WELLBORE AND SOIL THERMAL SIMULATION
FOR GEOTHERMAL WELLS:
DEVELOPMENT OF COMPUTER MODEL AND
ACQUISITION OF FIELD TEMPERATURE DATA*

PART I REPORT

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ABSTRACT

A downhole thermal simulator has been developed to improve understanding of the high downhole temperatures that affect many design factors in geothermal wells. This report documents this development and presents field temperature data for flowing and shut-in conditions.

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Nomenclature

English Symbols

A , area or subscripted coefficient
 a , denotes accumulation
 B , subscripted coefficient
 C , subscripted coefficient
 c , specific heat capacity of
denotes conduction
 D , subscripted coefficient
 E , energy rate or subscripted
coefficient
 F , subscripted coefficient
 f , denotes flow or friction factor
 G , subscripted coefficient
 g , acceleration of gravity
 H , rate of enthalpy change
 h , specific enthalpy
 i , radial level index
 K , thermal conductivity
 k , vertical level index
 M , power law fluid model
coefficient
 m , power law fluid model exponent
 N , number of nodes
 n , denotes time step
 P , pressure
 $\dot{Q}$, rate of heat generation
 q , rate of heat transfer
 r , radial coordinate
 S , Bingham plastic fluid model
coefficient
 s , Bingham plastic fluid model
intercept
 T , temperature
 T' , absolute temperature

Greek Symbols

β , volume coefficient of
thermal expansion
 $\dot{\gamma}$, shear strain rate
 Δ , denotes a change
 Π , 3.1415927
 ρ , density
 τ , shear stress
 $\bar{\mu}$, viscosity

English Symbols (continued)

t , time
 U , thermal conductance
 V , volume
 $\dot{V}$, volume flow rate
 v , velocity
 Y_P , yield point
 z , vertical coordinate
 PV , plastic viscosity

I Summary

Geothermal wells experience high temperatures during drilling, production, and injection. These high temperatures must be accounted for in casing design, logging, mud and cement design, and many other applications. To improve understanding of downhole temperatures, Enertech Engineering and Research Co. has contracted with Sandia Laboratories to:

1. Develop a thermal simulator for computing downhole temperatures.
2. Acquire field temperature data for flowing and shut-in conditions.
3. Test computer model with analytical solutions, laboratory data and field data.
4. Establish sensitivity of downhole temperatures to flow history, well design, material properties and other variables.

This report documents objectives 1 and 2.

A computer model has been developed to compute downhole temperatures in and around a geothermal well. There are three distinct parts of the model development, the flowing stream energy balance, the wellbore description, and the soil energy balance.

The flowing stream energy balance is a fully transient analysis with vertical heat convection, and radial heat conduction. Such a fully transient behavior has not previously been available for public use.

A composite of annular materials makes up the wellbore description, including the steel, cement, and fluids present in a well. A fully transient radial heat conduction model accounts for the wellbore region. Material heat capacities and natural convection in annular fluids are both included.

Radial and vertical heat conduction are the bases for the transient energy transfer in the soil. A key feature in the thermal simulator is the direct coupling of soil and well temperature calculations. An iterative solution between the soil and well regions is not required as in most thermal simulators, public or private.

As a result of these construction features, we believe the thermal simulator described herein is the best available for modeling drilling and circulation, and for injection and production of single phase fluids.

Field temperature data has been acquired from geothermal and petroleum wells for both flowing and shut-in conditions. These data provide well descriptions, flow histories, and surface and downhole temperature measurements. These data will be correlated with the thermal simulator in the next phase of the project.

The following conclusions are presented:

1. The thermal simulator is now operational for application to geothermal and other wells.
2. The general purpose thermal simulator is user oriented and can be operated with commonly available information.
3. Sufficient downhole and surface field temperature data has been acquired to evaluate the thermal simulator with field conditions.

II. INTRODUCTION

II-1 Background

As a result of the high temperatures experienced in geothermal wells, special design criteria are needed for drilling, completion, and injection or production. To ensure proper design of cement, casing, drilling and packer fluids, drill bits, downhole valves, packers and other downhole equipment, a firm understanding of the temperatures in and surrounding the well is necessary.

Determination of downhole wellbore and soil temperatures is a complex task. Many variables influence the temperatures, which are continuously changing with time. Temperature recording devices have been developed, but these provide only isolated data points for a transient quantity, and furthermore cannot provide sufficient information to establish the relative importance of the variables influencing temperatures. Therefore, a means of predicting downhole temperatures is needed to determine important design criteria such as maximum temperature and time of exposure to high temperatures. Many simplified analytical techniques and some correlations of experimental data have been constructed in the past, each with limited success in predicting downhole temperatures. However, experience has demonstrated that a computer model is needed to account for the complexities of heat transfer in a well.

II-2 Industry Needs

Many computer models have been constructed to analyze various well conditions. All the simulators have different capabilities and limitations. Some are available to the public, but most were developed by major oil companies and are not for public use. To assess the remaining needs of petroleum and geothermal industries with respect to temperature prediction, to review the capabilities of existing models, and to establish the availability of these models for public use, a literature study and survey of industry was conducted by Goodman [1]* in 1977.

The results of the industry and literature survey led to the five conclusions presented by Goodman:

1. The major need expressed by industry for wellbore temperature predictions in geothermal wells is for highly transient behavior during drilling and completion operations. The steady-state behavior during production is not a major concern.

*See References

2. The service and producing companies that were contacted indicated an interest and need for wellbore temperature predictions during and immediately following transient circulation. Cementing and logging problems were most frequently mentioned.

3. The producing companies are also interested in highly transient wellbore temperature predictions for monitoring geothermal gradients while drilling and for short-term production testing.

4. The companies indicated that their present thermal simulation capability was lacking and that they would use, if available at reasonable cost, a computer program to predict transient wellbore temperatures.

5. A computer program for predicting highly transient wellbore temperatures does not presently exist in the public domain. Existing capability is limited to long time behavior and is not accurate for early times (less than 24 hours) or short time periods (less than 7 days).

II-3 Simulator Construction

Computer models have been developed as needs arise for specific heat transfer analyses. For example several models appeared at the time interest in steam stimulation surged in the 1960's, and additional models were introduced in the early 1970's as a result of the interest in arctic completions.

Most existing wellbore thermal simulators have been designed for one of the following four applications.

1. drilling, cementing, circulation
2. single phase injection or production
3. two phase injection or production
4. permafrost thaw

The present model is planned for applications one and two.

Although some thermal simulators that have been developed are quite useful they are not available for public use. Those that are available lack the capabilities required to properly predict temperatures in geothermal wells. Therefore, the present project is being conducted to develop the necessary capabilities for geothermal applications.

There are basically three components of wellbore thermal simulators, flowing stream, well completion, and soil. The relative importance of each component depends on the particular application and the desired result. For predicting surface production temperature in high rate wells, the soil calculation is

much less important than in the flowing stream. Conversely, for predicting permafrost thaw around an arctic well the flowing stream is less important than the soil analysis.

II-3.1 Flowing Stream

Several methods have been used to determine flowing stream temperatures. Many of the early models assumed constant temperatures in the flowing fluid, or used experimentally determined correlations to compute temperatures. Later studies performed a steady state analysis to estimate fluid temperature, followed by models employing a pseudo steady state scheme whereby the temperatures in the flowing stream are assumed to be steady state at each time step. Recent models have included the full transient response of the flowing stream. For short time periods the transient response is very important, such as for drilling and cementing operations.

II-3.2 Well Completion

There is resistance to heat flow between the flowing fluid and the soil. During drilling in an open hole the resistance results simply from the convection coefficient between the drilling fluid and the soil. However, when circulating inside casing, or during production and injection, the heat transfer resistance is more complex. Indeed, arctic well completions use this resistance to advantage with varying methods of insulation.

Figure 1 shows a well completion for an injection or production well. It is clear from this figure that heat must pass through several materials to travel from the fluid stream to the soil. Moreover, the resistance to heat flow varies with depth as casing is set, introducing steel, cement, and annular fluids where there once was soil. Some previous work has neglected the wellbore resistance, but more recent papers have indicated a trend to detailed analyses of the well components.

II-3.3 Soil

In the soil heat is transferred both radially and vertically by conduction. Early attempts to model wellbore temperatures assumed a constant undisturbed geothermal temperature in the soil, or employed an analytical solution to the steady state or transient radial heat conduction problem. Later models included numerical solutions to transient radial heat flow, or both radial and vertical conduction.

III. Flowing Stream Energy Balance

III-1 Energy Terms

Energy is transferred in the flowing stream by various mechanisms. Some of the mechanisms are much more important than others, but the relative importance is dependent on the particular application. Figure 2 shows a cell in the flowing fluid through which energy is transferred and accumulated. The directions of energy flow shown are positive coordinate directions and coincide with injection for the tubing, denoted by $i = 1$, or reverse circulation for the tubing/casing annulus, $i = 2$. A dashed line outlines the cell to be considered.

In the radial direction heat conduction transfers energy into and out of the cell in Figure 2. Fourier's law of heat conduction establishes the rate of heat flow

$$q_r = - \frac{2 \pi K \Delta z}{\ln \frac{r_2}{r_1}} \Delta T, \quad (\text{III-1})$$

where ΔT denotes the radial temperature difference, r_2 and r_1 are the radii to the temperatures, and Δz is a length interval.

Let a heat conductance be defined by,

$$U_{k,i} = \frac{2 \pi K_k}{\ln \frac{r_i}{r_{i-1}}}, \quad (\text{III-2})$$

then the rate of heat conduction into the cell in Figure 2 is

$$E_{i-1}^C = \Delta z U_{k-\frac{1}{2},i-1} \Delta T_{k-\frac{1}{2},i-1}. \quad (\text{III-3})$$

Similarly, the rate at which heat is conducted out of the cell is

$$E_i^C = \Delta z U_{k-\frac{1}{2},i} \Delta T_{k-\frac{1}{2},i}. \quad (\text{III-4})$$

Energy transport in a flowing stream is more complex than simple conduction. A change in flow energy along a streamline is the sum of the change in enthalpy, potential energy and kinetic energy, [2],

$$\Delta E^f = \Delta H + \Delta PE + \Delta KE, \quad (\text{III-5})$$

where E^f is the rate of change of the flow energy. An expansion of the definition of enthalpy, as shown in the appendix, leads to

$$\Delta h = c \Delta T + \frac{\Delta P}{\rho} (1 - \beta T') , \quad (\text{III-6})$$

where c is the specific heat capacity, ρ is the density, and β is the volumetric coefficient of thermal expansion. For this application the temperature T' must be in absolute units, degrees Kelvin or Rankine. A rate of energy flow is obtained by multiplying the specific enthalpy, Δh , by the mass flow rate, $\rho \dot{V}$,

$$\Delta H = \dot{V} [\rho c (T_{k,i} - T_{k-1,i}) + (P_k - P_{k-1}) (1 - \beta T'_{k-\frac{1}{2},i})] \quad (\text{III-7})$$

Changes in potential and kinetic energy account for gravity and inertia. A change in vertical position effects the potential energy of the fluid,

$$\Delta PE = \rho \dot{V} g \Delta z , \quad (\text{III-8})$$

which is the rate of potential energy transfer, where g is the acceleration of gravity. Kinetic energy is altered by a change in velocity, v ,

$$\Delta KE = \dot{V} (v_k^2 - v_{k-1}^2) , \quad (\text{III-9})$$

which is the rate of kinetic energy transfer for a velocity change from v_{k-1} to v_k . The potential and kinetic energy, and the enthalpy are all discussed in more detail in the appendix with attention to units and relative importance.

In addition to energy being transferred through the cell in Figure 2, energy may be accumulated within the cell. The rate at which energy is accumulated, E^a , is the rate at which the sensible heat of the fluid in the cell changes,

$$\Delta E^a = \rho V c \frac{\Delta T}{\Delta t} , \quad (\text{III-10})$$

where V denotes the volume of the cell, ΔT is a change of temperature in the cell with time, and Δt is the time increment. This transient behavior term is a key feature missing in the thermal simulators available for public use.

Another energy form that must be considered is heat generation. Heat may be generated by fluid friction against pipe walls, by chemical reaction such as cement hydration, or by external sources such as bit work during drilling. Other phenomena such as electric resistance heating in an electrode well may be modeled with a heat generation term. The rate at which heat is generated is denoted $\dot{Q}$ in the energy balance to follow.

III-2 Energy Balance

All the energy terms discussed in the previous section must obey the first law of thermodynamics, an energy balance. Consider again the fluid cell in Figure 2. An energy balance requires that the sum of the energy flowing into a cell, plus the heat generated in the cell, less the energy flowing out of a cell must equal the energy accumulated in the cell,

$$\Delta E^C + \Delta E^f + \dot{Q} = \Delta E^a \quad (III-11)$$

Substituting the expressions given above for the terms in Equation III-11 yields

$$\begin{aligned} & \Delta z (U_{k-\frac{1}{2},i-1} \Delta T_{k-\frac{1}{2},i-1} - U_{k-\frac{1}{2},i} \Delta T_{k-\frac{1}{2},i}) \\ & + \dot{V} [\rho c (T_{k,i} - T_{k-1,i}) + (P_k - P_{k-1}) (1 - \beta T'_{k-\frac{1}{2},i})] \\ & + \rho \dot{V} g \Delta z + \rho \dot{V} (v_k^2 - v_{k-1}^2) + \dot{Q} = \rho V c \frac{\Delta T_{k,i}}{\Delta t} \end{aligned} \quad (III-12)$$

This equation is expanded in more detail in appendix IX-3.

Equation III-12 may be regrouped (see appendix) to write each term as a factor times a temperature, which results in an equation of the form

$$\begin{aligned} T_{k,i}^{n+1} = & A_{k,i} T_{K-1,i}^{n+1} + B_{k,i} T_{k+1,i}^{n+1} + C_{k,i} T_{K,i-1}^{n+1} + D_{k,i} T_{k,i+1}^{n+1} \\ & + E_{k,i} T_{k,i}^n + F_{k,i} T_{k-1,i-1}^{n+1} + G_{k,i} T_{k-1,i+1}^{n+1} \end{aligned} \quad (III-13)$$

This equation can be written for every position k, i in the wellbore to yield a system of simultaneous linear algebraic equations. The unknowns are the temperatures at each node at time step $n + 1$ and there is one independent equation at each node, for a total of $N_z \times 3$ equations and unknowns.

Here N_z is the number of nodes in the vertical direction. For certain nodes some of the coefficients in Equation III-13 are zero. For example, referring to Figure 2, tubing or drill string fluid temperatures are computed with $B_{k,i} = C_{k,i} = F_{k,i} = 0$. For $i = 2$ the flowing fluid temperature in the drill pipe/casing annulus is computed with $B_{k,i} = 0$, and for no flow conditions in the well $A_{k,i} = B_{k,i} = F_{k,i} = G_{k,i} = 0$. The relative importance of each of the neighboring temperatures in computing $T_{k,i}$ is determined by the relative magnitudes of the coefficients. Each of the coefficients is a function of the geometry and material properties used in Equation III-12, and the details of the definitions are included in appendix IX-3.

Reverse circulation and production may be modeled with a modified form of Equation III-13. This equation is developed from the heat flow directions of Figure 2. The vertical heat flow direction may be reversed by replacing $\dot{V}$ with $-\dot{V}$ in Equation III-12. Equation III-13 may then be rewritten to compute

$$T_{k-1,i}^{n+1} = \frac{1}{A_{k,i}} (T_{k,i}^{n+1} - B_{k,i} T_{k+1,i}^{n+1} - C_{k,i} T_{k,i-1}^{n+1} - D_{k,i} T_{k,i+1}^{n+1} - E_{k,i} T_{k,i}^n - F_{k,i} T_{k-1,i-1}^{n+1} - G_{k,i} T_{k-1,i+1}^{n+1}), \quad (\text{III-14})$$

where the coefficients are defined as in the appendix with the substitution of $-\dot{V}$ for $\dot{V}$. This formulation is applicable in the streams that flow vertically upward, such as production, reverse circulation for the inner stream, and forward circulation for the annulus.

III-3 Numerical Grid

A grid system has been selected whereby three temperatures are computed in the wellbore at each depth. Mathematical cells are constructed with radial boundaries at the well centerline, at the outside surface of the drill pipe or tubing, at the surface of the open hole or the outside of the first casing string run. Such a grid is illustrated in Figure 3 and shows the three wellbore temperatures. The location of the outer boundary of the third cell is at the radial position outside of the wellbore/soil interface. The centerline of the third cell is located at the well/soil interface.

Temperature nodes are located at the centers of the cells as shown in Figure 3. The first node is for the fluid in the drill pipe or tubing, which gives the flowing production, injection or circulating fluid temperature, or simply a temperature near the center of the well during shut-in. The second node is located to compute annular fluid temperature during circulation. The third node serves as an interface between the wellbore and soil calculations by being located at the boundary between the two regions.

Vertical heat conduction is neglected in the wellbore. Cell dimensions in the wellbore and the presence of vertical convection in the flowing streams make vertical conduction negligible. Each cell is selected to be 200 feet long and is generally a few inches to a few feet in radial width. A length to width ratio of 100 to 1000 clearly allows a much greater path to radial heat flow. Moreover, the vertical temperature gradients are much less than the radial temperature gradients for most applications. Finally, for a flowing stream the vertical forced convection transports heat at a much faster rate than vertical conduction.

III-4 Solution Scheme

As described in the previous section Equation III-13 or III-14 can be applied to every temperature node to form a system of simultaneous linear algebraic equations. Of course there are many solution techniques, of varying degrees of sophistication and speed, available for computing the new temperatures at each new time step, $n + 1$. A Gauss elimination direct solution or a Gauss-Seidel iteration solution are the two simplest methods to apply, with variations of each available for improving speed or efficiency of solution.

A Gauss-Seidel iteration is employed to solve for the new temperatures. Equations III-13 and III-14 are repetitively applied until two successive calculations of each temperature are sufficiently close. The computation time required for applications of the model does not justify any effort to speed up the process. However, as more complex problems are analyzed, it may become necessary to improve convergence.

Primarily two means of improving efficiency are planned if needed. First, some temperatures in the grid converge faster than others, but they are recomputed with every iteration. A check for local convergence can reduce wasted effort in subsequent iterations. Secondly, the convergence rate can be accelerated by a modification of the iteration procedure. Successive overrelaxation may be used to accelerate convergence, but it is well known that only a slight improvement is achieved unless the proper relaxation factor can be determined fairly accurately, to two or three digits. Since the value of the relaxation factor is

dependent on the geometry of the grid, some attention must be given to the computation of the proper factor, [3,4]. Another procedure to be considered is the implicit alternating direction method, which provides even greater convergence than successive overrelaxation. However, this method too requires the determination of an iteration parameter, [3,5].

III-5 Flowing Stream Options

Four options have been included for fluid flow in the wellbore. By definition of the appropriate value for an input variable one of the following is selected:

1. Injection
2. Production
3. Forward Circulation
4. Reverse Circulation

Flowing fluids are allowed in the first and second strings of pipe, but not in annuli outside of the second string. However, natural convection is accounted for in the outer strings, which is discussed further in Section IV of this report. Of course, in addition to fluid flow in the first two strings of pipe, shut in conditions may be modeled by setting $\dot{V} = 0$. For this condition the flowing option variable selected is irrelevant.

Drilling is a special case of circulation. Fluid is circulated under options 3 or 4 to a specified depth, which is allowed to change with time. This circulation occurs for a specified number of hours per day. The remainder of a day is modeled with the well shut-in. The rate of change in depth, i.e. the drilling rate, the circulation rate, and other variables are fixed for a specified period of time then may be changed as desired.

An added feature that has been incorporated allows fluid influx into the well during circulation. Option 2 is combined with 3 or 4 for this feature. The circulation option is selected and the secondary fluid production is specified. This secondary fluid enters the well at the depth of circulation at a specified temperature and flow rate. Calculations in the fluid return stream account for the added flow.

III-6 Preliminary Tests

Although testing of the thermal simulator was not included in the objectives for Part 1 of the project presented in this report, and is planned for subsequent work, some preliminary testing has been conducted. Basically three tests have been conducted,

- compare alternative energy formulations
- compare transient radial conduction analysis to exact solutions

- evaluate the importance of time step size on solution accuracy.

Four versions of the flowing stream energy balance have been formulated. Each was implemented on the computer and calculations by each were compared. The various formulations included different cell boundary locations and different approximations to the radial heat flow. Results were compared for speed of computation and accuracy. The formulation presented in this report is the most efficient and as accurate as any.

By setting the flow rate equal to zero, the flowing stream energy balance reduces to transient radial heat conduction. This simplification has been applied to an exact solution with a fixed temperature outer boundary. Figure 7 illustrates the exact solution temperature profiles at various times, and the thermal simulator predictions at four discrete points. Even though a rather coarse grid is used to approximate the field, an accurate approximation of the exact solution is obtained. Only at early times where the temperature gradients are large is there an appreciable error, but a denser grid in the region of large gradients would reduce the error. It is worth noting that for wellbore thermal simulation the temperature gradients are larger near the well. But Section III-3 and Figure 5 have illustrated that the grid is denser near the well, reducing the error revealed in the preliminary test. For an appropriate grid dimension relative to the temperature gradient, later times or away from the heat source, Figure 7 demonstrates close agreement between computer and exact solutions.

Also illustrated in Figure 7 is the effect of time step size on solution accuracy. Two conclusions are apparent from the figure. First, time step size does not significantly influence the solution, except near the large gradient region, which can be eliminated as described above for solution accuracy. Second, convergence is clear as the time step size is reduced. For smaller time step sizes the computer solution approaches the exact solution monotonically.

Many additional tests are planned during Part 2 of the project. These tests will be presented in the final report. However, the preliminary tests presented here indicate the computing capabilities hoped for in the thermal simulator.

IV Wellbore Description

IV-1 Coupling Soil and Well

Linking the flowing fluid temperature calculations in the well to the soil temperatures is an important part of the formulation of the thermal simulator. Presently, most thermal simulators, both public and private, perform computations in the two regions independently, with one acting as a boundary for the other. For example, at a given time step the wellbore temperatures are computed based on the existing soil temperatures fixed as a boundary condition. Then the soil temperatures are computed with the wellbore temperatures fixed. Of course an accurate solution can be achieved in this fashion by iterating between the well and soil temperatures, but considerable computation time would be required. As a result most simulators simply perform one iteration, with hope that the error introduced remains small.

Our geothermal well simulator is unique in that it does not decouple the soil and well calculations. Temperatures in the two regions are computed simultaneously from one set of algebraic equations. Comparison of equations III-13 and V-10 reveals the similarity in the equations produced by analyses of the two regions. This technique of simultaneously solving for soil and well temperatures improves speed and accuracy of computation as compared to most existing models.

To make the link between a well and the soil, a conductance must be formulated to transfer heat between the two. Consider again the wellbore description given in Section II and illustrated in Figures 1 and 3. For heat to flow between the well and soil it must pass through various combinations of steel, cement, fluid, and soil. The conductance is computed from the well geometry and the properties of these materials. Heat passes from temperature $k,2$ to $k,3$ through a composite radial system of various materials. The rate of heat flow through such a system may be written as,

$$q = \Delta z U \Delta T , \quad (IV-1)$$

where U is the thermal conductance, [6],

$$U = \frac{2\pi}{\frac{1}{hr_1} + \frac{\ln r_2/r_1}{K_1} + \frac{\ln r_3/r_2}{K_2} + \dots} \quad (IV-2)$$

This particular formulation is for a flowing fluid inside a pipe with convection coefficient h , surrounded by a composite of solids with conductivities $K_1, K_2, \dots$, which is the model used for the wellbore.

Determination of the components of Equation V-2 is dependent on the temperatures in the well and on depth. Since the convection coefficient, h , is a function of viscosity, it must be allowed to vary with temperature. Although the thermal conductivities of steel, cement, and soil do not vary significantly with temperature, the natural convection occurring in the fluid in some annular regions is temperature dependent. Both the rate of convection and the fluid-pipe surface coefficient are temperature dependent. As a result of all these viscosity effects, the coefficients in Equations III-13 and V-10 are temperature dependent, resulting in a system of non-linear equations. However, these equations are weakly non-linear and may be linearized by simply evaluating the coefficients at the temperatures for the previous time step. In fact, the coefficients need not be updated at every time step, particularly in view of the computation time required to reconstruct the conductance, U , and the coefficients in Equations III-13 and V-10.

As a result of casing strings cemented at various depths, the conductance of Equation IV-2 changes with depth. Figure 4 illustrates the five sections in a well with different formulations for conductance as a result of a different number of casing strings at each depth. Near the surface there are five casing strings and five annuli between the injected fluid and the soil. Near the bottom of the well there is only one casing string and one annulus between the fluid and soil. Three other intervals can be identified between these two extremes as indicated in Figure 4. Moreover, there is an interface between cement and fluid to be located in each annulus. Above this interface, fluid properties are used for the annulus. Below the interface, cement properties are used.

IV-2 Convection Coefficient

Convection in wellbore fluids can significantly influence the rate of heat transfer from a well to the surrounding soil. To accurately model a wellbore, two aspects of convection must be considered, the rate of heat transfer between the fluid and pipe, and the rate across a naturally convecting annulus. Heat transfer between a fluid and pipe is defined by the surface convection coefficient, h , which is a function of fluid properties and well dimensions. Similarly, laboratory studies have correlated the rate of heat transfer across a naturally convecting fluid in terms of the fluid properties, well dimensions, and the temperature difference across the fluid. Both of these phenomena have been incorporated into the wellbore model as terms in the formulation of the conductance, Equation IV-2.

IV-2.1 Surface Convection in Coefficient

Laboratory measurements have determined the rate of heat transfer between a fluid and solid surface, [7,8]. These measurements are correlated in terms of nondimensional groupings. The Nusselt number is a measure of the rate of heat flow at the fluid/solid interface,

$$Nu = \frac{hD}{K}, \quad (IV-3)$$

where D is the pipe diameter and K is the thermal conductivity of the fluid. The relative importance of inertia to viscous forces is defined by the Reynolds number,

$$Re = \frac{\rho v D}{\mu} \quad (IV-4)$$

where ρ and μ are fluid density and viscosity and v is the flow velocity. Velocity to temperature diffusion ratio, or the ratio of kinematic viscosity to the thermal diffusivity, is given by the Prandtl number,

$$Pr = \frac{\mu C}{K} \quad (IV-5)$$

For $Pr < 1$ the temperature profile develops more rapidly than the velocity profile and for $Pr > 1$ the reverse is true, [9].

Experimental results correlate the Nusselt number with the Reynolds and Prandtl numbers, [7,8],

$$Nu = 0.023 Re^{0.80} Pr^{0.35}, \quad (IV-6)$$

for $0.6 < Pr < 100$, which applies to most liquids and gasses. For some high viscosity oils $Pr > 100$, in which case another correlation is recommended, [10],

$$Nu = \frac{(f/8) Re Pr}{1.2 + 11.8(Pr - 1) Pr^{-0.33} \sqrt{f/8}}, \quad (IV-7)$$

where f is the nondimensional friction factor. Computation of the friction factor is related to the roughness of the surface and the fluid Reynolds number, [11]. For laminar flow the friction factor is simply related to the Reynolds number,

$$f = \frac{64}{Re}, \quad (IV-7)$$

or $Re < 2000$. For turbulent flow, $Re > 2000$, the determin-

ation of f is more complex and is detailed in the appendix.

IV-2.2 Natural Convection

Natural convection in a fluid filled annulus increases the rate at which heat is transferred across the annular region. The rate of heat flow during natural convection is a function of well dimensions, fluid properties, and temperature difference across the annulus. Laboratory measurements have determined the rate of heat flow and expressed the results in terms of an effective thermal conductivity. If heat is transferred by conduction through the annulus, a material with the effective thermal conductivity would transfer heat at the same rate as the convecting fluid. A correlation of experimental data for concentric vertical pipe provides an estimate of the effective thermal conductivity, [12,13],

$$K_{\text{eff}} = 0.049 K (\text{Gr Pr})^{0.333} \text{Pr}^{0.074}, \quad (\text{IV-8})$$

where Gr is the Grashoff number. The Grashof number is a nondimensional ratio of the inertial or buoyant forces to viscous forces in a fluid,

$$\text{Gr} = \frac{(r_o - r_i)^3 \rho^2 \beta g \Delta T}{\mu^2}, \quad (\text{IV-9})$$

where

β = volumetric coefficient of thermal expansion

g = acceleration of gravity,

ΔT = temperature difference across annulus,

μ = fluid viscosity.

Employing Equations IV-8 and IV-9 the flow of heat through a naturally convecting fluid is reduced to a problem of heat conduction. Equation IV-2 may be employed with the conductivities $k_1, \dots$ computed with Equation IV-8 for naturally convecting fluids.

IV-3 Fluid Properties

Material properties of the fluids in a well strongly influence the heat exchange between the well and soil. There

are two important contributions of the fluid to heat transfer. First, transport of energy up and down inside the well is accomplished by the fluid flow, which is dependent on fluid properties. Second, radial heat conduction from the well must pass through the annular fluids between casings. In this instance fluid properties govern the heat conduction, and determine the existence of natural convection, for which the properties again govern the heat flow.

Fluid properties that must be defined are as follows:

- viscosity
- density
- specific heat capacity
- thermal conductivity
- volume coefficient of thermal expansion

Viscosity strongly affects the convection heat transfer, density and specific heat account for the accumulation of energy, and thermal conductivity governs the transfer of heat through the fluid. The thermal expansion coefficient has only a minor importance as demonstrated in appendix IX-2.

A power law fluid model is used to compute viscosity for the non-Newtonian fluids present in most wells. An effective viscosity is defined by the ratio of the shear stress to the shear strain rate, $\dot{\gamma}$,

$$\mu_{\text{eff}} = M \dot{\gamma}^{m-1} \quad , \quad (\text{IV-9})$$

where M and m are fluid properties. The thermal simulator requires input of the plastic viscosity, PV, and the yield point, YP. These properties are used to compute viscometer readings R_{600} and R_{300}

$$\begin{aligned} R_{600} &= YP + 2 PV \\ R_{300} &= YP + PV \end{aligned} \quad (\text{IV-10})$$

From these readings the properties M and m are computed,

$$\begin{aligned} m &= 3.322 \log \frac{R_{600}}{R_{300}} \\ M &= \frac{5.109 R_{600}}{1021.8^m} \end{aligned} \quad (\text{IV-11})$$

The details of these formulations are supplied in appendix IX-5. The shear strain rate, $\dot{\gamma}$, is computed from the fluid velocity and flow path. For flow through a circular pipe of radius R the shear strain rate is,

$$\dot{\gamma} = \frac{v(3m + 1)}{mR} \quad (\text{IV-12})$$

where v is the fluid velocity. Flow through an annular region of radii R_1 and R_2 has a shear strain rate given by,

$$\dot{\gamma} = \frac{2v(2m+1)}{m(R_2 - R_1)} \quad (\text{IV-13})$$

Using equations IV-10 to IV-13 in equation IV-9 provides a fluid viscosity, [16].

Temperature is an important variable in determination of fluid viscosity. Equation IV-9 defines viscosity at the temperature of the PV and YP measurements, which is assumed to be 70°F in the thermal simulator. To account for variations in viscosity with temperature, the change in viscosity of water with temperature, from the 70°F base, is added to the well fluid viscosity at 70°F.

Fluid density is important in computing the flow energy of the fluid transported up and down a well. Density is a required input for all well fluids specified.

Specific heat capacity is used to determine the sensible heat and the energy accumulation in a fluid. The value of the heat capacity for a particular fluid is determined from the solids fraction in the fluid. It is assumed that all fluids are derived by adding solids to water. The solids fraction is computed as follows,

$$\begin{aligned} \text{SF} &= 0.00508 (\rho - 8.33), \\ &\quad \text{for } 8.33 < \rho < 10.3 \end{aligned} \quad (\text{IV-14})$$

$$\begin{aligned} \text{SF} &= 0.0317 (\rho - 10.3) \\ &\quad \text{for } \rho > 10.3 \end{aligned}$$

where ρ is the fluid density in lbm/gal. Having determined the solids fraction, the specific heat is computed,

$$c = 1. - 0.777 \text{ SF} \quad (\text{IV-15})$$

where c is in BTU/lbm °F,

Thermal conductivity controls the conduction of heat in the radial direction through the fluid. Like the heat capacity, it is computed from the solids fraction as defined in equation IV-14,

$$K = 0.399 + 9.60 SF, \quad (IV-16)$$

where K is in BTU/hr ft °F. Although the thermal conductivity is dependent on temperature, the dependence is weak and therefore has been neglected. If future sensitivity studies establish a need for accuracy in the thermal conductivity, the dependence on temperature will be accounted for in the same manner as that described for viscosity. A variation in water conductivity with temperature will be added to the computation for 70°F.

Volume coefficient of thermal expansion makes a small contribution to the flow energy. Although it is dependent on temperature and fluid content, its contribution is sufficiently small that a constant value is used, appendix IX-3.

IV-4 Solid Properties

Material properties of solids in and around a well must be determined to properly model the heat flow. Solids that must be modeled are,

- cement
- soil
- steel

Although there are only three materials listed, several types of each are possible. However, it is believed that the thermal properties may not be significantly different between two types of the same material. Therefore, the thermal simulator presently uses only one set of properties for each of the three materials listed. If it becomes apparent that significantly different types of one material may exist, and the sensitivity studies indicate a need for accuracy in the thermal properties, then alternative choices of one material will be offered.

Required thermal properties for the simulator are,

- density
- specific heat capacity
- thermal conductivity

Density and specific heat capacity determine the rate of

energy accumulation, and thermal conductivity controls the rate of heat conducted through the solid.

At present all solid properties in the thermal simulator are treated as constants. This is a good assumption for density and specific heat capacity, but thermal conductivity has a fairly weak dependence on temperature. If later studies indicate a need to incorporate this temperature dependence, it can easily be accomplished. Definition of the constant values assigned to the solids properties is a matter of the particular application, but it is hoped that one set can be selected to serve most purposes. Table IV-1 contains the properties presently assigned to each solid in the thermal simulator.

Table IV-1

Thermal Properties of Solids

Material	Density, lbm/ft ³	Specific Heat Capacity BTU/lbm ^o F	Thermal Conductivity BTU/hrft ^o F
Cement	104	0.20	0.6
Soil	140	0.30	1.0
Steel	490	0.11	26.2

V. Soil Energy Balance

V-1 Energy Terms

Heat conduction controls the transfer of energy in the soil surrounding a well. Figure 5 shows a mathematical cell used to analyze radial and vertical energy transfer in the soil. Contrast the cell location with that for the wellbore in Figure 2. Cell boundaries are located between nodes since only conduction is involved, whereas the wellbore cell boundaries are located along nodes to more easily analyze a flowing fluid. An analysis based on the soil cell is applied at $i = 4, \dots, N_r$.

Radial conduction in the soil is formulated much like that in the flowing stream presented in section III. Fourier's law for heat conduction in radial coordinates can be written in terms of a conductance

$$q_r = \Delta z U \Delta T \quad . \quad (V-1)$$

Applying this formulation to the cell in Figure 5 produces a heat flow into the cell,

$$E_{i-1} = \Delta z U_{k,i-1} \Delta T_{k,i-1} \quad , \quad (V-2)$$

and a heat flow out,

$$E_i = \Delta z U_{k,i} \Delta T_{k,i} \quad . \quad (V-3)$$

Details of the definition of the factors in Equations V-2 and V-3 are provided in the appendix IX-3.

Fourier's law governs the vertical transfer of energy also. However, the geometry is somewhat different involving rectangular coordinates, or linear flow as opposed to radial,

$$q_v = KA \frac{\Delta T}{\Delta z} \quad . \quad (V-4)$$

Based on the energy flow illustrated in Figure 5 the heat conducted into a cell vertically is,

$$E_{k-1} = -K_{k-\frac{1}{2},i} A_i \frac{(T_{k,i} - T_{K-1,i})}{\Delta z} \quad . \quad (V-5)$$

where A_i is the cross sectional area of cell i . Similarly, the heat flow vertically out of a cell is,

$$E_k = - K_{k+\frac{1}{2},i} A_i \frac{(T_{k+1,i} - T_{k,i})}{\Delta z} \quad (V-6)$$

Energy may be accumulated in a cell in the soil. Vertical and radial heat conduction allow for energy transfer through the cell, but the energy in is not necessarily equal to the energy out. As stated in section III for transient problems the rate of increase in sensible heat reflects the rate of energy accumulation,

$$\Delta E^a = \rho V c \frac{\Delta T}{\Delta t} \quad (V-7)$$

where $\Delta T/\Delta t$ is rate of temperature change and ρV is the mass in the cell.

Heat generation in a soil cell must be considered, just as for the well. However, sources of heat generation in the soil are rare. Yet phenomena such as a phase change, evaporation or condensation in geothermal formations, may be modeled with the heat generation term. As in the flowing stream formulation, the rate of heat generation in the soil is denoted by $\dot{Q}$ in the energy balance.

V-2 Energy Balance

An energy balance is required on each cell in the soil based on the geometry of Figure 4. Radial and vertical heat conduction into the cell plus the heat generated within the cell less the heat conducted out of the cell must equal the rate of energy accumulated. This energy equality is written on a rate basis with the terms described above,

$$\Delta E_r^c + \Delta E_v^c + \dot{Q} = \Delta E^a \quad (V-8)$$

Expanding this expression by employing equations V-2, V-3, V-5, V-6, and V-7 yields.

$$\begin{aligned} & \Delta z (U_{k,i-1} \Delta T_{k,i-1} - U_{k,i} \Delta T_{k,i}) \\ & + \frac{A_i}{\Delta z} (K_{k-\frac{1}{2},i} \Delta T_{k-1,i} - K_{k+\frac{1}{2},i} \Delta T_{k+1,i}) \\ & + \dot{Q} = \rho V c \frac{\Delta T_{k,i}}{\Delta t} \end{aligned} \quad (V-9)$$

This equation provides an understanding of the relation between the energy terms, but the details of the equality are presented in the Appendix IX-3.

Equation V-9 may be rearranged to identify the influence of the surrounding temperatures on a particular node. An equation like III-13 for the flowing stream results for the soil,

$$T_{k,i}^{n+1} = A_{k,i} T_{k-1,i}^{n+1} + B_{k,i} T_{k+1,i}^{n+1} + C_{k,i} T_{k,i-1}^{n+1} + D_{k,i} T_{k,i+1}^{n+1} + E_{k,i} T_{k,i}^n, \quad (V-10)$$

where $A_{k,i}, \dots, E_{k,i}$ are computed coefficients, and the $n+1$ superscript denotes the time step $t+\Delta t$ compared to superscript n at time t . The coefficients F and G in equation III-13 do not appear in equation V-10 since the temperature at $k-1, i-1$ and $k-1, i+1$ do not influence node k, i in the soil. Equation V-10 may be applied to all nodes in the soil to produce a system of $N_z \times (N_r-3)$ simultaneous algebraic equations, where N_r is the number of nodes in the radial direction. An equal number of unknowns exist in the temperatures at the nodes. Material properties and geometry are used to define the coefficients of Equation V-10, the details of which are supplied in the appendix.

V-3 Numerical Grid

Nodal points in the soil are located at the same vertical positions as those in the wellbore. The cell lengths have been set at 200 feet, although this dimension is arbitrarily selected. Cell length should be small enough to permit acceptable grid refinement to realistically model a well, yet sufficiently long to avoid unnecessary calculations by employing too many grid points. Experience indicates that 200 feet is a reasonable compromise. Two rows of soil cells are placed below the maximum depth for fluid flow in the well. One provides a transient response of the soil below the bottom of the well and the second serves as a fixed temperature boundary 400 feet below the flowing fluid.

In the radial direction temperature gradients are much greater near the well so nodal locations are concentrated in this region. A scheme has been written to generate the radial positions of the nodes. Cell width is exponentially increased with radius to produce a grid as illustrated in

Figure 5. The exponent chosen is determined from the specified maximum radius and number of radial points. Selection of the maximum radius and number of nodes requires some understanding of the expected rate of radial heat flow. High rates of heat flow should be modeled with a large maximum radius to locate an outside boundary that is sufficiently far from the well to be unaffected by the temperature disturbance caused by the well. Smaller rates of heat flow do not require as large a maximum radius. Similarly, large temperature gradients near a well indicate that many nodes are needed in the radial direction.

V-4 Solution Scheme

Applying equation V-10 to each of the soil nodes generates a system of $N_z \times (N_r - 3)$ simultaneous linear algebraic equations in the same number of unknowns, temperature at each node. As described in Section V either direct or iterative methods may be used to solve the system of equations.

The Gauss-Seidel iteration scheme described in Section III is extended to the soil region for solution of the equations. By combining the $N_z \times 3$ Equations III-13 with the $N_z \times (N_r - 3)$ Equations V-10, a system of $N_z \times N_r$ equations are constructed. Simultaneous solution of this set of equations provides a significant improvement over existing thermal simulators, which is discussed in Section IV.

VI Field Temperature Data

VI-1 Data Description and Purpose

Any computer model must be tested to verify its accuracy and to develop confidence by its users. Model computations may be tested by comparisons to analytical solutions, laboratory measurements, or field data. Analytical solutions and laboratory measurements are best suited for testing the components of a model, since they apply to relatively simple cases. Evaluation of the overall capability of a computer model to analyze physically meaningful cases can best be accomplished by comparison of computed results to measured field data.

Field temperature data has been sought to demonstrate that the thermal simulator presented in this report included all the important variables encountered in the field. Required data for model testing includes an inlet fluid temperature to give the model a starting point, and another fluid temperature for comparison to model predictions. Temperature measurements may include mud return temperatures during circulation, or surface temperatures during production. Downhole temperatures during flow are not commonly measured, but they provide excellent comparisons to predictions if available. Downhole temperatures are more commonly recorded during static, or shut-in, conditions. Although these static downhole temperatures are useful for evaluating the transient response of the soil, temperatures during flow are more valuable since they also allow evaluation of the flowing stream calculation. The flow history of a well is necessary for thermal simulation. Flow rates and directions of flow must be given at every time. Also, periods of shut-in must be accounted for in the calculations and therefore they must be described as input data.

Other data that are required are thermal material properties. The needed properties for all solids are,

- thermal conductivity
- specific heat capacity
- density .

These properties must be specified for the cement, soil, and casing steel. Values have been selected for each of these

properties, but additional studies are being conducted to determine the versatility needed in the model to change these values. In addition to those properties specified for solids, fluid properties must include,

- viscosity
- volume coefficient of thermal expansion .

Viscosity is based on the assumption of non-Newtonian power law behavior, and is determined from two commonly measured properties, yield point and plastic viscosity, which are required input data in the simulator. Also temperature effects on viscosity are included, but input data for this feature is not required. A more detailed discussion of properties is given in section IV.

An important part of the input data is the wellbore dimensions. Each string of pipe must be specified by the following dimensions,

- inside diameter
- outside diameter
- setting depth
- cement interval length .

The mathematical grid used in the simulator is generated from these dimensions. In addition, the thermal conductance between the well and soil is determined from the location of each material in the well, which is based on these dimensions.

Unfortunately, field data is not generally recorded for the purpose of comparison to a thermal simulator. Sufficient well data is not always available to allow a useful comparison to computer predictions. In fact, a complete set of temperature data and well description is rare. Where the data is available for a well, it is often recorded in many separate reports, located in different companies, e.g. logging company, cement company, mud company, or perhaps a consultant, well operator, or a partner. For our model correlation purposes we have contacted many companies in both the petroleum and geothermal industries to locate and acquire field temperature data.

IV-2 Petroleum Industry Contacts and Data

Many petroleum industry sources have been contacted to obtain field temperature data. Major and independent producing companies, service companies, and the American Petroleum

Institute have all been contacted. Data has been reviewed for drilling, shut-in, circulating, injecting, and producing wells. Although the difficulties described above were encountered, some valuable data has been acquired.

Petroleum industry companies contacted include the following:

1. Schlumberger (John Dewan)
2. Halliburton (Dwight Smith)
3. Dresser Industries (Art Tregesser)
4. Gulf Oil (Horace Beach)
5. IMCO Services (Vernon Neale)
6. Kirby Petroleum (Mike Mayell)

Some of these contacts provided excellent data for comparison to computer predictions, some claimed to have data but would not release it, and others simply did not have satisfactory data.

Interpretation of data obtained from well logs and design of well logging instruments are both dependent on downhole temperatures. Therefore, logging companies have an interest in and experience with downhole temperatures. Schlumberger claims to have downhole temperature measurements during circulation both inside and outside the drill pipe. These data were recorded using Schlumberger MWD (measurement while drilling), a recent advance in downhole temperature monitoring. As a result the data is proprietary and we were unable to obtain it. However, discussions with them indicated that Schlumberger may be willing to trade data for some limited use of the thermal simulator. Schlumberger will be approached again for temperature data when the comparisons to field data are performed, allowing presentation of simulator results.

Design of well casing cement requires knowledge of temperatures to be experienced by the cement during injection, setting, and throughout the life of the well. The American Petroleum Institute Cementing Practices Committee has conducted a study of downhole temperatures, [14,15]. Downhole temperatures were recorded during circulation and shut-in in fifty wells. Emphasis for the study was on design temperatures for cement in deep hot wells, deeper than 10,000 feet, yet some shallower temperatures were recorded. These hot wells provide good comparisons for future geothermal applications of the thermal simulator. Halliburton and Dresser have supplied a detailed list of the wells and companies participating in the project, along with the more important data on each well. Table VI-1 is a summary of that data. Figure 8 is a schematic of the

carrier used for recording the downhole temperatures in the API study. A similar device may be useful for recording downhole temperatures in geothermal wells. Use of such a device, particularly in a new area, would improve definition of some of the variables needed in the thermal simulator, and allow more accurate temperature predictions for many conditions other than those for which a measurement is available.

Gulf Oil Company participated in the API study and was contacted for more detailed information. On invitation from Gulf Research personnel, we reviewed their data and obtained very useful downhole temperatures recorded during circulation. One desirable feature of the circulation data is that a significant shut-in period (35 to 42 hours) preceded the tests, allowing surrounding soil temperatures to return very close to undisturbed conditions. Fluid inlet and outlet and bottom-hole temperatures are recorded at various times after circulation was initiated. Out of five available wells, two were selected for detailed study. Tables VI-2 and VI-3 show the temperatures recorded in the two wells. Well #1 had tests conducted at a depth of 15,375 feet and experienced temperatures as high as 309°F prior to circulation, and cooled to 218°F after two hours of circulation. As a result of a continued rise in inlet temperature from 95°F to 152°F, further circulation after two hours increased the bottomhole temperature to an equilibrium value of 222°F. Well #2 was tested at a depth of 13,023 feet with temperature recorders located on bottom and at 11,023 feet. Bottom hole temperature was cooled from 229°F prior to circulation to 149°F after three and one half hours, at which time circulation stopped. At 11,023 feet the temperature cooled from 190°F to 162°F. Since the circulation temperature at 11,023 feet is greater than that at 13,023 feet, it is clear that the shallower recorder was located in the return annulus. Mud inlet temperature increased from 90°F to 112°F during the three and one half hours of circulation, while the return outlet temperature increased from 98°F to 120°F. It will be interesting to compare the predictions of the thermal simulator to these measured field temperatures.

Properties of drilling muds are clearly affected by temperature. Several mud companies have expressed an interest in the capability of the thermal simulator to predict downhole temperatures. IMCO, a Halliburton company, has supplied us with one set of data from a deep well in south Louisiana, and will supply us with another set in January for an offshore Texas well. These data are recorded during drilling operations. The only temperature available for comparison to computer prediction is the mud outlet temperature. As a result, interpre-

tation of the data is more complex than for the API test data supplied by Gulf. Nevertheless, some comparisons are possible. Perhaps the entire drilling program can be modeled. The offshore Texas data will be supplied by permission of the operator, Kirby Petroleum Company, who is interested in the thermal simulator also.

VI-3 Geothermal Industry Contacts and Data

In addition to petroleum industry sources, an effort was made to obtain field temperature data from operators of geothermal wells. In view of the relatively few number of geothermal wells compared to the vast petroleum industry experience there is a much smaller base from which to obtain data. Yet, on the other hand, many of the geothermal projects tend to be research oriented, so more precise data is likely to be recorded. Our search for data revealed temperature records on several wells. Although there are complex temperature histories to be interpreted, some useful data has been derived.

Contact has been made with several companies including the following:

1. Battelle Northwest (William McSpadden)
2. Republic Geothermal (Robert Nicholson)
3. Los Alamos Scientific Laboratories (John Rowley)
4. General Crude

Some useful temperature records have been obtained, some data is not complete enough to simulate, and other companies simply did not have any data.

Battelle Pacific Northwest Laboratories has recorded temperature data on two geothermal projects. In addition to temperature records Battelle measured some other useful data such as thermal conductivity. The two wells of interest are the COSO Geothermal Well BDSH-1 and the Marysville, Montana Geothermal Well #1. These wells, particularly Marysville, have the most complete data of the geothermal projects. Considerable effort was required to extract the needed information from the extensive data recorded at these wells.

Data obtained from the COSO well will be useful for temperature comparison's and for determination of soil thermal conductivity. Although the thermal history of this well is complex, and circulation data is not complete, it is expected

that some simulator predictions can be constructed to provide comparisons to field temperature measurements. Table VI-4 gives temperature data from a depth of 317 feet to 1327 feet. Mud inlet and outlet temperatures and bottom hole temperatures have been recorded. Bottom hole temperatures vary from 127°F at 217 feet to 547°F at 1100 feet. Mud inlet temperature varied from 68°F to 95°F and outlet from 80°F to 105°F. Although considerable data is available, including downhole temperatures during circulation, the drilling and circulation history is not complete, so constructing a model of the thermal history requires some assumptions. In addition to temperature measurements, the COSO project recorded numerous thermal conductivity measurements from core samples. Table VI-5 lists the results of these measurements from many wells in the COSO field. Thermal conductivity values ranged from 3.2 to 9.1 ucal/cm-sec-°C. For the 37 samples from BDSH-1, an average of 5.5 was measured.

Additional data has been retrieved from the Marysville well. This well was drilled in the summer of 1974 near Marysville, Montana. Like the COSO well this well had many interruptions during drilling for coring, logging, pump testing, water analysis, etc. Although interruptions complicate the thermal history of this well, many temperature measurements were made, both at the surface and downhole. An additional complication in this well is the apparent flow communication at a depth of 4550 feet between the well and formation during circulation. This well reached a depth of 6791 feet with temperature sensors distributed throughout the well. Figure 9 shows temperature vs. depth profiles at various times following shut-in in August, 1974. These curves show the transient response of the well to surrounding soil temperatures. A more detailed example of this response is given in Figure 10 at depths of 1000 and 1500 meters. These data will permit evaluation of the thermal simulator providing a flow history can be established.

Republic Geothermal has temperature data from geothermal wells in Imperial Valley, California and other areas. They have recorded surface temperatures during production, and inlet and outlet surface temperatures during circulation. In addition we understand that temperature data has been recorded during shut-in and flowing conditions. The surface production temperatures may be particularly useful if single phase flow dominates, and the bottom hole temperature can be established. Although discussions of the data have been conducted with personnel at Republic, we have not been able to obtain the data, but expect to do so in the near future.

Temperature measurements and predictions have been made at two geothermal wells by Los Alamos Scientific Laboratory. These wells were drilled in connection with their hot dry rock project. Downhole temperatures have been recorded with a static temperature bomb, which was run shortly after circulation stopped. No transient data is available. We have reviewed some of their data, but feel other data may be more useful for our purposes.

A geothermal well is presently being drilled in Brazoria County, Texas by General Crude. Contact has been made with a mud supplier, Milchem, and with a representative of the Department of Energy concerning temperature data during drilling. They have indicated that downhole temperatures and mud inlet and outlet temperatures are not available.

Table VI-1

API Downhole Circulation Data

Circulating temperature data

Well no.	Circ. depth	Bomb depth	Mud wt & type	Circ. rate, bbl/min	Static period, hr	Circ. time	Max. temp.	Circ. temp.	Δ T	Temp. gradient
1	7,215	7,215	10.2 W	7.6	23	2:30	172	148	24	1.34
2	13,083	13,083	W	4.3	42	3:30	229	149	80	1.18
		11,083					190	162	28	1.04
3	18,000	18,000	17.2 O	3.2		4:00	412	360	52	1.87
		15,500					368	345	23	
		7,890					217	225	(7)	
4	9,160	9,160	12.5 W	9.5		3:00	199	152	47	1.35
		8,200					188	160	28	
5	8,030	8,030	10.5 W	6.0		4:00	207	147	60	1.65
		5,315					161	147	14	
		2,320					125	128	(3)	
6	12,340	12,310	9.5 W	8.0		3:00	239	191	48	1.33
		9,940					204	192	12	
		7,540					185	179	6	
7	9,318	9,308	12.5 W	5.6		3:00	196	153	46	1.30
		6,527					175	150	25	
8	19,852	19,850	17.8 O	N.A.		4:00	401	351	50	1.65
		12,000					284	282	2	
9	22,725	22,725	10.5 W	4.8		4:00	333	295	38	1.14
						5:00		286	47	
10 a	12,180	12,180	16.8 O	5.0		4:00	312	212	91	1.95
b	13,902	13,902	18.5 W	4.0		2:30	414*	294	120	2.44*
11	11,470	11,450	13.4 W	5.6	23.8	2:15	232	197	35	1.37
12	15,000	15,520	12.4 W	5.3		4:00	350	295	55	1.72
13	14,665	14,665	W	4.0	14.5	2:05	369	332	37	2.00
14	13,875	13,875	W	6.7	26.5	4:00	260	217	43	1.33
		12,875					247	197	50	
15	11,340	11,340	W		32.5	2:30	262	210	52	1.65
		9,340					223	203	20	
16 a	12,294	12,294	16.7 W	4.7		4:00	325	250	75	2.03
b	11,356	11,356	16.7 W	5.1		2:30	290	235	65	1.89
c	10,424	10,424	16.7 W	5.0		2:30	275	220	55	1.92
d	5,500	5,500	16.7 W	5.0		1:30	182	148	34	1.95
e	16,035	16,035	17.8 O	3.8	68.0	4:00	364	285	79	1.81
f	12,291	12,291	17.8 O	3.8		2:00	318	233	85	1.98
g	10,424	10,424	17.8 O	3.5		2:00	290	200	90	2.06
h	5,520	5,520	17.8 O	3.8		2:00	202	146	56	2.31
17	13,640	13,540	10.9 W	10.0	32.5	3:00	223	177	46	1.1
		10,540					187	169	18	
18	11,920	11,920	13.8 W	9.65	16.5	3:00	185	166	19	0.92
19	16,300	16,300				4:00	315	292	23	1.47
		14,995	18.9 W	3.3			299	290	9	
20	13,037	13,037	14.0 W	6.4		4:00	260	211	49	1.42
		10,451					229	209	20	
		3,903					159	162	(3)	
21	15,000	15,000	18.2 W	6.2		4:00	296	254	42	1.35
		14,009					275	254	21	
		10,005					227	224	3	
22	15,000	14,965	18.5 W	4.0		4:00	313	291	22	1.53
		12,177					265	266	(1)	
23	17,410	17,310	16.3 W	6.8	33	3:00	227	189	38	.83
		12,235					159	158	1	

* Est.)

Table VI-2

Field Temperature Data
Gulf Well #1

Mud - 11.5 lb/gal water base

Flow rate - 477 gal/min

Shut-in prior to circulation - 34.5 hours

Hours of Circulation	BHT, °F @15,375'	Mud Temperature, °F	
		Suction	Discharge
0	309	95	92
0.125	290	-	-
0.25	264	-	-
0.5	250	-	-
1.0	234	102	130
1.5	226	-	-
2	218	128	156
3	218	142	164
4	222	149	166
5	222	152	167
6	223	-	-
7	222	-	-

Table VI-3

Field Temperature Data
Gulf Well #2

Mud - 11.0-11.5 lb/gal

Flow rate - 180 gal/min

Shut-in prior to circulation - 42 hours
(slight circulation prior to test to wash seven joints
of drill pipe)

Hours of Circulation	BHT, °F @13,023'	UHT, °F @11,023'	Mud Temperature, °F	
			Suction	Discharge
0	229	190	90	98
0.25	190	183	-	-
0.5	178	179	-	-
1.0	170	176	96	104
1.5	157	170	-	-
2.0	152	167	100	110
2.5	150	165	-	-
3.0	149	162	112	120
3.5	149	162	-	-

Table VI-4
COSO Geothermal Well Data

<u>DATE</u>	<u>DEPTH</u>	<u>TEMPERATURE</u>	<u>MUD</u>
8/25	317'	53°C (max temp after ~10 hr shut-in following intermittent circulation during coring & tripping)	water + gel a 7 GPM
8/25	340'	62°C 2 hr circulation 4 hr shut-in	12 GPM in 30°C in / 31°C out
8/25	359'	58°C	
8/27	325'	60°C intermittent drilling/blocked hole	
8/30	364'	69°C appear to have 3 day shut down	
8/31	425'	75°C overnight shut down	
8/31	434'	67°C 5-1/2 hr circulation, 1 hr shut-in	
9/1	434'	75°C 18 hr shut-in	
9/1	434'	67°C 1 hr circulation	
9/2	450'	77°C total 5 hr circulation 18 hr shut-in	

PERIOD OF AIR DRILLING. SOME APPARENT DISCREPANCIES IN HOLE DEPTH.

9/25	Start to drill with mud		
10/15	514'	85°C 8 hr drilling, 8 hr shut-in	no data
10/19	536'	83°C	
10/19	561'	83°C	
10/20	590'	93°C 8 hr shut-in	no data
10/20	616'	95°C	
10/21	625'	95°C 8 hr shut-in	gel - water
10/21	641'	85°C 5 hr circulation, 15 min shut-in	
10/27	650'	86°C	
11/2	736'	93°C after 5 hr circulation	Mud in 29°C Mud Out 33°C
11/3	745'	98°C 8 hr shut-in	
11/8-10	Cement Casing and hole back to ~45'		
11/16	813'	96°C	Mud in 20°C (tank) Mud Out 33°C 30 sec Marsh
		100°C 7 hr circulation	Mud in 24°C Mud out 35°C

Table VI-4 (Continued)

<u>DATE</u>	<u>DEPTH</u>	<u>TEMPERATURE</u>	<u>MUD</u>
11/17	860'		10 GPM 28 sec Marsh Water Mud in = 18°C Mud Out = 30°C Change to 26 GPM Mud in = 29°C Mud Out = 37°C
11/18	873'	104 & 112°C ~1 hr shut-in	Mud in = 20°C Mud Out = 31°C
11/22-24	~1000'		20 GPM 8.6#/gal 33 sec Mud in = 75°F Mud Out = 90°F
11/25	1100'	250-276°F	Mud in = 95°F Mud Out = 105°F
11/25	1100'	286°F at least 6 hr shut-in	
11/30	1109'		Mud in = 60°F Mud Out = 105°F Marsh Vis = 42.5 sec $\theta_{600} = 44$ $\theta_{300} = 22$ PV = 16, YP = 4 WT = 8.4
12/1	1132'	Complete mud check (p 54)	
12/2	1190'	265°F 1 hr drilling, 1 hr shut-in	8.55 ppg 42 sec Marsh PV = 73, YP = 5 No flow rate
12/4			6.1 GPM; 8.6 ppg Mud in = 16°C Mud Out = 34°C
12/9	1327'		T _{in} = 23°C; 12 GPM T _{out} = 27°C; 8.6 ppg PV = 5; YP = 4 34 Marsh

Table VI-5
Thermal Conductivity Measurements
For
COSO Geothermal Wells

Borehole Number	Elevation (m)	Depth Range (m)	Geothermal Gradient (°C/km)	Thermal Conductivity (mcal/cm-sec-°C)	No. of Samples	Heat Flow (μcal/cm ² -sec)
COSO BHF-1	1606	25-89	28.8	6.9	18	2.0
COSO BHF-2	1123	58-84	48.1	5.6	13	2.7
COSO BHF-3	1079	25-92	51.2	5.7	10	2.9
COSO BHF-4	1055	25-75	75.0	6.3	13	4.7
COSO BHF-5	1184	25-90	55.6	5.0	6	2.8
COSO BHF-6	1512	20-103	40.4	5.7	9	2.3
COSO BHF-7	757	20-48	68.6	3.5	3	2.4
COSO BHF-8	977	45-94	63.5	4.6	5	2.9
COSO BHF-9	1084	40-76	111.	4.0	3	4.4
COSO BHF-10	1416	20-108	32.7	6.4	4	2.1
COSO BHF-11	1254	50-91	190.	4.9	10	9.4
COSO BHF-12	1515	25-133	24.0	7.5	7	1.8
COSO BHF-13	1266	50-92	186.	5.0	4	9.3
COSO BHF-14	1356	40-91	57.6	6.1	11	3.5
COSO BHF-15	1440	30-98	42.	6.0	7	2.5
COSO BHF-16	804	50-99	42.	(5.9)*	-	2.5
COSO BHF-17	899	42-95	28.	7.9	10	2.2
COSO BDH #1	1344	80-410	350.	5.5	37	19.

* Number in parenthesis is assumed thermal conductivity.

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VIII Figures

FIG. 1

COMPUTER MODEL OF WELLBORE COMPLETION FOR PRODUCTION AND INJECTION

FIG. 2

ENERGY EXCHANGE IN A FLOWING FLUID CELL

FIG. 3

MATHEMATICAL CELLS IN WELLBORE

FIG. 4

ENERGY EXCHANGE IN A SOIL CELL

FIG. 5

MATHEMATICAL GRID FOR SOIL NODES

WELLBORE

GROUND LEVEL

FIXED TEMPERATURES

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FIXED TEMPERATURES

FIG. 6

WELL INTERVALS FOR CONDUCTANCE FORMULATION

FIG. 7

MODEL COMPARISON TO EXACT SOLUTION AND SENSITIVITY TO SIZE AND TIME STEP

Figure 8
Temperature Bomb Carrier

Figure 9

Marysville Geothermal Well Temperature Profiles

* Note: Kelly Bushing is 1635 meters above mean sea level

Figure 10

Maryville Geothermal Well Transient Temperature Response

IX. Appendix

IX-1 Enthalpy Derivation

Section II presents an expression for a specific enthalpy change. This expression is derived from treating enthalpy as a function of temperature and pressure only,

$$h = h(T, P) \quad . \quad (IX-1)$$

Therefore the total derivative is,

$$dh = \left. \frac{\partial h}{\partial T} \right|_P dT + \left. \frac{\partial h}{\partial P} \right|_T dP \quad , \quad (IX-2)$$

or in terms of a finite change,

$$\Delta h = \left. \frac{\partial h}{\partial T} \right|_P \Delta T + \left. \frac{\partial h}{\partial P} \right|_T \Delta P \quad (IX-3)$$

By definition, the specific heat capacity at constant pressure is

$$c = \left. \frac{\partial h}{\partial T} \right|_P \quad , \quad (IX-4)$$

and a fairly straight forward derivation yields, [2],

$$\left. \frac{\partial h}{\partial P} \right|_T = v - T \left. \frac{\partial v}{\partial T} \right|_P \quad , \quad (IX-5)$$

where v is the specific volume.

The volume coefficient of thermal expansion is defined as,

$$\beta = \frac{1}{v} \left. \frac{\partial v}{\partial T} \right|_P \quad (IX-6)$$

Substitution of Equations IX-6 into IX-5, and IX-5 and IX-4 into IX-3 yields the enthalpy change,

$$\Delta h = c \Delta T + v (1 - \beta T') \Delta P, \quad (IX-7)$$

where T' denotes absolute temperature. This is the expression presented in Section II.

IX-2 Flow Energy Terms and Units

Equation III-5 presents an expression for flow energy and the terms in that equation are introduced in Section III. Before actual computations can be made, units must be checked for consistency, from which the relative importance of each term is apparent. Consider a specific flow energy,

$$\Delta e^f = \Delta h + \Delta ke + \Delta ke \quad . \quad (IX-8)$$

The first term is the specific enthalpy discussed above. Let the properties in Equation IX-7 have the values for water at 150°F,

$$\begin{aligned} c &= 1.0 \text{ BTU/lb}_m \text{ } ^\circ\text{F} \quad , \\ v &= 0.0163 \text{ ft}^3/\text{lb}_m \quad , \\ \beta &= 2 (10^{-4})/^\circ\text{F} \end{aligned} \quad (IX-9)$$

and assume $\Delta T = 5^\circ\text{F}$, $\Delta P = 5 \text{ lb/in}^2$ for a typical length of 200 feet. Therefore, the terms in Equation IX-7 are computed

$$\begin{aligned} c \Delta T &= 1 \frac{\text{BTU}}{\text{lb}_m \text{ } ^\circ\text{F}} (5^\circ\text{F}) \\ &= 5.000 \frac{\text{BTU}}{\text{lb}_m} \\ v \Delta P &= 0.0163 \frac{\text{ft}^3}{\text{lb}_m} (5 \frac{\text{lb}_f}{\text{in}^2}) (\frac{144 \text{ in}^2}{\text{ft}^2}) (\frac{\text{BTU}}{778 \text{ ft lb}_f}) \\ &= 0.015 \frac{\text{BTU}}{\text{lb}_m} \\ v \beta T' \Delta P &= 0.0163 \frac{\text{ft}^3}{\text{lb}_m} \frac{2(10^{-4})}{^\circ\text{R}} (\frac{\text{BTU}}{778 \text{ ft lb}_f}) (460 + 150)^\circ\text{R} \\ &\quad (5 \frac{\text{lb}_f}{\text{in}^2}) (\frac{144 \text{ in}^2}{\text{ft}^2}) \\ &= 0.002 \frac{\text{BTU}}{\text{lb}_m} \end{aligned} \quad (IX-10)$$

Adding all the terms yields,

$$h = 5.000 + 0.015 + 0.002 = 5.0152 \frac{\text{BTU}}{\text{lb}_m} \quad (IX-11)$$

It is clear from Equation IX-11 that the dominant contribution to the enthalpy is the sensible heat.

The second term represents the specific potential energy,

$$\Delta p_e = g \Delta z \quad (IX-12)$$

For a cell length of 200 feet the value of Δp_e is,

$$\begin{aligned} \Delta p_e &= 32.2 \frac{\text{ft}}{\text{sec}^2} (200 \text{ ft}) \left(\frac{\text{lb}_f \text{ sec}^2}{32.2 \text{ lb}_m \text{ ft}} \right) \left(\frac{\text{BTU}}{778 \text{ ft lb}_f} \right) \\ &= 0.257 \frac{\text{BTU}}{\text{lb}_m} \end{aligned} \quad (IX-13)$$

Specific kinetic energy is the third term in Equation IX-9 and is given by

$$\Delta k_e = v_k^2 - v_{k-1}^2 \quad (IX-14)$$

The most dramatic velocity change would be in flowing from the drill pipe to the annulus. Consider a 3-1/2" drill pipe, inside a 7" casing, with approximate areas of $A_{dp} = 0.049 \text{ ft}^2$ and $A_a = 0.13 \text{ ft}^2$. The resulting velocity change based on a flow rate of 10 gal/min, or 1.25 ft³/hr, results in the a change in kinetic energy,

$$\begin{aligned} \Delta k_e &= (20.4^2 - 9.6^2) \frac{\text{ft}^2}{\text{hr}^2} \left(\frac{\text{hr}}{3600 \text{ sec}} \right)^2 \left(\frac{\text{lb}_f \text{ sec}^2}{32.2 \text{ lb}_m \text{ ft}} \right) \\ &\quad \left(\frac{\text{BTU}}{778 \text{ ft lb}_f} \right) \\ &= 3.6 (10^{-6}) \frac{\text{BTU}}{\text{lb}_m} \end{aligned} \quad (IX-15)$$

Substituting Equations IX-11, IX-13, and, IX-15 into Equation IX-8, the relative importance of enthalpy potential energy and kinetic energy becomes clear,

$$\begin{aligned} \Delta e^f &= 5.015 + 0.257 + 3.6 (10^{-6}) \\ &= 5.272 \frac{\text{BTU}}{\text{lb}_m} \end{aligned} \quad (IX-16)$$

It is evident that the enthalpy change is substantially greater than the change in potential energy, and that the kinetic energy is totally negligible. In fact, considering Equations IX-16 and IX-11 it can be seen that a major reduction in the formulation of e^f can be accomplished with a minor error, approximately 5%,

$$e^f = c \Delta T \quad (IX-17)$$

This formulation has been adopted for the present version of the thermal simulator for efficiency of computation.

IX-3 Energy Balance Coefficients

As described in Section III an energy balance in the flowing stream yields

$$\Delta E^c + \Delta E^f + \dot{Q} = \Delta E^a \quad (IX-18)$$

Using the simplified formulation for the flow energy described in Appendix IX-2 and given in Equation IX-17, and the expressions in Section II for the heat conduction and accumulation, the following energy terms result:

$$\begin{aligned} E_{k-1}^f &= \dot{V}_\rho^f c^f T_{k-1,i}^{n+1} \\ E_k^f &= \dot{V}_\rho^f c^f T_{k,i}^{n+1} \\ E_{i-1}^c &= \frac{z_k - z_{k-1}}{2} [U_{k-1,i-1} (T_{k-1,i-1}^{n+1} - T_{k-1,i}^{n+1}) \\ &\quad + U_{k,i-1} (T_{k,i-1}^{n+1} - T_{k,i}^{n+1})] \\ E_i^c &= \frac{z_k - z_{k-1}}{2} [U_{k-1,i} (T_{k-1,i}^{n+1} - T_{k-1,i+1}^{n+1}) \\ &\quad + U_{k,i} (T_{k,i}^{n+1} - T_{k,i+1}^{n+1})] \\ E^a &= \frac{z_k - z_{k-1}}{2 \Delta t} [(\rho c A)^f + (\rho c A)^{st}] (T_{k,i}^{n+1} - T_{k,i}^n \\ &\quad + T_{k-1,i}^{n+1} - T_{k-1,i}^n) \end{aligned} \quad (IX-19)$$

where f denotes fluid and st denotes steel. Equation IX-18 may be rewritten in terms of the expressions of Equation IX-19,

$$E_{k-1}^f - E_k^f + E_{i-1}^c - E_i^c - E^a = 0 \quad (IX-20)$$

which is based on the mathematical cell of Figure 2.

An expansion of Equation IX-20 may be regrouped to obtain products of temperatures and coefficients,

$$\begin{aligned}
T_{k,i}^{n+1} = & A_{k,i} T_{k-1,i}^{n+1} + B_{k,i} T_{k+1,i}^{n+1} + C_{k,i} T_{k,i-1}^{n+1} \\
& + D_{k,i} T_{k,i+1}^{n+1} + E_{k,i} (T_{k,i}^n + T_{k-1,i}^n) \\
& + F_{k,i} T_{k-1,i-1}^{n+1} + G_{k,i} T_{k-1,i+1}^{n+1}
\end{aligned} \tag{IX-21}$$

where the coefficients are defined as follows:

$$\begin{aligned}
H &= \frac{2 \dot{V} \rho c}{H \Delta z} + U_{k,i-1} + U_{k,i} + \frac{(\rho c A)^f + (\rho c A)^{st}}{\Delta t} , \\
A_{k,i} &= \frac{2 \dot{V} \rho c}{H \Delta z} - \frac{U_{k-1,i-1}}{H} - \frac{U_{k-1,i}}{H} - \frac{(\rho c A)^f + (\rho c A)^{st}}{H \Delta t} , \\
B_{k,i} &= 0 , \\
C_{k,i} &= \frac{U_{k,i-1}}{H} , \\
D_{k,i} &= \frac{U_{k,i}}{H} , \\
E_{k,i} &= \frac{(\rho c A)^f + (\rho c A)^{st}}{H \Delta t} , \\
F_{k,i} &= \frac{U_{k-1,i-1}}{H} , \\
G_{k,i} &= \frac{U_{k-1,i}}{H} .
\end{aligned} \tag{IX-22}$$

For the soil analysis heat is transferred by conduction in both the radial and vertical directions. Also, the cell boundaries are located at different positions than for the flowing fluid. The soil geometry is in Figure 4. The energy terms in this figure are,

$$E_{k-1}^C = \frac{A}{4 \Delta z_{k-1}} (K_{k-1} + K_k) (T_{k-1,i}^{n+1} - T_{k,i}^{n+1}) ,$$

$$E_k^C = \frac{A}{4 \Delta z_{k+1}} (K_k + K_{k+1}) (T_{k,i}^{n+1} - T_{k+1,i}^{n+1}),$$

$$E_{i-1}^C = \Delta z_k U_{k,i-1} (T_{k,i-1}^{n+1} - T_{k,i}^{n+1}),$$

$$E_i^C = \Delta z_k U_{k,i} (T_{k,i}^{n+1} - T_{k,i+1}^{n+1}),$$

$$E^a = \Delta z_k \frac{\rho c A}{t} (T_{k,i}^{n+1} - T_{k,i}^n) \quad . \quad (IX-23)$$

Where A is the cross sectional area and,

$$\Delta z_{k-1} = z_k - z_{k-1} \quad ,$$

$$\Delta z_{k+1} = z_{k+1} - z_k \quad ,$$

$$\Delta z_k = \frac{z_{k+1} + z_k}{2} - \frac{z_{k-1} + z_k}{2} \quad .$$

An energy balance for the cell in Figure 4 requires

$$E_{k-1}^C - E_k^C + E_{i-1}^C - E_i^C - E^a = 0. \quad (IX-24)$$

Substituting Equations IX-23 into IX-24 results in an equation of coefficients and temperatures,

$$\begin{aligned} T_{k,i}^{n+1} = & A_{k,i} T_{k-1,i}^{n+1} + B_{k,i} T_{k+1,i}^{n+1} + C_{k,i} T_{k,i-1}^{n+1} \\ & + D_{k,i} T_{k,i+1}^{n+1} + E_{k,i} T_{k,i}^n \end{aligned} \quad (IX-25)$$

where the coefficients are:

$$\begin{aligned} H = & \frac{A}{4} [\Delta z_{k-1} (K_{k-1} + K_k) + \Delta z_{k+1} (K_k + K_{k+1}) + \Delta z_k (U_{k,i-1} \\ & + U_{k,i} + \frac{\rho c A}{\Delta t})] \quad , \end{aligned}$$

$$A_{k,i} = \frac{A}{4 H \Delta z_{k-1}} (K_{k-1} + K_k) ,$$

$$B_{k,i} = \frac{A}{4 H \Delta z_{k+1}} (K_k + K_{k+1}) ,$$

$$C_{k,i} = \frac{\Delta z_k}{H} U_{k,i-1} ,$$

$$D_{k,i} = \frac{\Delta z_k}{H} U_{k,i} ,$$

$$E_{k,i} = \frac{\Delta z_k}{H} \frac{\rho c A}{\Delta t} . \quad (IX-26)$$

IX-4 Friction Factor

A friction factor is needed for fluid flow in a pipe or annulus. This factor is specifically needed for computation of a surface convection heat transfer coefficient, as described in Section IV-2.

Friction factor is dependent on the Reynolds Number and the steel roughness, [11]. A commercial steel roughness is selected to eliminate this as a variable. For laminar flow, $Re < 2000$, the friction factor may be analytical derived,

$$f = \frac{64}{Re} . \quad (IX-27)$$

For fully turbulent flow, $Re > 350,000$, the factor is assumed to be constant,

$$f = 0.013 . \quad (IX-28)$$

However, in the transition region the definitions are more complex. Two linear approximations have been used to estimate the transition region,

$$f = \frac{64 + 0.007735 (Re - 2000)}{2000} , \quad (IX-29)$$

for $2000 < Re < 4000$, and

$$f = \frac{0.316}{\text{Re}^{0.25}} \quad (\text{IX-30})$$

for $4000 < \text{Re} < 350,000$.

IX-5 Fluid Viscosity Model

Viscosity is computed in the thermal simulator with a power law fluid model, [16]. This model relates the shear stress, τ to the shear strain rate, $\dot{\gamma}$,

$$\tau = M \dot{\gamma}^m, \quad (\text{IX-31})$$

where M and m are fluid properties. For a Newtonian fluid such as water M is the viscosity, μ , and $m = 1$. For calculations an effective viscosity is defined,

$$\mu_{\text{eff}} = \frac{\tau}{\dot{\gamma}} = M \dot{\gamma}^{m-1}. \quad (\text{IX-32})$$

Values of M and m are determined from two commonly measured properties of drilling and circulating fluids, plastic viscosity and yield point. These properties are derived from a Bingham plastic type fluid, [16], for which

$$\tau = S\dot{\gamma} + s, \quad (\text{IX-33})$$

where S and s are material properties. With this model S is viscosity and $s = 0$ for a Newtonian fluid. Common notation labels S as the plastic viscosity and s as the yield point.

Fluid viscometers are in wide spread use to measure the plastic viscosity, PV in centipoise, and yield point, YP in $\text{lb}_f/100 \text{ ft}^2$. Most viscometers have the following relations designed in,

$$\begin{aligned} \tau &= 5.109 R_N \text{ dynes/cm}^2, \\ &= 1.067 R_N \text{ lb}_f/100 \text{ ft}^2, \end{aligned} \quad (\text{IX-34})$$

where R_N is the viscometer reading at a rotation of N RPM, and

$$\dot{\gamma} = 1.703 N \text{ 1/sec}. \quad (\text{IX-35})$$

YP and PV are defined by a relation similar to Equation IX-33, but based on viscometer readings,

$$R_N = YP + PV \frac{N}{300} \quad . \quad (IX-36)$$

Common practice records dial readings, R_N , at $N = 300$ and 600 RPM, from which PV and YP are easily determined. From IX-36 it is evident that

$$\begin{aligned} R_{300} &= YP + PV \\ R_{600} &= YP + 2PV \end{aligned} \quad (IX-37)$$

Utilizing viscometer readings in a power law model requires some additional consideration of the viscometer relations. Let's assume that the R_{600} and R_{300} readings have been recorded. From Equation IX-34, IX-35 and IX-31 we get

$$5.109 R_{600} = M [1.703 (600)]^m \quad , \quad (IX-38)$$

$$5.109 R_{300} = M [1.703 (300)]^m \quad .$$

Dividing these equations, yields

$$\frac{R_{600}}{R_{300}} = 2^m \quad ,$$

which can be solved for m ,

$$m = \frac{\log \frac{R_{600}}{R_{300}}}{\log 2} \quad ,$$

$$m = 3.322 \log \frac{R_{600}}{R_{300}} \quad . \quad (IX-39)$$

Substituting m back into IX-38 yields

$$M = \frac{5.109 R_{600}}{1021.8^m} = \frac{5.109 R_{300}}{510.9^m} \quad . \quad (IX-40)$$

Equations IX-39 and IX-40 provide a value of M and m for computation of a viscosity with Equation IV-10.

Requiring viscometer readings at 600 and 300 RPM restricts input to the viscosity model. Using a measurement of PV and YP allows the user to have any two viscometer readings for application to Equation IX-36. Generally, plastic viscosity and yield point are recorded for any well fluid, whether R_{600} and R_{300} readings or two others are used. Using the determined values of PV and YP in Equation IX-37 results in the R_{600} and R_{300} dial readings. These readings can be substituted in Equation IX-39 and IX-40 to determine the power law parameters, from which the viscosity is computed in Equation IX-32. This is the fluid viscosity model used in the thermal simulator.

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