TRANSPORT LINE from the LINAC to the SXLS RING
(Beam transport to compact synchrotron light sources, where available space
is strongly limited)

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1. ABSTRACT

Design of a transport line with highly restrictive space and beam size requirements and with flexibility in the orientation of the injector is presented.

An analytic method, based on symbolic algebra is used to find optimal regions of the design parameters. The fine tuning of the parameters were then performed with a standard matrix program.

Special interest was paid to the question of dispersion vector matching large angle bend with very small bending radius using non-iron bending magnets with very small gap area.

2. INTRODUCTION

Injector to the SXLS ring is a 200 Mev LINAC. Injection takes places at one of the straight sections immediately before the ring quadrupole (see Fig. 1). The transport line has to match (i) the 1% momentum spread in the LINAC to the 0.1% momentum spread in the ring, (ii) the phase space ellipses and (iii) the significantly nonzero dispersion and its derivative in the ring. At the same time, it has to fit into the small space left by the beam line, the cryostat and other elements of the densely packed compact ring. On the other hand, the presence of iron close to the superconducting ring dipoles has to be avoided.

The design, as shown on Fig. 1, is such, that by reversing the direction of a 29° achromatic bend in the transport line, the LINAC can be lined up either with the long axis of the ring or with the closest beam line, thus making future installation flexible.

The transport line consists of a triplett, followed by an achromatic bend (29°) with horizontal/vertical waist in the middle to allow matching of 1% momentum spread from the LINAC to the 0.1% momentum spread in the ring. This is followed by a quintuplett, which basically serves to match the phase space ellipses into the ring, and finally a total of 151° bend. This last bend consists of two combined function bending magnets with small radius of curvature ($B \geq 2$ T) and a septum. There is also a quadrupole between the two bending magnets, separated by drift spaces from them. This last 151° bend will bring the beam parallel with the stored beam as well as match the dispersion and its derivative into the ring.

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Fig. 1: Layout of the SXLS ring and the transport line.

3. LINAC parameters

The 200 MeV Linac will be similar to the one built for the IBM compact superconducting light source. It's parameters were reported [1] to be

Final momentum	$P_o = 200MeV$
Momentum Spread	$\Delta p/p \approx 1 \%$
Emittance	$\epsilon_x \approx \epsilon_y = 10^{-6} mrad$
Beam size	$\sigma_x \approx \sigma_y = 5 \cdot 10^{-3} m$
Beam divergence	$\sigma_x' \approx \sigma_y' = .2 \cdot 10^{-3} rad$

4. MATCHING CONDITIONS TO SXLS-RING

The SXLS ring is a superconducting compact synchrotron/storage ring of 8.5 m circumference, running at 696 MeV and providing $\lambda_{critical}=10$ A for x-ray lithography. It is a two superperiod machine consisting of two superconducting combined function bending magnets, four warm quadrupoles and four warm sextupoles as shown on Fig. 1. The structure is FODO-like but instead of horizontally defocusing quadrupole, the field index is $0 < n=B'/B\rho < 1$ in the bending magnet which provides the vertical focusing [2]. The machine functions for one superperiod are shown on Fig. 2.

Fig. 2

Machine functions for one superperiod of the SXLS ring.

The septum magnet is in one of the straight sections immediately before the ring quadrupole. From the values of the machine parameters at that point of the ring, the matching conditions can be obtained:

$$\begin{aligned}
 \sigma_e &\approx 10^{-3} & \beta_y &= 1.785 \text{ m} \\
 \beta_x &= 2.156 \text{ m} & \alpha_y &= 1.020 \\
 \alpha_x &= -.952 & \gamma_y &= 1.143 \\
 \gamma_x &= .884 & & \\
 \eta &= 1.208 \text{ m}, \quad \eta' = .603 & & \\
 \sigma_x &= \sqrt{\beta_x \epsilon_x + \sigma_e^2 \eta_x^2} = 1.901 \text{ mm} & \sigma_y &= \sqrt{\beta_y \epsilon_y} = 1.336 \text{ mm} \\
 \sigma_{x'} &= \sqrt{\gamma_x \epsilon_x + \sigma_e^2 \eta_x'^2} = 1.117 \text{ mr} & \sigma_{y'} &= \sqrt{\gamma_y \epsilon_y} = 1.069 \text{ mr}
 \end{aligned}$$

It is desirable to limit the presence of iron in the proximity of the superconducting coils of the 180° ring dipole. The only element, where iron is needed (to confine the flux) is the pulsing septum. The maximum achievable bending field in the septum is 1.1 - 1.3 T, and this dictates a minimum bend of 9° to clear the vacuum pipe*.

After the septum (in the reverse direction), the *minimum bending field, in order to clear elements in the ring as well as the cryostat itself, is 2.5 - 2.0 T*, the lower field corresponding to the higher operating field in the septum. The larger the bending field is, the more compact the final injection can be made and the easier it is to clear all ring elements as well as the lithography beam lines. From purely geometrical considerations, *to keep as far as possible from the lithography lines, $B = 3$ T would be preferable to 2 T*. Calculations in the following sections will be shown for the two extreme cases; for $B = 2$ and 3 T.

The minimum total bend (including the septum) is 122° , dictated by the closest lithography beamline. Instead, we chose this bend to be 151° for the following two reasons. Since the SXLS machine is a prototype, it will be more flexible if the design allows future installations at the customer's site with two different linac orientation (see Fig. 3); parallel with either the long axis of the ring or with the closest lithography beamline (which will be about 10m long). This can be achieved if the magnitude of the total final bend, θ_{tot} , and the momentum spread matching achromatic bend, θ_{achr} , are chosen to be:

$$\theta_{tot} \pm \theta_{achr} = \begin{cases} 180^\circ \\ 122^\circ \end{cases}$$

These conditions yield $\theta_{tot} = 151^\circ$ and $\theta_{achr} = 29^\circ$. The second reason is that the larger the total bend is, the easier is to match the dispersion vector into the ring - as it will be discussed in section-5 and in Appendix-1.

Fig. 3: Geometry of the transport line.

*The beam at that point in the ring has a direction of $\chi=1^\circ$ to the outside, therefore a 10° septum provides a 9° bend with reference to the vacuum pipe.

5. DISPERSION MATCHING BEND

The final 151° bend has to provide dispersion vector matching into the ring besides bending the injected beam parallel with the stored beam. As a consequence, the bend after the septum (still in reverse direction) is broken into two bending magnets with a horizontally focusing quadrupole between them (see Figs.4). The strength and location of the quadrupole (that is the $\theta_3 - \theta_4$ split of the total $\Theta = \theta_{tot} - \theta_{septum} = 142^\circ$ bend provided by the two bending magnets) is chosen to match the $\eta_T = \eta_T' = 0$ in the transport line with $\eta_R = -1.2$, $\eta_R' = -0.6$ in the ring*.

Fig. 4

Geometry studies of the 151° final bend.

- (1) $B_{sept} = 1.3T$ (or $1.1T$), $B_3 = B_4 = 2T$ (or $3T$), no drift space, $L_1 = L_2 = 0$.

This geometry would safely clear all ring elements, cryostat and all beam lines

However, the dispersion vector can not be matched.

- (2) $B_{sept} = 1.1T - 1.3T$, $B_3 = B_4 = 2T$ or $3T$, nonzero L_1 , L_2 drift spaces.

This geometry is implementable. However, either the beam size is small enough to fit inside the magnet aperture or it is compact enough to clear all beam lines.

- (3) $B_{sept} = 1.1T - 1.3T$, $B_4 = 3T$, $B_3 = 2$ or $1T$, nonzero L_1 , L_2

drift spaces. This geometry is implementable.

One would like to make the bend as compact as possible (no or very short drift spaces and short quadrupole as shown on Fig. 4a) in order to clear the closest lithography beam line. It turns out, however, that *it is impossible to match the dispersion vector without drift space* with $B_3 = B_4 = B = 3T$ or with $B = 2T$ (see Appendix-2).

Next, we will allow drift spaces as on Fig. 4b. The matching quadrupole strength and θ_4 angle were calculated for a range of the L_1 , L_2 drift spaces. Some of the solution leads to impractically strong quadrupoles or to small θ_4 angles. This latter is a problem, since the available distance, d_1 , until the closest beam line is a function of the θ_4

* The reversal of the sign of η and η' is due to the reversal of the x-direction between the transport line's and the ring's coordinate system.

angle and B_2 bending field. The distance, d_2 , available between the two bending magnets for the drift spaces and the quadrupole is smaller than d_1 , and it also depends on the B_1 bending field. Both, d_1 and d_2 are plotted on Fig. 5 as a function of θ_2 for different bending fields. One have to select solutions with implementable quadrupole strength and L_1+L_2 , θ_4 such, that the total bend clears the beam lines.

Figs. 5

Available distance between the closest beam line and the upstream end of the second dipole (d_1) and between the two dipoles (d_2) as a function of the θ_4 bending angle.

Table-1 summarizes the k_q and θ_4 values, calculated with different L_1 and L_2 drift spaces at $B=3$ and $2T$. The calculations were performed with $l_q = .15$ m. The 1-st and 2-nd columns in each box correspond to $n=.0$ and $.6$, respectively. Only the solutions, which require less than $k_q=10$ ($B' < 67$ T/m) are shown. The solutions, which satisfy the space requirements are highlighted with bold characters.

One can see, that even though k_q is smaller if $L_2 > L_1$ for a given $L=L_1 + L_2$ (see also Figs.2 and 3 in Appendix-2), they are not good solutions since θ_4 is also smaller and therefore the space requirement can not be satisfied. At either $B=3T$ or $B=2T$ and $n=.0$, there is only one solution which (barely) clears the beamline, and even that requires $k_q \approx 10$. By including gradient into the bending magnets, the dispersion vector can be matched with larger θ_4 angles, such that the space requirements can be more readily satisfied. The matching quadrupole however, is slightly stronger. The $n=.6$ case is shown as an example in Table-1.

All of the solutions require strong quadrupole. The solutions which fit between the cryostat and the beam line require the strongest ones. In addition, without gradient in the dipoles, there is no vertically focusing element in the combined bend and the vertical beam size grows out of control. In order to control both, the horizontal and the vertical beam size, one must provide both, horizontal and vertical focusing in the dipoles; *there must be a gradient in the dipoles such that $0 < n < 1$* . This will further increase the strength of the matching quadrupole.

Table-1

Bending angle of the 2-nd dipole (θ_4) and quadrupole strengths (k_q) when the dispersion vector is matched to the desired $\eta_d = -1.2$ and $\eta'_d = -.6$ with different values of the drift spaces, and with $L_q = .15\text{m}$.
Solutions, which satisfy the space requirement are highlighted.

B=3T		L_2					
		.1		.2		.3	
		n=.0	n=.6	n=.0	n=.6	n=.0	n=.6
.1	k_q θ_4					8.8 30	9.3 53
L_1 .2	k_q θ_4			9.4 41	9.7 67	8.3 32	8.4 57
.3	k_q θ_4	9.9 57	10.6 83	8.7 43	8.8 71	7.8 33	7.7 61

B=2T		L_2							
		.0		.1		.2		.3	
		n=.0	n=.6	n=.0	n=.6	n=.0	n=.6	n=.0	n=.6
.0	k_q θ_4					10.0 45	10.5 65	8.7 37	9.1 57
L_1 .1	k_q θ_4			10.5 57	10.8 78	9.0 47	9.1 69	8.0 40	8.1 61
.2	k_q θ_4			9.4 61	9.5 81	8.3 50	8.2 72	7.4 42	7.4 64
.3	k_q θ_4	9.9 75	10.4 94	8.6 63	8.7 85	7.7 53	7.6 75	7.0 44	6.9 67

Let us now examine if there are solutions which also satisfy the constraint on the beam size: the 3σ beam sizes have to be within the beam pipe. The special, pulsing non-iron 3T (or 2T) bending magnets can be implemented with $\approx 1\text{cm}^2$ (or 2cm^2) gap area. Calculations, presented in Appendix-2 indicate, that $n \geq .6$ yield acceptable beam sizes.

Therefore we looked at solutions with drift spaces as indicated by the results summarized in Table-1 and with $n \geq .6$ field indices. In addition to calculating the matching θ_4 and k_q , the beam size was also calculated as shown in Appendix-2. Starting with the matched beam at injection and proceeding in the reverse direction, through the septum

and the 142° bend, the beam sizes were traced*. The results are shown in Table-2. At $B=3T$ or $2T$ either the beam size can be kept small enough ($\approx 1\text{cm}^2$) or all beam lines can be cleared, but not both at the same time. The solutions which clear the beam lines require stronger quadrupoles (therefore the resulting beam size is larger). By increasing the length of the quadrupole the strengths is decreasing, (but of course, at the same time the length of the bend is also increasing). The solution in line-3 is the best in this category, requiring $\approx 1.2\text{cm}^2$ full gap and preserving all beam lines.

By lowering the bending field to $B=2T$, weaker ($\delta \approx 1/\rho$) quadrupole is needed to match the dispersion vector, and therefore the horizontal focusing/vertical defocusing

Table-2
Parameters of the dispersion matching bend and the corresponding maximum 1σ beam size. $\Theta = 142^\circ$.

	L_1 m	L_2 m	$B_{3,4}$ T	θ_4	n_3	n_4	B' T/m	l_Q m	$\sigma_x^{\max}$ cm	$\sigma_y^{\max}$ cm	$(3\sigma)^2$ cm^2	cut
1	.1	.2	3	61	.80	.60	68	.15	.38	.42	1.44	0
2	.1	.3	3	51	.80	.60	50	.15	.32	.36	1.04	1
3	.15	.25	3	58	.80	.60	51	.15	.34	.38	1.16	0
4	.2	.2	3	65	.80	.60	53	.15	.38	.38	1.30	0
5	.2	.3	3	49	.80	.60	37	.20	.30	.40	1.08	1
6	.2	.2	3	63	.80	.60	39	.20	.35	.46	1.45	0
7	.1	.3	3	48	.80	.60	29	.25	.28	.43	1.08	1
8	.2	.2	3	60	.80	.60	30	.25	.34	.50	1.53	1
9	.2	.3	2	61	.80	.60	32	.15	.39	.42	1.47	1
10	.2	.2	2	69	.80	.60	39	.15	.46	.48	1.99	1
11	.1	.2	2	66	.90	.60	47	.15	.48	.56	2.42	0
12	.1	.2	2-3	62	.80	.55	54	.15	.42	.45	1.70	0
13	.1	.1	1-3	77	.87	.56	41	.20	.61	.71	3.85	0
14	.1	.1	1-2	84	.70	.80	44	.15	.77	.65	4.50	0

will be less. However, this is more then compensated for by the increase in the length (l increases as ρ) of the bending magnet. As a net result, the $\sigma_{x,y}$ beam size will even slightly increase, as can be seen from Table-2. The real benefit will be that the gap area of the dipoles and the quadrupole can be increased by the same ratio as the bending field is decreased. As with $B=3T$, none of the cases satisfy all criteria. If one can

* We used the TRANSPORT code [3] for final fine-tuning of the solutions The phase space ellipse and dispersion was matched and beam sizes calculated after each element. With equal bending fields ($B_3 = B_4$), θ_4 and k_q were chosen as variables for the fit (keeping the total lengths of the bend constant). With unequal bending fields, n_4 was used as variable instead of θ_4 since in this case the lengths of the bend could not be kept constant.

afford to loose one beam line, then *solutions in lines-5 and 7 for $B=3T$ and in line-9 for $B=2T$ can be implemented with relatively weak quadrupoles.*

The bending field in the D_4 dipole has to be 3 or 2 T (preferably the higher) in order to clear the cryostat, but the D_3 dipole can be operated at lower fields, for example at 2 or 1 T, thus increasing the achievable gap area in the pulsed non-iron magnet. Due to the lower bending field it is also possible to shorten the bend and clear all beam lines (with beam sizes within a gap). *Line-12 of Table-2 show such a solution. This is the only solution which can be implemented with non-iron magnets where the 3σ beam size is well within the gap area, and all beam lines are cleared. However, it requires $B' = 55$ T/m quadrupole.*

The $B_3=1T$ field could be implemented with standard iron magnet (though the presence of iron is *not desirable*) with still larger gap sizes and vertical edge focusing which helps to keep the vertical beam size small (see the last two lines in Table-2). The geometry of the $B_3 \neq B_4$ cases are shown of Fig. 4c.

After solving the most difficult part (crucial portion) of the transport line, let us now turn our attention to its overall design.

6. ACHROMATIC BEND / MOMENTUM DISPERSION MATCHING

The 29° achromatic bend serves to match the original 1% momentum dispersion to that of .1% in the ring. It consists of two 14.5° bending magnets operating at .582 T bending field ($l=.29$ m, $\rho=1.146$ m) and two 15 cm long quadrupoles separated from each other by 30 cm to accommodate the aperture (on which the beam is focused). The quadrupoles are separated from the dipoles by 25 cm drift spaces.

The beam is focused (the horizontal and vertical phase space ellipses are transformed into upright ellipses with small beam sizes) at the middle of the achromatic bend with the help of the $Q_{1,2,3}$ quadrupole triplet after the linac.

7. PHASE SPACE ELLIPSE MATCHING

After the achromatic bend, a quadrupole quintuplett is used to match the horizontal and vertical phase space ellipses from the slit through the second half of the achromat and through the 151° dispersion matching bend into the ring.

8. BEAM ENVELOPE and ACCURACY of MATCHING

The location and the strengths of the quadrupoles as well as the beam envelope through the whole transport line for lines-3,7 and 12 in Table-2 are shown on Figs. 6. The quadrupole strengths are somewhat different for the three cases, but the quadrupole locations were kept the same, and in all cases, the $r = 3$ cm, $l = 15$ cm phase-1 quadrupoles can safely be reused, except of course the non-iron pulsing quadrupole in the final bend. In all three cases the match of the dispersion vector and the phase space ellipses into the ring are good to the third decimal place (see desired values in

section-4). The dotted line on Fig. 6b shows the effect of $\delta p/p=1\%$ in the 29° achromatic bend and the effect of the $\delta p/p=1\%$ in the 151° final bend. The dispersion and its derivative is shown of Fig. 6d. The effect of $\delta p/p \neq 0$ and the behavior of η and η' are very similar for all cases.

Figs. 6

Horizontal and vertical beam envelopes through the whole transport line, for lines-3,7 and 12 (Figs. 6a,b,c). The location of the magnetic elements are shown as well. Horizontally focusing and defocusing quadrupoles are represented by $\curvearrowright$ and $\curvearrowleft$ and dipoles by $\curvearrowright$

For the above versions of the transport line, the magnet requirements are summarized in Tables 3 and 4. The quadrupoles are practically the same in either versions. Their lengths, gap radius, maximum gradient as well as the maximum full beam size in the magnets are given. For the bending magnets, their bending angle, bending field (also magnetic length and bending radius), field index, full horizontal and vertical gap size as well as the full beam size in the magnets are given.

10. BEAM STEERING AND MONITORING

Special care was exercised in the spacing of the triplet and quintuplett as well as other elements in the transport line to leave ample room for flags, beam monitors and

corrector magnets to be installed throughout the whole transport line.

Table-3
Quadrupoles

		l m	r cm	B' T/m	$3\sigma_x^{\max}$ cm	$3\sigma_y^{\max}$ cm
Q_{1-3}		.15	3	≤ 12	3.6	3.6
Q_{4-5}		.15	3	≈ 10	0.2	0.4
Q_{6-10}		.15	3	≤ 13	2.2	3.6
Q_{11}	(a)	.15	1	≈ 51	0.3	0.4
	(b)	.25	1	≈ 29	0.3	0.6
	(c)	.15	1	≈ 54	0.3	0.4

Table-4
Bending magnets

		θ Deg	B T	l m	ρ m	n	g_H cm	g_V cm	$3\sigma_x^{\max}$ cm	$3\sigma_y^{\max}$ cm
$B_{1,2}$ parallel edge iron		14.5	.58	.29	1.146	.0	2.5	1.25	1.2	1.2
B_3 noniron	(a)	84	3	.3267	.222	.80	.7	.8	1.1	1.2
	(b)	94	3	.3654	.222	.80	.7	.8	0.9	1.3
	(c)	80	2	.4673	.334	.79	.8	.9	1.3	1.4
B_4 noniron	(a)	58		.2244		.60				
	(b)	48	} 3	.1857	} .222	.60	} .7	} .6	} 1.1	} .9
	(c)	62		.2396		.55				
Septum parallell edge iron		10	1.3	.0897	.513	.0	2.5	1.25	} 1.2	} .8

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APPENDIX-1

Dependence of the quadrupole strength on the preceeding elements in dispersion matching bends.

Consider a thin quadrupole of strength $-\delta = -1/f_x$. If the dispersion and its derivative (dispersion vector) before the quadrupole are $\begin{bmatrix} \eta \\ \eta' \end{bmatrix}_i^Q$, then after the quadrupole their value will be:

$$\begin{bmatrix} \eta \\ \eta' \end{bmatrix}_f^Q = \begin{bmatrix} 1 & 0 \\ -\delta & 1 \end{bmatrix} \begin{bmatrix} \eta \\ \eta' \end{bmatrix}_i^Q = \begin{bmatrix} \eta_i^Q \\ \eta'_i{}^Q - \delta \eta_i^Q \end{bmatrix}.$$

The thin quadrupole leaves η unchanged but modifies η' . Its effect is "amplified" by the value of $1/\eta_i^Q$. For example, the quadrupole strengths which yields $\eta'_f = 0$ is $\delta = (\eta'/\eta)_i^Q$; that is the smaller the $(\eta'/\eta)_i^Q$ ratio is, the weaker quadrupole is needed. For $\eta'_f \neq 0$:

$$\delta = \frac{\eta'_i{}^Q - \eta'_f{}^Q}{\eta_i^Q},$$

that is δ is decreasing if either the $\eta'_i{}^Q - \eta'_f{}^Q$ difference is decreasing or η_i^Q is increasing, or both.

One way to decrease the ratio of $(\eta'/\eta)_i^Q$ before the quadrupole, and thus δ , is to *increase the drift space* before the the quadrupole, since it transports the dispersion vector as:

$$\begin{bmatrix} \eta \\ \eta' \end{bmatrix}_f^{dr} = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \eta \\ \eta' \end{bmatrix}_i^{dr} = \begin{bmatrix} \eta_i^{dr} + L \eta'_i{}^{dr} \\ \eta'_i{}^{dr} \end{bmatrix}.$$

Increasing the bending angle of a dipole before the quadrupole also have the effect of decreasing the $(\eta'/\eta)_i^Q$ ratio since a dipole with θ bending angle changes $\eta_i^B = \eta'_i{}^B = 0$ to

$$\begin{bmatrix} \eta \\ \eta' \end{bmatrix}_f^B = \begin{bmatrix} \rho(1-\cos \theta) \\ \sin \theta \end{bmatrix}, \quad (A1.1)$$

and thus the $(\eta'/\eta)_f^B$ ratio is decreasing with θ as can be seen on Fig. A1.1. On the other hand, *including gradient* into the bending magnet is not beneficial. In this case eq.(A1.1) becomes:

$$\begin{bmatrix} \eta \\ \eta' \end{bmatrix}_f^B = \begin{bmatrix} \frac{1}{\rho k^2} [1 - C(kl)] \\ \frac{1}{\rho k} S(kl) \end{bmatrix}, \quad (A1.2)$$

where $k^2 = |1-n| / \rho^2$ and $n = \rho B'/B$ and

$$S = \begin{cases} \sin \\ \sinh \end{cases} \quad \text{and} \quad C = \begin{cases} \cos \\ \cosh \end{cases} \quad \text{if } n \begin{cases} \leq 1 \\ > 1 \end{cases}$$

Both, η_f^B and $\eta'_f{}^B$ are plotted as a function of n on Fig. A1.2. It is clearly seen, that

$(\eta'/\eta)_f^B$ increases with increasing n (this increase is rapid for $n>0$), thus δ is also increasing with n .

Fig. A1.1

η , η' and η'/η as a function of the θ bending angle.

Fig. A1.2

η , η' and η'/η as a function of the n field index for a dipole with $\theta = 71^\circ$ bending angle and $B=3$ T bending field ($l=.276$ m, $\rho=.222$ m).

In summary; δ is decreasing with increasing (i) θ_1 bending angle, (ii) L_1 drift space and increasing with increasing field index.

APPENDIX-2

Dependence of the dispersion vector and its matching conditions on the parameters of a composite bend.

A) Composite bend.

Consider a horizontal composite bend (vertical can be treated similarly), consisting of two bending magnets with bending fields B_1 and B_2 , bending angles θ_1 and θ_2 ($\theta_1 + \theta_2 = \Theta$) and field indices of n_1 and n_2 and a horizontally focusing quadrupole between them. The quadrupole is separated from the dipoles by L_1 and L_2 drift spaces. The length and the strength of the quadrupole is l_q and k_q ($=B'/B\rho$). The total (horizontal or vertical) transport matrix of the bend is

$$M = M^{B_2} M^{L_1} M^Q M^{L_2} M^{B_1} = \quad (A2.1)$$

$$\begin{bmatrix} b_{11} & b_{12} & \eta_1 \\ b_{21} & b_{22} & \eta'_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & L_1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} q_{11} & q_{12} & 0 \\ q_{12} & q_{22} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & L_2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & \eta_2 \\ a_{21} & a_{22} & \eta'_2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \eta \\ c_{21} & c_{22} & \eta' \\ 0 & 0 & 1 \end{bmatrix},$$

where

$$\begin{aligned} a_{11} &= a_{22} = C(k_1 l_1) & \eta_1 &= a_{13} = \frac{1}{\rho k_1^2} [1 - C(k_1 l_1)] \\ a_{12} &= \frac{1}{k_1} S(k_1 l_1) & \eta'_1 &= a_{23} = \frac{1}{\rho k_1} S(k_1 l_1) \\ a_{21} &= -k_1 S(k_1 l_1) & k_1^2 &= \frac{|1-n|}{\rho^2} = \frac{h^2}{\rho^2}, \quad n = \frac{\rho B'}{B}. \end{aligned}$$

Similar expressions are valid for the second dipole. The non-zero matrix elements for the quadrupole are also similar with $k_q = \sqrt{B'/B} \rho = \sqrt{n} / \rho$ and l_q instead of k_1 and l_1 . The interpretation of the S and C functions are:

$$S = \begin{cases} \sin & \text{and} \\ \sinh & \end{cases} \quad \text{and} \quad C = \begin{cases} \cos & \text{if } n \leq 1 \\ \cosh & \text{if } n > 1 \end{cases}$$

Sin and cos are used for the quadrupole (since $B' \geq 0$ horizontally focusing quadrupole is used) and for the dipole if $n \leq 1$, sinh and cosh are used for the dipole if $n > 1$.

B) Dependence of the dispersion vector on the parameters of the composite bend.

The resulting η and η' can be calculated from eq.(A2.1) as:

$$\eta = \eta_2 + q_{12} b_{11} \eta'_1 + q_{21} (L_1 \eta'_1 + \eta_1) (b_{11} L_2 + b_{12}) + q_{11} [b_{11} \eta_1 + (b_{11} (L_1 + L_2) + b_{12}) \eta'_1] \quad (A2.2a)$$

$$\eta' = \eta'_2 + q_{12} b_{21} \eta'_1 + q_{21} (L_1 \eta'_1 + \eta_1) (b_{21} L_2 + b_{22}) + q_{11} [b_{21} \eta_1 + (b_{21} (L_1 + L_2) + b_{22}) \eta'_1] \quad (A2.2b)$$

Fig. A2.1.

η as a function of the quadrupole strengths for different values of θ_4 at a given B , n and l_q . (a) $B=3T, n=0, l_q=0.15\text{m}$, (b) $B=3T, n=0.5, l_q=0.15\text{m}$, (c) $B=3T, n=0, l_q=0.25\text{m}$, (d) $B=2T, n=0, l_q=0.15\text{m}$, (e) $B=2T, n=0.5, l_q=0.15\text{m}$.

Eqs.(A2.2) can be written in a simple format if $L_1=L_2=0$, $\rho_1=\rho_2$, $n_1=n_2$ and if the thin lens approximation for the quadrupole is valid ($q_{11}=q_{22}=1$, $q_{12}=0$ and $q_{21}=-\delta=-1/f_x$):

$$\eta = \frac{\rho}{h^2} \left\{ 1 - \delta \frac{\rho}{h} [1 - C(h\theta_1)] S(h\theta_2) - C(h\Theta) \right\} \quad (\text{A2.3a})$$

$$\eta' = \frac{1}{h} \left\{ -\delta \frac{1}{h} [1 - C(h\Theta)] S(h\theta_2) + 1 - C(h\Theta) \right\} \quad (\text{A2.3b})$$

It is not easy to see from the exact eqs.(A2.2) how η (and η') changes with increasing or decreasing Θ , ρ , n or k_q . Therefore we will use the simpler expressions, obtained for the thin quad approximation to get an insight on the behaviour of η (and η') before we would examine the case with the real quadrupoles. One can see from eqs.(3), that

a) η (and η') has a minimum as a function of Θ , and this minimum is reached at 180°

b) the δ value, which yields $\eta=0$ (or $\eta'=0$) is decreasing with increasing ρ due to the $(\delta \rho)$ factor in the first term.

c) Below/above this δ value, η is increasing/decreasing with ρ due to the overall ρ factor on the right hand side of eq.(A2.3a) (η' is always decreasing with ρ).

d) η (and η') is linearly decreasing with increasing δ . (For real quadrupoles, however, since then $\delta = k_q \sin(k_q l_q)$, η will have a minimum as a function of k_q -- as will be shown on Figs. A2.1).

Let us turn now to the realistic case with quadrupoles of length l_q . η was calculated from eq.(A2.2a) and is plotted on Figs. A2.1 as a function of k_q for $L_1 = L_2 = 0$, $B_1 = B_2 = B$, $n_1 = n_2 = n$, $\Theta = 142^\circ$ and different values of B , n , θ_4 and l_q .

One can see, that there is a minimum value of the $\eta(k_q)$ function which can be reached with a given set of parameters. *The minimum and the location of the minimum (k_q) slightly depends on the gradient in the bending magnet but strongly depends on the bending field, on the θ_1 , θ_2 split of the total bending angle and on the quadrupole lengths:*

- a) Lower bending field results in lower $\eta_{\min}$ at slightly smaller k_q 's
- b) Larger field index in the bending magnet results in lower $\eta_{\min}$ at slightly larger k_q 's
- c) Longer quadrupole results in higher $\eta_{\min}$ at smaller k_q 's
- d) Smaller θ_2 results in strongly lower $\eta_{\min}$, while the location of the minimum is slightly decreasing.

It is interesting to notice, that with 3T bending field and no drift space the minimum achievable dispersion is $\eta_{\min} \approx -.75$, while the desired dispersion in the SXLS ring at the injection point is $\eta_d = -1.2$!

C) Dispersion vector matching

Let us now turn our attention to the conditions of dispersion vector matching. The requirement to match $\begin{pmatrix} \eta \\ \eta' \end{pmatrix}_o = 0$ before the 1-st dipole to a desired $\begin{pmatrix} \eta \\ \eta' \end{pmatrix}_d$ after the 2-nd bend, implies that $\eta = \eta_d$ and $\eta' = \eta'_d$. There are ten parameters to be chosen:

for dipole-1: $\rho_1 = (\frac{B\rho}{B_1})$, $\theta_1 = (\frac{l_1}{\rho_1})$ and $n_1 = (\frac{\rho_1 B'}{B_1})$.

for dipole-2: ρ_2 , $\theta_2 = \Theta - \theta_2$ and n_2 .

for the quadrupole and the drift spaces: l_q , k_q , L_1 and L_2 .

The eqs.(A2.2) nonlinear equations have to be solved for the strength (k_q) and location (θ_4) of the quadrupole with a given set of the remaining eight parameters. As discussed in section-5, η_d and η'_d are -1.2 and -.6, respectively.

First, we will explore the most compact geometry, when $L_1=L_2=0$. The solutions as a function of the B bending field with different l_q quadrupole lengths are shown on Fig. A2.2. One can see, that already at $B=1.5T$, k_q reaches 9.5 - 11 ($B' \approx 60 - 80$ T/m), and -depending on the lengths of the quadrupole- beyond that bending field there is no solution at all. By increasing the length of the quadrupole, one can weaken it but at the same time, not only the θ_4 angle is also decreasing, but also the limiting bending field, where solution still exist, is decreasing.

Fig. A2.2

Location (θ_4) and strengths (k_q) of the matching quadrupole is shown as a function of the B bending field, while the quadrupole length (l_q) is used as parameter.

Incorporating gradient into the bending magnets helps in greatly increasing the θ_4 angle (thus increasing the distance from the beam line) but at the same time, the necessary quadrupole strength is increasing. This effect of the gradient is shown on Figs. A2.3, where δ , k_q and θ_4 are plotted as a function of the B bending field with $n=0$ and .7 field indices for a thin lens and for a quadrupole of length $l_q = 0.15m$.

Figs. A2.3.

Location (θ_4) and strengths (k_q) of the matching quadrupole is shown as a function of the B bending field for $n = 0$ and $.7$ and $l_q = 0$ and $.15$ m. $\Theta = 142^\circ$

For higher bending fields, drift spaces have to be allowed. The results are shown on Figs.4 and 5.

On Figs.4 the effect of the lengths of the quadrupole is shown. In this case $B = 3$ T, $\Theta = 142^\circ$ and $n = 0$ were used. k_q and θ_2 are plotted as a function of the L_1 drift space with L_2 as parameter and with $l_q = .10, .15, .20$ m (Figs.2b-d). As a comparison, the thin quadrupole case is shown on Fig.2a (for thin lenses, only δ and not k_q is meaningful). One can see, that there is a minimum required drift space, for which solution exists, and the longer the quadrupole is, the more drift space is needed. Even with $l_q = .1$ m, there is no solution below $L = L_1 + L_2 \approx .2$ m. This requirement grows to $.3$ m with $l_q = .2$ m. (That, there is no solution below a certain L is a consequence of the above described behaviour of η . At $B = 3$ T and with $L = 0$, $\eta_{\min}$ is larger than the desired η_d) The role of the two drift spaces is not symmetrical, shorter L_1 's with the same total L result in lower k_q 's and lower θ_2 's. Unfortunately, as was shown in section-5, small θ_2 prevents us from using these weaker quadrupole solutions, since in that case there is not enough room to accommodate the L total drift. In general, k_q decreases with increasing L_1 and L_2 , with the latter dependence being stronger. On the other hand, θ_2 slightly increases with increasing L_1 and strongly decreases with increasing L_2 .

Figs. A2.4.

Bending angle of the 2-nd dipole (θ_2) and quadrupole strengths (k_q) when the dispersion vector is matched to the desired $\eta_d = -1.2$ and $\eta'_d = -.6$ is shown as a function of L_1 , while L_2 is used as parameter. $B=3T$, $\Theta=142^\circ$ and (a) thin quad approximation, (b) $l_q = .1$ m, (c) $l_q = .15$ m, (d) $l_q = .2$ m.

Figs. A2.5.

Bending angle of the 2-nd dipole (θ_2) and quadrupole strengths (k_q) when the dispersion vector is matched to the desired $\eta_d = -1.2$ and $\eta'_d = -.6$ is shown as a function of L_1 , while L_2 is used as parameter. $\Theta=142^\circ$ $n=.6$, and $l_q=.15$ m.

Incorporating gradient into the bending magnets slightly decreases the strengths of the quadrupole but greatly affects the location of the quadrupole; θ_2 increases significantly. This effect of the gradient is shown on Figs. A2.5.

D) Beam Matrix

Next, we turn our attention to the beam size. Assuming the absence of horizontal-vertical coupling and assuming on-momentum particles, the beam matrix consists of two 2x2 matrices, Σ_x and Σ_y . The beam size, divergence and orientation of the phase space ellipse (Twiss α) are given by the matrix elements as:

$$x = \sqrt{\Sigma_{x,11}}, \quad x' = \sqrt{\Sigma_{x,22}}, \quad \text{and} \quad \alpha = \frac{r}{\sqrt{(1-r^2)}}, \quad \text{where} \quad r = \frac{\Sigma_{x,12}}{\sqrt{\Sigma_{x,11}\Sigma_{x,22}}}$$

and similar expressions for the vertical case. The Σ matrix transforms as $\Sigma^1 = M \Sigma^0 M^T$ which yields:

$$\begin{aligned} \Sigma_{11}^1 &= m_{12}^2 \Sigma_{22}^0 + 2m_{11}m_{12}\Sigma_{12}^0 + m_{11}^2 \Sigma_{11}^0 \\ \Sigma_{12}^1 &= \Sigma_{21}^1 = m_{12}m_{22}\Sigma_{22}^0 + (m_{11}m_{22} + m_{12}m_{21})\Sigma_{12}^0 + m_{11}m_{21}\Sigma_{11}^0 \\ \Sigma_{22}^1 &= m_{22}^2 \Sigma_{22}^0 + 2m_{21}m_{22}\Sigma_{12}^0 + m_{21}^2 \Sigma_{11}^0 \end{aligned}$$

Proceeding in the reverse direction, that is starting with the desired horizontal and vertical phase space ellipses after the 2-nd bending magnet, the Σ matrix was calculated before the 1-st bending magnet using the previously obtained solutions for the strengths and location of the dispersion matching quadrupole. The calculated H and V beam sizes are plotted on Figs. A2.6 as a function of L_1 with L_2 as parameter for $\Theta=142^\circ$, $B=3\text{T}$, $l_q=.15\text{m}$ and $n_1=n_2=.6$. One can see, that only solutions with $L_2 \geq .2\text{m}$ and $L_1 \geq .1\text{m}$ yield reasonably small horizontal and vertical beam sizes at the same time.

Fig. A2.6.

Horizontal and vertical beam size as a function of L_1 , while L_2 is used parameter. $\Theta=142^\circ$, $l_q=.15\text{m}$ and $n=.6$.

E) Summary

The requirements on the value of the different parameters, imposed on them by the desirable conditions (small enough beam size, relatively weak quadrupole and availability of all beam lines) are contradictory. The different conditions impose opposite requirements on each of the parameters. This "play the opposites" is summarized in Table-A2.1 The optimal range of the parameters are also given. Strong dependence of the conditions on the parameter are indicated by bold characters.

Table-A2.1

Requirements, imposed on the value of the parameters by the desirable conditions. Bold characters are used when the dependence is strong.

conditions parameters	large space	large θ_2	small k_q	small σ 's	
n	-	$n \geq .6$	$n = 0$	$n \geq .6$	$.6 \leq n \leq .85$
l_q	short	short	long	short	$l_q = .15\text{m}$
L_1	short	long	long	long	$.1 \leq L_1 \leq .2\text{m}$
L_2	short	short	long	long	$.2 \leq L_2 \leq .3\text{m}$
B	high	low	low	-	$2T \leq B \leq 3T$