MASTER

SEMI-ANNUAL REPORT
ON
DEVELOPMENT OF IMPROVED GEOTHERMAL WELL
LOGGING COMPONENTS: THE HEAT FLUX TRANSDUCER
AND THERMAL CONDUCTIVITY PROBE

Energy Research & Development Administration
Contract E(04-3)-1318

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Preface

This program for ERDA relates to the development of two separate transducer systems which are to be used to measure geothermal heat flux and earth thermal conductivities in boreholes. One of them, the geothermal heat flux transducer, consists of two long cylindrical sections made of materials with known thermal conductivities containing thermopiles which measure the axial temperature fields. The geothermal heat flux in the earth at some distance from the transducers can be extracted from the temperature measurements with the aid of mathematical heat flow models for the specific geometry involved. The second system, the thermal conductivity probe, consists of a long cylindrical section containing a surface heater and a surface temperature sensor. The time-temperature history of the surface temperature is recorded as a function of time. From this record, the constant heating rate and the system geometry, the thermal conductivity of the surrounding earth can be extracted using transient heat conduction functions. It is believed that quantitative information on borehole heat flux will be of significant value in making assessments of geothermal reserves during the exploration period.

A list of the major tasks being undertaken to complete the eighteen-month development effort for each transducer is as follows:

1. Mathematical sensor system modeling
2. Preliminary sensor design
3. Test sensor fabrications
4. Laboratory and shallow hole sensor tests and evaluations
5. Deep drillhole site survey and final selection

6. Deep drillhole sensor fabrication
7. Deep drillhole measurements
8. Reduction and presentation of data

During the first six months of this program, tasks 1 and 2 have been completed for both transducer systems; part of task 3 has been completed for both systems; part of task 4 has been completed for the thermal conductivity probe and part of task 5 has been completed for both systems.

The technical monitors for this work are Mr. Lawrence Ball from the Division of Geothermal Energy of ERDA, Washington, and Mr. Michael Lamers of Measurement Analysis Corporation.

Technical interim reports were prepared on the development work conducted during the first six months of this program. These documents are presented in this report under the task headings noted above.

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X Task 1. Mathematical Sensor System Modeling

X Task 2. Preliminary Sensor Design

GLR-178, "A Thin Rod Heat Flux Transducer Positioned in the Earth Having a Uniform Temperature Gradient: A Closed Form Solution," H. F. Poppendiek, D. J. Connelly, Feb 1977.

+ GLR-179, "A Thin Rod Heat Flux Transducer Positioned in the Earth Having a Uniform Temperature Gradient: A Numerical Two-Component Laplace Solution," H. F. Poppendiek, A. J. Sellers, Feb 1977.

X GLR-182, "A SINDA Program for Steady State Network Solutions Applicable to Geothermal Heat Flow," A. J. Sellers, Mar 1977.

X GLR-185, "Cylindrical and Spherical Transient Heat Transfer Solutions and Their Application to Thermal Conductivity Measurements in the Earth," H. F. Poppendiek, D. J. Connelly, Feb 1977.

X Task 3. Test Sensor Fabrication

X Task 4. Laboratory and Shallow Hole Sensor Tests and Evaluations

GLR-187, "Preliminary Thermal Conductivity and Thermal Capacity Measurements in the Laboratory," H. F. Poppendiek, D. J. Connelly, Mar 1977.

SUMMARY

Task 1. Mathematical Sensor System Modeling

Task 2. Preliminary Sensor Design

A THIN ROD HEAT FLUX TRANSDUCER POSITIONED
IN THE EARTH HAVING A UNIFORM TEMPERATURE
GRADIENT: A CLOSED FORM SOLUTION

For
ERDA
Contract E(04-3)-1318

H. F. Poppendiek
D. J. Connelly

February 1977

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INTRODUCTION

One of the geothermal sensors that is being developed by Geoscience for ERDA in connection with the measurement of the geothermal heat flux is a thin rod heat flux transducer. It is to be lowered in a drillhole where signals from this sensor will be measured and translated into a geothermal heat flux. A mathematical temperature solution has been developed for an idealized thin rod positioned in an infinite solid with a uniform temperature gradient. This solution approximates the actual temperature to be encountered in the transducer when located in a drill hole.

The solution is a closed form type for a thin rod wherein radial temperature differences are small compared to axial ones; the system can be classified as a fin with a variable environmental temperature gradient. The resulting temperature field is evaluated for a range of system parameters that are of interest in geothermal heat flux measurement.

A description of how the mathematical results obtained will be used to extract the unknown geothermal heat flux and earth thermal conductivity from the experimental temperature information obtained by the heat flux transducer is also presented. In addition, the solution is extended to the case where there is a water or air annular space between the transducer and the drill-hole.

DERIVATION

The thermal analysis presented below is based on a thin rod, positioned in an infinite solid which is characterized by a uniform temperature gradient (see Figure 1). The problem is to determine the temperature field in the thin rod. A heat balance on a differential element of the thin rod which is transferring heat to or from the surrounding infinite solid through a radial thermal resistance is given by the classical fin equation,

$$\frac{d^2 t}{dz^2} = \frac{P}{R_{\infty} k A} (t - t_{\infty}) \quad (1)$$

where

t , rod temperature (above the rod midpoint temperature datum)

z , distance along rod

P , perimeter of the rod

A , cross sectional area of the rod

k , thermal conductivity of the rod

R_{∞} , equivalent radial thermal resistance of the solid surrounding the rod

t_{∞} , the linear lateral temperature variation (above the rod midpoint temperature datum) in the solid at a radial distance sufficiently great so that the presence of the rod does not effect it.

Figure 1. Idealized thin rod transducer system

The temperature variation in the solid is given by,

$$t_{\infty} = bz \quad (2)$$

where the parameter, b , is the undisturbed vertical temperature gradient of the earth.

Thus, Equation 1 can be expressed as

$$\left(\frac{d^2}{dz^2} - C \right) t = -Bz \quad (3)$$

where

$$C = \frac{P}{R_{\infty} kA}$$

$$B = \frac{Pb}{R_{\infty} kA}$$

The complimentary solution of Equation (3), t_c , is

$$t_c = c_1 e^{-\sqrt{C}z} + c_2 e^{\sqrt{C}z} \quad (4)$$

The particular solution of Equation (3), t_p , can be obtained using the method of undetermined coefficients, namely, let

$$t_p = c_3 z^2 + c_4 z + c_5 \quad (5)$$

Substituting Equation (5) into (3) yields,

$$\left(\frac{d^2}{dz^2} - C \right) (c_3 z^2 + c_4 z + c_5) = -Bz$$

or

$$2c_3 - Cc_3 z^2 - Cc_4 z - Cc_5 = -Bz \quad (6)$$

Thus, the coefficients in Equation (5) became,

$$\left. \begin{aligned} c_3 &= 0 \\ c_4 &= \frac{B}{C} \\ c_5 &= 0 \end{aligned} \right\} \quad (7)$$

Therefore, the complete solution of Equation (3) is the sum of (4) and (5),

$$t = c_1 e^{-\sqrt{C}z} + c_2 e^{\sqrt{C}z} + \frac{B}{C} z \quad (8)$$

One boundary condition for this problem is $t = 0$ at $z = 0$. Thus,

$$c_2 = -c_1 \quad (9)$$

$$\text{and } t = c_1 \left(e^{-\sqrt{C}z} - e^{\sqrt{C}z} \right) + \frac{B}{C} z \quad (10)$$

The second boundary condition for this problem defines the heat loss from the end of the rod (at $z = l$), namely,

$$-k \left(\frac{dt}{dz} \right)_l A = \frac{1}{R_e} (t_l - b_l) A \quad (11)$$

where R_e is the equivalent end thermal resistance of the solid surrounding the rod. Upon substituting Equation (10) into (11), there results

$$-k \left[c_1 \left(-\sqrt{C} e^{-\sqrt{C}l} - \sqrt{C} e^{\sqrt{C}l} \right) + \frac{B}{C} \right] = \frac{1}{R_e} \left[c_1 \left(e^{-\sqrt{C}l} - e^{\sqrt{C}l} \right) + \frac{B}{C} l - b_l \right]$$

or

$$c_1 = \frac{-b}{-\sqrt{C} \left(e^{-\sqrt{C}l} + e^{\sqrt{C}l} \right) + \frac{1}{kR_e} \left(e^{-\sqrt{C}l} - e^{\sqrt{C}l} \right)} \quad (12)$$

It is also necessary to define the cylindrical and hemispherical thermal resistances of the solid surrounding the transducer, namely, R_∞ and R_e , respectively. The resistance for the annulus surrounding the rod is,

$$R_\infty = \frac{r_i}{K_\infty} \ln \frac{r_o}{r_i} \quad (13)$$

where

r_o , radius at which the undisturbed linear temperature field in the infinite solid exists

r_i , radius of the rod

k_∞ , thermal conductivity of the infinite solid

The thermal resistance of the hemispherical shell at the end of the rod is,

$$R_e = \frac{r_i^2 \left(\frac{1}{r_i} - \frac{1}{r_o} \right)}{k_\infty} \quad (14)$$

Thus, the complete temperature solution for the thin rod transducer is given by Equations (10), (12), (13) and (14).

The solution developed above has also been extended to a composite thin rod which consists of a sensor rod surrounded by an annulus of water or air (the space between the sensor and borehole). Again, it is postulated that radial temperature differences in the rod and annulus are small and negligible compared to axial values. From elementary parallel flow conduction concepts, it can be shown that the composite (or equivalent) thermal conductivity of a rod with an annulus, k_c , is

$$k_c = k_r \nu_r + k_a \nu_a \quad (15)$$

where

k_r , rod thermal conductivity

k_a , annulus thermal conductivity

ν_r , rod volume fraction of the composite rod

ν_a , annulus volume fraction of the composite rod

PARAMETRIC RESULTS

To illustrate the variation of the temperature field in a rod heat flux transducer positioned vertically in the earth, Equations (10), (12), (13) and (14) were evaluated for the following range of parameters:

Geothermal heat flux ($k_{\infty}b$) = 2×10^{-6} cal/sec cm² = 2 HFU

$2l$ = 6 feet (1.83 meters)

d = 4 inches (0.102 meters)

k range: 0.05 to 100 Btu/hr ft°F (0.0002 to 0.40 cal/sec cm°C)

k_{∞} range: 0.1 to 1.0 Btu/hr ft°F (0.0004 to 0.004 cal/sec cm°C)

r_0 range: 1 to 9 feet (0.305 to 2.743 meters)

The results of these evaluations can be seen in Figures 2 to 13.

Legend Information For

Figure 2 - 13

Curve	$k_{\text{rod}} \left(\frac{\text{Btu}}{\text{hr ft}^2 \text{ } ^\circ\text{F/ft}} \right)$	$k_{\text{rod}} \left(\frac{\text{cal}}{\text{sec cm}^2 \text{ } ^\circ\text{C/cm}} \right)$
A	0.05	0.0002
B	0.1	0.0004
C	0.5	0.002
D	1.0	0.004
E	5	0.02
F	10	0.04
G	50	0.2
H	100	0.4

Figure 2. Thin rod temperature profiles for $K_{\infty} = 0.1 \text{ Btu/hr ft}^2 \text{ } ^\circ\text{F/ft}$ and $r_o = 1 \text{ ft}$

Figure 3. Thin rod temperature profiles for $K_{\infty} = 0.3 \text{ Btu/hr ft}^2 \text{ } ^\circ\text{F/ft}$ and $r_o = 1 \text{ ft}$

Figure 4. Thin rod temperature profiles for $K_{\infty} = 1.0 \text{ Btu/hr ft}^2 \text{ } ^\circ\text{F/ft}$ and $r_0 = 1 \text{ ft}$

Figure 5. Thin rod temperature profiles for $K_{\infty} = 0.1 \text{ Btu/hr ft}^2 \text{ } ^\circ\text{F/ft}$ and $r_o = 3 \text{ ft}$

Figure 6. Thin rod temperature profiles for $K_{\infty} = 0.3\text{ Btu/hr ft}^2\text{ } ^\circ\text{F/ft}$ and $r_o = 3\text{ ft}$

Figure 7. Thin rod temperature profiles for $K_{\infty} = 1.0 \text{ Btu/hr ft}^2 \text{ } ^\circ\text{F/ft}$ and $r_o = 3 \text{ ft}$

Figure 8. Thin rod temperature profiles for $K_{\infty} = 0.1 \text{ Btu/hr ft}^2 \text{ } ^\circ\text{F/ft}$ and $r_o = 6 \text{ ft}$.

Figure 9. Thin rod temperature profiles for $K_{\infty} = 0.3 \text{ Btu/hr ft}^2 \text{ } ^\circ\text{F/ft}$ and $r_o = 6 \text{ ft}$

Figure 10. Thin rod temperature profiles for $K_{\infty} = 1.0 \text{ Btu/hr ft}^2 \text{ } ^\circ\text{F/ft}$ and $r_o = 6 \text{ ft}$

Figure 11. Thin rod temperature profiles for $K_{\infty} = 0.1 \text{ Btu/hr ft}^2 \text{ } ^\circ\text{F/ft}$ and $r_o = 9 \text{ ft}$

Figure 12. Thin rod temperature profiles for $K_{\infty} = 0.3 \text{ Btu/hr ft}^2 \text{ } ^\circ\text{F/ft}$ and $r_o = 9 \text{ ft}$

Figure 13. Thin rod temperature profiles for $K_{\infty} = 1.0 \text{ Btu/hr ft}^2 \text{ } ^\circ\text{F/ft}$ and $r_o = 9 \text{ ft}$

DISCUSSION

A. Application of Solution to Geothermal Heat Flux Measurement

The thin rod solution derived above can be used to relate rod transducer temperature measurements and the sought for geothermal heat flux in the earth where the transducer has been positioned. Specifically, the boundary value problem described above contains two unknowns, namely, the thermal conductivity of the earth, k_{∞} , and the vertical earth temperature gradient, b (the product of these two quantities being the geothermal heat flux). There are two ways in which the two unknowns can be evaluated from the prediction equations. Either two temperature points along a single transducer of known conductivity are used in the evaluation, or one temperature point each for two transducers of different but known thermal conductivity are required.

The results shown in Figures 2 to 13 have been translated into thermopile voltage outputs for $k_{\infty} = 0.3$ Btu/hr ft²°F (0.0012 cal/sec cm²°C/cm) and 1.0 Btu/hr ft²°F (0.004 cal/sec cm²°C/cm) for a range of practical junction sets, rod thermal conductivities and typical geothermal heat flux values. The resulting voltages fall in the 0.5 to 1.0 millivolt range and are large enough to be measured by potentiometers without amplification. Details of the thermopile design will appear in a subsequent report.

B. Comments on Model Accuracy

The thermal resistance R_{∞} is an important parameter in the rod solution. The variation of this quantity with r_0 for a given k_{∞} and r_i is shown in Figure 14, together with the corresponding R_e variation. Note that although R_e quickly becomes invariant with r_0 , the resistance R_{∞} continues to increase with r_0 but very slowly at larger r_0 values. A calculation shows, for example, that for $k = 10$ Btu/hr ft°F, the temperature at the end of the rod changes from 0.320°F for $r_0 = 9$ ft. to 0.297°F for $r_0 = 20$ ft. (a seven percent difference).

The limitations of this model will be further discussed in a forthcoming report where the complete heat transfer system is represented by a two-region Laplace equation set which is solved numerically.

Figure 14. R_{∞} and R_e versus r_0

**A THIN ROD HEAT FLUX TRANSDUCER
POSITIONED IN THE EARTH
HAVING A UNIFORM TEMPERATURE GRADIENT: A
NUMERICAL TWO-COMPONENT LAPLACE SOLUTION**

**Energy Research & Development Administration
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February 1977

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INTRODUCTION

This report is the second one which describes the thermal performance of a thin rod heat flux sensor being developed by Geoscience for ERDA. Specifically, a finite difference equation set has been derived which describes the two dimensional heat transfer in the transducer and the surrounding earth. Numerical evaluations of the two-component system are presented. In addition, the results are compared to those previously obtained by the simpler closed-form model.

FINITE DIFFERENCE EQUATION SET

A. The System Geometry and External Boundary Conditions

The transducer system consists of a vertical rod that is positioned in the earth that is bounded by a vertical, linear temperature field. The geometry involved together with the external boundary conditions are shown in Figure 1.

B. Differential Equations

The Laplace differential equation in polar coordinates with angular symmetry is,

$$\frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \frac{\partial t}{\partial r} + \frac{\partial^2 t}{\partial z^2} = 0 \quad (1)$$

The coordinates and the temperature datum are shown in Figure 1. By classical, numerical representations, Equation (1) can be expressed in the following finite difference form,

$$t_{n,m} = \frac{1}{4} (t_{n+1,m} + t_{n-1,m} + t_{n,m+1} + t_{n,m-1}) + \frac{\Delta r}{8r} (t_{n+1,m} - t_{n-1,m}) \quad (2)$$

where n is the increment number in the r direction and m is the increment number in the z direction. This finite difference equation applies to both the transducer and earth regions.

Figure 1. System geometry and external boundary conditions.

C. Internal Boundary Equations

1. Boundary $r = r_0$

A heat rate balance on this annular boundary is (see Figure 2),

$$q_1 + q_2 + q_3 + q_4 = 0$$

or

$$\begin{aligned} (k_a C_{1a} + k_b C_{1b})(t_1 - t_0) + k_b C_2(t_2 - t_0) + (k_a C_{3a} + k_b C_{3b})(t_3 - t_0) \\ + k_a C_4(t_4 - t_0) = 0 \end{aligned} \quad (3)$$

where

$$C_{1b} = C_{3b} = \frac{\left(\pi \left(r_0 + \frac{\Delta r}{2}\right)^2 - \pi r_0^2\right)}{\Delta z}$$

$$C_{1a} = C_{3a} = \frac{\left(\pi r_0^2 - \pi \left(r_0 - \frac{\Delta r}{2}\right)^2\right)}{\Delta z}$$

$$C_2 = \frac{2\pi\Delta z}{\ln \frac{r_0 + \Delta r}{r_0}}$$

Figure 2. Heat rate balance along internal boundary $r = r_0$.

and
$$C_4 = \frac{2\pi\Delta z}{\ln \frac{r_0}{r_0 - \Delta r}}$$

Thus, from Equation (3) the central temperature, t_0 , can be expressed in terms of the surrounding temperatures as,

$$t_0 = \frac{(k_a C_{3a} + k_b C_{3b})(t_1 + t_3) + k_b C_2 t_2 + k_a C_4 t_4}{k_b C_2 + k_a C_4 + 2(k_a C_{3a} + k_b C_{3b})} \quad (4)$$

2. Corner $r = r_0$, $z = l$

A heat rate balance on the corner $r = r_0$, $z = l$ is performed by again adding the four heat flow terms and setting them equal to zero. The resulting corner temperature, t_0 , in terms of the four surrounding temperatures is

$$t_0 = \frac{(k_a C_{3a} + k_b C_{3b}) t_1 + k_b C_2 t_2 + k_b f_3 t_3 + (k_a + k_b) \frac{C_4}{2} t_4}{(k_a + k_b) \frac{C_4}{2} + k_b t_3 + k_b C_2 + k_a C_{3a} + k_b C_{3b}} \quad (5)$$

where
$$f_3 = \frac{\pi(r_0 + \frac{\Delta r}{2})^2 - \pi(r_0 - \frac{\Delta r}{2})^2}{\Delta z}$$

3. Boundary $z = l$

A heat rate balance on the plane $z = l$ leads to the remaining finite difference internal boundary equation,

$$t_0 = \frac{k_{a3} f t_1 + (k_a + k_b) \frac{C_2}{2} t_2 + k_{b3} f t_3}{k_{b3} f + (k_a + k_b) \frac{C_2}{2} + k_{a3} f} \quad (6)$$

TYPICAL RESULTS

The finite difference equation set presented here was treated by the SINDA computer program.* Two illustrative network geometries were studied for an arbitrary earth thermal conductivity and several rod thermal conductivities. The geometries considered are shown in Figure 3.

Some results for Network 2 are shown in Figures 4 to 8. Figures 4, 5, and 6 show normalized isotherms in the rod sensor and earth regions for three different rod thermal conductivities and a constant earth thermal conductivity. The last of these three figures presents the results for the special case where the rod and earth conductivities are equal; note the horizontal isotherms which are a check on the solution process. Figure 7 shows normalized axial temperature profiles along the transducer for three different rod conductivities. Figure 8 shows radial temperature profiles at $z = l/2$ for the three different rod thermal conductivities and Figure 9 presents these profiles for $z = l$.

The results obtained for Network 3 are presented in Figures 10 to 15.

* Details of the computer work are presented in "A SINDA Program for Steady State Network Solutions Applicable to Geothermal Heat Flow," GLR-182, by A. J. Sellers.

NETWORK NO. 2

$$\begin{aligned}
 b &= 0.030 = 6 \Delta r \\
 r_o &= 0.010 = 2 \Delta r \\
 h &= 0.200 = 40 \Delta Z \\
 l &= 0.100 = 20 \Delta Z \\
 \Delta r &= \Delta Z = 0.005 \\
 \text{TOTAL NODE COUNT} &= 275
 \end{aligned}$$

NETWORK NO. 3

$$\begin{aligned}
 b &= 0.060 = 12 \Delta r \\
 r_o &= 0.010 = 2 \Delta r \\
 h &= 0.200 = 40 \Delta Z \\
 l &= 0.150 = 30 \Delta Z \\
 \Delta r &= \Delta Z = 0.005 \\
 \text{TOTAL NODE COUNT} &= 509
 \end{aligned}$$

Figure 3. Geometries for two illustrative tranducer-earth networks.

Figure 4. Normalized isotherms through transducer and earth for Network No. 2 ($k_a = 20 \text{ Btu/hr ft } ^\circ\text{F}$).

Figure 5. Normalized isotherms through transducer and earth for Network No. 2 ($k_a = 10 \text{ Btu/hr ft } ^\circ\text{F}$).

Figure 6. Normalized isotherms through transducer and earth for Network No. 2 ($k_a = 0.3 \text{ Btu/hr ft } ^\circ\text{F}$).

Figure 7. Normalized axial temperature profiles along the rod transducer at $r = r_o/2$ for three different rod thermal conductivities and $k_b = 0.3 \text{ Btu/hr ft } ^\circ\text{F}$.

Figure 8. Radial normalized temperature profiles at $z = l/2$ for three different rod thermal conductivities (Network No. 2).

Figure 9. Radial normalized temperature profiles at $z = l$ for three different rod thermal conductivities (Network No. 2).

Figure 10. Normalized isotherms through transducer and earth for Network No. 3 ($k_a = 20 \text{ Btu/hr ft } ^\circ\text{F}$).

Figure 11. Normalized isotherms through transducer and earth for Network No. 3 ($k_a = 10 \text{ Btu/hr ft } ^\circ\text{F}$).

Figure 12. Normalized isotherms through transducer and earth for Network No. 3 ($k_a = 0.3 \text{ Btu/hr ft } ^\circ\text{F}$).

Figure 13. Normalized axial temperature profiles along the rod transducer at $r = r_o/2$ for three different thermal conductivities and $k_b = 0.3$ Btu/hr ft °F (Network No. 3).

Figure 14. Radial normalized temperature profiles at $z = l/2$ for three different rod thermal conductivities ($k_a = 0.3 \text{ Btu/hr ft } ^\circ\text{F}$) (Network No. 3).

Figure 15. Radial normalized temperature profiles at $z = l$ for three different rod thermal conductivities ($k_a = 0.3 \text{ Btu/hr ft } ^\circ\text{F}$) (Network No. 3).

DISCUSSION

Comparisons were made between closed form solution^{*} and numerical solution predictions of the axial temperature profiles along a typical thin rod heat flux transducer. The results are shown in Figures 16, 17, and 18 for the geometrical conditions given for Network No. 2. Note that the temperature distributions are in close agreement for rod conductivities 0.1 Btu/hr ft °F and 10 Btu/hr ft °F; the two profiles for the rod conductivity of 20 Btu/hr ft °F differ by about 16 percent but the agreement is still reasonable.

It is felt that on the basis of the analytical work completed for the rod heat flux transducer, it will be possible to extract the geothermal heat flux from transducer temperature measurements made in the earth. Thus, preliminary designs have been prepared for such sensors to be used for geothermal well logging. One type consists of long, solid hemicylinders which are bolted together after axial thermopiles are inserted; high thermal conductivity candidate materials are aluminum, steel, and iron, and low conductivity candidates are Teflon, Bakelite, and stable elastomers. The second type of heat flux sensor (for low conductivity materials) consists of a thin wall stainless steel tube that is capped at its ends; an axial thermopile is affixed to its inner surface prior to filling with a high temperature foam. All sensors have thermopile output connector plugs at the upper ends so that they can be disconnected from the lowering and thermopile cables.

* "A Thin Rod Heat Flux Transducer Positioned in the Earth Having a Uniform Temperature Gradient: A Closed Form Solution," GLR-178, by H. F. Poppendiek and D. J. Connelly, February 1977.

Figure 16. Comparison of finite difference and closed form temperature profiles along rod ($k_a = 0.1$ Btu/hr ft $^{\circ}$ F and $k_b = 0.3$ Btu/hr ft $^{\circ}$ F).

Figure 17. Comparison of finite difference and closed form temperature profiles along rod ($k_a = 10$ Btu/hr ft $^{\circ}$ F and $k_b = 0.3$ Btu/hr ft $^{\circ}$ F).

Figure 18. Comparison of finite difference and closed form temperature profiles along rod ($k_a = 20$ Btu/hr ft °F and $k_b = 0.3$ Btu/hr ft °F).

**A SINDA PROGRAM FOR STEADY STATE
NETWORK SOLUTIONS APPLICABLE TO
GEOTHERMAL HEAT FLOW**

**Energy Research & Development Administration
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A. J. Sellers

March 1977

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INTRODUCTION

The purpose of this SINDA program is to determine the steady state temperature distribution in a cylindrical volume knowing prescribed external thermal boundary conditions. This volume consists of a rod heat flux sensor having a thermal conductivity, k_1 , surrounded by a cylinder of earth having a thermal conductivity, k_2 , as indicated in Figure 1. The cylindrical surface temperature varies linearly between the top and bottom temperatures.

NETWORK SOLUTION CONSIDERATIONS

The finite difference equation set (differential and internal boundary equations) for this two dimensional steady state heat conduction system was previously developed.⁽¹⁾ The solution of this problem involves the utilization of a grid network and the selection of an appropriate iteration technique to yield the steady state nodal temperature distribution. Because of the angular symmetry of the external boundary temperature conditions, the nodal network can be represented as a two dimensional network as depicted in Figure 1. It shows the two materials and their interface B-D-C which represents the internal boundary node locations. The centerline radial temperature gradient along line A-C-E is zero and the temperature distribution at the external boundary of the cylinder, F-G, is linear. As indicated previously, the upper and lower cylinder surface temperatures are uniform.

Network No. 2

$$R = 0.030 = 6\Delta R$$

$$R_1 = 0.010 = 2\Delta R$$

$$Z = 0.200 = 40\Delta Z$$

$$Z_1 = 0.100 = 20\Delta Z$$

$$\Delta R = \Delta Z = 0.005$$

Total Node Count = 275

Network No. 3

$$R = 0.060 = 12\Delta R$$

$$R_1 = 0.010 = 2\Delta R$$

$$Z = 0.200 = 40\Delta Z$$

$$Z_1 = 0.150 = 30\Delta Z$$

$$\Delta R = \Delta Z = 0.005$$

Total Node Count = 509

Figure 1. Thermal network data

To comply with the prescribed finite difference form of the Laplace partial differential equation, a grid network was constructed using equal radial and axial node distance increments, $\Delta r = \Delta z$. For the nodes within the rod and earth zones, the Laplace equation in the finite difference form is:

$$t_{n,m} = \frac{t_{n,m+1} + t_{n+1,m} + t_{n,m-1} + t_{n-1,m}}{4} + \frac{\Delta r}{8r} (t_{n+1,m} - t_{n-1,m}) \quad (1)$$

where

r - radial node coordinate

n } node identification subscripts, shown
 m } in Figure 2

For the nodes located on the cylindrical internal boundary surface B-D, the finite difference equation is:

$$t_{n,m} = \frac{\left((R_1 R_3 + R_2 R_4) (t_{n,m+1} + t_{n,m-1}) + R_2 R_5 t_{n+1,m} + R_1 R_6 t_{n-1,m} \right)}{\left(2 (R_1 R_3 + R_2 R_4) + R_2 R_5 + R_1 R_6 \right)} \quad (2)$$

Figure 2. Node coordinate and subscript representation in Equation 1 through 5

where

R_1 = thermal conductivity of the rod

R_2 = thermal conductivity of the earth

$$R_3 = \pi(r_{n,m}^2 - (r_{n,m} - \Delta r/2)^2) / \Delta z$$

$$R_4 = \pi((r_{n,m} + \Delta r/2)^2 - r_{n,m}^2) / \Delta z$$

$$R_5 = 2\pi\Delta z / \ln((r_{n,m} + \Delta r) / r_{n,m})$$

$$R_6 = 2\pi\Delta z / \ln(r_{n,m} / (r_{n,m} - \Delta r))$$

For the internal boundary node located at point D:

$$t_{n,m} = \frac{\left((R_1 R_3 + R_2 R_4) t_{n,m+1} + R_2 R_5 t_{n+1,m} + R_2 R_7 t_{n,m-1} + \left((R_1 + R_2) R_6 / 2 \right) t_{n-1,m} \right)}{(R_1 R_3 + R_2 R_4 + R_2 R_5 + R_2 R_7 + (R_1 + R_2) R_6 / 2)} \quad (3)$$

where

$$R_7 = \pi((r_{n,m} + \Delta r/2)^2 - (r_{n,m} - \Delta r/2)^2) / \Delta z$$

For the internal boundary node located at midpoint between C and D:

$$t_{n,m} = \frac{(R_1 R_{10} t_{n,m+1} + (R_1 + R_2) R_8 t_{n+1,m} + R_2 R_{10} t_{n,m-1})}{(R_1 R_{10} + (R_1 + R_2) R_8 + R_2 R_{10})} \quad (4)$$

where

$$R_8 = \pi \Delta z / \ln ((r_{n,m} + \Delta r) / r_{n,m})$$

$$R_{10} = \pi ((r_{n,m} + \Delta r/2)^2 - (r_{n,m} - \Delta r/2)^2) / \Delta z$$

THERMAL NETWORK SPECIFICATIONS

The thermal network specifications are depicted in Figure 1. It shows the geometry, dimensions and external boundary temperature conditions for two similar systems denoted by Network No. 2 and Network No. 3. Both networks are specified with identical grid size for nodal temperature representation. In both networks the cylindrical boundary surface temperature at radius R is linear, increasing in the z direction from $t_1 = 0$ to $t_2 = 100$ (which is a normalized temperature field).

SINDA SYSTEM DESCRIPTION

SINDA, the Systems Improved Numerical Differencing Analyzer, is the product of an intensive analytical, engineering and programming effort that surveyed numerous thermal analyzer-type programs and studied them in depth. It is an operating software system which possesses the capabilities which make it well suited for solving lumped parameter representations of physical problems governed by diffusion type equations. The SINDA system consists of two main parts: (1) the preprocessor, and (2) the library. The SINDA preprocessor is a program which accepts problems written in the SINDA language and converts them to the FORTRAN language. The SINDA library consists of many pre-written FORTRAN subroutines which perform a large variety of commonly needed actions that reduce the programming effort required to solve a given problem. These routines are fully compatible with the FORTRAN routines produced by the preprocessor from the user's input.

Detailed information on variety of different SINDA programming aspects is provided in Reference 2. Similarly, each of the numerical solution routines is detailed from a theoretical as well as from a computational view point in Reference 3. This reference also provides SINDA nomenclature and mnemonics as well as a review of lumped parameter equations and the basic numerical solutions.

The SINDA computer program can be tailored to suit the network solution conditions and specifications provided in the previous sections. The temperature points in the grid network can be treated as node centers and, except for the external and internal boundary node strings indicated in Figure 1, the nodes can be interconnected by the conductors of unit value. The assignment of these conductor values is based on the definition of Equation (1), which suggests a Cartesian nodal network with equal conductors supplemented by a variable impressed heat load term,

$$q_{n,m} = \frac{R_g}{r} (t_{n+1,m} - t_{n-1,m}) \quad (5)$$

where

$$R_g = \Delta r/8$$

Using the above interpretation of the Laplace finite difference equation form in cylindrical coordinates with angular symmetry, the SINDA program was written, compiled and executed for networks No. 2 and 3. A brief description of this program is provided in the following section.

SINDA PROGRAM LISTING

The listing of the SINDA program is depicted in Appendix A. It contains the input data representation and the execution program for the solution of steady state thermal network No. 3. The node identification of this network is shown in Figure 3. The external boundary temperature conditions are identified by T500 through T540. The internal boundary conditions are represented by T1 through T39, T69, and T79 through T108. The internal temperature conditions of rod are denoted by T40 through T68. Similarly, the temperature conditions of the earth are denoted by T70 through T78, and T109 through T468. Based on above nodal network representation, the SINDA program was written, compiled and executed.

CINDSL is the steady state network solution routine used in this program. Its theoretical background, computational procedure, control constant requirements and functional flow chart are provided in Appendix B. The listing of the CINDSL is in Appendix C.

Figure 3. Network No. 3

LEGEND:

○---○ $K_1 = 10.0$
 x---x $K_1 = 20.0$

SINDA PROGRAM OUTPUTS

The steady state temperature conditions were determined for both networks indicated in Figure 1. The temperature outputs are presented in Appendix D in column format. Each page is identified by a heading showing network number, case number and symbol (x/y), where x and y denote the thermal conductivities k_1 and k_2 for the rod and earth, respectively. Both networks were solved for the thermal conductivity values of the materials as shown in Table I.

Table I.

Case No.	Thermal Conductivity, Btu/hr ft °F	
	k_1	k_2
0	0.3	0.3
1	0.1	0.3
5	10.0	0.3
12	20.0	0.3

REFERENCES

1. Poppendiek, H. F.; Sellers, A. J.; "A Thin Rod Heat Flux Transducer Positioned in the Earth Having a Uniform Temperature Gradient: A Numerical Two-Component Laplace Solution," Geoscience Ltd Report GLR-179, ERDA Contract E(04-3)-1318, February 1977.
2. Smith, J. P., "SINDA User's Manual," 14690-H001-R0-00, June 1971, TRW Systems Group.
3. Ishimoto, T.; Fink, L. C.; Systems Improved Numerical Differencing Analyzer Engineering Program Manual, TRW Systems, 14690-H002-R0-00, April 1971.

APPENDIX

Appendix A - SINDA Computer Program Listing

Appendix B - Theoretical Background and Functional Description of CINDSL

Appendix C - Computer Listing of CINDSL

Appendix D - Steady State Temperature Printouts for Network No. 2 and 3.

DATA 74 02/17-04:02:42

```

1.  SAS, A SINJA=SINJAO=PREP.
2.  OLUP, K SINJA=LIO=DEVL.PRNTHA
3.  OXUT, C SINJA=SINJAO=PREP.SINJAB/NEW
4.
5.  M      OCU SIMERHAL LPCS
6.  OCU 9 GEUSCENCE LTD, NETWORK NO.3, CASE 0, (.3/.3)
7.  END
8.  OCU 3.4JOE DATA
9.  GEN 40,29,1,50.0,1.0
10. GEN 70,4,1,50.0,1.0
11. GEN 104,360,1,50.0,1.0
12. GEN -1,39,1,50.0,1.0
13. -09,50.0,1.0
14. GEN -79,30,1,50.0,1.0
15. -500,0.0,1.0
16. -501,2.5,1.0,-502,5.0,1.0,-503,7.5,1.0,-504,10.0,1.0
17. -505,12.5,1.0,-506,15.0,1.0,-507,17.5,1.0,-508,20.0,1.0
18. -509,22.5,1.0,-510,25.0,1.0,-511,27.5,1.0,-512,30.0,1.0
19. -513,32.5,1.0,-514,35.0,1.0,-515,37.5,1.0,-516,40.0,1.0
20. -517,42.5,1.0,-518,45.0,1.0,-519,47.5,1.0,-520,50.0,1.0
21. -521,52.5,1.0,-522,55.0,1.0,-523,57.5,1.0,-524,60.0,1.0
22. -525,62.5,1.0,-526,65.0,1.0,-527,67.5,1.0,-528,70.0,1.0
23. -529,72.5,1.0,-530,75.0,1.0,-531,77.5,1.0,-532,80.0,1.0
24. -533,82.5,1.0,-534,85.0,1.0,-535,87.5,1.0,-536,90.0,1.0
25. -537,92.5,1.0,-538,95.0,1.0,-539,97.5,1.0,-540,100.0,1.0
26. END
27. OCU SCUNDUCTION DATA
28. 1,500,40,1.0      SAXIAL, R=.005, Z=.0 TO Z=.20
29. GEN 2,38,1,40,1,41,1,1.0
30. 40,78,540,1.0
31. GEN 41,9,1,108,1,109,1,1.0      SAXIAL, R=.010, Z=.15 TO Z=.20
32. 50,117,540,1.0
33. 51,500,118,1.0      SAXIAL, R=.015, Z=.00 TO Z=.20
34. GEN 52,38,1,118,1,119,1,1.0
35. 90,158,540,1.0
36. 91,500,157,1.0      SAXIAL, R=.020
37. GEN 92,38,1,157,1,158,1,1.0
38. 130,195,540,1.0
39. 131,500,196,1.0      SAXIAL, R=.025
40. GEN 132,38,1,196,1,197,1,1.0
41. 170,234,540,1.0
42. 171,500,235,1.0      SAXIAL, R=.030
43. GEN 172,38,1,235,1,236,1,1.0
44. 210,273,540,1.0
45. 211,500,274,1.0      SAXIAL, R=.035
46. GEN 212,38,1,274,1,275,1,1.0
47. 250,312,540,1.0
48. 251,500,313,1.0      SAXIAL, R=.040
49. GEN 252,38,1,313,1,314,1,1.0
50. 290,351,540,1.0
51. 291,500,352,1.0      SAXIAL, R=.045
52. GEN 292,38,1,352,1,353,1,1.0
53. 330,390,540,1.0
54. 331,500,391,1.0      SAXIAL, R=.050
55. GEN 332,38,1,391,1,392,1,1.0

```

56.		370,429,540,1.0		
57.		371,500,430,1.0		SAXIAL, R=.055
58.	GEN	372,38,1,430,1,431,1,1.0		
59.		410,408,540,1.0		
60.	GEN	411,11,1,1,39,40,39,1.0		SRADIAL, Z=.005
61.		422,430,501,1.0		
62.	GEN	423,11,1,2,39,41,39,1.0		Z=.010
63.		434,431,502,1.0		
64.	GEN	435,11,1,3,39,42,39,1.0		Z=.015
65.		440,432,503,1.0		
66.	GEN	441,11,1,4,39,43,39,1.0		Z=.020
67.		450,433,504,1.0		
68.	GEN	451,11,1,5,39,44,39,1.0		Z=.025
69.		470,434,505,1.0		
70.	GEN	471,11,1,6,39,45,39,1.0		Z=.030
71.		482,435,506,1.0		
72.	GEN	483,11,1,7,39,46,39,1.0		Z=.035
73.		494,436,507,1.0		
74.	GEN	495,11,1,8,39,47,39,1.0		Z=.040
75.		500,437,508,1.0		
76.	GEN	501,11,1,9,39,48,39,1.0		Z=.045
77.		510,438,509,1.0		
78.	GEN	511,11,1,10,39,49,39,1.0		Z=.050
79.		530,439,510,1.0		
80.	GEN	531,11,1,11,39,50,39,1.0		Z=.055
81.		542,440,511,1.0		
82.	GEN	543,11,1,12,39,51,39,1.0		Z=.060
83.		554,441,512,1.0		
84.	GEN	555,11,1,13,39,52,39,1.0		Z=.065
85.		566,442,513,1.0		
86.	GEN	567,11,1,14,39,53,39,1.0		Z=.070
87.		578,443,514,1.0		
88.	GEN	579,11,1,15,39,54,39,1.0		Z=.075
89.		590,444,515,1.0		
90.	GEN	591,11,1,16,39,55,39,1.0		Z=.080
91.		602,445,516,1.0		
92.	GEN	603,11,1,17,39,56,39,1.0		Z=.085
93.		614,446,517,1.0		
94.	GEN	615,11,1,18,39,57,39,1.0		Z=.090
95.		626,447,518,1.0		
96.	GEN	627,11,1,19,39,58,39,1.0		Z=.095
97.		638,448,519,1.0		
98.	GEN	639,11,1,20,39,59,39,1.0		Z=.100
99.		650,449,520,1.0		
100.	GEN	651,11,1,21,39,60,39,1.0		Z=.105
101.		662,450,521,1.0		
102.	GEN	663,11,1,22,39,61,39,1.0		Z=.110
103.		674,451,522,1.0		
104.	GEN	675,11,1,23,39,62,39,1.0		Z=.115
105.		686,452,523,1.0		
106.	GEN	687,11,1,24,39,63,39,1.0		Z=.120
107.		698,453,524,1.0		
108.	GEN	699,11,1,25,39,64,39,1.0		Z=.125
109.		710,454,525,1.0		
110.	GEN	711,11,1,26,39,65,39,1.0		Z=.130
111.		722,455,526,1.0		
112.	GEN	723,11,1,27,39,66,39,1.0		Z=.135

[illegible]

```

170.      SPACE,39,END
171.      13  TEMPERATURE, NODE COLUMN 13
172.      SPACE,39,END
173.      14  ARRAY LINE IDENTIFICATION NUMBERS FOR PRNTMA
174.      SPACE,39,END
175.      15  LABEL FOR COLUMN 0, PRNTMA
176.      BCD 4   LINE      NO.
177.      END
178.      16
179.      BCD 4   TEMP.      COLUMN 1
180.      END
181.      17
182.      BCD 4   TEMP.      COLUMN 2
183.      END
184.      18
185.      BCD 4   TEMP.      COLUMN 3
186.      END
187.      19
188.      BCD 4   TEMP.      COLUMN 4
189.      END
190.      20
191.      BCD 4   TEMP.      COLUMN 5
192.      END
193.      21
194.      BCD 4   TEMP.      COLUMN 6
195.      END
196.      22
197.      BCD 4   TEMP.      COLUMN 7
198.      END
199.      23
200.      BCD 4   TEMP.      COLUMN 8
201.      END
202.      24
203.      BCD 4   TEMP.      COLUMN 9
204.      END
205.      25
206.      BCD 4   TEMP.      COLUMN 10
207.      END
208.      26
209.      BCD 4   TEMP.      COLUMN 11
210.      END
211.      27
212.      BCD 4   TEMP.      COLUMN 12
213.      END
214.      28
215.      BCD 4   TEMP.      COLUMN 13
216.      END
217.      29  SPRNTMA ARRAY FOR COLUMN 2
218.      SPACE,39,END
219.      30  SPRNTMA ARRAY FOR COLUMN 3
220.      SPACE,39,END
221.      31  RADIAL NODE CENTERS
222.      0.000,0.005,0.010,0.015,0.020,0.025,0.030
223.      0.035,0.040,0.045,0.050,0.055,0.060,END
224.      32  SHEAT SOURCE TERMS FOR NODE COLUMN 2, 040-068,070-078
225.      SPACE,39,END
226.      33  SHEAT SOURCE TERMS FOR NODE COLUMN 3, 009-0117

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221. SPACE,39,END
222. 34 SHEAT SOURCE TERMS FOR NODE COLUMN 4, 0118-0150
223. SPACE,39,END
224. 35 SHEAT SOURCE TERMS FOR NODE COLUMN 5, 0157-0195
225. SPACE,39,END
226. 36 SHEAT SOURCE TERMS FOR NODE COLUMN 6, 0190-0234
227. SPACE,39,END
228. 37 SHEAT SOURCE TERMS FOR NODE COLUMN 7, 0235-0273
229. SPACE,39,END
230. 38 SHEAT SOURCE TERMS FOR NODE COLUMN 8, 0274-0312
231. SPACE,39,END
232. 39 SHEAT SOURCE TERMS FOR NODE COLUMN 9, 0313-0351
233. SPACE,39,END
234. 40 SHEAT SOURCE TERMS FOR NODE COLUMN 10, 0352-0390
235. SPACE,39,END
236. 41 SHEAT SOURCE TERMS FOR NODE COLUMN 11, 0391-0429
237. SPACE,39,END
238. 42 SHEAT SOURCE TERMS FOR NODE COLUMN 12, 0430-0468
239. SPACE,39,END
240. END
241. BCU 3EXECUTION
242. F DU 1 I=1,39
243. A14+1=1
244. F 1 CONTINUE
245. PRINT
246. SMFTV(39,1501,A13+1)
247. CINDSL
248. END
249. BCU
250. $VARIABLES 1
251. SMFTV(39,I1,A1+1) $BOUNDARY NODE TEMP. COL.1
252. SMFTV(29,I40,A2+1)
253. SMFTV(1,I09,A2+30) $BOUNDARY NODE TEMP. COL.2, T69
254. SMFTV(9,I70,A2+31)
255. SMFTV(30,I79,A3+1) $BOUNDARY NODE TEMP. COL.3
256. SMFTV(4,I109,A3+31)
257. SMFTV(39,I118,A4+1)
258. SMFTV(39,I157,A5+1)
259. SMFTV(39,I195,A6+1)
260. SMFTV(39,I235,A7+1)
261. SMFTV(39,I274,A8+1)
262. SMFTV(39,I313,A9+1)
263. SMFTV(39,I352,A10+1)
264. SMFTV(39,I391,A11+1)
265. SMFTV(39,I430,A12+1)
266. REM HEAT SOURCE TERMS FOR NODES 40-68 AND 70-78
267. F DU 10 I=1,29
268. A32+I=K9/(A31+2)*((A3+1)-(A1+1))
269. F 10 CONTINUE
270. A32+30=0.0
271. F DU 11 I=31,39
272. A32+I=K9/(A31+2)*((A3+1)-(A1+1))
273. F 11 CONTINUE
274. SMFTV(29,A32+1,040)
275. SMFTV(9,A32+31,070)
276. REM HEAT SOURCE TERMS FOR NODES 109-117
277. DU 12 I=31,39
278. J=I-30

```

284. $A33+J=R9/(A31+3)*((A4+1)-(A2+1))$
285. F 12 CONTINUE
286. SHFTV(9,A33+1,U109)
287. REM HEAT SOURCE TERMS FOR NODES 118-156
288. F DU 13 I=1,39
289. $A34+1=R9/(A31+4)*((A5+1)-(A3+1))$
290. F 13 CONTINUE
291. SHFTV(39,A34+1,U118)
292. REM HEAT SOURCE TERMS FOR NODES 157-195
293. F DU 14 I=1,39
294. $A35+1=R9/(A31+5)*((A6+1)-(A4+1))$
295. F 14 CONTINUE
296. SHFTV(39,A35+1,U157)
297. REM HEAT SOURCE TERMS FOR NODES 196-234
298. F DU 15 I=1,39
299. $A36+1=R9/(A31+6)*((A7+1)-(A5+1))$
300. F 15 CONTINUE
301. SHFTV(39,A36+1,U196)
302. REM HEAT SOURCE TERMS FOR NODES 235-273
303. F DU 16 I=1,39
304. $A37+1=R9/(A31+7)*((A8+1)-(A6+1))$
305. F 16 CONTINUE
306. SHFTV(39,A37+1,U235)
307. REM HEAT SOURCE TERMS FOR NODES 274-312
308. F DU 17 I=1,39
309. $A38+1=R9/(A31+8)*((A9+1)-(A7+1))$
310. F 17 CONTINUE
311. SHFTV(39,A38+1,U274)
312. REM HEAT SOURCE TERMS FOR NODES 313-351
313. F DU 18 I=1,39
314. $A39+1=R9/(A31+9)*((A10+1)-(A8+1))$
315. F 18 CONTINUE
316. SHFTV(39,A39+1,U313)
317. REM HEAT SOURCE TERMS FOR NODES 352-390
318. F DU 19 I=1,39
319. $A40+1=R9/(A31+10)*((A11+1)-(A9+1))$
320. F 19 CONTINUE
321. SHFTV(39,A40+1,U352)
322. REM HEAT SOURCE TERMS FOR NODES 391-429
323. F DU 20 I=1,39
324. $A41+1=R9/(A31+11)*((A12+1)-(A10+1))$
325. F 20 CONTINUE
326. SHFTV(39,A41+1,U391)
327. REM HEAT SOURCE TERMS FOR NODES 430-468
328. F DU 21 I=1,39
329. $A42+1=R9/(A31+12)*((A13+1)-(A11+1))$
330. F 21 CONTINUE
331. SHFTV(39,A42+1,U430)
332. REM INTERNAL BOUNDARY NODE TEMPERATURES T79-T107
333. $A3+1=((R1*R3+R2*R4)*(1500+(A3+2)))+(R2*R5*(A4+1)+R1*R6*(A2+1))/$
334. $(2.*(R1*R3+R2*R4)+R2*R5+R1*R6)$
335. F DU 22 I=2,29
336. F J=1-1
337. F L=1+1
338. $A3+1=((R1*R3+R2*R4)*((A3+J)+(A3+L)))+(R2*R5*(A4+1)+R1*R6*(A2+1))/$
339. $(2.*(R1*R3+R2*R4)+R2*R5+R1*R6)$
340. F 22 CONTINUE

```

341.      REM INTERNAL BOUNDARY CORNER NODE T108
342.      A3+30=((K1*K3+K2*K4)*(A3+29)+K2*K5*(A4+30)+K2*K7*(A3+31)+
343.      *(K1+K2)*K0/2.*(A2+30))/((K1*K3+K2*K4)+K2*K5+K2*K7+(K1+K2)*
344.      *K0/2.)
345.      SHFTV(30,A3+1,179)
346.      REM INTERNAL BOUNDARY NODE TEMPERATURE T69
347.      T69=(K1*K10+100*(K1+K2)*K0+T108+K2*K10*(70)/
348.      *(K1*K10+(K1+K2)*K0+K2*K10)
349.      REM ASSIGN COLUMN 2 TEMPERATURES TO COLUMN 1
350.      SHFTV(29,140,A1+1)
351.      SHFTV(1,109,A1+30)
352.      SHFTV(9,170,A1+31)
353.      SHFTV(39,A1+1,T1)

```

END

BCD VARIABLES 2

END

BCD

SUBROUTINE CALLS

SHFTV(29,140,A29+1)

SHFTV(1,109,A29+30)

SHFTV(9,170,A29+31)

SHFTV(30,179,A30+1)

SHFTV(9,1109,A30+31)

IPRINT

PRNTMA(39,A15+1,A14+1,A16+1,T1,A17+1,A29+1,A18+1,A30+1

A19+1,T118,A20+1,T157,A21+1,T196,A22+1,T235,A23+1,T274

A24+1,T313)

PRNTMA(39,A15+1,A14+1,A25+1,T352,A26+1,T391,A27+1,T430

A28+1,T501)

END

BCD SEND OF DATA

#ADD SINDA*SINDAB-PREP.LIST\$MAP/NOPLJT

#XUT SINDA

END DATA.

#ASB,A SINDA*SINDAB-PREP.

-FAC WARNING 040200004000

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6.5.2 Subroutine: CINDSL

6.5.2.1 General Comments

Subroutine CINDSL is a steady state routine that requires the long pseudo-compute sequence (LPCS). Both diffusion- and arithmetic-node temperatures are calculated by a "successive point" iteration computational technique. Every third iteration a linear extrapolation is performed to accelerate convergence. CINDSL generally yields significantly faster solutions than CINDSS, but nonlinear problems such as those with radiation heat transfer can pose considerable convergence difficulties unless a large amount of damping (low values of DAMPA and DAMPD) is imposed.

A series of steady state solutions at various points in time can be generated by specifying control constants TIMEND and ØUTPUT. ØUTPUT is used both as the output interval and the computation interval; this requires appropriate calls in VARIABLES 1 to modify boundary conditions with time.

CINDSL can be followed by a call to one of the transient numerical solution routines which have the same LPCS requirements. Used in this manner the steady state solutions become the initial conditions for the transient analysis. Note that since CINDSL utilizes control constants TIMEND and ØUTPUT for the coupled steady state-transient problem, the user must specify the values of TIMEND and ØUTPUT in the execution block after the steady state call and prior to the transient analysis call.

Solution convergence is based upon a temperature relaxation criterion stored in control constants DRLXCA for diffusion nodes and ARLXCA for arithmetic nodes. Normally, identical values are specified for both DRLXCA and ARLXCA for lack of anything better. The damping factors DAMPD for diffusion nodes and DAMPA for arithmetic nodes are merely multipliers of "new" temperatures and the factor $1.0 - \text{DAMPD}$ (or $1.0 - \text{DAMPA}$) is a multiplier of the "old" temperatures. Normally, these damping factors are specified to be less than 1.0, but for a linear system the convergence probably could be accelerated by using a damping factor greater than one.

6.5.2.2 Finite Difference Approximation and Computational Algorithm

The set of steady state heat balance equations,

$$q_i + \sum_{j=1}^P a_{ij} (T_j - T_i) + \sum_{j=1}^P ob_{ij} (T_j^4 - T_i^4) = 0$$

$$i = 1, 2, \dots, N$$

$$T_j = \text{constant } N < j \leq p$$

is solved by a re-iterative scheme called a "successive point" iterative method here. Both diffusion-node and arithmetic-node temperatures are solved in this manner. The only difference between the two algorithms is that control constant DAMPD is used with diffusion nodes and control constant DAMPA is used with arithmetic nodes.

Diffusion Nodes (if any)

$$T_{i,k+1} = DD * T_{i,k} + DN * \frac{(q_{i,k} + \sum_{j=1}^1 G_{ij,k} T_{j,k+1} + \sum_{j=i+1}^P G_{ij,k} T_{j,k})}{\sum_{j=1}^P G_{ij,k}} \quad (6.5-4)$$

where, $i = 1, 2, \dots, NND$; $k = k$ th iteration

q_i, a_{ij}, b_{ij} = may be optionally specified (refer to Tables 6.2-1 - 6.2-4)

$T_{j,k}$ = constant, $(NND + NNA) < j \leq p$ (NNA is the number of arithmetic nodes and p is the total number of nodes)

DN = DAMPD (diffusion-node damping factor)

DD = 1.0 - DN

$$G_{ij,k} = a_{ij,k} + ob_{ij,k} (T_{j,l}^2 + T_{i,k}^2) (T_{j,l} + T_{i,k})$$

($l = k$ if $j \geq i$ and $l = k+1$ if $j < i$)

Arithmetic Nodes (if any)

$$T_{i,k+1} = AD * T_{i,k} + AN * \frac{(q_{i,k} + \sum_{j=1}^P G_{ij,k} T_{j,k+1} + \sum_{j=i+1}^P G_{ij,k} T_{j,k})}{\sum_{j=1}^P G_{ij,k}} \quad (6.5-5)$$

where, $i = (NND + 1), (NND + 2), \dots, (NND + NNA)$

q_i, a_{ij}, b_{ij} = may be optionally specified (refer to Tables 6.2-1 - 6.2-4)

$T_{j,k}$ = constant $(NND + NNA) < j \leq p$ (NNA is the number of arithmetic nodes and p is the total number of nodes)

AN = DAMPA (arithmetic-node damping factor)

AD = 1.0 - AN

$$G_{ij,k} = a_{ij,k} + ob_{ij,k} (T_{j,l}^2 + T_{i,k}^2) (T_{j,l} + T_{i,k})$$

($l = k$ if $j \geq i$ and $l = k+1$ if $j < i$)

6.5.2.3 Comments on the Computational Procedure

The important steps of the computational procedure used in the steady state subroutine CINDSL are indicated in Table 6.5-2. For a detailed procedural description, the user must examine the computer listing for CINDSL in Appendix C, but some general computational details are given in Section 6.2.5.3. A functional flow chart of CINDSL is shown in Figure 6.5-2.

The computational pattern of CINDSL is very similar to CINDSS with the differences being that CINDSL uses the long pseudo-compute sequence, whereas CINDSS uses the short pseudo-compute sequence, and that CINDSL contains the acceleration convergence feature, whereas CINDSS does not. The user is required to specify the maximum number of iterations to be performed via control constant NLØØP and the diffusion-node temperature change relaxation criteria DRLXCA and the arithmetic-node temperature change relaxation criteria ARLXCA. The iterations continue until either NLØØP is satisfied or both DRLXCA and ARLXCA are satisfied. If DRLXCA and ARLXCA are not satisfied with NLØØP, an appropriate message is printed. Acceleration of convergence is performed every third iteration if a temperature is converging over two time-steps.

6.5.2.4 Control Constants

Control constant NLØØP must be specified and control constants ARLXCA and DRLXCA must be specified if $NNA > 0$ and $NND > 0$, respectively; otherwise "run" will terminate with an appropriate error message. Control constants DAMPA and DAMPD may be optionally specified among others. Control constant characteristics are tabulated in Table 6.2-5 and description of these control constants is presented in Section 6.2.3.2. Specification of NLØØP is dependent upon the values of ARLXCA and DRLXCA and thus the accuracy of the solution. Since the type of problem will influence accuracy, it appears that a trial and error procedure is the only practical way of determining realistic control constant values.

6.5.2.5 Error and Other Messages

If control constants ARLXCA, DRLXCA and NLØØP are not specified, the following error message will be printed for each,

ARLXCA	"NØ ARLXCA"
DRLXCA	"NØ DRLXCA"
NLØØP	"NØ NLØØP"

If the long pseudo-compute sequence LPCS is not specified, the error message will be,

"CINDSL REQUIRES LONG PSEUDO-COMPUTE SEQUENCE"

If the dynamic storage allocation is not sufficient, (NDIM < 2* (NNA + NND)), the message will be,

"_____ LOCATIONS AVAILABLE"

Note that the number printed will be negative indicating the additional storage locations required.

"LOOPCT = _____ and ENGBAL = _____"

If both temperature change relaxation criteria, ARLXCA and DRLXCA, are not met with NLØØP iterations, the message will be,

"ITERATION COUNT EXCEEDED, LOOPCT = _____"

Checks on the control constants, the pseudo-compute sequence, and the dynamic storage allocation are made in the following order with the "run" terminating if a single check is not satisfied.

NLØØP, ARLXCA, DRLXCA, LPCS, and dynamic storage allocation.

Table 6.5.2 Basic Computational Steps for CINDSL

1. Specification of control constants. Control constants ARLXCA (if NNA > 0), DRLXCA (if NND > 0) and NLOOP must be specified. LPCS is required. (Refer to Table 6.2-5 for values and Section 6.2.3.2 for description.)
2. Sufficiency check on dynamic storage. Requirements = 2* (NND + NNA) (NND = diffusion nodes and NNA = arithmetic nodes).
3. Setting of TIMEN for first and succeeding iterations.

$$\text{TIMEN} = \text{TIME0}, \text{ first iteration}$$

$$\text{TIMEN} = \text{TIME0} + \text{0UTPUT}, \text{ succeeding iterations}$$
4. Setting of iterative loop for all nodes, kl = 1, NLOOP.
5. Setting of source locations to zero.
6. Calling of VARIABLES 1 (refer to Section 6.2.2.2 for description).
7. Calculation of diffusion-node temperatures by "block" iteration if NND > 0 (refer to Section 6.2.5.2 and 6.5.1.2).

$$T_{i,k+1} = DD * T_{i,k} + \frac{DN * (q_{i,k} + \sum_{j=1}^I G_{ij,k} T_{j,k+1} + \sum_{j=I+1}^P G_{ij,k} T_{j,k})}{\sum_{j=1}^P G_{ij,k}}$$

$$DN = \text{DAMPD and } DD = 1.0 - DN$$

8. Calculation of DRLXCC.
9. Calculation of arithmetic-node temperatures by "successive point" iteration if NNA > 0 (refer to Sections 6.2.5.3 and 6.5.1.2).

$$T_{i,k+1} = AD * T_{i,k} + \frac{AN * (q_{i,k} + \sum_{j=1}^I G_{ij,k} T_{j,k+1} + \sum_{j=I+1}^P G_{ij,k} T_{j,k})}{\sum_{j=1}^P G_{ij,k}}$$

10. Calculation of ARLXCC.
11. Checking of DRLXCC and ARLXCC against the relaxation criteria DRLXCA and ARLXCA, respectively, for convergence. If both ARLXCA and DRLXCA are satisfied, iterations cease, otherwise NLOOP iterations are performed.
12. Acceleration of convergence each third iteration, if linear extrapolation criterion is met (refer to Section 6.2.7).
13. Calculation of system energy balance which is stored in ENGBAL.
14. Call VARIABLES 2 and 0UTCAL, print ENGBAL and L00PCT.
15. Check if TIMEND = TIMEN.

Figure 6.5-2 Functional Flow Chart for CINDSL

APPENDIX C

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00101 1* SUBROUTINE CINDSL
00101 2* C STEADY STATE EXECUTION SUBROUTINE FOR SINDA FORTRAN V
00101 3* C THE LONG PSEUDO-COMPUTE SEQUENCE IS REQUIRED
00101 4* C DIFFUSION NODES RECEIVE A SUCCESSIVE POINT ITERATION
00101 5* C ARITHMETIC NODES RECEIVE A SUCCESSIVE POINT ITERATION
00101 6* C OVER-RELAXATION IS ALLOWED, THE DAMPING FACTORS ARE ADDRESSABLE
00103 7* INCLUDE COMM.LIST
00104 7* COMMON /TITLE/H(1) /TEMP/T(1) /CAP/C(1) /SOURCE/S(1) /COND/G(1)
00105 7* COMMON /PC1/NSQ1(1) /PC2/NSQ2(1) /KONST/K(1) /ARRAY/A(1)
00106 7* COMMON /FIXCON/KCN(1) /XSPACE/NDIM,NTH,X(1)
00107 7* COMMON /DIMENS/ NND,NNA,NNT,NGT,NCT,NAT,LSQ1,LSQ2
00110 7* DIMENSION CON(1),XK(1),NX(1)
00111 7* EQUIVALENCE (KCN(1),CON(1)),(K(1),XK(1)),(X(1),NX(1))
00112 7* END
00113 8* INCLUDE DEFF.LIST
00113 8* C***** CONTROL CONSTANT DEFINITIONS AND NAMES *****
00113 8* C CONTROL CONSTANT 1 CONTAINS THE NEW PROBLEM TIME (TIMEI)
00113 8* C CONTROL CONSTANT 2 CONTAINS THE TIME STEP USED (DTIMEU)
00113 8* C CONTROL CONSTANT 3 CONTAINS THE PROBLEM STOP TIME (TIMEND)
00113 8* C CONTROL CONSTANT 4 CONTAINS THE TIME STEP FACTOR, EXPLICIT (CSGFAC)
00113 8* C CC5 IS THE INPUT NUMBER OF ITERATION DO LOOPS, INTEGER (NLOOP)
00113 8* C CC6 CONTAINS THE DIFFUSION TEMPERATURE CHANGE ALLOWED (DTMPCA)
00113 8* C CC7 CONTAINS THE OUTPUT EACH ITERATION SWITCH (OPEITR)
00113 8* C CC8 CONTAINS THE MAXIMUM ALLOWED TIME STEP (DTIMEH)
00113 8* C CC9 CONTAINS THE NEW ARITHMETIC TEMP. DAMPING FACTOR (DAMPA)
00113 8* C CC10 CONTAINS THE NEW DIFFUSION TEMP. DAMPING FACTOR (DAMPD)
00113 8* C CC11 CONTAINS THE MAXIMUM ALLOWED ARITHMETIC TEMP. CHANGE (ATMPCA)
00113 8* C CC12 CONTAINS THE BACKUP SWITCH CHECKED AFTER VARIABLES (BACKUP)
00113 8* C CC13 CONTAINS THE PRESENT TIME OR PROBLEM START TIME (TIMEO)
00113 8* C CC14 CONTAINS THE MEAN TIME BETWEEN AN ITERATION (TIMEM)
00113 8* C CC15 CONTAINS THE DIFFUSION TEMPERATURE CHANGE CALCULATED (DTMPCC)

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00113 8* C CC16 CONTAINS ARITHMETIC TEMPERATURE CHANGE CALCULATED (ATMPCC)
00113 8* C CONTROL CONSTANT 17 IS RESERVED FOR THE C/SG MINIMUM (CSGMIN)
00113 8* C CONTROL CONSTANT 18 CONTAINS THE OUTPUT INTERVAL (OUTPUT)
00113 8* C CC19 CONTAINS THE ARITHMETIC RELAXATION CRITERIA ALLOWED (ARLXCA)
00113 8* C CC20 CONTAINS THE NUMBER OF RELAXATION LOOPS USED, INTEGER (LOOPCT)
00113 8* C CC21 CONTAINS THE MINIMUM ALLOWED TIME STEP (DTIMEI)
00113 8* C CC22 IS FOR THE INPUT TIME STEP IMPLICIT (DTIMEI)
00113 8* C CC23 CONTAINS THE C/SG MAXIMUM (CSGMAX)
00113 8* C CC24 CONTAINS THE C/SG RANGE ALLOWED (CSGRAL)
00113 8* C CC25 CONTAINS THE C/SG RANGE CALCULATED (CSGRCL)
00113 8* C CC26 CONTAINS THE DIFFUSION RELAXATION CRITERIA ALLOWED (DRLXCA)
00113 8* C CC27 CONTAINS THE DIFFUSION RELAXATION CHANGE CALCULATED (DRLXCC)
00113 8* C CC28 CONTAINS THE LINE COUNTER, INTEGER (LINECT)
00113 8* C CC29 CONTAINS THE PAGE COUNTER, INTEGER (PAGECT)
00113 8* C CC30 CONTAINS ARITHMETIC RELAXATION CHANGE CALCULATED (APLXCC)
00113 8* C CC31 IS INDICATOR, 0=THERMAL SPCS,1=THERMAL LPCS,2=GENERAL (LSPCS)
00113 8* C CC32 CONTAINS THE ENERGY BALANCE OF THE SYSTEM, IN - OUT (ENGRAL)
00113 8* C CC33 CONTAINS THE DESIRED ENERGY BALANCE, USER INPUT (BALENG)
00113 8* C CC34 CONTAINS THE HOCOPY SWITCH FOR MATRIX USERS (HOCOPY)
00113 8* C CC35 CONTAINS RELATIVE NODE NUMBER OF CSGMIN
00113 8* C CC36 CONTAINS RELATIVE NODE NUMBER OF DTMPCC
00113 8* C CC37 CONTAINS RELATIVE NODE NUMBER OF ARLXCC
00113 8* C CC38 CONTAINS RELATIVE NODE NUMBER OF ATMPCC
00113 8* C CC39-40-41-42-43 CONTAIN DUMMY INTEGER CONSTANTS (I-J-K-L-MTEST)
00113 8* C CC44-45-46-47-48 CONTAIN DUMMY FLOATING CONSTANTS (R-S-T-U-VTEST)
00113 8* C CC49 IS THE QUASI-LINEARIZATION INTERVAL FOR CINDSM (LAXFAC)
00113 8* C CC50 IS NOT USED AT PRESENT
00114 8* C END
00115 9* IF(KON(5).LE.0) GO TO 999
00117 10* IF(CON(9).LE.0.) CON(9) = 1.0
00121 11* IF(CON(10).LE.0.) CON(10) = 1.0
00123 12* IF(NNA.GT.0.AND.CON(19).LE.0.) GO TO 998
00125 13* IF(NND.GT.0.AND.CON(26).LE.0.) GO TO 997
00127 14* IF(KON(31).NE.1) GO TO 994
00131 15* PASS = -1.0
00132 16* NN = NND+1
00133 17* NNC = NNA+NND
00134 18* IE1 = NTH
00135 19* IE2 = IE1+NNC
00136 20* NLA = NOIM
00137 21* JJ = 2*NNC
00140 22* NTH = NTH+JJ
00141 23* NOIM = NOIM-JJ
00142 24* IF(NDIM.LT.0) GO TO 996
00144 25* CON(1) = CON(13)
00145 26* CON(2) = 0.0
00146 27* CON(14) = CON(13)
00147 28* GO TO 10
00150 29* 5 CON(1) = CON(13)+CON(18)
00151 30* IF(CON(1)-CON(13).GT.0.) CON(1) = CON(13)
00153 31* CON(14) = (CON(1)+CON(13))/2.0
00154 32* CON(2) = CON(1)-CON(13)
00154 33* C COMPUTE STEADY STATE TEMPERATURES
00155 34* 10 LAX = KON(5)
00156 35* JJ = 0
00157 36* DO 145 K1 = 1,LAX
00162 37* JJ = JJ+1
00163 38* KGM(20) = K1
00163 39* C ZERO OUT ALL SOURCE LOCATIONS
00164 40* DO 15 I = 1,NNC

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00167 41* 15 Q(I) = 0.0
00171 42* CALL VARBL1
00172 43* IF(PASS.GE.0.) GO TO 20
00174 44* CALL OUTCAL
00175 45* PASS = 1.0
00176 46* 20 J1 = 0
00177 47* J2 = 1
00200 48* RLXD = 0.0
00201 49* RLXA = 0.0
00202 50* IF(NND.LE.0) GO TO 75
00204 51* DN = CON(10)
00205 52* DD = 1.0-DN
00205 53* C DO A SUCCESSIVE POINT ITERATION ON THE DIFFUSION NODES
00206 54* DO 70 I = 1,NND
00211 55* GSUM = 0.0
00212 56* INCLUDE DUMC,LIST
00213 56* IF(FLD(1,1,NSQ1(J1+1)),EQ.0) GO TO 2000
00215 56* NTYPE = FLD(0,5,NSQ2(J2))
00216 56* GO TO (1999,1998,1998,1998,1999,1998,1998,1998,1999), NTYPE
00217 56* 1998 J2 = J2+1
00220 56* 1999 J2 = J2+1
00221 56* 2000 CONTINUE
00222 56* END
00223 57* INCLUDE VARQ,LIST
00224 57* IF(FLD(4,1,NSQ1(J1+1)),EQ.0) GO TO 5000
00226 57* NTYPE = FLD(0,5,NSQ2(J2))
00227 57* LA = FLD(5,17,NSQ2(J2))
00230 57* LK = FLD(22,14,NSQ2(J2))
00231 57* GO TO (4005,4010,4015,4020,4025,4030,4035,4040,4030), NTYPE
00232 57* 4005 Q(I) = XK(LK)+Q(I)
00233 57* GO TO 4999
00234 57* 4010 Q1 = 0.0
00235 57* 4012 CALL D1D1WM(T(I),A(LA),XK(LK),Q2)
00236 57* GO TO 4998
00237 57* 4015 Q1 = 0.0
00240 57* 4017 CALL D1D1WM(CON(14),A(LA),XK(LK),Q2)
00241 57* GO TO 4998
00242 57* 4020 CALL D1D1WM(CON(14),A(LA),XK(LK),Q1)
00243 57* 4022 J2 = J2+1
00244 57* LA = FLD(5,17,NSQ2(J2))
00245 57* LK = FLD(22,14,NSQ2(J2))
00246 57* GO TO 4017
00247 57* 4025 Q1 = XK(LK)+XK(LA)
00250 57* GO TO 4022
00251 57* 4030 CALL D1D1WM(CON(14),A(LA),XK(LK),Q1)
00252 57* J2 = J2+1
00253 57* LA = FLD(5,17,NSQ2(J2))
00254 57* LK = FLD(22,14,NSQ2(J2))
00255 57* Q2 = XK(LK)+XK(LA)
00256 57* GO TO 4998
00257 57* 4035 CALL D1D1WM(CON(14),A(LA),XK(LK),Q1)
00260 57* 4037 J2 = J2+1
00261 57* LA = FLD(5,17,NSQ2(J2))
00262 57* LK = FLD(22,14,NSQ2(J2))
00263 57* GO TO 4012
00264 57* 4040 Q1 = XK(LK)+XK(LA)
00265 57* GO TO 4037
00266 57* 4998 Q(I) = Q1+Q2+Q(I)
00267 57* 4999 J2 = J2+1
00270 57* 5000 CONTINUE

```

*NEW
**-1

*NEW
*NEW
*NEW
**-3

*NEW
*NEW
**-2

*NEW
*NEW
**-2

*NEW
*NEW
**-2

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00271 57* END
00272 58* 25 J1 = J1+1
00273 59* LG = FLD(5,16,NSQ1(J1))
00274 60* LTA = FLD(22,14,NSQ1(J1))
00275 61* INCLUDE VARG,LIST
00275 62* C CHECK FOR RADIATION CONDUCTOR
00276 62* IF(FLD(2,1,NSQ1(J1)).EQ.0) GO TO 3000
00300 62* NTYPE = FLD(10,5,NSQ2(J2))
00301 62* LA = FLD(5,17,NSQ2(J2))
00302 62* LK = FLD(22,14,NSQ2(J2))
00303 62* GO TO(2005,2010,2015,2020,2025,2030,2035,2040,2045,2050,2055,
00303 62* 2060,2065), NTYPE
00304 62* 2005 TM = (T(I)+T(LTA))/2.0
00305 62* 2007 CALL D1D1WM(TM,A(LA),XK(LK),G(LG))
00306 62* GO TO 2999
00307 62* 2010 TM = T(I)
00310 62* GO TO 2007
00311 62* 2015 CALL D1D1WM(T(I),A(LA),XK(LK),G1)
00312 62* 2017 J2 = J2+1
00313 62* LA = FLD(5,17,NSQ2(J2))
00314 62* LK = FLD(22,14,NSQ2(J2))
00315 62* CALL D1D1WM(T(LTA),A(LA),XK(LK),G2)
00316 62* GO TO 2998
00317 62* 2020 G1 = XK(LK)+XK(LA)
00320 62* GO TO 2017
00321 62* 2025 CALL D1D1WM(T(I),A(LA),XK(LK),G1)
00322 62* J2 = J2+1
00323 62* LA = FLD(5,17,NSQ2(J2))
00324 62* LK = FLD(22,14,NSQ2(J2))
00325 62* G2 = XK(LK)+XK(LA)
00326 62* GO TO 2998
00327 62* 2030 TM = (T(I)+T(LTA))/2.0
00330 62* 2032 CALL PLYAWM(A(LA),TM,A(LA+1),XK(LK),G(LG))
00331 62* GO TO 2999
00332 62* 2035 TM = T(I)
00333 62* GO TO 2032
00334 62* 2040 CALL PLYAWM(A(LA),T(I),A(LA+1),XK(LK),G1)
00335 62* 2042 J2 = J2+1
00336 62* LA = FLD(5,17,NSQ2(J2))
00337 62* LK = FLD(22,14,NSQ2(J2))
00340 62* CALL PLYAWM(A(LA),T(LTA),A(LA+1),XK(LK),G2)
00341 62* GO TO 2998
00342 62* 2045 G1 = XK(LK)+XK(LA)
00343 62* GO TO 2042
00344 62* 2050 CALL PLYAWM(A(LA),T(I),A(LA+1),XK(LK),G1)
00345 62* J2 = J2+1
00346 62* LA = FLD(5,17,NSQ2(J2))
00347 62* LK = FLD(22,14,NSQ2(J2))
00350 62* G2 = XK(LK)+XK(LA)
00351 62* GO TO 2998
00352 62* 2055 TM = (T(I)+T(LTA))/2.0
00353 62* CALL D2D1WM(TM,CON(14),A(LA),XK(LK),G(LG))
00354 62* GO TO 2999
00355 62* 2060 TM = T(LTA)
00356 62* GO TO 2007
00357 62* 2065 TM = T(LTA)
00360 62* GO TO 2032
00361 62* 2999 G(LG) = 1./(1./G1+1./G2)
00362 62* IF(FLD(3,1,NSQ1(J1)).EQ.1) G(LG) = G1+G2
00364 62* 2999 J2 = J2+1

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*NEW
*NEW
*NEW
**-3

*NEW
*NEW
**-2

*NEW
*NEW
**-2

*NEW
*NEW
**-2

*NEW
*NEW
**-2

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00365 62* 3000 CONTINUE
00366 62* END
00367 63* IF(FLD(3,1,NS01(J1)).EQ.0) GO TO 30
00371 64* T1 = T(I)+460.0
00372 65* T2 = T(LTA)+460.0
00373 66* GV = G(LG)*(T1+T1+T2+T2)*(T1+T2)
00374 67* GO TO 35
00375 68* 30 GV = G(LG)
00376 69* 35 GSUM = GSUM+GV
00377 70* O(I) = O(I)+GV*T(LTA)
00377 71* C CHECK FOR LAST CONDUCTOR TO THIS NODE
00400 72* IF(NS01(J1).GT.0) GO TO 25
00402 73* T2 = DD+T(I)+DN*O(I)/GSUM
00402 74* C OBTAIN THE CALCULATED TEMPERATURE DIFFERENCE
00403 75* T1 = ABS(T(I)-T2)
00403 76* C STORE THE NEW TEMPERATURES AND EXTRAPOLATION FACTORS
00404 77* GO TO(65,60,55),JJ
00405 78* 55 LE1 = IE1+I
00406 79* LE2 = IE2+I
00407 80* R1 = T2-T(I)
00410 81* X(LE1) = T(I)
00411 82* X(LE2) = R1/(R1-X(LE2))
00412 83* GO TO 65
00413 84* 60 LE2 = IE2+I
00414 85* X(LE2) = T2-T(I)
00415 86* 65 T(I) = T2
00415 87* C SAVE THE MAXIMUM DIFFUSION RELAXATION CHANGE
00416 88* IF(RLXD.GT.T1) GO TO 70
00420 89* RLXD = T1
00421 90* NI = I
00422 91* 70 CONTINUE
00424 92* CON(27) = RLXD
00425 93* IF(NNA.LE.0) GO TO 130
00427 94* 75 DN = CON(9)
00430 95* DD = 1.0-DN
00431 96* JJ1 = J1
00432 97* JJ2 = J2
00432 98* C DO SUCCESSIVE POINT ITERATION ON ARITHMETIC NODES
00433 99* DO 125 I = NN,NNC
00436 100* GSUM = 0.0
00437 101* L = I
00440 102* INCLUDE VRQ2,LIST
00441 102* IF(FLD(4,1,NS01(JJ1+1)).EQ.0) GO TO 6000
00443 102* NTYPE = FLD(0,5,NS02(JJ2))
00444 102* LA = FLD(5,17,NS02(JJ2))
00445 102* LK = FLD(22,14,NS02(JJ2))
00446 102* GO TO (5005,5010,5015,5020,5025,5030,5035,5040,5030), NTYPE
00447 102* 5005 O(L) = XK(LK)+G(L)
00450 102* GO TO 5999
00451 102* 5010 O1 = 0.0
00452 102* 5012 CALL D1D1WM(T(L),A(LA),XK(LK),O2)
00453 102* GO TO 5998
00454 102* 5015 O1 = 0.0
00455 102* 5017 CALL D1D1WM(CON(14),A(LA),XK(LK),O2)
00456 102* GO TO 5998
00457 102* 5020 CALL D1D1WM(CON(14),A(LA),XK(LK),O1)
00460 102* 5022 JJ2 = JJ2+1
00461 102* LA = FLD(5,17,NS02(JJ2))
00462 102* LK = FLD(22,14,NS02(JJ2))
00463 102* GO TO 5017

```

*NEW
*NEW
*NEW
**3

*NEW
*NEW
**2

CINDSL,CINDSL

00464 102* 5025 Q1 = XK(LK)*XK(LA)
 00465 102* 60 TO 5022
 00466 102* 5030 CALL DID1WM(CON(14),A(LA),XK(LK),Q1)
 00467 102* JJ2 = JJ2+1
 00470 102* LA = FLD(5,17,NSQ2(JJ2)) *NEW
 00471 102* LK = FLD(22,14,NSQ2(JJ2)) *NEW
 00472 102* Q2 = XK(LK)*XK(LA) **2
 00473 102* 60 TO 5998
 00474 102* 5035 CALL DID1WM(CON(14),A(LA),XK(LK),Q1)
 00475 102* 5037 JJ2 = JJ2+1
 00476 102* LA = FLD(5,17,NSQ2(JJ2)) *NEW
 00477 102* LK = FLD(22,14,NSQ2(JJ2)) *NEW
 00500 102* 60 TO 5012 **2
 00501 102* 5040 Q1 = XK(LK)*XK(LA)
 00502 102* 60 TO 5037
 00503 102* 5998 Q(L) = Q1+Q2+Q(L)
 00504 102* 5999 JJ2 = JJ2+1
 00505 102* 6000 CONTINUE
 00506 102* END
 00507 103* 80 JJ1 = JJ1+1
 00510 104* LG = FLD(5,16,NSQ1(JJ1))
 00511 105* LT = FLD(22,14,NSQ1(JJ1))
 00512 106* INCLUDE VRG2,LIST
 00512 107* C CHECK FOR RADIATION CONDUCTOR
 00513 107* IF(FLD(2,1,NSQ1(JJ1)),EQ,0) GO TO 4000
 00515 107* NTYPE = FLD(0,5,NSQ2(JJ2)) *NEW
 00516 107* LA = FLD(5,17,NSQ2(JJ2)) *NEW
 00517 107* LK = FLD(22,14,NSQ2(JJ2)) *NEW
 00520 107* 60 TO (3005,3010,3015,3020,3025,3030,3035,3040,3045,3050,3055,
 00520 107* 3060,3065), NTYPE **3
 00521 107* 3005 TM = (T(L)+T(LTA))/2.0
 00522 107* 3007 CALL DID1WM(TM,A(LA),XK(LK),G(LG))
 00523 107* 60 TO 3999
 00524 107* 3010 TM = T(L)
 00525 107* 60 TO 3007
 00526 107* 3015 CALL DID1WM(T(L),A(LA),XK(LK),G1)
 00527 107* 3017 JJ2 = JJ2+1
 00530 107* LA = FLD(5,17,NSQ2(JJ2)) *NEW
 00531 107* LK = FLD(22,14,NSQ2(JJ2)) *NEW
 00532 107* CALL DID1WM(T(LTA),A(LA),XK(LK),G2) **2
 00533 107* 60 TO 3998
 00534 107* 3020 G1 = XK(LK)*XK(LA)
 00535 107* 60 TO 3017
 00536 107* 3025 CALL DID1WM(T(L),A(LA),XK(LK),G1)
 00537 107* JJ2 = JJ2+1
 00540 107* LA = FLD(5,17,NSQ2(JJ2)) *NEW
 00541 107* LK = FLD(22,14,NSQ2(JJ2)) *NEW
 00542 107* G2 = XK(LK)*XK(LA) **2
 00543 107* 60 TO 3998
 00544 107* 3030 TM = (T(L)+T(LTA))/2.0
 00545 107* 3032 CALL PLYAWM(A(LA),TM,A(LA+1),XK(LK),G(LG))
 00546 107* 60 TO 3999
 00547 107* 3035 TM = T(L)
 00550 107* 60 TO 3032
 00551 107* 3040 CALL PLYAWM(A(LA),T(L),A(LA+1),XK(LK),G1)
 00552 107* 3042 JJ2 = JJ2+1
 00553 107* LA = FLD(5,17,NSQ2(JJ2)) *NEW
 00554 107* LK = FLD(22,14,NSQ2(JJ2)) *NEW
 00555 107* CALL PLYAWM(A(LA),T(LTA),A(LA+1),XK(LK),G2) **2
 00556 107* 60 TO 3998

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00557 107* 3045 G1 = XK(LK)*XK(LA)
00560 107*      GO TO 3042
00561 107* 3050 CALL PLYAWH(A(LA),T(L),A(LA+1),XK(LK),G1)
00562 107*      JJ2 = JJ2+1
00563 107*      LA = FLD(5,17,NSQ2(JJ2))
00564 107*      LK = FLD(22,14,NSQ2(JJ2))
00565 107*      G2 = XK(LK)*XK(LA)
00566 107*      GO TO 3998
00567 107* 3055 TM = (T(L)+T(LTA))/2.0
00570 107*      CALL D2D1WM(TM,CON(14),A(LA),XK(LK),G(LG))
00571 107*      GO TO 3999
00572 107* 3060 TM = T(LTA)
00573 107*      GO TO 3007
00574 107* 3065 TM = T(LTA)
00575 107*      GO TO 3032
00576 107* 3998 G(LG) = 1./(1./G1+1./G2)
00577 107*      IF(FLD(3,1,NSQ1(JJ1)).EQ.1) G(LG) = G1*G2
00501 107* 3999 JJ2 = JJ2+1
00602 107* 4000 CONTINUE
00603 107*      END
00604 108*      IF(FLD(3,1,NSQ1(JJ1)).EQ.0) GO TO 85
00606 109*      T1 = T(I)+460.0
00607 110*      T2 = T(LTA)+460.0
00610 111*      GV = G(LG)*(T1+T1+T2+T2)*(T1+T2)
00611 112*      GO TO 90
00612 113* 85  GV = G(LG)
00613 114* 90  Q(I) = Q(I)+GV*T(LTA)
00614 115*      GSUM = GSUM+GV
00614 116* C      CHECK FOR LAST CONDUCTOR TO THIS NODE
00615 117*      IF(NSQ1(JJ1).GT.0) GO TO 80
00617 118*      T2 = DO+T(I)+DN*Q(I)/GSUM
00620 119*      T1 = ABS(T(I)-T2)
00620 120* C      STORE THE NEW TEMPERATURES AND EXTRAPOLATION FACTORS
00621 121*      GO TO(120,115,110),JJ
00622 122* 110 LE1 = IE1+1
00623 123*      LE2 = IE2+1
00624 124*      R1 = T2-T(I)
00625 125*      X(LE1) = T(I)
00626 126*      X(LE2) = R1/(R1-X(LE2))
00627 127*      GO TO 120
00630 128* 115 LE2 = IE2+1
00631 129*      X(LE2) = T2-T(I)
00632 130* 120 T(I) = T2
00633 131*      IF(RLXA.GT.T1) GO TO 125
00635 132*      RLXA = T1
00636 133*      N2 = I
00637 134* 125 CONTINUE
00641 135*      CON(30) = RLXA
00641 136* C      SEE IF THE RELAXATION CRITERIA ARE MET
00642 137* 130 IF(RLXA.LE.CON(19).AND.RLXD.LE.CON(26)) GO TO 150
00644 138*      IF(JJ.LE.2) GO TO 140
00646 139*      JJ = 0
00647 140*      DO 135 I = 1,NMC
00652 141*      LE2 = IE2+1
00652 142* C      SEE IF THE EXTRAPOLATION CRITERIA ARE MET
00653 143*      IF(X(LE2).GE.0.) GO TO 135
00655 144*      IF(X(LE2).LT.-8.) X(LE2) = -8.
00657 145*      LL1 = IE1+1
00660 146*      T(I) = X(LE2)*X(LE1)+(1.0-X(LE2))*T(I)
00661 147* 135 CONTINUE

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*NEW
*NEW
**2

CINDSL,CINDSL

00663 148* 140 IF(KON(7).NE.0) CALL OUTCAL
00665 149* 145 CONTINUE
00667 150* WRITE(6,882)
00671 151* WRITE(6,885) LAX
00674 152* KON(28) = KON(28)+2
00675 153* 150 KON(37) = N2
00676 154* IF(RLXA.GT.RLXD) GO TO 155
00700 155* CON(30) = RLXD
00701 156* KON(37) = N1
00701 157* C CHECK THE ENERGY BALANCE OF THE SYSTEM
00702 158* 155 CALL NOHLIN
00703 159* QOUT = 0.0
00704 160* QIN = 0.0
00705 161* J1 = 0
00706 162* DO 195 I = 1,NMC
00711 163* QIN = QIN+Q(I)
00712 164* 165 J1 = J1+1
00713 165* LTA = FLD(22,14,NSQ1(J1))
00714 166* IF(LTA.LE.NNC) GO TO 175
00716 167* LG = FLD(5,16,NSQ1(J1))
00717 168* IF(FLD(3,1,NSQ1(J1)).EQ.0) GO TO 170
00721 169* T1 = T(I)+460.0
00722 170* T2 = T(LTA)+460.0
00723 171* QOUT = QOUT+G(LG)*(T1**4-T2**4)
00724 172* GO TO 175
00725 173* 170 QOUT = QOUT+G(LG)*(T(I)-T(LTA))
00725 174* C CHECK FOR LAST CONDUCTOR TO THIS NODE
00726 175* 175 IF(NSQ1(J1).GT.0) GO TO 165
00730 176* 195 CONTINUE
00732 177* CON(32) = ABS(QIN-QOUT)
00733 178* CALL VARBL2
00734 179* CON(13) = CON(1)
00735 180* CALL OUTCAL
00736 181* WRITE(6,882)
00740 182* WRITE(6,883) KON(20),CON(32)
00744 183* KON(28) = KON(28)+2
00745 184* IF(CON(3).GT.CON(1)*1.000001) GO TO 5
00747 185* NTH = IE1
00750 186* NDIM = NLA
00751 187* -RETURN
00752 188* 994 WRITE(6,884)
00754 189* GO TO 1000
00755 190* 996 WRITE(6,886) NDIM
00760 191* GO TO 1000
00761 192* 997 WRITE(6,887)
00763 193* GO TO 1000
00764 194* 998 WRITE(6,888)
00766 195* GO TO 1000
00767 196* 999 WRITE(6,889)
00771 197* 1000 CALL OUTCAL
00772 198* CALL EXIT
00773 199* 882 FORMAT(1H)
00774 200* 883 FORMAT(10H LOOPCT = I6,10H ENGBAL = E12.5)
00775 201* 884 FORMAT(45H CINDSL REQUIRES LONG PSEUDO-COMPUTE SEQUENCE)
00776 202* 885 FORMAT(35H ITERATION COUNT EXCEEDED, LOOPCT = , I10)
00777 203* 886 FORMAT(1A,20H LOCATIONS AVAILABLE)
01000 204* 887 FORMAT(10H NO DRLXCA)
01001 205* 888 FORMAT(10H NO ARLXCA)
01002 206* 889 FORMAT(14H NO LOOP COUNT)
01003 207* END

02/17/77 09:02:41 AJSGED 0041AA100 000041

3 100

DATE 021777

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SYSTEMS IMPROVED NUMERICAL DIFFERENCING ANALYZER
DATE: 17 FEB 77 TIME: 09:14:49- - SINDA - - UNIVAC-1108 EXEC-8 - - ISD/KAISER VERSION
SYSTEM: 32-K2**UPD-81SD-SB SEC USED: 117.246 PAGE: 12

GEUSCENCE LTD, NETWORK NO.3, CASE 0, (1.3/1.3)

LINE NO.	TEMP. COLUMN 1	TEMP. COLUMN 2	TEMP. COLUMN 3	TEMP. COLUMN 4	TEMP. COLUMN 5	TEMP. COLUMN 6	TEMP. COLUMN 7	TEMP. COLUMN 8	TEMP. COLUMN 9
1.00000	2.50243	2.50243	2.50236	2.50233	2.50215	2.50161	2.50146	2.50128	2.50104
2.00000	5.00471	5.00471	5.00452	5.00444	5.00351	5.00326	5.00291	5.00246	5.00198
3.00000	7.50680	7.50680	7.50641	7.50602	7.50546	7.50485	7.50418	7.50349	7.50277
4.00000	10.0087	10.0087	10.0082	10.0079	10.0071	10.0062	10.0053	10.0044	10.0035
5.00000	12.5103	12.5103	12.5098	12.5094	12.5081	12.5072	12.5062	12.5051	12.5041
6.00000	15.0119	15.0119	15.0104	15.0099	15.0090	15.0081	15.0069	15.0057	15.0045
7.00000	17.5109	17.5109	17.5108	17.5106	17.5097	17.5087	17.5074	17.5062	17.5049
8.00000	20.0116	20.0116	20.0114	20.0111	20.0101	20.0090	20.0077	20.0065	20.0051
9.00000	22.5124	22.5124	22.5118	22.5113	22.5102	22.5090	22.5078	22.5066	22.5045
10.0000	25.0127	25.0127	25.0117	25.0111	25.0099	25.0088	25.0077	25.0059	25.0047
11.0000	27.5123	27.5123	27.5113	27.5106	27.5096	27.5086	27.5074	27.5061	27.5049
12.0000	30.0117	30.0117	30.0108	30.0102	30.0093	30.0084	30.0073	30.0060	30.0049
13.0000	32.5109	32.5109	32.5101	32.5096	32.5089	32.5078	32.5068	32.5058	32.5044
14.0000	35.0099	35.0099	35.0093	35.0090	35.0080	35.0072	35.0064	35.0049	35.0036
15.0000	37.5089	37.5089	37.5084	37.5080	37.5074	37.5067	37.5057	37.5038	37.5033
16.0000	40.0078	40.0078	40.0074	40.0071	40.0066	40.0061	40.0047	40.0037	40.0031
17.0000	42.5067	42.5067	42.5063	42.5060	42.5057	42.5053	42.5042	42.5035	42.5029
18.0000	45.0056	45.0056	45.0053	45.0051	45.0048	45.0043	45.0037	45.0031	45.0025
19.0000	47.5045	47.5045	47.5043	47.5041	47.5039	47.5035	47.5030	47.5026	47.5021
20.0000	50.0033	50.0033	50.0032	50.0031	50.0029	50.0026	50.0023	50.0019	50.0016
21.0000	52.5022	52.5022	52.5021	52.5021	52.5019	52.5016	52.5016	52.5013	52.5011
22.0000	55.0012	55.0012	55.0011	55.0010	55.0010	55.0010	55.0008	55.0007	55.0006
23.0000	57.5002	57.5002	57.5002	57.5002	57.5002	57.5002	57.5002	57.5004	57.5003
24.0000	59.9993	59.9993	59.9993	59.9994	59.9994	59.9995	59.9999	60.0002	60.0000
25.0000	62.4985	62.4985	62.4985	62.4986	62.4988	62.4991	62.4996	62.4998	62.4997
26.0000	64.9976	64.9976	64.9980	64.9981	64.9986	64.9989	64.9992	64.9993	64.9994
27.0000	67.4977	67.4977	67.4980	67.4983	67.4982	67.4985	67.4988	67.4989	67.4991
28.0000	69.9963	69.9963	69.9980	69.9979	69.9979	69.9982	69.9984	69.9987	69.9990
29.0000	72.4962	72.4962	72.4976	72.4976	72.4977	72.4980	72.4983	72.4985	72.4988
30.0000	74.9978	74.9978	74.9976	74.9974	74.9976	74.9976	74.9981	74.9984	74.9987
31.0000	77.4976	77.4976	77.4973	77.4972	77.4974	77.4977	77.4980	77.4984	77.4991
32.0000	79.9971	79.9971	79.9971	79.9972	79.9975	79.9978	79.9981	79.9986	79.9990
33.0000	82.4971	82.4971	82.4972	82.4974	82.4977	82.4979	82.4982	82.4986	82.4989
34.0000	84.9973	84.9973	84.9974	84.9976	84.9979	84.9981	84.9984	84.9987	84.9989
35.0000	87.4976	87.4976	87.4978	87.4979	87.4981	87.4983	87.4985	87.4988	87.4991
36.0000	89.9981	89.9981	89.9982	89.9983	89.9985	89.9986	89.9988	89.9990	89.9992
37.0000	92.4986	92.4986	92.4986	92.4987	92.4989	92.4990	92.4991	92.4993	92.4994
38.0000	94.9991	94.9991	94.9991	94.9992	94.9992	94.9993	94.9994	94.9995	94.9996
39.0000	97.4995	97.4995	97.4995	97.4996	97.4996	97.4997	97.4997	97.4998	97.4998

SYSTEMS IMPROVED NUMERICAL DIFFERENCING ANALYZER

- - SINDA - - UNIVAC-1108 EXEC-8 - - ISD/KAISER VERSION

DATE: 17 FEB 77

TIME: 09:14:51

SYSTEM: 32-R2**UPD-BISD-SB

SEC USED: 117.757

PAGE: 13

GEOSCIENCE LTD, NETWORK NO.3, CASE 0, (1.37.3)

LINE NO.	TEMP. COLUMN 10	TEMP. COLUMN 11	TEMP. COLUMN 12	TEMP. COLUMN 13
1.00000	2.50076	2.50051	2.50025	2.50000
2.00000	5.00147	5.00095	5.00046	5.00000
3.00000	7.50204	7.50133	7.50065	7.50000
4.00000	10.0025	10.0017	10.0008	10.0000
5.00000	12.5030	12.5020	12.5010	12.5000
6.00000	15.0034	15.0022	15.0011	15.0000
7.00000	17.5037	17.5024	17.5012	17.5000
8.00000	20.0035	20.0024	20.0012	20.0000
9.00000	22.5035	22.5024	22.5012	22.5000
10.0000	25.0036	25.0024	25.0012	25.0000
11.0000	27.5037	27.5024	27.5012	27.5000
12.0000	30.0036	30.0024	30.0012	30.0000
13.0000	32.5034	32.5023	32.5011	32.5000
14.0000	35.0029	35.0020	35.0010	35.0000
15.0000	37.5027	37.5018	37.5009	37.5000
16.0000	40.0024	40.0017	40.0008	40.0000
17.0000	42.5022	42.5015	42.5007	42.5000
18.0000	45.0019	45.0013	45.0006	45.0000
19.0000	47.5016	47.5010	47.5005	47.5000
20.0000	50.0012	50.0008	50.0004	50.0000
21.0000	52.5006	52.5005	52.5003	52.5000
22.0000	55.0005	55.0003	55.0002	55.0000
23.0000	57.5002	57.5001	57.5001	57.5000
24.0000	60.0000	60.0000	60.0000	60.0000
25.0000	62.4997	62.4998	62.4999	62.5000
26.0000	64.9995	64.9997	64.9998	65.0000
27.0000	67.4994	67.4996	67.4998	67.5000
28.0000	69.9992	69.9995	69.9998	70.0000
29.0000	72.4991	72.4994	72.4997	72.5000
30.0000	74.9992	74.9995	74.9997	75.0000
31.0000	77.4993	77.4995	77.4997	77.5000
32.0000	79.9992	79.9995	79.9997	80.0000
33.0000	82.4992	82.4995	82.4997	82.5000
34.0000	84.9992	84.9995	84.9998	85.0000
35.0000	87.4993	87.4996	87.4998	87.5000
36.0000	89.9994	89.9996	89.9998	90.0000
37.0000	92.4996	92.4997	92.4999	92.5000
38.0000	94.9997	94.9998	94.9999	95.0000
39.0000	97.4998	97.4999	97.5000	97.5000

SYSTEMS IMPROVED NUMERICAL DIFFERENCING ANALYZER
DATE: 10 FEB 77 TIME: 00:31:10

- - SINDA. - - UNIVAC-1108 EXEC-8 - - ISU/KAISER VERSION
SYSTEM: 32-K2**UPU-BISU-SB SEC USED: 35.869 PAGE: 12

GEOSCIENCE LTD. NETWORK NO.3, CASE 1, (1.17.3)

LINE NO.	TEMP. COLUMN 1	TEMP. COLUMN 2	TEMP. COLUMN 3	TEMP. COLUMN 4	TEMP. COLUMN 5	TEMP. COLUMN 6	TEMP. COLUMN 7	TEMP. COLUMN 8	TEMP. COLUMN 9
1.00000	2.50070	2.50070	2.50077	2.50075	2.50072	2.50067	2.50062	2.50054	2.50045
2.00000	5.00159	5.00159	5.00160	5.00157	5.00151	5.00145	5.00130	5.00113	5.00093
3.00000	7.50255	7.50255	7.50257	7.50251	7.50245	7.50229	7.50207	7.50179	7.50148
4.00000	10.00330	10.00330	10.0037	10.0036	10.0035	10.0033	10.0029	10.0026	10.0021
5.00000	12.5052	12.5052	12.5051	12.5050	12.5047	12.5044	12.5040	12.5035	12.5029
6.00000	15.0069	15.0069	15.0067	15.0065	15.0062	15.0058	15.0052	15.0045	15.0041
7.00000	17.5088	17.5088	17.5086	17.5084	17.5082	17.5076	17.5068	17.5062	17.5052
8.00000	20.0111	20.0111	20.0113	20.0112	20.0106	20.0097	20.0087	20.0077	20.0065
9.00000	22.5142	22.5142	22.5142	22.5140	22.5132	22.5122	22.5109	22.5096	22.5080
10.0000	25.0177	25.0177	25.0175	25.0171	25.0163	25.0151	25.0136	25.0119	25.0099
11.0000	27.5218	27.5218	27.5216	27.5211	27.5202	27.5188	27.5169	27.5147	27.5121
12.0000	30.0209	30.0209	30.0206	30.0200	30.0249	30.0231	30.0208	30.0180	30.0150
13.0000	32.5334	32.5334	32.5327	32.5319	32.5305	32.5283	32.5255	32.5224	32.5183
14.0000	35.0411	35.0411	35.0399	35.0390	35.0372	35.0346	35.0314	35.0271	35.0222
15.0000	37.5498	37.5498	37.5485	37.5474	37.5453	37.5421	37.5376	37.5327	37.5269
16.0000	40.0602	40.0602	40.0588	40.0575	40.0548	40.0507	40.0455	40.0394	40.0324
17.0000	42.5728	42.5728	42.5712	42.5696	42.5661	42.5611	42.5552	42.5475	42.5389
18.0000	45.0881	45.0881	45.0859	45.0838	45.0797	45.0743	45.0663	45.0570	45.0467
19.0000	47.6003	47.6003	47.6035	47.6010	47.5962	47.5888	47.5794	47.5681	47.5557
20.0000	50.1202	50.1202	50.1250	50.1218	50.1156	50.1065	50.0948	50.0811	50.0660
21.0000	52.6557	52.6557	52.6509	52.6469	52.6388	52.6272	52.6127	52.5960	52.5779
22.0000	55.1896	55.1896	55.1823	55.1769	55.1665	55.1517	55.1337	55.1133	55.0915
23.0000	57.7313	57.7313	57.7208	57.7135	57.6996	57.6806	57.6580	57.6330	57.6007
24.0000	60.2835	60.2835	60.2682	60.2582	60.2399	60.2148	60.1859	60.1551	60.1236
25.0000	62.8517	62.8517	62.8260	62.8143	62.7877	62.7541	62.7172	62.6792	62.6415
26.0000	65.4407	65.4407	65.4061	65.3631	65.3445	65.2988	65.2513	65.2048	65.1594
27.0000	68.0680	68.0680	68.0101	67.9700	67.9113	67.8480	67.7872	67.7301	67.6776
28.0000	70.6177	70.6177	70.6575	70.5804	70.4869	70.3988	70.3210	70.2530	70.1932
29.0000	73.1283	73.1283	73.3851	73.2150	73.0631	72.9434	72.8462	72.7704	72.7042
30.0000	75.6255	75.6255	76.2924	75.8455	75.6159	75.4690	75.3624	75.2786	75.2090
31.0000	78.8301	78.8301	78.5503	78.2871	78.0479	77.9612	77.8576	77.7750	77.7063
32.0000	80.9500	80.9500	80.8257	80.6736	80.5380	80.4266	80.3356	80.2602	80.1961
33.0000	83.2096	83.2096	83.1467	83.0557	82.9622	82.8706	82.8017	82.7368	82.6799
34.0000	85.5409	85.5409	85.5052	85.4484	85.3840	85.3202	85.2610	85.2073	85.1587
35.0000	87.9124	87.9124	87.8405	87.8537	87.8092	87.7625	87.7170	87.6742	87.6344
36.0000	90.3083	90.3083	90.2944	90.2702	90.2596	90.2062	90.1723	90.1396	90.1064
37.0000	92.7200	92.7200	92.7112	92.6954	92.6750	92.6520	92.6281	92.6045	92.5816
38.0000	95.1417	95.1417	95.1365	95.1270	95.1144	95.1000	95.0848	95.0694	95.0544
39.0000	97.5694	97.5694	97.5670	97.5625	97.5565	97.5496	97.5422	97.5340	97.5272

SYSTEMS IMPROVED NUMERICAL DIFFERENCING ANALYZER - - SINDA - - UNIVAC-1108 EXEC-8 - - 1SD/KAISER VERSION
 DATE: 10 FEB 77 TIME: 06:31:12 SYSTEM: 32-K2*UPD-B1SD-SB SEC USED: 36.376 PAGE: 13

GEOSCIENCE LTD, NETWORK NO.3, CASE 1, (17.3)

LINE NO.	TEMP. COLUMN 10	TEMP. COLUMN 11	TEMP. COLUMN 12	TEMP. COLUMN 13
1.00000	2.50035	2.50024	2.50012	2.50000
2.00000	5.00072	5.00049	5.00025	5.00000
3.00000	7.50113	7.50077	7.50041	7.50000
4.00000	10.0016	10.0012	10.0006	10.0000
5.00000	12.5024	12.5016	12.5008	12.5000
6.00000	15.0032	15.0022	15.0011	15.0000
7.00000	17.5040	17.5028	17.5014	17.5000
8.00000	20.0051	20.0034	20.0017	20.0000
9.00000	22.5062	22.5042	22.5021	22.5000
10.0000	25.0076	25.0052	25.0026	25.0000
11.0000	27.5093	27.5063	27.5032	27.5000
12.0000	30.0115	30.0077	30.0039	30.0000
13.0000	32.5140	32.5094	32.5047	32.5000
14.0000	35.0170	35.0115	35.0058	35.0000
15.0000	37.5206	37.5139	37.5069	37.5000
16.0000	40.0247	40.0167	40.0083	40.0000
17.0000	42.5297	42.5200	42.5100	42.5000
18.0000	45.0356	45.0239	45.0119	45.0000
19.0000	47.5423	47.5284	47.5142	47.5000
20.0000	50.0501	50.0335	50.0167	50.0000
21.0000	52.5569	52.5393	52.5196	52.5000
22.0000	55.0689	55.0458	55.0228	55.0000
23.0000	57.5800	57.5530	57.5263	57.5000
24.0000	60.0920	60.0608	60.0301	60.0000
25.0000	62.6046	62.5688	62.5339	62.5000
26.0000	65.1173	65.0767	65.0377	65.0000
27.0000	67.6292	67.5840	67.5413	67.5000
28.0000	70.1393	70.0902	70.0441	70.0000
29.0000	72.6464	72.5943	72.5460	72.5000
30.0000	75.1491	75.0958	75.0467	75.0000
31.0000	77.6471	77.5944	77.5460	77.5000
32.0000	80.1403	80.0902	80.0439	80.0000
33.0000	82.6292	82.5832	82.5406	82.5000
34.0000	85.1146	85.0741	85.0362	85.0000
35.0000	87.5976	87.5633	87.5310	87.5000
36.0000	90.0791	90.0515	90.0253	90.0000
37.0000	92.5598	92.5390	92.5192	92.5000
38.0000	95.0400	95.0202	95.0129	95.0000
39.0000	97.5200	97.5131	97.5065	97.5000

SYSTEMS IMPROVED NUMERICAL DIFFERENCING ANALYZER
 DATE: 18 FEB 77 TIME: 06:41:36

- - SINDA - - UNIVAC-1108 EXEC-8
 SYSTEM: 32-R2**UPD-BISD-SB

- - 1SD/KAISER VERSION
 SEC USED: 36.556 PAGE: 13

GEOSCIENCE LTD, NETWORK NO.3, CASE 5, (10.07.5)

LINE NO.	TEMP. COLUMN 10	TEMP. COLUMN 11	TEMP. COLUMN 12	TEMP. COLUMN 13
1.00000	2.42880	2.45276	2.47657	2.50000
2.00000	4.85650	4.90454	4.95262	5.00000
3.00000	7.28175	7.35555	7.42744	7.50000
4.00000	9.70577	9.80325	9.90237	10.0000
5.00000	12.1247	12.2508	12.3762	12.5000
6.00000	14.5401	14.6950	14.8485	15.0000
7.00000	16.9500	17.1356	17.3191	17.5000
8.00000	19.3464	19.5677	19.7856	20.0000
9.00000	21.7402	21.9977	22.2512	22.5000
10.0000	24.1276	24.4238	24.7146	25.0000
11.0000	26.5078	26.8452	27.1756	27.5000
12.0000	28.8799	29.2605	29.6334	30.0000
13.0000	31.2409	31.6684	32.0879	32.5000
14.0000	33.5908	34.0697	34.5388	35.0000
15.0000	35.9308	36.4637	36.9832	37.5000
16.0000	38.2609	38.8451	39.4283	40.0000
17.0000	40.5787	41.2272	41.8698	42.5000
18.0000	42.8766	43.6011	44.3086	45.0000
19.0000	45.1611	45.9742	46.7459	47.5000
20.0000	47.4859	48.3470	49.1836	50.0000
21.0000	49.7920	50.7210	51.6215	52.5000
22.0000	52.0990	53.0936	54.0587	55.0000
23.0000	54.4004	55.4666	56.4974	57.5000
24.0000	56.7146	57.8535	58.9439	60.0000
25.0000	59.0337	60.2596	61.3996	62.5000
26.0000	61.4493	62.6926	63.8682	65.0000
27.0000	63.8808	65.1538	66.3506	67.5000
28.0000	66.3775	67.6538	68.8518	70.0000
29.0000	68.9365	70.1972	71.3741	72.5000
30.0000	71.5826	72.7915	73.9200	75.0000
31.0000	74.3058	75.4316	76.4875	77.5000
32.0000	77.0900	78.1107	79.0738	80.0000
33.0000	79.9190	80.8188	81.6743	82.5000
34.0000	82.7757	83.5463	84.2847	85.0000
35.0000	85.6464	86.2848	86.9010	87.5000
36.0000	88.5217	89.0283	89.5203	90.0000
37.0000	91.3961	91.7729	92.1406	92.5000
38.0000	94.2671	94.5165	94.7606	95.0000
39.0000	97.1345	97.2587	97.3806	97.5000

5-2-75 APPROVED NUMERICAL DIFFERENCING ANALYZER
 DATE: 18 FEB 77 TIME: 08:47:35

- - SINDA - - UNIVAC-1108 EXEC-8 - - ISD/KAISER VERSION
 SYSTEM: 32-K2*UPD-BISU-SB SEC USED: 38.248 PAGE: 12

GEOSCIENCE LTD, NETWORK NO.3, CASE 12, (20.07.3)

LINE NO.	TEMP. COLUMN 1	TEMP. COLUMN 2	TEMP. COLUMN 3	TEMP. COLUMN 4	TEMP. COLUMN 5	TEMP. COLUMN 6	TEMP. COLUMN 7	TEMP. COLUMN 8	TEMP. COLUMN 9
1.00000	2.11105	2.11105	2.11104	2.15710	2.19432	2.23996	2.28016	2.31822	2.35020
2.00000	4.21450	4.21450	4.21279	4.31055	4.34535	4.47751	4.55623	4.63400	4.71041
3.00000	6.30265	6.30265	6.30272	6.44395	6.57843	6.70508	6.82714	6.94308	7.05725
4.00000	8.38414	8.38414	8.38503	8.57669	8.75057	8.92591	9.00221	9.23030	9.39154
5.00000	10.4551	10.4551	10.4542	10.6982	10.9226	11.0873	11.3112	11.5291	11.7307
6.00000	12.5016	12.5018	12.4967	12.8001	13.0102	13.2873	13.5543	13.8200	14.0700
7.00000	14.5100	14.5100	14.5133	14.8070	15.1462	15.4804	15.7984	16.1022	16.3703
8.00000	16.5028	16.5028	16.5015	16.8908	17.2787	17.6572	18.0178	18.3577	18.6939
9.00000	18.4497	18.4497	18.4442	18.9335	19.3803	19.8055	20.2170	20.6096	20.9988
10.0000	20.3427	20.3427	20.3459	20.9109	21.4313	21.9197	22.3410	22.8464	23.2930
11.0000	22.2208	22.2208	22.2208	22.8614	23.4432	24.0013	24.5392	25.0616	25.5707
12.0000	24.0707	24.0707	24.0799	24.7747	25.4252	26.0537	26.6614	27.2535	27.8295
13.0000	25.8815	25.8815	25.8787	26.6596	27.3807	28.0822	28.7645	29.4290	30.0747
14.0000	27.0212	27.0212	27.0270	28.4935	29.3084	30.0902	30.8512	31.5895	32.3071
15.0000	29.3308	29.3308	29.3450	30.3075	31.2098	32.0768	32.9184	33.7340	34.5260
16.0000	31.0130	31.0130	31.0185	32.0877	33.0833	34.0386	34.9639	35.8618	36.7296
17.0000	32.0179	32.0179	32.0159	33.8172	34.9137	35.9687	36.9859	37.9556	38.8894
18.0000	34.1200	34.1200	34.1322	35.4694	36.6968	37.8618	38.9586	39.9830	41.0513
19.0000	35.5781	35.5781	35.5914	37.0720	38.4342	39.7085	40.8464	42.0410	43.1978
20.0000	36.9805	36.9805	36.9928	38.6286	40.1291	41.4491	42.7905	44.0636	45.3499
21.0000	38.2970	38.2970	38.3034	40.1288	41.7454	43.2710	44.7170	46.1501	47.5208
22.0000	39.5020	39.5020	39.5187	41.5502	43.3678	45.0085	46.6485	48.2131	49.6972
23.0000	40.6392	40.6392	40.6638	42.9159	44.9570	46.7745	48.5884	50.2843	51.8730
24.0000	41.7110	41.7110	41.7375	44.2500	46.4822	48.5892	50.5524	52.3701	54.0606
25.0000	42.6037	42.6037	42.7056	45.5495	48.0867	50.4212	52.5505	54.5058	56.2989
26.0000	43.5207	43.5207	43.5028	46.8234	49.7200	52.3127	54.6278	56.7033	58.6065
27.0000	44.2805	44.2805	44.3312	48.1325	51.4507	54.3247	56.8099	59.0524	60.9898
28.0000	44.9391	44.9391	45.0053	49.5799	53.3979	56.5378	59.2005	61.4807	63.4810
29.0000	45.4025	45.4025	45.5730	51.3848	55.7498	59.1151	61.8391	64.1254	66.1110
30.0000	45.7094	45.7094	46.0519	54.1490	58.8147	62.1602	64.7931	66.9850	68.8800
31.0000	54.7709	54.7709	56.0404	54.6548	62.9978	65.7310	68.0554	70.0493	71.7938
32.0000	62.2553	62.2553	63.2451	65.2991	67.4898	69.6341	71.5601	73.2694	74.8172
33.0000	68.5095	68.5095	69.2590	70.5127	72.0575	73.6548	75.1770	76.6014	77.9205
34.0000	74.0717	74.0717	74.5412	75.3821	76.4062	77.6408	78.8288	79.9537	81.0834
35.0000	79.0151	79.0151	79.3372	79.9183	80.6771	81.5498	82.4628	83.3722	84.2573
36.0000	83.5775	83.5775	83.7954	84.1911	84.7296	85.3074	86.0448	86.7398	87.4227
37.0000	87.8704	87.8704	88.0197	88.2776	88.6558	89.0915	89.5783	90.0781	90.5817
38.0000	92.0048	92.0048	92.0851	92.2538	92.4803	92.7599	93.0732	93.3995	93.7293
39.0000	96.0202	96.0202	96.0601	96.1442	96.2530	96.3915	96.5438	96.7043	96.8672

SYSTEMS IMPROVED NUMERICAL DIFFERENCING ANALYZER
 DATE: 18 FEB 77

TIME: 08:47:36

- - SINDA - - UNIVAC-1108 EXEC-8
 SYSTEM: 32-R2**UPD-BISD-SB

- - ISD/KAISER VERSION
 SEC USED: 38.797

PAGE: 13

GEOSCIENCE LTD, NETWORK NO.3, CASE 12, (20.0/.3)

LINE NO.	TEMP. COLUMN 10	TEMP. COLUMN 11	TEMP. COLUMN 12	TEMP. COLUMN 13
1.00000	2.39239	2.42907	2.46491	2.50000
2.00000	4.78471	4.85777	4.92790	5.00000
3.00000	7.17221	7.28073	7.39152	7.50000
4.00000	9.55384	9.70496	9.85391	10.0000
5.00000	11.9341	12.1270	12.3153	12.5000
6.00000	14.3021	14.5410	14.7688	15.0000
7.00000	16.6686	16.9436	17.2247	17.5000
8.00000	19.0242	19.3536	19.6796	20.0000
9.00000	21.3819	21.7605	22.1337	22.5000
10.0000	23.7316	24.1614	24.5846	25.0000
11.0000	26.0677	26.5541	27.0313	27.5000
12.0000	28.3904	28.9383	29.4713	30.0000
13.0000	30.7029	31.3093	31.9108	32.5000
14.0000	33.0021	33.6811	34.3465	35.0000
15.0000	35.2952	36.0438	36.7769	37.5000
16.0000	37.5729	38.3986	39.2014	40.0000
17.0000	39.8269	40.7438	41.6325	42.5000
18.0000	42.0694	43.0867	44.0543	45.0000
19.0000	44.3294	45.4194	46.4736	47.5000
20.0000	46.5801	47.7590	48.8945	50.0000
21.0000	48.8425	50.1015	51.3171	52.5000
22.0000	51.1028	52.4475	53.7416	55.0000
23.0000	53.3712	54.7982	56.1692	57.5000
24.0000	55.6551	57.1623	58.6037	60.0000
25.0000	57.9729	59.5464	61.0494	62.5000
26.0000	60.3443	61.9692	63.5120	65.0000
27.0000	62.7742	64.4303	65.9950	67.5000
28.0000	65.2633	66.9422	68.5023	70.0000
29.0000	67.6850	69.3113	71.0370	72.5000
30.0000	70.5833	72.1395	73.5997	75.0000
31.0000	73.3694	74.8216	76.1878	77.5000
32.0000	76.2347	77.5523	78.7991	80.0000
33.0000	79.1607	80.3229	81.4308	82.5000
34.0000	82.1270	83.1199	84.0739	85.0000
35.0000	85.1073	85.9294	86.7257	87.5000
36.0000	88.0935	88.7462	89.3808	90.0000
37.0000	91.0777	91.5629	92.0367	92.5000
38.0000	94.0561	94.3772	94.6918	95.0000
39.0000	97.0294	97.1892	97.3461	97.5000

- SYSTEMS IMPROVED NUMERICAL DIFFERENCING ANALYZER - - SINDA - - UNIVAC-1108 EXEC-8 - - ISD/KAISER VERSION
 DATE: 04 FEB 77 TIME: 08:46:01 SYSTEM: 32-R2**UPD-BISD-SB SEC USED: 72.950 PAGE:

GEOSCIENCE LTD - NETWORK NO.2, CASE 0 (1.3/1.3)

[illegible]

SYSTEMS IMPROVED NUMERICAL DIFFERENCING ANALYZER - - SINDA - - UNIVAC-1108 EXEC-8 - - ISD/KAISER VERSION
 DATE: 04 FEB 77 TIME: 09:03:27 SYSTEM: 32-R2**UPD-BISD-SB SEC USED: 74.355 PAGE: 5

GEOSCIENCE LTD, NETWORK NO.2,CASE 1 (0.1/0.5)

LINE NO.	TEMP. COLUMN 1	TEMP. COLUMN 2	TEMP. COLUMN 3	TEMP. COLUMN 4	TEMP. COLUMN 5	TEMP. COLUMN 6	TEMP. COLUMN 7
1.00000	2.50033	2.50033	2.50029	2.50025	2.50019	2.50010	2.50000
2.00000	5.00072	5.00072	5.00062	5.00055	5.00041	5.00021	5.00000
3.00000	7.50120	7.50120	7.50105	7.50093	7.50069	7.50036	7.50000
4.00000	10.0018	10.0018	10.0016	10.0014	10.0011	10.0006	10.0000
5.00000	12.5028	12.5028	12.5024	12.5022	12.5016	12.5008	12.5000
6.00000	15.0042	15.0042	15.0036	15.0032	15.0024	15.0013	15.0000
7.00000	17.5062	17.5062	17.5054	17.5048	17.5035	17.5019	17.5000
8.00000	20.0091	20.0091	20.0079	20.0070	20.0052	20.0027	20.0000
9.00000	22.5134	22.5134	22.5116	22.5103	22.5076	22.5040	22.5000
10.0000	25.0197	25.0197	25.0171	25.0151	25.0112	25.0059	25.0000
11.0000	27.5290	27.5290	27.5251	27.5221	27.5163	27.5086	27.5000
12.0000	30.0428	30.0428	30.0368	30.0325	30.0239	30.0125	30.0000
13.0000	32.5633	32.5633	32.5540	32.5475	32.5349	32.5182	32.5000
14.0000	35.0942	35.0942	35.0795	35.0696	35.0508	35.0264	35.0000
15.0000	37.6418	37.6418	37.6172	37.6017	37.5735	37.5380	37.5000
16.0000	40.2171	40.2171	40.1736	40.1484	40.1056	40.0540	40.0000
17.0000	42.8414	42.8414	42.7590	42.7156	42.6493	42.5750	42.5000
18.0000	45.5584	45.5584	45.3922	45.3101	45.2055	45.1005	45.0000
19.0000	48.4625	48.4625	48.1118	47.9350	47.7687	47.6267	47.5000
20.0000	51.7610	51.7610	51.0202	50.5638	50.3172	50.1444	50.0000
21.0000	55.5729	55.5729	53.2867	53.0122	52.8050	52.6415	52.5000
22.0000	59.7067	59.7067	55.5768	55.4134	55.2605	55.1243	55.0000
23.0000	57.9869	57.9869	57.9185	57.8174	57.7084	57.6018	57.5000
24.0000	60.3440	60.3440	60.3037	60.2380	60.1606	60.0798	60.0000
25.0000	62.7465	62.7465	62.7207	62.6764	62.6211	62.5609	62.5000
26.0000	65.1781	65.1781	65.1607	65.1299	65.0902	65.0457	65.0000
27.0000	67.6293	67.6293	67.6172	67.5954	67.5666	67.5339	67.5000
28.0000	70.0940	70.0940	70.0855	70.0699	70.0490	70.0250	70.0000
29.0000	72.5665	72.5665	72.5624	72.5511	72.5359	72.5184	72.5000
30.0000	75.0499	75.0499	75.0455	75.0373	75.0263	75.0135	75.0000
31.0000	77.5364	77.5364	77.5332	77.5272	77.5192	77.5098	77.5000
32.0000	80.0265	80.0265	80.0241	80.0196	80.0140	80.0072	80.0000
33.0000	82.5192	82.5192	82.5175	82.5144	82.5102	82.5052	82.5000
34.0000	85.0139	85.0139	85.0126	85.0104	85.0073	85.0038	85.0000
35.0000	87.5099	87.5099	87.5090	87.5074	87.5052	87.5027	87.5000
36.0000	90.0069	90.0069	90.0063	90.0052	90.0037	90.0019	90.0000
37.0000	92.5047	92.5047	92.5043	92.5035	92.5025	92.5013	92.5000
38.0000	95.0029	95.0029	95.0026	95.0022	95.0015	95.0008	95.0000
39.0000	97.5014	97.5014	97.5012	97.5010	97.5007	97.5004	97.5000

SYSTEMS IMPROVED NUMERICAL DIFFERENCING ANALYZER - - SINDA - - UNIVAC-1108 EXEC-8 - - ISO/KAISER VERSION
 DATE: 04 FEB 77 TIME: 09:16:02 SYSTEM: 32-K2**UPD-BISD-SB SEC USED: 82.084 PAGE: 6

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LINE NO.	TEMP. COLUMN 1	TEMP. COLUMN 2	TEMP. COLUMN 3	TEMP. COLUMN 4	TEMP. COLUMN 5	TEMP. COLUMN 6	TEMP. COLUMN 7
1.00000	2.07206	2.07206	2.07393	2.18977	2.29733	2.40065	2.50000
2.00000	4.13307	4.13307	4.13905	4.37225	4.59051	4.79920	5.00000
3.00000	6.18404	6.18404	6.19372	6.54654	6.87708	7.19433	7.50000
4.00000	8.21974	8.21974	8.23175	8.70334	9.15369	9.58539	10.0000
5.00000	10.2280	10.2280	10.2417	10.8512	11.4233	11.9720	12.5000
6.00000	12.1981	12.1981	12.2175	12.9700	13.6748	14.3497	15.0000
7.00000	14.1348	14.1348	14.1594	15.0603	15.9065	16.7177	17.5000
8.00000	16.0334	16.0334	16.0616	17.1217	18.1190	19.0782	20.0000
9.00000	17.8783	17.8783	17.9087	19.1493	20.3124	21.4276	22.5000
10.0000	19.6549	19.6549	19.6932	21.1287	22.4734	23.7016	25.0000
11.0000	21.3710	21.3710	21.4171	23.0579	24.6037	26.0814	27.5000
12.0000	23.0221	23.0221	23.0737	24.9443	26.7048	28.3861	30.0000
13.0000	24.5873	24.5873	24.6429	26.7752	28.7684	30.6734	32.5000
14.0000	26.0477	26.0477	26.1140	28.5298	30.7923	32.9431	35.0000
15.0000	27.4120	27.4120	27.4910	30.2287	32.7858	35.2006	37.5000
16.0000	28.6729	28.6729	28.7629	31.8786	34.7602	37.4529	40.0000
17.0000	29.8036	29.8036	29.9072	33.4933	36.7424	39.7186	42.5000
18.0000	30.7753	30.7753	30.9117	35.1288	38.7961	42.0333	45.0000
19.0000	31.5712	31.5712	31.7780	36.9625	41.0583	44.4615	47.5000
20.0000	32.1155	32.1155	32.3435	39.4723	43.7721	47.0414	50.0000
21.0000	34.0609	34.0609	40.8543	44.1258	47.2215	49.9752	52.5000
22.0000	45.8359	45.8359	46.7854	48.6804	50.8275	52.9550	55.0000
23.0000	50.9221	50.9221	51.5884	52.8122	54.3170	55.9126	57.5000
24.0000	55.2521	55.2521	55.7127	56.5470	57.6172	58.7984	60.0000
25.0000	59.0589	59.0589	59.3798	59.9659	60.7347	61.6036	62.5000
26.0000	62.4980	62.4980	62.7262	63.1439	63.6997	64.3370	65.0000
27.0000	65.6779	65.6779	65.8415	66.1420	66.5456	67.0122	67.5000
28.0000	68.6717	68.6717	68.7897	69.0073	69.3010	69.6422	70.0000
29.0000	71.5309	71.5309	71.6166	71.7748	71.9888	72.2381	72.5000
30.0000	74.2930	74.2930	74.3555	74.4706	74.6264	74.8085	75.0000
31.0000	76.9647	76.9647	77.0300	77.1135	77.2272	77.3601	77.5000
32.0000	79.6249	79.6249	79.6577	79.7185	79.8013	79.8981	80.0000
33.0000	82.2277	82.2277	82.2514	82.2956	82.3557	82.4260	82.5000
34.0000	84.8033	84.8033	84.8203	84.8523	84.8957	84.9465	85.0000
35.0000	87.3591	87.3591	87.3715	87.3944	87.4254	87.4617	87.5000
36.0000	89.9011	89.9011	89.9098	89.9259	89.9477	89.9731	90.0000
37.0000	92.4334	92.4334	92.4392	92.4500	92.4647	92.4819	92.5000
38.0000	94.9590	94.9590	94.9625	94.9692	94.9783	94.9888	95.0000
39.0000	97.4805	97.4805	97.4822	97.4853	97.4896	97.4947	97.5000

CYLINDRICAL AND SPHERICAL TRANSIENT HEAT
TRANSFER SOLUTIONS AND THEIR APPLICATION TO
THERMAL CONDUCTIVITY MEASUREMENTS IN
THE EARTH

Energy Research & Development Administration
Contract E(04-3)-1318

H. F. Poppendiek
D. J. Connelly

February 1977

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INTRODUCTION

One of the geothermal transducers that is being developed by Geoscience for ERDA in connection with measurement of the geothermal heat flux in drillholes is a thermal conductivity probe; this sensor also measures the vertical temperature gradient in the earth. Upon measuring the thermal conductivity of the earth and the vertical temperature gradient, their product, the heat flux, is established.

Classical mathematical transient temperature solutions for cylindrical and spherical transducers exposed to step function surface heat releases are described and the corresponding surface temperature time functions evaluated for computational use.

The mathematical solutions are also extended to include the presence of water or air annuli that space the transducers from the drillhole surfaces. Finally, preliminary design information for a cylindrical thermal conductivity probe system is outlined.

MATHEMATICAL TRANSIENT HEAT TRANSFER SOLUTIONS FOR CYLINDRICAL AND SPHERICAL TRANSDUCERS

A. Cylindrical System

The report describes a thermal conductivity probe which is to be used in drillholes; a step function surface heat flux is added to the probe system and then the thermal conductivity of the earth is extracted from the time-temperature measurements (see Figure 1). It is clear that for this transient heating process, a second thermal parameter, namely the thermal capacity per unit volume (product of heat capacity and density) also controls the time dependent temperature field. Thus, to be more correct, the probe should probably be called a thermal conductivity-thermal capacity per unit volume probe.

The classical solution which defines transient heat flow in a cylindrical system with constant surface heating is:

$$\frac{\partial t}{\partial \theta} = \frac{k_{\infty}}{\gamma_{\infty} c_{p_{\infty}}} \left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \frac{\partial t}{\partial r} \right] \quad (1)$$

where

t , temperature at distance r

θ , time

k_{∞} , thermal conductivity of the earth

γ_{∞} , density of the earth

Figure 1. Cylindrical thermal conductivity probe concept.

c_p , specific heat of the earth

r , radial distance from the cylinder centerline

Equation (1) is first solved for an instantaneous line source which involves an integral representation.¹ The resulting temperature distribution caused by a source, M , at x' , y' is

$$t = \frac{M}{4\pi a_\infty \theta} e^{-\frac{(x-x')^2 + (y-y')^2}{4a_\infty \theta}} \quad (2)$$

where

$Q = \gamma_\infty c_p M$ (the heat generated instantaneously per unit length)

$a_\infty =$ thermal diffusivity, $\frac{k_\infty}{\gamma_\infty c_p}$

$$r = \sqrt{x'^2 + y'^2}$$

Equation (2) for a line source can be extended to a cylinder if an instantaneous source of strength M is equally distributed over the surface of the cylinder. Specifically, the temperature distribution may be found by placing instantaneous line sources of strength $\frac{M}{2\pi} d\phi$ around the cylinder of radius r_o , where ϕ and r are the polar coordinates.² The derived temperature distribution is

$$t = \frac{M}{4\pi a_\infty \theta} e^{-\frac{r_o^2 + r^2}{4a_\infty \theta}} J_0\left(\frac{r_o r}{2a_\infty \theta}\right) \quad (3)$$

The instantaneous source temperature solutions was extended to the case of a cylinder with constant surface heating.³ Specifically, the source strength M is now a function of time requiring integration over that variable. The result of this extension is,

$$\frac{4\pi k_{\infty}}{2.30 q} t = \int_{-\infty}^{\log \frac{k_{\infty}}{\gamma_{\infty} c p_{\infty}} \theta} e^{-\frac{(r_o^2 + r^2)}{4 \frac{k_{\infty}}{\gamma_{\infty} c p_{\infty}} \theta}} J_0 \left(\frac{r_o r}{2 \frac{k_{\infty}}{\gamma_{\infty} c p_{\infty}} \theta} \right) d \left(\log \frac{k_{\infty}}{\gamma_{\infty} c p_{\infty}} \theta \right) \quad (4)$$

where q is the constant heat release per unit length.

This equation may be integrated graphically or numerically.

B. Spherical System

Upon using a solution method similar to that for cylinders, the time-temperature distribution in the earth as a result of a continuous spherical heat source is:¹

$$t = \frac{q'}{4 \gamma_{\infty} c p_{\infty} \pi^{3/2} a_{\infty}^{3/2}} \int_{\frac{1}{\sqrt{\theta}}}^{\infty} e^{-\frac{r^2}{4 a_{\infty}^2 \theta}} d \frac{1}{\sqrt{\theta}} \quad (5)$$

where q' is the constant heat source coming from the sphere.

CALCULATION RESULTS

A. Cylindrical System

The accuracy of the resulting calculations for Equation (4) which were found in the literature³ was not known. The derivation was rather early (1931) and there was a question as to how many terms had been used in the series expansion of the Bessel function. Therefore, Geoscience spent a little time recalculating the curve (using eight terms in the Bessel function expansion). It was found that the Geoscience calculations did not differ from the literature values. The plot of $4\pi k_{\infty} t / 2.30 q$ versus $\text{Log } (a_{\infty} \theta / r_0^2)$ is shown in Figure 2 for $r/r_0 = 1.0$.

B. Spherical System

A number of specific time-temperature calculations were made for a spherical system (using Equation (5)). Some illustrative results are shown in Figure 3 where constant heat releases per unit length are related to times and temperature rises at a radius of four inches.

No further work is anticipated for the spherical geometry at this time.

Figure 2. Generalized time versus temperature curve for a cylindrical step function heat release system.

Figure 3. Some time versus heat release curves for a spherical system.

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q' (watts)

$$\begin{aligned} k_{\infty} &= 1.0 \text{ Btu/hr ft}^2 \cdot \text{F/ft} \\ \gamma_{\infty} &= 200 \text{ lb/ft}^3 \\ c_{p\infty} &= 0.24 \text{ Btu/lb} \cdot \text{F} \end{aligned}$$

100

90

80

70

60

50

40

30

20

10

0

t = 15°F

t = 10°F

t = 5°F

 θ (min)

10

20

30

40

50

60

DISCUSSION

A. Solution Extensions to Include Air and Water Annuli

Exact two-region transient cylinder solutions (to account for air and water annuli around a thermal conductivity probe) can be performed analytically (see, for example, reference 4), but the complexity of the effort appears not to be worth it. Instead, two approximate solutions, one for an air annulus and one for a water annulus, are used.

1. Air Annulus System

A stagnant air annulus can be represented satisfactorily as a layer that has thermal resistance but no thermal capacitance (because of the low density of air). Therefore, it is proposed that the temperature drop across the air annulus is obtained from the classical logarithmic annulus conduction equation (using the constant, known heat release and the known radii and air thermal conductivity). The temperature drop so determined is then used to extrapolate the probe surface temperature to the drill-hole temperature.

2. Water Annulus System

A stagnant water (or mud) annulus has both thermal resistance and thermal capacitance. Therefore, two equivalent earth annuli are determined, one having the same thermal resistance and the other having the same thermal capacitance as the water annulus. These two annuli have different thicknesses. Then a mean annulus thickness is obtained (by a simple average) and it used to establish a new effective radius of the cylindrical thermal conductivity probe.

The approximate thermal representations of the heat transfer in air and water annuli described here are expected to be good after short time periods have elapsed after initiation to the step function heating when the controlling thermal resistances and capacitances are in the earth layers.

B. Preliminary Cylindrical Probe Designs

The curve in Figure 2 was used to produce design graphs for a cylindrical transducer which would measure the temperature rise produced by pulses of heat of different strengths and durations. The transducer dimensions were held fixed and it was assumed to be in contact with the walls of the borehole. Graphs were produced for a range of k_{∞} and $\gamma_{\infty} c_p$ values.

A number of such working design graphs are shown in the Appendix.

These design curves were used to verify the physical interpretation previously made that at short time periods after the beginning of the heating process, the thermal capacity per unit volume has a more pronounced effect on the time-temperature function than does the thermal conductivity. On the contrary, for long time periods after heating initiation, the thermal conductivity is more important. These facts are used in the process of extracting the thermal conductivity from downhole measurements using the theory.

A preliminary design for a thermal conductivity probe system has been completed. It consists of a long cylindrical component that has a cylindrical surface heater at one end. A surface thermopile is positioned along the cylinder; one of the thermopile junction sets is under the surface heater and the other junction set in the unheated zone. When using the transducer, the vertical temperature gradient is first measured (with no heat being applied to the cylindrical surface heater). After that is accomplished, only then is the surface heater activated and the transient experiment performed.

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APPENDIX

Working Design Curves for a Four-Inch Diameter Cylindrical Thermal Conductivity Transducer

$P \text{ (watts/Ft)}$

P (watts/ft)

Task 3. Test Sensor Fabrication

Task 4. Laboratory and Shallow Hole Sensor Tests and Evaluations

**PRELIMINARY THERMAL CONDUCTIVITY AND
THERMAL CAPACITY MEASUREMENTS IN
THE LABORATORY**

**For
Energy Research & Development Administration
Contract E(04-3)-1318**

**H. F. Poppendiek
D. J. Connelly**

March 1977

**GEOSCIENCE LTD
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III. LABORATORY TEST SYSTEM AND PROCEDURE	3
A. System	3
B. Procedure	3
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I. INTRODUCTION

In order to obtain early information on the performance of a thermal conductivity probe for down hole application as well as verification of the procedure of extracting the thermal conductivity from the transient heat transfer measurements and the corresponding theory, preliminary tests were performed in the laboratory.

This report outlines the laboratory test system, the procedure and the results of the experiments.

II. METHOD

In a previous report,^{*} mathematical functions for cylindrical and spherical thermal conductivity probes were presented. The time-temperature expressions for step-function surface heat releases were evaluated for representative conditions. It was pointed out that the short-time and long-time temperature histories (after the start of heating) can be related to the thermal conductivity and thermal capacity per unit volume of the surrounding earth. The thermal capacity per unit volume is most important in controlling the short-time temperature variation and the thermal conductivity is most important in controlling the long-time temperature variation.

^{*} "Cylindrical and Spherical Transient Heat Transfer Solutions and their Application to Thermal Conductivity Measurements in the Earth," GLR-185, Feb 1977.

III. LABORATORY TEST SYSTEM AND PROCEDURE

A. System

The probe used in this test consisted of a nine-inch long piece of thin walled one-quarter inch diameter stainless steel tubing. Heavy copper electrodes were soldered to each end of the tube so that an electrical current could be passed through it generating heat in the tube wall. The probe was instrumented with a thermocouple placed inside the tube at the axial center near the tube wall. AC power was supplied through a Variac. A voltmeter and an ammeter were in the electrical circuit so that the power applied could be measured accurately.

This transducer was placed in the center of a cylindrical container fourteen inches in diameter and eighteen inches deep. The container was then filled with sand.

B. Procedure

Power was applied to the transducer and the time-temperature curve was recorded on a graphic recorder. The power input was also continuously monitored during the run.

IV. RESULTS

The time-temperature measurements obtained during the test are shown in Figure 1.

A number of mathematical time-temperature curves were calculated for a range of thermal conductivities and thermal capacities per unit volume. Each successive calculation was made by varying the two parameters so that the mathematical curve more closely fit the experimental curve. The procedure is accomplished quickly with little difficulty. A reasonable curve fit is shown in Figure 2 for the properties:

$$k = 0.18 \text{ Btu/hr ft}^2 \text{ } ^\circ\text{F/ft} \text{ (0.00074 cal/sec cm}^2 \text{ } ^\circ\text{C/cm) and}$$

$$\gamma c_p = 22 \text{ Btu/ft}^3 \text{ } ^\circ\text{F} \text{ (0.35 cal/cm}^3 \text{ } ^\circ\text{C)}.$$

Figure 1. Experimental time-temperature curve for the sand experiment.

Figure 2. Analytical time-temperature curve fit to the experimental data.

V. DISCUSSION

The thermal conductivity, density, and heat capacity of the sand used in the test effort described above were also measured separately in Geoscience's property laboratory. The thermal conductivity was measured by a classical annulus apparatus under steady state conditions and found to be $0.19 \text{ Btu/hr ft}^2 \text{ }^\circ\text{F/ft}$ ($0.00079 \text{ cal/sec cm}^2 \text{ }^\circ\text{C/cm}$). The density was measured by weighing a given volume of sand and it was found to be 1.35 gm/cc . The method of mixtures was used to determine the heat capacity and it was determined to be $0.215 \text{ cal/}^\circ\text{C gm}$.

Thus it is seen that the thermal conductivity value determined via the transient probe was within six percent of the laboratory steady state measurement; the thermal capacity per unit volume value determined by the transient probe was within 20 percent of the laboratory steady state value. This agreement is thought to be satisfactory at this point in the program.

SUMMARY

SUMMARY

The following progress has been made in the first six months of Geoscience's heat flux sensor and thermal conductivity probe development for ERDA:

1. A closed form solution was derived that describes the temperature field in a rod heat flux transducer located vertically in the earth. This solution can be used to relate rod transducer temperature measurements and the geothermal heat flux in the earth where the transducer has been positioned. The boundary value problem contains two unknowns, namely, the thermal conductivity of the earth and the vertical temperature gradient in the earth (the product of these two quantities being the geothermal heat flux). There are two ways in which the two unknowns can be evaluated from the prediction equations. Either two temperature points along a single transducer of known conductivity are used in the evaluation, or one temperature point each for two transducers of different but known thermal conductivity are required.

The solution was also extended to the case where there is a water or air annular space between the transducer and the drillhole.

2. A finite difference equation set was derived which describes two dimensional heat transfer in a transducer and the surrounding earth. Numerical evaluations of the two-component Laplace system were presented. In addition, the results were compared to those previously obtained by the simpler, closed-form model. The closed-form and numerical temperature fields for the rod heat flux transducer were found to be in reasonable agreement. As a result of the mathematical work for this transducer system, its quantitative performance has now been described.

3. Classical mathematical transient temperature solutions for cylindrical and spherical thermal conductivity transducers exposed to step function surface heat releases were described and the corresponding surface temperature-time functions evaluated for computational use.

The mathematical solutions were also extended to include the presence of water or air annuli that space the transducer from the drillhole surfaces. A stagnant air annulus was represented as a layer that has thermal resistance but no thermal capacitance (because of the low density of air). Therefore, the temperature drop across the air annulus is obtained from the classical logarithmic annulus conduction equation, thus making it possible to extrapolate the probe surface temperature to the drillhole temperature. A stagnant water (or mud) annulus has both thermal resistance and thermal capacitance. Therefore, two equivalent earth annuli can be determined, one having the same thermal resistance and the other having the same thermal capacitance as the water annulus. Then a mean annulus thickness is obtained (by a simple average) and it used to establish a new effective radius of the cylindrical thermal conductivity probe.

4. Design curves for a cylindrical thermal conductivity probe were computed and the results used to verify the physical interpretation previously made that for short time periods (after the beginning of the heating process) the thermal capacity per unit volume has a more pronounced effect on the time-temperature function than does the thermal conductivity. On the contrary, for long time periods the thermal conductivity is more important. These features are used in the process of extracting the thermal conductivity from downhole measurements.

5. Preliminary downhole heat flux sensor designs were completed.
6. Preliminary downhole thermal conductivity probe designs were completed.
7. A test model thermal conductivity probe was fabricated and installed in a sand volume in the laboratory. The resulting time-temperature recording for the probe was evaluated using the mathematical heat transfer functions; the thermal conductivity and thermal capacity per unit volume were extracted from the data. The former property differed from the known (laboratory) value by six percent and the latter property differed from the known value by twenty percent.