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Two consequences of charge symmetry breaking (CSB) in the n-p system are examined. In n-p elastic scattering, CSB nuclear forces cause a difference between the polarizations of the neutron and the proton scattered in opposite directions in the center of mass system. The expected differences in polarizations due to one boson exchanges, particularly a photon exchange, isospin mixed mesons, and a pion (with the n-p mass difference taken into account), are computed. The calculated polarization difference is typically of the order of several tenths of a percent. In $np + d\pi^0$, the CSB of nuclear forces can be tested by measuring the asymmetry of the angular distribution of deuterons about 90° in the center of mass frame. The expected asymmetry is calculated for one boson exchange CSB mechanisms, as in the elastic scattering. The asymmetry caused by the mixed $n\pi^0$ exchange is dominant, but all of the mechanisms contribute asymmetries with similar angular dependences. The maximum asymmetry is about 0.8% at 0° ; the average is about 0.2%.

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1. INTRODUCTION

Nuclear interactions can be divided into hadronic and electromagnetic sectors. The dominant hadronic sector possesses SU(2) (or isospin) invariance as a symmetry which is broken by electromagnetic forces. In the absence of the symmetry breaking forces, SU(2) invariance is exact and nuclear forces would be charge independent and charge symmetric. Although electromagnetic effects are small, a study of the symmetry breaking mechanisms is useful in understanding the complicated character of nuclear forces.

A system's interactions are charge independent if the Hamiltonian H commutes with the total isospin operator $\vec{T}$. Charge symmetry is a weaker symmetry which only requires that H be invariant under reflections in the x-y plane in isospin space, if the z-axis is the charge axis.

Charge independence has been shown to be violated in low energy nucleon-nucleon scattering. A comparison of neutron-proton and proton-proton scattering lengths in the $^1S_0(I=1)$ state shows an unambiguous difference [1]. This difference in scattering lengths can be attributed primarily to the mass splitting of neutral and charged mesons exchanged between nucleons [1,2]. On the other hand no direct violation of charge symmetry of nuclear forces has yet been detected. There is indirect evidence for charge symmetry breaking forces, mainly from the comparison of binding energies of mirror nuclei. However, this comparison is plagued by some uncertainties in the knowledge of nuclear

structure. Most of the past tests of charge symmetry have concentrated on the examination of the equality of neutron-neutron and proton-proton forces in free and quasi-free two-body scatterings, and results from these tests show no violation of charge symmetry. For instance, after Coulomb corrections, the neutron-neutron and proton-proton scattering lengths are equal within experimental errors [1].

Theoretically there are two classes of charge symmetry breaking forces, namely those of class III and class IV, in the nomenclature of Ref. 1. (Class I and class II forces refer to those which are charge independent and charge symmetric respectively.) Class III forces have the form

$$V_{III} = D[\tau_3(1) + \tau_3(2)] \quad (1.1)$$

and thus commute with the total isospin operator T^2 in a two-body system. Note that V_{III} vanishes for the neutron-proton system. On the other hand, class IV forces cause isospin mixing. These forces are of the form

$$V_{IV} = F[\tau_3(1) - \tau_3(2)] + G[\vec{\tau}(1) \times \vec{\tau}(2)]_3 \quad (1.2)$$

and vanish for neutron-neutron and proton-proton systems. The most general forces of this type are

$$\begin{aligned} & [\tau_3(1) - \tau_3(2)] (\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \vec{L} f(r^2, p^2, L^2) \\ & [\vec{\tau}(1) \times \vec{\tau}(2)]_3 (\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \vec{L} g(r^2, p^2, L^2) \end{aligned} \quad (1.3)$$

where $\vec{L}$ is the angular momentum operator, $\vec{r} = \vec{r}_1 - \vec{r}_2$ and $\vec{p} = 1/2 (\vec{p}_1 - \vec{p}_2)$ are the relative coordinate and momentum of the two

nucleons, f and g are arbitrary functions of the given quantities. Note that Eq. (1.3) is odd under exchanges of nucleon isospins or spins. This is not so for classes I, II and III forces.

It has been pointed out by two of us [1] that the comparison of a_{nn} and a_{pp} is sensitive only to V_{III} , and thus does not fully explore the properties of charge symmetry. Few experiments carried out so far have examined effects of V_{IV} . In this work we investigate theoretically two consequences of charge symmetry breaking in the neutron-proton system, both caused by class IV forces shown in (1.3).

The first reaction we consider is neutron-proton elastic scattering. As long ago as 1956, Wolfenstein [3] remarked that in this reaction, isospin conservation implies the equality of the polarization of neutrons scattered at an angle θ with that of protons scattered at $\pi - \theta$ in the center of mass frame (see Fig. 1a), that is

$$P_n(\theta) = P_p(\pi - \theta) \quad (1.4a)$$

or equivalently (see Fig. 1b)

$$P_n(\theta_n) = P_p(\theta_p) \quad (1.4b)$$

As explained in Ref. 4, this equality is broken by class IV forces which violate isospin conservation. We calculate the difference between neutron and proton polarizations on the basis of one boson exchange mechanisms in Section 2. We neglect two boson exchange mechanisms. The results are typically of the order of several tenths of a percent. A brief report of this work is given in

Ref. 4. Here we correct an error, and extend the calculation to include effects of the neutron-proton mass difference. Significant changes in the calculated polarization differences are obtained.

The second reaction we study for charge symmetry breaking effects is pion production in $np \rightarrow d\pi^0$. In the absence of class IV forces, isospin is conserved and one consequence is that the angular distribution of deuterons (or pions) produced in the reaction is symmetric about 90° in the center of mass frame:

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{d\sigma}{d\Omega}(\pi - \theta) \quad (1.5)$$

The presence of class IV interactions would cause the angular distribution to be slightly asymmetric about 90° . In Section 3, the magnitude of this asymmetry is investigated in the $A_{33}(1232)$ resonance energy region on the basis of one boson exchange mechanisms. We predict a front-to-back asymmetry of about 0.8%, which arises primarily from the $\eta - \pi^0$ mixing mechanism.

Lists of notations and conventions used here are given in the Appendix. A version of this work which contains extensive explanations of the various calculations is also available [5]. A brief report of the $pn \rightarrow d\pi^0$ study appears in Ref. 6.

2. CHARGE SYMMETRY BREAKING IN NEUTRON-PROTON ELASTIC SCATTERING

In the neutron-proton system, the simplest reaction that can occur is elastic scattering; it provides a situation where charge symmetry breaking effects might be detected without comparing results of two different experiments, such as the comparison of neutron-neutron and proton-proton scattering lengths.

As can be seen from (1.3), class IV forces involve spin operators that are odd under the exchange of particles 1 and 2, and thus break the equality (1.4). When these forces are included, the transition operator T can be considered as the sum of a dominant charge symmetric term T_s and a small charge symmetry violating term δT . That is

$$T = T_s + \delta T \quad (2.1)$$

where δT involves spin-odd operators. The neutron-proton polarization difference is described by $\Delta(\theta)$:

$$\Delta(\theta) = P_n(\theta) - P_p(\pi-\theta) \quad (2.2)$$

It arises from the interference between T_s and δT . We calculate $\Delta(\theta)$ for three types of charge symmetry breaking mechanisms illustrated in Fig. 2(a) - (c). They are: (a) the direct electromagnetic effect (γ), (b) the ρ^0 - ω mixing mechanism, and (c) the nucleon mass difference (δ). We write

$$\Delta(\theta) = \Delta_Y(\theta) + \Delta_{pw}(\theta) + \Delta_S(\theta) \quad (2.3)$$

The exchange of a π^0 - n mixed boson does not give rise to a class IV force [1] and therefore does not contribute to $\Delta(\theta)$. Other mechanisms involve two boson (including photon) exchanges, and are neglected here.

2.1 Charge Symmetry Breaking Mechanisms and Class IV Potentials

(a) The direct electromagnetic effect

A direct electromagnetic force is one that is transmitted by photons alone; it exists even when hadronic forces are absent. Thus the reliable estimation of its contribution to the polarization inequality $\Delta(\theta)$ is crucial in learning about other more interesting terms. The class IV part of the one-photon exchange potential is given by

$$V^{IV} = -\frac{e^2}{4\pi} \frac{\kappa_n}{4N^2} [\tau_3(1) - \tau_3(2)] (\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \vec{L} \frac{1}{r} \frac{d}{dr} \left(\frac{1}{r} \right) \quad (2.4)$$

This expression differs by an overall minus sign from the corresponding expression, Eq. 3 of Ref. 4, and corrects this equation.

We note that the class IV potential V_Y^{IV} arises from terms involving the current (γ_μ) coupling with one nucleon (proton) and the anomalous magnetic ($c_{\mu\nu} q^\nu$) coupling with the other nucleon (neutron), therefore V_Y^{IV} depends on κ_n and not on κ_p . In addition, an isoscalar interaction is required at one vertex with an isovector interaction at the other.

(b) The ρ^0 - ω mixing mechanism

Particle mixing is a well known phenomenon in the theory of elementary particles (see e.g., Refs. (7) and (8)). Mixing occurs when a perturbing Hamiltonian H' violates some quantum numbers which are conserved by the unperturbed Hamiltonian H_0 . H' then can introduce nonzero off-diagonal matrix elements into the mass matrix of the particles. One then finds that the states that diagonalize the full Hamiltonian $H_0 + H'$ are mixtures of eigenstates of H_0 . Direct experimental evidence for ρ^0 - ω mixing comes from the analysis of the $\omega \rightarrow 2\pi$ decay process in, say, e^+e^- annihilation. It is found that the ω to ρ^0 transition is required to explain the relatively large $\omega \rightarrow 2\pi$ decay rate, which is observed despite the fact that the reaction violates G-parity conservation [8, 9].

The ρ -nucleon and ω -nucleon interaction Hamiltonians are given by

$$H_{\rho NN} = g_\rho \bar{\psi} \left(\frac{1}{2} \gamma^\mu - \frac{K^V}{2M} \sigma^{\mu\nu} \nabla_\nu \right) \hat{\tau} \cdot \hat{\rho}_\mu \psi \quad (2.5)$$

$$H_{\omega NN} = g_\omega \bar{\psi} \left(\frac{1}{2} \gamma^\mu - \frac{K^S}{2M} \sigma^{\mu\nu} \nabla_\nu \right) \omega_\mu \psi \quad (2.6)$$

where $\hat{\rho}_\mu$ and ω_μ are the ρ and ω field operators, and ∇_ν operates on vector meson field operators only. The coupling constants g_ρ , g_ω and the anomalous magnetic couplings K^V , K^S can be obtained through the use of Vector Dominance Model and e^+e^- annihilation data [9]. We use the following values:

$$\begin{aligned} g_\omega^2/4\pi &= 18.4, & g_\rho^2/4\pi &= 2.4 \\ K^V &= 1.85, & K^S &= -0.06 \end{aligned} \quad (2.7)$$

The potential due to ρ^0 - ω mixing has recently been recomputed by McNamee, Scadron and Coon [10], by Henley and Miller [1], as well as by Friar and Gibson [11]. The class IV part of the ρ^0 - ω mixing potential is given by

$$V_{\rho\omega}^{IV} = + \frac{g_\rho g_\omega}{4\pi} \frac{K_n}{4M^2} \frac{\langle \rho^0 | H_{em} | \omega \rangle}{m_\omega^2 - m_\rho^2} [\tau_3(1) - \tau_3(2)] (\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \vec{L} \\ \times \frac{1}{r} \frac{d}{dr} \frac{1}{r} (e^{-m_\rho r} - e^{-m_\omega r}) \quad (2.6)$$

where the mixing matrix element $\langle \rho^0 | H_{em} | \omega \rangle$ has been determined by McNamee, Scadron and Coon [10] to be -3400 MeV^2 , and by Friar and Gibson to be -6000 MeV^2 . We adopt the more conservative value, that is $\langle \rho^0 | H_{em} | \omega \rangle = -3400 \text{ MeV}^2$. This value is consistent with the theoretically expected up-down current quark mass difference [12].

(c) The nucleon mass difference

The difference of nucleon masses can introduce charge asymmetric terms into the pion-nucleon interactions Hamiltonian [13]. On the other hand, there is no contribution from the kinetic energy difference. Consider the pseudoscalar πNN coupling

$$H_{\pi NN} = g_\pi \bar{\psi} i\gamma_5 \vec{\tau} \cdot \vec{\phi} \psi \quad (2.9)$$

If we take the nucleon mass difference into account we obtain for the non-relativistic reduction of $H_{\pi NN}$

$$H_{\pi NN}(N.R.) = H_0 + H_1 + H_2, \quad (2.10)$$

with

$$H_0 = - \frac{g_\pi}{2M} \vec{\sigma} \cdot \vec{\sigma} \cdot \vec{\sigma} \quad (2.11)$$

$$H_1 = - \delta \frac{g_\pi}{2M} \vec{\sigma} \cdot \vec{\sigma} \cdot \vec{\sigma}_3 \quad (2.12)$$

$$H_2 = - \delta \frac{g_\pi}{2M} \vec{\sigma} \cdot (\vec{p}' + \vec{p}) (\vec{\tau} \times \vec{\sigma})_3 \quad (2.13)$$

and

$$\delta = \frac{m_n - m_p}{m_n + m_p} \quad (2.14)$$

H_0 is the usual charge independent p-wave interaction, while H_1 and H_2 are the induced charge symmetry breaking terms. When H_1 is included in the evaluation of Fig. 2, then only a class III potential results. This is similar to the case of the $\eta - \pi^0$ mixing potential [1]. The class IV potential due to H_2 is given by

$$V_6^{IV} = - \frac{g_\pi^2}{4\pi} \frac{\delta}{2M^2} (\vec{\tau}_1 \times \vec{\tau}_2)_3 (\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \vec{L} \frac{1}{r} \frac{d}{dr} \left(\frac{1}{r} e^{-m_\pi r} \right) \quad (2.15)$$

2.2 Results

From the transition matrix T , one can obtain the polarization of the neutron or proton by the standard formula,

$$P_{n,p}(\theta_{n,p}) = \frac{1}{4} \text{Tr}(\vec{\sigma} \cdot \vec{y} T) / (d\sigma/d\Omega) \quad (2.16)$$

where $\hat{y} = \hat{p}_i \times \hat{p}_f$ according to the Madison convention. Then $\Delta(\theta)$ as defined in Eq. (2.2) is given by

$$\Delta(\theta) = \frac{1}{2} \text{Re } T_x (T_g^\dagger (\vec{\sigma}_n - \vec{\sigma}_p) \cdot \hat{y} \delta T) / (d\sigma/dn) \quad (2.17)$$

In order to calculate the transition matrix $T = T_g + \delta T$, we need to solve a two potential scattering problem, with

$$V = V_g - V^{IV} \quad (2.18)$$

where V_g is the charge symmetric nucleon potential and V^{IV} is the sum of all class IV potentials. Since V^{IV} is much weaker than V_g , we can use the distorted wave Born approximation (DWBA) result

$$\delta T_{fi} = - \frac{M}{4\pi} \langle \phi_f^- | V^{IV} | \phi_i^+ \rangle \quad (2.19)$$

The charge symmetric part T_g of T is determined from the phenomenological Livermore phase shifts [14]. To compute δT_{fi} from Eq. (2.19), it is necessary to have a model for the strong interaction in order to compute the distorted initial and final wave functions, $\phi_i^{(+)}$ and $\phi_f^{(-)}$; the super-soft-core de Tourreil-Rouben-Sprung [15] and the hard-core Hamada-Johnson [16] potentials are used for this purpose.

The polarization of the particle scattered at c.m. angle θ and the polarization difference $\Delta(\theta)$ of Eq. (2.17) are shown in Figs. 3-4 for laboratory energies of 160 and 300 MeV for the

de Tournell-Rouben-Sprung soft core potential. The contributions due to V_Y^{IV} , $V_{\rho\omega}^{IV}$ and V_δ^{IV} are plotted separately. It is apparent that the effect due to V_Y^{IV} dominates the magnitude of $\Delta(\theta)$ for small momentum transfers, because V_Y^{IV} has the longest range among the three class IV potentials.

The overall magnitude of $\Delta_{\rho\omega}(\theta)$ is smaller than that of $\Delta_Y(\theta)$ or $\Delta_\delta(\theta)$; this is primarily due to the short range of $V_{\rho\omega}^{IV}$ and the effect of the hadronic repulsion between nucleons at these distances. Because ρ - ω mixing has the shortest range, the soft-core potential gives a polarization difference that is somewhat larger than that obtained with the hard-core Hamada-Johnson potential; $\{\Delta_{\rho\omega}[\text{S.C.}] - \Delta_{\rho\omega}[\text{H.C.}]\} / \Delta_{\rho\omega}[\text{S.C.}]$ is approximately 10% near the angle where the magnitude of $\Delta_{\rho\omega}(\theta)$ is largest. $\Delta_Y(\theta)$ and $\Delta_\delta(\theta)$ are less sensitive to the choice of distorting potential, because they are due to relatively longer range effects. The angular dependences of $\Delta_Y(\theta)$ and $\Delta_{\rho\omega}(\theta)$ are similar, which seems to preclude the possibility of easily distinguishing the two effects experimentally. On the other hand, the angular dependence of $\Delta_\delta(\theta)$ is quite different from that of the other two contributions; this is due to the difference in the forms of class IV potentials. (Compare Eqs. (2.4), (2.8) and (2.15)).

The polarization difference $\Delta_Y(\theta)$ and $\Delta_\delta(\theta)$ shown in Figs. 3-4 are those with electromagnetic and pion-nucleon form factors set to unity. With the inclusion of form factors, we find that $\Delta_Y(\theta)$ and $\Delta_\delta(\theta)$ decrease by less than 10%. This is expected since the distortion of the initial and final state wave functions can provide some momentum transfer between the

two nucleons; contributions from high momentum photons and pions are not important.

Theoretically, the best angular region to search for a polarization difference seems to be around $\theta_{cm} = 25^\circ$ where $\Delta(\theta_{cm})$ is large. However, because the polarization P is also big there, it is probable that $\Delta(\theta_{cm})$ would be hidden within error bars. In practice, a null experiment can be done more accurately [17] and precise measurements of where $P_n(\theta_n)$ and $P_p(\theta_p)$ cross zero would serve our purpose. Charge symmetry is broken if experiments show that $P_n(\theta_n)$ and $P_p(\theta_p)$ cross zero at different angles in the center of mass frame. Note that for "high" energies ($E_{lab} \approx 300$ MeV), Δ_δ dominates near the angle where P crosses zero. Therefore the neutron-proton mass difference effect may be distinguishable from other contributions at these energies.

3. CHARGE SYMMETRY BREAKING IN THE $np \rightarrow d\pi^0$ REACTION

In the absence of class IV forces, the symmetry of the angular distribution of deuterons (or pions) produced in the $np \rightarrow d\pi^0$ reaction can be understood as follows. The final state of the reaction ($d\pi^0$) has total isospin $I_f = 1$. Therefore, if isospin is conserved, the relevant component of the initial state also must have isospin $I_i = 1$. The generalized exclusion principle requires that, for isospin one states, even (odd) angular momentum be associated with singlet (triplet) spin configurations(s). If the initial state is unpolarized, singlet and triplet sectors do not interfere, so that the angular distribution involve only even powers of $\cos(\theta_{cm})$, and thus symmetry about 90° results. This is an example of the Barshay-Temmer theorem [18].

Class IV forces do not respect isospin conservation, and thus break the angular symmetry. When these forces are included, we can consider the transition operator, T , for pion production in neutron-proton collision to be the sum of two terms,

$$T = T_S + T_A \quad (3.1)$$

in which T_S is the dominant charge symmetric contribution, which connects only the isospin one component of the initial neutron-proton state and the final $d\pi^0$ state. On the other hand, the small charge asymmetric part T_A of T has nonzero matrix elements

between the isospin zero component of the initial state and the isospin one final state.

The asymmetry $\Delta(\theta)$ in the angular distribution of deuterons is defined by

$$\Delta(\theta) = \frac{d\sigma}{d\Omega}(\theta) - \frac{d\sigma}{d\Omega}(\pi-\theta) \quad (3.2)$$

It arises from the interference between T_S and T_A , and is proportional to $\text{Re}(T_S T_A^*)$. We calculate $\Delta(\theta)$ for all one-boson-exchange charge symmetry breaking mechanisms, namely the direct electromagnetic effect (γ), ρ^0 - ω mixing, η - π^0 mixing and the neutron-proton mass difference (δ); they are discussed in subsections 3.2, 3.3, 3.4 and 3.5 respectively. The reason that η - π^0 mixing contributes here, even though it does not give rise to a class IV potential, will be made clear in subsection 3.4c. We write,

$$T_A = T_A(\gamma) + T_A(\rho\omega) + T_A(\eta\pi) + T_A(\delta) \quad (3.3)$$

$$\Delta(\theta) = \Delta_\gamma(\theta) + \Delta_{\rho\omega}(\theta) + \Delta_{\eta\pi}(\theta) + \Delta_\delta(\theta)$$

Other charge symmetry breaking mechanisms with two boson intermediate states are neglected.

3.1 The Pion Production Differential Cross Section

Since the asymmetry $\Delta(\theta)$ is proportional to T_S , we must

have a satisfactory model for calculating the reaction cross section for the purely hadronic interaction. At neutron incident energies in the range of 500 to 700 MeV in the laboratory frame, it has been known for a long time [19] that the pions are produced mainly by mechanisms in which the resonant pion-nucleon $\Delta_{33}(1232)$ interaction is involved. There have been many recent treatments of the $pp \rightarrow d\pi^+$ reaction in this energy region. Here we adopt the model of Alberg et al. [20]. In that work, pion production is considered to be dominated by the mechanism depicted in Fig. 5. The right half of Fig. 5 is just the off-the-energy-shell pion-nucleon scattering T-matrix in the (3,3) channel. A careful evaluation of the behavior of this T-matrix off the energy shell leads to the inclusion of a factor of $E/(\vec{q}^2 + m_\pi^2)^{1/2}$ in the propagator of the exchanged pion of momentum $\vec{q}$, where E is the initial kinetic energy in the center of mass frame. This somewhat unconventional factor serves as a part of the cut-off function for high momentum transfers; its origin lies in the treatment of the (3,3) resonance as a composite system of a pion and a nucleon. The off-energy shell dependence of the crossed Born graph and its iterates (which cause the (3,3) resonance in that model) leads directly to the $E/(\vec{q}^2 + m_\pi^2)^{1/2}$ factor [21, 22]. With $H_0 = -g_\pi/2M \vec{\sigma} \cdot \vec{\nabla} \vec{\tau} \cdot \vec{\phi}$ as the fundamental pion-nucleon interaction, a Low equation can be derived for $pp \rightarrow d\pi^+$. When this equation is solved in the one-pion intermediate state approximation, T_S is given by (see Fig. 5)

$$T_S = \sum_{i \neq j} \int \frac{d^3 q}{(2\pi)^3} t_1(E, k, q) \cdot \frac{E}{\omega_q} \frac{1}{(E/2)^2 - \omega_q^2 + i\epsilon} \cdot 2\omega_q B_j(q) \quad (3.4)$$

where i, j are particle labeling indices, B is the pion production operator defined as

$$B_j(q) = i \frac{g_\pi}{2M} \tau_q^\dagger v(q) \hat{\sigma}(j) \cdot \hat{q} \sqrt{\frac{1}{2\omega_q}} \exp[-i\hat{q} \cdot \hat{r}(j)] \quad (3.5)$$

and $t(E)$ is the pi-nucleon (3,3) channel T-matrix given by

$$t_1(E, k, q) = 4\pi v(k) v(q) h(E) [Q_{3/2}(k, q) P_{3/2}(\hat{k}, \hat{q})]_{(1)} \sqrt{\frac{1}{4\omega_k \omega_q}} \exp[i(\hat{q} - \hat{k}) \cdot \hat{r}(1)] \quad (3.6)$$

where $Q_{3/2}(P_{3/2})$ is the isospin (spin) 3/2 channel projection operator. The quantity $h(E)$ is related to the experimental (3,3) phase shift δ_{33} through

$$h(E) = -e^{i\delta_{33}} \sin \delta_{33} / (p_{cm}^3 v^2(k)) \quad (3.7)$$

in which p_{cm} is the π -nucleon center of mass momentum,

and the pion-nucleon form factor $v(q)$ is taken to be[†]

$$v(q) = [1 + (m_\pi^2 - q^2)/Q^2]^{-2} \quad (3.8)$$

and a cut-off momentum of $Q = \sqrt{95} m_\pi$ is required to obtain reasonable agreement with experiment. In the treatment of Ref. 20, the initial state is approximated by a plane wave, and the deuteron D-wave component is neglected. These simplifications will be assumed throughout this section.

A comparison of the calculated angular distribution (with T_S only) with the data of Bartlett et al. [23] is given in Fig. 6. Quite a good representation of the experiment is achieved.

3.2 The Electromagnetic Contribution

Of the various charge symmetry breaking mechanisms to be considered, the electromagnetic interaction is of least interest. Therefore in order to learn about other kinds of charge symmetry breaking mechanisms which have a meson exchange origin, it is important to be able to calculate the direct electromagnetic effects in a reliable manner.

We consider the one-photon exchange process shown in Fig. 7, and describe it by the emission of a virtual photon by one nucleon combined with the off-shell resonant photon-pion production amplitude on the other. We expect this process to

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[†]In the original form of Ref. 20, the pi-nucleon form factor is set to unity. However, after correcting a sign mistake in Eq. (63) of that paper, that is $[2\vec{k} \cdot \vec{q} - (\vec{\sigma} \cdot \vec{q} \times \vec{k})]$ should read $[2\vec{k} \cdot \vec{q} - (\vec{\sigma} \cdot \vec{k} \times \vec{q})]$, a momentum cut-off factor is needed to fit experimental data.

dominate the one photon exchange contribution to the angular asymmetry, $A_Y(\theta)$, for energies in the range 500-700 MeV, because it involves the $\Delta(3,3)$ resonance and the exchange of a virtual photon provides a mean of sharing momentum among the two nucleons.

In order to connect the photon and the nucleon to the $\Delta(3,3)$ resonance, we need to have the isovector electromagnetic current acting at the right hand vertex of Fig. 7; then the relevant contribution of Fig. 7 to $T_A(\gamma)$ is that part in which the isoscalar current acts at the left hand vertex. The photo-production T-matrix is proportional to the π -nucleon scattering T-matrix with $\vec{q}$ replaced by $\vec{q} \times \hat{\epsilon}$ where $\hat{\epsilon}$ is the polarization of the photon [24,25]. Thus we have

$$t_{\gamma\pi}(k, f; \vec{q}, \hat{\epsilon}, i) = \frac{e}{g_\pi} \sqrt{\frac{\omega q}{|\vec{q}|}} \frac{1}{v(p)} (F_1^V + F_2^V) \times t(k, f; \vec{q} \times \hat{\epsilon}, i) \quad (3.9)$$

where F_1^V and F_2^V are the current and magnetic nucleon electromagnetic form factors in the notation of Ref. 26. The dipole electromagnetic form factors of Ref. 27 are used in our calculation. In our case, the photoproduction process is off the energy shell, and in the model of Alberg *et al.* [20], this is accounted for by the inclusion of a factor of $E/|\vec{q}|$ in the propagator of the intermediate photon with momentum q .

The charge symmetry breaking T-matrix due to one photon exchange $T_A(\gamma)$ is given by the expression

$$T_A(\gamma) = \int_{ij} \int_{\lambda} \frac{d^3q}{(2\pi)^3} T_{\gamma\pi}^{(i)}(E, k, \vec{q}, \hat{\epsilon}_\lambda) \frac{E}{q} \frac{1}{(E/2)^2 - q^2 + i\epsilon} \times 2q B_{ij}^S(\vec{q}, \hat{\epsilon}_\lambda) \quad (3.10)$$

where E is the c.m. kinetic energy, q is the intermediate photon momentum (here $q \equiv |\vec{q}|$), $\vec{k}$ is the outgoing pion momentum. The function $B^S(\vec{q}, \hat{\epsilon}_\lambda)$ is the isoscalar photon production operator given by

$$B_{(j)}^S(\vec{q}, \hat{\epsilon}_\lambda) = \frac{e}{2M} \left\{ (F_1^S + F_2^S) i\vec{\sigma}(j) \times \vec{q} \cdot \hat{\epsilon}_\lambda - F_1^S(\vec{p}_j + \vec{p}_j') \right\} \\ \times 1/\sqrt{2q} e^{-i\vec{q} \cdot \vec{r}(j)}, \quad (3.11)$$

where $\vec{p}_j + \vec{p}_j'$ is the sum of the momenta of nucleon j before and after the photon emission and F_1^S and F_2^S are the isoscalar nucleon form factors (notation of Ref. 26).

3.3 The ρ^0 - ω Mixing Contribution

The ρ^0 - ω mixing mechanism is discussed in section 2; its effect in $np + d\pi^0$ is considered here. Figure 8a illustrates the process in which an ω -meson is emitted by one nucleon, transformed via electromagnetic interactions to a ρ^0 -meson which then interacts resonantly with the other nucleon. The same process with the ρ^0 and ω interchanged (see Fig. 8b) gives a negligible contribution, because it requires a second violation of isospin conservation at the " $\omega N\Delta$ vertex" and thus contributes only via a higher order electromagnetic effect.

In calculating the term of Fig. 8a, the $\rho^0 N + \Delta(3,3) + \pi^0 N$ process is taken to be an off energy shell T-matrix, $t_{\rho\pi}$. Since $H_{\pi NN}$ and $H_{\gamma NN}$ have similar forms, the $\rho^0 N + \pi^0 N$ T-matrix may be obtained in the same manner as the photo-production

($\gamma N \rightarrow \pi^0 N$) T-matrix is found. We take

$$t_{\rho\pi}(E, \vec{k}, \vec{q}, \hat{\epsilon}_\lambda) = g_\rho / g_\pi \sqrt{\frac{\omega}{W_q}} \frac{1}{v(q)} (1/2 + K^V) t(E, \vec{k}, \vec{q} \times \hat{\epsilon}_\lambda) \quad (3.12)$$

with $W_q = (m_\rho^2 + \vec{q}^2)^{1/2}$. A factor of $E/(m_\rho^2 + \vec{q}^2)^{1/2}$ is included in the propagator of the exchanged rho meson of momentum q . This factor accounts for the off energy shell behavior of the $\rho^0 N \rightarrow \pi^0 N$ T-matrix according to the model of Alberg et al. Thus the ρ - ω exchange contribution to the T-matrix, $T_A(\rho\omega)$ is given by the relation

$$T_A(\rho\omega) = \sum_{i,j,\lambda} \int \frac{d^3q}{(2\pi)^3} t_{\rho\pi}^{(i)}(E, \vec{k}, \vec{q}, \hat{\epsilon}_\lambda) \frac{E}{W_q} \times \frac{1}{(E/2)^2 - W_q^2 + i\epsilon} 2W_q B_{(j)}(\vec{q}, \hat{\epsilon}) \quad (3.13)$$

where

$$B_{(j)}(\vec{q}, \hat{\epsilon}_\lambda) = \frac{g_\omega}{2M} \left[(1/2 + K^S) i \vec{\sigma}_j \times \vec{q} \cdot \hat{\epsilon}_\lambda - \frac{1}{2} (\vec{p} + \vec{p}') \right] \times \langle \omega | H_{em} | \rho^0 \rangle \frac{1}{(E/2)^2 - m_\omega^2 - \vec{q}^2} \frac{1}{\sqrt{2W_q}} e^{-i\vec{q} \cdot \vec{r}_j} \quad (3.14)$$

Further details of the calculation are provided in Ref. 5.

3.4 The η - π^0 Mixing Contribution

The neutral pion (π^0) has isospin one, and the η -particle is an isotopic singlet. Because of electromagnetic effects or the up-down quark mass difference, the physical π^0 contains a

small admixture of η and vice versa. Charge symmetry breaking due to η - ρ^0 mixing, like ρ^0 - ω mixing, is an indirect electromagnetic effect.

Evidence for η - π^0 mixing comes from the unexpectedly large $\eta \rightarrow 3\pi$ decay rate, see e.g. Ref. 28. Unlike the case of ρ^0 - ω mixing, the η - π^0 mixing matrix element $\langle \eta | H_{em} | \pi^0 \rangle$ has not been measured directly in any experiment. One can, however, infer $\langle \eta | H_{em} | \pi^0 \rangle$ from the tadpole dominance model of electromagnetic mass splittings [10], which gives

$$\langle \eta | H_{em} | \pi^0 \rangle \simeq -4000 \text{ MeV}^2 \quad (3.14)$$

More recent calculations [29,30], which start from the light quark (up, down) mass differences and use current algebra, obtain similar values for $\langle \eta | H_{em} | \pi^0 \rangle$.

While the η - π^0 mixing mechanism does not contribute to the polarization difference in neutron-proton elastic scattering (because it gives rise to a class III potential only) [1], it does contribute to the angular asymmetry in $np \rightarrow d\pi^0$. In the $\Delta(3,3)$ resonance energy region, we expect the process shown in Fig. 9a to dominate the η - π^0 mixing contribution to the angular asymmetry, $\Delta_{\eta\pi}(\theta)$. The terms of Fig. 9b contribute only to second order electromagnetic effects, as the " η NA vertex" is zero if isospin is conserved.

It is amusing to note that if the $\Delta(3,3)$ is replaced by a nucleon, a simple counting of isospin operators would show that the sum of figures like 10a and 10b does vanish. The η -nucleon interaction Hamiltonian is defined by

$$H_{\eta NN} = g_{\eta} \bar{\psi} i \gamma_5 \psi \eta \quad , \quad (3.15)$$

in which η is the field operator of the η -meson. Since $H_{\eta NN}$ and $H_{\pi NN}$ (Eq. (2.9)) have similar forms, the two amplitudes corresponding to Fig. 10a and Fig. 10b are identical except for isospin operators. These operators are $\tau_3(2)\tau_3(2)$ and $\tau_3(1)\tau_3(2)$ for Fig. 10a and Fig. 10b respectively. In the neutron-proton system, $\tau_3(2)\tau_3(2) = 1$ and $\tau_3(1)\tau_3(2) = -1$, thus they cancel.

The contribution of the terms of Fig. 9 to the CSB T-matrix, $T_A(\eta\pi)$, is given by

$$T_A(\eta\pi) = \int \frac{d^3q}{(2\pi)^3} t^{(i)}(E, k, q) \frac{E}{\omega_q} \frac{1}{(E/2)^2 - \omega_q^2 + i\epsilon} \\ \times 2\omega_q B'_{(j)}(\vec{q}) \quad , \quad (3.16)$$

where $B'_{(j)}(\vec{q})$ is defined by

$$B'_{(j)}(\vec{q}) = i \frac{E_{\eta}}{2M} \frac{1}{(E/2)^2 - m_{\eta}^2 - q^2 + i\epsilon} \langle \eta | H_{em} | \pi^0 \rangle \\ \times \vec{\sigma}(j) \cdot \vec{q} \frac{1}{\sqrt{2}\omega_q} e^{-i\vec{q} \cdot \vec{r}(j)} \quad . \quad (3.17)$$

There is a problem in evaluating $T_A(\eta\pi)$ because the magnitude of the η -nucleon coupling constant g_{η} is uncertain. From various one-boson-exchange (OBE) models for nucleon-nucleon interactions [31], it is found that

$$7.60 \geq \frac{g_\eta^2}{4\pi} \geq 3.68 \quad (3.16)$$

On the other hand, SU(3) symmetry predicts that [32]

$$g_\eta = g_\pi \sqrt{3} (1 - \frac{4}{3} \alpha) \quad (3.17)$$

where $\alpha/(1 - \alpha)$ is the ratio of D-type to F-type coupling strengths; and with the generally accepted value of $\alpha \simeq 2/3$ [10], Eq. (3.17) gives

$$\frac{g_\eta^2}{4\pi} \simeq 0.5 \quad (3.18)$$

However, α is not precisely determined, as the SU(3) symmetry is not a perfect one. We note that a 10% decrease in α leads to an increase of g_η^2 by a factor of 3.24. Here we use the value

$$\frac{g_\eta^2}{4\pi} = 3.68 \quad (3.19)$$

Since the contribution to the asymmetry due to π^0 - η mixing is linear in g_η , it is straightforward to scale our results.

3.5 The Nucleon Mass Difference Contribution

The charge asymmetric terms, induced by the neutron-proton mass difference (Eqs. (2.10)-(2.14)) can lead to an asymmetry in the $np \rightarrow d\pi^0$ angular distribution. The CSB T-matrix for π production is defined as $T_A(\delta)$.

First consider the charge symmetry breaking process illustrated in Fig. 11a, in which the vertex where H_1 or H_2 (see. Eqs. (2.12) and (2.13)) acts is marked by "x". Note that except for the charge asymmetric vertex, Fig. 11a is identical to Fig. 5 which represents the charge symmetric process. Thus the contribution of the term of Fig. 11a to $T_A(\delta)$ can be computed by replacing the operator B_j of Eq. (3.5) by the sum of the pion production operators resulting from H_1 and H_2 .

The nucleon mass difference can also affect the $\Delta(3,3)$ resonance; this is illustrated in Fig. 11b. In our model, the $\Delta(3,3)$ is treated as a pi-nucleon scattering T-matrix; thus effects of the nucleon mass difference on the $\Delta(3,3)$ can be calculated to first order in δ .

To carry out this calculation, note that the solution of the pi-nucleon Low equation in the (3,3) channel may be obtained by solving the integral equation [33]

$$t(E) = \frac{v}{E} (1 + g(E)t(E)) \quad , \quad (3.20)$$

where

$$g(E) = \frac{E^2}{h_0^2} \frac{1}{E + h_0} \quad , \quad (3.21)$$

with h_0 representing the total pi-nucleon center of mass energy (minus the nucleon mass). In the (3,3) channel v/E is the crossed Born graph of Fig. 12a. The CSB part of $t(E)$ may be obtained by replacing v/E by $v/E + \delta v/E$ where δv is obtained by allowing either of the two vertices to be given by $H_1 + H_2$, Fig. 12b. Defining the total pi-nucleon T-matrix as $t + \delta t$,

we have to first order in δt

$$\delta t(E) = [1 + t(E)g(E)] \frac{\delta v}{E} [1 + g(E)t(E)] \quad (3.22)$$

The computation of (3.22) is carried out in Ref. 5. Here only the charge symmetry violating part of the resulting pi-nucleon T-matrix, Δt , is presented. We find

$$\begin{aligned} \Delta t(E, k, q) = & t'(E, k, q) p_{3/2}(\vec{k}, \vec{q}) \frac{\delta}{2} \left[-2\beta(c, 0) Q_{3/2}(k, q) \right. \\ & \left. + \int_l \beta(l, 0) Q_{3/2}(k, l) Q_{1/2}(l, q) + Q_{3/2}(k, 0) \tau_q \right] \end{aligned} \quad (3.23)$$

where

$$t'(E, k, q) = 4\pi v(k)v(q)h(E)/[4\omega_k \omega_q]^{1/2} \quad (3.24)$$

and $\beta(k, q)$ is a function defined in the Appendix. Note that Δt differs from the charge symmetric (3,3) channel T-matrix only in isospin projection operators. The contribution of Δt to $T_A(\delta)$ is calculated by using Δt of (3.20) instead of t in Eq. (3.4).

3.7 Results and Discussions

The angular asymmetry in $np + d\pi^0$ is shown in Fig. 13 for $E_{lab} = 577$ MeV. The contributions due to one-photon exchange (Δ_Y), ρ^0 - ω mixing ($\Delta_{\rho\omega}$), η - π^0 mixing ($\Delta_{\eta\pi}$) and the np mass

difference ($\Delta\delta$) are plotted separately in the figure. It is apparent that the sum (not shown) of all the asymmetry effects is dominated by the $\eta\text{-}\pi^0$ mixing mechanism. $\Delta_{\eta\pi}(\theta)$ is largest at $\theta = 0^\circ$, and the front to back asymmetry is

$$\frac{\Delta_{\eta\pi}(\theta)}{(d\sigma(0)/d\Omega)} = -0.73\% \quad (3.25)$$

The contribution of $\Delta_\delta(\theta)$ is about 1/5 of $\Delta_{\eta\pi}(\theta)$ at $\theta = 0^\circ$, while the effects of one-photon exchange and $\rho^0\text{-}\omega$ mixing are even smaller and are negligible compared to $\Delta_{\eta\pi}$.

At a laboratory energy of 577 MeV, the center of mass momentum is $|\vec{p}| \simeq 500$ MeV. Since the nucleons inside the deuteron have relatively small momenta, we can assume that the momentum carried by the exchanged particle(s) is approximately $|\vec{q}| \simeq 500$ MeV. The exchange of a high momentum pion is an efficient means of sharing momentum, because of the relatively small pion mass and large pi-nucleon coupling constant. The exchange of a mixed $\eta\text{-}\pi^0$ can also be considered as one pion exchange with an effective charge symmetry breaking pi-nucleon form factor; hence we can understand why this term gives a large contribution to $\ell(\theta)$. We note again that $\Delta_{\eta\pi}(\theta)$ is proportional to g_η so that the result (Fig. 13) obtained with (3.19) would be reduced by a factor of about three if (3.18) were used. The term resulting from the nucleon mass difference is also a one pion exchange term but Δ_δ is only about 1/5 of $\Delta_{\eta\pi}$ because the nucleon mass difference parameter $\delta (\simeq 0.65 \times 10^{-3})$ is small. The exchange of a mixed $\rho^0\text{-}\omega$ meson

is not an efficient way of sharing momentum between the two nucleons, since the energy denominator is very large; furthermore, the inclusion of the off-shell factor $E/(m_\rho^2 + q^2)^{1/2}$ in the propagator of the exchanged ρ decreases the asymmetry, $\Delta_{\rho\omega}$, by about a factor of 4. The smallness of $\Delta_{\rho\omega}$ is consistent with the fact that the ρ -exchange mechanism is not an important part of the pion-production process in the treatment of Ref. 20. The one-photon exchange contribution is insignificant primarily because of the small coupling constant ($e^2 = 1/137$), but also because the nucleon electromagnetic form factors reduce Δ_γ by about a factor of 1/3.

From Fig. 13, we note that the shapes of the asymmetry curves are similar to one another, and since the η - π^0 mixing contribution is dominant, it would be difficult (if not impossible) to separate the effects due to the other charge symmetry breaking mechanisms.

We can also calculate the integrated asymmetry, A_{fb} , from $\Delta(\theta)$

$$A_{fb} = \frac{1}{\sigma} \int_0^{\pi/2} \Delta(\theta) d\Omega \quad (3.15)$$

where σ is the total cross section for $np \rightarrow d\pi^0$; we find $A_{fb} = -0.19\%$. This parameter has been measured in the past, and limits of $(1 \pm 3)\%$ [23] and $-(0.36 \pm 0.66)\%$ [34] have been obtained for an energy region spanning 300-700 MeV.

At this point we may assess the model dependence of the results shown in Fig. 13. The calculation of $\Delta(\theta)$ depends on T_S and T_A . If one ignores the $E/\sqrt{q^2 + m_\pi^2}$ factor of Ref. 20 one must

include the suppression due to ρ -exchange (absent in Ref. 20) in order to obtain a T_S consistent with the $pp \rightarrow d\pi^+$ data (e.g. Ref. 35). This ρ -exchange phenomenology provides alternate models for both T_S and T_A . However the quantity T_S would be (by construction) essentially the same as the T_S of Ref. 20. What about T_A ? If one ignores the factor $E/\sqrt{q^2 + m_\rho^2}$ which suppresses ρ -exchange terms (in Ref. 20), one finds that $\Delta_{\rho\omega}(0^\circ)$ is increased by less than a factor of four (depending on the ρ form factor used). Furthermore, if one removes the $E/\sqrt{q^2 + m_\pi^2}$ from the calculation of $\Delta_{\rho\pi}(0^\circ)$, one finds that it increases by about a factor of two. Thus even with models in which ρ -exchange plays a crucial role, $\Delta_{\pi\pi}$ is much larger than $\Delta_{\rho\omega}$.

In addition to the results of Fig. 13, we have also studied the energy dependence of the asymmetry. Close to the resonance, results very similar to Fig. 13 are obtained. At lower energies π -nucleon s-wave scattering and additional CSB mechanisms must be included. At higher energies p-wave channels other than the (3,3) enter also, and d-wave rescattering may be important. In some rough estimates we found that A_{fd} is about the same order of magnitude throughout the energy region of 400 to 800 MeV.

4. SUMMARY AND CONCLUSIONS

We have examined two types of measurable effects which result from violations of charge symmetry in the neutron-proton system. In neutron-proton elastic scattering, charge asymmetry causes an inequality of the polarizations of neutrons and protons scattered in opposite directions in the center of mass frame. The direct electromagnetic (Δ_Y) and the ρ^0 -mixing $\Delta_{\rho\omega}$ contribution have the same sign and similar angular dependence; $\Delta_{\rho\omega}$ is smaller than Δ_Y at all angles and energies. The averaged magnitude of the nucleon mass difference contribution [$\Delta_\delta(\theta)$] is smaller than that of $\Delta_Y(\theta)$, however, their angular dependence are quite different. Our calculations yield a typical value of $\Delta(\theta)$ of several tenths of a percent, which may be measurable. We have omitted two boson exchange effects and effects which cannot be calculated reliably, such as radiative corrections of the pion-nucleon coupling constant [36]. It will be easier to carry out a time reversed experiment, in which a polarized neutron beam is scattered from a polarized hydrogen target. By determining the magnitudes, and by appropriately reversing the directions of polarizations of the beam and target, it is possible to determine the desired asymmetry in a single set-up.

In the case of $np \rightarrow d\pi^0$, charge symmetry violations result in the asymmetry of angular distribution ($d\sigma/d\Omega$) about 90° in the center of mass frame. The calculated asymmetry due to one boson exchange effects is largest in the forward direction and decreases smoothly to zero at 90° . Our results at an

energy of 577 MeV are summarized in Table 1. The largest front to back asymmetry arises from the η - π^0 mixing mechanism. The effect of the nucleon mass difference is about 1/5 of $\Delta_{\eta\pi}$, and the contributions due to the direct electromagnetic interaction (one-photon exchange) and the ρ^0 - ω mixing mechanism are an order of magnitude smaller than $\Delta_{\eta\pi}$. The integrated asymmetry, A_{fb} , defined by Eq. (3.14), is also calculated. We find $A_{fb} = -0.19\%$. A measurement of A_{fb} or $\Delta(\theta)$ would demonstrate the charge asymmetry of nuclear forces. In addition, if our theoretical framework is correct, the measurement of an asymmetry would allow a determination of the η - π^0 mixing matrix element, $\langle \gamma | H_{em} | \pi^0 \rangle$, and might thereby set a limit on the up-down quark mass difference.

Experimental observations of the polarization difference in neutron-proton elastic scattering, or the asymmetry about 90° in $np \rightarrow d\pi^0$ would represent the first direct measurement of charge symmetry breaking effects in nuclear interactions.

APPENDIX: Notations, Definitions and Conventions

1. Notations and Definitions

(a) e : Absolute value of electronic charge

(b) $\vec{\sigma}$: Spin operator; $\sigma_x, \sigma_y, \sigma_z$ are Pauli matrices.

S : Spin quantum number

$P_{3/2}$: Spin 3/2 projection operator;

$$P_{3/2}(\vec{k}, \vec{q}) = 3 \vec{k} \cdot \vec{q} - \vec{\sigma} \cdot \vec{k} \vec{\sigma} \cdot \vec{q}$$

$P_{1/2}$: Spin 1/2 projection operator;

$$P_{1/2}(k, q) = \vec{\sigma} \cdot \vec{k} \vec{\sigma} \cdot \vec{q}$$

(c) $\vec{\tau}$: Isospin operator; τ_x, τ_y, τ_z are Pauli matrices.

I : Isospin quantum number

$Q_{3/2}$: Isospin 3/2 projection operator;

$$Q_{3/2}(k, q) = \delta_{kq} - 1/3 \tau_k^\dagger \tau_q$$

$Q_{1/2}$: Isospin 1/2 projection operator;

$$Q_{1/2}(k, q) = 1/3 \tau_k^\dagger \tau_q$$

(d) m_a : Mass of particle a

We use (in MeV): $m_n = 939.57$, $m_p = 938.28$

$m_\pi = 137.27$, $m_\eta = 548.8$, $m_\rho = 770$,

$m_\omega = 782.6$

M : $1/2 (\pi_n + m_p)$

ω_q : $(m_\pi^2 + q^2)^{1/2}$

(e) A_μ : Electromagnetic field operator

ψ : Nucleon field operator; $\bar{\psi} = \psi^\dagger \gamma_0$

- $\vec{\phi}$: Pion field operator.
- (f) κ_n : Neutron anomalous magnetic moment;
 $\kappa_n = -1.91$.
- κ_p : proton anomalous magnetic moment;
 $\kappa_p = 1.79$.
- μ_p : Proton magnetic moment; $\mu_p = 1 + \kappa_p$
- κ^V : Isovector anomalous magnetic coupling;
 $\kappa^V = (\kappa_p - \kappa_n)/2 = 1.85$.
- κ^S : Isoscalar anomalous magnetic coupling;
 $\kappa^S = (\kappa_p + \kappa_n)/2 = -0.06$.
- (g) Δ_{33} : Same as $\Delta(3,3)$ or $\Delta(1232)$.
- (h) $v(q)$: pion-nucleon form factor.
- (i) $\beta(i,j)$: $\beta(i,j) = \begin{cases} +1 & i \geq j \\ 0 & i = j \end{cases}$; $i, j = 1, 0, -1$
- (j) $\vec{p}, \vec{p}'$: As operators (in Hamiltonians or whenever specified), $\vec{p} = -i\vec{\nabla}$ and $\vec{p}' = i\vec{\nabla}$ operating on nucleon field operators (or wave functions) only.

2. Conventions

- (a) Natural units are used. That is $\hbar = c = 1$.
- (b) Dirac matrices ($\gamma_\mu, \gamma_5, \sigma_{\mu\nu}$) are defined in Ref. 26.
- (c) $A \cdot B = A_\alpha B_\alpha = \vec{A} \cdot \vec{B}$,

where A, B are 4-vectors.

$$(d) \quad \tau_{\pm} (\tau_{\pm 1}) = (\tau_x \pm i\tau_y) / \sqrt{2}$$

$$\tau_0 = \tau_3 = \tau_z$$

$$\tau_3 |p\rangle = |p\rangle, \quad \tau_3 |n\rangle = -|n\rangle,$$

$$\tau_+ |n\rangle = \sqrt{2} |p\rangle, \quad \tau_- |p\rangle = \sqrt{2} |n\rangle.$$

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TABLE 1

Charge Symmetry Breaking Effects	$\Delta(0^\circ)/[d\sigma(0^\circ)/d\Omega]$
One photon exchange (Δ_γ)	0.036%
ρ^0 - ω mixing ($\Delta_{\rho\omega}$)	0.024%
η - π^0 mixing ($\Delta_{\eta\pi}$)	-0.73%
Nucleon mass diff. (Δ_δ)	-0.16%

FIGURE CAPTIONS

- Fig. 1a) np elastic scattering.
- 1b) np elastic scattering.
- Fig. 2 Charge symmetry breaking mechanisms in np elastic scattering.
- Fig. 3 Polarization difference at $E_{\text{lab}} = 160$ MeV.
- Fig. 4 Polarization difference at $E_{\text{lab}} = 300$ MeV.
- Fig. 5 $np \rightarrow d\pi^0$ via charge symmetric hadronic interactions.
- Fig. 6 Comparison of the calculated angular distribution for $np \rightarrow d\pi^0$ with experimental data.
- Fig. 7 Charge symmetry breaking in $np \rightarrow d\pi^0$ due to one-photon exchange.
- Fig. 8 Charge symmetry breaking in $np \rightarrow d\pi^0$ due to the ρ^0 - ω mixing mechanism.
- Fig. 9 Charge symmetry breaking in $np \rightarrow d\pi^0$ due to the η - π^0 mixing mechanism.
- Fig. 10 Charge symmetry breaking in $np \rightarrow d\pi^0$ due to the η - π^0 mixing mechanism (without the $\Delta(3,3)$ resonance).
- Fig. 11 Charge symmetry breaking in $np \rightarrow d\pi^0$
  - 11a) the nucleon mass difference.
  - 11b) effect of the nucleon mass difference on the $\Delta(3,3)$.
- Fig. 12 Pi-nucleon Born terms.
- Fig. 13 Angular asymmetry in $np \rightarrow d\pi^0$ at $E_{\text{lab}} = 577$ MeV.

Figure 1

(a)

(b)

(c)

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7

(a)

(b)

Figure 8

(a)

(b)

(c)

Figure 9

Figure 10

(a)

(b)

Figure 11

Figure 12

Figure 13