}}] \\ \quad \text{if } \delta^{n-\frac{1}{2}}\Delta_{j+\frac{1}{2}} < 0 \\ \beta_1, \text{ otherwise} \end{cases} \quad (\text{I-3})$$

For the calculations discussed here, $\beta_1 = 1.02$, $\beta_2 = 0.25$ and $\alpha_1 = 4.0$.

Note II--Aliasing Error and the Smoothing Operator

Equation (4) frequently leads to a form of computational instability known as aliasing error. This instability is completely independent of the transport process and occurs even when the mesh is everywhere Lagrangian, i.e., when $S_j^{n+\frac{1}{2}} = U_j^{n+\frac{1}{2}}$ for all j . Aliasing is associated with the exchange of energy between Fourier components. Normally, the mesh is defined such that the smallest wavelength accurately resolved is several mesh-element

lengths because, as is well known, the first few Fourier components are dispersive. (Hence, the characteristic rounding of steep wave fronts.) Sometimes, especially when the particle velocity is small compared to characteristic wave speeds, the components interact in such a way that energy cascades from the long wavelengths to the short. If appropriate dissipative mechanisms are not present, the amplitude of the short wavelengths can grow without bound. A physical analogy is the energy of turbulence that generally moves from large to small eddies, and then is eventually dissipated or degraded into internal energy via friction. The artificial viscosity does not sufficiently discriminate over wavelength to allow selective damping over one- to two-zone waves.

An effective method of eliminating this difficulty is the introduction of a smoothing operator:

$$U_j^{n+1/2} = 2U_j^n - \phi_j^{n-1/2}, \quad (\text{II-1})$$

where

$$\phi_j^{n-1/2} = \frac{1}{2} (U_j^n + U_j^{n-1}).$$

Equation (II-1) is employed in place of Eq. (4) whenever $U_j^{n+1/2}$ derived from the latter satisfies the inequality

$$|U_j^{n+1/2} - U_j^{n-1/2}| > \beta_3 + |U_j^n|, \quad (\text{II-2})$$

where typically $\beta_3 = 0.1$.

This technique has worked very well in a wide variety of problems; a similar scheme was successfully employed by Grammeltoedt to solve the

barotropic vorticity equation.⁽¹²⁾ Its disadvantage lies in its nonconservative character. If Eq. (66) is summed over all j in the interval $t^{n-1} \leq t \leq t^n$ and Eqs. (II-1) and (II-2) are used, it can be shown that, in place of zero, the right-hand side has a residual value of

$$R^{n-1/2} = (\delta^{n-1/2} t)^{-1} \sum_j \frac{1}{2} \phi_j^{n-1/2} [m_j^n (U_j^{n+1/2} - \phi_j^{n+1/2}) - m_j^{n-1} (U_j^{n-1/2} - \phi_j^{n-1/2})] \quad (\text{II-3})$$

where it is understood that the terms $U_j^{n+1/2}$ and $U_j^{n-1/2}$ derive from Eq. (4) and not Eq. (II-1).

In practice, the residual sum, $\sum_n R^{n-1/2} \delta^{n-1/2} t$, is normally insignificant when compared with the total energy $\sum_j H_j^n$.

Note III—Grid Motion

The blow-off from the inner fall surface to the cavity interior is controlled by the short-range energy deposition, accurate resolution of which required extremely fine zoning. The initial zone thickness at the inner fall radius for all problems was of the order of 1 μm , or less. The thickness of each successive radially increasing zone was typically $\sim 2\%$ wider than its predecessor and with ~ 500 zones employed, the outermost zone thickness was of the order of 10 mm. The inner and outer fall radii were always Lagrangian surfaces ($S_j^{n+1/2} = U_j^{n+1/2}$) so that no lithium was transported across these. Also, the entire region in which the specific internal energy initially exceeded the cohesive energy ($e_{j+1/2}^0 > e_c$) was taken to be Lagrangian, along with three or four adjacent zones in the two-phase fluid. The outer grid line in this region then constituted an artificial Lagrangian interface, with all grid lines contained between it and the outer fall surface gradually allowed to move towards equal spacing as the motion progressed. Whenever a fluid zone on the outer side of the artificial interface expanded (nominally 10%) below

normal density, the interface counter was incremented to include the expanded material within the Lagrangian core. This dynamic rezoning technique enabled good resolution to be maintained in the core of the cavity and in the shock pulse moving out towards the outer annular boundary without requiring an excessively small timestep.

Note IV--Transport Density Field

The choice of spatial differencing methods for transport of extensive variables affects both the solution accuracy and stability. Weighted central differencing, as in Eq. (26a), minimizes numerical diffusion and is, therefore, preferred to donor-cell (backward) differencing. The former method, however, frequently leads to numerical instability. A heuristic demonstration of the (extreme) problem with central differencing is provided by examining the phenomenon of depletion. Consider two adjoining zones centered at $j-\frac{1}{2}$ and $j+\frac{1}{2}$, and let the direction of grid motion be such that w_j is negative. Mass then will be transported from the cell at $j+\frac{1}{2}$ to the cell at $j-\frac{1}{2}$. Now suppose that $(\rho_{j+\frac{1}{2}})_L \ll (\rho_{j-\frac{1}{2}})_L$. Equation (26a) shows that, in this case, the transport density $\hat{\rho}_j$ is effectively independent of the density in the donor cell. From Eq. (24) we then see that the mass out rate out of the donor cell is independent of the mass contained therein so that it is possible for the donor cell to contribute more mass than it actually contains in a single timestep.

Instability resulting from central differencing is eliminated with an empirically developed smoothness test based on the sign of the second difference of the Lagrangian density. Defining

$$\psi_{j+\frac{1}{2}} \equiv \text{sgn} [\nabla^2 (\rho_{j+\frac{1}{2}})^{n+1}_L] = \text{sgn} [(\rho_{j-\frac{1}{2}})^{n+1}_L - 2(\rho_{j+\frac{1}{2}})^{n+1}_L + (\rho_{j+\frac{3}{2}})^{n+1}_L],$$

and letting Ψ_j represent the sequence

$$\psi_{j-3/2}, \psi_{j-1/2}, \psi_{j+1/2}, \psi_{j+3/2}$$

we choose centered transport [Eq. (26a)] across j unless Ψ_j matches the following sequence.

Row					
1	$\bar{+}$	$\underline{+}$	$\bar{+}$	$\underline{+}$	$\omega_j^{n+1/2} < 0$
2		$\underline{+}$	$\bar{+}$	$\underline{+}$	$\omega_j^{n+1/2} < 0$
3	$\bar{+}$	$\underline{+}$	$\bar{+}$		$\omega_j^{n+1/2} > 0$
	$j-2$	$j-1$	j	$j+1$	$j+2$

The rule is implemented by testing for a match in Row 2 if $\omega_j^{n+1/2} < 0$ and in Row 3 if $\omega_j^{n+1/2} > 0$. If no match is found, centered transport is chosen, except that no match is attempted and backward transport automatically is used if any of the ψ values making up the sequence Ψ_j includes density values from adjacent sides of a Lagrangian material interface.

Note V—Outer Boundary Condition for Radiation Transport

Whether the diffusion approximation is an adequate model for radiation transport everywhere in the plasma is uncertain; what is certain is that this approximation cannot be used at all in the relatively cool, expanded two-phase fluid. Figure 12 shows the variation of the radiation mean free path with density at a temperature of 0.125 eV (1450 K), the lowest temperature for which we have tabulated data. These data derive from calculations by Huebner⁽⁶⁾. Accurate data near the liquid reference density are not available, but it is clear that λ increases extremely rapidly as ρ decreases away from

the reference value. (The exponential extrapolation shown by the dashed curve is merely a simple means for obtaining a smooth fit between the very small value of λ that must apply to the condensed phase and the several hundred meter value Huebner derives at $\rho = 0.02 \text{ Mg/m}^3$. The actual λ - ρ isotherm may be even steeper if a sharp transition occurs near the critical density--estimated to be 0.091 Mg/m^3 for lithium).

Throughout the entire period when radiation transport in the lithium has important influence on the motion there exists an expanded liquid-vapor layer between the hot core gas and the main body of the fall. To take account of this (largely transparent) layer in the emission model is relatively simple. Once the density is defined (assumed) at which significant absorption occurs, we proceed outward from the core and find the first liquid-vapor zone ($e < e_c$) with greater density, implying $\lambda < \lambda_c$. All the radiant energy emitted by the hot gas zones ($e \geq e_c$) is assumed to be absorbed in this one zone, bounded on the outer edge by the surface j^* . All zones in between the last hot gas zone, bounded on the inner edge by the surface j^{**} , and the absorber zone neither emit nor absorb any radiant energy. Moreover, as the absorber zone heats and expands, λ will eventually increase above λ_c , at which point no further radiant absorption is allowed and the next outer adjacent two-phase or liquid zone acts as the radiation sink. This process continues until the radiation flow no longer influences the motion.

For the diffusion model, we calculate the boundary flux from a radiation potential-weighted mean opacity, considering the absorber zone at $j^{*-1/2}$ and the last hot gas zone at $j^{**+1/2}$ (by (56), $j^{**+1/2} < j^{*-1/2}$). The potential gradient at the boundary is measured between the surfaces j^* and j^{**} and the radiant flux across all surfaces $j^{**+2} \leq j \leq j^*-1$ is set to the flux computed across j^{**+1} so that no temperature change occurs in the zones bounded by j^{**+1} on the inside and j^*-1 on the outside. For all practical purposes, however, the precautions taken to account for the optically thin layer between the hot gas and the

condensed phase (in the diffusion approximation) were unnecessary for the example problem considered. In the time period of interest, virtually no radiant energy escaped from the gas, so that an adiabatic boundary condition would have sufficed.

APPENDIX B - Nomenclature

A	Area
a	Radiation density constant
C	Speed of sound
c	Speed of light
e	Specific internal energy
e_c	Cohesive specific energy*
E	Total energy
K	Opacity
L	Radiant energy flux
M	Momentum
m	Mass
M	Coefficient, defined by equation (51)
N	Coefficient, defined by equation (52)
P	Pressure
Q	Artificial viscosity
S	Grid velocity
T	Temperature
t	Time
U	Particle velocity
v	Specific volume
X	Position coordinate (radius)
α_1	Artificial viscosity coefficient (typically = 4.0)
α_2	Artificial viscosity coefficient (typically = 0.25)
α	Coefficient, defined by equation (46)
β	Coefficient, defined by equation (47)
γ	Coefficient, defined by equation (48)
Δ	Cubical dilation

*The cohesive specific energy is equivalent to the heat of sublimation at 0° K and is approximately the dividing line between gas and liquid, liquid/vapor regions in the energy-volume plane.

δ	Difference operator
ϵ	Emissivity coefficient
ζ	Residual, defined by equation (49)
θ	Radiation potential = T^4
λ	Radiation mean free path (mfp)
λ^*	Radiation-potential-weighted mfp
$\bar{\lambda}$	Flux-limited radiation mfp
λ_c	mfp value above which (cold) lithium is considered transparent
ρ	Density
ω	Grid velocity relative to particle velocity

Notes:

1. The subscript L denotes Lagrangian.
2. The superscript 0 denotes the reference or normal state.
3. The symbol $\sim$ above a variable denotes an advection-related quantity or an intermediate state related to advection.
4. The symbol $\sim$ above a variable denotes the value derived from the hydrodynamic increment, prior to applying the radiation transport operator.

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LIST OF FIGURES

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- FIG. 2 The Planck mean opacity for lithium as a function of temperature and density.
- FIG. 3 Lagrangian motion within the Eulerian reactor cavity as a function of time, as calculated with the emission approximation.
- FIG. 4 The ratio of the total energy radiated to the total energy initially contained in the core gas as a function of time, as calculated with the emission approximation.
- FIG. 5 Radial distribution of pressure in the Eulerian reactor cavity at $t = 60 \mu s$ as calculated with the emission approximation.
- FIG. 6 Radial distribution of temperature in the Eulerian reactor cavity at $t = 60 \mu s$, as calculated with the emission approximation.
- FIG. 7 Radial distribution of particle velocity in the Eulerian reactor cavity at $t = 60 \mu s$, as calculated with the emission approximation.
- FIG. 8 Lagrangian motion within the Eulerian reactor cavity as a function of time, as calculated with the diffusion approximation.
- FIG. 9 Radial distribution of pressure in the Eulerian reactor cavity at $t = 60 \mu s$, as calculated with the diffusion approximation.
- FIG. 10 Radial distribution of temperature in the Eulerian reactor cavity at $t = 60 \mu s$, as calculated with the diffusion approximation.
- FIG. 11 Radial distribution of particle velocity in the Eulerian reactor cavity at $t = 60 \mu s$, as calculated with the diffusion approximation.
- FIG. 12 Variation of the radiation mean free path in lithium with density at a temperature of 0.125 eV (1450 K). The Planck and Rosseland means are essentially indistinguishable in this range.

FIGURE 1

FIGURE 2

Emission limited radiation flow

FIGURE 3

FIGURE 4

FIGURE 5

FIGURE 6

FIGURE 7

FIGURE 8

FIGURE 9

FIGURE 10

FIGURE 11

FIGURE 12