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AN ANALYSIS OF THE MADARAS ROTOR POWER PLANT—
AN ALTERNATE METHOD FOR EXTRACTING LARGE
AMOUNTS OF POWER FROM THE WIND

Volume 2. Technical Report

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AN ANALYSIS OF THE MADARAS ROTOR
POWER PLANT ---- AN ALTERNATE METHOD
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VOLUME 2 TECHNICAL REPORT

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PROJECT SUMMARY

The purpose of the program was to analyze and up-date the design of the Madaras Rotor Power Plant concept that had been developed in the 1930's to determine the technical and economic feasibility of this system to be competitive with conventional horizontal axis wind turbines. The Madaras concept uses rotating cylinders, vertically mounted on flat cars, to react with the wind like a sail and propel an endless train of connected cars around a closed track at constant speed. Electricity is generated by alternators on each car that are geared to the wheels. Electrical power is transmitted from each car to the power house by a trolley system.

A four-task program consisting of a series of wind tunnel tests, an electro-mechanical analysis, a performance analysis, and a cost analysis was conducted. Wind tunnel tests were conducted to validate rotating cylinder data in the literature and to obtain non-existing data that relates aerodynamic performance to rotating cylinder (rotor) geometry. Supporting studies included structural design, mechanical component design, and an electrical system design to provide a realistic set of inputs to a performance trade study which provided the basis for selecting a single rotor configuration and set of operating conditions which were considered to represent a good, but not necessarily an optimum design. Studies to determine the minimum spacing between cars governed by inter-rotor aerodynamic interference also were conducted.

Drawings and specifications of the system were used to estimate costs, and parametric cost/performance computer runs were made to correlate plant cost, annual output, and energy cost as a function of plant size parameters. These cost studies incorporated appropriate learning curves to account for reduction in cost resulting from improved production techniques and increased productivity.

Primary results of this study are:

- Madaras plants having circular track plan-forms probably will not be economically attractive, but those having racetrack plan-forms appear to be of interest.
- Madaras racetrack plants appear to be competitive with a farm of MOD-1 horizontal axis machines of equal total power generation capability (to within the overall accuracy of estimation for both schemes) in cost per unit rated power and cost per unit of energy delivered. Cost comparisons were based upon estimates made in this study and estimates made by Hightower¹ and General Electric² for the MOD-1 machine. A substantial economic benefit favoring the Madaras scheme does not appear, although this possibility may exist (under more "optimal" designs determined through more detailed analysis).
- Madaras racetrack plants having rated capacities as high as 228 MW and annual energy outputs of 975×10^6 kW-hr/year were studied, and analyses indicated that even larger, more efficient plants could be developed. Thus, Madaras plants can produce energy in the quantities of interest to the electric utility industry.
- Madaras plants are more complex, have higher electrical and mechanical losses, and hence will require higher operation and maintenance costs than horizontal axis machines. Further, a racetrack plant configuration for Madaras plants limits them to regions having generally unidirectional winds (including reciprocal directions) in which large expanses of relatively flat land are available. (However, similarly-sized HA-WTG plants require twice the land area as Madaras plants.)
- At this stage of the investigation, it is not possible to state conclusively whether or not the Madaras system will significantly out-perform a similarly-sized horizontal axis wind turbine system. The Madaras system seems to outperform horizontal axis systems in the areas of structural durability, economy of scale, amount and efficiency in use of land, and possibly energy cost. Therefore, in view of this potential, it appears that an in-depth optimal design study of the system is warranted.

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FOREWORD

This final report describes the study conducted by the University of Dayton Research Institute during the period from October 1976 to February 1978 on Contract E(49-18)-2554, sponsored by the Wind Energy Conversion Branch, Division of Solar Energy, United States Energy Research and Development Administration. Dr. Robert Thresher was Project Monitor.

The study was under direction of Mr. Dale H. Whitford, who, with Dr. John E. Minardi, conducted the preliminary performance analyses, planned the wind tunnel tests, and analyzed the wind tunnel test data. Mr. Levere F. Starner and Mr. Blaine S. West were responsible for the structural analysis study, and Mr. Robert J. Dominic conducted the electrical analysis.

The authors wish to express their appreciation for the excellent contribution to this program made by the personnel of the Gas Dynamics Laboratory, Aerospace Engineering Department, University of Michigan. Overall guidance, assistance in developing the test plan, development of the method for generating the simulated atmospheric boundary layer, and coordinating all aspects of the University of Michigan effort was provided by Professor William W. Willmarth. Test supervision and scheduling of support for the tests was provided by Mr. David R. Glass. Mr. Danile O. Scharf was responsible for directing tests and processing the data, and Mr. Charles Hogan, Mr. Leo Griffin, and Mr. Cletus Iott were responsible for instrumentation, model installation, and operation of the wind tunnel.

We also want to acknowledge other major contributions: the vortex analysis study conducted by Professor Harold Larsen of the Air Force Institute of Technology; the detailed structural and mechanical design layout done by Mr. Francis Shannon of Maier and Associates; the economic analysis conducted by

Mr. John L. McClellan; and the aerodynamic consultation provided by Dr. Frank L. Wattendorf.

Finally, we are especially appreciative of the support of the Detroit Edison Company and the assistance given by Mr. Walker L. Cisler, former Chairman of the Board of that Company; and the consultation provided by Mr. Russell F. Hardy, former Chief Engineer of the Madaras Rotor Power Project; and Dr. E.E. Lapin of the Aerospace Corporation.

SECTION I

INTRODUCTION

The Madaras Rotor Power Plant concept was originally developed in the period from January 1930 to April 1934. This concept, conceived by Julius D. Madaras, made use of the Magnus effect¹ on a rotating circular cylinder to generate electrical power.

1.1 CONCEPT DESCRIPTION AND BACKGROUND

An artist's rendering of the Madaras Rotor Power Plant, obtained from the January 1932 issue of Popular Science, is presented in Figure 1.1, and a copy of Madaras' patent is presented in Figure 1.2.

The Madaras project ranks as one of the more significant wind power generation projects conducted in the United States because: (a) it was uniquely different from other wind generation projects; (b) a full-scale rotor was designed, built, tested, and proven successful; and (c) it showed the promise of generating the large quantities of electric power (e.g., 100 megawatts electric) in a single power plant. In view of the unusual method of wind energy conversion of this system and its promise of generating large amounts of power at energy cost below that of coal-fired plants in 1934, this study was conducted to determine if a modernized Madaras design would be economically competitive with horizontal axis wind energy conversion systems.

The original concept of the Madaras plant consisted of a circular track, an endless train of rotor units mounted on streamlined flat cars, a power collection and distribution system, and a control and maintenance building, all located on a 323,752m² (80-acre) site. Design capacity was 18,000 kW for a 13.4 m/s rated wind speed and a 8.9 m/s constant track speed.² A brief description of the system proposed by Madaras is given as follows.

Big Electric Plant Run by Wind

AN AMAZING "merry-go-round" power plant to harness the wind for electrical energy is soon to be built, somewhere in the East, if final tests now under way, prove successful. Twenty spinning cylinders will rumble around a track 3,000 feet in diameter in an endless train. They will propel flatcars and turn dynamos geared to the axles.

Fantastic as the project sounds, prominent engineers have indorsed it, and six of the most important power companies in the United States have financed the build-

ing of the first \$100,000 experimental cylinder. Now rising at West Burlington, N. J., this seven-story-high shaft of aluminum recalls the cylinders that propelled Germany's famous "rotor ship" of a few years ago. Like them, it operates on the principle of the so-called "Magnus effect"—that a cylinder spun in the wind tends to move at right angles to the breeze.

The inventor, Julius D. Madaras, a Hungarian engineer living in Detroit, revealed the details of the proposed power station in an interview with **POPULAR**

SCIENCE MONTHLY. Electricity from dynamos on the rotor cars, moving at a fixed speed under automatic control, will be collected by a third rail and conducted to a substation for distribution to the surrounding country. Since the rotors must be spun artificially, a small electric motor keeps each one turning, but it uses only a small proportion of the power generated.

With a twenty-eight-mile wind blowing, Madaras estimates, the completed plant would supply enough electricity for a city of 150,000 inhabitants.

JANUARY, 1932

Figure 1.1. Artist's Conception of Madaras Plant, Popular Science Monthly. January 1932.

June 8, 1937.

J. D. MADARAS

2,082,966

WIND MOTOR

Original Filed Feb. 9, 1931

3 Sheets-Sheet 1

JULIUS D. MADARAS

BY Whittome Hallist Whittome
& Belknap ATTORNEYS

June 8, 1937.

J. D. MADARAS

2,082,966

WIND MOTOR

Original Filed Feb. 9, 1931

3 Sheets-Sheet 3

Fig. 6

JULIUS D. MADARAS

BY Whittome Hallist Whittome
& Belknap ATTORNEYS

Figure 1.2. Madaras Patent Diagram.

1.1.1 System Description

Track: A two-rail, steel track of 11 m gage, 457.3 m in diameter was used to guide the train of rotor cars. A spur track permitted the removal and exchange of rotor car units for maintenance.

Cars: Flat cars about 12.2 m long, were mounted on four, two-wheel trucks. The axles were mounted radially with respect to the center of the track, and the inner wheels were smaller than the outer wheels to eliminate wheel slippage. Side forces were reacted by idler wheels which rolled on the rail sides to further reduce track friction. The cars were streamlined to minimize drag, and they contained ballast to prevent overturning. Weight of each car and rotor was about 63,503 kg. The cars were connected in an endless train by steel cables with a 80.2 m center-to-center distance between the rotors.

Rotors: Eighteen rotors, 27.4 m high by 6.8 m in diameter, having a projected area of about 185.9 m^2 , were used in the basic plant. The general construction of the rotor and its support tower is shown in Figure 1.3. Each rotor was provided with a motor for revolving it at the proper speeds in either direction. Twice each revolution of the track (at track positions $\pm 90^\circ$ relative to the wind) each rotor was braked by the motor acting as a generator and then was rotated in the opposite direction in order that the component of the Magnus force tangent to the track would continuously propel all vehicles in a given direction. In this way, most of the rotational energy dissipated during braking was assumed to be conserved for use in spinning up other cars on the opposite side of the circular track. Change of rotational direction was initiated automatically by a weather-vane-activated control system mounted on top of the rotor.

A rotating end plate having a diameter of about 10.1 m was mounted on top of the cylinder to improve the aerodynamic performance (by increasing circulation, and hence lift; and by reducing induced drag by increasing the effective aspect ratio of the cylinder).

Figure 1.3. Rotor and Support Tower Used in Madaras' Tests, Burlington, New Jersey, 1932.

Power Generation: One induction generator was carried on each car, gear-connected to one of the axles. Generated power was delivered to contact wires arranged concentrically with the track, and through feeders to the switch mat. A synchronous condenser in the control house maintained a high power factor.

Control and Maintenance Building: A small building on an 80-acre plant site housed, in addition to the synchronous condenser and a motor-generated set, the control switch board, instruments, supplies, and accommodations for the operators.

1.1.2 History of the Madaras Project

The Madaras Rotor Power Corporation was incorporated under the laws of the state of Michigan in August 1928. By the end of 1929, seven public utility companies agreed to finance the development of the rotor plant. The utilities forming this group were:

The Detroit Edison Co.- Detroit, Michigan
Middle West Utilities Co.- Chicago, Illinois
North American Light and Power Co.-Chicago, Illinois
Public Service Electric and Gas Co.-Newark, New Jersey
Standard Gas and Electric Corporation-Chicago, Illinois
United Gas Improvement Co.-Philadelphia, Pennsylvania
The United Light and Power Co.-Chicago, Illinois

A three-step program was planned: (1) conduct wind tunnel experiments on the Magnus effect and perform plant performance and cost analyses based upon test results; (2) construct a full-sized rotor unit and test its performance; and (3) construct and test a commercial-sized plant of not less than 6,000 kW capacity.²

The wind tunnel experiments were conducted at the New York University Guggenheim School of Aeronautics under the direction of Professor Alexander Klemin. Additional studies were conducted by Professor Klemin; Professor Felix Pawlowski of

the University of Michigan, and Professor Theodore von Karman reviewed and concurred with the aerodynamic and performance analysis. Step 1 was completed on January 23, 1931 at a cost of \$10,136.³

In May 1931, the design, fabrication, erection, testing, and analysis of data from a full-sized rotor began. The rotor was mounted on a stationary platform on the property of the Public Service Electric and Gas Corporation in Burlington, New Jersey. Principal contributors to this work were Mr. Walker L. Cisler, Public Service Electric & Gas Company (recently retired as Chairman of the Board, Detroit Edison Co.), Mr. Russell F. Hardy (Chief Engineer of the Madaras Rotor Power Corporation), and Messrs. Hirshfeld and Madaras. Consulting assistance in structural design was obtained from Professor John D. Akerman, University of Minnesota, and Professor Stephen Timoshenko, University of Michigan. Construction started in October 1931, and was completed ten months later; but a ten-month delay occurred as a result of difficulty in developing a satisfactory force-measuring system. Tests were conducted from June to October, 1933. The cost of the second phase was \$161,691. Thus total project cost was \$171,827.⁴

Because of the severity of the depression and the lack of venture capital, the project was abandoned before the prototype plant could materialize.⁵ Thus, the economics and complete operation of the Madaras concept was never fully evaluated.

Very little information and no technical reports were published in the open literature on the project because of the strong desire of Madaras to protect his patents and the equally strong desire of the utility companies to make sure that the system was completely proven and that only completely reliable data be released publicly. They did not want information on cost of power at the plant boundary to be interpreted as cost to the consumer. Consequently, only carefully-controlled news releases were disseminated. Thus, the scientific community had

nothing with which to evaluate the program until recently when the University of Dayton was given the entire report file by the Detroit Edison Company.

1.1.3 Prior Research-Magnus Effect

A significant amount of work has been done to study the Magnus effect. After the work of Magnus,¹ the first quantitative data on rotating cylinders was obtained by Lafay from 1910-1912.^{6,7} As a result, Anton Flettner became interested in the possibilities of this device for propelling ships, and convinced Prandtl and others at Göttingen to conduct wind tunnel tests on cylinders. This work resulted in one of the classic sets of data on the subject.^{8,9,10} Flettner was encouraged by the results and convinced the directors of the Friedrich Krupp A.G. Germania-Werft to outfit a boat with two vertically-mounted cylinders. These cylinders were 15.6 m high by 2.8 m in diameter, and were outfitted with end caps having a diameter 1.4 times that of the cylinder. Each cylinder was turned by its own 11 kW electric motor at a speed of 120 rpm.¹⁰ Flettner had two successful ocean voyages, one in the North Sea and one round trip across the Atlantic to New York. The rotors performed well, and even in four or five days of dangerously stormy weather performed without malfunction or damage. Flettner said after the tests, "The trials of the Baden Baden have proved that all expectations which were based on the Göttingen experiments have been entirely fulfilled." A second ship, the Barbara, having three rotors also was built and successfully demonstrated.

Probably the most complete work on rotating cylinders was done by Thom at the University of Glasgow between 1925 and 1935. He investigated effects of Reynolds numbers, end plates, surface roughness, end conditions, and other variables. (Reference 11-16 inclusive.)

The first significant contribution in the United States was that of Reid¹⁷ in 1924. Later, in 1930, Klemin was retained by the Madaras Company to conduct tests for use in predicting the performance of the Madaras power plant.

Klemin's work, described in Reference 2, included wind tunnel studies of 0.20 m diameter cylinders having lengths of 0.89, 1.14, 1.40, and 1.65 m. Tests were conducted without end plates and with end plates having diameters 0.38 and 0.45 m. Peripheral-speed to wind-speed ratios from 0 to 6.6 at Reynolds numbers up to 3×10^5 were used in the tests. Klemin concluded that the cylinder performance agreed well with that of Göttingen, Reid, and Thom, and that the disturbance between adjacent cylinders is negligible beyond six diameters. He concluded that end plates were very beneficial and should be used for the Madaras system. Unfortunately, Klemin's data were not published and we have found only that for the 1.65 m cylinder having only the 0.45 m end plates.

In 1960, rotating cylinder studies at Reynolds numbers between 1.3×10^5 and 6.1×10^5 with end plate-to-cylinder diameter ratios of 2.0 were conducted by Matthews¹⁸ and by Griffiths in 1968 at the University of Wales.¹⁹ Both of these studies agree well with the early data. An excellent survey of the old and recent data was reported by Swanson in 1960.²⁰ Figure 1.4 is a reproduction of lift coefficient curves showing all types of data with and without end plates. We have added Klemin's data, labeled 1, for comparison. Swanson's contribution is probably the first truly infinite aspect ratio result that has been generated. He noted that the addition of end discs will yield maximum lift coefficients that are greater than those of a two-dimensional infinite aspect ratio cylinder. This result is attributed to the super circulation caused by the superposition of the flow around a finite rotating cylinder with that of a rotating end disc.

Thom's work has been recently validated by Dean R. Stuart in the laboratories of the University of Waterloo, Ontario, Canada.²¹

We infer from Swanson's and Stuart's modern studies that the work of Klemin agrees with the modern data trends. Since Swanson's findings indicate that Reynolds number effects on

lift and drag coefficients are negligible at cylinder rotational surface speeds greater than the wind speed, above the critical Reynolds number, and that it simulated very well the performance of the full-scale cylinder (27.4 m high x 6.8 m diameter) which operated at Reynolds numbers on the order of 6×10^6 .

1.1.4 Prior Research- Tracked Vehicle Airfoil Concept

Stalker²² compared a wing with a rotating cylinder for powering a tracked vehicle. The conclusion was that the winged vehicle will extract five times more power from the wind than a cylinder. This conclusion was based upon inappropriate simplifying geometric and aerodynamic assumptions which permitted him to solve his equations of motion in a simple closed form.

The basis for our conclusion on Stalker's work can be seen from the following equation derived from the power output of a vehicle moving down a track:

$$\frac{P}{1/2 \rho A V_w^3} = C_L \left(1 - \frac{C_D V_t}{C_L V_w} \right) \left(\frac{V_R}{V_w} \right) \left(\frac{V_t}{V_w} \right) \quad (1)$$

where P is the output power, A is the projected area of the wing or cylinder, V_R is the resultant velocity, V_w the wind speed, V_t the track speed, ρ is the air density, and, of course, C_L and C_D are the lift and drag coefficients respectively. The only difference in performance between a wing and a cylinder having a given area, A, for given values of V_w and V_t is the lift coefficient and the C_D/C_L ratio. The C_L term of a cylinder overpowers that of a wing to such an extent (10:1 ratio) that the superior C_D/C_L ratio of a wing over that of a cylinder does not compensate for this difference in C_L at low values of track speed. As the values of V_t become large, the first bracketed term overcomes the cylinder's C_L advantage and the wing generates more power than a cylinder. The C_D term could include the losses due to rotating the cylinder as well as the other common losses

to both systems; so a direct comparison of the lift-drag polars for the lifting surface does not tell the whole story. To further substantiate this performance comparison, our computer data is compared with that of a wing²³ in Figure 1.5 for low track speeds. A recent, excellent study by Lapin²⁴ also confirmed the superiority of a cylinder over a wing at low track speeds.

We believe that this analysis clearly demonstrates why a cylinder-powered vehicle is a low speed translator and that while in its low speed regime, it should be superior to the tracked wing vehicle. This low-speed, high-performance feature is the key to the promising potential of the Madaras concept.

1.2 UNIVERSITY OF DAYTON PRELIMINARY STUDIES OF MADARAS SYTEM

The University first became interested in the Madaras system as a result of discussions with Mr. Russell F. Hardy, who was the Chief Engineer responsible for the design, erection, and testing of the full-sized rotor system. The potential of the system looked so attractive that the University of Dayton agreed to finance a study of the concept. This study included: (a) a literature search and evaluation; (b) the writing of a computer program for simulating the performance of both the Madaras system and a tracked vehicle-airfoil system; (c) the running of some parametric studies to check Madaras' results; and (d) a study of the data in technical reports obtained from Mr. Walker Cisler of Detroit Edison. The reports include the final reports for the Phase I wind tunnel tests and performance analyses and the Phase II full-scale rotor test, as well as other supporting documents of consultants. (These documents are listed as References 2, 3, 4, and 25 through 36.)

We have also discussed the Madaras project with Mr. Walker L. Cisler, who was responsible for the full-scale test support. He and Mr. Hardy believed that the system was proven to be technically feasible, and that a modernization of the design and a current economic analysis of its potential should be conducted. They both have served as advisors on the present program.

Comparison of Power Output from Rotating Cylinder and Wind each with a Projected Area of 186m².

Curve	Investigator	Aspect Ratio	Reynolds Number	Remarks
a	Faundl	"	"	Ideal Fluid
b	Thom	12.5, 26	(5.3 to 8.8) × 10 ³	End Plates 3 × cyl.-dia.
c	Rold	13.3	(3.9 to 11.6) × 10 ⁴	
d	Göttingen	4.7	5.2 × 10 ⁴	End Plates 1.7 × cyl.-dia.
e	Thom	8	1.6 × 10 ⁴	
f	Swanson	"	(0.35 to 2) × 10 ⁵	
g	Thom	5.7	(3 to 9) × 10 ⁴	Rough Sanded Surface
h	Thom	5.7	(3 to 9) × 10 ⁴	Smooth Surface
i	Göttingen	4.7	5.2 × 10 ⁴	
j	Schwartzberg	4.5	(5.4 and 18) × 10 ⁴	Unpublished (Case Inst.)
k	Swanson	2	5 × 10 ⁴	Continuous End Sections
l	Klemin	4.4 to 8.1	(0.9 to 3.1) × 10 ⁵	End Plates 2.25 × cyl.-dia.
m	Matthews	5.7	1.9 to 6.1 × 10 ³	Stationary End Plates 1 × cyl.-dia. *(same as curve c)

Figure 1.4. Comparison of Rotating Cylinder Data from Various Wind Tunnels (Reference 20).

Figure 1.5. Performance Comparison of a Rotating Cylinder with a Wing (Reference 23) for Translation Machines.

1.2.1 Performance Analysis

Figure 1.6 contains power output curves we calculated for various wind and track speeds for a circular track and a straight track. We found that we had sufficient data to follow and critique the Madaras computations, and our independent computations using Madaras' data agrees closely with Madaras' results, as shown in Figure 1.7. As a result of our preliminary studies, we were unable to find any statement or evidence that the system had been proven either technically or economically unsound.

1.2.2 Preliminary Economic Comparisons

The foundation for our preliminary analysis of the Madaras system was the cost and performance analysis conducted by Madaras and the utilities in 1931 and our computer study. In the 1931 study, the consulting firm of Stevens and Wood³ was retained to work with engineers from the public utility companies, and not under the direction of Madaras. A set of checks and balances were established by utilities to prevent the inventor from being carried away by his enthusiasm. The estimators prepared preliminary designs, obtained equipment quotes, and then priced all manufactured parts. The utility companies reviewed the costs, and reported that "the estimates are liberal and that the total would easily cover the plant cost."³

The Madaras performance analysis predicted the gross rotor power generation performance based on Klemin's wind tunnel tests at various wind speeds, and deducted track losses; axle, bearing, and friction losses; electrical generator losses; and losses due to the power required to rotate and reverse the rotors.

These data on net power generated were then used by Madaras in conjunction with three wind duration curves, of which the most favorable curve was nearly identical to the $\bar{V} = 8.0$ m/s curve specified by NASA for the General Electric and Kaman studies. Estimates also were made for annual operating and maintenance charges, which were developed and approved by the utility companies.

Figure 1.6. Performance Comparison of a Madaras System Used on a Straight or a Circular Track.

Figure 1.7. Comparison of Madaras and UDRI Performance Computations.

From this analysis it was predicted that a plant with a rated output of 18,000 kW at a rated wind speed of about 13 m/s could be constructed at a cost of \$38.50/kW, and that electric power cost at the busbar would be 1.22 mills/kWh. This was based upon the wind duration curve which yielded a specific output of $4425 \frac{\text{kWh/yr}}{\text{kW}}$. It should be noted that in 1934 this cost was considerably lower than that of coal-fired steam plants in the Pittsburgh area, where the costs for the Duquesne Light Company varied from 4 to 5 mill/kWh for newer plants to 6 to 7 mills/kWh for the total system. In fact, coal costs in 1934 were from 1.25 to 1.5 mill/kWh.²⁹

By scaling this early Madaras cost data to the 1945 and 1975 time periods, we were able to compare the Madaras system economic predictions with those of the Smith-Putnam³⁷ and General Electric-Kaman³⁸ horizontal axis wind turbine. These comparisons indicated that the Madaras system could provide energy at lower cost than all of the above horizontal axis machines, and hence the Madaras system was potentially attractive from an economic standpoint.

1.3 POTENTIAL FEATURES OF THE MADARAS SYSTEM

- System shows potential for power production in large quantities at least in the range of 10-100 MW.
- Concept can capitalize on economy of scale; large systems are generally more economical than small systems.
- Simplicity and ruggedness of rotor structure will permit scaling up to large sizes with fewer structural problems than for long, flexible rotor blades
- Rotor has extremely high lift coefficients (10 times greater than an airfoil).
- Rotor can generate high power output at low track speeds.
- System does not need expensive towers to elevate and structurally support both the conversion unit and the heavy transmission-generation equipment high above the ground.

- Previous full-scale tests on the 27.4 m high by 6.8 m diameter rotor as well as wind tunnel tests have proven the technical feasibility of the concept.
- The cost of further development has been reduced because of the wealth of technical information that has been completed, the understanding of the system and its problems by the proposed staff, subcontractors, and consultants, and the computer software that already has been developed.
- Historical cost estimates by independent consultants and the one included herein indicate the system is competitive with fossil-fueled systems.

1.4 MAJOR ISSUES

Potential issues that have caused some people to question the Madaras system are:

- The concern that the early wind tunnel data on the Magnus effect were not valid and that performance estimations based on this data may be overstated.
- The opinion that the total of aerodynamic, mechanical, and electrical losses would be excessive.
- The opinion that the Burlington tests did not provide the economic basis for successful competition with other forms of power generation.
- The fact that rotors are mounted close to the ground in the zone of friction retardation of the winds.
- The fact that there has been little published technical data in the open literature concerning the early experiments and analysis and the absence of a hardware demonstration of a total Madaras electric power generation system.
- The concern that since the Flettner rotor ship was not considered an economic success, the Madaras system also would not be a success.

1.5 UNANSWERED QUESTIONS

We believe that we answered most of the above issues in our preliminary study. However, four important questions remained that had not been answered to our satisfaction. These questions were:

- What is the fewest number of rotors required to extract as nearly as possible the theoretical maximum amount of power per unit area from the wind?
- What aerodynamic performance can be expected from full-sized cylinders operating in the lower levels of the atmospheric boundary layers in terms of the various geometric and operational design variables which affect performance.
- What are the design requirements for structural, electrical and mechanical subsystems; and what performance of a Madaras system can be expected when modern, commercially-available electrical and mechanical components are used?
- What is the capital cost and the cost of electric power generated by modernized Madaras plants of different sizes in various climatic areas of the United States?

SECTION II

OBJECTIVES AND SCOPE

2.1 OBJECTIVES AND SCOPE

The primary objective of this program was to demonstrate the degree in which Madaras power plants having capacities in the 10 MW to 200 MW range are competitive with horizontal axis wind turbines. This objective was achieved by addressing the four unanswered questions stated in Paragraph 1.5.

These objectives and the major questions were addressed in a four-phase study. These phases were:

- An aerodynamic study which included wind tunnel testing to determine the aerodynamic performance of a rotating cylinder as a function of geometry, free stream flow conditions, and boundary layer profile.
- An electromechanical study aimed at updating the structural, mechanical, and electrical components to present-day technology and design criteria.
- A performance analysis involving the use of various computer simulation codes to determine the performance of various sizes of Madaras plants for various geometries and wind conditions.
- An economic analysis to determine the cost of installation, and the cost of electric power.

2.2 GROUND RULES

The following ground rules, as stated at the beginning of the study, will further define the scope and nature of this investigation.

- The project was a conceptual design study which analyzed the basic Madaras system with only those changes which could be incorporated simply with no major development efforts nor efforts to develop an optimized design.
- Analysis techniques, design approaches, and equipment will reflect the current state-of-the-art that is commercially available.

- Madaras system performance will be compared with the best available studies of horizontal-axis wind turbines without storage.
- Wind siting, wind duration curves, and design load and life criteria will be in accordance with that data specified for General Electric studies.
- Costs will be generated in accordance with accepted public utility practice, and costs will be based on production lots of 1, and 100 plants.

SECTION III

WIND TUNNEL TESTS

The performance of the Madaras rotor is complicated by the interaction of the atmospheric boundary layer with the flow field of the spinning rotor. The interaction region represents a complicated problem in three-dimensional, nonuniform flow. The separated flow interaction is greater near the ground at the base of the cylinder. The flow involves the interaction, stretching, and diffusion of the vorticity in the atmospheric boundary layer and the vorticity produced by the flow field of the spinning rotor.

Although references to the major contributions to rotating cylinder performance literature were given in Paragraph 1.1.3, a study of these data indicate that a complete set of data with aspect ratios and end plate sizes measured in modern facilities did not exist. Further, all of the available data were measured under uniform, upstream flow conditions free from a boundary layer; hence no information on the effects of the earth's boundary layer profile on cylinder performance was available. Lapin²⁴ was particularly concerned about both the quantity and validity of the data obtained in the 1920-1935 time period, and recommended that new wind tunnel tests be run. He was particularly concerned that the values of C_D in the older data seemed to be too small.

Since the three-dimensional flow field around a rotating cylinder immersed in the lower levels of the earth's boundary layer is so complex that it cannot be efficiently analyzed by analytical techniques, and since no wind tunnel data has been developed for this condition, it was decided to model the boundary layer flow field and perform measurements of the pertinent aerodynamic forces in a wind tunnel test program designed specifically to analyze the Madaras system. Existing analytical techniques such as the vortex analysis technique developed by Professor Larsen of the Air Force Institute of Technology

(described in Reference 39), were then used to study the mutual interference and minimum spacing question that was raised in Paragraph 1.5. This analysis will be described in Section 6.

3.1 WIND TUNNEL STUDY OBJECTIVES

The objectives of the aerodynamic study were to:

- Obtain a complete set of free-stream rotating cylinder aerodynamic data as a function of ranges of geometrical parameters pertinent to the Madaras system.
- Obtain a complete set of aerodynamic geometric data on rotating cylinders in a simulated atmospheric boundary layer profile.
- Obtain data on power required to rotate the cylinder under all flow and geometric conditions.
- Determine the validity of existing rotating cylinder data, especially that used by Madaras for his performance calculations.
- Determine by analytical methods the relationship between various boundary layer profiles and aerodynamic coefficients.

In all of the above tests, flow conditions were designed to represent adequately those of a full-size Madaras rotor.

3.2 WIND TUNNEL TEST PLAN

The wind tunnel program consisted of three parts:

- Developing methods for simulating atmospheric boundary layers for two sizes of cylinders.
- Conducting exploratory free-stream and boundary-layer tests on a 2-inch (51 mm) diameter cylinder having a geometric aspect ratio of 4 and end plate to diameter ratios (e/d) of 1, 2, and 3.
- Conducting extensive free-stream and boundary-layer tests on a series of 6-inch (152 mm) diameter cylinders having geometric aspect ratios of 3, 4, 5, and 6; and end plate to diameter ratios (e/d) of 1.25, 2, and 3.

One of the important features of this test plan was that each test was conducted at a constant wind speed (V) throughout the U/V range (U is the peripheral speed of the cylinder resulting from its rotation). This is a unique feature of this test plan. In the past, experimenters have decreased V in order to obtain higher U/V values because of motor speed and power limitations. However, for this program, motor selection was based upon its ability to provide sufficient power and speed so that maximum U/V values of 6 or more could be obtained at all test wind speeds.

The test plan is described in the following paragraphs.

3.2.1 Test Matrix

The basic wind tunnel test matrix for the 6-inch (152 mm) cylinder is presented in Tables 3.1 and 3.2 which shows all combinations of test, model, and flow configurations tested. A total of 106 basic tests were conducted, reduced, and analyzed, not counting reruns or special tests. The resulting data is the most complete set of rotating cylinder data available in the literature.

The 51-mm cylinder tests were primarily exploratory investigations to define the design and test requirements, problem areas, and the ranges of variables we would want to test in our more complete six-inch cylinder test series. Thus, primary emphasis was placed upon the six-inch cylinder tests, whereas the two-inch cylinder data were used as a back up and as an independent means for checking results and trends in some of the six-inch cylinder data.

3.2.2 Wind Tunnel Description

The subsonic wind tunnel of the University of Michigan's Aerospace Engineering Department was used for this program. This tunnel is a closed circuit, single return tunnel with an essentially rectangular test section 2.1 m wide by 1.5 m high by 7.6 m long. The corners of the test section are filleted

TABLE 3.1
SIX-INCH (152 mm) DIAMETER CYLINDER
FREE STREAM WIND TUNNEL TEST MATRIX

Test Description	Run Number	Test Configuration				Run Conditions					
		Model Location	Struts		Number End Plates	e/d	AR	Ref. Vel: m/sec	Rx 10 ⁻⁵	Max U/V	Max RPM
Boundary Layer Profiles	200-05	No Model						4.6	0.48	0	0
	200-14							13.7	1.45	0	0
	200-29							27.4	2.90	0	0
Vibration Test	242-00	Free Stream, Lower End Plate 30.5 cm	X		2	2.0	4.0	0	0	-	18,000
Static Tare	242-00A		X					0	0	-	18,000
Reynolds Number Survey	242-05		X					4.6	0.48	8	5,000
	242-14		X					13.7	1.45	6	10,000
	242-29		X					27.4	2.90	6	16,800
Static Pull Calibration	242-00A	Above Floor	X					0	0	0	0
	242-00B		X		↓	↓	↓	0	0	0	0
Free Stream Tests Top and Bottom end plates Spinning	342-14		X		2	2.0	4.0	13.7	1.45	6	10,000
	342-14A		X	X		2.0					
	341-14		X			1.25					
	341-14A		X	X		1.25					
	343-14		X			3.0					
	343-14A		X	X		3.0					
	351-14		X			1.25	5.0	13.7	1.45	6	10,000
	351-14A		X	X		1.25					
	352-14		X			2.0					
	352-14A		X	X		2.0					
	353-14		X			3.0					
	353-14A		X	X		3.0					
	361-14		X			1.25	6.0	13.7	1.45	6	10,000
	361-14A		X	X		1.25					
	362-14		X			2.0					
	362-14A		X	X		2.0					
	363-14		X			3.0					
	363-14A		X	X		3.0					
	331-14		X			1.25	3.0	13.7	1.45	6	10,000
	331-14A		X	X		1.25					
	332-14		X			2.0					
	332-14A		X	X		2.0					
	333-14		X			3.0					
	333-14A		X	X		3.0					

TABLE 3.2
SIX-INCH (152 mm) DIAMETER CYLINDER
SIMULATED ATMOSPHERIC BOUNDARY LAYER
WIND TUNNEL TESTS

Description	Run Number	Test Configuration				Run Conditions						
		Model Location	Cyl Car	Track	No. End Plates	e/d	AR	Approx. ref. vel. height cm	ref. vel. m/s	R 10 ⁻⁵	U/V	Q, Cyl Height cm
Comparable to runs for Aspect Ratio	3 400-14A	None ↓	None	None	None	None	None	30	13.7	1.43	-	-
	4 400-14B		↓	↓	↓	↓	↓	38	↓	↓	-	-
	5 400-14C		↓	↓	↓	↓	↓	46	↓	↓	-	-
	6 400-14D		↓	↓	↓	↓	↓	53	↓	↓	-	-
Same as Above	3 400-14E	1/2 Car in Front of Rakes ↓	↓	↓	↓	↓	↓	30	↓	↓	-	-
	4 400-14F		↓	↓	↓	↓	↓	38	↓	↓	-	-
	5 400-14G		↓	↓	↓	↓	↓	46	↓	↓	-	-
	6 400-14H		↓	↓	↓	↓	↓	53	↓	↓	-	-
Same as Above	3 600-14A	1/2 Car on 1/2 of Track in Front of Rakes ↓	↓	↓	↓	↓	↓	30	↓	↓	-	-
	4 600-14B		↓	↓	↓	↓	↓	38	↓	↓	-	-
	5 600-14C		↓	↓	↓	↓	↓	46	↓	↓	-	-
	6 600-14D		↓	↓	↓	↓	↓	53	↓	↓	-	-
Tare Test Boundary Layer Tests	531-00	Cyl. & Car on Balance Free From Track or Floor	X	X	2	1.25	3	30	0	0	6	38
	531-14		X	X	2	↓	3	30	13.7	1.43	↓	38
	531-14A		X	X	1	↓	↓	↓	↓	↓	↓	38
	531-14B		X	None	1	↓	↓	↓	↓	↓	↓	30
Tare Test Boundary Layer Tests	532-00		X	X	2	2.0	↓	↓	0	0	↓	38
	532-14		X	X	2	↓	↓	↓	13.7	1.43	↓	38
	532-14A		X	X	1	↓	↓	↓	↓	↓	↓	38
	532-14B		X	None	1	↓	↓	↓	↓	↓	↓	30
Tare Test Boundary Layer Tests	533-00		X	X	2	3.0	↓	↓	0	0	↓	38
	533-14		X	X	2	↓	↓	↓	13.7	1.43	↓	38
	533-14A		X	X	1	↓	↓	↓	↓	↓	↓	38
	533-14B		X	None	1	↓	↓	↓	↓	↓	↓	30
Tare Test Boundary Layer Tests	541-00	↓	X	X	2	1.25	4	38	0	0	↓	46
	541-14		X	X	2	↓	↓	↓	13.7	1.43	↓	33
	541-14A		X	X	1	↓	↓	↓	↓	↓	↓	46
	541-14B		X	None	1	↓	↓	↓	↓	↓	↓	38
Tare Test Same B/L as in Tests 541-XX	542-00	Cyl Elevated Car on Floor	X	None	2	2.0	↓	38	0	0	↓	56
	542-14D		X	None	2	↓	↓	↓	13.7	1.43	↓	56
	542-14C		X	None	2	↓	↓	↓	↓	↓	↓	46
	541-XX											
Boundary Layer Tests	542-14	Cyl & Car on Balance Free from Track & Floor	X	X	2	↓	↓	↓	↓	↓	↓	46
	542-14A		X	X	1	↓	↓	↓	↓	↓	↓	46
	542-14B		X	None	1	↓	↓	↓	↓	↓	↓	38
Tare Same B/L Ref. Height as Tests 541-XX	543-00	Cyl. Elev., Car on Floor	X	None	2	3.0	4	38	0	0	6	56
	543-14D		X	None	2	↓	↓	↓	13.7	1.43	↓	56
	543-14C		X	None	2	↓	↓	↓	↓	↓	↓	46
Boundary Layer Tests	543-14	Cyl. & Car on Balance Free From Track & Floor	X	X	2	↓	↓	↓	↓	↓	↓	46
	543-14A		X	X	1	↓	↓	↓	↓	↓	↓	46
	543-14B		X	None	1	↓	↓	↓	↓	↓	↓	38
	551-14	Track & Floor	X	X	2	1.25	5	46	13.7	1.43	↓	53
	551-14A		X	X	1	↓	↓	↓	↓	↓	↓	53
	551-14B		X	None	1	↓	↓	↓	↓	↓	↓	46
	552-14		X	X	2	2.0	↓	↓	↓	↓	↓	53
	552-14A		X	X	1	↓	↓	↓	↓	↓	↓	53
	552-14B		X	None	1	↓	↓	↓	↓	↓	↓	43
	553-14		X	X	2	3.0	↓	↓	↓	↓	↓	53
	553-14A		X	X	1	↓	↓	↓	↓	↓	↓	53
	553-14B		X	None	1	↓	↓	↓	↓	↓	↓	46

TABLE 3.2 (Concluded)
SIMULATED ATMOSPHERIC BOUNDARY LAYER
WIND TUNNEL TESTS

		Test Configuration					Run Conditions						
Test Description	Run Number	Model Location	Cyl Car	Track	No. End Plates	e/d	AR	Approx. Ref. Vel. Height	Ref Vel m/s	R 10 ⁻⁵	Max U/V	Q, Cyl Height	
↓	561-14	↓	X	X	2	1.25	6	53	13.7	1.43	6	61	
	561-14A		X	X	1							61	
	561-14B		X	None	1							53	
	562-14		X	X	2	2.0						61	
	562-14A		X	X	1							61	
	562-14B		X	None	1							53	
	563-14		X	X	2	3.0						61	
	563-14A		X	X	1							61	
	563-14B		X	None	1		↓	↓	↓	↓	↓	↓	53
Cylinder Static C _D Versus R runs in Boundary Layer	642-05	Cylinder and Car on Balance Free from Floor	X	None	1	2.0	4	38	4.6	0.48	-	38	
	642-10		X						9.1	0.96	-		
	642-15		X						13.7	1.44	-		
	642-20		X						18.3	1.92	-		
	642-25		X						22.9	2.40	-		
	642-30		X						27.4	2.88	-		
	642-35		X						32.0	3.36	-		
	642-40		X						36.6	3.84	-		
	642-45		X						41.1	4.32	-		
	642-50		X		↓	↓	↓	↓	42.7	4.80	-	↓	
2 Spinning end plates	642-14	↓	X		2	2.0	4	38	13.7	1.44	6	38	
	643-14		X		2	3.0	4		13.7	1.44	6	38	
one end plate on top, one end plate in center of cylinder	662-14X	↓	X		2	2.0	6	53	13.7	1.44	6	38	
	662-14X		X		2	3.0	6	53	13.7	1.44	6	38	
		↓		↓									

0.2 m up each wall, thus reducing the cross-sectional area to about 3.2 m^2 . Several fine mesh screens in the settling chamber combined with the high contraction ratio (15:1) result in unusually low turbulence in the test section.

The tunnel is capable of continuous operation at test section velocities of up to 67 m/sec (150 mph) with a model having an effective blockage of about 0.28 m^2 (3 ft^3). Somewhat higher velocities are attainable with less blockage.

The front side of the tunnel has many windows in it and a few windows are in the top of the tunnel. The side windows especially provide relatively good opportunity for viewing or photographing the model. Additional windows can be provided where specifically needed.

The wind tunnel facilities have instrumentation systems for measurement and recording of all necessary quantities. The force and moment data are obtained from the wind tunnel pyramidal strain gage balance system. This balance is mounted below the tunnel test section with the sting (7.6 mm in diameter) passing through the tunnel floor for attachment to the model. Force capacity is 2670 N (600 lb) lift, 980 N side, and 980 N drag. The side and drag force and all three components of moment were used for this test series. A hollow sting is used to facilitate the routing of flexible tubing from the model pressure taps to the pressure measuring system. This balance can support a model of gross weight over 136 kg. It provides for the measurement of all three moments, each having an accuracy of $\pm 2 \text{ in-lb}$ (0.226 N-m) by means of various combinations of strain gages. The six strain gage signals, corresponding to each of the six components, are digitized and fed into variable gain amplifiers. The outputs are displayed by digital panel voltmeters. The tunnel dynamic pressure and temperature are also displayed on digital panel meters. All these displayed values can be recorded manually if desired, but all the meters are equipped to output to a Digitec punched paper tape recording system. Thus, simultaneous readings of all meters can be

recorded on paper tape whenever a record of test data is desired. The paper tape can then be fed into a computer to provide a tabular printout or graph of the desired parameters. A picture of the wind tunnel, the balance system, and the data console is presented in Figure 3.1. One of the 152 mm models being tested can be seen through the window.

The boundary layer profiles were measured by five pressure rakes mounted behind the model at various stations across the tunnel as shown in Figure 3.2. Velocity measurements were obtained at nine heights above the tunnel floor at each rake position. Velocity data from all 45 readings were recorded instantaneously on a scanner valve using a Satra systems pressure transducer and then was punched into paper tape, and reduced automatically on the computer.

Data obtained during each test included tunnel speed, dynamic pressure, atmospheric conditions, cylinder rotational speed (by means of a photoelectric cell), and measurements of lift, drag, moments due to lift and drag, and motor torque (yaw moment) from the six component balance.

3.2.3 Test Model Development

The general requirements for the design of both the 51 mm and the 152 mm diameter test model designs were that they must be: (1) adaptable to the University of Michigan six component balance system; (2) capable of being raised and supported at various heights above the tunnel floor for free stream tests and the earth's boundary layer tests; (3) dynamically balanced over a speed range from 0 to 20,000 rpm (0 to 12,000 rpm for the $e/d = 3$ end plates); (4) able to be configured for end plate to cylinder diameter ratios of about 1, 2, and 3; and (5) that the cylinders be driven by motors housed within them to minimize model whipping which could occur from an externally-mounted motor. In addition, the 152 mm cylinder design was required to provide for aspect ratio variations of 3, 4, 5, and 6; whereas the 51 mm cylinder design was fixed at an aspect ratio of 4.

Figure 3.1. External Picture of University of Michigan Aerospace Department Wind Tunnel.

Figure 3.2. Boundary Layer Profile Pressure Rake Array.

An assembly drawing of the 152 mm diameter cylinder is presented in Figure 3.3. This cylinder was configured in much the same way as the 51 mm cylinder, but its size, the tremendous centrifugal forces generated on it, and the requirement to change sleeves in order to make aspect ratio changes required more care in both design and manufacture in order to insure safety and vibration-free operation. Both manual stress computations and finite element computer analyses were used to confirm the structural integrity of this large cylinder.

A Tech Development, Incorporated pneumatic motor was used to drive the cylinder. This is a four-stage turbine capable of delivering 37.3 kW at 15,000 rpm. Under lighter loads, maximum speeds of 20,000 rpm were attained. This motor, which weighed 4.5 kg and was 82.5 mm diameter by 16.5 mm long, was leased by the University of Dayton for the tests. The air supply and exhaust was fed through the tunnel sting extension, and the air hoses were routed so that no net force was exerted on the balance system by the air impulse. Maximum compressed air flow rate required was 0.45 kg/s (1 lb/sec) at 1.38 MPa (200 psi). Air supply was provided by the University of Michigan's filtered, pressurized tank system which was capable of providing air for one day of testing before repressurizing was required.

Four, 152 mm diameter sleeves were made having varying lengths from 457 mm to 914 mm. Thus, geometric aspect ratios of 3, 4, 5, and 6 were available for tests. These lengths were selected to bracket the AR of 4 used for the original Madaras cylinder design and to bridge between the AR = 8 cylinder tested by Klemin for Madaras.

Three sets of end plates having diameters varying from 29.5 mm to 456 mm provided end plate to diameter ratios of 1.25, 2, and 3. All rotating components were made of aluminum and the support structures are made of steel. High-speed, precision ball bearings were used at each end of the cylinder

Figure 3.3. Assembly Drawing of 152 mm Diameter Cylinder.

and these were oiled by an oiling device which automatically pumped oil into each bearing every 20 seconds. The oil lines were led inside the support sting beside the air hose.

The 152 mm cylinder was manufactured by Tech Development, Incorporated in Dayton, the manufacturer of the air turbine motor used to drive the cylinders. Tolerances and concentricity were carefully held; however, the larger size, centrifugal forces, and the requirement to utilize one set of end plates on each of the four cylinder sleeves necessitated dynamically balancing all end-plate, cylinder combinations. Care was taken in providing match marks to assure the cylinders were kept in the balanced configuration during reassembly.

After balancing, each cylinder sleeve with all end plate combinations was instrumented and spun up in a test cell to check operation under still air conditions. These tests were completely successful, and the cylinders were then released for wind tunnel testing.

The model car was rectangular in plan form with dimensions of 383 mm long, 295 mm wide, by 38 mm thick. All sides were streamlined with approximately a 45° wedge angle rounded at the top surface. As with the 51 mm cylinder, the car was capable of being mounted to the balance system, free from the floor, so that the influence of car drag would be included in the measured results.

3.2.4 Model Mounting in Wind Tunnel

Since for the free stream tests, it was necessary to elevate the cylinder about 30.5 mm above the tunnel floor with a relatively large-diameter, rigid sting, it was necessary to provide a streamlined housing around but free from the sting in order to prevent drag loads on the sting from being transmitted to the balance system. A typical photograph of the lower fairing used for the 152 mm cylinder is shown in Figure 3.4

In view of the size of the housings in relation to the cylinders, it was thought that the test data would be affected by the presence of the housing. Therefore, a mirror image streamlined strut was designed to extend from the ceiling of the test section down to, but not touching the cylinder. Each streamlined housing and mirror strut was carefully aligned relative to the tunnel centerline as well as relative to each other. Clearance between the streamlined members and the cylinder was of the order of 3 mm or less. A picture showing this test setup is presented in Figure 3.5.

Tests were run for each cylinder and plate combination with the lower housing only and with both the lower housing and the mirror strut. Any differences in the data noted was then used to correct the aerodynamic data for the effect of the lower housing so that free stream cylinder data would result.

A much simpler procedure was used for mounting the cylinder and car for the atmospheric boundary layer tests. The car was constructed so it could be clamped to the sting extension of the balance system at the point where the sting extension entered the tunnel floor. About 3 mm clearance between the car and the floor was provided to prevent friction between the car and the tunnel floor. The cylinder also was mounted on the sting with minimal clearance between the car and the cylinder. Figure 3.6 depicts this mounting arrangement for typical 152 mm cylinders. Both top plate only and top plus bottom end plate configurations were used for these tests.

The preceding mounting configurations were those used for the primary data acquisition effort. Additional tests not listed in the test matrix, were used to obtain background data. Some of these tests included the use of one revolving and one stationary end plate under free stream conditions; attaching the car to the floor but letting it float free from the cylinder, mounting the cylinder on the floor without the car; mounting the car alone in front of the pressure rakes

Figure 3.4. Side View, 152 mm Cylinder in Tunnel With Lower Streamlined Fairing Only.

Figure 3.5. Front View, 152 mm Cylinder in Tunnel With Lower Streamlined Fairing and Mirror Strut.

Figure 3.6. 152 mm Diameter Cylinder and Car Ready for Simulated Boundary Layer Test, $e/d = 3$ End Plates, $AR = 6$.

at various angles of attack to study the effect on the velocity profile of the car; mounting an elevated track model in front of the rakes; and mounting the cylinder and car on an elevated track to determine the amount of low altitude velocity enhancement this arrangement would make.

3.2.5 Calibrations and Operational Checks

Standard force calibration techniques were used to calibrate the balance systems. A cylindrical extension was mounted on the balance system sting, and varying static loads were applied at various heights and directions by means of a loading fixture as shown in Figure 3.7. Outputs of each data channel were recorded on the automatic data acquisition system, and matrix analysis techniques were used to correct for load interactions. The calibration data obtained were found to be linear and repeatable over the entire range of interest.

After all calibrations were completed and all computerized data processing and plotting routines were checked, an overall operational check was conducted. With one of the 152 mm cylinders mounted on the balance system, loads of the order of 222 N and 155 N were applied individually at the top, center, and bottom of the cylinder in both the lift and drag directions. The balance response to each load was recorded on the data acquisition system, punched into paper tape, and then the tape was computer reduced and plotted. The results indicated accuracy of about 1 percent and demonstrated that the entire system was operating satisfactorily.

No further checks were done for the 51 mm cylinder, but the test procedure for the 152 mm cylinder was approached cautiously, one step at a time because of the potential safety hazard associated with structure failure of this cylinder.

After the 152 mm cylinder was mounted on the balance system, the cylinder was spun throughout its range with no tunnel air blowing to make sure that the cylinder did not

Figure 3.7. Calibration Loading Fixture for Wind Tunnel Balance System.

excite any resonances in the balance system, which was less rigid than the floor of the test cell in which the cylinder had been run-up previously. These tests showed that cylinder operation over the range from 0 to 19,000 rpm was entirely satisfactory. During this run, data were recorded and processed to assure ourselves that no undesirable tare loads resulted from the incoming and exhausting air used in driving the turbine. No problems were observed.

Finally, a cylinder having an aspect ratio of 4 and end plate to cylinder diameter ratio of 2 ($e/d = 2$) was given a preliminary test over its entire rotating speed range at free stream Reynolds numbers varying from 0.5×10^5 to 2.9×10^5 . Results indicated that a test Reynolds number of 1.45×10^5 would provide data that could be extrapolated reliably to full-size cylinders. This is comparable to a tunnel speed of 45 ft/sec (13.7 m/s).

3.2.6 Atmospheric Boundary Layer Simulation Study

During the period when the models were being developed, University of Michigan personnel developed the methods to be used for generating an artificially thickened boundary layer beginning at the tunnel which simulated the earth's atmospheric boundary layer.

The profile selected, obtained from Reference 40, was that typical of wind blowing over flat, grassy plains in which wind speed varies at $h^{0.18}$. Although this profile is slightly different from the $h^{0.17}$ profile listed in our design criteria, we believe it simulates real conditions closely, and hence is satisfactory for this study.

The methods used for boundary layer simulation were similar to those used by Professor Willmarth at the University of Michigan for studies of wind loads on buildings. A photograph of the experimental setup for the 152 mm cylinders is presented in Figure 3.8. Boundary layers of up to 0.6 m thick were obtained by this method for the wind tunnel speeds planned for the tests.

Figure 3.8. Method of Simulating Atmospheric Boundary Layers for 152 mm Cylinder.

As can be seen in Figure 3.8, beginning at the point where the entrance cone enters the test section, three rectangular strips having a 127 mm high by 12.7 mm wide cross-section were spaced on 457 mm centers across the tunnel floor. From there on, twelve, 12.7 mm diameter rods followed by six, 19 mm quarter round moldings spanned the tunnel floor at 0.3 m intervals.

In order to assure a uniform boundary layer profile across the entire tunnel width, auxiliary fans directed toward the tunnel walls were placed in the entrance cone of the tunnel. Excellent results were achieved. Tests were conducted and data were reduced in a form that would permit the test director to specify a value of V_{∞} which would provide the desired profile and the desired reference wind velocity at a given reference height above the floor. The reference height selected was that at the midpoint of the cylinder when mounted on a car.

Typical results are plotted in Figures 3.9 and 3.10a. Figure 3.9 depicts the standard tunnel boundary layer which demonstrates that our free-stream tests for both the 51 mm and 152 mm cylinder were conducted in a uniform velocity field. In all free-stream tests the bottom of the cylinder was elevated 12-inches (305 mm) above the tunnel floor. The reader should note that although the 51 mm cylinder and the 152 mm cylinders having aspect ratios of 3 and 4 were completely immersed in the variable boundary layer, the longer aspect ratio cylinders (5 and 6) were partially in free stream flow. This test condition was beyond the control of the University of Michigan because a 0.6 m thick boundary layer was the maximum that could be generated.

One of the primary reasons for conducting the boundary layer test series was to help validate the air load model used in our Madaras system performance simulation computer program. The following paragraphs describe this air load model.

Figure 3.9. Tunnel Boundary Layer Profile Prior to Modification, 11 m/s Free Stream Velocity, .97 m Above Floor. Curve 3 is Centerline Profile.

Figure 3.10a. Simulated Atmospheric Boundary Layer for 152 mm Cylinder Having AR = 3 and AR = 6.

Figure 3.10b. Ekman-like Spiral Resultant Wind Relative to Different Sections of the Rotor Produced by Vector Addition of Translatory Motion of Rotor Car and Wind Boundary Layer Near Earth's Surface.

Two airload distributions are imposed on a rotor which is moving along a track at a speed, V_t : (1) a uniform airload versus rotor height caused by rotor car motion along the track; and (2) a nonuniform atmospheric boundary layer airflow distribution versus rotor height caused by the wind blowing over the surface of the earth at an angle ϕ relative to the track. The combination of these two flows yields a resultant velocity envelope along the height of the rotor that somewhat resembles an Ekman spiral* as shown in Figure 3.10b. Since this combination of air flows cannot be obtained in a conventional wind tunnel, it became necessary to develop an empirical model to predict the spiral-like airload distribution on the rotor.

Our computational procedure first included the determination of the magnitude and direction of the resultant local wind velocity vector V_R versus rotor height as a function of: (1) the free-stream wind velocity, V_W relative to the track at any given track location; (2) the wind boundary layer distribution; and (3) the constant track speed of the rotor car, V_t .

Then, for a given rotor size (aspect ratio, rotor diameter, end plate size, and number of end plates) and the appropriate value of U/VR_h (ratio of rotor surface speed to resultant wind speed at height h) we computed the spiral-like airload vector distributions of lift and drag as a function of height. These distributions were integrated relative to rotor height to obtain the total lift and total drag forces, and then the centroids and lines of action of the total lift and total drag forces relative to the line of tangency to the track were computed.

* Ekman - Theory first applied to slow currents at ocean floor - large scale plus coriolis effect "Über die Beeinflussung von Windbahnen durch Gegirge" - Beitr. Phys. Atmosphäre 19: 272-274; 1932.

The final step was to resolve the total lift and total drag vectors to determine the net normal and tangential force components relative to the track caused by the two airload distributions. These normal and tangential values were then used in our Madaras performance simulation computer program to compute rotor loads and power output.

The aerodynamic data obtained from our boundary layer tests were used as an independent means for validating our ability to reliably compute the total lift force, the total drag force, and the centroid of these forces caused by the nonuniform boundary airload distribution alone.

Using actual measured boundary layer velocity profiles for a given cylinder size (such as that in Figure 3.10a for a cylinder having an aspect ratio = 6) and using C_L and C_D versus U/V data from freestream wind tunnel tests, we computed resultant values of C_L , C_D , and centroid locations of these forces using the approach described above for the condition of a stationary rotor car (i.e., $V_t = 0$). Direct comparisons were made of the computed C_L , C_D , and centroid data with respective data measured under comparable conditions during the simulated boundary layer flow tests. This comparison indicated that errors in the computed force and centroid data were less than 3 percent. Therefore, the model described above was considered to be satisfactory for use in our performance computations.

3.2.7 Typical Cylinder Test Procedure

A brief description of a typical free stream test is given to assist the reader in understanding the test conditions and the resulting data. Nearly the same procedure was used for the boundary layer tests.

After the cylinder was mounted in the tunnel and all fasteners were checked, the instrumentation amplifiers were adjusted to their pretest positions. Instrumentation circuits were normally left energized at all times, even overnight, so that all systems would be thoroughly warmed up and stabilized prior to the test.

Barometric pressure readings were taken from a precision mercury barometer, and pretest zero readings were then recorded on punched paper tape. A test log was kept throughout the test by the test director. Two other operators were required: one to operate the tunnel and one to regulate the air flow which drove the cylinder turbine.

Next, tunnel air was accelerated to the desired speed, stabilized, and data readings from all channels were recorded on paper tape. This test point represented the zero rpm force measurement on the cylinder. The instrumentation system "freezes" all readings at the instant a "Read" button is pressed in order to provide chronologically coincident readings from several measurements, which were then punched into the paper tape. Digital readouts on panel volt meters were also available to visually monitor each measurement at any time.

From this point on, the cylinder motor air was turned on, and cylinder speed was carefully regulated and stabilized near a speed corresponding to $U/V = 0.5$. (The velocity, U , is the circumferential speed ($2\pi rn$) and V is the free stream tunnel speed.) When the rps counter indicated speed had stabilized, the "Read" button was pressed, and data were recorded as before.

The cylinder speed was successively increased in U/V increments of 0.5 up to at least $U/V = 5$ or possibly higher if it appeared from the panel readings that cylinder stall had not occurred. During this entire procedure, the tunnel operator carefully regulated his propeller speed to maintain constant q , as read from a manometer. Thus, each data set was

obtained at nearly constant q over the entire U/V range. Careful attention had to be paid to tunnel q , because the 152 mm cylinders caused considerable turbulence, especially when the $e/d = 1.25$ end plate was used. As end plate size increased, the turbulence over the top of the cylinder decreased, and tunnel fan blade stall was not as much of a problem.

Upon recording data at the maximum rpm, the cylinder was permitted to decelerate to zero rpm, and a complete data set at zero rpm and at test wind speed was taken. The tunnel was then shut down, and when q dropped to zero, a post-test zero was recorded.

Configuration changes such as adding or removing the mirror strut or changing the end plates were made, and then the procedure was repeated.

After all runs at a given aspect ratio and all end plate sizes were completed, the data were computed, plotted, and analyzed. If all appeared satisfactory, approval was given to change to the next aspect ratio.

3.3 FREE STREAM TEST RESULTS

In this and the following paragraphs, we will present the more important results of the wind tunnel tests.

The primary conclusions drawn from these tests were:

- All test models operated satisfactorily.
- The general arrangement and test procedure was satisfactory, and the method for generating the simulated atmospheric boundary layer was satisfactory.
- Test results were satisfactory, and C_L and C_D curves correlated well with those in the literature.
- The data are satisfactory for computing forces on full-sized cylinders.

Since the 152 mm data set was the most reliable and extensive, only this data will be presented.

3.3.1 Typical Set of Raw Free-Stream Data

The initial output of these tests from the computer consisted of a number of data tabulations and Calcomp plots of the data. During data processing, the data were read into the computer by means of punched paper tape, and the results were studied by the operator using a CRT display. At this stage additional identifying inputs were entered, and data editing was done. The approved edited data was then stored, final computations were made, and output was tabulated and plotted. The final results were also stored on disc.

A set of final raw data output includes:

- Final Data Tabulation
- Plot of C_L versus U/V
- Plot of C_D versus U/V
- Plot of HP versus U/V

A typical set of these data for the free-stream test 363-14 and 363-14A are presented in Tables 3.3 and 3.4 and in Figures 3.11 and 3.12. Only one rotational horsepower plot is presented (Run 363-14) because there was no difference between it and 363-14A. These and all other tests are listed in Tables 3.1 and 3.2.

From the data presented, the reader can see that the expected large C_L values have been verified, and a typical drag bucket is also evident. The data trends seem smooth and orderly.

The raw data were corrected for the strut effect to obtain the net free stream data by adding or subtracting the difference between ordinates of two comparable curves at a given U/V , as appropriate. Thus, for the C_L curve at $U/V = 5$, since the addition of the second (mirror) strut (363-14A) caused an apparent increase in C_L data from that when only the lower fairing was used (363-14), the difference between the two curves at $U/V = 5$ was subtracted from the value of 363-14 to obtain the corrected free stream curve.

TABLE 3.3

FREE-STREAM DATA FOR CYLINDER WITH BOTTOM FAIRING ONLY

RUN 36314

PM 12/32 12:??:??

ATM PRESSURE 74.80 CM HG

CYLINDER HEIGHT 36.000 IN
 DIAMETER 6.000 IN
 E/D 3.000

CENTER LINE HEIGHT 30.500 IN
 VELOCITY 45.000 FT/SEC
 REYNOLDS NUMBER 145161

U/V	CL	CD	CML	CMD	Y(ML)/D	Y(MD)/D	HP	REV/SEC
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.00	0.
0.00	0.050	0.926	-0.346	-1.579	-6.892	1.705	0.00	0.
0.50	0.049	0.608	-0.368	-0.456	-7.486	0.749	0.00	15.
0.99	0.839	0.531	0.633	-0.512	0.754	0.964	0.06	29.
1.50	2.479	0.414	0.433	-0.923	0.175	2.229	0.19	44.
1.97	4.445	0.432	0.576	-1.365	0.130	3.161	0.41	58.
2.45	6.602	0.601	0.586	-2.240	0.088	3.730	0.84	72.
2.92	8.706	0.970	0.927	-3.267	0.106	3.367	1.28	86.
3.52	11.237	1.664	1.267	-4.028	0.113	2.422	1.94	101.
3.86	12.050	1.938	1.355	-4.080	0.112	2.106	2.82	115.
4.53	13.719	2.553	1.899	-4.691	0.138	1.837	4.04	130.
5.02	14.262	2.800	2.021	-4.447	0.142	1.589	5.24	144.
0.00	0.169	1.070	-0.921	-0.654	-5.452	0.611	0.00	0.
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.00	0.

U/V	LIFT LB	DRAG LB	ML IN-LB	MD IN-LB	TEMP F	DENSITY	F3 VEL
0.00	0.000	0.000	0.000	0.000	75.9	0.002267	0.000
0.00	0.182	3.352	-7.102	-37.630	77.2	0.002261	46.203
0.50	0.182	2.255	-7.778	-12.382	77.4	0.002260	46.771
0.99	3.036	1.922	20.630	-13.026	77.5	0.002260	46.215
1.50	8.975	1.500	29.761	-21.549	77.5	0.002260	46.215
1.97	16.093	1.563	49.038	-31.197	77.8	0.002259	46.228
2.45	23.902	2.174	66.962	-50.816	77.9	0.002258	46.233
2.92	31.516	3.513	91.649	-74.455	78.1	0.002257	46.241
3.52	38.695	5.729	113.970	-88.928	78.1	0.002257	45.099
3.86	44.688	7.185	131.556	-97.933	78.0	0.002258	46.797
4.53	47.241	8.793	146.436	-105.652	78.0	0.002258	45.095
5.02	49.113	9.640	153.200	-101.471	78.0	0.002258	45.095
0.00	0.567	3.590	-17.248	-16.724	78.3	0.002257	44.526
0.00	0.757	0.732	-19.615	20.744	77.4	0.002260	0.000

TABLE 3.4

FREE STREAM DATA FOR CYLINDER WITH
BOTH MIRROR STRUT AND BOTTOM FAIRING

RUN 36314A
PM 12/3? 12:??:??
ATM PRESSURE 74.80 CM HG

CYLINDER HEIGHT 36.000 IN
DIAMETER 6.000 IN
E/D 3.000

CENTER LINE HEIGHT 30.500 IN
VELOCITY 45.000 FT/SEC
REYNOLDS NUMBER 145161.

U/V	CL	CD	CML	CMD	Y(ML)/D	Y(MD)/D	HP	REV/SEC
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.00	0.
0.00	-0.014	0.863	0.053	-1.686	-3.698	1.952	0.00	0.
0.51	-0.014	0.573	0.021	-0.697	-1.499	1.217	0.04	15.
1.00	0.913	0.493	0.612	-0.525	0.671	1.066	0.07	29.
1.48	2.435	0.237	0.340	-1.357	0.139	5.737	0.21	43.
2.00	4.430	0.271	1.166	-1.587	0.263	5.850	0.51	58.
2.51	6.848	0.359	1.077	-2.719	0.157	7.574	0.77	72.
3.00	9.242	0.738	1.219	-3.170	0.132	4.295	1.21	86.
3.57	11.885	1.372	1.519	-3.200	0.128	2.333	2.01	101.
4.01	12.613	1.645	1.827	-3.160	0.145	1.921	2.99	115.
4.53	14.062	2.076	2.113	-3.112	0.150	1.499	3.77	130.
4.96	14.470	2.221	2.300	-3.115	0.159	1.402	5.01	144.
0.00	0.106	1.051	-0.551	-0.562	-5.215	0.535	0.00	0.
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.00	0.

U/V	LIFT LB	DRAG LB	ML IN-LB	MD IN-LB	TEMP F	DENSITY	FS VEL
0.00	0.000	0.000	0.000	0.000	73.3	0.002278	0.000
0.00	-0.051	3.050	1.015	-38.755	74.9	0.002271	45.538
0.51	-0.050	2.075	0.338	-17.207	75.4	0.002269	46.125
1.00	3.224	1.740	20.291	-12.865	75.7	0.002267	45.572
1.48	8.602	0.835	26.717	-29.589	75.7	0.002267	45.572
2.00	15.645	0.958	60.198	-34.574	76.2	0.002265	45.593
2.51	23.581	1.236	75.755	-57.409	76.2	0.002265	45.020
3.00	31.824	2.542	97.399	-68.023	76.6	0.002264	45.036
3.57	39.878	4.602	121.072	-68.988	76.5	0.002264	44.451
4.01	43.432	5.663	136.291	-70.917	76.6	0.002264	45.036
4.53	48.424	7.148	153.538	-71.400	76.5	0.002264	45.032
4.96	51.106	7.846	164.698	-73.812	76.5	0.002264	45.606
0.00	0.373	3.710	-10.822	-15.599	76.8	0.002263	45.619
0.00	0.436	0.822	-11.160	24.122	75.6	0.002268	0.000

Figure 3.11. C_L and C_D versus U/V for $AR = 6$, $e/d = 3$, Lower Fairing only and Lower Fairing Plus Mirror Strut.

Figure 3.12. HP versus U/V for $AR = 6$, $e/d = 3$. Neither Fairing Nor Strut Has any Effect on Data.

A similar technique was used to correct the C_D curve, but the process was reversed: the difference was added to the 363-14 C_D curve.

No corrections for the horsepower curve were required. Note that it took about 5.2 hp to drive this cylinder at 8,640 rpm (184 rps). Our method for developing equations for scaling these power data to a rotor of any geometry and size is presented in Paragraph 3.3.7.

3.3.2 Selection of Test Reynolds Number

Swanson²⁰ determined that free stream Reynolds number, R , was not the governing parameter for flow similarity for a rotating cylinder as it is for an airfoil. His experiments on an infinite aspect ratio cylinder included acquisition of data at values of R ranging from 0.36×10^5 to 5×10^5 and for U/V values from 0 to 17. He determined that beyond U/V values of about 1.0, there appeared to be virtually no difference in the C_L versus U/V and C_D versus U/V data as a function of R . As the U/V increases beyond 1, the rotating cylinder is gradually immersed in an airflow that is significantly higher in speed than that in the free stream, and hence it is this local flow region that governs flow separation and transition.

In view of Swanson's experience, we anticipated that it would not be necessary to conduct our experiments at a free-stream Reynolds number at or beyond the critical Reynolds number of a stationery cylinder in an airstream (of the order of 3.5×10^5). Tests at this speed would have pushed the speed and power requirements, and the wind tunnel balance system beyond practical limits. Thus, we conducted a simple series of tests to check Swanson's observations and to determine the value of free

stream R beyond which C_D versus U/V data would be relatively unaffected by further increases in R .

The results of this study are plotted in Figure 3.13. Data from both the 51 mm diameter and 152 mm diameter cylinders were used where available. We also overlaid C_L and C_D data from adjacent data sets to check effects of changes in R . From this study we determined that adequate test results would be obtained at $R = 1.45 \times 10^5$, which was equivalent to a tunnel speed of about 13.7 m/s. Thus, the entire test program was run at a Reynolds number of about 1.45×10^5 . Further, to preserve this desired running condition, care was exercised to run all tests at constant dynamic pressure.

Therefore, since Reynolds number effects have been properly considered, we will be able to scale the wind tunnel data directly to full-sized configurations over the entire test ranges of U/V , e/d , and AR .

3.3.3 Basic Free Stream Data

The complete set of free stream aerodynamic data is summarized in Table 3.5 and Table 3.6. This data is unique in that it is the most complete information on rotating cylinder aerodynamic performance in terms of geometric variations that exists in the literature today. The addition of the boundary layer data (described in Paragraph 3.4) makes the data even more extensive.

A typical C_D versus C_L^2 plot and its accompanying C_L versus U/V plot are presented in Figures 3.14 and 3.15, respectively, for end caps having diameters twice that of the cylinder diameter ($e/d = 2$). The numbered points on the curve represent the corresponding U/V values of those points.

Figure 3.13. Study to Determine Test Reynolds Number C_D versus R for Various Values of U/V .

TABLE 3.5
FREE STREAM C_L DATA

U/V \ e/d	AR=3			AR=4			AR=5			AR=6		
	1.25	2	3	1.25	2	3	1.25	2	3	1.25	2	3
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.5	0.00	0.00	-0.05	-0.10	-0.10	-0.10	0.10	-0.10	-0.10	-0.20	-0.20	-0.10
1.0	0.75	1.00	1.00	0.90	1.00	0.95	0.80	1.00	0.95	0.70	0.60	0.85
1.5	2.30	2.40	2.60	2.25	2.50	2.50	2.40	2.50	2.60	2.25	2.30	2.50
2.0	3.85	4.30	4.70	3.30	4.35	4.50	3.80	4.40	4.80	3.75	4.20	4.40
2.5	4.40	6.60	7.05	4.60	6.65	7.00	4.80	6.70	7.10	5.20	6.50	6.70
3.0	3.95	8.20	9.30	5.20	8.45	9.00	5.80	9.00	9.50	6.65	8.70	9.20
3.5	3.80	9.10	10.75	5.00	9.25	10.90	6.50	10.40	11.40	6.95	10.40	10.90
4.0	3.70	9.40	11.95	4.90	9.75	12.30	6.70	11.20	12.70	7.10	11.80	12.15
4.5	3.65	9.55	12.55	4.80	9.90	13.25	6.70	11.80	13.30	7.30	12.50	13.20
5.0	3.55	9.60	13.35	4.80	10.00	13.60	6.70	12.10	13.50	7.50	13.00	13.90
5.5	3.55	9.60	13.60	4.80	10.30	13.85	6.70	12.30	13.55	7.65	13.10	14.20
6.0	3.50	9.50	14.05	4.80	10.80	14.00	6.65	12.30	13.60	7.85	13.10	14.30

TABLE 3.6
FREE STREAM C_D DATA

U/V \ e/d	AR=3			AR=4			AR=5			AR=6		
	1.25	2	3	1.25	2	3	1.25	2	3	1.25	2	3
0	0.85	0.80	0.80	0.80	0.70	0.80	0.80	0.75	0.75	0.70	0.85	0.85
0.5	0.50	0.55	0.65	0.60	0.50	0.60	0.55	0.55	0.60	0.55	0.70	0.70
1.0	0.60	0.50	0.55	0.55	0.45	0.55	0.55	0.50	0.55	0.50	0.65	0.55
1.5	1.10	0.70	0.75	0.90	0.65	0.65	0.85	0.60	0.65	0.55	0.75	0.50
2.0	2.35	1.50	1.25	1.70	1.30	0.90	1.40	1.05	0.90	0.85	0.95	0.60
2.5	3.45	2.70	2.10	2.75	2.35	1.80	2.30	1.95	1.50	1.35	1.30	0.85
3.0	3.55	4.50	3.25	3.40	3.85	2.75	3.45	3.20	2.50	2.15	2.00	1.35
3.5	3.60	5.70	4.60	3.75	5.05	3.75	4.00	4.35	3.35	2.70	2.70	1.95
4.0	3.65	6.75	5.40	3.95	5.70	4.75	4.15	5.35	4.15	2.90	3.50	2.55
4.5	3.65	7.50	6.10	3.95	6.30	5.55	4.20	5.80	4.65	3.05	4.10	2.95
5.0	3.50	7.95	6.60	3.90	6.65	6.00	4.25	5.90	4.80	3.15	4.30	3.30
5.5	3.55	8.10	6.85	3.95	6.95	6.20	4.30	5.95	4.90	3.20	4.35	3.45
6.0	3.55	8.15	7.00	4.15	7.25	6.35	4.30	6.00	4.90	3.25	4.40	3.55

Figure 3.14. Freestream C_D versus C_L^2 for Various AR, Two End Caps, $e/d = 2$.

Figure 3.15. Freestream C_L versus U/V for Various AR, Two End Caps, $e/d = 2$.

All data seem to fit smooth curves with minimal scatter except for the $AR = 5$ curve at U/V values of 4 and 5. Since these most inaccurate points are either within ± 6 percent of the corresponding C_D value or within ± 3 percent of the corresponding C_L value, we believe that the data set is well within acceptable accuracy limits. Further, the faired data trends were checked in other cross plots as a function of AR and e/d ratio, and these trends were found to be consistent within the complete data set. It is believed that the larger errors mentioned above were the result of inconsistent behavior during the onset of stall and the beginning of the super-circulation phenomena. Error bands due to the instrumentation were no larger than ± 1 percent. The uniformity of the plotted raw data trends in Figures 3.11 and 3.12 also is indicative that the data are reliable.

In view of the satisfactory trends in the data set as a whole, we believe the data can be used confidently in performance simulations of full-sized Madaras systems. Other data cross plots will be presented in paragraphs which follow.

3.3.4 Effect of e/d Ratio Variation on C_L and C_D

Typical plots of C_L versus C_D , C_L versus U/V , and C_D versus U/V for all values of e/d ratio are presented in Figures 3.16 and 3.17. Data for aspect ratios 3 and 6 were selected to show the effect of e/d ratio for the limiting values of the aspect ratio range studied.

One of the first observations one makes after studying Figures 3.16 and 3.17 is that C_L max dramatically increases as e/d ratio increases, and that the increase in C_L max caused by changing e/d ratio from 1.25 to 2.0 is much more significant than for changing e/d ratio from 2.0 to 3.0. Further as aspect ratio increases, the increase in C_L max caused by e/d changes from 2.0 to 3.0 becomes less significant.

Figure 3.16. Freestream C_L versus C_D for AR of 3 and 6 and e/d Ratios of 1.25, 2, and 3, $d = 152$ mm.

Figure 3.17. Freestream C_L and C_D versus U/V , AR of 3 and 6, and e/d Ratios of 1.25, 2, and 3, $d = 152$ mm.

Thus it would appear that an optimum design point may occur near $e/d = 2$ in view of the tradeoff between the small C_L max versus the large increase in power required to spin the cylinder when end plate ratio is increased from 2 to 3. (Power data will be presented later.)

For the C_D curves, the effect of aspect ratio or induced drag is apparent, and one can see a small reduction in C_D with increased e/d up until stall occurs. Beyond stall, both lift and drag of the $e/d = 1.25$ curves is essentially unchanged. Although C_D for the $e/d = 1.25$ curves looks small at U/V values beyond stall, the L/D ratio is small and so is power output. It should be noted that the maximum C_D value for an e/d ratio of 3 is less than that for an e/d ratio of 2 for all aspect ratios, but that the incremental change in C_D is about the same for each aspect ratio studied. This difference in C_D also gives weight to our previous observation that there may not be enough improvement in aerodynamic performance to justify end plate ratios much greater than 2.

3.3.5 Effect of Aspect Ratio Variation on C_L and C_D

Figure 3.18 presents for e/d ratios of 1.25, 2, and 3, a set of curves representing the variation of C_L versus U/V and C_D versus U/V for each of the aspect ratios tested.

The lift curve slopes are consistent, and the pronounced drag buckets observed in existing rotating cylinder data are present in our data.

There is a definite increase in C_L max with AR increase for a given value of e/d . However, it is interesting to note that the effect of aspect ratio on C_L max decreases as e/d increases until there is very little effect on C_L max caused by aspect ratio for $e/d = 3.0$. These curves again show that C_L max is affected more strongly by e/d ratio than by aspect ratio.

On the other hand, as would be expected, aspect ratio increases in the range tested caused marked reductions in

Figure 3.18. Freestream C_L and C_D versus U/V , AR of 3 to 6, and e/d Ratios of 1.25, 2, and 3; $d = 152$ mm.

C_D as a result of decreasing induced drag. Later, we will show that the increased power required to rotate the cylinder with increased aspect ratio is not appreciable, so that a good aerodynamic design point should favor use of a higher aspect ratio. The improved wind speed near the top of taller cylinders also favors a high aspect ratio; however, the resulting increase in car weight to compensate for the resulting increase in overturning moment may limit the magnitude of the aspect ratio.

It is an interesting historical note that the original Madaras design proposed by Professor Klemin of New York University was for a cylinder 125 feet high by 16 feet diameter ($AR = 7.8$)² However, Professor Pawlowski of the University of Michigan convinced Madaras to reduce the cylinder proportions to 27.4 m high by 6.8 m in diameter in order to reduce bending moments and overturning moments.²

Since we showed (Paragraph 1.16, Equation 1) that the Madaras system is influenced largely by C_L and not so much by C_D/C_L because the system is a low speed translator, design points favoring the higher lift coefficients at the expense of higher drag coefficients could result in a more favorable overall system design. This is the type of compromise Professor Pawlowski probably made in 1931. Since aerodynamic performance of the cylinder with $AR = 3$ is so much below that of the other cylinders, an aspect ratio of 3 probably will not be competitive. However, a cylinder having an aspect ratio in the 4 to 6 range with an e/d ratio of 2 appeared to be attractive.

3.3.6 Comparison of UDRI Aerodynamic Data with Pertinent Existing Data

We believe that the wind tunnel data generated by Klemin for Madaras and that generated by Betz, et al, (Göttingen) for the Flettner rotor ship are the most pertinent sets of existing data with which to compare our data. Unfortunately, only one of Klemin's curves has been found, and it is for $AR = 8.1$ with e/d ratio of 2.25. The Göttingen data were for

three configurations all of $AR = 4.7$ with e/d ratios of 1.0, 1.7, and 2.0.

As a means of providing checks of the data reported herein, we selected Klemin's data and the Göttingen data having the e/d ratio of 2 for comparison. These comparisons are presented in Figures 3.19 and 3.20.

The comparison with Klemin's data is in the form of lift-drag polars of Klemin's data and UDRI data extrapolated to aspect ratio of 8 for $e/d = 2$. For simplicity, our extrapolated method accounted only for the decreases in induced drag caused by aspect ratio. No account was made for the increased C_L which would result from the e/d increase from 2 to 2.25 and due to the aspect ratio increase from 6 to 8. Nor was C_D decreased due to the difference in e/d ratio. Thus, we believe our extrapolation is conservative.

Our extrapolation was accomplished by two independent methods. First, a cross plot was made of the C_D data versus AR for various values of U/V for all $e/d = 2$. This is shown in Figure 3.21. From this plot, appropriate trends of C_D variation versus AR at various U/V levels were established.

Secondly, the decrease in induced drag caused by changing aspect ratio from 6 to 8 was computed by the well known induced drag equation:

$$C_{D_8} = C_{D_6} + \Delta C_{Di} = C_{D_6} + \frac{C_L^2}{\pi} \left[\frac{1}{8} - \frac{1}{6} \right] \quad (2)$$

In this computation, the C_L value used for each value of U/V from 0 to 6 was that obtained from Run 362 ($AR = 6$, $e/d = 2$).

After computing the values of C_D (aspect ratios of 8) for U/V values from 0 to 6 by Equation 2, the computed data were plotted on the cross plot, Figure 3.21. Then the data trends and the computed data were cross checked. As can be seen, the trends and the computed data agree rather well. The computed data then were used to plot the extrapolated lift-drag polar on Figure 3.19.

Figure 3.19. Comparison of UDRI Extrapolated Data with Klemin's Measured Data.

Figure 3.20. Comparison of Interpolated UDRI Data with Göttingen Data.

Figure 3.21. Cross Plot of C_D versus AR for $e/d = 2$. Used to Extrapolate UDRI Data for AR = 6 to AR = 8 Data for Comparison with Klemin's Data.

The results in Figure 3.19 are quite encouraging. Klemin's data shows lower drag than the UDRI data in the region below $U/V = 2.5$. This result is valid because Klemin ran his tests at varying Reynolds numbers in order to obtain the desired U/V values because of motor speed limitations. His lower U/V ranges started at $R = 3.1 \times 10^5$ compared to our test Reynolds number of 1.45×10^4 . Thus at zero or low rotational speeds, his C_D data should be lower than ours. On the other hand, our conservatively extrapolated data has higher L/D ratios than Klemin's above U/V values of 2.5.

A second comparison of our data was made with the Göttingen rotating cylinder data derived from tests of a cylinder having an aspect ratio of 4.7 and an e/d ratio of 2.0. This comparison is shown on Figure 3.20. UDRI data were interpolated from the crossplots shown in Figures 3.21 and 3.22 to obtain the curve which would be comparable to the Göttingen data. Thus, correction of both UDRI's C_L and C_D data were made, and therefore a direct comparison was possible.

The interpolation procedure was straightforward. Figure 3.21 was used to interpolate values of C_D versus U/V at $AR = 4.7$, and C_L versus U/V data at $AR = 4.7$ was obtained in a similar manner from Figure 3.22.

We believe the data correlate very well, and as in the Klemin comparison, our data appears to have lower C_D values and higher C_L values than Göttingen's, especially at higher U/V values. The fact that our data were obtained at a Reynolds number which was three times higher than that of the Göttingen tests may account for the lower drag below $U/V = 1.5$.

3.3.7 Power Required to Rotate Cylinder

Plots of horsepower required to rotate cylinders having AR of 3 and 5 are presented in Figure 3.23. Both graphs present power required at the test wind speed (13.7 m/s) versus U/V . In addition, the $AR = 3$ graph presents curves for power required to rotate the cylinders in still air (refer to the rpm

Figure 3.22. Cross Plot of C_L versus AR for $e/d = 2$. Used to Interpolate UDRI Data for $AR = 4$ to $AR = 4.7$ for Comparison with the Göttingen Data. (The apparent curvature shown by experimental points for $C_L = C_L(AR)$ for $U/V > 3.0$ could exist, or it could represent data scatter. More study is required to solve this anomaly.)

Figure 3.23. Power Required to Rotate Cylinder versus U/V and rpm for $AR=3$ and 6 and for e/d of 1.25, 2, and 3. Both Power Required in Still Air as Well as Power Required at $V_w = 13.7$ m/s are Given for the $AR = 3$ Configuration, $d = 152$ mm.

scale). Note that more power is required in still air than is required in the wind for a given value of rpm. This result is consistent with data from Reid.¹⁷

An important trend to notice in the data is the marked amount of power required to rotate a cylinder with an $e/d = 3$ end plate ratio versus a cylinder with an $e/d = 2$ end plate ratio.

Based on our calculations, the power required to turn a cylinder without end plates at a speed of 10,000 rpm varies from 224 W for a 152 mm cylinder having an aspect ratio of 3 to only 447 W for the same diameter cylinder having an aspect ratio of 6.

The calculated variation of power required with end plate size is another matter, as is shown in Figure 3.4. For our experiment, the calculated power for two disc (end plates) without a cylinder revolving at 10,000 rpm is as follows:

$e/d = 1.25$	$e/d = 2.0$	$e/d = 3.0$
112W	1007W	6229W

So, it can be seen that it is definitely the end plates and not the cylinder that absorb most of the power.

This comparison substantiates our earlier contention that the designer pays a large penalty in power absorbed in turning the cylinder for a small gain in performance when one selects an e/d of 3.0 instead of an e/d of 2.0 for end plate size.

Our computer program which simulates Madaras power plant performance requires input data to account for the viscous and bearing losses caused by rotating the cylinder. This input must be capable of accounting for full-sized cylinder power losses for any geometry desired. Although the power-required data previously described are adequate for model-sized cylinders, we needed to develop a reliable method for simulating performance

of full-sized cylinders. Our method made use of a set of empirical equations developed by Theodorsen and Regier⁴¹ and to independently check the prediction capability of these equations by our wind tunnel results and the full-sized cylinder results obtained by Madaras in the 1930's.

Theodorsen and Regier conducted a series of analytical studies and tests to determine the torque required to revolve discs, cylinders, and streamlined rods at constant speed. The purpose of this work was to obtain data at speed regimes higher than those studied earlier by Von Karman, Prandtl, Ackeret, and Taylor. Theodorsen and Regier's data verified the earlier work and also provided information applicable to Mach numbers up to 1.7 and Reynolds numbers up to 7,000,000. Since these tests extended the earlier results up to and beyond the range of Reynolds number of interest to the Madaras study, we felt that the resulting equations should be valid and useful for our computer simulation.

These equations for computing power required to overcome viscous losses are:

- For one end plate (disc) in turbulent flow:

$$\text{Power required} = C_M \frac{1}{2} \rho a^5 \omega^3 \quad (3)$$

where

$$C_M = 0.146 R^{-1/5} = 0.146 \left(\frac{\omega a^2 \rho}{\mu} \right)^{-1/5} \quad (4)$$

- For one cylinder in turbulent flow:

$$\text{Power required} = C_D \pi \rho r^4 \omega^3 \ell \quad (5)$$

where

$$\frac{1}{\sqrt{C_D}} = -0.6 + 4.07 \log_{10} R \sqrt{C_D} \quad (6)$$

In these equations, C_M is the disc moment coefficient, ρ is the air density, a is the disc radius, ω is the angular velocity, C_D is the cylinder drag coefficient, R is Reynolds number based on free stream velocity and cylinder (or disc) radius, r is cylinder radius, ℓ is the cylinder length, and M is the coefficient of viscosity.

By appropriately combining these equations, an empirical relationship for a cylinder of any diameter and length and having either 0, 1, or 2 end plates was generated.

These equations were used to compute power required to turn each of our 152 mm diameter rotor models, and the computed results were compared with the wind tunnel test data. Comparisons for all cylinder AR and e/d combinations were made using both 1 and 2 end plates mounted on a rotor car in a simulated atmospheric boundary layer. The comparison showed good agreement between measured and computed values, with a 3 σ error of less than about 10 percent for all U/V values and for all of the 24 configurations studied.

One final check was made to validate the use of the preceding equations to predict power required to rotate full-sized rotors. The equations were used to compute power required to turn the full-sized Madaras rotor (27.4 m high by 6.8 m diameter) and to compute power required for the rotors to rotate on three of Flettner's rotor ships. The Madaras data, obtained from Reference 4, is plotted with our computed data in Figure 3.24. The Flettner data were obtained from several sources, principally from Reference 10, the accounts by Flettner of his work.

Power required data from all of these large cylinders (including one of the large configurations analyzed for this present study) were scaled to a rotor configuration having 2 end caps with an e/d ratio of 1.35 and a 120 rpm motor speed. These scaled results were plotted in terms of power/unit length of rotor versus rotor diameter and are presented in Figure 3.25.

Figure 3.24. Comparison of Measured Power Required to Spin the Full-Sized Madaras Rotor with Predicted Power Required Based on Equations (2) and (3).

Figure 3.25. Power/Unit Length versus Diameter of Rotor Having Two End Caps, $e/d = 1.35$, and Rotational Speed of 120 rpm.

The Madaras data were measured after the end cap was blown off by high winds, so the computation in Figure 3.24 was made for the condition of $AR = 4.05$ and $e/d = 1.0$.

One can see in Figure 3.24 that the computed results predict lower power requirements than those measured. We believe this difference is primarily due to the bearing friction and drive gear losses of the large Madaras cylinder. These losses were negligible for our 152 mm diameter test models which used ABEC-7 miniature precision ball bearings and no drive gearing. Since bearing manufacturers have assured us that friction in large diameter bearings will be much greater (relatively) than our small bearings, we assumed that the bearings we would use on our present study would have friction of the magnitude required to shift our computed curve upward until it coincides with the Madaras curve. Thus, our computed power required curve would coincide with the original Madaras results.

We believe this adjustment will yield conservative performance predictions in the light of the advancements in bearing manufacturing that have been made since 1933. This increment of bearing power required versus rpm obtained from Figure 3.24 was added to each cylinder we analyzed during our performance computations.

The comparison of powers required to rotate the various Flettner and Madaras rotors, shown in Figure 3.25 fits very well to an orderly pattern that approximates a function which varies as the fourth power of the diameter, as theory predicts. Therefore, we believe that the use of the previous power-required equations with appropriate additive factors for mechanical and electrical losses is a reliable method for predicting rotational power requirements for a large Madaras-type rotor of any geometry.

3.4 BOUNDARY LAYER TEST RESULTS

The boundary layer tests conducted were those having 500 series numbers in the test matrix in Table 3.2 which

follows Paragraph 3.2.1. The primary purpose of this test series was to obtain aerodynamic data for checking our empirical model which describes the combined air flow that strikes the cylinder (see Paragraph 3.2.6). The tests also enabled us to determine whether or not one of the Madaras' basic assumptions were correct: i.e., Madaras assumed that his wind tunnel aerodynamic coefficients could be used without modifications to predict the performance of rotor plant, even though the conditions were not the same in both cases. A comparison of comparable conditions is shown below.

<u>Item</u>	<u>Wind Tunnel</u>	<u>Rotor in Power Plant</u>
Air Flow	Constant V_w with rotor height	Variable (boundary layer) V_w with rotor height
Geometric AR	8	4
e/d Ratio	2	1.3
No. Rotating End Plates	2	1 on Top
Gap Between Rotor and Ground Plane	N/A Free stream	Assumed none Actually 10 ft

This set of tests also provided insight into the effect of the trade-off between aerodynamic coefficient magnitude and power required to spin the cylinder in a boundary layer flow as a function of number of rotating end plates.

These boundary layer tests were run at a free stream Reynolds number of 1.45×10^5 , the same as that used for the free stream tests.

Typical boundary layer data are presented in Figures 3.26 through 3.28, inclusive. Also, a typical data tabulation showing forces, coefficients, and points of application of each force as a function of U/V is shown in Table 3.6.

As with the free stream data, the data trends and accuracy are satisfactory. Also, the data verified our prediction of the lift and drag force and their centroids obtained from our model which combines the boundary layer wind distribution with the uniform velocity caused by rotor car speed down the track.

Figure 3.27. C_L , C_D , Power versus U/V for Boundary Layer Tests for e/d Ratios of 1.25 and 2; one and two end Caps; and Aspect Ratios of 3 and 6, $d = 152$ mm.

- AR= 6, e/d = 2, 2 Moving end plates, free stream
 --- AR= 3, e/d = 2, 1 Moving end plate at top, plus car, boundary layer

Figure 3.28. Comparison of Aerodynamic Coefficients for Configurations Madaras Considered to be Equivalent.

Figure 3.28 as well as Figures 3.26 and 3.27 indicate that he could use directly his Madaras' assumption wind tunnel data for his plant performance computation was in error, and hence his performance computations based on these assumptions were overstated. The end plates function as air pumps which not only reduce induced drag in the classical end plate sense; but also they aid in developing additional circulation. Thus, at a given value of U/V , generally in the range of $U/V > 2$) a rotor with a larger end plate will have a higher life coefficient. This effect is more pronounced at the smaller aspect ratios than at larger ones. See Figures 3.17 and 3.26 which demonstrate this observation.

Most of the conclusions drawn from the free stream data apply to the boundary layer data; however, as expected, the maximum lift coefficient in the boundary layer flow is lower than that in free stream flow for similar geometrical configurations. Some of the more important observations are:

- The use of two end plates instead of one provides significantly higher lift coefficients at the expense of higher drag coefficients and higher motor power to spin the cylinder.
- The small increase in C_L resulting from the use of end plates having an e/d ratio greater than 2 is not justified in view of the power consumed.
- Aspect ratio has very little effect on power consumption but increasing aspect ratio markedly improves lift/drag ratio and C_L max at e/d ratios of about 2 (see Figure 3.27). It appears that the optimum geometry in the range tested will be at an aspect ratio of 6 and at an e/d ratio of 2 or less.

SECTION IV

STRUCTURAL AND MECHANICAL DESIGN

The objectives of this structural-mechanical design study were to:

- Define loads the structure must withstand.
- Develop a realistic design based on the original Madaras configuration which will fulfill all design criteria specified in Paragraph 2.3, and withstand all loads defined herein.
- Specify structural configuration, fabrication techniques, and purchased components.

The design resulting from this study is being developed as a means for defining system feasibility, and as such is not the level of design which one would develop for plant fabrication. Hence, the development of an optimum design configuration and detailed specifications was beyond the scope of this study.

4.1 GROUND RULES

The objective of this portion of the study was to define the structure and the methods of fabrication. The study was governed by the ground rules listed below. The dimensions of the rotor were similar to the rotor tested in 1934; however, these dimensions do not necessarily represent the optimum rotor aspect ratio.

1. Rotor

- Rotor Height = 27.4 m (90 ft)
- Rotor Diameter = 6.8 m (22.2 ft)
- Cap Height = 0.91 m (3 ft)
- Cap Diameter = 10.2 m (33.3 ft)

2. Design Wind and Operating Conditions

- Wind duration curve having a mean wind of 8.1 m/s at an altitude of 9 meters above the terrain. This curve will be up-rated by the 0.167 power law to

provide a design duration curve having a mean wind speed of 8.8 m/s at rotor mid-height (16.8 m above terrain), as shown in Figure 4.1.

- Structure must withstand steady winds of 53.6 m/s with rotor in the nonoperational state.
- Structure must withstand wind gusts of 26.8 m/s while operating at rated wind speed.
- The assumption is made that the rotational velocity of the rotor will be decreased for winds exceeding the mean rated wind velocity such that the power output for the mean rated conditions is maintained (i.e., the aerodynamic loading at any steady wind operating condition will not exceed the aerodynamic loading for the mean design operation condition).
- System land speed will be nominally constant at 8.9 m/s (20 mph).
- The rotational velocity of the rotors will be varied as a function of track position and wind velocity to optimize power output.
- For the rated wind speed and nominal car speed defined above, the maximum rotational velocity of the rotor is approximately 185 rpm.

3. Life and Environmental Criteria

- Design life of the car excluding the rotating cylinder and bearings is 50 years.
- Design life of the rotating cylinder is 30 years.
- Design life of the bearings is 30 years.
- Design life of the track, road bed, and restraining system is 50 years.
- The service life loading spectrum shall be defined on a basis of design wind conditions and wind duration curves.
- Ultimate and yield factors of safety will be applied to all design loads. These factors will be consistent with applicable design code requirements.
- The system must withstand hail up to 1-inch in diameter, operate in a temperature range of -51°C (-60°F) to 49°C (120°F), and operate in snow, rain, lightning, icing, salt vapor, windblown sand, and dust.

Figure 4.1. Wind Duration Curve Updated from 8 m/s Speed at 9 m Above Ground to 8.8 m/s at 16.8 m Above Ground.

4. Sytem Criteria

- Track configuration - circular or racetrack.
- Power output compatible with requirements of existing public utility networks.
- Failsafe automatic plant operation.
- No power storage capability.
- One complete spare rotor car per plant.
- Method for removing a car from the track for maintenance and repairs.

4.2 DESIGN LOADS

The imposed forces acting on the system have been categorized as follows:

- Aerodynamic forces resulting from the Magnus effect.
- Body forces resulting from the rotation of the cylinder and transverse motion of the car.
- Structure dead weight.
- Loads due to snow and ice accumulation.

In a design effort directed toward actual plant construction, considerable attention will have to be given to the dynamic response of the system. However, such an analysis is considered beyond the scope of this feasibility study.

4.2.1 Aerodynamic Loading on the Cylinder

The variation in wind velocity with altitude is illustrated in Figure 4.2 for the rated wind speed at the rotor midheight of 8.8 m/s (19.7 mph).

The structure also must withstand gusts to 26.8 m/s (60 mph) while operating at rated conditions. For the worst-case conditions, the direction of the gust vector was assumed to differ by 180° from the direction of the rotor car vector as the car traveled along the track. Thus, the resulting maximum relative wind speed for an operating rotor car would be 35.6 m/s (8.8 m/s

Figure 4.2. Wind Velocity Profile.

wind + 26.8 m/s gust) the normal and tangential forces are listed below. These loads could occur at any track location, depending upon wind direction variation, as shown in Figure 4.3.

$$F_{\text{norm}} = 580,900 \text{ N (130,500 lb)}$$

$$F_{\text{tan}} = 59,250 \text{ N (13,320 lb)}$$

$$F_{\text{resultant}} = 583,930 \text{ N (131,280 lb)}$$

These forces are also dependent on the linear velocity of the cylinder surface (U) as shown in Figure 4.4, which is a simplification of the spin schedule shown in Figure 5.1. The nonoperational wind condition (53.6 m/s) results in a drag load of 114,180 N. This load is significantly less than the operational gust condition. Figure 4.5 shows the effect of cylinder rotation on the cylinder pressure coefficients. This data illustrates the relative insignificance of the nonoperational condition for the static analysis of the system. Figure 4.5 is based on data taken from Reference 13. These comments on the insignificance of the nonoperational condition apply only to the static analysis of the system. Any analysis relating to the dynamic response of the system should reconsider the nonoperational condition.

4.2.2 Aerodynamic Loading on the Cap

The peak positive and negative pressures on the underside of the cap have been defined as a function of radial location for both the mean design wind condition and the operational gust condition. The nonoperational gust condition is judged to be less severe than the operational gust condition and will not be considered further as a static load condition.

These pressure distributions are shown in Figure 4.6. The peak positive and peak negative pressures occur at circumferential locations which are diametrically opposed. The peak pressures have been assumed to act on all or a portion of the cap in such a manner as to create a most critical load condition with regard to the structural members in the cap and the attachment of the cap to the cylinder.

Figure 4.3. Operational Design Loads for 8.0 m/s (18 mph) Mean Design Wind Speed. Derived from Preliminary Spin Schedule of Figure 5.1, in which U/V was Varied at Each 5° Incremental Position Around Orbit, Such that F_{tan} is Maximized at Each Increment.

Figure 4.4. Ratio U/V versus Track Locations for 8.8 m/s Rated Wind Speed.

U = Resultant Wind Vector
 v = Linear Velocity of the Cylinder Surface
 ρ = Mass Density of Air
 P = Pressure

Figure 4.5. Pressure Distribution Around the Center of a Rotating Cylinder at Various Values of U/V (Taken from Reference 13).

Figure 4.6. Madaras Rotor Cap Peak Pressure versus Radius.

4.2.3 System Acceleration Loads

The system acceleration loads are those loads associated with the rotational motion of the cylinder and the transverse motion of the car along the track.

The normal acceleration associated with the rotation of the cylinder is equal to the product of the radius and the square of the rotational velocity. At a rotational velocity of 186 rpm, the normal acceleration at the surface of the cylinder is 130.8 g, and at the outer radius of the rotor cap, 196.2 g. The maximum rate of rotor spin-up, for design purposes, is defined by a constant acceleration from 0 rpm to 186 rpm over a 15-degree arc of the car track assuming the car is traveling at a rate of 8.9 m/s. This spin-up results in a tangential acceleration of 0.97 g. The normal or radial acceleration associated with the motion of the car around a 457.3 m (1500 ft) diameter circular track is 0.036 g. The tangential accelerations associated with changes in car velocity are negligible and will not be considered as a design load.

4.2.4 Snow and Ice Accumulation

The likelihood of locating the Madaras Power Plant in a temperate zone is rather certain; thus, loading due to snow and ice accumulation is a concern. Snow accumulation during operation will probably not be significant for the following reasons.

- The cap is located in a relatively windswept environment while in operation.
- The radial acceleration due to rotor rotation is quite high. At 186 rpm, the radial acceleration of the outer extremity of the rotor cap ($R = 5.1$ m), the normal acceleration is 196.2 g. Thus, the radial acceleration should prevent snow accumulation except in the extreme center of the rotor cap.
- The outer portion of the rotor cap will be sloped like a roof at an angle of 20 to 30 degrees, which will aid in the prevention of snow accumulation.

In the case of severe storms during which the power plant might be shut down, a significant snow accumulation could occur. As the system is restarted, the point at which the motion of the rotor would remove most of the snow is somewhat hard to define because of the tendency of snow to adhere tenaciously to a structure. At the same time, operation for moderate lengths of time with significantly unbalanced loads on the thrust bearings could have a very detrimental effect on bearing life. Snow loads could reach 2.39 kN/m^2 (50 psf) to 3.83 kN/m^2 (80 psf) in some regions of the United States (as defined in Reference 42). In light of the detrimental effect high snow loads can have on bearing life, the following positions have been taken regarding snow accumulation.

- The system will be designed to accommodate a moderate snow load of 0.48 kN/m^2 . This amount of snow accumulation would permit restarts after severe snow storms without provisions for snow removal in most areas of the United States where this type of power plant is likely to be situated.
- In those areas where snow accumulation could exceed 0.48 kN/m^2 during a severe storm, provisions for snow removal prior to restart must be provided.

A comment will be made here regarding snow accumulation in the nonoperational state. The snow load capability of the rotor cap while stationary will be significantly greater than 0.48 kN/m^2 since the structural members in the rotor cap will be sized primarily by the body forces associated with the normal acceleration resulting from rotor rotation. Limitation of snow loads in the operational state is required to provide a more desirable bearing life; however, significantly greater snow accumulations will be permitted in the nonoperational state.

Ice accumulation on the cap and cylinder must be considered. The likelihood of cap ice accumulation exceeding 0.48 kN/m^2 is rather remote; however, a rather small depth of ice accumulation can induce significant loads into the structural elements of the cylinder and cap when considering the radial acceleration associated with rotor rotation. Also, some concern exists relative to the dynamic unbalance which would result if

the cap or cylinder would shed a portion of the accumulated ice. The possibility of rotor unbalance would have to be considered in a dynamic analysis.

4.2.5 Car Weight, Wheel Loads, and Restraining Loads

The aerodynamic loading on the cylinder results in a significant overturning moment about the base of the car. When the overturning moment exceeds the restoring moment of the car, stability will be maintained. This overturning stability can be provided either by vertical restraint and/or by the weight of the rotor car.

The design restraining loads, wheel loads, and car weight are governed by one or more of four design requirements.

- a. Operation at the design wind condition (8.3 m/s with the car traveling at a track speed of 8.9 m/s).
- b. Operation at the gust condition which in this worst condition analysis was assumed to be equivalent to a resultant constant wind speed of 35.8 m/s along the track with the rotor turning at 186 rpm. (Lift load normal to wind causes the greatest overturning load.)
- c. Nonoperation of the plant while satisfactorily withstanding winds having speeds of 53.6 m/s.
- d. Provision of wheel traction under the most severe wind loads which will provide sufficient torque to drive a 1 MW generator geared to the wheels.

Our studies indicated that condition d (the traction required to turn the generator) governs the design car weight and wheel loads; that this design weight will be sufficient to overcome all overturning conditions; and that no vertical restraint will be required. Since the car wheels will have insufficient capability to react, the radial aerodynamic force (wheels without flanges will be used) front and rear lateral restraints in the form of idler wheels acting on the sides of the rails will be required.

Assuming an eight wheel configuration (1 pair of wheels at each corner of the car), and assuming that each pair of wheels will drive a 250 kW generator, a gross car mass of 327,950 kg

(723,000 lb) is required to develop the necessary tractive force. (This is based on a coefficient of friction of 0.07 attributed to a steel wheel rolling on a steel track.) For this car, maximum wheel loads will be 556,000 N (125,000 lb), and maximum horizontal restraining forces will be 580,900 N (130,600 lb). This horizontal restraint will be divided between four reaction points, two located at the front and two at the rear of the car.

4.2.6 Fatigue

The Madaras Power Plant must operate with a high degree of reliability. On the other hand, various structural members will undergo an enormous number of stress cycles in the life of the system. Consider the rotor. Assuming the average rotational velocity of the rotor is 90 rpm, the load cycle associated with one rotor revolution will be applied 5.6×10^8 times in a 30 year life, based on a 40 percent plant factor. Of course, these load cycles will be applied at various load levels. The loads associated with rotor spin-up will be applied twice per track revolution. Assuming constant operation at 8.9 m/s, the number of cycles for the 30 year life of the rotor is 2.3×10^6 . The corresponding number of cycles for the 50 year life of the car is 3.9×10^6 .

Existing design codes such as the "Steel Construction Manual" published by the American Institute of Steel Construction are not based on structures which experience such a high number of load applications. These existing codes can be used in a feasibility study of this type; however, to insure a high degree of reliability in the case of a final design, a more sophisticated approach is recommended. A cumulative fatigue damage (durability) or a crack propagation study should be conducted based on the service life loading spectrum. Statistical methods should be applied to determine a required mean life to insure adequate reliability within the service life of the structural components.

4.3 STRUCTURE

Initial costs for construction and assembly are expected to represent a significant portion of the costs for the Madaras Power Plant. These costs might very well have the strongest influence on the feasibility of such a system. The rotor cars are large enough that complete assembly in a factory is not realistic. On the other hand, field fabrication and assembly of the entire car would result in a much more costly structure. This cost picture suggests the use of a modular construction approach whereby segments of the structure would be fabricated in the factory and only final assembly would be required at the plant site. This concept does not eliminate the necessity for a field shop; however, it does minimize the assembly effort required at the construction site. The modules should be made as large as possible while keeping shipping costs at a reasonable level.

4.3.1 Rotor Cap

General details relating to the rotor cap are illustrated in Figure 4.7. The primary load carrying members are 8 truss structures which are positioned radially on 45 degree centers. The trusses are attached to a stiff cylindrical shell at the center of the rotor cap. The outside diameter of the cylindrical shell is 0.9 m. Both the upper and lower chords of the truss are attached to a vertical rotor truss at the rotor-cap interface. This is the only connection which attaches the rotor cap to the rotor structure.

The upper and lower chords of the cap truss will consist of a "T" section. The diagonal and vertical members will be attached to the stem of the "T" sections using welded connections. A typical connection is illustrated in Figure 4.8. The vertical and diagonal members consist of structural tubing. The joints will be detailed such that the centerlines of the vertical and diagonal members intersect at the centroid of the truss chord which will minimize bending moments in the truss members.

Figure 4.7. Rotor and Cap Construction.

Figure 4.8. Rotor Cap Truss Joint.

As shown in Figure 4.7, the modular construction of the rotor cap will consist of 8 sections, each section consisting of a 45-degree segment. A structural angle will be located at the boundary of each module on both the upper and lower surfaces. The flange of the angle will be turned outward so that it mates with the corresponding angle of the adjacent module to form a "T" section. Adjacent modules will be connected with bolts through the "T" section stem and/or by a welded connection. Additional members which have structural significance are shown in Figures 4.9 and 4.10. Figure 4.9 illustrates the location of circumferential members. A ring will be located at each joint on the upper and lower chord of the truss. Members $\overline{AB}$, $\overline{CD}$, $\overline{EF}$, and $\overline{GH}$ (Figure 4.10) will form a truss network in the circumferential direction which will provide a load path to the radial truss. Members $\overline{AB}$ and $\overline{CD}$ will be structural tubing. Members $\overline{EF}$ and $\overline{GH}$ will be structural angles. A segment of the heavy cylindrical shell will be attached to each module at the inner radius. This concept is illustrated in View B on Figure 4.7.

The material alternatives considered in this feasibility study are steel and aluminum. Some advantages and disadvantages of each are listed below.

1. Steel is more easily welded.
2. Aluminum weighs less which is advantageous from the viewpoint of bearing loads and rotor spin-up efficiency.
3. Radial body forces associated with rotor rotation are significantly less for aluminum (approximately 1/3 of the value for steel).
4. Some grades of steel would require periodic maintenance to provide corrosion protection.
5. Steel costs less than aluminum.

Aluminum material costs are approximately five times the cost of ASTM A-36 steel and about four times the cost of a corrosion resistant steel such as ASTM A-242. Thus, from a material cost standpoint, steel is much more desirable. However, the candidate

Figure 4.9. Location of Circumferential Rings.

Figure 4.10. Cap Module Radial View.

structural grade steels and aluminums have relatively comparable strengths yet the density of steel is approximately 2.85 times the density of aluminum.

Body forces resulting from cylinder rotation represent by far the most significant source of structural loading. Since one prime design goal is to provide a highly reliable structure, an aluminum structure can be designed with lower nominal stresses as compared to a steel structure and, as a result, will be less susceptible to fatigue problems. In addition, the lower weight aluminum structure will reduce the axial load on the upper bearing and the power requirements for rotor spin-up. Initial cost of aluminum should be offset somewhat by reduced maintenance and repair costs. In light of these design considerations and the lower maintenance cost of aluminum, the material selected for the cap was 2024-T4 Alclad aluminum alloy.

4.3.2 Rotor Cylinder

Details relating to the rotor construction also are shown on Figure 4.7. The rotor will have 8 longitudinal truss stiffeners which extend the entire length of the cylinder. These longitudinal members will be located on 45-degree centers. The cylinder will also have 9 circumferential stiffeners. The lower 25.6 meter portion of the cylinder will contain 8 stiffeners located on 3.66 meter centers. The remaining stiffener will be located at the top of the cylinder with a 1.83 meter spacing between the stiffener and the cylinder's top end.

Both the longitudinal and circumferential stiffeners will be truss structures. The chords of the trusses will be either tubing or angles. Utilizing a "T" section for the truss chord has three distinct advantages: the outer flange provides a means for attaching the skin; the inner flange provides a means for attaching internal structural members which transmit loads to the bearings; and the stem of each "T" section provides a means for attaching the radial and diagonal members. The radial and diagonal members will consist of structural tubing when possible; however, the modular fabrication will prohibit the use of structural tubing in some instances.

The modular fabrication consists of fabricating semi-cylindrical segments 3.66 meters in height with the exception of the top cylindrical section which will be 1.83 meters (6-ft) in height. Field assembly consists of mating two semi-cylindrical sections to form a cylindrical section. Each 180 degree segment will have vertical truss structures located at the 45, 90, and 135 degree locations. These truss locations are illustrated schematically in Figure 4.11. The trusses are enlarged in the figure relative to the cylinder diameter to show greater detail. The angles at 0 and 180 degrees will mate with the angles of the adjacent semi-cylindrical module to form a truss chord at the juncture. The radial and diagonal members at the 45, 90, and 135 degree locations will be tubing; however, angles are required at the 0 and 180 degree locations. The "T" section of the circumferential stiffener will be formed by mating the angles of adjacent cylindrical sections. The radial and diagonal members of the cylindrical stiffeners will be angles throughout. Some details of the circumferential stiffeners are illustrated in Figure 4.7. An additional flange could be sandwiched between angles of adjacent modules where additional structural capability is required. The semi-circular portion of a circumferential stiffener could be assembled from subassemblies of 45 or 90 degree segments. A typical 45 degree segment is illustrated in Figure 4.12.

The skin of the cylinder will be circumferentially corrugated because this configuration provides efficient circumferential stiffening and because it is a cost/effective production technique. Where longitudinal seams are required, the overlap will be sufficient so that the corrugation continues to act as a hoop. In addition, corrugation overlap in the longitudinal direction may be required at the juncture of adjacent cylindrical sections. The corrugation could be either welded or riveted to the stiffeners.

Figure 4.11. Rotor Cylinder Internal Structure.

Figure 4.12. Typical 45° Segment of Circumferential Stiffener.

Practical fabrication tolerances could result in some rotor ovalization, and the aerodynamic loading also could cause rotor ovalization. If ovalization becomes a problem, a spoke assembly could be incorporated inside the rotor to maintain the required circular cross-section. However, it was decided to control ovalization by stiffening the circumferential truss members, since this approach seems less labor intensive and therefore should reduce cost.

As for the rotor cap, 2024-T4 Alclad aluminum alloy was selected for the rotor because the source of structural loading to the rotor cylinder is the body force resulting from cylinder rotation.

4.3.3 Rotor Bearings

Rotor bearings need to be very large, and hence represent a significant cost to the Madaras Power Plant. Thus, it was necessary to select this component carefully. The Madaras test and both Flettner ship rotors used two ball bearings.

The bearing locations and internal cylinder structure are illustrated in Figure 4.13. The upper and the lower bearings are located at elevations of 21.9 m and 7.3 m above the base of the cylinder, respectively. An early decision was made to transmit all of the rotor weight to the tower through the upper bearing. If the weight were transmitted through the lower bearing, the structural weight and the aerodynamic suction loads on the under side of the cap would combine and subject the longitudinal stiffeners to combined compressive and bending loads. However, if the upper bearing supports the rotor weight, a significant portion of the longitudinal stiffener will be subjected to combined tensile and bending loads, and improved elastic stability and reduced weight is the result. The axial location of the bearings was governed primarily by the need to minimize the radial loading, but also to provide a suitable load path for transmitting loads from the cylinder to the bearing.

Figure 4.13. Preliminary Assembly Sketch of Rotor and Support Tower.

Two factors limited the size range of the lower bearing. The lower limit on the bearing diameter is governed by the bending stresses in the tower due to the aerodynamic loading on the cylinder. The upper limit on the lower bearing diameter is dependent on the diameter of the cylinder. Thus, in order to maintain a reasonable thickness of the cylindrical tower shell, the minimum inside diameter should be about 3 m. The maximum practical outside diameter should be about 4.6 m, a diameter which is dependent upon the cylinder diameter and the size of the circumferential truss structure.

The upper bearing shares the same upper limit on the outside bearing diameter; however, the tower stresses due to aerodynamic loading are significantly less at the upper location. Thus, the inside diameter of the upper bearing could be less than 3 m.

The specified bearing life at both locations is 30 years. However, with regard to fatigue damage, this life can not be met with a spherical roller bearing. Only two bearing types are acceptable for this use: a cylindrical roller bearing or a double row tapered roller bearing. Based on cost estimates, the cylindrical roller bearing is found to be the most economical choice for the lower bearing location. At the upper location, the more flexible inner diameter size permits the use of a double row tapered roller bearing which can be selected from stock sizes currently in production. This selection results in a significant cost saving. A B-10 life of at least 20 years is believed to be easily attainable. Whether or not a B-10 life of 30 years is attainable is somewhat less certain. Manufacturers indicated bearing life may be governed more by environmental factors than by fatigue.

Very little information is available on 3 m diameter bearings rotating at an angular speed of 186 rpm. Most bearings of this size are used in heavy construction equipment (cranes, etc.) which rotate at a relatively low speed. Vibration

and other dynamic problems could exist at the high rotational speeds. However, data on successful use of smaller diameter bearings at higher speeds, and hence higher linear speeds than those of the Madaras application are encouraging. Nevertheless, bearing selection has been designated as an area needing further study prior to constructing a plant. Full scale bearing tests may be necessary.

As an indication of the state of the art of large bearing manufacturing and design, the design torque equations of two major manufacturers did not contain the same variables (in fact, one equation did not contain a speed term). Also, one manufacturer said actual bearing torque could vary by a factor of five from his equations, and the variability of torque between bearings of a given type was large.

4.3.4 Support Tower

The rotor is supported on a nonrotating tower which is attached to the deck of the car. The tower must support axial loading, transverse loading, and a torsional moment. The axial loading is a result of rotor structure dead weight, snow and ice accumulation on the cap, and the summation of the aerodynamic forces on the upper and lower surfaces of the rotor cap. Rotor spin-up will induce a torsional moment in the tower, and aerodynamic loading on the cylinder surface is the source of transverse loading. The transverse loading was found to be the most significant in sizing the tower cross-section.

4.3.5 Rotor Tower

The rotor and bearings are supported by a nonrotating tower which is attached to the deck of the car. The tower must support axial loading, transverse loading, and a torsional moment. The axial loading is a result of rotor structure dead weight, snow and ice accumulation on the cap, and the net aerodynamic forces on both sides of the rotor cap. Rotor spin-up will induce a torsional moment in the tower, and aerodynamic loading on the cylinder surface is the source of

transverse loading. The transverse loading is most significant in sizing the tower cross-section.

A two-piece tower is envisioned. The tower would consist of a welded cylindrical monocoque structure supported at the base by a truncated conical section. Since weight was of less concern for this nonrotating structure, it was decided to construct the tower from ASTM A-242 steel. Although ASTM A-242 steel costs about 25 percent more than ASTM A-36 steel, the A-242 alloy was considered the better choice since its corrosion resistance would reduce life cycle cost. The total tower height will be about 23 m, with the cylindrical section being 3m in diameter by 17 m high. The truncated conical section will be about 6 m in height and 4.6 m in diameter at the base. The thickness of the cylinder and cone will be in the range of 9.5mm to 12.7 mm. Additional stiffness may be required in the region of the bearings to prevent ovalization and undue stresses on the inner bearing races.

The cylindrical section of the tower can be completely fabricated in a factory and shipped to the site in one piece. However, the truncated conical section would be shipped to the construction site in three pieces. An illustration of the fabrication breakdown of the tower base is shown in Figure 4.14.

Field assembly of the tower would consist of welding the conical segments to form the truncated cone followed by welding the conical portion to the cylindrical portion. The tower could be either welded or bolted to the car deck. A sketch of the tower is shown in Figure 4.13.

4.3.6 Car Layout

External views of the car are shown in Figure 4.15. With the streamlined contour of each edge overhanging the wheels, the overall dimensions of the car are:

Figure 4.14. Tower Base Segmentation Plan.

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Figure 4.15. Three-View Drawing of Rotor Car.

Length: 19.2 m (63 ft)

Width: 17.4 m (57 ft)

Height: 2.3 m (9.5 ft)

Deck Height Above Track: 3.8 m (12.5 ft)

The suspension system consists of four, 2-wheel trucks located at each corner of the car. The longitudinal spacing of the truck centerline is 11m and the two 1.2 m diameter wheels in each truck are spaced 1.8 m apart. The wheels, which have a 279 mm wide tread, are forged from AISC 1045 steel hardened to a depth of 25 mm to Rockwell C-46. The lateral spacing of the trucks is 11 meters, which will permit the use of a 11-meter track gage (36 ft) as was used by Madaras. Each truck is free to pivot about its vertical centroidal axis to facilitate negotiating curved parts of the track.

The wheels in each truck are coupled together by a chain drive system, and from there through a flexible coupling to a 9.3:1 speed increaser, through a second flexible coupling, and finally to a 250 kW, 4160 V, induction generator having a rated speed of 1150 rpm at an 8.9 m/s track speed. Some of the other details of the end truck assembly are shown in Figure 4.16.

The inclined wheel housing on the forward and rear sides of the front and rear trucks serves two functions: it provides a means for scraping foreign objects off the track (like the cow catcher on a steam locomotive) and it serves as a mount for four idler wheels, two on each side of the rail for each truck. There are sixteen of these 127 mm-diameter, idler wheels (four idlers on each of four wheel housings), hence eight of these idlers are always available to react the radial aerodynamic load for all wind directions relative to the power plant track. More analysis on the sizing and support of these wheels is needed.

Each wheel is supported by its own 203 mm-diameter axle and two pillow blocks, so that wheels on the inside and outside rail can turn independently.

TOP VIEW (LESS DRIVE SUPPORT BRACKET)

Figure 4.16. Rotor Car End Truck Assembly.

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4.3.7 Car Structure

The structural plan of the car is illustrated in Figure 4.17. This layout is based upon the use of a series of deep, built-up, longitudinal and lateral box beams positioned in such a way to provide the required bending stiffness. Closed section box beams made from welded steel plates were selected because the closed section yields considerable torsional stiffness compared to an open section and because use of a closed section eliminates the need for inter-beam bracing. Two beam sizes were selected. The side beams, those to which the tower is fastened, and other longitudinal interior members are made from 0.9 m deep by 0.4 m wide steel plate (Section C-C). The forward and rear members, which transmit the radial bending moments to the wheels, will be even larger: 1.22 m deep by 0.41 m wide (Section D-D). All of these beams will be fabricated from 19 mm thick steel plate welded along all edges. Sectional elevations of this structural member layout are presented in Figure 4.18.

The streamlined fairing of the car extends at a 45° angle downward from the main frame on all four sides. This fairing is stiffened by eight closed tube sections on each side that are 305 mm deep, 76 mm wide, and 6 mm thick. The fairing is reinforced at each corner by 12.7 mm x 12.7 mm by 6.3 mm thick diagonal tubes. The bottom perimeter of the fairing is stiffened by a 152 mm-diameter, Schedule 46 steel pipe. The top deck and fairing framework is completed with a network of longitudinal and lateral stringers made of rectangular tubes varying in depth from 152 mm to 25 mm. The frame is covered by a stressed skin covering of the tape deck and fairing by panels fabricated from 3 mm-thick Corten steel, which will provide a corrosion-resistant weather surface. The use of Corten steel for the tower and car was recommended by U.S. Steel for this project. Figure 4.19 contains some of these miscellaneous skin and stiffener sections.

The bottom floor of the car provides the final structural detail. The floor, not shown on the drawings, will be flush with the bottom of the streamlined housing, and will provide

Figure 4.17. Car Structural Plan.

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Figure 4.13. Sectional Elevations of Car Structural Layout.

Figure 4.19. Miscellaneous Skin and Stiffener Section.

an enclosed compartment about 2.9 m high in areas that are not obstructed by main beams, etc. The floor will be stiffened by tubes, as was the top deck, and appropriate cut-outs for trucks, etc. will be provided.

The compartment thus formed will have three major functions. First, it will provide environmental protection for the expensive motors, generator, gearing, and other electrical and electronic control and instrumentation equipment. It also will provide safe working space which can be occupied by maintenance personnel even while the car is in motion. Russell F. Hardy, Madaras' Chief Engineer, indicated noise inside the rotor was not objectionable. Finally, the floor will provide additional structural rigidity as well as a means for supporting a pre-cast concrete slab if ballast is required.

The mechanical drive system for the rotor (partly shown in Figure 4.15) will be described in Paragraph 4.4.2. The electrical and electronic components mounted in the car will be described in Section 5.

4.3.8 Power Transfer Trolley

The electrical power output from each car will be transferred to an overhead trolley rail by one, spring-loaded, trolley arm attached to each car as shown in Figure 4.20. The trolley arm, located on the side of the car facing the inside of the track, provides support for three collector brackets and shoes, one for each electrical phase. Each of these shoes, which have two contactors, ride on collector rails and the contactor rails are fastened to expanded steel joists which are 0.4 m deep. The joists are supported by municipal-type, metal lighting poles spacing 12.2 m apart (see Figure 4.20). An electrical description of the trolley rails is presented in Paragraph 5.6.2.

4.3.9 Inter-Car Coupling

The cars will be coupled together into an endless train by steel cables attached by a pin-and-clevis arrangement at the mid-point of the front and rear of each car.

Figure 4.20. Power Transfer Arm.

The cable is attached, as shown in Figure 4.18, to the main, 1.22 m deep lateral beam with cable loads fed into the 0.91 m deep longitudinal beams by a closed tube having a rectangular cross section of 203 mm by 305 mm and a wall thickness of 10 mm. The point of cable attachment is 3 m above the top of the track. This is ample height to allow for the 1.22 m catenary deflection expected in the cable. Each inter-car connection cable will consist of two wire ropes, each having a diameter of 70 mm. The length of the cable will be determined by the minimum spacing between cars, which is discussed in Section 6, Performance Analysis. Cables will be tensioned after train assembly to improve rigidity of the train system.

4.3.10 Dynamic Balancing of the Rotor

The rotor will be balanced dynamically after it is assembled in the on-site assembly building, which is described later in Paragraph 4.5.4. The procedure used will be essentially the same as that used by Madaras.⁵

Assembly of the structure will be checked at all stages to insure that required circular concentricity of the tower and rotor is maintained. This maintenance of concentricity will be particularly important at the bearing locations to improve bearing life.

Dynamic balancing then will be accomplished by slowly rotating the rotor by its motors measuring the degree of imbalance, and then distributing lead weights along the length of the rotor in the longerons and frames as required. This process will be repeated until the rotor is operating smoothly above the rated speed. The Madaras rotor was so well balanced that, at a speed of 160 rpm, oscillations on a machinist's dial indicator applied to the measuring apparatus were barely noticeable.⁵

4.4 DESIGN MODIFICATIONS

During the course of the program, results from our various studies and information received at the ERDA Third Wind Energy Workshop indicated that it would be advisable to change some of the original design criteria presented in Paragraph 4.1. These

changes and the basis for these changes are as follows:

1. Rotor Configuration: changed from Madaras configuration to Aspect Ratio 6, e/d Ratio = 2, two end caps, area unchanged. A rotor having the following size:

- Height: 33.4 m (109.5 ft)
- Cylinder Diameter: 5.6 m (18.25 ft)
- End Cap Diameter: 11.1 m (36.5 ft)

Basis: Results of wind tunnel tests and performance analyses indicated gross output would improve significantly.

2. Rated Wind Speed: Change from 8.18 m/s to 13.4 m/sec

Basis: Madaras selected this rated speed figure and we wanted to correlate with his computations. Also, our performance analyses indicated this level would be attractive.

3. Gust Criteria: changed from 26.8 m/s to 15.6 m/sec. This is the incremental gust over and above rated operating conditions.

Basis: This condition designed the structure and car weight. Inasmuch as GE in their MOD 1 design reduced their gust speed to 15.6 m/s, we also changed to have a more equitable basis for comparing the Madaras and the GE systems.

4. Track Configuration: changed from circular to racetrack.

Basis: Computer studies indicated improved performance for winds that fluctuated no more than $\pm 45^\circ$ from the design direction. Since there is no penalty for reciprocal wind direction changes, we noted that a racetrack configuration could be used in many parts of the country quite effectively. Also, the larger amount of track will permit use of more cars and will enable plant capacity to be increased at an improved economy of scale.

The effects of these criteria changes were evaluated, and the results of these evaluations are presented in the following paragraphs.

4.4.1 Revised Rotor Configuration

The revised rotor mounted on its rotor car is shown in Figure 4.21. Other than the geometric aspect ratio and e/d ratio of the rotor, the most important change is the addition

Figure 4.21. Revised Rotor Car Configuration.

of the lower end cap. Note that the lower end cap is inverted in order to permit a continual, orderly flow of air over the streamlined fairing of the car into the rotor cylinder. This same configuration was used by Flettner on all of his rotor ships. The lower cap was made 37% thinner than the top cap in order to smooth air inflow even more. Notice also that the tower diameter was decreased from 3.04 m to 1.82 m in order to reduce rotor inertia, bearing size, bearing outer race speed, bearing lubrication problems, bearing inertia, and cost.

No changes were made to the design of the rotor car or described in Paragraphs 4.3.5 and 4.3.6.

After reanalyzing the loads for the new conditions, it was concluded that the load changes would have no effect on the governing design structural concept. The required modifications were made to provide realistic data for costing and appropriate inputs to our simulation program. The car weight previously computed was sufficient to prevent overturning and to provide traction for power generation; hence, no change in car weight was made.

4.4.2 Rotor Drive System

After studying the initial rotor-spin-motor, drive system, the decision was made to relocate the motor from the position near the upper bearing, which was about 32 m above ground to the position shown in Figures 4.15 and 4.21 on the floor of the rotor car. This position was selected to facilitate motor and drive unit changes and to simplify routine maintenance and servicing of these components. Although this second approach will require the addition of a long drive shaft, which will increase first costs, it was believed that the life cycle cost will be reduced by incorporating this change. Drive methods which eliminate the shaft should be considered further, however.

The drive system consists of a 450 kW, 1150 rpm, 500 vdc motor coupled to a right-angle speed reducer having a 2.25/1 gear ratio. The speed reducer is then connected to a 29 m long by 203 mm diameter, vertical, drive shaft, which is

composed of 5 sections, each 5.8 m long. The drive shaft is supported along its length by six bearings. Flexible couplings will be used to join all drive shaft and motor-gearbox connections. The top end of the drive shaft is connected to a 254 mm thick, hardened-steel, pinion, spur gear having a 0.6 m pitch diameter. Through a mating internal spur gear, which has a 1.66 m pitch diameter and which is fastened to the rotor structure, torque from the drive shaft at a further speed reduction of 2.77/1 is transferred to drive the rotor. Thus, through the bottom and top speed reducers, an overall motor speed reduction of 6.22/1 is achieved.

4.4.3 Rotor Bearings

In view of the decrease in support tower diameter and the increase in rotor height over the original design, bearing changes were made. The new bearing locations, shown on Figure 4.21 are 13.9 m and 32 m above ground. As before, the upper bearing, a 2.6 m OD roller bearing, reacts all of the rotor weight and part of the radial load; whereas a 3.4 m diameter lower bearing reacts only radial loads. The rated capacities of these bearings is well above the load requirements of the governing design criteria.

4.5 SITE DESIGN

The layout plan for our version of a Madaras Rotor Power Plant utilizing a race track rail configuration is presented in Figure 4.22. For the purpose of this drawing, a track having a 229 m (750 ft) end radius and a 612 m straight-away was selected. The top of each track rail is 1.2 m above mean ground level. The actual size of the track recommended will be described in Section VI, since the track radius affects spin motor performance significantly. An elevation drawing of the plant with one rotor car on the track is shown in Figure 4.23.

The total area of 0.65 km^2 (160 acres) is required for the configuration shown. The plant consists of the 11 m (36-ft) gauge track with a spur track that can be used to transfer a rotor car from the active track area to the maintenance building and vice versa.

Figure 4.22. Plant Layout.

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Figure 4.23. Typical Elevation of Madaras Plant, One Car on Track.

Surrounding the outside of the track is a 7.6 m wide, reinforced concrete road which will enable service vehicles to gain access to any rotor car in the train. A 4.6 m clearance space is provided between the inside edge of the road and the outer rail of the track.

The power collection trolley, presented as Figure 4.20 and also shown in Figure 4.23, is located 5.5 m inboard of the inside track, supported by metal, municipal-type light poles that are 6.5 m high and form a complete loop inside of and parallel to the track loop. This power collection system is described in Paragraphs 5.6.2 and 5.6.3.

The area inside the track is available for any use that does not disturb the airflow. Obviously, since the top of the rotor car deck (the bottom of the rotor) is 5 m above ground, the use of the internal area for any farm crop or grazing would not interfere with plant operation. The formation of a lake or water reservoir in this area also could be attractive. However, most uses of this land would require a tunnel under the track for access. Effective use of this internal area, or the siting of a plant at a location where the land is not useful will improve plant economics.

4.5.1 Track and Roadbed Design

Figure 4.22 illustrates some details of the track and roadbed structure. A well-constructed road bed is an important factor influencing the life of the track. Railroad research personnel whom we contacted recommended strongly that every precaution should be taken to provide a sound track foundation in order to minimize repair and replacement costs.

The car wheels run on flat steel rails made of plates having a width of 51 cm and a thickness of 10 cm. Each steel plate will rest on a separate, continuous, steel-reinforced, concrete pavement that is 2.4 m wide by 35.5 cm thick. In order to assure that level track surface is achieved, the track will be shimmed by a 5 cm-thick layer of grout having strength of

68 MPa (10,000 psi). The steel rail will then be bolted to the concrete road bed by two countersunk bolts, 3.1 cm in diameter by 43 cm long every 1.2 m of track length.

The supporting pavement will be steel reinforced concrete. Since replacement of all or part of the pavement would be a very costly operation in terms of material, labor, and lost revenue, a factor of safety of five is recommended.

The ballast layer which supports the concrete pavement consists of a mixture of coarse aggregate and fines that must be properly balanced. An insufficient proportion of fines could result in excessive abrasion of the aggregate with repeated load applications, whereas the aggregate tends to float in the fines in a case of excessive fines. This latter situation could result in poor compaction of the ballast layer.

In addition to a proper mixture of aggregate and fines, a soil stabilization admixture should be mixed with the fines to insure a maximum degree of compaction, to retard capillary action, and to avoid potential problems with frost. The stabilized ballast should have an ultimate bearing capacity of 958 KPa ($20,000 \text{ lb/ft}^2$). Assuming a point load on the track and a shear lag angle of 45 degrees, the bearing pressure on the ballast would be 197 KPa (4125 lb/ft^2). Thus, for a factor of safety of 5.0, the allowable pressure would be 191 KPa (4000 lb/ft^2). The ballast layer for each rail is about 6.3 m wide by 2.1 m deep with a 1 in 2 slope for drainage. The bottom of the ballast rests on a soil foundation excavated 0.9 m below grade.

The foundation bed should be well-compacted, granular material. As in the case of the ballast layer, a factor of safety of 5.0 is desirable. If the plant site is located in a region of clay and/or silty soils, an adequate portion of this material should be excavated and replaced with a suitable granular material. The susceptibility of clay to capillary action when exposed to moisture can result in a loss of bearing capacity, excessive hydrostatic pressure, and volume changes all of which

could be very detrimental to the life of the track and to the operation of the facility. The compacted granular material should have an ultimate bearing capacity of 479 KPa (10,000 lb/ft²). Two, 30 cm porous drain tiles will be used to drain water from each track rail foundation.

4.5.2 Plant Buildings

Two types of buildings are required: an assembly-maintenance building and a control building.

The assembly-maintenance building will be built during the early stages of plant construction in order to provide the necessary facilities to assemble the rotor and rotor cars from the factory-fabricated segments that are shipped to the site. The plan form of this building will be about 30 m wide by 50 m long.

The assembly building will have two roof heights. The first 25 m of building length will have a clear ceiling height of about 42 m to permit the complete assembly of a rotor on a car inside the building. The second half of the building will have a ceiling height of about 9 m in order to provide room for assembling parts of a car alone. Thus, there is room for a two-car assembly line inside the building. While one car is being assembled, the rotor can be assembled and mounted on the second car. The cars will be assembled on an extension of the spur track inside the building to permit easy transfer of completed rotor cars to the oval track. An overhead, 30-ton bridge crane and two 15-ton jib cranes as well as welding and other fabrication equipment will compliment the building. Office space, tool rooms, and supply rooms also will be provided.

After construction of the plant is completed, this building will then serve as a repair facility and will house one complete spare rotor car.

The control building will be 15 m long by 9 m wide by 4 m high. This building will be built against the edge of the assembly building nearest the track. This building will

house the operator's control station, the central processor, and the telemetry system. An external substation will contain the equipment required to interface the power from the Madaras plant with the utility system power grid. See Section V for a description of these control and interface components.

Provisions have been made in the cost analysis for a prime mover to push the cars around the plant area during car assembly, train assembly, and maintenance.

4.6 MASS SUMMARY OF ROTOR CAR

The following table presents a summary of the mass of the major components of each rotor car.

COMPONENT	Mass	
	kg	lb
Car, Motor, Speed Reducer	232,700	513,000
End Trucks, Speed Reducers, Generators, (4 each)	32,700	72,000
Tower	37,200	82,000
Rotor Cylinder	8,600	18,900
Rotor End Caps (2 each)	4,500	10,000
Miscellaneous Electrical Components	1,400	3,000
Ballast (Concrete on Car Floor)	10,900	24,100
TOTAL MASS	328,000	723,000

SECTION V

ELECTRICAL DESIGN

The objectives of this study were to define electrical requirements, define and specify electrical components, and determine electrical losses. The electrical design includes consideration of the following components:

- a. Rotor spin system
- b. Generator system
- c. Control and instrumentation system
- d. Electric circuitry to interface these three systems with each other and to deliver output power to the outside power grid.

5.1 DESIGN GUIDELINES

Guidelines given were to design a system of maximum efficiency and minimum cost. Considerations under cost would include both initial capital investment and maintenance cost. Whenever possible, standard marketplace components were to be utilized.

5.2 DESIGN CONDITIONS

The following design criteria had a major effect on the designs:

- a. Circular track of 1500 ft (approximately 457m) diameter (later increased to 3000 ft or greater).
- b. Eighteen equally spaced cars linked on the track.
- c. Track speed of 8.9 m/s.
- d. Rated gross generator output of 1 MW per car
- e. Rotor rotational inertia of $61,820 \text{ Kg-m}^2$
- f. Maximum rotor rotational speed limit of 186 rpm
- g. Speed control of rotor required throughout the car's traverse of the circular track.
- h. Reversal of the rotor's direction of rotation at two diametrically opposite points on the track.

i. Wind direction and velocity measurements required at various points along the track.

j. Rotor viscous friction and rotor bearing loss load in accordance with equations in Paragraph 3.5.

k. Preliminary motor rpm schedule shown in Figure 5.1 obtained by varying speed at each 5° increment of track position (θ) such that power extracted is maximum at each track position.

l. Compatibility of power generated with utility company power grid.

5.3 ROTOR SPIN SYSTEM

5.3.1 Concept Selection

The rotor spin system has some very serious constraints imposed on it by the design conditions. The most serious constraints are the requirements: (1) for continuously variable speed control; (2) for a direction reversal approximately every 80 seconds for the given track size and track speed; and (3) the high rotational inertia of the rotor. These constraints limit the system to a direct current motor or to a constant speed motor driving through a continuously variable speed transmission and reversing gear system. The latter probably would have to include a brake system to stop the rotor motion for reversal. The transmission and brake system was discarded because of cost, inefficiency, and maintenance considerations for a system of adequate size.

A direct current motor drive has advantages in that regenerative braking is available to recover part of the inertial energy of the rotor during spindown and because motor hardware and controller hardware are commercial catalog items. However, the 80 second period for spinup, spindown, and direction reversal (based on track speed of 8.9 m/s and track diameter of 457 m) requires the motor to operate in its most inefficient mode of operation: a constant start-stop duty cycle which causes very high motor internal heat load and high power losses. Several alternatives were considered to replace or supplement an electric motor drive.

Figure 5.1. Preliminary Estimate of Aerodynamically Optimum Rotational Speed Schedule.

Two alternatives to an electric drive were considered: an airmotor and a hydraulic motor drive. Each would be best implemented by having a large electric motor driving a compressor. The air storage tanks required for the air volume needed to spin up the rotor would be large and expensive. In addition, the cost of the large airmotor and compressor, a valving system to reverse rotational direction, and the cost of increased inefficiency of an air system over an electrical system lead to the conclusion that an air system would be inferior to an electric system. The same reasoning holds true for a hydraulic motor spin system. In addition, a hydraulic system might require heating for start-up on cold winter days and cooling on warm or hot days. Further, no hydraulic motors as large as the size required are available as standard items, and a developmental program would be needed to obtain them. Thus, cost and technical uncertainties seemed to rule these alternatives out as solutions at this time.

Flywheel systems were considered as a load leveling means to improve the efficiency of the electrical spin motor system. A flywheel much larger than any previously constructed would be required for the expected spinup load. The inertia of the Madaras system rotor is very high, but a flywheel would be operated at a much higher rotational velocity. Modern flywheels have been designed for such uses as load leveling and deceleration load recovery on commuter trains and buses, and on helicopter mounted high-capacity lifting cranes. Most such devices have used an electric motor-generator to transfer energy into and out of the inertia wheel. Reliability of these devices is very much in question with a few hundred hours at best being quoted as the mean time between failures. Energy recovery efficiency is not very high, 30 to 40 percent, because of the same problem that plagues an electric motor spin system; i.e., low efficiency of electric motors or generators at speeds very much below or above their design speeds. The development cost, purchase price, and maintenance problems and costs of flywheels seemed sufficient reason to rule them out of consideration during this conceptual design study.

Consequently the decision was made to select the system utilizing a direct current spin motor driving the rotor shaft through a gear train. The use of regenerative braking for energy recovery during the rotor spindown segment of the spin cycle was also considered for this system. This is by far the simplest and lowest cost system to implement in the required power range.

5.3.2 Spin System Sizing

The spin system motor size is sensitive to the angular acceleration demanded and the motor speed at which this acceleration occurs. Although our calculations indicated the rotor's steady-state maximum load was 350 kW at 186 rpm, the load actually varies from 0 kW at 0 rpm to the upper 350 kW limit. However, the rotor provides a constant steady-state motor load at any constant rotational speed. Thus, as a minimum, the spin system must be able to deliver a 350 kW load to the rotor structure. This load will be applied to the rotor through a gearbox or an equivalent gearing system; and a gearing loss must be assigned to this function. If a four percent gearing loss is estimated, in accordance with other wind turbine design programs,³⁹ the limit steady-state load becomes 365 kW at the motor shaft.

The acceleration load requirement on the spin drive system is caused by the rotor inertia, the rotational speed schedule, and the maximum rotational velocity. At a track speed of 9 m/sec a time interval of 40 seconds is required to transverse 90 degrees of a 457-meter-diameter, circular track. Figure 5.1 shows a rotor accelerating to approximately 186 rpm in this time period. For a constant acceleration schedule, the rotational acceleration would be 0.472 rad/sec^2 ; and the input power required to accelerate the rotor to 186 rpm would be 568 kW. Adding the four percent gearing loss, a motor which delivers 592 kW to the motor shaft is required. Thus, for the given design conditions (schedule in Figure 5.1), the motor must be sized to overcome not only this 592 kW acceleration load, but also it must provide the steady-state viscous and bearing load of 365 kW at

186 rpm. Consequently, under these conditions, a 958 kW motor is required to accelerate and maintain rotation of each cylinder according to the schedule in Figure 5.1.

The time history of the spin motor load can be deduced from the characteristics of the two previously-described rotor loads. The first load (steady-state viscous and bearing load) increases as a cubic function of rpm and rotor bearing load increases linearly with rpm. For our purposes here, the test data curve developed by Madaras (Figure 3.24) for a rotating but not translating cylinder, as modified by our own wind tunnel test data will be used (see Paragraph 3.3.7). The second load (acceleration load) increases as a quadratic function of rpm during spinup and spindown.

In order to simplify this discussion of the rotor's rotational schedule, we will modify the Figure 5.1 rotational schedule in the form of a linearly increasing or decreasing rpm (constant acceleration and deceleration) for each quadrant of track, as shown in Figure 5.2. It should be recognized that this simplified spin rotational schedule will generate considerably less power from the cylinder than the optimum rotational schedule presented in Figure 5.1; hence the linearized system would not be used in an operating system.

The steady-state load requirement for a 180° traverse of the track with the simplified spin schedule would approximate that shown in Figure 5.3. The acceleration load and ideal inertial energy recovery would approximate that shown in Figure 5.4, and would ideally result in no net energy expended during each revolution of the track. Thus, for this simplified spin schedule the total motor shaft load would be that shown in Figure 5.5, (summation of Figure 5.3 and Figure 5.4) which accounts for the viscous energy losses during spindown.

Unfortunately, to maintain this rotational spin schedule, a very large and costly spin system motor must be used, and there are very serious additional spin system motor inefficiency losses. These will be considered later in this section.

Figure 5.2. Simplified Spin Scheduled Used to Estimate Spin Motor Size.

Figure 5.3. Rotor Steady-State Viscous and Bearing Friction Load for Simplified Spin Schedule.

Figure 5.4. Inertial Power Load, Ideal Performance, No Losses, Regenerative Braking, Simplified Spin Schedule.

Figure 5.5. Total Power Load, Including Viscous Losses During Spindown, Regenerative Braking, Simplified.

It can be seen from Figure 5.5 that no more than 42 percent of the total spin motor shaft input power can be recovered by regenerative braking during spindown for the simplified spin schedule, assuming no motor inefficiency losses during spinup and spindown.

For the aerodynamically-optimized spin schedule (Figure 5.1) the result is less encouraging as shown in Figure 5.6. This figure shows recovery of only 20 percent of the spinup energy during regenerative braking because of the higher losses (compared to those of the Figure 5.2 simplified schedule). These added losses are caused by the higher viscous-friction-power, load-schedule during the major portion of the spindown period. Thus, the viscous-friction-power load consumes most of the inertial energy input to the rotor, even without considering motor electrical losses during both the spinup and spindown portions of the schedule.

The analysis above shows that use of a 457 m diameter track requires a large, expensive, spin motor which consumes an unacceptable amount of power. Further, we find that very little spinup power is recoverable, especially for the aerodynamically optimum spin schedule (Figure 5.1). Therefore, it appears that we must minimize both the inertial load and viscous load requirements.

Unfortunately, given a rotor size and rpm schedule, we have little control over the steady-state load other than by reducing structural inertia. However, we cannot reduce cylinder inertia appreciably because of strength requirements. Therefore, the only means for reducing acceleration losses are by increasing the track diameter and by changing the spinup schedule. We evaluated the influence of these concepts in a computerized study of the Madaras system operation; but before we discuss these results, it is appropriate to discuss the efficiency losses in the spin motor.

Figure 5.6. Total Power Load, Including Viscous Losses During Spindown, Regenerative Braking, Aerodynamically Optimum Spin.

Figure 5.7 is an efficiency versus power output curve for a shunt-wound dc motor, the type of spin system motor that must be used in order to recover energy by regenerative braking. This curve is for a motor running near rated speed. However, for the speed schedule of a motor which would be used in the Madaras spin system, the efficiency versus power output curve would be that shown in Figure 5.8. If we plot the corresponding speed versus power output data with that of Figure 5.8, Figure 5.9 is obtained. Then, if efficiency versus speed at given power levels are cross-plotted from Figure 5.9 the efficiency for a complete Madaras speed schedule appears as in Figure 5.10. The speed-efficiency range for normal usage of large motors also is shown in Figure 5.10. Large motors are rated for this usage at no more than two starts per hour; whereas, the Madaras spin motor would have to start 45 times per hour (based on a 457-m diameter, circular track, and a 8.9 m/s track speed). Thus, the Madaras spin motor is used in a very severe and unfavorable mode from a motor design and efficiency standpoint and would require coding.

The magnitude of the power losses for a regenerative spin schedule are also stated on Figure 5.10. For this schedule, 100 percent power is immediately supplied to the motor and speed builds up as shown in the curve. Speed increases as motor power overcomes inertia and viscous loads. Unfortunately, high motor losses in the form of heat also are realized. Consequently, about 56 percent of the electrical energy input to the motor will be lost through heating of the motor, and an additional four percent of the shaft output will be lost because of rotor gearing. Thus as shown in Figure 5.10, regenerative braking during spindown of the rotor can recover only 40 percent of the inertial energy in the spinning rotor less the power consumed by the gradually decreasing steady-state viscous load during spindown. Since only 40 percent of the power input to the motor remains after spinup, and since this remaining inertial energy is regenerated at only 40 percent efficiency, only 0.40×0.40 or 16 percent of the electrical energy initially input to the motor

Figure 5.7. Percent Efficiency versus Percent Rated Output Power for a Shunt-Wound dc Motor, Normal Operation.

Figure 5.8. Percent Efficiency versus Percent Rated Output Power for a Shunt-Wound dc Motor.

Figure 5.9. Percent Efficiency and Percent No-Load Speed versus Percent Rated Output Power.

Figure 5.10. Percent Efficiency versus Percent No-Load Speed Motor Losses for a Shunt-Wound dc Motor.

during spinup can be saved by regeneration, and then this amount must be decreased by the steady-state spindown energy demand. With only 20 percent of the spinup energy available for regenerative braking because of viscous losses during spindown, as was shown in Figure 5.6, then only 20 percent of 16 percent, or 3.2 percent of the total spinup energy could be recovered during spindown. Therefore, it is obvious that the Figure 5.1 spin schedule with regenerative braking is not practical for a 457-m-diameter track (which governs the spinup, spindown acceleration time period), or for the rotor geometry studied. Thus, a series of computer runs were made to determine a better design.

The Madaras system computer program evaluates the power output from a spinning cylinder at 5° increments around a circular track for any rotor configuration and weight, track diameter, wind speed, and track speed. Power and aerodynamic forces are computed at each point, and then power is accumulated and averaged to give the average power output level per circuit of the track. This program also contains routines to evaluate the spin motor power requirements and to assess losses for other mechanical and electrical components of the system. Losses other than those of the spin motor system amount to 27 percent of the cylinder output power. These losses include:

- 15 percent generator losses
- 4 percent generator gearing losses
- 1 percent wheel bearing losses
- 5 percent synchronous condenser losses
- 2 percent trolley, control, and other miscellaneous losses.

In addition, losses due to mutual interference, aerodynamic drag, and rail friction are incorporated in the aerodynamic computations and are additive to the spin motor losses and the above miscellaneous losses.

The following design conditions were studied in our spin motor computer analysis.

- Track speed: 8.9 m/s
- Wind speed: 8.9 and 13.4 m/s
- Track diameter: 457, 915, 1372, 1830 m
- Spin motor size: 450, 675, 900, 1200 kW
- Motor efficiency - speed curve as in Figure 5.10
- Motor spin schedule
 - Constant spin motor power at rated limit applied during spinup and spindown
 - Maximum rotor speed of 186 rpm at power level required to maintain constant rotor speed
 - Regenerative braking beginning at the point where $\theta = 240^\circ$.
 - See Figure 5.11 for power and speed schedules used for various motor sizes. This is not an optimum schedule, but it is satisfactory for motor sizing studies. The motor is sized for the initial acceleration at $\theta = 90^\circ$.

These computer runs yielded data which were plotted in the form shown in Figure 5.12. The optimum spin motor size and the maximum power output for each set of conditions was then selected from Figure 5.12 and these optimum points were plotted versus track diameter for the 13.4 m/s (30 mph) and 8.9 m/s (20 mph) wind speeds as shown in Figures 5.13 and 5.14 respectively. Figures 5.13 and 5.14 show that the original Madaras design for a 457-m-diameter track would either consume power or at best yield negligible net power.

The only design condition that can be changed without directly reducing the power output for a cylinder is track diameter. That is, track speed or rotational speed reductions reduce output power directly, and reduction of rotational inertia requires reduction of rotational speed because of strength requirements. Increasing track diameter, however, reduces cylinder acceleration requirements directly with no effect on cylinder aerodynamic operation. Track diameter increases impact unit plant power cost only slightly if number of cars increases with track size, provided that the proper car spacing is maintained; i.e., car cost per unit of track remains the same and plant cost per unit of power is increased only slightly because of the cost

Figure 5.11. Modified Spin Schedule for Various Spin Motor Sizes.

Figure 5.12. Plant Power Output versus Spin Motor Size, 3,000-ft Diameter Track, 186 rpm for Various Wind Speeds.

Figure 5.13. Optimum Motor Size and Maximum Power Output/Rotor versus Track Diameter, 30 mph Wind Speed.

Figure 5.14. Optimum Motor Size and Maximum Power Output/Rotor versus Track Diameter, 20 mph Wind Speed.

of the greater area of unused land in the center of the track. However, spin motor size, cost, and power assessment against the power output of the cylinder are greatly reduced because acceleration power requirements become a decreasing proportion of the spin motor power load as track size increases.

Figure 5.13 shows that for the given operating conditions, optimum spin motor size appears to be approaching an asymptote of 425 kilowatts at very large track diameter, say 3 km, or 10,000-feet, and maximum output power is approaching an asymptote of about 350 kilowatts. This means the greatest power that can be obtained from a circular track for a 13.4 m/s wind and the given rotor configuration, speed limit, and structural strength is 350 kilowatts per car. Similarly, Figure 5.14 shows an asymptote approaching 400 kilowatts for the optimum spin motor size and an asymptote approaching zero net output power from the negative power level for a 8.9 m/s wind. That is to say, for a wind speed of 8.9 m/s, no net power can be produced by the rotor design on any circular track size. It is expected that performance improvement could be obtained on large diameter tracks by allowing the rotor to coast down in rpm on the upwind section of the track and thereby conserving a portion of the steady-state power requirement. Then, a small amount of dynamic braking, instead of regenerative braking, would be applied just prior to spin direction reversal since most of the angular momentum would have been dissipated during the coastdown.

Figure 5.13 shows the track diameter should be greater than 1220 m (4000 ft) for reasonable operation of the given system in a 13.4 m/s wind; and a track diameter of 1830 m (6,000 ft) is required to obtain maximum power output with a 450 kW (600 hp) motor. Figure 5.14 shows a track diameter of 915 m (3000 ft) should be the minimum diameter considered for a 8.9 m/s wind, and that this is nearly the minimum diameter for which optimum operation can be obtained with a 450 kW spin motor. Larger diameter tracks will give slightly better performance with that same motor.

Thus, we concluded that a 450 kW, 500 volt dc spin motor would be used for our final analyses, and that the minimum track diameter should be 1372 m. Studies of the spin schedule are presented in Section VI.

This study also confirmed initial findings that regenerative braking was of negligible value to the Madaras system. We confirmed that regenerative braking under any of the speed schedules studied would recover less than 5% of the energy input to the spin motor. Further, regenerative braking is not necessary to decelerate the cylinder, because viscous forces are so large that power actually must be supplied to maintain the desired spin schedule during spin-down. Finally, a cost penalty is realized through use of regenerative braking. This penalty results from the need for more expensive motor controls (a dc-ac converter), larger circuit breakers, and larger trolley bus bars to distribute the regenerated electricity to the system. Thus, we decided to use controlled motor spin-down followed by resistor-activated dynamic braking at very low speeds to bring the cylinder to a complete stop.

5.3.3 Spin System Controller

A commercial dc motor controller was selected to control the rotor spin motor. Units selected will provide both speed and dynamic braking control on each car. The control system consists of a three-phase transformer feeding a set of Silicon Controlled Rectifiers (SCR's). The SCR's can be fired by control circuits over any portion of their conductive half of the input alternating current cycle to control the motor armature power. Dynamic braking will be resistor controlled, and direction reversal will be achieved by reversing the polarity of power leads to the armature.

Speed control will be achieved by balancing the input power level to the armature against the demand of the control function signal. A motor voltage, current, power, or

speed profile time history can be used as a motor control function, or an external control signal can be furnished to the controller. Since the control function can not obtain more power from the transformer-rectifier power supply than it is capable of delivering, the control units must be sized greater than the maximum demand for motor terminal input power. Therefore, we selected control units sized for a 450 kW, 500 volt motor.

One controller will be mounted on each rotor car. Modern controllers represent a significant advance in motor use technology. Their losses are significantly less than the losses incurred by the former methods of resistor voltage control, autotransformer-rectifier voltage control, or motor-generator voltage control. They are more flexible, respond more quickly to control inputs, and cost no more than many of the older control methods.

In addition to the controller system, a magnetic disconnect and overload circuit breaker or fusing would be required for the spin motor. These elements would normally be placed on the power line side of the controller input transformer.

5.4 GENERATOR SYSTEM

The generator system was sized at 1 MW rated output by the electrical system design requirements. The following paragraphs discuss the generator operating conditions and the approach to generator selection.

The generator selection was limited to three basic types of generators: synchronous generators of the type used by most utility companies; induction generators as planned for use by Madaras; or direct current generators. The first two types are 60 Hz alternating current machines. The third would require some type of inverter system to convert its output to power grid frequency with a concurrent cost and efficiency penalty.

Synchronous generators could be operated at 90 to 95 percent efficiency, but with considerable penalty in operational problems and cost. The operational problems would consist of aligning and maintaining all the generators precisely in phase with the output power grid frequency. Since current plans are to install four 250 kW generators on each car, the initial alignment of the total system would be a large problem. Speed control of the system, speed changes between cars due to stretching and contraction of interconnecting linkages, and speed changes on a car due to wheel slippage would all cause circulating current power losses. Because of these problems, synchronous generators were eliminated from consideration for this study.

Induction generators are not in wide usage at the present time, however a number of smaller wind turbines are using them. The generator consists of an induction motor connected to a power grid with the shaft being driven above synchronous speed by an external mechanical power source. The generator/motor then feeds power into the grid at synchronous frequency and at a leading power factor. An induction generator would act as a governor on the Madaras system. As more power becomes available from the rotors, the cars would speed up slightly, produce more power from the generators, and hence load down the input shaft to limit the speed increase. It is believed the generators would operate at 80 to 85 percent efficiency at a power factor of 0.8 to 0.9 leading. The power factor would be corrected by a synchronous reactor system at the control house distribution station. Induction generators would be purchased as 250 kW induction motors, four to a car, and 72 for an 18-car plant. As wind decreases from the rated value, individual generators could be dropped off the line to enable the remaining connected units to operate at peak efficiency. Magnetic contactors and fuses or circuit breakers would be required for each generator, four sets to a car. No controls would be required except the magnetic contactor for each generator. Each 250 kW generator would provide 4160V, 60 Hz, 3Ø power.

5.5 POWER PLANT

The control system consists of the following components:

- A minicomputer-based primary controller in the control house.
- A microcomputer-based controller on each car.
- A two-way radio telemetry system to link all the car units to the primary controller.
- A wind sensor network dispersed around the track and hard wired underground to the primary controller.
- Monitoring instruments and control actuator circuits on each car and on system network components.
- An operators station in the control house consisting of monitoring instruments and manual over-rides of the primary controller.

These components are discussed in the sections which follow.

5.5.1 Primary System Controller

The primary controller would be based on a minicomputer of the PDP 11 family, probably one of the medium capability sizes. The computer would be programmed for complete automatic control of the entire system including rotor speed control and actuation of control disconnects on the cars; the interconnecting power network within the system; and the interface with the external power grid. Rotor speed control software would be flexible to permit simple modification to obtain optimum rotor performance characteristics based on car position and wind speed and direction around the track. However, it is believed that an improved version of Figure 5.11 spin-up profile would be used during the major portion of spin-up to minimize spin motor size. Wind data from a network of wind sensors dispersed around the track would be routed into the computer through a dedicated multiplexer, as would data from the interconnecting power network sensors. Data from and to the moving cars would be communicated through a central radio telemetry station which would have its own multiplexer interface.

The control function would include system safety monitoring and control as well as performance monitoring and control. Parameters such as voltages, currents, phase angles, bearing and motor temperatures, noise and vibration levels, disconnect status, and possibly others would be monitored. Appropriate steps, such as disconnecting equipment, dropping load by limiting rotor rpm, or shutting down the system would be performed as specified in the control program by either safety or operational algorithms. Control output signals would be transmitted by telemetry to the cars and by hard wire to the remainder of the system. An operator would have control overrides to shut down the system in segments and in its entirety for safety reasons as indicated by his own set of monitoring instruments. The primary controller would be backed up by a spare system or an alternate operating mode, otherwise the total system would be nonfunctional when a controller failure occurred.

5.5.2 Secondary on-Car Controller

Each car would have a microcomputer-based, on-board controller to transmit data signals from the car to the central controller and to receive control signals and act on them. Multiplexed two-way radio telemetry would provide the communication link. Primary data from the car, such as location, rotor rpm, motor and generator and line voltages and currents would be digitized and transmitted at discrete intervals. Safety functions such as temperature and vibration levels might be examined by the local processor and only maximum values or selected samples would be sent out. Sufficient capacity could be provided in the processor to control rotor rpm on the car based on wind data for the car location, as received from the central controller. The division of functions between the two controllers would depend on their relative capacities and the capacity of the telemetry link. Complete definition of these systems is not possible without a comprehensive study which is much beyond the scope of the present program.

The controller will have to include control output interfaces and control actuator drive circuits to operate the spin motor speed control system and the operational and safety disconnects. Disconnects will be provided for the spin motor controller and for each of the four generators as a minimum. Manually operated circuit breakers will also be provided for each of these functions and for the car primary lines. Controller circuits would be organized on a modular basis so that circuit boards or equipment components could be readily replaced from a stock of spares. The onboard controller program would be resident in a ROM or EPROM memory board in the processor which could be replaced or reprogrammed.

5.5.3 Radio Telemetry System

The radio telemetry system would consist of a central receiver-transmitter station located at the center of the track and hardwired to the central processor. A receiver-transmitter would be mounted on each car and connected to the onboard controller. It has not been determined yet if a single or multiple carrier radio beam would be required. In either case, the signal from the central station would be transmitted on a full-time basis, with coded digital command transmitted virtually simultaneously to all cars. The transmitted beam would include a synchronizing code to activate the transmitters on the various cars serviced by the carrier on a sequential basis. The signal received back would then be a continuous signal from a series of adjacent cars. The central antennas could be controlled to rotate in synchronism with the cars, if necessary.

It is expected that seven or eight-bit digital data signals would give sufficient resolution of transmitted analog data. Some data might be transmitted in fewer bits, for instance, disconnect or switch closures could be transmitted in a single bit. Data transmittal rate requirements are not expected to be high at the planned car speed of 8.9 m/s. Telemetry system overall requirements cannot be finalized until

track size and configuration and car spacing are established. Commercial equipment is readily available for most components of the system, but the multiplexing scheme might require development depending on the final size of the system. A spare central unit should be provided.

5.5.4 Wind Sensor Network

The wind sensor network would consist of an array of standard vane and cup instruments mounted on poles around the inside and outside periphery of the track line, but outside the immediate field of influence of the rotating cylinders. A spacing between 152 m and 304 m, depending on plant size, should provide adequate coverage of the wind pattern impinging on the upwind sides of the track. The direction and velocity sensors would be hard wired to a multiplexer input station of the central controller. The sensor data would be processed to define the changing wind flow field, and the individual rotor speeds would be controlled accordingly to produce optimum power output from each car. The wind sensor network would provide the primary data for startup and shutdown of the system and for controlling it during its operation. Commercial sensors adequate for use in the network are available.

5.5.5 Monitoring Instruments and Actuators

Monitoring instruments and actuators needed for the system are readily available as commercial items. The complement of actuators would consist only of magnetic contactors. At least no other type has been identified as a requirement to date. Monitoring instruments would include voltmeters, ammeters, phase angle or power factor meters, vibration and noise sensors, thermocouples or thermistors, microswitches, and possibly smoke detectors.

The use of these sensors is straightforward and requires very little explanation. Microswitches would be used to measure wheel rotational velocity, track speed, track position,

actuator closures, rotor rpm, and occupancy of a car. Processing of microswitch codes or closure rates would be performed by the onboard computer to develop required signals or control functions. Photocell circuits might be used more cheaply and reliably for some of these event detection operations.

It is expected that the system or onboard computers would have no difficulty handling the signals from any of the required sensors if suitable signal conditioning is provided. Many of the instruments can be purchased with accessory digitally coded outputs. Algorithms can be readily provided for the processor operations required to analyze the data signals and to transmit a selected subset and a critical subset to the central processor. A selected set of data also would be provided to onboard digital display instruments for use in maintenance and repair operations. A controller interrogation set with display could be provided to access all the data in the controller at not too great a cost. These options would be defined in the final detailed design of the operational system.

5.5.6 Operator's Control Station

An operator's control station console would be provided in the system control house. This station would have instruments displaying all the system operational parameters, including monitors of the central processor and telemetry system. Also, any data identified by the central controller as questionable or critical from the cars, the system network, or the wind sensor network would be displayed with identification data on a set of warning digital readout displays. A computer control console with CRT display and hardcopy capability would be furnished to interface with the central controller and to obtain detailed information on the operation of the system. A running analysis of the entire system operation could be provided on the hardcopy unit as the major part of the bookkeeping operation.

In addition to the displays described above, the operator's station would contain switches to over-ride the controller to shut down any car or the entire system. Switches would be available also to replace the controller and central telemetry systems with spare units.

5.6 SYSTEM NETWORK CIRCUITRY AND COMPONENTS

The system network circuitry and components are the electrical components required to interface the electrical units described in the previous sections of this section with each other and with the output commercial electrical power line grid.

5.6.1 System Network Elements

The system network elements include the following:

- a. Car trolleys and trolley feeder bus
- b. Distribution circuit to the trolley feeder bus
- c. Synchronous reactors for power factor correction
- d. Utility feeder circuits
- e. Substation

The system network components listed in items a,b,c, above would all be 4160V, 60 Hz, 3Ø equipment. Transformers to interface the 4160V system network to the commercial power grid and to produce 480V, 3Ø and 120V, 1Ø utility circuits for the lighting and equipment circuits of item d would be included in the substation listed under item e above.

5.6.2 Car Trolleys and Trolley Feeder Bus

The car trolleys would consist of three sliding shoe trolleys operating on a 4160V, 3Ø trolley bus. The car ground would consist of ground brushes and the car wheels running on the grounded steel track. Each of the car trolleys would require a current capacity of 70 amperes minimum for incoming current to energize the rotor spin motors during system startup. This figure assumes current limiting in the spin motor controller to limit

motor current to 1000 amperes RMS, and no use of the induction generators as motors to assist in startup. The limiting design condition, however, would occur with the generators at full output during a high wind condition with the spin motor coasting. The trolley current then would be 140 amperes out of the car. We have decided to use a 500 ampere trolley arm manufactured by Insul-8, Inc. because of the high voltage capability of their trolley feeder bus. This will provide long-lived trolley shoes as well as considerable capacity for regenerative braking, if desired.

A 4160V, 3 ϕ , 500 ampere trolley bus system manufactured by Insul-8, Inc. would be supported from poles around the inside periphery of the track system. This 3 ϕ bus would form a complete loop inside and parallel to the track loop. The trolley shoes will be supported from the car by articulated supporting arm devices also manufactured by Insul-8, Inc. A sketch of the trolley arm is shown in Figure 4.20.

5.6.3 Distribution Circuit to the Trolley Feeder Loop

A distribution circuit is required to connect the trolley feeder bus loop to the system 4160V, 3 ϕ transformer circuit. With a maximum car output current capacity of 140 amperes and a 500 ampere trolley loop, the distribution circuit will have to be connected to the trolley loop conductor bus bars within a distance of three and one-half car spacings. A three car spacing for inter-connections would provide a capacity for 1.2 MW output per car, which should be more than adequate.

The distribution circuit connectors around the trolley feed loop could be supported from the poles that support the trolley feed bus bars. Supporting them from the opposite side of the poles would tend to balance the side overturning moment load on the poles. The distribution circuit would be fed into the loop at a point on the loop across the track from the 4160V substation. The feed would best be placed underground

to cross the track. It would then branch right and left around the loop, each side carrying half of the system output current or startup current. The conductors could be tapered down from the branch point to the opposite point on the loop at a rate of 500 ampere decrease in conductor current capacity as each interconnection point to the trolley loop is passed. The main feeder would have to carry both branch currents and should be rated at 170 amperes per phase for each car on the system. The branch conductors and the 4160V transformer and its switchgear can all be sized from the final capacity of the feeder circuit. The feeder circuit should be protected with a circuit breaker. Also, a magnetic contactor should be provided to turn the entire trolley system on and off.

5.6.4 Synchronous Reactors

The requirements for synchronous reactors seems to be a dubious proposition. If the Madaras power became a significant portion of the grid power, correction would be necessary to balance the line more closely to the normal power load. In this case it would be better to furnish this correction in order to preempt resistance to the use of utility company lines.

Synchronous reactors provide the capability to produce large values of reactive loading for a small expenditure in power losses. Further, the reactive load is easily controllable by controlling the field of what is generally an in-line, self-excited, reactor field DC generator. If reactors are provided they can be sized from the capacity of the system and the fact that the induction generators would normally be operated at 0.8 to 0.85 leading power factor. At 0.8 P.F. it would require a synchronous reactor capacity of 75 percent of the system rated power output capacity to correct the system output to 1.0 P.F. The reactors would operate at about 98 percent efficiency, producing about 1.5 percent efficiency penalty against the system rated output. They would be sealed, hydrogen filled units. An exciter field control pot should be provided on the operator's

console and a disconnect magnetic contractor operable by the operator or by the automatic controller also should be provided. An overload circuit breaker should be installed in the 3Ø reactor lines.

5.6.5 Utility Feeder Circuits

Feeder circuits to operate lights and equipment should be provided for the control house and repair areas of the system. These circuits must provide power for lighting, heating, air-conditioning, controls and for power tools and other utility equipment. The circuits must provide 480V-3Ø and 120V-1Ø power. If a sizable crane is needed for the repair area, a higher voltage circuit might be provided for that purpose. These utility circuits could all be powered as branch circuits from transformers loaded on the secondary of the system 4160V-3Ø transformer in parallel with the trolley feeder distribution circuit. Each transformer would require a primary circuit breaker. Secondary circuits would be protected in distribution panels located in the control house and repair area.

In addition to the building utility circuits lighting, control, and utility equipment power would be required on the operating cars of the system. A lighting transformer and distribution panel would be provided on each car. This would be the last major component of the car electrical equipment. An electrical schematic of the car circuitry is shown in Figure 5.15.

5.6.6 Substation

The system substation will provide the interface between the system and the high voltage commercial power grid lines. The direct interface components would be a high voltage circuit breaker protecting the input to the primary of a step down transformer which would provide 4160V-3Ø power for the operation of the Madaras system. The transformer would be wye-configured with a grounded centertap. The rail system and all stationary equipment should be tied to that grounding system and a heavy earth ground should be provided.

Figure 5.15. Electrical Schematic of Circuitry on Each Rotor Car.

The secondary of the 4160V transformer would be fed to the trolley feeder distribution circuit through a circuit breaker and magnetic contactor sized for the maximum capacity of the generator system. The synchronous reactors would be attached to the transformer secondary with their own circuit breakers and magnetic contactors. Likewise, the utility circuit transformer would be fed at that point through its own primary circuit breaker or fusing. If a high voltage crane circuit were required, it would also emanate from the substation. Control circuits for the magnetic contactors and for the synchronous reactor field exciter would have to be fed to the substation.

The substation should be located outside the track and adjacent to the control house. This will simplify and shorten all the feeder circuits.

SECTION VI

PERFORMANCE ANALYSIS

The objectives of this analysis were to:

- Predict the performance of the Madaras system as a function of geometry, major operating parameters, and losses inherent in the system.
- Conduct trade studies to determine candidate plant configurations and sizes for use as inputs to the economic analysis.
- Compute rated power output and annual energy output for various plant configurations using the best system selected.

As was stated before, the determination of an optimum design for a Madaras system is beyond the time and budgetary scope of this program. However, it is believed that the results of trade studies conducted in this section of the report will isolate the important design parameters and will provide a sufficiently efficient design to permit an objective evaluation of the Madaras Power Plant concept.

Three basic studies were included in this performance analysis:

- A vortex analysis conducted to determine the mutual interference between rotor cars and hence define the effect of rotor car spacing on gross plant output.
- A trade study of the geometric and operational variables of a Madaras plant and their combined effects on aerodynamic, electrical, and mechanical losses and on net plant output.
- The determination of net power output versus wind speed and annual energy production for various plant sizes.

6.1 MUTUAL INTERFERENCE STUDY

The mutual interference study made use of the vortex analysis technique developed by Professor H.C. Larsen of the Air Force Institute of Technology for the Giromill vertical axis wind turbine currently under study by the McDonnell Aircraft Company. Since the Giromill and the Madaras system are essentially comparable in

system concept, and since McDonnell's wind tunnel tests have recently verified the analytical technique, the proven analysis was modified for use on this study.

A detailed description of Professor Larsen's analysis as it was developed for the Giromill and as it was modified and used for this Madaras program is given in Appendix A. Thus, the balance of this section will be used to present the results of using the vortex analysis to determine gross power output of Madaras plants. By gross power output we mean that the predicted power data obtained from Professor Larsen's computer program accounts only for the losses attributed to the aerodynamic drag of the rotor (but not the rotor car) and the losses attributed to the induced airflow caused by the interaction of the wind and the vortices shed by each cylinder as it travels around the track.

All other types of losses associated with plant performance are addressed in the trade studies of Paragraph 6.2, and the computation of net power output which accounts for the combination of all types of losses is presented in Paragraph 6.3.

A list of the variables and ranges of the variables studied to determine mutual interference effects is presented in Table 6.1.

TABLE 6.1
VARIABLES STUDIED IN MUTUAL INTERFERENCE, VORTEX ANALYSIS

AR	e/d	d	Track Diameter	Range No. Cylinders	Solidity s	Range V_t/V_w	Range V_w	Rotor Speed
		m	m	n	nd/D		m/s	rpm
6	2	5.6	915	1	0.006	0.1-2.5	8.9-13.4	183
6	2	5.6	915	6-18	0.037-0.110	0.1-1.5	8.9-13.4	183
6	2	5.6	1220	6-16	0.028-0.073	0.1-1.5	8.9-13.4	183
6	2	5.6	1524	8-14	0.029-0.051	0.0-1.5	8.9-13.4	183
6	2	5.6	1829	16-18	0.049-0.055	0.8-1.5	8.9-13.4	183
6	2	5.6	2392	16	0.038-0.041	0.1-1.5	8.9-13.4	183
8	2	4.9	1372	1-20	0.004-0.071	0.5-1.5	7.8-13.4	186
8	2	4.9	1524	1-20	0.003-0.064	0.3-1.5	4.5-13.4	186

Since the prohibitive number of over 161,000 computer runs would be required to study all combinations of all variables in the range of interest, we conducted a number of preliminary runs of boundary cases to determine the sensitivity of the various variables. This work, conducted by Professor Larsen, is described in Appendix A. We also used the results of our trade study runs as input to the selection of the variables listed in Table 6.1.

A typical plot of the complete performance map of a single rotor spinning at a constant speed of 183 rpm is presented in Figure 6.1. This plot shows the optimum value of λ (V_t/V_w) for each wind speed. One can see that optimum value of λ increases as V_w decreases; however, at least for this case, operation at a constant track speed of about 10.6 m/s (35 mph), would permit operation near optimum λ for all wind speeds.

We want to emphasize that the results from the Appendix A analysis (Figures 6.1, 6.2, and 6.3) account only for the rotor drag losses and the interference losses caused by the influence on the wind by the vortices shed by the various rotors in a Madaras plant. Thus, the output from the vortex analysis has been defined as the gross output.

By defining the location and nature of the peak value of λ on the set of runs in Figure 6.1, it was not necessary to run complete performance maps for all conditions. Some examples of the other data from the first set of runs is presented in Appendix A.

From these studies as well as our studies conducted on the various loss mechanisms (Paragraph 6.2), we concluded that a 1372-m (4500 ft) track diameter was nearly optimum, and that probably no more than 20 rotors should be used on a 1372-m track. This density is equivalent to an inter-car spacing of 215 m. We also learned that the power output/rotor car was essentially constant when from one to eight rotors are used on a 1372-m track; and that the power per rotor drops as the number of cars increases beyond eight. Thus, our later studies were concentrated on track diameters of 1372 m and 1524 m and with inter-car spacing varying from that associated with eight to twenty cars on a given track

Figure 6.1. Gross Power Output versus λ for Various Wind Speeds, One Rotor Spinning at 183 rpm, 915 m Diameter Track. Professor Larsen's Vortex Analysis Accounts Only for Aerodynamic Drag and Interference Losses.

diameter. For each track diameter, the performance of a 1-rotor plant was first computed to provide a basis for defining a maximum output condition.

Our subsequent runs indicated that there was virtually no difference in performance between plants utilizing 1372-m and 1524-m diameter tracks. Therefore, since the larger diameter track would increase plant cost without performance benefits, the smaller diameter track was selected for all subsequent studies.

Typical results of our later computations are shown in Figures 6.2, 6.3, and 6.4. Figure 6.2 presents the gross power generated by one car, and data from it is used in developing the mutual interference loss factor in Figure 6.4.

Gross power generated by plants utilizing from 8 to 20 cars, inclusive, on a 1524-m diameter circular track is presented in Figure 6.3. The effect of mutual interference on cut-in speed is quite pronounced as the number of cars increases. Note that cut-in wind speed for 1 car (Figure 6.2) is about 3.3 m/s; whereas for 10 cars and 18 cars, cut in wind speed is about 4.4 m/s (10 mph) and 7.8 m/s (17.5 mph), respectively. Thus, as one would expect, as the number of cars increases (while track diameter and track speed are held constant) the cut-in wind speed increases and the mutual interference loss increases.

The effect of increasing the number of cars, N , on power loss is demonstrated more dramatically in Figure 6.4, where mutual interference loss factor f_n for a given wind speed is defined by

$$f_n = \frac{P_n/N}{P_1} \quad (7)$$

where P_n is the gross power generated by an N -car plant at wind V_w and track speed V_t (from Figure 6.3), and

P_1 is the gross power generated by a 1-car plant at the same wind speed V_w (From Figure 6.2) and track speed V_t .

Separate sets of f_n loss factors were computed for each value of V_t , hence f_n was considered to be a function of both V_w and V_t .

Figure 6.3. Gross Power Output versus Number of Rotors for Various Wind Speeds for a Constant Rotor Speed of 186 rpm and Constant Track Speed of 13.4 m/s.

Figure 6.4. Mutual Interference Loss Factor versus Wind Speed for Various Numbers of Cars. Constant Rotor Speed and Track Speed of 186 rpm and 13.4 m/s, respectively.

The cut-in speed in Figure 6.4 for each value of N is the V_w value at which each curve terminates near the x-axis. At wind speeds lower than these terminal points, the power rapidly becomes negative.

The loss in power for an 8-car plant is negligible until V_w decreases below 5.6 m/s (12.5 mph), and 75 percent of the power per car of a 1-car train will be available at wind speeds of about 3.6 m/s (8 mph). However, a 20-car plant does not operate as efficiently. At a wind speed of 13.4 m/s (30 mph) each of the 20 cars will produce only 90 percent of the power of a one-car train; and at the cut-in speed of 8.0 m/s (18 mph) only 5.5 percent of the power from a 1-car plant would be generated by each car of the 20-car plant.

Interference loss factors like those shown in Figure 6.4 will be used in the computations described in Paragraph 6.3 to develop the power duration curves and to determine net rated power output of a power plant. Values of f_n for the 1372 m (4500 ft) track diameter are presented in Figure 6.4.

6.2 PARAMETRIC TRADE STUDY

The computer program used in Section V to determine the spin motor power requirements was used for this trade study. This program simulates the performance of a 1-rotor plant taking into account all losses except for the mutual interference losses. Thus, by combining the results from the vortex interference analysis with those from this trade study analysis, net Madaras plant output accounting for all losses was obtained.

The program was written in very general format to permit flexibility in conducting parametric studies of both cylinder-driven and wing-driven cars as well as provisions for analyzing both circular and race track configurations.

The program is designed for automatic, interactive computations of average power output. Input to the program includes:

- Cylinder and end cap geometry in terms of AR , e/d , cylinder diameter, and cylinder projected area.
- Aerodynamic lift, drag, and center of pressure data corresponding to the geometry as a function of U/V_R where U is the surface speed of the cylinder and V_R is the resultant wind derived from the wind speed and track speed at all points around the track orbit.
- Frontal area of car and height of the car above ground.
- Track gage and diameter.
- Design rpm of the cylinder.
- Spin schedule - rotor rpm can be varied or held constant.
- Spin motor size.
- Spin motor speed-efficiency characteristic curves.
- Track speed.
- Wind speed.

Internal to the program are special routines which compute:

- Rotor weight and moment of inertia corresponding to inputs.
- Total vehicle weight.
- Wind speed distribution with height representing the wind boundary layer flow.
- Mean resultant aerodynamic velocity vector obtained by combining the uniform flow along the length of the cylinder caused by rotor motion down the track with the nonuniform boundary layer distribution.
- Gross power and loads on the rotor at any point on the track.
- All losses except for mutual interference losses at any point on the track.
- Net power (excluding interference effects) at any point on the track.

- Spin history of the rotor.
- Average net power per cycle for a circular track.
- Average net power per cycle for race tracks having sides varying in length from 610 m to 18,300 m with wind direction varying at 7 different angles between 0° to 90° relative to the straight track section. Thus, power for racetracks having all combinations of 40 lengths of straight tracks and 7 wind directions (280 racetrack length-wind combinations) is computed on each computer run.

Losses accounted for in the program are:

- Cylinder aerodynamic drag loss.
- Rotor car aerodynamic drag loss.
- Vertical and lateral friction losses of the rotor car main wheels and idler wheels rolling on the track.
- Wheel bearing losses.
- Gearing losses associated with the step-up gear box required to drive the generators by the wheels.
- Generator losses.
- Power conditioning, conversion, and transmission losses described in Section 5.
- Rotor spin motor efficiency vs. speed (Fig. 5.10) during acceleration, steady state, and deceleration operational modes.
- Motor power required to overcome the inertia of the rotor during rotor spin-up.
- Motor power required to overcome viscous drag bearing, and rotor drive system losses at all times during plant operation.

A summary of the computer runs made during this trade study is presented in Table 6.2, which indicates the general types of analyses that were conducted:

- Cylinder geometry
- Spin motor size
- Track speed and cylinder rpm
- Spin motor schedule
- Plant configuration selection.

TABLE 6.2
TRADE STUDY VARIABLES

Study Category	AR	e/d	d	A	Track Dia.	Rotor Speed	Range V_t/V_w	Range V_w	Range Viscous Braking	Range Regen. Braking	Spin Motor Size	Trans- mission
			m	m ²	m	rpm		m/s	degrees	degrees	kW	
Cylinder Geometry	4	1.25	6.8	186	1372	186	1.5	13.4	—	240	450	none
	5	2,3	6.1	↓	↓	↓	↓	↓	—	↓	↓	↓
	6	↓	5.5	↓	↓	↓	↓	↓	—	↓	↓	↓
	8	↓	4.9	↓	↓	↓	↓	↓	—	↓	↓	↓
	8	1.25	6.1	297	↓	150-200	0.6-0.8	3.4	—	↓	↓	↓
	8	↓	5.5	241	↓	↓	↓	↓	—	↓	↓	↓
	8	↓	4.9	186	↓	↓	↓	↓	—	↓	↓	↓
	8	↓	4.3	146	↓	↓	↓	↓	—	↓	↓	↓
Spin Motor Size	6	2	5.5	186	457	100-200	1-1.5	8.9-13.4	—	240	450-1200	none
	6	↓	↓	↓	609	↓	↓	↓	—	↓	↓	↓
	6	↓	↓	↓	1372	↓	↓	↓	—	↓	↓	↓
	6	↓	↓	↓	1524	↓	↓	↓	—	↓	↓	↓
	6	↓	↓	↓	1829	↓	↓	↓	—	↓	↓	↓
Track Speed and Cylinder rpm	6	1.25	5.5	186	1372	120-186	0.7-2.3	6.7-13.4	—	240	450	none
	6	2	5.5	↓	↓	150-200	↓	↓	—	↓	↓	↓
	8	1.25	4.9	↓	↓	↓	↓	↓	—	↓	↓	↓
	8	2	4.9	↓	↓	↓	↓	↓	—	↓	↓	↓
Spin Motor Schedule	6	1.25	5.5	186	1372	150-200	0.3-2.7	6.7-13.4	none	220-260	450	none
	6	2	5.5	↓	↓	↓	↓	↓	↓	↓	↓	↓
	8	1.25	4.9	↓	↓	↓	↓	↓	↓	↓	↓	↓
	8	2	4.9	↓	↓	↓	↓	↓	↓	↓	↓	↓
	6	1.25	5.5	↓	↓	↓	↓	↓	220-260	none	↓	↓
	6	2	5.5	↓	↓	↓	↓	↓	↓	↓	↓	↓
	8	1.25	4.9	↓	↓	↓	↓	↓	↓	↓	↓	↓
	8	2	4.9	↓	↓	↓	↓	↓	250	none	↓	3-step
Plant Configuration Selection	8	2	4.9	186	1372	186	0.5-2.8	5.6-13.4	250	none	450	none
	8	2	4.9	186	1524	186	0.5-2.8	5.6-13.4	250	none	450	none

Note: For each of the above runs, net power was computed for a circular track as well as for a racetrack having straight sections varying from 610 m to 18,300 m and wind direction relative to the straight sections varying from 90° to 180°.

The results of these computer studies are presented in the paragraphs which follow.

6.2.1 Rotor Geometry and Size

Three studies were conducted. The first study analyzed rotor power output as a function of 15 combinations of aspect ratio (3,4,5,6, and 8), and end plate size ratio (e/d values of 1.25, 2, and 3). All cylinders had an area of 186 m^2 , and track and wind speed were held constant at 8.9 m/s and 13.4 m/s, respectively. The results of this study indicated that the large end plate required more power than was available to turn it by a 450 kW motor, and hence the consideration of an e/d ratio of 3 was dropped from the study. We also learned that an aspect ratio from 6 to 8 was of greatest interest and that e/d ratios of 1.25 and 2 should be considered further. See Appendix C for rotor size and operational condition sealing used in all parametric studies.

The second study considered the variation of diameter and cylinder rpm while holding area constant. The results of this study are presented in Figure 6.5, which shows power output versus cylinder diameter for rotors having e/d ratios of 1.25 and 2.

The decreasing of diameter has three effects. First, as diameter decreases with area being held constant, AR and hence aerodynamic efficiency increases. This effect improved power output, especially for $AR = 8$, $e/d = 2$. The second effect in decreasing diameter is to reduce rotor weight and moment of inertia which improves net power output. The third effect causes power loss: as diameter decreases so does cylinder surface speed, U , and hence C_L decreases with the resulting decrease of U/V . This last effect is beginning to show itself for the $e/d = 1.25$ data, whereas the larger end plate ($e/d = 2$) is improving aerodynamic efficiency sufficiently well to overcome the effect of decreased U/V . Apparently, an $AR > 8$ would yield better performance, however since the $AR = 8$ data were extrapolated, we decided to stop this study at that point.

Figure 6.5. Net Power for One Rotor versus Cylinder Diameter as a Function of e/d Ratio and Cylinder rpm for Constant Projected Area, Wind Speed and Track Speed; $e/d = 1.25$ and 2.

The decrease in performance as diameter exceeds 18 ft (5.5 m) is severe, and losses exceed output as AR approaches 5. This loss is caused by increased cylinder weight and inertia resulting from diameter increase as well as loss of aerodynamic efficiency. Thus, cylinders having aspect ratios less than 6 were eliminated from further consideration.

The effect of varying cylinder rpm was not as marked, although the reduced U for AR = 8 (small diameter cylinder) resulted in loss of power output whereas a small increase in power occurred at AR = 6 as rpm was decreased. This latter observation was caused by reduced cylinder weight and inertia, because rpm influences these variables markedly. Although the larger diameter cylinder causes U to increase, the decreased rpm decreases U by the same percentage. Thus, the improved performance at AR = 6 is the result of decreased weight caused by lower structural stresses in a cylinder designed to operate at 150 rpm versus 186 rpm.

Since an aspect ratio of eight looked promising, the third study analyzed the effect of varying cylinder diameter and cylinder projected area while holding constant aspect ratio. Since the end plate absorbs a large amount of motor power and increases inertia significantly, we selected an e/d ratio of 1.25 thinking that the improved aerodynamic efficiency of AR=8 would offset some of the loss caused by the small end plate. This compromise combined with the improved weight and inertia of the smaller end plate seemed attractive.

The results of this study are presented in Figure 6.6. All of the effects of changing diameter and speed mentioned previously are exemplified. In addition, one can observe that the power output increases with increased diameter (and area) at the lower rotational speeds (150 rpm). However, as rpm increases, the effect of increased cylinder weight, inertia aerodynamic drag, and viscous drag that occurs as diameter increases results in higher spin-motor loads, and the result is reduced power output.

Figure 6.6. Net Power for One Rotor versus Cylinder rpm for Constant Aspect Ratio of 8 but with Variable Diameter and Variable Projected Area.

This effect is significant for the 20-ft (6.1 m) diameter cylinder; however, the 16-ft (4.9 m) and 18-ft (5.5 m) diameter cylinders have reached peak performance at about 186 rpm. It appears that peak performance of the largest cylinder will be at rotational speeds below 140 rpm.

The loss in area resulting from reducing cylinder diameter below 16 ft (4.9 m) is so significant that no further consideration was given to a cylinder this small. Thus, the 16 ft, 18 ft, and 20 ft cylinders having $AR=8$ and $e/d = 1.25$ were retained for further study.

6.2.2 Spin Motor and Track Size

The computer trade study conducted to determine spin motor size is described thoroughly in Paragraph 5.3.2, and the graphical results of these studies are presented in Figures 5.12, 5.13, and 5.14. The Figure 5.13 plot of motor size and net power output vs track diameter is particularly significant. If track diameter is too small, the motor power requirements become excessive in order to accelerate the rotor rapidly enough to effectively generate power before it is time to decelerate the rotor. As track diameter increases, more time is available for rotor spin-up (assuming a given track speed) and hence motor power requirements decrease.

At a track diameter of 1372 m, a near optimum motor size is shown to be about 450kW for plant operation at the 13.4 m/s (30 mph) rated plant capacity. Although further reductions in motor size could be achieved by increasing track diameter to 1830 m, it was thought that the increased cost of track would offset motor cost savings, and that the minor improvement in plant output would be insufficient to offset this added cost.

Therefore, it was decided to restrict further studies to track diameter in the 4500 ft (1372 m) to 5000 ft (1524 m) range and to use a 450kW motor in all further studies.

6.2.3 Track Speed and Cylinder RPM

A summary of our results from studying combined track speed and cylinder RPM effects for a 4500 ft (1372 m) track diameter and a 450 kW spin motor are presented in Figures 6.7 and 6.8.

The improvement in performance of the rotor having an aspect ratio of 8 over that having an aspect ratio of 6 is significant. Also, the indication that a 150 rpm rotational speed produces significantly more power than a 186 rpm speed for the aspect ratio 6 cylinder, again shows the importance of the diameter-speed relationship discussed previously. The improved low wind speed performance of the aspect ratio 8 cylinder is particularly important because this power output improvement will increase significantly the annual energy yield of the plant.

As in Figure 6.1, the data shows that peak power occurs at approximately the same track speed (25 mph) for both of the wind speeds studied even though the peak occurs at a different value of λ . This feature is quite advantageous in that it is necessary to operate the plant at a given track speed for all wind speeds. Although peak power occurs at different rpm values as wind speed decreases from its rated value of 13.4 m/s, our control system is designed to sense wind speed and adjust rotational speed to its optimum value.

It also should be noted that when we indicate a particular run was made at n rpm, we also infer that the structure was designed to withstand a rotational speed of n rpm. Thus, once this design variable is fixed, it cannot be increased. Therefore, one cannot operate a rotor at design rpm at one wind speed and then operate the rotor above the design rpm at another wind speed. Of course, one can always operate below the design rpm, if desired. In fact, decreasing rpm is the means used for "feathering" the rotor during wind conditions greater than the 13.4 m/s rated speed.

Figure 6.7. Net Power Output for One Rotor versus λ and Cylinder rpm for Various Cylinder Rotational Speeds, Aspect Ratio = 6, $e/d = 2$.

Figure 6.8. Net Power Output for One Rotor versus λ and Cylinder Rotational Speed, AR = 8, e/d = 2.

This study shows a clear superiority of an aspect ratio 8 configuration over that of aspect ratio 6, not only because of the improved power output at rated wind speed, but also because of vastly improved performances at low wind speed even when operating at the design rpm (186) which provides maximum power at rated wind speed (13.4 m/s). It also was evident that maximum power at both wind speeds will be achieved while the plant is operating at a track speed of about 25 mph (11.2 m/s).

6.2.4 Spin Motor Schedule

The characterization of the spin motor schedule, the motor's performance requirements, and losses were discussed in Paragraph 5.3. Since spin motor losses are so significant, a brief trade study was conducted to determine ways in which the spin schedule could be varied in order to improve performance. The study was restricted to the use of an electric motor although other sources of power (hydraulic, air motors, and flywheels) were considered in a cursory manner. Within these ground rules, then, we studied the use of different forms of braking to improve the deceleration portion of the cycle, and we simulated the use of a three-step transmission (gear box and clutch) in our computer program to improve the acceleration portion of the cycle. The results of our studies of the mid-portion of the cycle were presented earlier in Paragraphs 6.2.1, 6.2.2, and 6.2.3.

A typical performance plot in terms of motor power required, motor rpm, and instantaneous net power output as a function of rotor position on a circular track is presented in Figure 6.9. The cycle shown in this figure represents only one-half of an entire cycle, however the remaining half is similar to the portion of the cycle shown in Figure 6.9.

Our initial cycle incorporated regenerative braking beginning at 235° (shown by the solid lines). Then, we modified the cycle by turning off the motor at various points between 215° and 265° and then letting viscous friction decelerate

Figure 6.9. Net Power Output for One Rotor, Spin Motor Power, and Motor rpm versus Rotor Position on Track as Affected by use of Viscous Braking, Regenerative Braking, and a Three-Step Transmission to Vary the Spin Schedule.

the cylinder naturally (shown by the dashed lines). Similar studies were conducted for regenerative braking. Note that both of these studies affect only the deceleration portion of the cycle. The results of using the transmission during the spin-up portion of the cycle are depicted by the dotted lines.

The results of the braking studies are shown in Figure 6.10. As we noted before, at the 8.9 m/s wind speed, more power can be obtained at the 170 rpm rotational speed; and a significant improvement in spin cycle performance is obtained by the proper application of either viscous or regenerative braking. Viscous braking at the 245 degree point appears to be the better solution not only because of its 6-kW edge in power output, but also because of the added cost of the heavier trolley components, increased complexity in circuit design, and wear on the motor which would result in use of regenerative braking. The poor performance that occurs when regenerative braking is initiated earlier than its optimum time is attributed to its ability to stop the rotation too quickly (shown in Figure 6.9). By applying regenerative braking at 235°, the rotor is stopped by the time it reaches 250°, and from there on the non-spinning rotor presents itself as a large drag producer which must be driven by the train. This situation was improved considerably by initiating regenerative braking at 260°.

As can be seen in Figure 6.9, the net extra energy regenerated is very small (the small triangular section in the negative region on the power curve) as was predicted in Section 5, since viscous drag and motor losses consume much of the power regenerated. The increase in average power per orbit of the track of only 2 kW (0.5%) was gained from the regenerative cycle shown, and an average increase per orbit of only 11 kW (3%) was developed for the best regenerative cycle. This best case adds only 1 percent to the total energy (kW-hr) generated in this best cycle.

Figure 6.10. Net Power Output for One Rotor for Various Spin Motor Speeds versus Track Position at Which Regenerative Braking or Viscous Braking was Initiated.

Therefore, it was decided to utilize viscous braking at 245° for all future runs.

The use of a transmission in conjunction with optimum viscous braking resulted in an increase of average power generated of about 16 percent. This is a significant improvement in performance, and the reason for a transmission's effectiveness is shown in Figure 6.9. The primary benefit derived from use of a transmission is to reduce the start-up load on the motor. This load reduction permits the rotor to accelerate more rapidly and thereby increases cylinder surface speed, U . The resulting increase in U/V ratio over the 90° - 160° portion of the orbit produces increased lift and consequently produces increased power as shown in Figure 6.9. In addition, since a rotor equipped with a transmission reached operating RPM (186 rpm in this case) earlier (140° instead of 160°) there is a small reduction in power consumed by the motor (See the motor power curve in Figure 6.9).

Unfortunately, time was not available to study the effects of various types of transmissions or performance as well as to evaluate the life cycle cost and maintenance problems associated with transmission use. All that can be said at this time is that transmissions can have a significant effect on performance, as shown in Figure 6.9, and that further study of transmissions is in order.

One interesting result of our study of transmissions was that transmissions are justified only for Madaras plants having a circular track configuration. For racetrack-type plants, the power generated on the straight section is so much greater than that on the circular ends, that the improved performance on the ends is hardly discernable in the overall output.

6.2.5 Rotor and Plant Configuration Selection

A total of six rotor configurations, selected as candidates for the final analysis, are itemized in the upper section of Table 6.3. These were the rotor-operating-condition combinations which appeared to be the most competitive of all that were studied.

The figure of merit upon which initial screening was based was the rotor mass per kW generated at rated wind speed, listed in the last column of Table 6.3. This figure of merit was selected as a cost/performance indicator because of the close relationship that exists between cost and weight of semi-monocoque structure like the rotor.

Of the six rotors listed, Configurations 1, 2, and 4 were considered to be the most attractive. Of the three, Configuration 2 had clearly the better performance figure of merit for a circular track (18.3 kg/kW vs. 22.2 kg/kW for Configuration 4). However, Configuration 2 is a very large rotor having a height of 48.8 m (160 ft), a diameter of 6.1 m (20 ft), and a projected area of 298 m^2 . Since this cylinder was about 60 percent larger than that for which loads and stress analyses had been conducted, and since time was not available to conduct another iteration of our design and concepts, the decision was made to eliminate Configuration 2 from further consideration on this study. Should the Madaras system look attractive, an investigation of larger rotor of this size certainly would be recommended.

This first screening left Configurations 1 and 4. Both are the same size as the rotors analyzed; however Configuration 1 has an end cap ratio of 2, whereas Configuration 4 has an end cap ratio of 1.25. These two Configurations were re-run using viscous braking instead of regenerative braking (used for the initial comparisons) and the results are listed in the middle section of Table 6.3. Notice that this second comparison considered

TABLE 6.3

COMPARISON OF CANDIDATE ROTOR CONFIGURATIONS
1372 m (4500ft) TRACK DIAMETER, 450 KW SPIN MOTOR

Configuration Number	AR	e/d	d m	h m	Area m ²	V _w m/s	V _t m/s	n rpm	P kW	Mass kg	I kg-m ²	kg/kW Circle
<u>With Regenerative Braking</u>												
1	8	2	4.9	38.1	186	13.4	11.2	186	380	8773	41,663	23.1
2	8	1.25	6.1	48.8	298	13.4	8.9	150	526	9646	60,350	18.3
3	8	1.25	5.5	43.9	241	13.4	8.9	186	400	10284	52,050	25.7
4	8	1.25	4.9	38.1	186	13.4	15.6	186	324	7189	28,556	22.2
5	6	1.25	5.5	33.5	184	13.4	11.2	186	222	8162	40,761	27.8
6	6	2	5.5	33.5	184	13.4	11.2	150	280	7164	43,787	25.6
<u>Screened Configurations - Viscous Braking</u>												
1	8	2	4.9	38.1	186	13.4	11.2	186	431	8773	41,663	20.3
						8.9	11.2	170	214			41.0
4	8	1.25	4.9	38.1	186	13.4	15.6	186	407	7189	28,556	17.7
						8.9	15.6	186	110			65.3
<u>Selected Configurations, Optimum Viscous Braking*</u>												
1	8	2	4.9	38.1	186	13.4	11.2	186	436	8773	41,663	20.1
						8.9	11.2	170	216			40.6

*Optimum Viscous Braking is initiated at the 245° point in the orbit.

both the 13.4 m/s rated speed performance as well as performance at the wind speed of 8.9 m/s.

This second comparison showed that Configuration 4 had a 15 percent lower value of kg/kW at the rated wind speed than Configuration 1; although Configuration 1 provided about 6 percent more power at the rated condition of 13.4 m/s. Configuration 1 was clearly superior at the 8.9 m/s speed with a kg/kW value that was about 60 percent smaller than that of Configuration 4. This means that a Configuration 1 rotor would generate more power at low wind speeds, and since the lower wind speeds occur more often during the period of a year, Configuration 1 probably would provide considerably more kW-hr of energy per year than would Configuration 4. Finally, our computer studies indicated that Configuration 1 would generate more power than Configuration 4 at rated wind speed if used in a racetrack plant layout. Thus, in view of its superior low-wind-speed performance and the high probability that a racetrack layout would prove to be the more cost effective, we selected Configuration 1 for the final analysis. Slight improvements in Configuration 1 performance were obtained by optimizing the viscous braking routine, as shown in Figure 6.10. The results of this optimization are shown in the lower part of Table 6.3.

Having selected the rotor configuration, we next directed our attention to the selection of the most appropriate track configuration and size. Figure 6.11 presents a performance plot of the power output from one rotor car of the size previously selected ($AR=8$, $e/d=2$, $d=4.9\text{m}$, $\text{Area} = 186\text{ m}^2$) vs track diameter for various wind speeds. Performance on both circular tracks and racetracks having a 4878-m (16,000 ft) straight sides is presented. For illustration, these data were plotted for operation at a design rotor speed of 186 rpm for all wind speeds at a V_t/V_w ratio (λ) that will optimize performance for the racetrack. This selection was made because it is very likely that a racetrack configuration will be the more feasible. Should a circular-track-

plant appear attractive, the optimum track speed data are available, so this selection is not irreversible.

Figure 6.11 demonstrates clearly the trend that we have noted in our previous analyses of this data: the most attractive track size is in the 1372 m (4500 ft) to 1524 m (5000 ft) diameter range. Further, one can see that there is very little difference in performance in selecting either of the two track sizes. The 1372-m size is slightly favored for a racetrack configuration, and a 1524-m circular track will provide a small increase in output over that having a 1372-m diameter. Our cost analysis will govern the final selection; however, at this time it appears that a 1372-m diameter racetrack will prove to be the most cost-effective.

It is also important to note again here that mutual interference effects described in Paragraph 6.1 become more pronounced as track diameter decreases, and from these studies we determine that interference losses would become excessive if track diameter were decreased below 1219 m (4000 ft).

Figure 6.12 presents an overall performance map of the selected rotor operating on a 1372-m (4500 ft) diameter circular track and on a racetrack having 1372-m diameter ends and 4878-m (16,000 ft) straight sections. Figure 6.12 depicts two principal advantages of the racetrack over the circular track: (1) power output at a given V_w is nearly 2.5 times greater; and (2) power can be generated at lower wind speeds. Of course, the racetrack has two principal disadvantages: (1) for optimum operation, the wind direction should remain constant; and (2) it requires much more land.

The 4878-m straight length has been selected arbitrarily at this point. Our computer program currently provides power vs straight section length in 150-m increments from 610 m to 18,300 m, and it can be modified easily to provide any length desired. In our cost analysis we will study the effect of varying straight section length on plant efficiency and energy cost.

Figure 6.11. Net Power Output from One Rotor versus Track Diameter as a Function of Wind Speed. Performance for Both Cylinder and Racetrack Configurations.

Figure 6.12. Net Power Output from One Rotor versus λ as a Function of Wind Speed. Performance is Presented for Both a 1372-m Diameter Circular Track and a Racetrack having 1372-m Diameter Ends and 4878 m Straight Sections.

6.3 NET POWER AND ANNUAL ENERGY OUTPUT

The results and discussion pertaining to plant performance that have been presented prior to this section of the report have considered the mutual interference effects for plants composed of N rotor cars and the net power output from one rotor car after considering all losses except for mutual interference losses.

The purpose of this portion of the report is to:

- Develop wind duration curves appropriate for the mid-height of the rotor selected for this study.
- Describe the methods used to determine the net output power of an N-rotor-car plant as a function of wind speed.
- Demonstrate how power duration curves were developed from wind duration curves.

The results of this study will then be used in the economic analysis in Section VII.

6.3.1 Wind Duration Curve

The rotor selected for this final analysis has the following geometric characteristics:

- $AR = 8$
- Diameter = 16 ft (4.9 m)
- Cylinder Length = 125 ft (38.1 m)
- e/d ratio = 2
- End Cap Diameter = 32 ft (9.8 m)

Figure 4.23 indicates that the top of the track rail is 4 ft (1.2 m) above mean terrain level, and the distance from the top of the rail to the bottom of the cylinder, shown in Figure 4.15, is 15.5 ft (4.7 m). Here, we define the bottom of the cylinder as the lower point of intersection between the cylindrical surface and the inclined, inside surface of the end cap. Thus, according to Figure 4.15, the bottom of the cylinder is 3 ft (0.9 m) above the top surface of the rotor car.

Using the above dimensions, the height above ground of the mid-point of the rotor is:

$$h_m = \frac{125}{2} + 15.5 + 4 = 82 \text{ ft (25 m)}$$

The design wind duration curve is that used by Brulle³⁹ in his Giromill study. This curve, which has a mean wind speed, $\bar{V} = 18 \text{ mph (8.1 m/s)}$ at a height 29.5 ft (9.0 m) above ground, was specified by ERDA for use in the design of the MOD-1 WTG. This design curve was specified for only a 9-meter height, whereas the center of the proposed Madaras rotor design is 25 meters above ground. Thus, the wind duration curve corresponding to the 25-meter height was ratioed from the basic design curve using the conventional method:

$$V_2 = V_1 \left[\frac{h_1}{h_2} \right]^{0.167} \quad (8)$$

Where wind speeds V_2 and V_1 are functions of t hours, and the subscripts 1 and 2 refer, respectively, to the original 9-meter design curve and the modified 25 m design curve. The modified design curve, having a $\bar{V} = 9.6 \text{ m/s}$ at 25 m, also is presented in Figure 6.13.

The third wind duration curve presented in Figure 6.13 is that corresponding to average wind conditions at a 25-meter height at Medicine Bow, Wyoming. These data were obtained from the paper by Hightower and Watts contained in the Proceedings of the Third Wind Energy Workshop.⁴³ The Medicine Bow data are presented because that location would be an ideal candidate site for a Madaras plant.

6.3.2 Determination of Net Plant Power Output

The net power output as a function of wind speed was computed for about 100 differently-sized Madaras plants using the data developed in Paragraphs 6.1 and 6.2 in the following equation:

Figure 6.13. Modified Design Wind Duration Curve to Represent Wind Conditions at a Rotor Mid-Height of 25 m (82 ft) Above Mean Terrain Level.

$$P_V = (p_1 f_n N)_V$$

where P_V is the net plant output for wind speed V ,

p_1 is the net power output for one car not corrected for mutual interference effects,

f_n is the mutual interference correction factor for p_1 , which varies with the spacing between cars and with wind speed, and track speed, and

N is the total number of cars in a Madaras plant.

Values of p_1 were obtained as a function of V_w and V_t for all combinations of the following conditions:

- Rotor configuration described at beginning of Paragraph 6.3.1
- Track diameter = 1372 m (diameter of circle and of racetrack ends)
- Rotor speed = 186 rpm
- Track speed = 8.9 and 13.4 m/s
- Wind speeds varying from 3 m/s to 13.4 m/s
- Circular track as well as racetracks having straight sections varying from 610 m to 18,300 m.

The appropriate mutual interference factors were obtained from Figure 6.4 for the above rotor and operating conditions, and for rotor spacings varying from 215 meters to 540 m. Since specific mutual interference factors correspond to a particular combination of V_w , V_t , and b (rotor spacing), our computation of net power as a function of wind speed does not assume that power varies as a function of V_w^3 , as is usually assumed for most WTG computations.

This assumption does not hold for the Madaras system (and probably does not hold strictly for some other wind turbine systems) because the mutual interference and other system losses cause a power variation different from a V_w^3 ratio variation. Figure 6.14 is indicative of the variation of net power output vs wind speed for two, large, racetrack-configured

Madaras plants. Both of these plants are of the same physical size and have the same number of cars. The only difference in capacity is caused by differences in track speed. Notice that a lower cut-in speed and a lower rated capacity is realized as a result of reducing track speed. Thus, Plant 1a would be more effective than Plant 1 in a region having a wind duration curve with a lower mean wind speed. Both plants are designed to operate up to a cut-out speed of 29.1 m/s (65 mph). Constant power output in the region from 13.4 m/s to 29.1 m/s is maintained by means of reducing rotor rpm to "spill the wind". This practice is comparable to varying blade angle on horizontal axis WTG.

In computing the data for the net plant power output vs wind speed curve, the assumption was made that the mutual interference factor f_n for a given wind speed, track speed, and inter-rotor spacing would be the same for both a circular track and a racetrack having the same end diameters. Actually, the f_n factors were derived from Professor Larsen's vortex analysis for a circular track. No attempt was made to revise the vortex analysis to consider a racetrack because both the time required to revise the programs and computer run times would be prohibitive.

After studying the interference problem, we believe that our assumption probably will yield smaller power values than one would compute from a more rigorous analysis, and hence, the end results of the study should be conservative. The vortex analysis for the circular track includes effects of the interaction of the vortices shed from each car in the train on every other car in the train. Thus, all combinations of the vortices shed from N cars are considered. However, when a racetrack pattern is adopted, mutual interference should be reduced because the rotors on the average are more widely separated than those in a circular pattern. The rotors on one semi-circular end are much farther from the rotors on the other

semi-circular end than they would be if the track were circular. Also, all of the rotors on the straight sections are at the maximum upwind-downwind distance that rotor cars would reach in a circular pattern. Since for large plants, many more of the cars would be on the straight sections than on the curved sections, it is reasonable to expect interference would be, at most, equal to that for a circular track and probably would be less. Finally, in comparing the 1372-m separation distance proposed for this Madaras study with the separation proposed for the 3.9 MW MOD-2 WTG, which has a 107 m diameter blade, the Madaras separation is greater. A separation distance of 10 diameters in a triangular (HEX)" arrangement was proposed for "parks" of these large machines in the Lockheed study.⁴⁴

On the other hand, the upwind-downwind spacing between rotors on the straight track sections of the proposed Madaras plant would be 36 rotor heights, and the minimum lateral spacing would be 44 rotor cylinder diameters (or 22 end cap diameters). Therefore, we believe the rotors on the straight track sections are realistically spaced and that our method for computing mutual interference for the present analysis of a racetrack-configured Madaras plant will not cause an overstatement of the results.

6.3.3 Development of Power Duration Curves and Energy Output

Power duration curves like those presented in Figure 6.15 were developed in the usual manner by cross-plotting the data from Figures 6.13 and 6.14, and the annual amount of energy generated was obtained by integrating each curve. The characteristics of the two plants shown are the same as those described for Figure 6.14, the power-versus wind speed curves. In addition, we found that Plant 1 will generate 343 million kW-hr of energy annually: about 6.7 percent more energy than that of Plant 1a, whereas the load factor for Plant 1a is about 56 percent compared to 49 percent for Plant 1. Since both plants would cost

Figure 6.14. Typical Net Power versus V_w Curve for Two Racetrack-Configured Plants.

Figure 6.15. Power Duration Curves for the Two Plants Shown in Figure 6.14 Based on the $\bar{V} = 9.6$ m/s Design Wind Duration Curve at 25-m Height (Figure 6.13).

the same (the only difference being that Plant 1 is operated at a 13.4 m/s track speed compared to an 8.9 m/s track speed for Plant 1a), it appears that Plant 1 would provide the lowest energy cost. These are only two of the 97 plant configurations studied. Our comparison of the power output and annual energy production of all plants analyzed is presented in Section VII, which follows.

SECTION VII

ECONOMIC ANALYSIS

A specific geometric configuration and operating condition for a single rotor and our method for combining these rotors into complete power plants were discussed in Section VI. The purpose of this section of the report is to present the methods used to compute the cost of a Madaras plant as a function of plant size variables, to present the results of the cost trade studies that were computed to investigate the effects of land cost and learning curves on plant efficiency, and to compare the plant configuration that appears to be the most efficient Madaras plant with conventional horizontal axis wind turbine plants. Finally, a discussion of the feasibility of the Madaras concept will be presented.

7.1 COST ESTIMATING PROCEDURE

Our basic approach to cost estimating was to develop unit costs for the variable items which affected plant size and energy capacity; to combine these items so that a reasonable estimate of Madaras plant costs will be obtained; and to present the costs in a form that would facilitate comparison with conventional Horizontal Axis Wind Turbine Generator (HA-WTG) plant.

The basic approach to cost estimating was to develop unit costs for a Madaras plant and then to ratio the unit costs by appropriate size factors to obtain costs of various sizes of Madaras plants. The following criteria were used to develop the unit cost data:

- Wind duration curve specified by ERDA for the large HA-WTG (Figure 6.13) up-rated to the appropriate height for the Madaras Rotor.
- Rotor having an aspect ratio of 8, an e/d ratio of 2, a 4.9-m diameter by 39.2-m height, a design midheight of 25 m, a rotor rotation speed of 186 rpm, a 450-kW spin motor incorporating viscous breaking at the 245° point on the orbit.

- Overall structural and electrical design details as specified in Section IV and V.
- Detailed plant and mechanical design layout as specified in the drawings included as Figures 4.15 to 4.23, inclusive.
- Initial plant size as shown in Figures 4.22 and 4.23 having a race-track pattern, 457 m diameter ends, 610 m straight sections, and 28 rotor cars.

Basic cost estimating work was done by M.L. McClellan and Company, a professional engineering firm which specializes in estimating cost of large plants, building construction, and machinery fabrication. In addition to using the bases for cost estimating outlined above, quotations were obtained from vendors for off-the-shelf items, manufacturers quotations were obtained for fabricated items, and supplemental estimates were developed based on standardized construction costs and pertinent standardized material-weight-fabrication cost data.

In developing his cost data, McClellan used the following major estimating assumptions:

- Basis of cost was 1978 prices.
- No architectural or engineering costs included.
- No transmission line costs beyond boundary.
- No railroad siding available--- all freight deliverable by available roads to the plant.
- Plant roadway construction included.
- Site reasonably level and not heavily wooded.
- One extra rotor car included as a spare.
- Cost for two viaducts under track to permit access to insure area in center is readily accessible for agriculture.
- Cost of constructing a permanent fabrication-assembly-maintenance building and a plant control building included.
- No learning curve utilized.
- Time of construction estimated to be 36 months.

The detailed cost estimate developed for this program is presented in Appendix B. This information is summarized in Table 7.1 by four major cost categories in the unit cost modular form used for the cost-performance trade studies which will be described in Paragraph 7.2. The cost of purchasing land is not included in Table 7.1 in order to permit comparison of Madaras results with HA-WTG studies which do not include land. This approach also will permit the flexibility of adding any land cost deemed appropriate by the reader. We believe land cost must be included when considering wind turbine "farms" or "parks", and hence we will address this subject in Paragraph 7.3.

The generalized plant layout and the equations used to relate plant geometry, performance, and cost are presented in Figure 7.1. The generalized cost equation:

$$C = a_1 N + a_2 A_n + a_3 T + a_4 \quad (9)$$

utilizes the equations of Figure 7.1 and the a_n constants developed and listed in Table 7.1. Other inputs required for this computation were the $p_1(S, V_w, V_t)$ values of power/car excluding interference losses and the $f_n(b, V_w, V_t)$ interference loss factor data. These data were obtained from the computer runs described in Paragraph 6.2 and 6.1, respectively. Of course, these p_1 and f_n values depend on other variables, but since we have fixed rotor geometry, rotor rpm, and track diameter, the number of variables affecting the values of p_1 and f_n is reduced.

The final cost computation, not indicated in Figure 7.1 is that of determining the average cost of energy delivered by a given plant. This computation was done in the usual manner by integrating the power duration curve for a given plant (Figure 6.15) to obtain the kW-hr/yr obtained from a plant; computing annual costs of plant operation; and then computing the cost per kW-hr.

Annual costs for the basic 28-car plant depicted in Figures 4.22 and 4.23 were divided into categories of annual fixed charges and operating and maintenance costs. These costs, which

TABLE 7.1
SUMMARY OF UNIT COST DATA⁽¹⁾

		<u>Unit Cost</u>
I.	COST/ROTOR CAR	
●	Rotor Car Direct Cost, 28 cars	\$18,850,420
(2)●	Indirect Mechanical Cost at 22.7%	4,276,400
●	Construction Financing at 11.7% of Direct Costs	<u>2,205,500</u>
	Total Cost	\$25,332,320
	Cost/Car:	$a_1 = \underline{\underline{\$904,730}}$
II.	COST SITE PREPARATION	
●	Based on Net Land Area of 70 Acres (Net Acreage excludes land enclosed by race track.)	
●	Total Site Preparation Direct Cost	\$219,500
●	Indirect Construction Cost at 19.1%	41,810
●	Construction Financing at 11.7% of Direct Costs	<u>25,680</u>
	Total Cost	\$286,990
	Cost/Net Acre:	$a_2 = \underline{\underline{\$ 4,100}}$
III.	COST UTILITIES, TRACK, TROLLEYS	
●	Total Track Length = 8,712 ft	
●	Total Direct Cost	\$ 10,649,530
●	Indirect Construction at 19.1%	2,028,630
●	Construction Financing at 11.7% of Direct Cost	<u>1,246,000</u>
	Total Cost	\$ 13,924,160
	Cost/ft of Track:	$a_3 = \underline{\underline{\$ 1,600}}$
IV.	FIXED COSTS	
●	Direct Cost Buildings and Viaduct	\$ 2,374,900
●	Indirect Construction at 19.1%	452,400
●	Direct Cost Power and Control System plus 1 Spare Rotor Car	1,090,280
●	Indirect Mechanical at 22.7%	247,340
●	Subtotal Direct Cost	\$3,465,180
●	Construction Financing at 11.7% of Direct Cost	<u>405,420</u>
	Total Fixed Cost	$a_4 = \underline{\underline{\$4,570,340}}$

GRAND TOTAL PLANT COST \$44,113,810

(1) From Appendix B

(2) Indirect cost percentage values of 22.7 and 19.1 are rounded values; numerical cost values are exact and are in agreement with Appendix B.

1. $A = L \times W, \text{ m}^2$
2. $A_n = [L \times W - \frac{\pi}{4} (D-22)^2 - K_b (D-22)], \text{ m}^2$
3. $L = D + 5 + 190.9, \text{ m}$
4. $W = D + 96.4, \text{ m}$
5. $T = \pi D + 2 K_b, \text{ m}$
6. $S = K_b, \text{ m}$ K is number of cars on straight track
7. $N_c =$ Number of cars on circular part of track.
8. $b = D \sin \frac{360}{2N_c}$, inter-car spacing based on chordal distance on circle, m .
9. $N = N_c + 2K$
10. $\text{Cost} = a_1 N + a_2 A_n + a_3 T + a_4$, Dollars
11. $P_R = p_1 N f$: p_1 is power for 1 rotor $= p_1 (S, V_w, V_t)$
 f_n is interference loss factor $= f_n(b, V_w, V_t)$.
12. $\text{Cost/kW} = \frac{\text{Cost}}{P_R} = \text{Installed Cost/Rated Power Capacity}$

Figure 7.1. Generalized Geometric Plant Layout and Equations for Computing Plant Cost, Plant Output, and Installed Cost Rated Plant Power Capacity.

are itemized in Table 7.2, were estimated to be about 16.5 percent for operation and maintenance costs. Thus, in the cost trade studies to be presented in Paragraph 7.2, the cost of energy generated by a Madaras plant will be based upon an annual cost of 16.5 percent of the original plant cost.

7.2 COST PERFORMANCE/ANALYSIS

Having selected the rotor size, track diameter, and certain operating conditions, the remaining variables and their ranges which were considered in the plant cost performance analysis were:

- Track configuration: circle or racetrack
- Track speed: 8.9 m/s (20 mph) 11.2 m/s (25 mph), and 13.4 m/s (30 mph)
- Number of cars: 8 to 190
- Inter-Car, Spacing: 44 to 108 rotor diameters [4.9 m (16 ft) diameter]
- Length of straight track section: 0-18,300 m.

A total of 97 different plants representing 97 combinations of the above variables were selected as a representative sample of the many possible plant combinations for our first analysis. This set of plants consisted of three basic groupings:

- 22 plants having circular tracks
- 28 plants having racetracks with rotor-cars traveling at 8.9 m/s track speeds
- 47 plants having racetracks with rotor-cars traveling at 13.4 m/s track speeds.

All plants were designed in terms of a 13.4 m/s (30 mph) rated wind speed and the up-rated ERDA $\bar{v} = 8.1$ m/s (18 mph) wind duration curve shown in Figure 6.13.

7.2.1 Unit Cost of Circular Track-Plant Configurations

These initial computations indicated that circular track plant configurations are noncompetitive with horizontal axis wind turbines. The circular-track-plant study utilized a track speed of 11.2 m/s because earlier computations indicated optimum power occurred at this speed. Results of this study are presented in

TABLE 7.2
COMPUTATION OF MADARAS PLANT ANNUAL COSTS
(Based upon 28-Car Plant)

I. ANNUAL CHARGES

● Mortgage payments: \$44,123,370, 30 years at 8.75% per annum	\$4,171,700
● Depreciation: \$44,123,370, at 30 years	1,470,800
● Real Estate Taxes	455,500
● Income Taxes	300,000
Annual charges = 14.5% Plant Cost	<u>\$6,398,000</u>

II. OPERATING AND MAINTENANCE COSTS

● Wages	
● Secretary	\$13,000
● Station Manager	50,000
● Outside Labor	50,000
● Electricians (2)	50,000
● Millwrights (2)	50,000
● Labor Insurance and Taxes	30,000
	<u>\$243,000</u>
● Utilities-Heat and Light	
● Control Building	3,000
● Assembly Building	9,000
	<u>\$ 12,000</u>
● Expendable Parts	
● Bearings (616)* 60 at \$3500=	\$21,000
● Wheels (224) 8 at 3000=	24,000
● Reducers (140) 4 at 1500=	60,000
● Lube Oil Pumps (56) 2 at 1500 =	3,000
● Couplings (392) 16 at 450 =	7,200
● DC Brushes (56) 56 at 600 =	33,600
● Contactors (504) 504 at 80 =	40,320
● Trolley Shoes (84) 84 at 300 =	25,200
● Collector Rails (13,180) 1200 at 40=	<u>48,000</u>
	\$262,320
● Miscellaneous,	115,000
● Insurance	250,000
Annual O&M Cost = 2% of Plant Cost	<u>\$882,320</u>
(Total Annual Cost = 16.5% Plant Cost	<u>\$7,280,320</u>

*Number in (xxx) is total number of items in a 28-car Plant.

+Note that in addition to these parts, 1 complete spare rctor car is included in the Plant cost of \$44,123,370.

Figure 7.2. In addition to analyzing the standard 1372-m (4500 ft) diameter track, a 1524-m (5000 ft) diameter track was analyzed to determine if a larger track diameter would improve performance. However, although the larger diameter track improved rated plant output slightly, it still appears that use of a circular track will not be economical. Thus, it can be seen from Figure 7.2 that the plant having the lowest unit cost is one having 20 rotors on either a 1372-m or a 1524-m diameter track. The specific data pertaining to these two least expensive plants studied is presented in Table 7.3.

TABLE 7.3
COST AND PERFORMANCE OF A 20-ROTOR MADARAS
PLANT HAVING CIRCULAR TRACKS
(Land Cost not Included)

V_w m/s	V_t m/s	Track Diameter m	Rated Power (MW)	Cost 10^6 \$	Unit Cost \$/kW
13.4	11.2	1524	8.31	48.25	5806
13.4	11.2	1372	7.85	45.65	5815

From Figure 7.2 one can see that as the number of cars increases, the diameter has decreasing effect on unit cost and that the increased rated power output advantage of the larger plant is modified by its increasing cost. After including the cost of land, the larger plant would undoubtedly be less attractive than the smaller plant.

One can see in Figure 7.2 that our study did not identify the minimum cost circular plant; however, the 20-car plant was rapidly approaching the minimum, which probably occurs at about 22 rotors, and minimum cost is probably not less than \$5600/kW. Limitations in time did not permit running higher than 20-car configurations with the vortex analysis computer program.

This result is quite different from that predicted by Madaras. For a similar number of cars having the same area (but

Figure 7.2. Plant Cost versus Number of Rotors for Madaras Plants having Circular Track Configurations. Rated Power in Megawatts Indicted for Each Plant. Plants Having 1372-m and 1524-m Diameter Tracks were Studied.

poorer aerodynamic efficiency) he predicted plant output of over 20 MW at 13.4 m/s wind speeds, and he specified only a 457-m diameter track for this plant. The primary differences between the results of our analysis and those of Madaras are:

- Madaras did not realize the magnitude of spin-motor power losses caused by motor inefficiency, rotor inertia, and viscous friction. He predicted he needed only a 37 kW motor to turn his 27.4 m high by 7.8-m diameter rotor compared to the 450 kW motor we have specified for our rotor.
- Madaras did not account for mutual interference between cylinders.

The motor loss is severe, and is the only one of the two losses that can be reduced by engineering; thus, if a more efficient means for rotating the cylinder could be developed, a circular track could again be attractive. At the rated wind speed level, mutual interference losses on the 1372-m track were of the order of 10 percent, but motor losses of over 50 percent would be expected. Conversely, at lower wind speed levels, the mutual interference loss exceeds motor losses. It will be shown later that the plant energy output was not sufficient to overcome the high-first-cost of the Madaras circular plant, and hence the cost of energy also would be expected to be excessive.

Finally, since the unit cost of the circular Madaras plant is more than a factor of three greater than the predicted cost of the MOD-1 WTG's, the Madaras circular plant concept was eliminated from further consideration. The elimination of this circular concept is unfortunate in that the plant design cannot take advantage of the inherent wind direction independence of the circular plant.

7.2.2 Unit Cost of Racetrack Plant Configuration

Our original performance computations indicated that best performance (without regard for interference losses) for a racetrack plant would be obtained at a track speed of 13.4 m/s. However, after studying the mutual interference data, we determined that cut-in speeds for a given rotor spacing decreased and wind plant cost increased as track speed decreased.

Therefore, plants utilizing two track speeds were studied in order to determine whether or not the lower cut-in speeds would enable a plant to increase its annual energy yield.

Results of our unit cost analyses of 56 of the racetrack plant concepts are presented in Figures 7.3 and 7.4, which represent, respectively, plants operating at track speeds of 8.9 m/s (20 mph) and 13.4 m/sec (30 mph). Each of these curves shows the relationship between the number of rotors, inter-rotor spacing, and the length of the straight section of the track. Identical plant configurations were analyzed at both track speeds. Thus, a comparison of the effect of track speed can be made by comparing a given set of coordinates in Figure 7.3 with its counterpart set in Figure 7.4.

The data trends were as expected, with unit plant cost decreasing as N (number of rotors) and S (length of straight section) increase and as b (inter-rotor spacing) decreases. Thus, from a unit plant cost standpoint, it is better to provide a very long racetrack and maximize the number of cars on the track. Also, by comparing Figures 7.3 and 7.4, it is evident that unit plant cost increases as track speed decreases from 13.4 m/s to 8.9 m/s.

The effect of variables S , b , and N on cost and rated plant power is shown in Figure 7.5. As expected, rated power increases markedly as N and S increase, and one can see that the effectiveness of each rotor decreases as b increases. Additional improvement in plant performance can be achieved by decreasing b and increasing N and S ; however, reasonably good design seems to have been achieved for plants having rotor spacing of $44d$ and at combinations of $S > 3000$ m and $N > 76$. Of the three variables, the least benefit in increasing rated power would be achieved by decreasing b , and the most benefit would occur from adding track and rotors at the maximum density studied. Since the cost of 1 rotor is equivalent to 173 m (565 ft) of track, a break-even trade point would seem to occur when $b = 173$ m or 35 rotor diameters ($35d$). This distance is equivalent to the spacing of

Figure 7.3. Unit Plant Cost versus Length of Straight Section of Racetrack Plant Configuration as a Function of Number of Cars and Inter-Car Spacing. Track Speed: 8.9 m/s (20 mph).

Figure 7.4. Unit Plant Cost versus Length of Straight Section of Racetrack Plant Configuration as a Function of Number of Cars and Inter-Car Spacing.
 Track Speed: 13.4 m/s (30 mph).

Figure 7.5. Unit Plant Cost versus Rated Power for Racetrack Configuration as a Function of Inter-Rotor Spacing, Length of Straight Section of Track, and Number of Rotors. DOE Design Wind Duration Curve: $\bar{V} = 8.1\text{ m/s}$ @ 9 m.

25 rotors around a 1372-m diameter track. If more rotors can be added to a given amount of track without incurring excessive interference losses, a more economical design should be achieved. However, it appears that a 25-d spacing would be counter productive.

As noted in Figures 7.3, 7.4, and 7.5, all calculations up to this point have been based upon the assumption that the wind is perpendicular to the track. We will address the effect of nonoptimum wind direction on plant performance later in this section of the report.

After studying the various plants represented in Figures 7.3, 7.4, and 7.5, it appeared that those operating at a track speed of 13.4 m/s (Figure 7.4) were superior to those operating at the 8.9 m/s speed. Thus, only the best of the 8.9 m/s plants ($N = 76$, $S = 20,000$, $b = 44d$, Unit Cost = \$2389/kW) was retained for the energy cost study.

All of the 13.4 m/s racetrack plants covering the range of operating conditions included in Figure 7.5 and the best plant using a circular track configuration were retained for the energy cost analysis.

7.2.3 Energy Cost

The energy cost was computed from all plant configurations studied utilizing the usual equation:

$$\text{Cost/kW-hr} = \frac{\text{Plant Cost} \times \text{Annual Cost}}{\text{Annual Energy Output}}$$

where the annual cost was 16.5 percent of plant cost (developed in Table 7.2) and the annual energy output in kW-hr/year was obtained by integrating the power duration curve for each plant, typical of those shown in Figure 6.15.

Four sets of energy cost data were computed; one set for each of the following conditions:

1. Circular track plant, track diameter of 1372m, track speed of 8.9 m/s and up-rated $\bar{V} = 8.1$ m/s at 9m wind duration curve.

2. Racetrack plant, track diameter of 1372 m, track speed of 8.9 m/s and up-rated $\bar{V} = 8.1$ m/s duration curve.
3. Racetrack plant, track diameter of 1372 m, track speed of 13.4 m/s, and up-rated $\bar{V} = 8.1$ m/s wind duration curve.
4. Racetrack plant, track diameter of 1372 m, track speed of 13.4 m/s and up-rated $\bar{V} = 9.7$ m/s wind duration curve (Medicine Bow, Wyoming).

7.2.3.1 Racetrack Configurations

The results of the third set of these computations are presented in Figure 7.6. These energy cost data correspond to the same 47 plants used to compute the plant cost/kW data presented in Figure 7.5. Although the characteristics of the curves in Figures 7.5 and 7.6 are similar, there are two major differences in the cost/kW-hr data in Figure 7.6: (1) a definite minimum cost point as a function of straight track distance, S , was realized; and (2) the effect of inter-rotor spacing is much more pronounced in Figure 7.6. Actually, both of these differences are influenced by inter-car spacing, b .

In Figure 7.6, one can see a major drop in energy cost for a given value of rated power as b decreases from 108 rotor diameters ($108d$, where $d = 4.9$ m) to $b = 87d$. From that point on, the effect of decreasing b is less pronounced. However, between $b = 87d$ and $b = 44d$, a minimum energy cost is realized. Thus, for a plant defined by a given value of rated plant power, the cost of energy is slightly higher for an inter-rotor spacing of $44d$ than the minimum energy cost which occurs at a larger value of rotor spacing. This trend can be seen in Figure 7.6 by following the b -values at a constant rated power level of 110 MW (i.e. energy cost decreases as b progresses from $108d$ to $87d$, to $73d$, and to $63d$. From that point on, energy cost increases as b decreases to its minimum value of $44d$.

This cost/performance trend is caused by: (1) an increase in mutual interference and losses and hence a decrease in annual energy yield of a plant having a $44d$ rotor spacing compared to a plant having a $63d$ spacing, and (2)

Figure 7.6. Energy Cost versus Rated Power for Racetrack Configuration as a Function of Inter-Rotor Spacing, Length of Straight Section of Track, and Number of Rotors. DOE Design Wind Duration Curve: $\bar{V} = 8.1\text{ m/s}$ @ 9 m.

an increase in plant cost (and hence increased operating cost) of a 44d plant over that of a 63d plant having the same value of rated power. Thus, since the annual energy of a 63d plant is greater than that of a 44d plant, and since the annual cost of operating a 63d plant is less, the energy cost of the 63d plant was smaller than that of the 44d plant. Note also that the optimum value of $\underline{b}$ at which the trend reversal occurs decreases as P_R increases.

The other interesting result of this study is that for all plant sizes studied, the minimum cost of energy for a given value of S is obtained for those plants having inter-rotor spacing of 49d. We did not plot the $b=49d$ curve in order to improve reading clarity in this region where the b -curves are compressed due to the previously-mentioned trend reversal. Thus, a rotor spacing of 49d will provide the minimum cost of energy for a given value of S , a mean wind speed of 8.1 m/s, and no consideration of learning curves.

Another minimum energy cost condition also was found: i.e., the optimum rotor spacing of the number of rotors, N , is fixed. This trend can be seen by following the trends of the $N = 50, 75, 100$, and 150 curves plotted in Figure 7.6. The energy cost of a plant having 75 rotors, for example, decreases rapidly as rotor spacing decreases, a minimum value is reached at some value of b , and then the energy cost increases as rotor spacing is further compacted. This effect is, of course, caused by rotor interference. Unlike the locus of energy cost minima for various values of S , the optimum rotor spacing varies with number of cars. Thus, for plants limited to 50, 75, 100, and 150 rotors, the inter-rotor spacing which will provide minimum energy cost would be 73d, 63d, 55d, and 49d, respectively. Of course, since varying the number of cars also increases S , and since power output increases as S increases, ($P_{\max}$ occurs at $S = \infty$) it would appear that minimum energy cost will be achieved, by increasing S to the maximum length practicable and then providing the number of rotors required to achieve the optimum inter-rotor spacing for the

selected track length. This approach suggests the definition of an optimum solidity factor, one of the objectives of this study; i.e., the definition of the minimum number of rotors that can produce the lowest energy cost in a given length of land perpendicular to the wind. For a racetrack plant, the length perpendicular to the wind is $S + D$, where D is the end diameter. Thus, the proportion of this length occupied by N rotors having diameter, d is:

$$\sigma = \frac{Nd}{S + D} ,$$

where σ is the solidity factor. Since the optimum value of S for N rotors can be approximated from Figure 7.6, the optimum values of σ and the associated values of N and S have been tabulated and plotted in Figure 7.7. This figure shows the marked economic advantage of larger plants and indicates that small decreases in energy cost may be obtained by increasing solidity for plants having values of $S < 18,300$ m (60,000 ft). Since Figure 7.6 also indicates that our computations have not been extended sufficiently far to obtain an absolute minimum, it may be advisable to investigate larger plant configurations. Of course the cost of land is not included in the computations at this point, so the effect of land cost on plant size could alter the size-efficiency relationship. This land factor will be evaluated in Paragraph 7.3.

7.2.3.2 Energy Cost Comparisons, Best Plant Combinations

The four different sets of plant design conditions analyzed in this program were itemized in Paragraph 7.2.3. These conditions provided the opportunity of comparing the energy cost of circular versus racetrack plants, the effect of track speed on energy cost for racetrack plants, and the effect of wind duration curves on energy cost ($\bar{V} = 8.1$ m/s vs $\bar{V} = 9.7$ m/s at Medicine Bow, Wyoming). The plants having the lowest energy cost and others which indicate the effect of the variables on plant economy are presented in Table 7.4.

Figure 7.7. Optimum Rotor Solidity Factor versus Length of Straight Track Section.

TABLE 7.4

COMPARISON OF PLANT PERFORMANCE AND COST - CIRCULAR AND RACETRACK CONFIGURATIONS

Plant No.	Track End	V_t	b	N	S	A_n	P_R	Annual Energy	Plant Factor	Cost	Unit Cost	Energy Cost
	Dia. m	m/s	Rotor Diameters	Nc. Rotors	m	Acre	MW	10^6 kW-hr/yr		10^6 \$	\$/kW	¢/kW-hr
BEST CIRCULAR PLANT - $\bar{V} = 8.1$ m/s												
8a	1372	8.9	44	20	Circle	213	7.85	32.76	0.48	45.65	5815	22.99
RACETRACK PLANT - TRACK SPEED EFFECT, $\bar{V} = 8.1$ m/s												
1	1372	13.4	44	76	6034	390	79.75	340.36	0.49	160.97	2018	7.80
1a	1372	8.9	44	76	6034	390	66.45	321.82	0.55	160.97	2422	8.25
BASIC RACETRACK PLANT - $\bar{V} = 8.1$ m/s												
49-10	1372	13.4	49	44	13,112	304	41.37	180.95	0.50	100.99	2441	9.21
49-20	1372	13.4	49	69	6106	392	74.95	330.22	0.50	155.39	2073	7.76
49-30	1372	13.4	49	95	9219	483	109.82	484.80	0.50	211.98	1930	7.21
49-40	1372	13.4	49	120	12,212	569	143.41	640.08	0.51	266.38	1857	6.86
49-50	1372	13.4	49	146	15,325	661	178.28	788.33	0.51	322.97	1812	6.76
49-60	1372	13.4	49	170	18,198	745	210.59	931.25	0.51	375.20	1781	6.65
44-60	1372	13.4	44	190	18,318	748	227.77	975.43	0.49	394.57	1732	6.67
MEDICINE BOW SEA LEVEL PLANTS (MBSL) - $\bar{V} = 9.7$ m/s (4)												
49-60M	1372	13.4	49	170	18,198	745	210.59	1103.35	0.60	375.20	1781	5.61
44-20M	1372	13.4	44	76	6034	390	79.75	408.52	0.58	160.97	2018	6.50
44-45M	1372	13.4	44	148	13,792	616	173.29	889.78	0.59	308.50	1780	5.72
44-50M	1372	13.4	44	190	18,318	748	227.72	1170.17	0.59	394.57	1732	5.56

BASES OF COST

- (1) Cost of Land Not Included
- (2) Cost of First Item --- No Learning Curves
- (3) Annual Charge of 16.5%
- (4) Medicine Bow, Wyoming Wind Duration Curve with Sea Level Density to show only effect of Wind Duration Curve Duration.

The superiority of the racetrack over the circular-track plant is striking. The circular plant selected was that found to have the highest performance in Figure 7.2. Although our studies indicated that a slightly lower cost/kW could be obtained by using more closely-spaced rotors, and Figure 6.11 indicates that a small amount of additional power might be obtained from a larger diameter circle, we believe that Plant 8a, the most efficient circular plant studied, typifies the best performance one might expect from a circular plant. Since cost of energy from this plant is grossly noncompetitive with that from horizontal axis wind machines and about twice as expensive as the cost of energy from the most inefficient Madaras racetrack plant (see Figure 7.6), we again concluded that this plant type was not worthy of further consideration.

The effect of track speed on plant economics can be determined by comparing Plants 1 and 1a. This is a comparison of the effect of track speed, V_t , on the operating characteristics of one plant. The only difference in Plants 1 and 1a is the track speed. The effect of track speed on cost/kW was presented in Figures 7.3 and 7.4, and there it was concluded that the decreased track speed decreased the performance of all plant sizes studied, and hence increased the cost/kW of each "low speed" plant. However, since the power duration curves of the "high speed" and "low speed" plants (e.g., Plants 1 and 1a) were significantly different in both the low and high wind speed regions (see Figure 6.15), we continued our study to determine if the lower track speed and the resulting lower cut-in wind speed of Plant 1a would produce more energy annually than Plant 1. Table 7.4 indicates the reverse is true, and that Plant 1 not only has a higher power rating, but it also will produce energy at lower cost. Therefore, since Plant 1a was the best "low-speed" plant studied, it was concluded that, for the design wind duration curve used for this study, the 13.4 m/s (30 mph) track speed would provide lower cost energy for any plant configuration. However, for lower mean-speed wind duration curves, a lower track speed may be advisable.

One interesting possibility should be investigated in any future analyses of the Madaras system: consider the use of a variable gear ratio drive from the rotor-car wheels to the generators in order to permit operating at two or more track speeds. Refer to Figure 6.15. Here, one can see the cross-over of the power duration curves of Plants 1 and 1a at a wind speed of about 9.8 m/s (22 mph). If a variable-speed drive were utilized, one could shift into the high gear ratio and operate at $V_t = 8.9$ m/s (20 mph) when winds were in the range between 6.5 m/s (14.5 mph) and 9.8 m/s. Then, as the 9.8 m/s speed was approached, the gear ratio would be reduced and track speed increased to 13.4 m/s (30 mph). In this way, the plant could benefit from "the better of two worlds" and add the area between the two curves below 9.8 m/s to its annual energy output. If the added energy offsets the added cost of the variable speed drive, the cost trade would be attractive.

The racetrack plants having the lowest energy cost of those studied are listed in the third grouping of plants. As can be seen, each of Plants 49-10 through 49-60 represents the optimum configuration for a given value of S obtained from Figure 7.6. As noted before, each optimum plant utilized rotor spacing of 49 diameters. In addition, this grouping includes the largest of all 47 plants studied: Plant 44-60 which has a rated capacity of over 227 MW and will produce 975 million kW-hr/yr at an energy cost of 6.67¢/kW-hr (.002¢/kW-hr higher than Plant 49-60). The data in Table 7.4 indicates that plants having values of $S < 3112$ m (10,000 ft) probably will not be cost effective even when mass-produced components are used. Also, a significant drop in energy cost occurs up to Plant 49-30, and from there on the performance improvement is slower. There is not much difference in energy cost for Plants 49-50, 49-60, and 44-60. The economy of scale effect is quite evident from the ever-decreasing unit cost (\$/kW) of the installed plant.

It also is particularly interesting to note that Plants 49-60 and 44-60 are quite competitive with the

cost and energy cost of a single MOD-1 wind turbine not wired into a plant. This comparison will be made later after appropriate application of similar ground rules and conditions.

The last group of plants listed in Table 7.4 were operated in a region having a mean wind speed of 9.7 m/s and sea level density. Since this wind duration curve is like that at Medicine Bow, Wyoming, one could think of these as Medicine Bow Sea Level (MBSL) plants. These are presented to show the effect on plant performance of varying only the wind duration curve. The effect of air density at Medicine Bow, Wyoming, on performance will be shown in Paragraph 7.5. All 47 basic plants were run for this condition.

The first observation of importance is that for the higher wind duration curve, optimum performance for a given value of S was obtained from plants having inter-rotor spacing of 44d instead of 49d. This can be seen by comparing Plant 49-60M with Plant 44-60M. The marked increase in annual yield by Plant 44-60M more than offsets the added cost of added rotor cars and the decreased efficiency of reducing inter-rotor spacing. Since cost and rated power was the same for the Medicine Bow plants and the basic plants, the installed unit plant costs remained unchanged. The effect of the higher wind duration curve is also seen in the increase of about 20 percent in plant factor for the MBSL plants and in the 15.6 percent reduction in energy cost of Plant 49-60M over Plant 49-60.

The particular MBSL plants listed in Table 7.4 were selected to illustrate some of the design trades available with the increased wind duration curve. We already have mentioned the ability to decrease rotor spacing to obtain optimum energy cost. Another interesting comparison is Plant 44-45M with 49-60. Here are two plants of different rated capacity having the same installed cost/kW. Yet, the MBSL plant, though 22 percent smaller in rated capacity, will produce energy at a cost that is 14 percent lower than that from the larger plant. A similar result is obtained when comparing Plant 44-20M with Plant 49-60. Here are two plants having approximately equal energy

costs, and yet this same performance by the MBSL Plant can be achieved with a straight track section of only 6034 m (20,000 ft) compared to a length of 18,198 m (60,000 ft) for Plant 49-60. Thus for MBSL winds, only about one-half the land and only 43 percent of the plant cost are required to produce energy at about the same cost as that in the lower wind region. Of course, the basic plant's rated capacity is almost three times as large as the Medicine Bow plant.

From plant comparisons in Table 7.4, we conclude that the most efficient basic plant is Plant 49-60, which has a rated capacity of 210 MW, an installed unit cost of \$1781/kW, an annual energy yield of 931 million kW-hr, and an energy cost of 6.65¢/kW-hr. The largest basic plant is Plant 44-60, which has 227.77 MW rated capacity, an installed unit cost of \$1732/kW, an annual yield of 975 million kW-hr, and an energy cost of 6.67¢/kW-hr. These two plants along with their MBSL counterparts were considered the best Madaras plants of those analyzed. However, all plants listed in Table 7.4 will be considered in the analyses that follow in order to assure that the effects of land cost, learning curves, and off-axis wind conditions will not alter the basic Madaras plant economic trends that have already been noted.

We have no doubt, that given sufficient space and high wind conditions, even larger Madaras plants than Plant 44-60 could be built. Further, we again want to emphasize that the plant designs in Table 7.4 do not represent an overall optimized system design, but instead represent good conceptual designs. Hence, given the opportunity to conduct a more extensive design analysis, we believe basic system efficiency could be improved over that stated in Table 7.4.

7.3 EFFECT OF LAND COST AND LEARNING CURVES ON ENERGY COST

All computations of power and energy cost made previously have included the cost of site preparation, grading, and general earthwork. However, the cost of purchasing or leasing the land has not been included. In addition, cost estimates have not taken into account the cost reduction resulting from mass production: a factor that must be considered before the Madaras system can be compared with modern horizontal axis WTG's.

This section of the report will address both of these cost factors.

7.3.1 Land Cost Effect

Three approaches to land acquisition and management were considered:

- Long term land lease.
- Purchase of all land required both inside and outside the racetrack in conjunction with leasing the inside track area for agricultural use.
- Purchase of only a right-of-way to a tract of land; i.e., the A_n defined in Figure 7.1 as that area outside the track plus land occupied by the track and trolleys. The land inside the track then would remain the property of the original land owner to use for agriculture or other noninterfering use. Two viaducts, one at each end, would be built of sufficient size to permit access to the inner area.

After consulting with personnel at the Dayton Power & Light Company, as well as a local realtor, it was recommended that the third approach be taken: purchase outright only the net area. The Dayton Power & Light Company has used this approach successfully many times. This concept is the same as that prepared by Lockheed for their Wind Energy Mission Analysis,⁴⁴ which suggested that large wind turbines be spaced 10 to 15 diameters apart in a hexagonal pattern that only 15 acres around each wind turbine would be purchased, and that the rest of the land would be retained by the original owner and used for agriculture. Thus, large "parks" of wind turbines would be developed, and only 6.4 percent of the

land required for such a "park" would have to be purchased.⁴⁴
(This study did not include the land required for access roads.)

Since the area inside the racetrack is completely open, free of obstructions, and can be very large (6500 acres for the largest plant), a large-scale farmer would consider farming this tract of land as an attractive business venture.

Consequently, for Madaras plants, one would purchase only that are designated A_n in Table 7.4 for each of the plants being considered. This amounts to the purchase of from 745 to 748 acres for the two largest plants being considered for this study. This land would provide ample area for the erection of the buildings, a power substation, a fenced-in, clear perimeter area, as well as space for the circumferential service road, the track, the power distribution system, and the wind velocity sensing system.

Our land cost analysis was designed to bracket a wide range of agricultural land costs that might be prevalent in areas which would be suitable for Madaras plants: i.e., a large open expanse of land with essentially unidirectional winds. Generally the Great Plains and certain Texas Gulf Coastal areas are typical of the sites one might select. It is questionable whether attractive land quality, wind strength and duration, and wind direction combinations would be found in the North East, the Midwest, or most of the Cornbelt States where land prices are at premium levels. Thus, we selected the following four unit land price levels which we believe will encompass the geographical areas of interest to Madaras plants. These unit land costs were: \$500/acre; \$1000/acre; \$1500/acre; and \$3000/acre. These costs were input to our cost analysis program, and the total energy costs were computed again using an annual cost rate of 16.5 percent of plant cost (including land) to obtain energy costs.

These computations were made for all 47 racetrack plants, both for the basic and Medicine Bow wind duration curves. No changes in trends noted in Table 7.4 were observed, and again

the plants which were best for the basic analysis were also the best where land costs were included.

The effect of land costs on these best plants is shown in Table 7.5. For the planned land purchase approach these data indicate that land cost has an insignificant effect on plant cost and energy cost for large Madaras plants. Both the plant cost (\$/kW) and the energy cost (¢/kW-hr) had a maximum increase of about 0.6 percent. This cost increase was essentially the same for all of the plants in Table 7.5. For the smallest Madaras plants studied (one with 20 rotors and a 3050 m straight track section) the maximum increase due to land cost was about 1.1 percent. Therefore, it appears that land cost will have little effect on Madaras plant performance even at \$3000/acre land costs. However, when learning curves are applied to the cost analysis, land costs become more prominent.

7.3.2 Learning Curve Effect

In preparing the cost estimate for the basic plant, no provision for learning curves or mass production were included. Since each of the large plants we are contemplating requires laying about 268,000 ft (81,800 m) of nonconventional track rails and the building of 170-190 rotor cars, it is readily apparent that a learning-curve-type of cost reduction will be realized in building even one plant. This cost reduction will be even greater if more than one plant is built in a given production lot.

Learning curves are based on the concept that each time production is doubled, the average cost of $2n$ production items is a specified fraction of the average cost of n items. Thus, the ratio of the average cost of $2n$ items to the average cost of n items defines the learning curve. Thus, for an 85 percent learning curve, the average cost of two would be 85 percent of the first item cost, and the average cost of the four items would be .85 percent of the average cost of two items [i.e., $(.85)^2$ times the cost of the first item]; etc. Thus:

TABLE 7.5
EFFECT OF LAND COST ON PLANT AND ENERGY COST

Plant No.		49-60	44-60	49-60M*	44-60M*
Number Rotors		170	190	170	190
A _n Acres		745	748	745	748
P _R MW		210.59	227.77	210.59	227.77
Energy Output 10 ⁶ kW-hr/yr		931.25	975.43	1103.35	1170.17
No Land Cost	\$/kW	1781	1732	1781	1732
	¢/kW-hr	6.648	6.674	5.611	5.564
\$500 Acre	\$/kW	1783	1734	1783	1734
	¢/kW-hr	6.654	6.681	5.616	5.569
\$1000 Acre	\$/kW	1785	1736	1785	1736
	¢/kW-hr	6.661	6.687	5.622	5.574
\$1500 Acre	\$/kW	1787	1737	1787	1737
	¢/kW-hr	6.668	6.693	5.628	5.579
\$3000 Acre	\$/kW	1792	1742	1792	1742
	¢/kW-hr	6.687	6.712	5.644	5.595
Max Increase+	\$/kW	0.62%	0.58%	0.62%	0.58%
	¢/kW-hr	0.59%	0.57%	0.59%	0.56%

* Plant Designation Indicates Size

49 = Rotor Spacing = 49d = 784 ft (239 m)

60 = Straight Track Length = 60,000 ft (18,300 m)

No Letter Following = Basic Wind Duration Curve

M = Medicine Bow Wind Duration Curve (MBSL), Sea Level Density
d = 16 ft (4.9 m)

+ % Increase for \$3000/acre land over no land cost

$$K_n = n^e \quad (10)$$

$$C_n = [(n)^{1+e} - (n-1)^{1+e}] \quad (11)$$

where

$$b = \frac{Q_m LC}{Q_m 2} \quad (12)$$

LC is the learning curve ratio (e.g., 0.85)

K_n is the average cost of n units of the cost of constructing only one item

C_n is the cost of the n th item of the cost of constructing only one item

These equations were obtained from Hightower, who used them for his HA-WTG plant design in Reference 45.

Thus, in order to use the learning cost reduction concept, it was first necessary to define learning curves for various parts of the plant and then to define the corresponding production units to which the curves apply.

Our initial cost equation (Equation (9) Paragraph 7.1) contains four terms: (1) cost of rotor cars, (2) cost of general site preparation, (3) cost of track, utilities, and roads, and (4) the cost of the assembly and maintenance building, the control building, the power switching and conditioning equipment, and the control equipment. The fourth term also included the spare rotor, which will be handled differently in the new equation that will be developed.

After analyzing the site preparation and miscellaneous components, items (3) and (4), we concluded that this technology would be conventional and that skilled earth moving and construction personnel would already be high on the learning curve. Therefore, for our analysis, we assumed that components (3) and (4) (less the spare rotor) would not benefit from learning or mass production, and hence no learning curves were applied to these cost components.

On the other hand, we believe that it is definitely appropriate to apply learning curves to the rotor cars and the track laying. Different approaches were taken to develop appropriate learning curves for each of these components.

The manufacture and assembly of the rotor cars is quite similar to the manufacture of large transport aircraft. The rotor is a semi-monocoque structure designed for high strength-weight ratio and fabricated from aluminum and other light-weight parts. Although the rotor support tower car and wheels are very massive, all use materials and manufacturing techniques like those used for aircraft. In addition, each rotor and car is equipped with electronic controls, electrical equipment, gearing, and bearings, just as an aircraft. Thus, since the manufacture of rotor cars does not have a manufacturing cost experience base of its own, we elected to utilize an 85 percent learning curve for the rotor car, based on learning curve experience of aircraft.

The selection of a learning curve for the track laying was not as straightforward. First, as can be seen in Table 7.1, we have included a number of other items in the track category: i.e., track, trolleys, drains, and other utilities, power collection and distribution, and the service road. These were combined with the track because they ran parallel to the track, and hence increased in size and cost in direct proportion to the length of the track. These types of work represent a mixed group of technologies, some of which are well developed, and some (like the track road bed and trolleys) are highly non-conventional. Thus it was assumed that a 90 percent learning curve would be a reasonable average learning curve for the track, et al.

The next step was to develop an appropriate unit of production for the track system. One could consider a unit of production to be the laying of one 6-m (20 ft) length of rail, or the building of a 305-m (1000 ft) segment of roadways. However, another method served to be more convenient: that is

to develop a track, roadbed, etc., unit equal to that length of track associated with each rotor car. Thus, since there are N rotors in a given plant with a track perimeter, T , there is one length of track and roadbed, t , equal to T/N that is added each time one rotor car is added to increase the size of a plant. Thus, for an N -rotor plant, there are N track units, each having a length t . This unit length, t , is not constant for all plants but increases as inter-rotor spacing increases. Thus, an N -rotor plant having rotor spacing of $49d$ will have a larger value of t than that of an N -rotor plant having rotor spacing of $44d$. In each case, however, the number of production units would be the same as the number of rotors. (Since all of our cost calculations were carried out for the $49d$ and $44d$ spacing, the effect on the cost of the track resulting from the different size t is less than 2 percent.)

Equation (9), our basic cost equation, had to be modified slightly to accommodate the learning curve terms as well as to provide a better basis for comparing Madaras plants with horizontal axis WTG plants. Our initial equation scaled the number of rotors, the amount of site preparation, and the length of track directly as these items varied from one plant size to another. The fourth item, which was held constant, included the cost of buildings, power conditioning and switching equipment, system control equipment, and one complete, spare rotor car. Although for the original study it was recognized that the power conditioning and switching equipment would increase with plant size, the cost of these items was found to be small relative to the complete plant cost. However, for the learning curve study, it was decided that a better cost estimate would be obtained if the power conditioning and switching equipment were scaled with plant size and if the cost of the spare rotor [which is the $(N+1)$ th rotor car purchased for a given plant] were developed as a part of the learning curve experience of the total rotor purchase. Thus, two new terms were added to Equation (9) such that:

(12)

$$\text{Plant Cost} = a_1(N+1)K_{N+1} + a_2A_n + a_3NtK_t + a_5P_R + a_6. \quad (12)$$

In Equation (12), the constants a_1 , a_2 , and a_3 were assumed to be the identical unit cost rates defined in Table 7.1, and:

$$a_5 = \frac{\text{Cost of Power Factor and Switching Equipment for 28 MW Plant}}{28 \text{ MW}}$$

$$= \$6120/\text{MW}$$

$$P_R = \text{Rated power of the scaled plant in MW}$$

$$a_6 = \text{Cost of buildings, viaducts, control systems, which is assumed constant for all plant sizes studied} = \$3,497,610.$$

Further, in defining terms of Equation (12):

N is the number of rotors in one plant

t is the length of each track unit in one plant = T/N

T = total track perimeter length

K_N is the average cost of N rotor units expressed as a fraction of the cost of the first unit and is a function of a given learning curve (85 percent in this case).

K_T is the average cost of R track units expressed as a fraction of the cost of the first unit and is a function of a given learning curve (90 percent in this case).

Equation (12) was used to compute plant costs and energy costs for 14 plants thought to be of the most interest for this study. Plants considered in this study included all of those in Table 7.5, some of those in Table 7.4, and others that would be of interest in comparing the effect of rotor

spacing and in comparing performance of Madaras plants with HA-WTG plants. These computations were made for the following conditions:

- Number of Plants: 1, 2, 3, 10, 100, 500
- Learning Curves: 85% for rotors, 90% for track units.
- Annual Cost: 16.5% Plant Cost
- Land Cost/Acre: 0, \$500, \$1000, \$1500, and \$3000/acre.

By adding the land cost to the a unit cost constant, we were able to include the land purchase cost, as described previously in Paragraph 7.3.1.

The results of the learning curve computation for all of these plants are presented in Tables 7.6 and Table 7.7, for plants sited at mean wind speeds of 8.1 m/s and 9.7 m/s, respectively.

As was expected, the learning curve reduced plant and energy costs dramatically. In addition, the learning curve effect caused a shift in the optimum plant configuration from 49-60 to 44-60. As can be seen, this trend was present for all of the plant sizes studied, and is not just a peculiarity of the largest plant. This shift in optimal configuration occurs because the plants having the smaller rotor spacing require more rotors, and hence are able to capitalize more on the learning curve cost reduction. This same trend is also noted for the Medicine Bow Plants in Table 7.7.

The results from Plant 8a, the best circular plant, still indicated that circular plants were not cost effective, even for 500 plant production lots sited at MBSL regions (the most favorable condition). The lowest energy cost obtainable was 5.17 ¢/kW-hr, and minimum installed plant cost was \$1539/kW, which is completely beyond the area of economic interest. The low annual energy output, the high electrical losses, and small plant size combine to cause poor performance of circular plants.

TABLE 7.6

EFFECT OF LEARNING CURVE AND LAND COST ON ENERGY COST
FOR PLANTS SITED WHERE $\bar{V} = 8.1$ m/s AT 9-m HEIGHT

Plant No.	P_R	Annual Output	$A_n^{(1)}$	No. Plants	No. (2) Rotor	Unit (3) Cost	Rotors - 85% LC; Track -90% LC				
							ϵ /kW-hr (4)				
							Land Cost, \$/Acre				
-	MW	10^6 kW-hr	Acres	-	Units	\$/kW	0	500	1000	1500	3000
49-25	92.4	407	437	1	82	923	3.45	3.46	3.47	3.48	3.50
				10	820	632	2.37	2.37	2.38	2.39	2.41
				100	8200	444	1.66	1.67	1.68	1.69	1.71
49-50	178.3	788	661	1	146	755	2.82	2.82	2.83	2.84	2.85
				10	1460	515	1.92	1.93	1.94	1.94	1.96
				100	14600	359	1.34	1.35	1.35	1.36	1.37
49-60	210.6	931	745	1	170	722	2.69	2.70	2.71	2.71	2.73
				10	1700	492	1.84	1.84	1.85	1.86	1.88
				100	17000	342	1.28	1.28	1.29	1.30	1.32
44-25	99.2	424	437	1	91	873	3.37	3.38	3.39	3.39	3.42
				2	182	776	3.00	3.00	3.01	3.02	3.05
				3	273	725	2.80	2.81	2.82	2.82	2.85
				10	910	596	2.30	2.31	2.32	2.33	2.35
				100	9100	417	1.61	1.62	1.63	1.63	1.66
44-50	191.4	820	660	1	162	713	2.75	2.76	2.76	2.77	2.79
				2	324	634	2.44	2.45	2.46	2.46	2.48
				3	486	592	2.28	2.29	2.29	2.30	2.32
				10	1620	485	1.87	1.88	1.88	1.89	1.90
				100	16200	337	1.30	1.30	1.31	1.32	1.34
44-60	227.8	975	748	1	190	681	2.62	2.63	2.64	2.64	2.66
				2	380	605	2.33	2.34	2.34	2.35	2.37
				3	570	565	2.18	2.18	2.19	2.20	2.22
				10	1900	463	1.78	1.79	1.80	1.80	1.82
				100	19000	321	1.24	1.24	1.25	1.25	1.27
8a	78.5	33	213	1	20	3575	14.13	14.19	14.24	14.30	14.46
				10	200	2519	9.95	10.00	10.01	10.01	10.28
				100	2000	1858	7.35	7.40	7.45	7.51	7.67
				500	10000	1539	6.09	6.14	6.19	6.25	6.41

(1) Net area purchased.

(2) Number of Track Units = Number of Rotor Units. Number shown corresponds to number of plants.

(3) Installed cost per plant. Does not include land cost.

(4) Based on 16.5% of Plant Cost = Annual Cost.

TABLE 7.7
EFFECT OF LEARNING CURVE AND LAND COST ON ENERGY COST
FOR PLANTS SITED WHERE $\bar{V} = 9.7$ m/s AT 9-m HEIGHT
(MEDICINE BOW SEA LEVEL PLANTS)

Plant No.	P_R	Annual Output 10^6 kW-hr	$A_n^{(1)}$ Acres	No. Plants ⁽²⁾	No. Rotors ⁽³⁾	Unit Cost \$/kW	Rotors -85% LC; Track -90% LC				
							ϵ /kW-hr (4) (5)				
							Land Cost, \$/Acre				
	MW			-	Units		0	500	1000	1500	3000
49-25M	92.4	483	437	1	82	923	2.91	2.92	2.93	2.94	2.96
				10	820	632	2.00	2.00	2.01	2.02	2.04
				100	8200	444	1.40	1.41	1.42	1.42	1.45
49-50M	178.3	934	661	1	146	755	2.38	2.38	2.39	2.39	2.41
				10	1460	515	1.62	1.63	1.63	1.64	1.66
				100	14600	359	1.13	1.14	1.14	1.15	1.16
49-60M	210.6	1103	745	1	170	722	2.27	2.28	2.28	2.29	2.31
				10	1700	492	1.55	1.56	1.56	1.57	1.58
				100	17000	342	1.08	1.08	1.09	1.09	1.11
44-25M	99.2	509	437	1	91	873	2.81	2.81	2.82	2.83	2.85
				2	182	776	2.50	2.50	2.51	2.52	2.54
				3	273	725	2.33	2.34	2.35	2.35	2.38
				10	910	596	1.92	1.92	1.93	1.94	1.96
				100	9100	417	1.34	1.35	1.36	1.36	1.38
44-50M	191.4	985	660	1	162	713	2.29	2.29	2.30	2.30	2.32
				2	324	634	2.03	2.04	2.04	2.05	2.07
				3	486	592	1.89	1.90	1.91	1.91	1.93
				10	1620	485	1.56	1.56	1.57	1.57	1.59
				100	16200	337	1.08	1.08	1.09	1.10	1.11
44-60M	227.8	1170	748	1	190	681	2.19	2.19	2.20	2.20	2.22
				2	380	605	1.94	1.95	1.95	1.96	1.97
				3	570	565	1.81	1.82	1.83	1.83	1.85
				10	1900	463	1.49	1.49	1.50	1.50	1.52
				100	19000	321	1.03	1.04	1.04	1.05	1.06
8aM	78.5	38	213	1	20	3575	12.06	12.11	12.15	12.20	12.33
				10	200	2519	8.50	8.54	8.59	8.63	8.77
				100	2000	1858	6.27	6.31	6.36	6.41	6.54
				500	10000	1539	5.19	5.24	5.28	5.33	5.47

- (1) Net area purchased.
- (2) Number of Track Units = Number of Rotor Units. Number corresponds to number of plants.
- (3) Installed cost per plant. Does not include land cost.
- (4) Based on 16.5% of Plant Cost = Annual Cost.
- (5) Does not include engineering design cost or contingencies; 3 year construction period.

The 100-plant data are interesting, but whether or not the 85 percent learning curve cost reduction could continue for as many as 17,000 to 19,000 units needs to be determined. In Reference 44, Lockheed estimated that their minimum cost would be reached at 10,000 units. Therefore, until further analysis can be conducted, the validity of the 100-plant cost figures certainly should be considered with caution. We do believe that the cost reduction shown for 10 plant lots is reasonable, and the energy cost of 1.78 to 1.82 ¢/kW-hr for ten of the 44-60 type Madaras plants is certainly in the economic range of interest. The cost range of from 1.49 ¢/kW-hr to 1.52¢/kW-hr for ten 44-60M (MBSL) plants is even more attractive.

7.4 EFFECT OF NONOPTIMUM WIND DIRECTION

The most versatile form of a Madaras plant is one having a circular track pattern because its performance is independent of wind directions. However, we demonstrated above that for several reasons, this plant type is completely beyond economic consideration, even if 500 plant units were to be built. We also indicated in Table 7.7 that large racetrack-type plants appear to be economically attractive. Unfortunately, racetrack plants have one important weakness: they must be located at a site that has essentially unidirectional wind (including the reciprocal direction) to perform at full potential.

To illustrate the sensitivity of a racetrack Madaras plant to nonoptimum wind direction, the following example is given. To simplify the illustrations it was assumed that X percent of the time during the year, the wind will be blowing at a given nonoptimum angle $\pm\beta$ measured from a perpendicular to the straight track (Figure 7.8). For the remainder of the year [(1-X) of a year's time] the wind will be blowing normal to the straight track, i.e., at an angle $\beta = 0$. Then, the total amount of energy generated during the year, E_T , would be:

Figure 7.8. Effect of Nonoptimum Wind Duration on Cost of Madaras Racetrack Plant 49-60.

$$E_T = X(E)_{\beta \neq 0^\circ} + (1-X)(E)_{\beta=0^\circ} \quad (13)$$

where $X(E)_{\beta \neq 0^\circ}$ is the total energy generated during X percent of the year when the wind vector is not normal to the track at an angle $\pm\beta$

$(1-X)(E)_{\beta=0^\circ}$ is the total energy generated during the balance of the year when $\beta=0^\circ$; i.e., the wind vector is normal to the track.

Then, since average annual energy cost is inversely proportional to the total annual energy output, energy costs then increase by a factor, F, caused by partial off-optimum wind direction during a year relative to energy cost for wind direction during an entire year is:

$$\frac{(E)_{\beta=0^\circ}}{X(E)_{\beta=0^\circ} + (1-X)(E)_{\beta=0^\circ}} \quad (14)$$

where $(E)_{\beta=0^\circ}$ is the annual energy output when $\beta=0^\circ$ for an entire year.

Typical trends for this simplified case are presented for $\beta = 0^\circ, \pm 15^\circ, \pm 30^\circ$, and $\pm 45^\circ$ for Plant 49-60 in Figure 7.8. The extreme at $X = 0$ represents optimum plant operation, and hence the cost increase factor would be 1.0 and the other extreme at $X = 100$ percent represents the condition in which the wind would blow all year at an angle where $\beta \neq 0^\circ$.

The sensitivity to large deviations of the wind from the optimum is quite evident from Figure 7.8. Small angular deviations of the wind vector are probably acceptable, as are large deviations for a small portion of a year (e.g., 20 percent of the year at $\beta = \pm 45^\circ$ would increase cost of energy by only 14 percent). However, large deviations of β from the optimum can affect energy cost substantially. In fact, although not shown in Figure 7.8,

for plants having straight track sections as long as Plant 49-60 (18,300 m), if β becomes as large as $\pm 60^\circ$, and as the time that this wind direction is maintained approaches one year, the plant energy output will approach zero and the cost will, of course, approach infinity.

Another factor, not shown in Figure 7.8 is of interest: as the straight track length, S , decreases the plant becomes less sensitive to β (the angle β increases for a given value of F and X). As an example, if S is decreased from 18,300 m to 6100 m, the value of β for $F = 1.55$, $X = 100$ would be larger than $\pm 30^\circ$. Thus, if a racetrack Madaras plant is considered for a location at which small variations in wind direction occur, an optimum plant size would be smaller than that of Plant 49-60, and the energy cost would be higher.

The example plotted in Figure 7.8 is an example of the average annual energy cost increase factor utilizing only two wind directions. The energy cost factor or the total energy for an actual site can be obtained by a double integration over wind direction and speed. Because of the sensitivity of this type of plant to both wind direction and speed, it will be necessary for one to conduct a site wind survey using instruments which provide detailed annual wind direction data at the site in increments much more precise than those obtained from usual weather stations. Values of β in increments of $\pm 5^\circ$ would be appropriate.

Fortunately, there are a number of essentially unidirectional wind sites in the United States that are suitable for a Madaras racetrack plant. The caution we want to offer here is that one must select and survey the site with care to assure that the plant is optimally oriented and sized.

7.5 COMPARISON OF MADARAS PLANTS WITH HORIZONTAL AXIS WIND TURBINE GENERATOR (HA-WTG) PLANTS

The purpose of this portion of the report is to compare the cost and performance characteristics of the Madaras concept

and conventional HA-WTG plants in order to determine whether or not the Madaras concept shows promise of producing energy at lower cost than that from HA-WTG plants.

Since our Madaras cost estimates include provisions for interconnecting the output of a number of rotors into a complete plant as well as provisions for roads, utilities, buildings and other plant facilities, it is not meaningful to compare Madaras plant performance with that of a single HA-WTG, or with a number of isolated HA-WTG's required to provide an amount of power equal to that of a Madaras plant.

Mr. S.T. Hightower and A.W. Watts of the Bureau of Reclamation, U.S. Department of the Interior, Denver, Colorado, have developed a design for a 49-unit array of MOD-1 HA-WTG's to be installed next to a hydroelectric facility in Medicine Bow, Wyoming.⁴⁵ We selected this plant for comparison with the Madaras plant for several reasons:

- A complete plant design has been conducted, which included all components of a complete wind-powered electrical power plant.
- Up-rated, MOD-1 WTG's proposed for the plant, have had the benefit of considerable design and production study, and hence are representative of soon-to-be available large HA-WTG's.
- Medicine Bow's geographic and unidirectional wind characteristics are ideal for Madaras plants.

Our comparison of Madaras and HA-WTG plants is divided into two parts: (1) Medicine Bow siting ($\bar{V} = 9.7$ m/s at 9 m height, an air density ratio of 0.81 corresponding to the 7000 ft (2134 m) height of the Medicine Bow plant site above sea level), and Federal financing; and (2) siting at a location having mean wind speeds of 8.1 m/s at 9 m height, sea level air density, and private financing. Since the cost information for the Medicine Bow HA-WTG plant is readily available, it will be used as a basis for comparison for both conditions studied.

The primary ground rules for the comparison are as follows:

- Cost estimates represent cost at the plant boundary. No substation or transmission line costs are included.
- Costs based on 1978 dollars.
- Five-year construction period, with construction interest at 7 percent.
- Thirty-year plant life.
- Land cost at Medicine Bow is \$200/acre⁴⁵ and at other sites, land cost was assumed to be \$3000/acre.
- Only that land required for the operation of the plant will be purchased. Thus, for Madaras plants the net land area, A_n (Reference Figure 7.1) would be purchased, and for the HA-WTG plant a 457-m diameter circle of land would be purchased. This size was recommended by Hightower in a telephone conversation.
- Capital fixed charge rate equal to 8.41 percent (excluding O&M) in accordance with Federal financing procedures⁴⁶ for Medicine Bow plants. Fixed charges of 15 percent will be used for plants privately financed in accordance with a personal communication with the Economic Planning section of the Detroit Edison Company.
- Construction and O and M costs in accordance with methods deemed most appropriate for each system.
- Identical learning curve equations are used for both systems.

7.5.1 Medicine Bow Plant Comparison

The WTG array planned for the Bureau's HA-WTG site consists of three parallel, staggered rows having 16, 17, and 16 turbines in each row, respectively. The turbines are spaced in an equilateral triangular array, with the sides of the triangle being 15 rotor diameters long (920 m), as recommended by the General Electric Company.

Each row of turbines is spaced about 795 m apart. Thus, the overall plant layout is about 14,700 m long (North-South perpendicular to the prevailing West wind) and about 1590 m wide. A total of 49, up-rated, 2 MW, MOD-1 HA-WTG's are proposed. A plant capacity of 98,000 kW and an annual output of 409.6 kW-hr per year are predicted for this plant. Hightower and Watts also

contemplated a second (and even a third) plant of the same size to be installed later should the need arise. We have selected two HA-WTG plants for this study. One having 49 HA-WTG's and one having 98 HA-WTG's. The Madaras plants most nearly comparable to these conventional plants are itemized below.

<u>Madaras Plant</u>	<u>HA-WTG Plant</u>
<ul style="list-style-type: none"> ● 44-25M 99,180 kW Rated 	49 WTG 98,000 kW Rated
<ul style="list-style-type: none"> ● 44-50M 191,430 kW Rated 	98 WTG 196,000 kW Rated.

In addition the performance of the largest Madaras plant studied (44-60M) will be itemized for reference purposes.

The Madaras cost estimates developed in Paragraph 7.3.2 were re-combined in order to facilitate as nearly as possible the line-by-line comparison with the Bureau of Reclamation plants. Further, these estimates were adjusted to meet the study ground rules and the criteria specified by Hightower. The methods used for both plant types are presented in Table 7.8 where the cost breakdown for Madaras Plant 48-25M and the 49-rotor Bureau of Reclamation plant are compared. The criteria for computing the various line items for the HA-WTG plant are listed where known. The bases for the Madaras cost estimates are in accordance with the information presented in Paragraph 7.1, 7.2, and 7.3 as well as the detailed cost breakdown presented in Appendix B.

All costs for the HA-WTG plants were obtained from Reference 45 and from further clarification provided by Hightower. Since his estimates were made in 1977, all HA-WTG costs were increased by 7.5 percent to reflect the inflation during the last year. This inflation rate was recommended by the Detroit Edison Company.

TABLE 7.8

INSTALLED COST BREAKDOWN OF MADARAS PLANT 44-25M AND 98,000 kW BUREAU OF RECLAMATION
PLANT PROPOSED FOR MEDICINE BOW, WYOMING

ITEM	MADARAS 44-25M -- (99,180 kW)	BUREAU OF RECLAMATION HA-WTG PLANT (98,000 kW)
	10 ⁶ Dollars (1978)	10 ⁶ Dollars (1978)
1. ROTORS: 91 ea (85% Learning Curve)	21.27	49 ea WTG = 55.52 ⁽¹⁾
2. TRACK: 91 ea, 707 ft long units (90% learning curve)	26.48	N/A
3. ELECTRICAL CONNECTION <ul style="list-style-type: none"> • Trolleys, Power Collection • Power Conditioning, Switching • Control Systems • Instrumentation TOTAL	6.63 0.44 0.29 <u>0.25</u> 7.61	6.85
4. SITE FACILITIES AND CONTINGENCIES a. Facilities <ul style="list-style-type: none"> • Utilities and Roads • Buildings and Viaducts • Site Preparation TOTAL b. Contingencies <ul style="list-style-type: none"> • 1 Spare Rotor Car • General Contingencies TOTAL {10% of all Costs Above}	6.24 2.37 <u>1.38</u> 9.99 0.18 <u>6.55</u> 6.73	15% WTG = 8.33 15% Elec.Conn. = <u>1.02</u> 9.35 Not Broken Out Included Above
5. DESIGN, MANAGEMENT, ENGINEERING, OVERHEAD (22.7% Mechanical, 19.1% Construction, 2.5% Direct Costs for Design)	14.87	15% WTG = 8.33 30% Elect.Conn. = 30% (6.85+1.02) = <u>2.37</u> 10.70
6. LAND COST	437 Acres 0.09	1988 acres 0.40 ⁽²⁾
7. ALLOWANCE FOR FUNDS USED FOR LAND DURING CONSTRUCTION (AFUDC) (7% Compounded over 5 years)	0.03	0.16
8. CONSTRUCTION INTEREST (1/2 E Items 1,2,3,4,5) x 7% x 5 years)	14.67	14.42
9. TOTAL PLANT COST	<u>\$101.74</u>	<u>\$97.40</u>
10. RATED CAPACITY	99,180 kW	98,000 kW
11. INSTALLED CCST	\$1026/kW	\$994/kW

(1) Based on cost of \$1410.94/kW for the first production unit.

(2) Overall Madaras plant dimensions are 9213 m long by 1468 m wide; BofR plant dimensions are 14,700 m long by 1590 m wide.

The costs associated with the Madaras plant in Table 7.8 were computed in accordance with the data in Table 7.1 and Equation 12. However, since Hightower's cost breakdown elements were different from those in Table 7.1, some of the Madaras cost elements had to be subdivided in order to be directly comparable with Hightower's cost elements. Although Table 7.8 is fairly self-explanatory, some additional comments are in order. These will be presented in an item by item basis.

Item 1 - This item represents the direct cost of 91 rotors. From Table 7.1 the direct cost of 28 rotors is \$18,850,420, and the direct cost per rotor is \$673,229. Thus, the cost of 91 rotors based on an 85 percent learning curve is: $91 \times 673,229 \times 0.34727 = \21.27×10^6 , where $K_N = 0.34727$. The 22.7 percent indirect costs were included in Item 5 in Table 7.8, and construction interest was computed in accordance with methods used by Hightower.

Items 2 and 3 - The Madaras plant cost breakdown combined costs of track, roads, utilities, and instrumentation into one cost element because these were all associated with track length. However, for the HA-WTG plant, track was not necessary, and Hightower presented electrical connection and roads and utilities in separate cost elements.

Thus, the cost for utilities, track, and trolleys for the Madaras system from Table 7.1 and Equation 12 would normally be:

$$\text{Cost}_{u,t,t} = a_3 NtK_t$$

However, since only direct costs are to be compared at this point, a_3 from Table 7.1 was redefined as:

$$(a_3)_d = \frac{\$10,649,530}{8712} = \$1.222 \times 10^{-3}/\text{ft}$$

Thus, since the number of track units equals the number of rotor cars (91), and since t equals 707 ft and K_t for a 90 percent learning curve equals 0.50376 for 91 units, the cost of utilities, track, and trolleys would be $(a_3)_d NtK_t = \$39.6 \times 10^6$.

In order to develop cost elements comparable to those of Hightower, this cost of utilities, track, and trolleys was broken down further as follows:

PLANT 44-25M

<u>Element</u>	<u>% of Total</u>	<u>10⁶ Dollars</u>
a. Track	66.87	26.48
b. Trolleys and Power Collection	16.74	6.63
c. Instrumentation	0.63	0.25
d. Utilities and Roads	<u>15.76</u>	<u>6.24</u>
TOTAL	100.00	\$39.60 x

The percentage of each element in the above table was computed from the direct cost breakdown data in Appendix B, Section B.1, item D. Then, these percentages were used to compute the dollar value of each of the above elements for each plant size. The above example is for Plant 44-25M. In Table 7.8, Element a is entered in Item 2, Elements b and c are included in Item 3, and Element d is included under Item 4.

The remaining elements of Item 3 in Table 7.6 were obtained in the following manner.

- Power conditioning and switching cost - this was computed as shown in Equation 12 except that $(a_5)_d$ is used instead of a_5 in order to obtain only the direct cost per MW of power conditioning and switching equipment. The value $(a_5)_d$ was obtained from Appendix B, Section B.1, Item F. Thus:

$$(a_5)_d = \frac{\$29,000 + 96,000}{28 \text{ MW}} = \$4464/\text{MW}$$

Since $P_R = 99.18$ MW for Plant 44-25M, then the cost of this conditioning and switching equipment would be $\$0.44 \times 10^6$ as shown in Table 7.8. As before, the indirect cost and construction interest costs are included in items 5 and 8, respectively.

- Control Systems- This direct cost is of computer control equipment, a line item in Appendix B, Section B.1, Item F and is equal to $\$0.29 \times 10^6$ as shown in Table 7.6.

Item 4 - Site facilities and contingencies were included as one item in the Bureau of Reclamation study. These were estimated as a fixed percentage of WTG cost and electrical connection cost. The Madaras estimate for site facilities comes directly from our basic cost estimating procedure, and as a contingency, one complete, spare rotor car had been planned at the outset. In addition, we added a general contingency value of approximately 10 percent of all direct costs excluding land. Thus, a total contingency of \$6.73 million has been provided for Madaras plants in the interest of realism and conservatism. Thus, a total amount of \$16.72 million has been proposed for site facilities and contingencies of the Madaras plant, which we believe is a reasonable and conservative number to use in comparing the two plant types. One should recognize that the Madaras plant, while more complex, encompasses a smaller land area than the Bureau of Reclamation plant. Thus, the cost of electrical connection of WTG units dispersed in three 14,700-m-long rows would probably exceed similar costs of the Madaras system.

Item 5 - Management, field engineering and overhead was provided for in our basic cost estimate. However, since Hightower indicated his study included plant design layout, an amount of about 2.5 percent of direct construction and mechanical costs was added to our estimate to cover this expense in accordance with recommendations from our cost estimating consultant.

Item 6 - Land cost based on Medicine Bow estimates by Hightower were used for both plants. The estimate of 1988 acres for the WTG was derived from Hightower's recommendation stated in the ground rules. Thus, only that land indicated in Item 5 would be purchased; the balance would be used for agriculture. A primary advantage of the Madaras system in this case is that the unused area (the "infield" of a racetrack) would provide the owner with a 2904-acre, unobstructed area of land (about 87 percent of the original amount) which could be efficiently farmed. By way of contrast, not only does the HA-WTG plant require more land, but

only 66 percent of the involved area would be available from a "park"⁴⁴ of distributed HA-WTG's, and this land would be broken and criss-crossed by roads.

Item 7 - Allowance for Funds Used During Construction (AFUDC) - this item was computed in accordance with Hightower's analysis. The land would be purchased outright, and thus the funds would be unavailable to earn interest during the five-year construction period. Thus, a compound interest charge at 7 percent per year was charged. The Detroit Edison Company agreed that this rate was reasonable.

Item 8 - Construction Interest - this item also was computed in accordance with Hightower's approach. It was assumed that during construction funds would be committed gradually, and that the effect would be simple interest at 7 percent on half the total plant cost (excluding land, which was covered in Item 7) over the five year construction period.

The remainder of Table 7.8 is self explanatory. The cost of the two plants is nearly the same, with the Madaras plant cost being about 4.4 percent higher, and the cost/kW being about 3.2 percent higher. One also will note that low cost land at \$200/acre has negligible impact on plant cost.

The method used for computing annual costs for each type of plant based on Federal financing rates is shown in Table 7.9. This procedure proposed by Hightower and Watts for the Medicine Bow area, is in accordance with economic analyses used in Government hydroelectric projects.⁴⁶

The annual charges were divided into three areas: (1) capital recovery on land investment; (2) annual fixed charges; and (3) operation and maintenance costs.

The capital recovery on land investment is simply a levelized annual charge based on 7 percent compound interest over a 30 year plant life. This charge is based not only on the land cost but also on the charge for loss in interest on the money tied up in land purchase during construction (AFUDC - Item 7, Table 7.8).

TABLE 7.9

ANNUAL COST BREAKDOWN OF MADARAS PLANT 44-25M AND 98,000 kW BUREAU OF RECLAMATION
HA-WTG PLANT PROPOSED FOR MEDICINE BOW, WYOMING

ITEM	MADARAS PLANT 44-25M (99,180 kW)	BUREAU OF RECLAMATION HA-WTG PLANT
	10 ⁵ Dollars (1978)	(98,000 kW) 10 ⁶ Dollars 1978
1. CAPITAL RECOVERY ON LAND INVESTMENT 7 PERCENT COMPOUND INTEREST, LEVEL PAYMENT OVER 30 YEARS	CR = 0.0806 (0.087+0.035)= 0.01 (Includes land cost + AFUDC cost incurred during 5-year con- struction period).	Same Criteria 0.05
2. ANNUAL FIXED CHARGES, FEDERAL FINANCING.	<ul style="list-style-type: none"> Interest Rate 7.00% Depreciation (Sinking Fund) 1.06% Interim Replacement 0.35% 8.41% (Plant Cost Less Land Cost) 8.55	Same Criteria 8.14
3. OPERATION AND MAINTENANCE	O&M BASE <ul style="list-style-type: none"> Direct Costs (Items 1, 2, 3, 4a, Table 7.8) 65.35 Contingencies 6.73 Engineering and Overhead 14.87 Land Cost 0.09 TOTAL BASE 87.04 2% of O&M Base 1.74	O&M BASE <ul style="list-style-type: none"> WTG Cost @ Cost of 100th Unit Based on 85% Learning Curve 36.00 Electrical Connections 6.85 Land Cost 0.40 Site Facilities <ul style="list-style-type: none"> 15% x 36.00 (WTG) 5.40 15% x 6.85 (Elec. Conn.) 1.03 TOTAL BASE 49.68 2% of O&M Base 0.99
4. TOTAL ANNUAL COST	<u>10.30</u>	<u>9.18</u>
5. PERFORMANCE AND ENERGY COST		
<ul style="list-style-type: none"> Annual Energy Yield Plant Factor Energy Cost 	412.0 x 10 ⁶ kW-hr/yr 0.47 <u>25.0 mills/kW-hr</u>	409.6 x 10 ⁶ kW-hr/yr 0.48 <u>22.4 Mills/kW-hr</u>

*Medicine Bow Wind Duration Curve and Air Density Ratio of 0.81, Standard Atmosphere.

The annual fixed charge and its components is described fully in Table 7.9, and it is applied to total plant cost less land.

The basis used by Hightower and Watts for computing O&M is shown in Table 7.9. They assumed that O&M costs would be based on all 49 WTG's having a unit cost equal to the 100th item using an 85 percent learning curve. They also included land cost, electrical connections, and site facilities (also reduced from Table 7.8 values due to the reduction in WTG cost). Their estimate that O&M costs should be 2 percent of WTG costs was obtained from General Electric, and for conservatism, O&M costs should also include 2 percent of the other items listed.

Our previous estimates indicated that this approach to developing the base for O&M would not be meaningful for the Madaras system. Instead, we assumed that all direct, indirect, and land costs (less interest) were appropriate for an O&M base. Further, to account for unforeseen costs, the \$6.73 million contingency amount was included in the base. We also used a 2 percent factor for O&M relative to this base, in accordance with our basic cost study results. We believe this approach is proper, and certainly correct relative to the HA-WTG, which should not require as large an annual expenditure for O&M as the Madaras system.

The computation of the performance and energy cost completes Table 7.9. Here, the actual air density ratio of 0.81, corresponding to an altitude of 7000 ft (2134 m) above sea level, was used to compute the annual output and plant factor of both types of plants. Thus, the performance results in Table 7.9 represent the combined effects of the Medicine Bow wind duration curve and air density ratio, whereas results in Table 7.7 reflect the effect of the Medicine Bow wind duration curve alone. Since the annual cost of Madaras Plant 44-25M are about 12 percent higher and the annual yield is only slightly higher (0.6 percent) than those of the HA-WTG plant, the 25.0 mill/kW-hr energy cost

predicted for Plant 44-25M is about 12 percent higher than that of the HA-WTG plant. In view of this small difference in energy cost difference and the uncertainties involved in predicting the cost and performance of both the Madaras and the HA-WTG systems we believe that these results do not necessarily indicate superiority of one system over the other.

The data for the comparison of the three Madaras and two HA-WTG plants previously mentioned is presented in Table 7.10. Data pertaining to plant geometry, performance, and installed cost, annual cost, and energy cost are presented. The table is divided into three parts: (1) plant ID numbers 1 to 3 which pertain to Medicine Bow plants; (2) plant ID numbers 4 to 6 which pertain to the same plants designed for Medicine Bow but situated at sites where $\bar{V} = 8.1$ m/s; and (3) plant ID numbers 7 and 7a which are the same as Plants 5 and 5a but do not include land cost in their cost analysis. The first part of Table 7.10 will be discussed here. The other parts will be discussed in Paragraph 7.5.2.

In Table 7.10 all plants having only a numerical ID number are Madaras plants, and those with the letter a suffix are HA-WTG plants. Plants to be compared directly have the same numerical part of the ID number. Thus, Plant 1 and 1 a are to be compared, etc.

The methods used to compute the cost and energy cost of all of these plants were identical to those described in Tables 7.8 and 7.9. We already have compared Plants 1 and 1a (those having $P_R = 98$ MW) and noted that the Madaras plant energy cost was numerically about 12 percent higher than those of the HA-WTG plant having 49 wind turbines at Medicine Bow, Wyoming.

Plants 2 and 2a are twice the size as Plants 1 and 1a, with the rated capacity of Plant 2a being about 2 percent larger than Plant 2. Because of economy of scale, both plants provided energy at reduced cost over Plants 1 and 1a; however, the Madaras Plant 2 energy cost of 20.5 mills/kW-hr is only 4 percent higher than HA-WTG Plant 2a. Thus, as the size of the

TABLE 7.10
OVERALL COMPARISON OF SEVERAL MADARAS AND
HA-WTG PLANTS AT TWO WIND REGIONS

	PLANT ID NO.	PLANT DESIGNATION	PLANT GEOMETRY						PERFORMANCE					
			PLANT LENGTH m	PLANT WIDTH m	PLANT AREA Acre	NET AREA PURCHASED Acre	TRACK PERI-METER m	NUMBER ROTORS/ WTG	ROTOR/ WTG (1) SPACING m	$\bar{V}$ @ 9 m m/s	RATED CAPACITY MW	(7) ANNUAL OUTPUT 10 ⁶ kw-hr	PLANT FACTOR -	V_R @ 9 m m/s
MEDICINE BOW PLANTS	①	44-25M (4) 99.18MW	9213	1468	3341	437	19,610	91	215	9.7	99.18	412.0	0.47	13.4
	1a	HA-WTGM 98 MW	14,700	1590	5772	1988	N/A	49	920	9.7	98.0	409.6	0.48	11.2
	②	44-50M 191.43 MW	16,863	1468	6115	660	34,910	162	215	9.7	191.43	797.9	0.48	13.4
	2a	HA-WGTM 196 MW	14,700	3975	14,432	3976	N/A	98	920	9.7	196.0	819.2	0.49	11.2
	③	44-60M 227.77 MW	19,880	1468	7209	748	40,940	190	215	9.7	227.77	947.9	0.48	13.4
	3a	HA-WTGM 196 MW	14,700	3975	14,432	3976	N/A	98	920	9.7	196.0	819.2	0.49	11.2
PLANTS WHERE $\bar{V} = 8.1$ m/s	④	44-25 99.18 MW	9213	1468	3341	437	19,610	91	215	8.1	99.18	423.9	0.49	13.4
	4a	HA-WTG 98 MW	14,700	1590	5783	1988	N/A	49	920	8.1	98.0	362.6	0.42	11.2
	⑤	44-50 191.43 MW	16,863	1468	6115	660	34,910	162	215	8.1	191.43	819.5	0.49	13.4
	5a	HA-WTG 196 MW	14,700	3975	14,457	3976	N/A	98	920	8.1	196.0	725.2	0.42	11.2
	⑥	44-60 227.77	19,880	1468	7209	748	40,960	190	215	8.1	227.77	975.4	0.49	13.4
	6a	HA-WTG 196.0 MW	14,700	3975	14,457	3976	N/A	98	920	8.1	196.0	725.2	0.42	11.2
PLANTS WHERE $\bar{V} = 8.1$ m/s No Land Cost	⑦	44-50 191.43 No Land Cost	16,863	1468	6115	660	34,910	162	215	8.1	191.43	819.5	0.49	13.4
	7a	HA-WTG 196.0 MW No Land Cost	14,700	3975	14,457	3976	N/A	98	920	8.1	196.0	725.2	0.42	11.2

	PLANT ID NO.	ROTOR OR WTG	TRACK	INSTALLED COST						TOTAL PLANT COST \$KW	FIXED CHARGE (6)	ANNUAL AND ENERGY COST		
				ELEC. CONN.	SITE AND FACILITIES	CONTINGENCIES (2)	MGMT. ENGR. OVERHEAD	LAND COST (3)	CONST. INTEREST			O&M	TOTAL ANNUAL	ENERGY COST Mills KW-hr
MEDICINE BOW PLANTS	①	21.27	26.48	7.61	9.99	6.73	14.87	0.12	14.67	101.74	1026	8.56	1.74	25.0
	1a	55.52	N/A	6.85	9.35	(5)	10.70	0.56	14.42	97.40	994	8.19	0.99	22.4
	②	33.08	43.18	12.37	14.61	10.50	23.56	0.18	24.03	161.51	844	13.58	2.75	20.5
	2a	94.38	N/A	13.70	15.54	(5)	18.89	1.12	24.94	168.57	860	14.17	1.98	19.7
	③	37.38	49.44	14.15	16.35	11.90	26.76	0.21	27.29	183.48	806	15.43	3.12	19.6
	3a	194.38	N/A	15.54	16.35	(5)	26.76	1.12	27.29	183.48	806	15.43	3.12	19.6
PLANTS WHERE $\bar{V} = 8.1$ m/s	④	←	←	SAME AS ①	←	←	←	1.84	Same as ①	103.45	1043	15.52	Same as ①	40.7
	4a	←	←	SAME AS 1a	←	(5)	←	8.37	Same as 1a	100.21	1074	15.78	Same as 1a	46.2
	⑤	←	←	SAME AS ②	←	←	←	2.70	Same as ②	164.10	857	24.62	Same as ②	33.4
	5a	←	←	SAME AS 2a	←	←	←	16.73	Same as 2a	184.18	940	27.63	Same as 2a	40.8
	⑥	←	←	SAME AS ③	←	←	←	3.15	Same as ③	186.42	818	27.96	Same as ③	31.9
	6a	←	←	SAME AS 3a	←	←	←	3.15	Same as ③	186.42	818	27.96	Same as ③	31.9

- (1) WTG spaced in three rows, staggered, equilateral triangular array at 15 rotor diameter ($d = 61$ m)
- (2) Contingencies in HA-WTG plant inched in site and facilities figure.
- (3) Includes allowance for funds used during construction, due to outright purchase of land.
- (4) The suffix letter 'M' refers to Medicine Bow plants; all others for wind duration curve $\bar{V} = 8.1$ m/s.
- (5) Contingencies for HA-WTG plants are included in the electrical connection cost.
- (6) Fixed charges for Medicine Bow plants based on Federal financing at fixed charge of 8.41 percent. All others, fixed charge = 15 percent.
- (7) Based on air density ratio of 0.81—7000 ft(2134m) above msl.

plant doubled, the energy cost advantage of the HA-WTG plant dropped from 12 percent to 4 percent. One also can see that as the Madaras plant size increases further to 227.77 MW (Plant ID 3), the energy cost decreases to 19.6 mills/kW-hr. Since no limitation in Madaras plant size has been found, further reduction in energy cost seems possible.

Thus it appears from this comparison that Madaras plants show promise of producing energy at a cost nearly equal to and possibly lower than comparable HA-WTG plants.

The reasons for the promise of better economy of the Madaras plants are their nonlinear improvement of annual energy yield with size, their better ability to benefit from cost reduction through mass production, and their requirement for less land for a given rated plant capacity.

Madaras plants having racetrack plant layouts increase in performance and economy nonlinearly as the length of the straight section increases. This trend is shown in Figure 7.5. As the length of the straight track increases, the losses at the circular ends become a smaller and smaller part of the average power per rotor per orbit, and plant power increases nonlinearly as shown in the following equation:

$$P = p_1 f_n N$$

where

N is the number of rotors

p_1 is the net power output of a single rotor at a given wind and track speed,

f_n is the mutual interference factor.

The term p_1 increases to a maximum as the length of straight track approaches infinity. If car spacing is kept constant, then f_n does not change, and P is then a function of N and p_1 . Thus, as the straight track length increases at intervals which permit the addition of cars without affecting f_n , not

only will the power increase because of the increase in N , but also the power per car, P_1 , will increase. These two increases then will combine to increase not only rated power, but also annual energy output. Comparison of Plants 1 and 2 demonstrates this effect well. Although the straight track length of Plant 2 is twice that of Plant 1, the track perimeter and number of cars of Plant 2 are 78 percent greater than those of Plant 1; however, rated capacity and annual energy output of Plant 2 are 93 percent greater than their counterparts in Plant 1. Obviously, this effect decreases as track increases. On the other hand, HA-WTG plant output is strictly linear with number of rotors (provided they are not spaced too closely together), and the HA-WTG plant capacity and energy output cannot benefit from economy of scale from the performance standpoint as can the Madaras plant.

One of the major reasons for the cost improvement of the Madaras plants over that of the HA-WTG plants was the cost decrease resulting from application of learning curves. Although for these cost studies, only one plant of each type was planned, the largest number of Madaras rotors (91 in Plant 1 compared to 49 WTG's in Plant 1a) enabled the learning curve to work more toward the benefit of the Madaras plant. We believe that this learning curve cost improvement is valid for the number of units proposed for this study, and probably could improve much more if a major assembly line could be established to build rotor cars for a number of plants. However, the extent to which such improvement could continue will require further study as well as the location of a sufficient number of acceptable sites for building a number of Madaras plants. However, if many more HA-WTG plants than Madaras plants are ordered, a reversal of the trend could occur.

The final economic advantage of a Madaras plant over an HA-WTG plant is that a Madaras plant uses less land per MW of rated plant capacity, and the Madaras plant land use is more efficient and conducive to coexisting operations with agriculture (and even industry and/or commerce). This feature was mentioned before; however, using data from Table 7.10 the comparative values of acre/MW are shown in Table 7.11.

TABLE 7.11
MADARAS VERSUS HA-WTG LAND USAGE EFFECTIVENESS

PLANT ID	NET ACRE/MW	LAND PURCHASED NET ACRES	RATED MW
1	4.41	437	99.18
1a	20.29	1988	98.00
2	3.45	660	191.43
2a	20.29	3976	196.00
3	3.28	748	227.77

Thus, not only does this data indicate the Madaras plant is less land intensive, the data show again how economy of scale improves for Madaras plants but not for HA-WTG plants. Table 7.11 shows another interesting comparison of land use effectiveness. Note that Plant 3 can produce 2.32 times the power of Plant 1a on only 37.6 percent of the land purchased for Plant 1a. However, at Medicine Bow, where the land is very inexpensive, the land cost is only about 0.6 percent of Plant 1a cost, and hence has negligible effect on energy cost at Medicine Bow.

7.5.2 Comparison of Plants at Sites Having Mean Wind Duration Curves of 8.1 m/s at 9-m Height

The basic ground rules for this program required the use of a wind duration curve having $\bar{V} = 8.1$ m/s at a 9-m height to predict Madaras plant performance. This was done in Section 6 and 7, and here we want to compare Madaras and HA-WTG plants at these sites. In addition, since the Medicine Bow study was based on use of Federal financing and inexpensive land, it was decided to determine the effect of using 15 percent annual fixed charges as recommended by the Detroit Edison Company and price the land at \$3000 per acre. Finally, sea level air density was used.

With these changes, it was believed that results would apply to a wider range of sites. The results of this study are presented in the second part of Table 7.10 (ID Numbers 4 to 6).

all cost analysis procedures were the same as before; however, only the different values based on the new criteria are presented in Table 7.10. Note that it was assumed O&M costs would be the same as at Medicine Bow, and that the 15 percent fixed charge was charged against the total plant cost. Further, annual output for the WTG's was obtained directly from the General Electric paper on the 2000 kW MOD-1 WTG presented at the Third Wind Energy Workshop.⁴³

In Reference 43, General Electric predicted annual energy output from the MOD-1 HA-WTG would be 7.4×10^6 kW-hr/year for each unit, assuming 90 percent WTG availability. Thus, annual output for a 49 unit plant would be 362.6×10^6 kW-hr.

The larger percentage difference between annual energy output of Plants 4 over 4a (16.9 percent) compared to that of Plant 1 over Plant 1a (0.6 percent) is mostly due to the 90 percent availability (10 percent derating) specified by General Electric for their MOD-1 in the $\bar{V} = 8.1$ m/s region.⁴³ Hightower and Watts assumed 100 percent availability in their Medicine Bow study, and 100 percent availability for the Madaras system was assumed at both the Medicine Bow and the $\bar{V} = 8.1$ m/s regions. In both regions, we used the annual outputs without modification that were predicted for the MOD-1 by Hightower and Watts and by General Electric. Thus, the source of the remaining 6 percent difference in the MOD-1 energy yield prediction between Medicine Bow and the other region is unknown. Possibly the fact that the General Electric study reflected a more recent analysis explains the difference. Apparently, General Electric believed it was realistic to assume a 90 percent availability for the MOD-1. However, because one completely-assembled, operational spare rotor car and ample financial provisions for spare parts and preventative maintenance have been provided for each Madaras plant, we believe it is realistic to assume 100 percent availability for the Madaras plants.

It is interesting to note that the air density effect on annual output was such that the annual energy output in the 8.1 m/s region was greater than that at Medicine Bow where $\bar{V} = 9.7$ m/s.

Although the increased costs used for the $\bar{V} = 8.1$ m/s region increased the energy cost of each plant, the comparative results were reversed from those at Medicine Bow. Madaras Plant 4 energy cost was predicted to be about 14 percent less than energy cost of Plant 4a and Madaras Plant 5 energy cost was predicted to be 22 percent less than energy from HA-WTG Plant 5a. The cost of land had a marked effect on HA-WTG plant cost (9 percent of plant cost for Plant 5a) whereas the land cost for Plant 5 was only 1.7 percent of total plant cost. Although the installed cost/kW of Plant 1 was slightly higher than that of Plant 1a at Medicine Bow, the Madaras Plant 4 had a lower installed cost/kW than Plant 4a —a reversal of trend brought about by land cost. Thus, the more effective use of land by Madaras plants helped to improve their economic position relative to HA-WTG plants.

Since the actual price one would have to pay for land for a wind-powered electric plant is speculative, we conducted one more comparison to determine the sensitivity of energy cost from these two plant types to land cost in a $\bar{V} = 8.1$ m/s region. The results of this study are presented in the bottom section of Table 7.10: Plants 7 and 7a. Here, we assumed no land cost, a \$3000 per acre decrease. These two plants are comparable to Plants 5 and 5a, respectively. One can see that the 40.8 mill per KW-hr energy cost of Plant 5a is 22.2 percent higher than the energy cost for Plant 5. By eliminating land cost, this spread was reduced to 13.7 percent. Thus, land cost is a more important factor in energy cost of an HA-WTG plant than in a Madaras plant. Since Madaras plants utilize land more efficiently, their competitive position relative to HA-WTG plants will improve as land costs are included in the analysis.

7.5.3 Summary of Madaras versus HA-WTG Plant Comparison

The study previously described was conducted on an equitable basis for both plants for two wind regimes, three land cost levels, and two methods of financing, Federal and private.

We have sought expert professional assistance in developing our basic cost estimate, and we believe we have been conservative yet realistic in developing the cost for the Madaras plants. Further, we believe that our costs and contingencies are comparable to or more conservative compared to those developed by others for HA-WTG plants. Finally, we have had several communications with Hightower, and have been assured that we were interpreting his results correctly and equitably.

For this study we have used our best estimates on financing and land costs, based upon advice received from our consultants and advisors. We realize that higher fixed charge rates and financing schemes are proposed by other investigators. Yet, we have applied the financing formulas identically to both systems, and a study of the data indicate that since the plant costs of the two system types are similar, neither one will be particularly favored by the financing scheme selected, even if higher fixed charges are assessed.

Thus, we conclude that the study was adequate to compare the cost effectiveness of one plant against the other, which was the study's main purpose. On the other hand, we believe that the values of energy cost contained in Table 7.10 should be looked upon as representative and comparative costs more than as absolute values until a more detailed study is conducted.

The results of this study indicate that the Madaras system could be an attractive alternative, from an energy cost standpoint, to horizontal axis wind turbines. We were particularly encouraged by the favorable way in which our non-optimized Madaras design seems to have compared with the MOD-1 WTG, which has had the benefit of several intensive design and manufacturing studies. Nevertheless, we believe it is premature to state definitely that the Madaras system is more or less cost effective than HA-WTG systems —a much more detailed system design and manufacturing analysis is required before such a statement could be made with confidence.

SECTION VIII

CONCLUSIONS

This conceptual design study of the Madaras Rotor Power Plant included an analysis of all major components of the Madaras system, has addressed each of the four unanswered questions itemized in Paragraph 1.5, and has fulfilled the basic study objective delineated in Paragraph 2.1. As a result of this analysis, it is believed that a reasonably efficient conceptual design, which is consistent from criteria and loads to cost, has been developed. It also is believed that the analyses have provided a satisfactory critique of Madaras' early work, and has discussed all major problem areas as well as all primary advantages of the system. The more significant conclusions drawn from this study are itemized below.

1. The Madaras Rotor Power Plant concept using a race-track plant configuration appears at least to be economically competitive with horizontal axis wind turbine generators, and the concept shows promise of out-performing horizontal axis systems from a number of standpoints: structural durability, economy of scale, energy yield, efficient use of land, and energy cost. The results of this study indicate that, although at this stage of analysis, the concept does not appear to offer a substantial economic improvement in wind energy conversion technology, Madaras racetrack-plant energy cost varied from 12 percent higher to 22 percent lower than the energy cost of MOD-1 plants. It is believed that these results are conservative since we believe our methods of analyses have overstated electrical spin-up losses and mutual interference losses for racetrack plants. This potential Madaras cost advantage, although attractive, is diminished by the Madaras system's limited application arising from a possible scarcity of large, flat, land areas having sufficiently unidirectional wind velocities. Although more efficient HA-WTG systems (MOD-2) are being developed, it is believed that more efficient Madaras systems can be developed given the opportunity to conduct the

design and development studies comparable to those that have been conducted for horizontal axis machinery. It is believed that the 125-ft high (38-m) Madaras rotor offers a superior structural alternative to the 200-ft to 300-ft (61 m to 91 m) diameter, flexible, wind-turbine blades when subjected to wind, gust, and tower loads. Thus, the Madaras system should be analyzed further in order to determine more accurately how well it fulfills the promise indicated in this present study.

2. Capital costs of installed Madaras plants are about the same as those of horizontal axis wind turbine plants. Neither the Madaras plant nor the horizontal axis wind turbine plant seemed to gain an advantage over the other in terms of energy cost as fixed annual charge rate was varied.

3. Madaras plants having rated capacities varying from 7.9 MW to over 227 MW with annual energy output varying from 32×10^6 kW-hr to 975×10^6 kW-hr are feasible. No limitations were noted that would restrict maximum plant capacity to 227 MW. In fact, improved efficiency is highly probable as plant size increases. Thus, Madaras plants are capable of providing plant capacities of interest to electric utility companies.

4. The most efficient Madaras plant studied is described as follows:

Rotor and Car Geometry

- Aspect Ratio = 8
- Rotor Diameter = 16 ft (4.9 m)
- Cylinder Length = 125 ft (38.1 m)
- End Cap Diameter = 32 ft (9.8 m)
- e/d Ratio = 2
- Rotor Car Weight = 723,000 lb (328,600 kg)
- Rotor Mid-Height = 82 ft (25 m) above ground

Plant Geometry

- Racetrack, two rails, 36 ft (11 m)
 - Track end diameter = 4500 ft (1372 m)
 - Track straight section = 60,000 ft (18,300 m)
- Area Encompassed = 7209 acres
- Net Area Purchased = 748 acres (Balance of racetrack infield used for agriculture.)
- Number of Rotor Cars = 190
- Spacing between Rotors = 705 ft (215 m)

Operational Conditions

- Track Speed = 30 mph (13.4 m/s)
- Rated Wind Speed = 30 mph (13.4 m/s)
- Cut-Out Windspeed = 65 mph (29 m/s)
- Rotor Rotational Speed = 186 rpm
- Viscous Braking

Electrical Equipment

- Spin Motor, 450 kW, 500 Volt dc
- Generator = 1 MW per car, Induction Generator, 4160 V, 60 Hz, 30

Performance

- Rated Capacity = 227.8 MW
- Annual Energy Yield = 947.7×10^6 kW-hr @ $\bar{V} = 9.7$ m/s
@ 9 m height and an elevation of 2130
 975.4×10^6 kW-hr @ $\bar{V} = 8.1$ m/s
@ 9 m height at sea level.

Cost

- $\bar{V} = 9.7$ m/s, 8.41% Federal Financing, Land Cost @ \$200/acre
 - Plant Cost = $\$183.5 \times 10^6$
 - Unit Cost = \$806/kW
 - Total Annual Cost = $\$18.08 \times 10^6$
 - Energy Cost = 31.4 mills/kW-hr
- $\bar{V} = 8.1$ m/s, 15% Private Financing, Land Cost @ \$3000/acre
 - Plant Cost = $\$186.4 \times 10^6$
 - Unit Cost = \$818/kW
 - Total Annual Cost = $\$31.08 \times 10^6$
 - Energy Cost = 31.4 mills/kW-hr

5. Madaras plants having circular track configurations were not sufficiently efficient to compete with horizontal axis wind machines. Circular track plants are limited primarily by the large electrical losses of the spin motor during spin-up and spin-down. These large electrical spin-up and spin-down losses constitute a major performance penalty for circular-track Madaras plants, and the study of these losses deserves considerably more attention than was possible to allot in the study reported herein. Thus, it appears that useful Madaras plants could be restricted to racetrack configurations unless means for substantially reducing electrical spin-motor losses are found. Possible methods for reducing spin-motor losses include:

- Development of optimized spin-schedules
- Improved motor efficiency during spin-up
- Reduction in rotor weight, radius of gyration, and end plate size
- Improved rotor bearings
- Reduced rotor design rpm
- Use of transmissions during spin-up
- Development of a method for conserving inertial energy.

6. The potential problem areas and disadvantages of the Madaras system relative to horizontal wind turbine plants are:

- The Madaras system is more complex, has higher losses, and will require higher operation and maintenance costs than a horizontal axis wind turbine system.
- Unless electrical and aerodynamic design improvements are made, the use of a racetrack plant configuration is necessary for optimum Madaras plant performance. Thus, Madaras plants will be limited to regions having nearly unidirectional winds or to those regions in which off-axis winds have an angular variation of less than $\pm 45^\circ$ and which occur only a small portion of a total year.

7. The advantages of a Madaras Plant over a comparably-sized horizontal axis wind turbine plant are:

- A rotating cylinder rotor structure is simpler and can be built to have greater structural strength, durability, and reliability compared to large, flexible rotor blades exposed to a wind and gust environment.
- Madaras plants show higher sensitivity to economy of scale.
- Madaras plants make more efficient use of land and use less land than HA-WTG plants.

8. Although the losses of a Madaras plant are significantly larger than those of a horizontal axis wind turbine plant; the large area swept by the rotors overcomes the total losses to such an extent that the Madaras system appears to compete with horizontal axis wind machine plants on a cost per annual energy output bases.

9. Power required to rotate the rotor is markedly influenced by the end plate diameter rather than by cylinder length. Therefore, a significant reduction in electrical spin-up power

requirements, and hence a reduction of electrical losses may be achieved by one or more of the following cylinder and end plate design approaches:

- Rotating cylinder with a fixed end plate
- Large diameter, non-rotating end plate in which a smaller-diameter, flush-mounted, rotating end plate is housed.
- Rotating cylinder with nonpowered, free wheeling end plates
- Fixed cylinder and end plates with boundary layer blowing and suction on the cylinder and/or end plates.

Wind tunnel tests would have to be conducted to determine whether or not any of these approaches will provide an improved cost-performance ratio.

10. A complete set of free stream and boundary layer rotating cylinder data in terms of rotor geometry and operating conditions was obtained. Data included lift, drag, moments caused by lift and drag, and rotor power required as a function of wind speed and rotor rpm. These data are the most extensive and complete set of information on rotating cylinder performance versus geometry and wind speed profile available in the literature. The data set appears consistent within its own parametric boundaries and also is in agreement with recent and early data obtained in the mid-to-late 1920's which was used by Madaras.

11. The wind tunnel aerodynamic data obtained during this program are capable of being used directly to predict full-sized cylinder performance in a free stream flow which creates a uniform wind velocity distribution along the height of the cylinder. This direct utilization is possible because free stream Reynolds number not govern scaling of model data to full-scale results when the rotor rotational surface speed to resultant wind speed ratio (U/V) is greater than 1.0. For predicting rotor performance in the atmospheric boundary layer, the wind tunnel data must be combined with a model which develops the resultant velocity profile along the height of the cylinder. The required model yields

a resultant spiral velocity distribution versus cylinder height after the vector combination of: (1) the uniform height-wise velocity distribution caused by rotor movement along the track; and (2) the nonuniform velocity distribution versus cylinder height of the wind in the atmospheric boundary layer region. The boundary layer data obtained in the wind tunnel provided an independent means for validating this model.

12. Standard off-the-shelf electrical, mechanical, and structural components as well as materials and fabrication techniques seem to fulfill requirements for building the Madaras system proposed herein. Other than the electrical spin motor loss problem previously mentioned, the only other equipment area that should receive attention is the life of the main rotor bearings. The bearing manufacturers were not able to predict with any reliability the life of the large bearings elected for the rotor, and it was questionable if 30-year bearing life could be achieved.

13. The track roadbed design quality is of utmost importance to minimize maintenance costs. A track system including proper consideration of subsoil stabilization, good ballast, steel reinforced concrete pavement, and good steel track is required. These studies indicate that such a system can be designed for the loads and life required by the design criteria. Also, consideration should be given to the cost effectiveness of terrain modification along the roadbed in order to increase the wind velocity distribution along the lower portions of the rotor.

14. Other structural techniques should be investigated to take advantage of modern designs and fabrication concepts in order to decrease the cost and weight of the rotor. Filament optimization techniques are obvious approaches.

15. The Madaras system performance was found to be lower than that predicted by Julius Madaras and his consultants, particularly in the areas of net power generated per rotor, power

generated per unit area of land, and annual energy output. This lower performance is attributed to the following incorrect assumptions made by Madaras, et al.:

- Although the problem was addressed, Madaras erroneously assumed that mutual interference effects were negligible for rotor spacing as close as 5 diameters on a circular track diameter of 1500 ft (457 m). Thus, for a 457-m diameter track, Madaras predicted a rated plant capacity of about 47 MW. On the present study, minimum rotor spacing was found to be 44 rotor diameters, and the optimum track diameter was found to be 1372 m. Thus, for a circular plant this size, we predicted the maximum rated plant capacity would be only 7.8 MW.

- Madaras also grossly under-estimated the electrical losses required to spin up the rotor, and incorrectly assumed he could recover nearly all of the kinetic energy of the spinning rotor during regenerative braking. He also under estimated the size of the motor required to accelerate the rotor in accordance with his selected spin schedule and track speed that would yield optimum power output at each point along the track.

- Madaras assumed incorrectly that the C_L and C_D values for a rotating cylinder having an aspect ratio of 8, two large rotating end plates, and immersed in uniform free-stream flow were identical to the C_L and C_D values of a rotating cylinder having an aspect ratio of 4, one smaller rotating end plate, one stationary end plate, and immersed in nonuniform atmospheric boundary layer flow.

16. At this stage of the investigation, it is not possible to state conclusively whether or not the Madaras system will significantly out-perform a similarly-sized horizontal axis wind turbine system. However, the results of the present study, which are thought to be conservative, are sufficiently encouraging to warrant further investigation. Areas requiring further study include:

- Definition of mutual interference for racetrack plants and for circular track plants having track end diameters greater than 8000 ft (2439 m)
- Development of optimal spin schedules which include modulating rotor speed at all points along the track such that the propulsive force is optimized at all times
- Further consideration of different types of electrical equipment, transmissions, and braking techniques
- Analyses of different end plate designs which promise to reduce viscous friction and inertia loads, and hence reduce spin power, without a proportionate decrease in aerodynamic performance
- Reduction of rotor weight and inertia by optimized design and use of new materials and construction techniques that are cost effective. Included in this analysis should be the consideration of larger rotors and filament winding techniques
- In-depth studies of power collection distribution as well as system control
- A thorough study of manufacturing techniques to develop the most cost-effective methods for producing Madaras plants, and the determination of detailed costs of mass-produced units
- A life cycle cost study to include system reliability as well as maintenance, operation, and depreciation costs of the system

APPENDIX A

CYCLOGIRO VORTEX APPLIED TO THE MADARAS SYSTEM TO DETERMINE
MUTUAL INTERFERENCE LOSSES BETWEEN ROTORS

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APPENDIX A
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LIST OF SYMBOLS

A	Aspect ratio b^2/S , also area	
C_F	Force coefficient, general	$C_F = F/(1/2\rho V^2 S)$
C_D	Drag coefficient	$C_D = D/(1/2\rho V^2 S)$
C_L	Lift coefficient	$C_L = L/(1/2\rho V^2 S)$
C_M	Pitching moment coefficient	$C_M = M/(1/2\rho V^2 S)$
C_R	Resultant force coefficient	$C_R = R/(1/2\rho V^2 S)$
CFX	Rotor X force coefficient	$CFX = FX/(1/2\rho V^2 bD)$
CFZ	Rotor Z force coefficient	$CFZ = FZ/(1/2\rho V^2 bD)$
CFQ	Rotor torque force coefficient	$CFQ = FQ/(1/2\rho V^2 bD)$
CQ	Rotor torque coefficeint	$CQ = Q/(1/2\rho V^2 bD R)$
CFR	Rotor radial force coefficient	$CFR = FR/(1/2\rho V^2 bD)$
CP	Rotor power coefficient	$CP = P/(1/2\rho V^3 bD)$
CT	Rotor thrust coefficient	$CT = T/(1/2\rho V^2 bD)$
D	Rotor diameter	$D = 2R$
D	Drag, component of aerodynamic force parallel to relative velocity	
F	Force, general	
FX	Rotor X force. Time average value of sum of x component of force on the blade.	
FR	Rotor radial force. Instantaneous value of force on blade in radial direction.	
FQ	Rotor torque force. Time average value of sum of tangential component of force on blade along the orbit or path of rotation.	
FZ	Rotor Z force. Time average value of sum of z component of forces on the blades.	
I	Moment of inertia about pivot axis of blade.	

LIST OF SYMBOLS (Continued)

J	Advance ratio $J = V/nd$
L	Left component of aerodynamic for perpendicular to relative velocity
(L/D)	Lift to drag ratio
M	Pitching moment. Aerodynamic moment about the y axis
NB	Number of blades in the rotor
NR	Number of rotor radii equal to length of value used in calculation.
P	Power
Q	Torque
R	Rotor radius. Distance from axis of rotation to pivot in blade or center of rotating cylinder.
R	Resultant aerodynamic force.
S	Area. Planform area of blade, or blade element or projected area of rotating cylinder, or general area.
T	Thrust
U	Rotating cylinder peripheral velocity at surface = w_r and also the induced velocity along the x-axis of the rotor.
V	Freestream velocity, or track speed for Madaras rotor.
VP	Peripheral velocity of blade on the orbit. $VP = 2\pi nR = \Omega R$.
VR	Resultant velocity. Vector sum of all velocity components.
VX	X component of the resultant velocity.
VZ	Z component of the resultant velocity.
W	Wind speed or induced velocity.
WX	X component of the induced velocity
WZ	Z component of the induced velocity

LIST OF SYMBOLS (Continued)

X	x coordinate axis identified, or x coordinate
Y	y coordinate axis identifier, or y coordinate
Z	z denote axis identifier, or z coordinate
a	Slope of the left coefficient versus angle of attack curve.
b	Span of the blade or rotor, span in general.
c	Chord of the blade or blade element
d	Diameter of propeller or windmill
h	Normal distance from vortex segment to general point
n	rpm, rps, or revolutions per unit time
r	Radius
r	Radius
s	Arc length
t	Time
u	x component of velocity local perturbation or induced component.
v	y component of velocity local perturbation or induced component.
w	z component of velocity local perturbation or induced component.
x	x coordinate
y	y coordinate
Z = z	z coordinate
Γ	Circulation $\Gamma = \frac{C_L SV}{2b}$ free and bound vortex strength.
Δ	Acceleration-deceleration ramp angle to reverse angle of attack.

LIST OF SYMBOLS (Continued)

ψ	Phase angle of blade on orbit measured from the x axis, or the angle between the plane of rotation and resultant velocity at propeller disk.
χ	Blade rocking angle - angle between chord line of blade and X-axis
ϕ	Resultant velocity orientation angle measured from the x-axis on the angle between the plane of rotation and resultant velocity far from the disk
Ω	Angular velocity, radians/sec
α	Angle of attack - angle between reference line in airfoil to resultant velocity.
α	Angle of attack for zero lift. Angle measured from reference line to resultant velocity when lift is zero.
β	Eccentricity angle - angle of diameter joining flip points measured from the axis, or blade angle of measurement measured from plane of rotation.
γ	Ratio of specific heats or $\gamma = \tan^{-1} (D/L)$.
ζ	$\zeta = \omega/V$ nondimensional velocity ratio.
η	$\eta = v/V$ nondimensional velocity ratio or efficiency of propeller or windmill
λ	Tip speed ratio, $\lambda = 2\pi nr/v = \Omega r/V$.
v	Kinematic velocity of fluid
ξ	$\xi = u/V$ nondimensional induced velocity ratio
ρ	Mass density of the fluid
σ	$\sigma = C_R S/A = \frac{C_{Rc}}{\pi d}$ solidity
ϕ	Velocity orientation axis measured from x-axis or plane of rotation.
ψ	Phase angle for rotor, or resultant velocity at disk oriented angle.
ω	Angular velocity.

LIST OF SYMBOLS (Concluded)

Subscripts

crit	Critical
max	Maximum
min	Minimum
1,2,3,4..	Specific point variable designator
m	point for maximum
pot.	potential value
Qseen	Oseen vortex value
r,R	resultant
t	track
w	wind

SECTION A.1

HISTORICAL DEVELOPMENT OF THE CYCLOGIRO

Horizontal axis wind turbines are inherent devices that must be operated at high tip speed to wind speed ratios to obtain good efficiency. However, though the tips may operate at high efficiencies, the inner two-thirds portion of each blade operates at low efficiency. Since this region comprises four-ninth of the total disk area, the integrated disk efficiency is only 30 to 40 percent. This low integrated efficiency relative to the limiting Betz efficiency of 59 percent has prompted efforts to find alternate higher-efficiency means of extracting energy from the wind. If the entire blade could be made to operate at the same efficiency, the integrated efficiency could approach the Betz limit. The cyclogiro, a vertical-axis wind turbine is one configuration which shows promise of achieving this objective.

A.1.1 Introduction to the Cyclogiro

A typical four-blade modern cyclogiro is illustrated in Figure A.1. The cyclogiro consists of a group of controllable pivoted blades with the pivot axis parallel to the axis of rotation about which the blades rotate on supporting arms, with the axis of rotation perpendicular to the freestream velocity, V . The blades are assumed to rotate at a constant angular velocity $\Omega = 2\pi n$, so that peripheral or orbit velocity of the blade is ΩR . The resultant velocity, VR , which drives the blades is the vector sum of V and ΩR or

$$\overline{VR} = \overline{V} + \overline{\Omega R} .$$

It is noted that VR is the same magnitude all along the blade in contrast to a propeller blade where VR varies from root to tip. This vector sum is shown in Figure A.1 for the blade positions shown. VR varies in magnitude and direction, as can be seen. The individual blades are controlled (or modulated as it is

Figure A.1. Typical Cyclogiro Windmill at a Speed Ratio of $\lambda = 2$.

called) to be placed at an angle of attack relative to VR. Depending upon whether the angle of attack is positive or negative and its location on the orbit relative to VR, the cyclogiro can be made to generate a positive or negative thrust and a positive or negative side force. Both can be zero. A negative thrust corresponds to a windmill type of operation, and the blades in Figure A.1 have been so-oriented. Associated with VR and the angle of attack, α , is a blade lift force which is

$$L = C_L \frac{\rho}{2} SVR^2.$$

The lift force associated with each blade has been shown as the heavy arrow perpendicular to the local resultant velocity, VR. The lift force has been shown proportional to VR^2 and is approximately correct in magnitude. There is also a drag force, but since this is from one-twentieth to one-fiftieth the magnitude of the lift, it is too small to be shown on this scale. The variable magnitude and direction of the lift force indicates that cyclogiro flow is very unsteady. If the radius is large and the rpm is low, the flow may be considered quasi-steady. To produce useful lift, and thrust, or to act as a windmill, the angle of attack must be reversed at two points on the orbit, usually at opposite ends of a diameter. By changing the angle of attack reversal point, the resultant rotor force developed can be varied continuously to generate a pure positive thrust and a small side force, a pure side force and no thrust, or a pure negative thrust and small side force. The angle at which the angle of attack reversal occurs is called the eccentricity angle, β , and the reversal point is called the blade flip point. For blades having an L/D equal to or greater than 20 and the lift coefficient of the blade is in the linear range, the thrust and side force are also linear with angle of attack.

The local lift and drag force at each point on the orbit can be resolved into thrust and side force components, and into radial and tangential or torque force components. If a

useful thrust is produced, the torque is positive which requires power to turn the rotor. This is the propeller state. If the pure lift state exists, power must still be put into the rotor. In the windmill state the torque is negative and power is extracted from the airstream.

The modulated cyclogiro is self starting and self limiting. The rigid blade cyclogiro cannot start by itself, but once beyond a certain rpm or tip speed ratio, it runs well and is self limiting. The Darrieus rotor is a rigid blade cyclogiro of special blade shape. The cyclogiro blades does not have to be straight or constant cord. Even delta (Δ) shaped rotors have been proposed.

It also is not necessary to use blades for a cyclogiro. Rotating cylinders, which can produce lift, and jet flap augmented lift can be used. The Madaras rotor which uses rotating cyclinders is a type of cyclogiro and can be treated by the theory developed for a blade.

A.1.2 Technical Background

The cyclogiro may be the oldest windmill known. The ancient Persian panemone was in principle a cyclogiro operated by drag forces instead of by lift forces. To improve its performance it was placed on a tower which shielded the advancing blades and helped divert flow to the retreating blades. It was actually a variation of the undershop water wheel mounted vertically, and it was used to pump water.

The first recorded cyclogiro was a patent by Congreve⁴⁷ in 1828. This patent was for a man-powered, man-carrying aircraft called the aerial carriage. Since that time, many individuals in many countries around the world have contributed to the theory and experimental development.⁴⁸ Interest in the concept have occurred in intervals of from 20 to 40 years since Congreve's patent. The primary concept has been in aircraft propulsion and lift, but is has been successfully applied to ship propulsion and as a wind wheel to power a sawmill. Madaras proposed to use the concept using rotating cylinders to drive a train of carts

around a circular track to generate large amounts of power from the wind. A prototype rotating cylinder was built and tested in the wind in a stationary site in 1933, as was described in Section 1 of this report.

Investigators in France, Germany, Sweden, and Japan studied cyclogiro applications to aircraft and ship propulsion and constructed machines which were tested. Only the Voight-Schneider⁴⁹ system and the Flettner rotor systems were actually used for ship propulsion, and they were quite successful.

One of the outstanding contributors in the United States in both theory and experimentation was Dr. Kirsten⁵⁰ of the University of Washington who was an active researcher from 1922 to 1950. He showed that the path of the blade moving about a circle and translating at a uniform velocity relative to still air was a prolate cycloid, a cycloid, or a curtate cycloid dependent upon the ratio of the peripheral velocity of the blade about its axis of rotation to the uniform translation velocity. This ratio is known as the tip speed ratio, λ . Thus, $\lambda = VP/W = 2\pi nR/W$, where VP is the peripheral speed, W is the translational speed, R is the radius from the axis of rotation to the pivot in the blade, and n is the number of revolutions per unit time. The fact that the path of the blade relative to a fixed axis system is a cycloid has given the device the name of cyclogiro.

Kirsten correctly formulated the "ideal blade theory" determined the ideal blade modulation schedule (continuous orientation of the blade), devised ways to produce it, and showed the necessity of angle of attack reversal. He showed how lift and thrust would be controlled by varying the position of angle of attack reversal. He then tested a device to validate the theory and combined the ideal theory with the momentum theory to improve its capability. The NACA⁵¹ tested the cyclogiro and concluded it was a feasible method of producing lift and propulsion, that it could be useful for vertical takeoff and landing aircraft, that high propulsive efficiency could be achieved, and that it

was competitive with propellers. However, it exhibited severe vibration, and the lack of an adequate theory limited its usefulness and hampered development at that time.

During World War II the cyclogiro again was proposed as a means of propulsion and lift. Kirsten was active in the development of the aircraft designated the XP-77.^{52,53} After two years of development and testing of the propulsion system, the project was abandoned because the mechanical vibration and structural problems were considered insurmountable at the time.

Mr. R.V. Brulle, while attending the Air Force Institute of Technology in 1954, learned of the cyclogiro. He developed and patented a cyclogiro pump. Later as an engineer with the McDonnell-Douglas Company, he conceived the idea of using fly-by-wire techniques to control the blade modulation of the cyclogiro and determined this scheme would solve the vibration problems associated with mechanical blade modulation. He designed a control system to accomplish the task and then designed a small cyclogiro aircraft.

Brulle knew that to develop an adequate blade modulation control system for fly-by-wire it would be necessary to have accurate data on the inflow and sidewash velocity at every point on the orbit for all conditions. Late in 1971 he appealed to Professor H.C. Larsen for assistance to solve the problem. Since the Air Force was interested in possible applications to aircraft, permission was granted and research support was provided.

Larsen realized that the existing theories were merely an application of wing theory (more properly airfoil theory), and that they did not attempt to solve the fundamental problem as Prandtl had done for three-dimensional wing theory and Goldstein had done for propeller theory. In these theories, the wing and propeller blade are replaced by a bound vortex distribution with an assumed rigid trailing vortex sheet. Prandtl

and Betz⁵⁴ assumed the trailing vortex sheet was flat and established integrals for the lift and drag of the wing. He then solved the problem to minimize the drag. He was successful in finding a closed solution for the elliptical load distribution, which enabled him to write equations for the lift and drag coefficients of the wing. All other load distributions require numerical analysis, but the results were shown by Glauert to be reducible to a factor times the elliptical load distribution results. Prandtl's theory required six years to develop, was highly successful in correlating the drag of wings, and introduced the concept of aspect ratio. It was considered one of the triumphs of theoretical hydrodynamics.

Prandtl, Betz, and others attempted to extend the wing theory to propellers. The trailing vortex sheet was assumed to be a rigid helical vortex sheet. Integrals were established to try to solve the blade circulation distribution, but the integrals were intractable. Prandtl succeeded in finding an approximate solution which was used in design. The results were in good agreement with measurements for lightly loaded propellers, and improvements over the simple blade element theory were achieved. However, it was realized that the solution was not exact.

In 1928, Sidney Goldstein⁵⁵ published his theory of propellers. Like Prandtl, he assumed a blade circulation distribution and a trailing vortex sheet. To obtain a solution it was necessary to assume a lightly-loaded propeller. There was no slipstream contraction and the trailing sheet became a rigid helical sheet. Goldstein obtained a solution with the aid of a potential function which satisfied the equations of motion when a potential difference existed across the vortex sheet. Goldstein obtained a solution for the potential function for any number of blades, and showed that it was a Lommel function, a complex form of the Bessel function. Unfortunately, only even number of blades

could be computed at that time. Goldstein related the potential to the circulation distribution function, and ultimately the local lift coefficient and solidity which enabled the local blade chord and twist distribution to be determined for the design condition. He then evolved a design procedure and indicated how to compute off design performance.

Theodorsen⁵⁶ extended the Goldstein theory to heavily-loaded propellers and contraction effects. He used an electrical analogue to evaluate the integrals and introduced solutions for counter rotating propellers. He developed design charts which have been applied to the development of improved successful propellers. This is considered another triumph for theoretical analysis.

Both Prandtl and Goldstein assumed rigid wakes which do not occur. It is relatively easy to show that a free wake distorts. The wing wake rolls-up into two concentrated vortices located at the center of vorticity on each semiwing to form essentially the famous horseshoe vortex which Prandtl developed and assumed was valid for a uniformly loaded wing. Roll-up is quite pronounced within one semispan of the wing. Glauert investigated this problem. Westwater solved the roll-up problem with numerical integration. The propeller wake also distorts, and is thought to form a series of toroidal rings far downstream of the propeller. Vortex wakes are ultimately damped out by the action of viscosity and turbulence, yet they persist for suprisingly long times. Near the wing and propeller blade the vortex sheet is almost identical to the potential solution, while far from the wing or propeller, the wake is highly distorted and does not resemble the potential solution. This presents a dilemma. In the cyclogiro wake, strong concentrated vortices exist at the blade flip points and dominate the flow. They cause distortion and sheet roll-up to proceed almost immediately. How to attack this problem was a difficult decision, as the analysis used would depend upon the basic assumptions made.

Larsen then decided to start with first fundamentals and to attempt to find a potential solution. Figure A.2 represents the rigid trailing vortex sheet shed from the trailing edge of a single cyclogiro operating at a tip speed ratio of $\lambda = 2$. This sheet is a prolate cycloidal surface, and the vorticity in the sheet varies both across the span (due to three-dimensional loading) and continuously along the sheet due to the varying resultant velocity at the blade. The parallel straight lines diagonally oriented in Figure A.2 represent the continuously shed spanwise vorticity due to the changing velocity. The heavy diagonal lines represent intense concentrated vorticity due to an assumed instantaneous reversal in angle of attack. The streamwise prolate cycloidal lines represent spanwise variation in the vorticity shed in a streamwise direction associated with the variable loading across the span. In the center the vorticity is nearly constant in the spanwise direction, but as the tip is approached the spanwise vorticity changes rapidly due to shedding, and the streamwise lines are closer together. There is a strong streamwise vortex at each tip due to the very rapid change of spanwise vorticity near the tip required to satisfy the tip pressure conditions. This sheet is modeled after Prandtl's three-dimensional wing theory.⁵⁷ The problem is to find a potential function which satisfies the boundary conditions and which gives the correct lift and drag of the system.

A rigid cycloidal wake was assumed since its shape could be computed. If a prolate cycloidal wake occurred, the wake intersected itself for a single blade, while multiple blades had multiple intersections. Each intersection was a singular point, and the potential would be multivalued with multiple connected regions. In addition, the potential would be time dependent. If the wake was a curate cycloid, a single blade would be single valued, but multiple blades would have multiple connected regions and singular points at each intersection. All solutions would be time dependent. The prospect of finding a potential solution was remote. If it could be found, it would have to be extended piecewise from region to region which would present enormous computation problems.

Figure A.2. Trailing Vortex Sheet Array of One Blade.

A second approach was to start with the Helmholtz vortex laws⁵⁸ and to write integrals as Prandtl had done in wing theory. In this approach, the rotor would be started impulsively and the wake allowed to develop in a step-by-step manner. Differentio-integral equations would result which could be solved by finite difference techniques. The complete problem required a spanwise distribution of vorticity to be used. An elliptical loading or a Schrenk approximation would be used, but there was no justification for either assumption. The problem was formulated, but when a computer program was written, the computing time was found to be excessive and not feasible. To simplify the problem a uniform loading was assumed. Even this simplification required excessive computing time. Because of the need for a usable theory in as short a time as possible, an acceptable approximation was needed.

An examination of the strength of the vorticity shed in the wake as the blade moved around the circle was made using ideal blade theory. At the blade flip points, the vorticity shed is twice the local value, and is given by

$$\Gamma_r = \frac{C_L \quad SW \sqrt{1 + \lambda^2 - 2\lambda \sin\beta}}{b},$$

(which is represented in normalized form in Figure A.3) and the ratio of the local vorticity shed to the vorticity shed at blade flip becomes in magnitude:

$$\frac{1}{\Gamma_r} \frac{d\Gamma}{d\psi} = \left| \frac{-\lambda \cos\psi}{2 \sqrt{1+\lambda^2 - 2\lambda \sin\psi} \sqrt{1+\lambda^2 - 2\lambda \sin\beta}} \right|$$

where λ is the local phase angle and β is the phase angle at blade flip, or eccentricity angle. For the ideal windmill, $\beta = \pm\pi/2$ for the flip points. Choosing $\beta = -\frac{\pi}{2}$ to obtain the maximum, this reduces to the following for constant C_L , which is plotted in Figure A.4:

Figure A.3. Nondimensionalized Ideal Bound Vortex Strength as a Function of Phase Angle Around the Orbit for $\lambda = 2$.

Figure A.4. Nondimensionalized Ideal Bound Vortex Rate of Shedding as a Function of Phase Angle due to Change in Resultant Velocity at $\lambda = 2$.

$$\frac{1}{\Gamma_{r_{\max}}} \frac{d\Gamma}{d\psi} = \left| \frac{-\lambda \cos\psi}{2(1+\lambda) \sqrt{1 + \lambda^2 - 2\lambda \sin\psi}} \right|.$$

This shed vorticity is small compared to the concentrated vorticity and is referred to as the distributed vorticity. It may be shown that the distributed vorticity has a maximum for $\lambda < 1$, when $\sin \psi = \lambda$, and for $\lambda > 1$ when $\sin \psi = 1/\lambda$. The corresponding value of the maximums are $\lambda < 1$,

$$\left| \frac{1}{\Gamma_r} \frac{d\Gamma}{d\psi} \right|_{\max} = \left| \frac{-\lambda}{2\sqrt{1 - \lambda^2}} \right| \quad \text{and } \lambda > 1,$$

$$\left| \frac{1}{\Gamma_r} \frac{d\Gamma}{d\psi} \right|_{\max} = \left| \frac{1}{2(1+\lambda)} \right|.$$

These occur in the lower half of the circle near the lower concentrated vortex. It is symmetrical with respect to the Z axis. The distributed vorticity is zero at $\psi = -\frac{\pi}{2}$ and gradually increases in magnitude to the maximum in the lower half of the circle, and then decreases in magnitude until it is zero at $\psi = \pi/2$. A plot of the magnitude of the distributed vorticity shows that there is more in the lower half of wake than in the upper half. Depending on β , the distributed vorticity is usually of the same sign as the lower concentrated vorticity. When $\beta = -\frac{\pi}{2}$ or $\frac{\pi}{2}$, the sum of the lower concentrated vorticity and the distributed vorticity is equal in magnitude and opposite in sign to the upper concentrated vorticity. This satisfies the irrotationality condition for the undisturbed flow in two dimensions. For other values of β , there are some regions where the distributed vorticity has the same sign as the upper concentrated vorticity, and the magnitude of the sums of opposite sign vorticity is again equal so that irrotationality is again satisfied.

The motion of the two unequal vortices in two dimensions in a fluid at rest is in circular paths about a common center whose location depends upon the magnitude and sign of vorticity. If they are equal in magnitude and opposite rotation, the vortex pair will translate at uniform velocity along straight parallel lines. It can be shown that if there is a finite number of discrete vortices in an undisturbed fluid at rest, then each vortex will tend to rotate in a circular path about the centroid of vorticity and this can be used to deduce how a cycloidal-shaped wake will distort.

If the total vorticity is divided into two parts which have a sum of equal magnitude taking into account the algebraic sign, the upper half will have vorticity of both signs while the lower half will have vorticity of the same sign. Then, calculate the centroid of vorticity of each half, and it will be found to lie very close to concentrated vorticity of each half. The resulting motion will tend to be in circular paths around each centroid. The motion will not be circular, since each half will tend to influence the motion of the other half. As a result, the vorticity tends to roll-up around the centroid of each half, to form two equal and opposite vortices concentrated at the centroid, but joined by spiraling threads of vorticity which decrease in strength as the vortices roll-up. This pair will then tend to translate back toward the rotor which shed them, but the freestream translates them downstream. The net result is the rolled-up pair does not translate downstream at the wind speed, but at a somewhat lesser speed.

The rotor is in three dimensions, so this pair will be joined at the tip to form an essentially rectangular ring which will tend to distort to a toroidal shape far downstream. Because of strength of the concentrated vorticity, roll-up begins soon after the concentrated vortex is shed, usually within one diameter of the rotor.

Larsen deduced this structure, later verified by detailed calculations, and formulated his "Primitive Vortex Theory of the Cyclogiro" on this model of the wake. It was realized that this was not an exact model, but represented an approximation to the actual structure of the wake.

McDonnell-Douglas, under the direction of Brulle, used this theory to analyze the performance of the cyclogiro which used the wake model and the bound vorticies to compute the individual velocity at each blade. A wind tunnel model was built and tested, and the results were in excellent agreement with the theory.⁵⁹

The following sections describe the basic theories behind the development of the Primitive and the Improved Primitive Vortex theory of the Cyclogiro, as well as the application of this theory to rotating cylinders in order to study the Madaras system.

SECTION A-2

IDEAL BLADE ELEMENT THEORY OF THE CYCLOGIRO

The ideal blade element theory of the cyclogiro assumes that a lifting surface is rotating at constant angular velocity, Ω , in a circular path about an axis of rotation in a constant uniform parallel flow of magnitude, W , as illustrated in Figure A.5. It is assumed that aerodynamic coefficients obtained in a uniform parallel flow are applicable to the rotating lifting surface. Since these coefficients are functions of the angle of attack of the lifting surface relative to the resultant velocity at the blade element, it is necessary to establish the resultant velocity and angle of attack at the blade element. The resultant velocity is the vector sum of the relative wind, W , and the rotational velocity, $V = \Omega R$, of the blade element or the peripheral velocity, V .

It is convenient to use the axis system shown in Figure A.5 with the origin at the axis of rotation at the mid span location of the blade, x pointing into the relative wind, W , z , downward, and y out the right side of the rotor. A blade is located instantaneously at the angle ψ from the x axis. The angle ψ is referred to as the phase angle or the blade orbit angle, and the circle on which the blade moves is referred to as the orbit. The X and Z components of the resultant relative velocity to the blade are:

$$V_X = V \sin \psi - W, \text{ and}$$

$$V_Z = -V \cos \psi.$$

The resultant velocity, V_R , is then:

$$\begin{aligned} V_R &= \sqrt{(V_X)^2 + (V_Z)^2} = \sqrt{(V \sin \psi - W)^2 + (-V \cos \psi)^2} \\ &= \sqrt{V^2 + W^2 - 2 VW \sin \psi} = W \sqrt{1 + \lambda^2 - 2 \sin \psi}, \end{aligned}$$

where $\lambda = V/W$ the tip speed ratio.

Figure A.5. Ideal Blade Vector Diagram.

The direction of the resultant velocity from the x axis is the angle, ϕ , or,

$$\phi = \tan^{-1} (VZ/VX) = \tan^{-1} (-V \cos \psi / (V \sin \psi - W)).$$

The resultant velocity is the aerodynamic velocity which produces the lift and drag forces on the blade. The lift force is at right angles to the resultant velocity, and the drag force is parallel to and in the direction of the resultant velocity. The direction of the lift force depends upon the angle of attack, α , of the blade relative to the resultant velocity. A positive angle of attack is measured counterclockwise from the chord line of the blade to the resultant velocity. This corresponds to the lift leading the drag by 90° for positive α , and lagging by 90° for negative α . The lift and drag forces acting on the blade are illustrated in Figure A.5 for a positive angle of attack.

Determination of the performance of the rotor requires the establishment of the dependence of the lift force generated by the rotor or Z force normal to the relative wind, the thrust force generated by the rotor, or x force parallel to the relative wind, and the torque force normal to the radius of the blades or t force along the orbit as functions of the primary variables. Since power is the product of force times velocity, the power absorbed or generated by the rotor is the torque force times the peripheral velocity. The instantaneous contribution of a blade to each of these forces can be computed from the geometry in Figure A.5. They are:

$$F_x = L \cos \left(\phi + \frac{\pi}{2} \right) + D \cos \phi = -L \sin \phi + D \cos \phi,$$

$$F_z = L \sin \left(\phi + \frac{\pi}{2} \right) + D \sin \phi = L \cos \phi + D \sin \phi,$$

$$F_r = L \cos \left(\psi - \phi - \frac{\pi}{2} \right) + D \cos (\psi - \phi)$$

$$= L \sin (\psi - \phi) + D \cos (\psi - \phi),$$

$$FQ = L \sin (\psi - \phi - \frac{\pi}{2}) + D \sin (\psi - \phi) \\ = - L \cos (\psi - \phi) + D \sin (\psi - \phi).$$

Since ϕ is a function of ψ , these forces are all functions of ψ . Also, since $L = C_L \frac{\rho S}{2} VR^2$, and $D = C_D \frac{\rho S}{2} VR^2$, and

$VR = W \sqrt{1 + \lambda^2 - 2\lambda \sin \psi}$, and $C_L = \frac{dC_L}{d\alpha} (\alpha - \alpha_{LO})$, and $C_D = C_D(C_L)$, these forces are functions of ψ , w , λ , and α or C_L . Also,

$$\sin \phi = VZ/VR = - \lambda \cos \psi / \sqrt{1 + \lambda^2 - 2\lambda \sin \psi}, \\ \text{and } \cos \phi = VX/VR =$$

$(\lambda \sin \psi - 1) / \sqrt{1 + \lambda^2 - 2\lambda \sin \psi}$, it is possible after substitution of these definitions and simplification of the results to obtain explicit expression for the instantaneous values of these forces. They are:

$$FX = \frac{\rho S}{2} W^2 \cdot C_L \cdot [\lambda \cos \psi + (C_D/C_L)(\lambda \sin \psi - 1)] \sqrt{1 + \lambda^2 - 2\lambda \sin \psi},$$

$$FZ = \frac{\rho S}{2} W^2 \cdot C_L \cdot [\lambda \sin \psi - 1 - (C_D/C_L)\lambda \cos \psi] \sqrt{1 + \lambda^2 - 2\lambda \sin \psi},$$

$$FR = \frac{\rho S}{2} W^2 \cdot C_L \cdot [\lambda \sin \psi - (C_D/C_L) \cos \psi] \sqrt{1 + \lambda^2 - 2\lambda \sin \psi},$$

$$FQ = \frac{\rho S}{2} W^2 \cdot C_L \cdot [\cos \psi + (C_D/C_L)(\lambda - \sin \psi)] \sqrt{1 + \lambda^2 - 2\lambda \sin \psi}.$$

For constant C_L , C_D , and λ , the instantaneous values of these forces are functions ψ . Since $d\psi = \Omega dt$, the time average value of these forces can be obtained by integrating around the orbit, and from the mean value theorem of the integral calculus. In the ideal theory no blade interference is assumed so that the mean value is directly proportional to the number of blades, NB. The time average values become:

$$\overline{FX} = \frac{NB}{2\pi} \int_0^{2\pi} FX d\psi = \frac{NB}{2\pi} \frac{\rho S}{2} W^2 \int_0^{2\pi} C_L [\lambda \cos \psi + (C_D/C_L)(\lambda \sin \psi - 1)]$$

$$[f(\lambda, \psi)] d\psi$$

where $f(\lambda, \psi) = \sqrt{1 + \lambda^2 - 2\lambda \sin \psi}$,

$$\overline{FZ} = \frac{NB}{2\pi} \int_0^{2\pi} FZ d\psi = \frac{NB}{2\pi} \frac{\rho S}{2} W^2 \int_0^{2\pi} C_L [(\lambda - \sin \psi - 1) - (C_D/C_L) (\cos \psi)] [f(\lambda, \psi)] d\psi$$

$$\overline{FR} = \frac{NB}{2\pi} \int_0^{2\pi} FR d\psi = \frac{NB}{2\pi} \frac{\rho S}{2} W^2 \int_0^{2\pi} C_L [(\lambda - \sin \psi) - (C_D/C_L) \cos \psi] [f(\lambda, \psi)] d\psi$$

$$\overline{FQ} = \frac{NB}{2\pi} \int_0^{2\pi} FQ d\psi = \frac{NB}{2\pi} \frac{\rho S}{2} W^2 \int_0^{2\pi} C_L [\cos \psi + (C_D/C_L) (\lambda - \sin \psi)] [f(\lambda, \psi)] d\psi$$

Even with constants C_L , C_D , and λ , these integrals cannot be evaluated in closed form due to the presence of the radical, and numerical methods must be used. The integral of $\cos \psi$ times the radical can be evaluated in closed form, while the integral of $\sin \psi$ times the radical cannot be evaluated in closed form and must be evaluated numerically. Inspection shows that certain terms can be evaluated in closed form. Also, the mean value theorem for the integral of the product of two functions can be used to obtain approximate relations which can indicate trends. If C_L is a constant, C_D is a constant, and the mean value would be directly proportional to C_L . Thus, to obtain high lift and thrust, high lift coefficients should be used, and this also means high power is required to turn the rotor, or high power would be extracted by the rotor. Finally, the term $NB \frac{\rho S}{2} W^2$ multiplies each integral.

Since a force divided by $\frac{\rho S}{2} \cdot S^2$ gives an aerodynamic coefficient which is independent of size or actual wind speed, in coefficient form these integrals imply that coefficients can be obtained which depend only on C_L , C_D , and λ . Size and wind speed can be removed as long as geometrical similarity is maintained. Thus, rotors can be scaled in size from one computation.

Because of the difficulty in evaluating the integrals, and since most investigators assumed that air foils would be used for blades so that $(C_D/C_L) \ll 1.0$, the integrals were simplified by ignoring the drag contribution to the integrals. Numerical evaluation was still required in most cases. However, with this simplification, $\overline{FX}$ and $\overline{FQ}$ can be evaluated in closed form, and lead to an understanding of the behavior of the rotor performance and the necessity of lift coefficient reversal to obtain useful thrust and torque. These integrals will be evaluated, and then the mean value theorem will be utilized to show the effect of the drag term. This is particularly important for the Madaras rotor which uses spinning cylinders which can generate very large values of C_L , but C_L/C_D is of the order of 3 to 5, which makes the drag term important in each integral.

Neglecting the drag term and expressing the integral in coefficient form, the integral for the thrust coefficient behavior becomes:

$$\begin{aligned}\overline{CFX} &= \frac{\overline{FX}}{NB \frac{\rho g}{2} W^2} = \frac{1}{2\pi} \int_0^{2\pi} \lambda C_L \sqrt{1 + \lambda^2 - 2 \sin \psi} \cos \psi d\psi \\ &= -\frac{2}{3} \frac{C_L}{2\pi} [1 + \lambda^2 - 2\lambda \sin \psi]^{3/2} \Big|_0^{2\pi} \\ &= -\frac{C_L}{3} [(1 + \lambda^2)^{3/2} - (1 + \lambda^2)^{3/2}] = 0, \text{ if } C_L \text{ is}\end{aligned}$$

assumed constant. Thus, if the lift coefficient is constant around the orbit no thrust is developed. If it is assumed that the lift coefficient is reversed in sign at $\psi = \beta$ and $\psi = \pi + \beta$ so that $C_L > 0$ if $\pi + \beta < \psi < 2\pi + \beta$, the expression for $\overline{CFX}$ becomes with

$$| - C_L | = | C_L | \text{ and constant}$$

$$\begin{aligned} \overline{CFX} &= \frac{|C_L|}{2} \left[\int_{\beta}^{\pi + \beta} -\lambda \sqrt{1 + \lambda^2 - 2\lambda \sin \psi} \cos \psi d\psi \right. \\ &\quad \left. + \int_{\pi + \beta}^{2\pi + \beta} \lambda \sqrt{1 + \lambda^2 - 2\lambda \sin \psi} \cos \psi d\psi \right] \\ &= \overline{CFX} = \frac{|C_L|}{3\pi} \left[\{1 + \lambda^2 + 2\lambda \sin \beta\}^{3/2} \right. \\ &\quad \left. - \{1 + \lambda^2 - 2\lambda \sin \beta\}^{3/2} \right] \end{aligned}$$

It is seen that if $\beta > 0$ a positive thrust is developed, while if $\beta < 0$ a negative thrust is developed. Thus, if the sign on C_L is reversed at some angle β , thrust can be developed, and the control of thrust is through the angle β . To reverse the lift coefficient, the angle of attack must be made negative for $\beta \leq \psi \leq \pi + \beta$, and the angle of attack must be made positive for $(\pi + \beta) \leq \psi \leq (2\pi + \beta)$. To obtain a constant $|C_L|$, the blade must be maintained at a constant angle relative to ϕ . Since $\phi = \tan^{-1} (VZ/VX) = \tan^{-1} (-V \cos \psi / (V \sin \psi - W))$, the orientation of the blade must be continuously varied as the blade traverses the orbit. At $\psi = \beta$ and $\psi = \pi + \beta$, the angle of attack must be reversed. This continual variation of blade orientation is referred to as blade modulation, and the reversal of blade angle of attack is referred to as blade flip. Kirsten showed that the ideal blade modulation could be achieved by the so called "swingin' sliding block mechanism." He devised mechanical means to accomplish this.

In a similar manner, it can be shown that the torque force coefficient can be found to be

$$\overline{CFQ} = \frac{|C_L|}{3\pi\lambda} \left[\{1 + \lambda^2 + 2\lambda \sin\beta\}^{3/2} - \{1 + \lambda^2 - 2\lambda \sin\beta\}^{3/2} \right]$$

If $\beta > 0$, $\overline{CFQ} > 0$, and power must be supplied to the rotor, while if $\beta < 0$, $\overline{CFQ} > 0$, and power is extracted by the rotor. Thus if $\beta > 0$ $\overline{CFX} > 0$, $\overline{CFQ} > 0$, and the device is a propulsive device, while if $\beta < 0$ $\overline{CFX} < 0$, $\overline{CFQ} < 0$, and the device is a windmill which extracts power from the airstream.

Since $V \cdot \overline{FQ}$ is a power, the power coefficient becomes

$$\begin{aligned} \overline{CP} &= \frac{P}{ND \frac{\rho S}{2} W^3} = \frac{|C_L|}{3\pi} \left[\{1 + \lambda^2 + 2\lambda \sin\beta\}^{3/2} - \{1 + \lambda^2 - 2\lambda \sin\beta\}^{3/2} \right] \\ &= \lambda \overline{CFQ} = \overline{CFX} \end{aligned}$$

This ideal analysis provides some insight into the performance of the rotor. The integral for $\overline{CFZ}$ which is the lift force or side force of the rotor cannot be evaluated in closed form, but if $\beta > 0$, $\overline{CFZ} < 0$, and if $\beta < 0$, $\overline{CFZ} > 0$. This is found from numerical computation. It may also be shown that $\overline{CFZ}$ is a maximum in magnitude for $\beta = \pm \frac{\pi}{2}$. Thus, a lifting rotor could take off vertically at $\beta = 0$, and by making $\beta > 0$, could produce both lift and thrust with power input. The cyclogiro would then behave much like a helicopter. It can be shown that in the event of power failure, the rotor will autorotate, and a safe landing is possible. Wind tunnel tests have verified this analysis.

The integrals for the aerodynamic coefficients can be evaluated with the aid of the second mean value theorem of the integral calculus. Because of blade modulation and mathematical restriction on the mean value theorem, piecewise integration is required. The resulting expressions indicate the trends of the coefficients, but are not convenient for application as numerical integration is still required. The theorem states

$$\int_a^b f(x) g(x) dx = f(\delta_i) \int_a^b g(x) dx \text{ where } f(\delta_i) = \frac{1}{(b-a)} \int_a^b f(x) dx. \text{ Where } f(x) \text{ and } g(x) \text{ are integrable,}$$

and $g(x)$ is always positive or always negative on the interval $(b-a)$. Now:

$$\overline{CFX} = \frac{1}{2\pi} \int_0^{2\pi} C_L [\lambda \cos \psi + \left(\frac{C_D}{C_L}\right) (\lambda \sin \psi - 1) f(\lambda, \psi)] d\psi$$

$$\text{where } f(\lambda, \psi) = \sqrt{1 + \lambda^2 - 2 \sin \psi} \text{ and}$$

$$g(\psi) = [\lambda \cos \psi + \left(\frac{C_D}{C_L}\right) (\lambda \sin \psi - 1)], \text{ with blade modulation } C_L < 0$$

if $\beta \leq \psi \leq (\pi + \beta)$, and $C_L > 0$ if $(\pi + \beta) \leq \psi \leq (2\pi + \beta)$. It is necessary to consider four intervals, $\beta \leq \psi \leq \pi$, $\pi \leq \psi \leq (\pi + \beta)$, $(\pi + \beta) \leq \psi \leq 2\pi$, and $2\pi \leq \psi \leq (2\pi + \beta)$. $f(\psi)$ on each interval is evaluated by numerical integration to give $f(\psi_1)$, $f(\psi_2)$, $f(\psi_3)$ and $f(\psi_4)$. The algebraic sign of C_L is used on each interval. The expression for $\overline{CFX}$ becomes with $|C_L|$ being constant and C_D being positive and constant

$$\overline{CFX} = \frac{|C_L|}{2\pi} [f(\psi_1) \int_{\beta}^{\pi} -g(\psi) d\psi + f(\psi_3) \int_{\pi+\beta}^{2\pi} g(\psi) d\psi + f(\psi_4) \int_{2\pi}^{2\pi+\beta} g(\psi) d\psi]. \text{ Substituting for } g(\psi), \text{ carrying out the}$$

indicated integration, substituting limits, and simplifying, the expression for $\overline{CFX}$ becomes

$$\overline{CFX} = \frac{|C_L|}{2\pi} [f(\psi_1) \left\{ \lambda \sin \beta - \left| \frac{C_D}{C_L} \right| [\pi - \beta - \lambda(1 + \cos \beta)] \right\} + f(\psi_2) \left\{ \lambda \sin \beta - \left| \frac{C_D}{C_L} \right| [\beta + \lambda(1 - \cos \beta)] \right\} + f(\psi_3) \left\{ \lambda \sin \beta - \left| \frac{C_D}{C_L} \right| [\pi - \beta + \lambda(1 + \cos \beta)] \right\}$$

$$+ f(\psi_4) \{ \lambda \sin \beta - \left| \frac{C_D}{C_L} \right| [\beta - \lambda (1 - \cos \beta)] \} \}].$$

Inspection reveals that the drag term reduces the thrust $\beta \geq 0$. When $\beta = 0$, $f(\psi_2) = f(\psi_4) = 0$, and $\overline{CFX}$ becomes

$$\overline{CFX} = - \left| \frac{C_L}{2\pi} \right| \{ f(\psi_1) + f(\psi_3) \} \left| \frac{C_D}{C_L} \right| (\pi + 2\lambda).$$

This shows that drag causes a negative thrust at $\beta = 0$.

When $\beta = \frac{\pi}{2}$, $f(\psi_1) = f(\psi_4)$ and $f(\psi_2) = f(\psi_3)$, and $\overline{CFX}$ becomes

$$\overline{CFX} = \left| \frac{C_L}{2\pi} \right| \{ f(\psi_1) \{ 2\lambda - \left| \frac{C_D}{C_L} \right| (\pi - 2\lambda) \} + f(\psi_2) \{ 2\lambda - \left| \frac{C_D}{C_L} \right| (\pi + 2\lambda) \} \}$$

Again, the drag term reduces the thrust from what it would be at the ideal maximum thrust condition. Since $f(\psi_1)$ and $f(\psi_2)$ increase with λ , it is seen that $\overline{CFX}$ increases with λ and C_L and is positive for $\beta > 0$. It is negative for $\beta < 0$ which can be determined from a similar computation.

The expression for $\overline{CFZ}$ is found similarly and is

$$\begin{aligned} \overline{CFZ} = & \left| \frac{C_L}{2\pi} \right| \{ f(\psi_1) \{ -\lambda(1 + \cos \beta) + \pi - \beta + \lambda \left| \frac{C_D}{C_L} \right| \sin \beta \} + f(\psi_2) \\ & \{ \lambda(1 - \cos \beta) + \beta + \lambda \left| \frac{C_D}{C_L} \right| \sin \beta \} \\ & + f(\psi_3) \{ -\lambda(1 + \cos \beta) - \pi + \beta - \lambda \left| \frac{C_D}{C_L} \right| \sin \beta \} \\ & + f(\psi_4) \{ \lambda(1 + \cos \beta) - \beta - \lambda \left| \frac{C_D}{C_L} \right| \sin \beta \} \} \end{aligned}$$

Note that at the same value of β , $f(\psi_1)$, $f(\psi_2)$, $f(\psi_3)$, and $f(\psi_4)$ are the same for all the coefficients. This simplifies evaluation of the coefficients.

$$\text{When } \beta = 0, \overline{CFZ} = \left| \frac{C_L}{2\pi} \right| [f(\psi_1)(-2\lambda + \pi) + f(\psi_3)(-2\lambda - \pi)] < 0 \sim$$

Note that the drag term does not affect $\overline{CFZ}$ at $\beta = 0$.

When $\beta = \frac{\pi}{2}$,

$$\overline{CFZ} = \frac{|C_L|}{2\pi} [f(\psi_1) \{ -\lambda + \frac{\pi}{2} + \lambda \left| \frac{C_D}{C_L} \right| + \lambda - \frac{\pi}{2} - \lambda \left| \frac{C_D}{C_L} \right| \} + f(\psi_3) \{ \lambda + \frac{\pi}{2} + \lambda \left| \frac{C_D}{C_L} \right| - \lambda \left| \frac{C_D}{C_L} \right| \}] = 0.$$

It may be inferred from this that the drag term does not affect the lift of the rotor.

The torque force coefficient, and hence the power coefficient is found similarly for $\beta > 0$ as:

$$\begin{aligned} \overline{CFQ} = & \frac{|C_L|}{2\pi} [f(\psi_1) \{ \sin\beta + \left| \frac{C_D}{C_L} \right| (\lambda(\pi-\beta) - (1 + \cos\beta)) \} \\ & + f(\psi_2) \{ \sin\beta + \left| \frac{C_D}{C_L} \right| [\lambda\beta - (1 + \cos\beta)] \} \\ & + f(\psi_3) \{ \sin\beta + \left| \frac{C_D}{C_L} \right| [\lambda(\pi - \beta) + (1 + \cos\beta)] \} \\ & + f(\psi_4) \{ \sin\beta + \left| \frac{C_D}{C_L} \right| [\lambda\beta + (1 + \cos\beta)] \}] > 0. \end{aligned}$$

The drag always makes a positive contribution to the torque force, and hence increases the power required or reduces the power extracted by a windmill.

$$\text{When } \beta = 0, \overline{CFQ} = \frac{|C_L|}{2\pi} [f(\psi_1) \left| \frac{C_D}{C_L} \right| (\lambda\pi - 2) + f(\psi_3) \left| \frac{C_D}{C_L} \right| (-2)]$$

$$\text{When } \beta = \frac{\pi}{2}, \overline{CFQ} = \frac{|C_L|}{2\pi} [f(\psi_2) + f(\psi_2)] (2 + \lambda\pi \left| \frac{C_D}{C_L} \right|)$$

$$\text{When } \beta = \frac{\pi}{2}, \overline{CFQ} = \frac{|C_L|}{2\pi} [f(\psi_1) + f(\psi_2)] (-2 + \lambda\pi \left| \frac{C_D}{C_L} \right|).$$

This analysis shows that when drag is present that the drag contribution causes a negative increment in the thrust, a positive increment in the torque, and hence power, and has no effect on the lift at $\beta = 0$ or $\pm\frac{\pi}{2}$, but may have an effect on the lift at other values of β . If $|C_D/C_L|$ is large and cannot be neglected, the performance of the rotor is seriously degraded.

The fact that thrust and lift are generated by the rotor shows that the ideal theory is incorrect. The momentum theorem of fluid mechanics requires that if a force is generated on a body immersed in a fluid that the fluid passing around the body be given an increment of velocity in the opposite direction of the force generated. The flow field is distributed in all directions out to infinity, and the assumption of a uniform parallel flow through the rotor is not valid. The disturbances created by the body are not uniform or constant, but vary throughout the flow.

The momentum theory of propellers assumed a thin actuator disc and assumed a wake to exist behind the disc. The stream tube which passed through the disc was assumed to contract in the direction of the flow. It was found that one-half of the final velocity increment in the wake occurred prior to the flow reaching the disc, and one-half after passing through the disc. Similarly in Prandtl's wing theory, one-half the induced velocity occurred prior to the wing and one-half after the flow passed the wing. This led researchers in cyclogiro theory to make a similar assumption about the flow through a cyclogiro. If the force on a body is known, the final velocity increment in the wake can be computed easily from the momentum theorem. Because of the linearity of the equations, it is possible to use components of the force.

The first correction to the ideal theory used these concepts. $\overline{FX}$ and $\overline{FZ}$ were computed from the ideal theory with no correction. These forces were then used to compute the X and Z components of the final velocity increment in the wake. It was assumed that one-half the final velocity increment in the wake occurred at the rotor. It was further assumed that the velocity increment at the rotor was uniform and constant throughout the rotor. If WX is the X component of velocity at the rotor and WZ the Z component of velocity at the rotor, then the resultant velocity components at the rotor become

$$VX = V \sin \psi - W + WX, \text{ and } VZ = -V \cos \psi + WZ \text{ so that}$$

$$VR^2 = W^2 + V^2 + WX^2 + WZ^2 - 2V(W - WX) \sin \psi - 2(W WX + V WZ) \cos \psi.$$

Let $\lambda = V/W$, $\xi = WX/W$, $\zeta = WZ/W$. Then,

$$VR^2 = W^2 [(1-\xi)^2 + \lambda^2 + \zeta^2 - 2\{(1-\xi) \sin\psi + \zeta \cos\psi\}] \quad \text{and}$$

$$\phi = \tan^{-1} \frac{-\lambda \cos\psi + \zeta}{-(1-\xi) + \sin\psi} \quad \text{and} \quad \sin\psi = \frac{VZ}{VR} \quad \text{and} \quad \cos\psi = \frac{VX}{VR} \quad \text{as}$$

before. Following the same procedures as before, one finds:

$$\overline{CFX} = \frac{1}{2\pi} \int_0^{2\pi} [C_L (\lambda \cos\psi - \zeta \sin\psi) - C_D ((1-\xi) \cos\psi - \zeta \sin\psi)]$$

$$\sqrt{(1-\xi)^2 + \lambda^2 + \zeta^2 - 2\lambda [(1-\xi) \sin\psi + \zeta \cos\psi]} \quad d\psi$$

$$\overline{CFZ} = \frac{1}{2\pi} \int_0^{2\pi} [C_L (\lambda \sin\psi - (1-\xi)) - C_D (\lambda \cos\psi - \zeta)]$$

$$\sqrt{(1-\xi)^2 + \lambda^2 + \zeta^2 - 2\lambda [(1-\xi) \sin\psi + \zeta \cos\psi]} \quad d\psi$$

$$\overline{CFQ} = \frac{1}{2\pi} \int_0^{2\pi} [C_L ((1-\xi) \cos\psi - \zeta \sin\psi) + C_D (\lambda - (1-\xi) \sin\psi + \zeta \cos\psi)]$$

$$\sqrt{(1-\xi)^2 + \lambda^2 + \zeta^2 - 2\lambda [(1-\xi) \sin\psi + \zeta \cos\psi]} \quad d\psi$$

$$\overline{CFR} = \frac{1}{2\pi} \int_0^{2\pi} [C_L (\lambda - (1-\xi) \sin\psi + \zeta \cos\psi) - C_D ((1-\xi) \cos\psi - \zeta \sin\psi)]$$

$$\sqrt{(1-\xi)^2 + \lambda^2 + \zeta^2 - 2\lambda [(1-\xi) \sin\psi + \zeta \cos\psi]} \quad d\psi$$

These integrals are considerably more complex than in the ideal theory, and none can be evaluated in closed form. The mean values theorem can be used as before, but the results are somewhat more complicated and will not be shown. Basically, the rotor performance has the same character as before, but the performance is degraded. Thrust is less, lift is less, and power required is

greater. The reason is that the lift and drag forces are rotated from the ideal orientation in a direction which produces those effects.

An iterative procedure was used to find the aerodynamic coefficients as defined by these integrals. For a given λ , β , C_L , and C_D , $\overline{FX}$ and $\overline{FZ}$ were computed from the ideal theory. WX and WZ were then computed from the momentum theorem, and ξ and η determined. The aerodynamic coefficients were then computed from the integrals with ξ and η corrections. Since $\overline{FX} = \overline{CFX} \frac{\rho S}{2} W_{xNB}^2$ and $\overline{FZ} = \overline{CFZ} \frac{\rho S}{2} W_{zNB}^2$; FX and FZ were then computed. A second value of WX and WZ were computed, and new values of ξ and η computed. The procedure was repeated until WX and WZ were computed to an acceptable accuracy. It required five to six iterations to achieve 1 percent accuracy. This procedure was accomplished by manual calculations and was extremely laborious. Eight to 16 points were used on the orbit to reduce the amount of effort required. The results were in good agreement with test results both qualitatively and quantitatively, but over estimated the performance actually achieved.

It was recognized that there were serious deficiencies in this attempt to compute the performance. The assumption of constant uniform values of WX and WZ around the orbit was not a valid assumption. Because of the finite rotor radius WX and WZ would be different at all points on the orbit. The flow is curvilinear and the use of C_L and C_D obtained in uniform parallel was questionable. The flow was clearly nonsteady and tended to be oscillatory through the rotor. In addition, mutual interference and interference of the wake was neglected. The rotor is actually three dimensional, and these theories assumed that all of the blade experienced the same velocity. This assumed two dimensional flow, or strip theory. Finally, the effect of any blade support structure had been neglected. Kirsten assumed cantilever blades from the side of a fuselage.

In spite of all these deficiencies, the results were encouraging enough that the cyclogiro was seriously considered

for a fighter aircraft design in World War II, but mechanical vibration and structural problems terminated the effort after two years.

SECTION A-3
PRIMITIVE VORTEX THEORY OF THE CYCLOGIRO

Larsen reviewed the existing theory of the cyclogiro, and concluded that it would be necessary to use vortex theory to simulate the blades as Prandtl had done in wing theory and Goldstein had done in propeller theory. This would permit computation of the induced velocities at all points on the orbit. The problem was exceedingly complicated. Kirsten had shown that the path of the blade relative to the air was a prolate cycloid, cycloid, or curate cycloid dependent upon the value of λ . In addition, since a number of blades would be used, the cycloids intersected. This created singularity and multivalued functions. Prandtl had assumed that the vortex sheet shed from the wing was plane and rigid in violation of the Helmholtz vortex laws. This was justified on the argument that the induced velocities were small, and in order to linearize the equations it was necessary. Analysis shows that the centroid of vorticity remains fixed, but the vortex sheet is unstable and rolls-up to form two concentrated trailing vortices located at the spanwise centroid of vorticity. Flow visualization techniques clearly demonstrate this phenomena in wind tunnel tests.

Goldstein assumed that the vortex sheet shed from the blades of a propeller was rigid, translated at uniform velocity, and being lightly loaded did not contract. Basically, his assumptions are valid far downstream from the propeller. The vortex sheet trailing from a propeller is not stable, and will roll-up similar to that trailing behind a wing. Because of the three dimensional nature of the problem, the final configuration is not known. The rigid sheet would be a helix. If the sheet rolled-up to the centroid of vorticity, it would form a concentrated helical vortex, one for each blade. How these multiple helical vortices would interact is unknown, but there is some experimental evidence that the wake ultimately formed is a series of equally spaced toroidal vortex rings joined by helical

vortices of vanishing strength as time progresses. This structure occurs far from the propeller and is the result of the curvilinear axis of the vortex, viscosity, turbulence, and compressibility. Unfortunately, the equations cannot be solved because they are nonlinear, but solutions obtained from steady state linear equations suggest that all these effects are present and affect the final structure.

The structure of vortex wake for a cyclogiro is complicated because it is nonsteady, nonrigid, curvilinear, nonuniform, and has regions of highly concentrated vorticity at the lift coefficient reversal or blade flip points. If the wake structure is known, the induced velocities can be computed from the geometry and the Biot-Savart law. If the structure is unknown, the Helmholtz vortex laws, the equations of motion, and the Biot-Savart law can be used to compute the structure and the induced velocities as functions of time. Hopefully, a steady-state solution would result after the transients had died out. It is necessary to review vortex properties in order to understand the problem.

A.3.1. Vortex Properties

Associated with each increment of length of a vortex filament there is an induced velocity at every point in the field. The induced velocity and the vortex filament co-exist. One does not create the other. The induced velocity is computed from the Biot-Savart law at a point P, as

$$d\vec{w} = \frac{\Gamma}{4\pi} \frac{\vec{r} \times d\vec{s}}{r^3} ,$$

where $\vec{r}$ is the radius vector from the element of vorticity, $d\vec{s}$ is the incremental vector length of the element of vorticity and Γ is the strength of the element of vorticity. In scalar form

$$dw = \frac{\Gamma}{4\pi} \frac{ds}{r^2} \sin\beta$$

which gives only the magnitude and not direction. β is the angle between the element of vorticity and the radius vector, r . This equation shows that the induced velocity field associated with the element of vorticity $\Gamma d\bar{s}$ is concentric circles about the axis $d\bar{s}$, the maximum velocity occurs when $\beta = \frac{\pi}{2}$ and decreases to 0 at $\beta = 0$. This means that if the vortex is curvilinear, the vortex will induce velocities on itself at any point where $\beta \neq 0$.

Helmholtz systematically studied vortices and developed theorems which describe their properties. They are:

1. The strength of a vortex filament is constant along its length.
2. A vortex filament cannot end in a fluid, it must extend to the boundaries of the fluid or form a closed path.
3. In the absence of rotational external forces, a fluid that is initially irrotational remains irrotational.

A corollary follows from Stokes theorem. In the absence of rotational external forces, if the circulation around a path enclosing a definite group of particles is initially zero, it will remain zero.

This also means that in the absence of rotational external forces vorticity remains attached to the same particles.

Vortices have the additional property that in the absence of any translatory force they move with the local velocity in the flow.

If the vortex filament is an infinitely long straight line, the flow is two-dimensional, and the induced velocity at any radius, r , in a plane normal to the vortex filament is for an incompressible flow

$$w = \frac{\Gamma}{2\pi r}$$

The flow is concentric circles about the vortex filament. As $r \rightarrow 0$, $w \rightarrow 0$, so that at the filament the velocity approaches infinity. It can be shown that the flow is irrotational everywhere except at the filament. To construct the vortex sheet trailing behind a wing, it was necessary for Prandtl to compute the induced velocity at a point associated with an arbitrary straight line finite length vortex filament segment. This is obtained by integrating the Biot-Savart law over the length of the segment. The result is:

$$w_p = \frac{\Gamma}{4\pi h} (\cos\alpha + \cos\beta),$$

where w_p is the induced velocity at the point, p , h is the normal distance from p to the vortex filament, and α and β are the angles between the line of the vortex filament, and lines joining the point p to the extremities of the filament. This is illustrated in Figure A.6. A finite length segment violates the Helmholtz vortex theorems, but can be used to represent the bound vortex in a wing, or to construct a polygonal arc to approximate curve-linear flow with a finite number of segments. Of particular interest is the problem of determining the velocity at the end of a semiinfinite straight line segment of a vortex filament for which $\alpha = 90^\circ$ and $\beta = 0$. The result is:

$$w_p = \frac{\Gamma}{4\pi h}$$

which is one-half the value for two-dimensional flow.

This result was used by Prandtl in computing the downwash at a wing due to the trailing plane rigid vortex sheet by integrating across the span.

The infinite velocity at the filament implies that for incompressible flow and constant total energy that at a certain radius the pressure would be zero, and for smaller radii the pressure would be negative or the fluid would have to accept a

$$\omega_p = \frac{\Gamma}{2\pi h} (\cos p + \cos \beta)$$

$\overline{AB}$ - length of vortex segment

h = normal distance

P = arbitrary point

ω_p = induced velocity

Γ = circulation

Figure A.6. Induced Velocity at an Arbitrary Point Associated with a Finite Straight Line Segment of Circulation, Γ

tension. This is impossible in a real fluid. The potential vortex cannot actually simulate a real vortex, but is an excellent approximation for the radius being large enough that the flow may be considered incompressible.

Oseen⁶⁰ found a solution for the two-dimensional incompressible viscous vortex in 1903. The solution is

$$w_p = \left(\frac{\Gamma}{2\pi r}\right) [1 - e^{-r^2/4\nu t}].$$

The velocity at a point P, w_p , is seen to be a function of r and t . At a given t , the velocity has a maximum at a radius r_m , when

$$r_m^2 = 2\nu (r_m^2 + 1)$$

or

$$r^2/4\nu t = 1.256431209.$$

That is, $r = 1.120906423 \sqrt{4\nu t}$. It may also be shown that at a given t $w_{\max}(r_m)/w_{\max \text{ pot}}(r_m) = 0.715331863$. This is, the maximum velocity in the Oseen vortex is 0.715331863 the velocity at the same radius, circulation, and time as a potential vortex at the radius for maximum velocity in an Oseen vortex. It may also be shown that when $r = 4.798525912 r_m = RVO$ that $w_{\text{pot}}/w_{\text{Oseen}} = 1 \times 10^{-0}$. Thus, for $r = 4.7985259128 r_m$, the velocity in the Oseen vortex may be considered to be the same as that in the potential vortex. All the effects of viscosity may be considered confined within the radius RVO. Outside that radius the flow may be considered to be nonviscous. Figure A.7 is a nondimensional plot of the velocity in an Oseen vortex and, for comparison, the potential vortex. It is seen that near the axis of the vortex the velocity increases nearly linearly with radius, but the curve is concave downward with gradually increasing curvature. A maximum velocity is reached at $r/r_m = 1.0$. The curve continues to curve downward and asymptotically approaches an equilateral hyperbola. At $r = 4.798525912 \bar{r}_m$, the curve may then be considered to be an equilateral hyperbola.

Figure A.7. Non-dimensionalized Velocity Distribution in an Oseen Vortex and Potential Vortex.

The Oseen vortex is an excellent approximation to a real vortex. Inside of RVO the flow is viscous and rotational. Outside of RVO, the flow may be considered potential and the velocity computed from the potential vortex. Viscosity limits the maximum velocity. Evidence from developed tornados indicate that the maximum velocity in tornados may reach the order of 400 mph. It may be shown that the maximum velocity for a given circulation, Γ , decreases as $\sqrt{t}$ and the radius at which the maximum velocity occurs increases as $\sqrt{t}$. If Γ is constant, the Oseen vortex gradually decays as the effects of viscosity propagate farther out into the fluid as RVO also increases as $\sqrt{t}$. Calculations show that if only viscosity is acting, Oseen vortices are persistent. It requires as much as 48 hours for the maximum velocity to decay to 10 percent of that which occurs shortly after the vortex is started.

In nature, real vortices are dissipated much more rapidly. This is believed to be due to turbulence which is much more powerful than viscosity in dissipating the vortex. Prandtl's mixing length theory of turbulence and data from turbulent mixing solution in wakes and jets can be used to estimate the effect of turbulence. When turbulence is used in place of viscosity in the Oseen solution, the time for the maximum velocity decay to 10 percent of that which occurs shortly after starting is from two to four hours. Measurements of the persistence of trailing vortices left by large aircraft in flight verifies that persistence is of the order of two to four hours. Thus, even with turbulence the vortices will persist for long periods of time, and extend far downstream from their point of origin. The Oseen solution is very useful.

The maximum velocity which occurs in tornados raises the question of the influence of compressibility on vortex structure. At 400 mph, the effects of compressibility are beginning to become important. G.I. Taylor⁶¹ obtained the solution for the non-viscous compressible vortex in 1930. The result is:

$$v^2 = v_{\max}^2 [1 - (\rho/\rho_0)^{\gamma-1}],$$

and

$$r^2 = \frac{\Gamma^2}{4\pi^2 v_{\max}^2} \frac{1}{[1 - (\rho/\rho_0)^{\gamma-1}]}$$

so that

$$\left(\frac{r}{r_1}\right)^2 = 1/[1 - (\rho/\rho_0)^{\gamma-1}],$$

where v is the velocity corresponding to r or ρ , $v_{\max}$ is the velocity that gas would achieve when exhausting into a vacuum, Γ is the circulation, and r is the radius corresponding to v_1 , and $v_{\max}$ when $\rho = 0$. For $r < r_1$, a perfect vacuum would exist, $r = r_1$, the maximum velocity would exist, and $p = \rho = 0$. At some critical radius, r , the sonic velocity is reached. For $r_1 < r < r^*$ the flow is supersonic, and for $r > r^*$ the flow is subsonic. When $r \gg r^*$ the flow may be considered incompressible, and the incompressible potential solution may be used to compute the velocity in the flow. Taylor's compressible vortex cannot exist in nature because of the effects of viscosity and turbulence, but the solution shows that the effects of compressibility may be considered to be confined to a region near the axis, and at distances far from the axis that the potential solution may be used to compute the velocity. If $(\rho/\rho_0) = 0.99$, $r/r_1 = 15.78759774$.

The solution for a viscous, compressible vortex has not been found, but its character may be inferred from the Oseen solution and the Taylor solution. Viscosity would not permit the creation of a vacuum at the core and the velocity distribution would resemble the Oseen solution. If the sonic velocity were reached in the flow, it would probably occur at the maximum velocity; otherwise, there would be subsonic core, a supersonic annular region, and a subsonic outer region. Pressure disturbance could propagate axially and radially only in the

subsonic regions, and these pressure disturbances would create shock waves which would destroy the supersonic region. This may be the phenomena known as "vortex busting." The stable configuration would be subsonic flow everywhere with the effects of viscosity, turbulence, and compressibility confined to a region near the axis of the vortex. Outside this region, the potential solution would be used to compute the induced velocity associated with the circulation. The vortex would decay with time, grow in size, but persist for relatively long periods of time.

The distributed and concentrated vorticity shed at blade flip points is distributed across the rotor from the upstream side to the downstream side. The blades in traversing the downstream side of the rotor periodically pass through the distributed vorticity and interact with the concentrated vorticity. If a vortex filament occurred, infinite velocities would occur as the blade passed through the wake or interacted with the concentrated vorticity. This would create both physical and computing problems. The bound vortex in the blade must follow the orbital or circular path, while the vortices in the wake are free to move. The interaction between the bound vortex and any free vortex is such that the free vortex would tend to rotate about the bound vortex in a circular path relative to the bound vortex. The free vortex never comes in contact with the bound vortex, but may pass quite close to the bound vortex. The free concentrated vortices can be twice as strong or stronger than the bound vortex. It can thus induce very high velocities on the bound vortex which affects the magnitude and direction of the resultant force on the blade. This can cause large local variations in the local forces as well as the integrated rotor characteristics.

The Oseen vortex and the Taylor vortex are useful in assessing this problem. A real vortex is a combination of both. Compressible effects can and do exist in real vortices, but would be confined to a region near the axis. The Oseen vortex shows that viscosity, also turbulence, limits compressibility effects and

prevents the formation of a vacuum as a core; that viscous effects are confined to a finite area near the axis; and that the maximum velocity is limited. The Oseen vortex also shows that the vortex grows in area with time, decreases in velocity with time, and is persistent.

The Oseen and Taylor vortices can be used to estimate the physical dimensions and velocities in the vicinity of the free concentrated vortices in the wake of a cyclogiro device. The bound vortex has a circulation of

$$\Gamma = \frac{C_L S V}{2b} ,$$

and the concentrated vorticity has a value of twice that amount, and the free vorticity has a circulation of

$$\Gamma_c = \frac{C_L S V}{b} .$$

The value of V at the instant the vorticity is shed when $\beta = \pm \frac{\pi}{2}$ $V=W(\lambda \pm 1)$.

To consider the worst case, the Madaras rotor can achieve $C_L = 12$, $S/b = 5.55$ m and $W = 11$ m/s. The induced velocity in a potential vortex is $w = \frac{\Gamma}{2\pi r}$, or $r = \frac{\Gamma}{2\pi w}$.

First, compute the radius, r , for the Taylor vortex at standard atmospheric conditions, of $T_o = 518.6^\circ R$. The maximum velocity air can achieve when exhausting into a vacuum is $V_{\max} = \sqrt{2C_p T_o}$, with $C_p = 557$ joules/Kg- $^\circ R$. Then $V_{\max} = 760$ m/s. In the Taylor vortex, r occurs when $\rho/\rho_o = 0$, so that $r = \frac{\Gamma_c}{2\pi r_{\max}}$. But $\Gamma_o = 12 \times 5.56 \times 11 = 733$ m²/sec so that $r_1 = 0.15$ m. Thus, if these conditions would ever be achieved, there would be an absolute vacuum core approximately 0.15m in radius. The radius, r_c , at which $\rho/\rho_o = 0.99$, that is where compressibility could be neglected $r_c = 15.79 r_1 = 2.4$ m. The velocity at this radius is 48 m/s. Similar calculations indicate the potential velocity at this radius is essentially identical in magnitude.

Thus, for this very powerful vortex, compressible effects can be ignored at radii greater than 2.4 m from the center of the vortex.

The region of influence of viscosity for the Oseen vortex will now be computed. In the Oseen vortex the maximum velocity reached is only 0.715 times the velocity in a potential vortex of the same strength at the same radius, r_m . Thus, $w = 0.715 w_{\text{pot}}$. Thus, if we assume $w_{\text{Oseen}} = 152 \text{ m/s}$, then

$$r_m = \frac{0.715 \Gamma_c}{2\pi w_{\text{Oseen}}} = 0.54 \text{ m.}$$

This is about 23 percent of the radius at which compressible effects can be ignored. However, the radius at which viscous effects can be safely ignored is $RVO = 4.80 r_m = 2.6 \text{ m}$, which is approximately the radius of the rotating cylinder used for the Madaras rotor. The velocity at that radius would be 44.5 m/s. Further, at a distance of 91.5 m from this vortex, the velocity induced by it would be 1.28 m/s.

The free concentrated vortices are not small, cause winds having tornadic velocity, and are persistent. The velocity fields associated with these vortices extend over a considerable radius before the induced velocities are small. The persistent wake could present a hazard far downstream. There is one factor which probably will alleviate this problem, and that is time. Taylor's solution does not involve time, but assumes the vortex exists in a compressible fluid, and describes relations in such a vortex. The Oseen solution does involve time; it assumes a potential vortex and that the associated flow exists prior to time $t = 0$; and describes how vorticity is altered by viscosity as a function of time. Neither solution describes how the flow is actually initiated or formed. That solution has never been found as it requires the solution for an unsteady compressible viscous flow.

One can only speculate on the sequence of events. After a vortex is shed from the surface of a body, a stagnation point is established at a point on the body determined by the combined flow around the body and the recently shed adjacent free vortex. The

fluid carrying vorticity generated in the boundary layer tends to accumulate at this newly-formed stagnation point, and it forms a vortex which is held at that position by the external pressure in the flow since the interior of the vortex has low pressure. As more fluid is accumulated it creates a bump on the body, and pressure signals propagated at the speed of sound alter the total flow pattern and establish the induced velocity field associated with the vortex. Fluid accumulates and the center of the vortex gradually moves away from the body, and the vortex begins to translate to the ultimate separation point. At a critical size it separates, and the core is a compressible-viscous Oseen vortex. It may continue to accumulate fluid and vorticity from the boundary layer while in the vicinity of the separation point until it is finally pushed away by pressures. The core is probably very intense, and the sonic velocity may be reached in the vortex. Viscosity is powerful enough due to the large velocity gradients to dominate compressible effects, and the free vortex decays similar to an Oseen vortex.

Prandtl studied the problem using photographic techniques which shows the process just described. A collection of his photographs are shown in Reference 62. The starting process is shown in the Reference in Figures 24 to 33 on pages 288 to 292. The complete set of photographs from pages 279 to 306 give a clear description of the vortex flow associated with separation of flow from bodies and the generation of lift.

The evaluation made for RVO shows that this radius can be fairly large, and that outside that radius potential flow can be used safely without viscous or compressible effects being important enough to consider. If the wake is considered rigid, RVO is the closest approach a body can make to a vortex for the wake and potential flow used. If the distance is $r_m \leq d \leq RVO$, the Oseen vortex should be used, and compressibility can still be neglected. If a body approaches r_m , it is safe to assume that it can get no closer. This is based upon the physical fact that in the presence of bodies, vortices move in the flow at the local velocity, that they

never actually touch a body, and actually move around it. This fact and the Oseen vortex model was used to compute the induced velocity on the bound vortex when in close proximity to a free vortex in the wake. In this manner singular points were awarded.

A.3.2 Kutta-Joukowski Law

Kutta⁶³ and Joukowski⁶⁴ showed independently that in two dimensional flow that if a circulation existed about a body in a uniform flow that there was a force at right angles to the relative velocity associated with the circulation. That is:

$$\vec{F} = \rho \vec{V} \times \vec{\Gamma}.$$

The magnitude of the force is a scalar at right angles to V and Γ and is $F = \rho V \Gamma$.

Joukowski⁶⁵ had developed airfoil shapes by transforming a circle into an airfoil, but found that there was no lift or drag. When a circulation (corresponding to a vortex at the center of the circle) was added, a lift was generated, but no drag existed. This law has been extended to three dimensions.

Assume a wing has uniform loading of span b . Then

$$L = \rho V \Gamma b = C_{L\frac{1}{2}} \rho V^2 S, \text{ or}$$

$$\Gamma = \frac{C_L}{2b} \frac{SV}{\alpha} = \frac{\frac{dC_L}{d\alpha}}{2b} \alpha \frac{SV}{\alpha} = \frac{\frac{dC_L}{d\alpha}}{2b} \alpha \frac{SW}{\alpha} \sqrt{1 + \lambda^2 - 2\lambda \sin \psi}$$

for a wing used as a lifting element of a cyclogiro.

It is seen that the magnitude of the circulation depends upon C_L or α and V . Thus, a change in α or V will change the circulation. Now,

$$VR = W \sqrt{1 + \lambda^2 - 2\lambda \sin \psi}$$

on the orbit of the cyclogiro, so that if C_L is held constant around the orbit, the circulation is continually changing. To satisfy the Helmholtz vortex laws, spanwise vorticity must be continually shed and pass downstream at the local velocity in the flow. Also, to satisfy the Helmholtz vortex laws, this shed vortex cannot end in the fluid but must form a closed path, and must be joined to the bound vortex located in the wing. Also, at the blade flip points, where the angle of attack is reversed, a strong concentrated vortex would be shed. This shed vortex has a magnitude equal to twice that of the bound vortex just prior to the time of shedding, and it has an opposite rotation. In between the flip points, the rate of shedding is small compared to the shedding at the flip point. The vortex shedding will be referred to as the distributed vorticity, and the shedding at the flip points as the concentrated vorticity. The ratio of the concentrated vorticity at the flip points is

$$\Gamma(\pi+\beta)/\Gamma(\beta) = \frac{\sqrt{1+\lambda^2+2\lambda \sin|\beta|}}{\sqrt{1+\lambda^2-2\lambda \sin|\beta|}}$$

It is seen that if $\beta \neq 0$, the magnitude of the concentrated vorticity shed is unequal and the difference becomes progressively larger as β approaches $\pm \frac{\pi}{2}$. Since each element of vorticity will be inducing velocity on every other element of vorticity, the distributed and concentrated vorticity will move along with the local velocity which is the vector sum of the free stream velocity and the total induced velocity at point. This causes the vortex sheet to distort in a spanwise and streamwise direction. It is possible to write differentio-integral equations which are time dependent to describe this motion, but a closed solution is not possible. An approximate solution can be generated using finite element techniques, but the cost in

computing time is prohibitive, especially when spanwise loading is considered. The time increases as $n^2 + n$, where n = number of blades x number of spanwise stations x number of points in the wake. For example, a three-bladed rotor using uniform loading, and 120 points in the wake per blade required 6,000 seconds of CDC 6600 computer time to generate a wake solution. This solution used points 15° apart on the orbit, and was not considered accurate enough to compute the performance. If 1° increment on the orbit had been used, 240 times the amount of computing time would have been required.

This solution revealed considerable qualitative information on the structure of the wake. For the first turn near the rotor the wake of each blade was nearly cycloidal downstream to the position of the first lower concentrated vortex at which point roll-up of the closely spaced points was beginning to become apparent. The upper part of the wake had points more widely spaced and distortion due to roll-up was not as great. After the first turn, distortion from the cycloidal shape becomes progressively greater as the wake tended to roll-up around the concentrated vorticity with the distributed vorticity tending to form straight lines with widening spacing between points as the roll-up progressed. After two turns the wake was nearly concentrated about the location of the concentrated vortex points with nearly uniform spacing between the rolled-up concentrated vortices. The wake was forming highly concentrated vortices which occurred in pairs closer spaced with a slightly larger distance between pairs in the streamwise direction. The wake became wider as it progressed downstream and the mean velocity in the wake decreased. This satisfied continuity.

The behavior of the wake in the spanwise direction was particularly interesting. If the wake had been rigid, the tip joining vortices would intersect. This did not happen in the free wake. As the wake approached the point of intersection, the wake from the previous blade (or if it formed a loop as at the lower part of the wake for the prolate cycloid) moved inward

spanwise, and the most recently formed wake moved outward spanwise. This occurred at each wake intersection. The wake widened in the spanwise direction as it progressed downstream.

The wake tends to roll-up in the spanwise direction around the points of concentrated vorticity, and the tip-joining vortices tend to roll-up along straight lines joining the spanwise concentrated vortices. This tends to form a series of gradually increasing rings of vorticity as the wake progresses downstream at a velocity which is less than the freestream velocity. Near the rotor the wake of each blade is pseudo cycloidal in shape, while far from the rotor the wake is a series of essentially equally spaced rings.

Larsen deduced the far field structure of the wake from considerations of vortex motions and the dominance of the concentrated vorticity shed at blade flip. He developed what he called the "Primitive Vortex Theory of the Cyclogiro" from this model of the wake, which he assumed to be a series of rigid rings of equal size and strength which translated downstream as the rotor rotated at a uniform velocity computed from an application of the momenteum theorem.

Figure A.8 is a sketch of the rotor and wake model used in the primitive theory, and shows the rectangular rings used to simulate the wake. These are essentially the concentrated vortices shed at angle of attack reversal. The segments parallel to the axis of the rotor are the vortices shed, and the segments at right angles to these at either end are referred to as the tip joining vortices. The bound vortices for this four-bladed rotor are the spanwise liner between the circles. The bound vortices were closed with tip joining vortices along radial lines. No distributed vortices were considered.

Figure A.9 is a flow field example computed with the primitive theory. The orbit of the rotor is the circle at the center of the axes. Figure A.9 is a cross section through the plane of symmetry at the center of the rotor. The concentrated

Figure A.8. Primitive Vortex Theory Rotor and Wake Array Model.

Figure A.9. Flow Field of a Giromill at Maximum Power.

vorticity is the array of circular arc arrows which form the wake similar to a von Kaiman vortex strut. Five velocity profiles were computed one rotor radius apart. These profiles begin one rotor diameter upstream, and extend to one diameter downstream of the rotor. The magnitude of the free stream velocity is the distance between the two vertical parallel lines at each profile. The induced velocity due to the wake is the horizontal distance between the left vertical line at each profile and the heavy line of the profile. The Z component of induced velocity can be inferred from the slope of each of the resultant velocity vectors shown at each profile.

At Station A the wake decreases the x component of velocity only a few percent. This effect is felt far to the side of the rotor. At approximately three radii to either side the x component of velocity is equal to freestream. At further lateral distances, it is greater than freestream as demanded by continuity.

Station B is just at the front of the rotor. The velocity defect is still small, but at approximately two radii laterally the velocity becomes greater than freestream. Slight divergence of the flow is apparent.

Station C is on the rotor's axis of rotation. Here the velocity defect is quite large at the center of the rotor. At the intersection of Station C and the rotor orbit, the velocity is almost freestream in magnitude, but the velocity vector has been rotated so that the angle of attack of the airfoil has been seriously altered. In the rotor the flow is divergent so that part of the flow passes outside the wake between adjacent vortices to satisfy continuity. Outside the wake, the velocity is considerably greater than freestream. This effect extends to infinity but dies out asymptotically.

At Station D, the rear of the rotor, the velocity defect is large, and flow divergence is quite pronounced. High local distortion near vortices in the wake is apparent. The velocity defect is nearly constant all across the wake.

At Station E, two radii from the center of the rotor, the velocity defect is still increasing, but distortion of the profile near the concentrated vorticity in the wake is occurring. This is due to the sidewash associated with the side force on the rotor.

Calculations show that the final velocity in the wake is nearly reached in one-diameter downstream from the center of the rotor. This figure illustrates that the velocity defect is changing continually through the rotor and that serious changes in the angle of attack occur. Thus, the assumption of uniform and constant velocity defect through the rotor is not justified.

The flow in Figure A.9 is for one instant of time. As the wakes continue to pass downstream, conditions are changing continuously at each point in the flow. The theory translates the wake, and computes the instantaneous velocity and angle of attack at each blade. When the forces are integrated around the orbit, a time average value is obtained.

Figure A.10 presents results of using the Primitive Theory to predict the X axis of a three-bladed giromill as a function of tip speed ratio, λ . This rotor had a span-to-diameter ratio of 1.5, and the chord to radius ratio of $C_R = 0.1$. The data was computed for a lift coefficient of $C_L = 1.5$ which is near the maximum achievable at Reynolds numbers which would occur in large rotors. The data has been nondimensionalized so that size and actual wind speed are not important. The data would apply to geometrically similar rotors at all wind speeds. Since $\lambda = 2\pi nR/W$, as W increases, n must increase proportionally if λ is to be constant.

The coordinate axes are chosen with the origin at the axis of rotation at the midspan station, the X-axis is in the direction of the wind far in front of the rotor, the Y-axis is along the axis of rotation, and the Z-axis is to the side. The flip point is assumed to be at $\beta = \pi/2$ or along the Z-axis. The theory and model used for computation was the Primitive Vortex Theory.

Figure A.10. Variation of the X-Component of the Resultant Velocity as a Function of λ for a Three-Bladed Giromill.

Thus a series of rectangular vortex rings whose strength is the mean value of vorticity at the flip points forms a nested ring of vortex rings extending downstream with a spacing between adjacent rings of $S = (2\pi/\lambda NB)$ where NB is the number of blades. These rings induce a velocity forward along the X-axis of magnitude u . The wind is blowing toward the negative axis with a speed of W . Since these two add vectorially, the resultant velocity along the X-axis is $V_x = u - W$. This is nondimensionalized by dividing by W so that

$$\frac{V_x}{W} = \frac{u}{W} - 1,$$

and if V_x is negative, the wind blows along the positive X-axis. If $V_x = 0$, all the energy in the wind would be extracted, but as energy is extracted, less mass flow passes through the rotor so that less total energy is extracted. Theoretically, the greatest energy is extracted at the Ritz limit when the final velocity in the wake is one-third the wind speed and hence 16/27 of the energy is recovered.

At any given X/R station on any of the curves of Figure A.10 (e.g., at $\frac{X}{R} = -.05$, $\lambda = 3.5$) the ordinate distance from the $\frac{X}{R}$ axis to the curve is equal to the magnitude of the local resultant velocity ratio along the X-axis at the given $\frac{X}{R}$ station; i.e., the distance equals V_x/W . Also, the distance from the horizontal line where $\frac{V_x}{W} = 1.0$ to the point on the curve ($\frac{X}{R} = -0.5$, $\lambda = 3.3$) is the magnitude of the local induced velocity ratio, V/W . The dotted lines show the leading and trailing edge of the rotor. It is seen that for each value of λ , the rotor effects the wind speed far upstream from the rotor. At the leading edge of the rotor, $V_x < W$ which means the "rocking angle" of the blade must be larger than the ideal theory would predict. Since V_x/W decreases very rapidly as the wind passes through the rotor and since Figure A.9 indicates that the resultant velocity ratio, V_x/W is almost constant across the rotor in the Z direction at a given X/R , the induced velocity changes the rocking angle

differently all around the orbit. This is equivalent to Prandtl's induced angle of attack in three-dimensional wing theory. If the change in rocking angle can be computed, two-dimensional airfoil data can be used to compute the lift and drag.

It is noted that at each value of λ , the final velocity in the wake has been nearly reached at the downstream edge of the rotor. As λ increases the rings in the wake are packed much closer since the spacing is inversely proportional to λ . In addition, the mean circulation in the wake increases directly as λ . The combined effects cause the rapid decrease of V_x/W as λ increases. At $\lambda = 3.80$, the final velocity in the wake is zero. As λ increases the curves become smoother because of the close packing. At low values of λ , V_x/W fluctuates in the wake since the spacing is quite large. The actual flow is very similar to the von Karman vortex street which causes the X component of velocity to fluctuate (e.g., examine the curves for $\lambda = 1, 1.5$).

Since $V_x < W$ at all values of X/R , then by continuity, this flow must go somewhere, as less fluid is in the wake than in the freestream. This fluid flows around the rotor as it would flow around a solid object, and the velocity outside the wake is greater than the freestream velocity. Since the velocity continues to decrease in the wake downstream of the rotor, the fluid continues to flow out of the wake between adjacent vortex rings. Detail calculations of the y and z components of velocity, v and w, verifies this.

When U is computed along the line of vortices at $Z = R$ and $y = b/2$, it is found that the rings do not move at the freestream velocity, W, nor at the velocity V_x , but something close to V_x . There is also a velocity which causes the wake to expand. This is very small before the flow out of the wake dominates. The wake is very nearly rigid.

The wake translates at a velocity relative to the rotor. The curves in Figure A.11 are for one instant of time and correspond to the flip point time, so the first ring is at the Z axis.

Figure A.11. Rotor and Wake Structure Assumed for Primitive Vortex Theory for a Typical Cyclogiro.

As the blades rotate the complete wake translates essentially as a rigid body, and the distance between the point and the wake increases, and the local velocity at a point decreases. This is equivalent to the point moving upstream if the wake were stationary. When the next blade flips, the flow abruptly returns to conditions as in Figure A.11. This is unsteady flow similar to helicopter blade flapping.

The theory accounts for this unsteady effect by translating the wake continuously, and because of the finite ramp angle in the lift reversal smooths out the abrupt blade flip.

The primitive theory was used by McDonnell-Douglas to analyze the performance of the giromill. The results were in excellent agreement with wind tunnel tests. Larsen recognized the limitations of the theory and continued to try to improve it. In the process he developed an exact theory including all three-dimensional effects, ground effect, and wind shear or nonuniform velocity profile. Blade mutual interference was included in the primitive theory as well as in the exact theory. However, the prohibitive computation time for the exact theory make its application impractical. Therefore, in an effort to produce a more exact and useful theory than the primitive theory, Larsen combined features of the primitive and the exact theory to produce a new approximate theory which he called the Improved Primitive Theory. This theory used an approximation to the near field, the far field ring approximation, and an approximation for the widening of the wake. The computation time was found to be only slightly greater than that of the primitive theory, and the results were believed to be more accurate. The improved theory has not been verified experimentally, but it corrects some of the deficiencies of the primitive theory.

This improved theory will be described in the sections which follow.

SECTION A-4

IMPROVED PRIMITIVE VORTEX THEORY OF THE CYCLOGIRO

The improved primitive vortex theory of the cyclogiro assumes that the wake is a series of equal-strength, equally spaced concentrated vortices arranged on an empirically widening wake based on results of the exact theory. The widening is in both the spanwise and lateral direction. Each bound vortex is joined to a concentrated vortex in the wake corresponding to the position it has translated from the flip point. The upstream bound vortices are joined to concentrated vortices in the wake on the negative Z side of the wake on top of the rotor, and the downstream bound vortices are joined to concentrated vortices on the positive Z side of the wake, or bottom of the rotor. The joining vortices are assumed to have a strength equal to the local bound vortex strength, and has a spanwise closing vortex coincident with the concentrated wake vortex. Each vortex segment in the wake and in the bound vortex ring system is assumed to be a straight line. All vortices in the wake translate downstream at a velocity computed from application of the momentum theorem. This array is an approximation to the configuration obtained from the exact theory, and does not satisfy all the requirements of the Helmholtz Vortex laws.

The strength of the bound and semibound system can be computed, while the strength and location of the free vortices in the wake is unknown, but is found from an iterative computation. This is accomplished by computing $\overline{FX}$ and $\overline{FZ}$ from the ideal theory. The momentum theorem is then used to compute the final velocity in the wake. The wake widening is computed, and the empirical wake boundary determined. One-half the final wake velocity increment is added to the free stream velocity to compute the translational velocity of the wake and the spacing of the free vortices in the wake. This locates the free vortices in the wake, but the strength of the free vortices is as yet unknown.

To compute the strength of the free vortices, the mass flow and momentum flux through the capture area are computed. The capture area is defined as the product of the span and the chordline joining the trailing edge of the blades at the flip point. The mass flow and momentum flux through the capture area is computed at a number of points across the capture area and integrated across the capture area, and the mean value of the velocity increment computed. This mean value must be equal to the value computed from the forces on the blades, $\overline{FX}$ and $\overline{FZ}$, and the momentum theorem. The strength of the free vortices is adjusted, and the problem iterated until the increment of velocity computed from the momentum theorem is within a prescribed tolerance of that computed from the induced velocity computed from the free wake and the bound-semi-bound vortex system. When this is achieved, the wake structure and strength has been determined. This wake structure is translated downstream as the rotor rotates. The induced velocity at each point on the orbit can then be computed. The local forces on each blade are then computed including the induced velocities of the wake and the mutual induced velocities due to the bound-semibound vortex system. Iteration is required to include the mutual induced velocities, and this iteration converges rapidly. Once convergence to a predetermined tolerance has been achieved, the final performance is computed. This whole procedure has to be repeated for each value of λ , w , α , β , and Δ used.

Subroutines were written which computed specific data used in the program. These subroutines in some instance had to be modified for use of airfoil blades or spinning rotors and possible options.

The blade modulation scheme assumes that positive lift occurred if $(\pi+\beta) \leq \psi \leq (2\pi+\beta)$ and negative lift if $\beta \leq \psi \leq (\pi + \beta)$. Since it is impossible to flip the blade or reverse the direction of rotation of spinning cylinders instantaneously, a constant deceleration-acceleration ramp occurred at each flip point, β and $\pi + \beta$. This occurred over a finite

phase angle, ψ , of width 2Δ . Δ is called the ramp angle. For blades, the angle of attack is modulated and the subroutine which specified the modulation was the subroutine ALFA. This subroutine also permitted constant geometric angle of attack with parabolic variation in the acceleration-deceleration ramp, constant effective angle of attack, sinusoidal variation of angle of attack, or rigid blade setting as in the Darrieus rotor.

The Madaras rotor used spinning rotors, and the lift of the rotor depends upon the ratio of the peripheral rotational velocity of the cylinder, U , to the relative wind speed; i.e. U/V

$$VR = W \sqrt{1 + \lambda^2 - 2\lambda \sin\psi}$$

Since UR varies, the rotor velocity U had to be modulated. The subroutine UØVSR controlled the rotor spin modulation with appropriate deceleration-acceleration ramps. Two options were available: constant U/V or constant U .

The lift coefficient C_L and drag coefficient C_D are functions of the angle of attack α . The variation of C_L and C_D for 360° angle of attack has been found from wind tunnel tests for the NACA0015 and other airfoils. Beyond the stall in normal flow and reverse flow, a nearly universal curve occurs for all airfoils. An empirical fit was found to fit the data to within less than 1 percent. To allow for input, the lift coefficient at the end of the linear range and maximum lift coefficient in normal flow and reverse flow, the zero lift drag coefficient, and increment of drag coefficient between zero lift and end of the linear region are all that is necessary. A subroutine CONS computes the constants used in the subroutine TDCLCD which then computes C_L and C_D for any angle of attack positive or negative. That is, $-180^\circ \leq \alpha \leq 180^\circ$.

The lift coefficient, C_L , and the drag coefficient, C_D , of spinning cylinders was obtained from wind tunnel tests conducted at the University of Michigan. They were found to depend upon the ratio of the cylinder peripheral velocity, U , to the relative wind speed, V , or U/V , and the aspect ratio, A , and

the end plate ratio, e/d . It was found that a piecewise curve fit could be used to fit the test data in three regions. For $U/V \leq U/V_{crit}$ a parabolic approximation could be used. For $U/V_{crit} \leq U/V \leq U/V_{max}$ a quartic approximation could be used. For $U/V_{max} \leq U/V$, C_L and C_D were essentially constant. U/V_{crit} is different for C_L and C_D . Eleven constants have to be determined for both C_L and C_D . These are input data to the program. In use, U/V is determined, and the subroutine MRCLCD computes C_L and C_D for the absolute value of U/V , and the sign on U/V determines the algebraic sign of C_L . C_D is always positive.

The subroutine VSIVCP (Vortex Segment Induced Velocity Component at the point P) computes the Cartesian induced velocity components at an arbitrary point P associated with a straight line vortex segment of known strength whose end points are known in the rotor axis system. It uses $w = \frac{\Gamma}{4\pi h} (\cos \psi_1 + \cos \psi_2)$. This includes the Oseen vortex for points which have $h \leq RVO$. This subroutine is the heart of the program.

The subroutine NASVR (Number and Strength of Vortex Rings) computes the number, strength, and spacing of the free vortex rings which form the wake with wake widening included. It includes the free system and the bound-semibound system, and computes the wake structure based on flow through the capture area.

The subroutine TRIVCO (Translating Ring Induced Velocity Components on the Orbit) computes the Cartesian induced velocity components at each blade on the orbit as the rotor rotates. The free wake is translated downstream and widened as the rotor rotates. It assumes the wake structure computed from NAVSR, and relocates each concentrated vortex on the wake boundary as the free vortices translate downstream with rotor rotation.

The subroutine TBIVCO (Translating Bound Induced Velocity Components on the Orbit) computes the Cartesian induced velocity components of the mutual induced velocities associated with the bound-semibound vortex system at each point on the orbit as the

rotor rotates. This uses the bound vortices and the rings formed by joining free vortex locations in the translating wake computed from NASVR including the wake widening effect.

The subroutine BRING (Bound Ring) computes the Cartesian induced velocity components at an arbitrary point associated with the vortex ring formed by a bound vortex, its associated free vortex in the wake, and the tip joining vortices, the end points of each segment being known. It is used by TBIVCO to compute the mutual induced velocities on the orbit associated with the bound-semibound vortex system.

The subroutine RING computes the Cartesian induced velocity components associated with a free vortex ring in the wake whose strength and end points of each segment are known. It is used with TRIVCO to compute the induced velocity components associated with the free wake.

With minor modification TBIVCO and TRIVCO can be modified to compute the induced velocity at an arbitrary point instead of the orbit. An arbitrary curve or line can be developed and the induced velocities computed on that path. This has not been included in the program to reduce the core memory requirements. If it is desired it can be done easily. The complete flow field can then be mapped for one instantaneous position of the rotor. By computing the flow field for successive rotor positions, non-steady effects could be determined if desired. It would be an expensive and time consuming undertaking. It is not believed to be of significant importance in large radii rotors, but may be important in small radii rotors. This has been investigated in a few examples which is the basis for the opinion expressed.

The primary difference between the exact theory and the improved primitive theory is that in the improved primitive theory a semi-rigid wake structure based on results of the exact theory is assumed, while in the exact theory the wake is formed step-by-step by computing the resultant velocity and displacement of the end points of each straight line vortex segment as the rotor is rotated in finite increments. The displacements are computed as

the sum of displacements along circular arcs associated with every vortex segment in the system. The computing time increases as $n^2 + n$ where n is the number of segments in the system.

Once the induced velocities have been found, computation of the rotor performance is relatively straightforward.

The program uses a series of nested DO loops to compute the performance for each of the primary variables β , w , λ , and α or U/V or U . Delta, Δ , the acceleration ramp, is a simple input variable. The initial value of β , w , λ , and α or U/V or U and the range on each DO loop are input parameters which enable computation to start and end at any selected range of the parameters. The rotor geometry and modulation schedule to be used are also input variables. The number of points on the capture area and the orbit, number of blades, and length of the wake in number of rotor radii are also the input variables. In addition, the necessary constants used to define the lift coefficient and drag coefficient functions are also input variables. Printing options are available to restrict the output to only rotor performance, or if desired, detailed information at each point on the orbit; including blade lift and drag, rotor x force, Z force, torque force, radial force, induced velocities due to mutual interference and wake, ϕ , ϕ_r , C_L , C_D , α , α_i , U/V , U , cylinder rps and rpm, depending on the program used.

The first DO loop starts with the initial value of β and computes the performance for a range of β desired. The second DO loop starts with the initial wind speed and computes the performance over a range of wind speeds. The third DO loop starts with the initial value of λ and computes the performance over the range of λ . The fourth DO loop starts with the initial angle of attack, α , or U/V or U and computes the performance over the range of the variable used. With the range of each DO loop specified as input variables, it is possible to restrict each DO loop to a single value or to cover any desired range. This allows flexibility in selecting the data desired as well as the range desired. Once β , w , λ , and α , or U/V have been

determined, nominal values of C_L and C_D are computed to determine the wake structure. NASVR is called, and the wake structure is computed for the current values of β , w , λ , α , or U/V and C_L and C_D . TRIVCO is then called to compute the wake associated induced velocities on the orbit which need to be computed only one time. A DO loop is then used to compute the bound-semibound induced velocities on the orbit. Subroutine ALFA or UOVSR is used to compute the local values of C_L and C_D with TDCLCD or MRCLCD to include the induced velocities and TBIVCO called with the circulation, Γ , determined iteratively to a desired tolerance at each point on the orbit. After convergence, the performance, with the induced velocities included, is finally computed. The DO loop returns to complete the computation for the range specified. Usually, α is held constant, and a new value of λ computed. After the λ DO loop has been satisfied, control returns to the w DO loop which is incremented, and the computation repeated for the range of w . Control then passes to the β DO loop.

The output is in British or SI units. The final output is in both units while the detailed data is in which ever system has been selected for computation. Input data may be in either units, but must be consistent. All input angular data is in degrees, but the program computes solely in radians which are converted to degree measure for output.

Tolerances are provided on the induced velocities computed and the circulation Γ . These can be adjusted, but as the tolerances are reduced, the computing time is increased. Usually 1 percent accuracy is used. Limits are placed on the number of iterations to conserve computing time if convergence does not occur. If convergence does not occur, the program progresses to the next data point when the limit is reached and no data is computed for that value of the current value of λ . This has occurred rarely when the limiting induced velocity in the wake has been approached. The tolerance on Γ is not a percentage but has to be in ft^2/sec (m^2/sec). This value is usually selected

as 1 percent of the mean value of Γ or

$$GTOL = \frac{C_L SW \sqrt{1 + \lambda^2}}{200b}$$

This has been found to be quite satisfactory.

SECTION A-5

BLADE MODULATION SCHEDULE

As was shown in the ideal blade theory, the rotor coefficients are proportional to the lift coefficient, C_L , and that this lift coefficient must be reversed in algebraic sign at two points on the orbit, β and $\beta + \pi$, if useful power is to be extracted by the rotor. The largest lift coefficient should be used if the drag coefficient is not too large. If the drag coefficient can be neglected, the largest lift coefficient should be used. Ideally, the maximum power will result if the lift coefficient is reversed instantaneously at the blade flip points. Physically this is impossible since the blade and the rotating cylinder have a moment of inertia about their axis of rotation. A symmetrical airfoil has zero moment about its aerodynamic center in a steady uniform flow, but it can be shown that in curvilinear flow and nonsteady oscillatory flow that a pitching moment exists. The rotating cylinder has a viscous torque which is proportional to square of the angular velocity due to the effect of viscosity on the exposed surface of the cylinder and the end cap. The effect of inertia usually dominates, but the aerodynamic torque can be large and should not be neglected. The equation of motion for the lift coefficient reversal may be written as:

$$I \ddot{\alpha} = Q - Q_a$$

where I is the moment of inertia, α is the angle of attack, Q is the torque applied to reverse the lift coefficient, and Q_a is the aerodynamic torque. Neglecting the aerodynamic torque, and integrating twice with time when assuming the applied torque is constant results in the following

$$I\alpha = \frac{Q}{2}t^2, \text{ or } \alpha = Qt^2/2I, \text{ or } t = \sqrt{2I\alpha/Q}, \text{ where } \alpha \text{ is}$$

the angle of attack change. Note also that $\alpha = Qt^2/2I$. It is seen that the angle of attack variation would be parabolic with time, which can be expressed as the phase angle through which the blade moves while the angle of attack reversal is occurring.

The angular velocity of the blade in changing angle of attack is $\dot{\alpha}$, and is seen to vary with time. Since the angle of attack must go from a constant positive value to a constant negative value, the blade must first be accelerated to zero angle of attack, and the decelerated to a constant $-\alpha$. Thus, an acceleration-deceleration ramp must exist. Now $dt = R d\psi / V = R d\psi / (\lambda W)$ so that $t = R \Delta / (\lambda W) = 2I\alpha / Q$, where Δ = the increment of phase angle the blade moves through during acceleration and deceleration at a constant torque which must be reversed in sign when $\alpha = 0$. Thus,

$$\Delta = \frac{\lambda W}{R} \sqrt{\frac{2I}{Q}} \quad \text{or} \quad Q = \left(\frac{\lambda W}{R}\right)^2 \frac{2I\alpha}{\Delta}$$

The acceleration-deceleration ramp is illustrated in Figure A.12.

Either the torque may be specified and the ramp angle Δ computed, or the ramp angle specified and the required torque computed. A similar analysis for the cylinder rpm merely replaces the angle of attack by the rpm. This simple analysis neglects the aerodynamic torque which also increases the required ramp angle or the applied torque.

The torque required to reverse the lift coefficient for a blade is quite small, and amplidyne servos which have an almost instantaneous and rapid reversal characteristic are available in almost any size required. However, electric motors are used to spin the rotors, and the power required to reverse the direction of rotation can be a very significant fraction of the power developed by the rotor. Some power regeneration is possible during the deceleration phase, but it is only a small fraction of that required to accelerate.

Figure A.13 is a modulation schedule used in the program to show the angle of attack, α , schedule of the blade for one revolution. In each of the acceleration/deceleration ramp regions, Δ , the angle of attack variation is a parabola. In between blade reversal, the angle of attack is constant. For the spinning cylinders, U/V or U follows the same schedule as the cylinder peripheral velocity $U = 2 \pi r n$ is changed in ramp angle interval Δ , and the rpm held constant between reversals.

β = ECCENTRICITY PHASE ANGLE
 Δ = ACCELERATION-DECELERATION RAMP ANGLE
 R = ORBIT RADIUS

Figure A.12. Acceleration-Deceleration Ramp Angle.

σ_v IS CONSTANT ANGLE OF ATTACK USED
 V IS THE CONSTANT VALUE OF V USED
 (V/V_v) IS THE CONSTANT VALUE OF (V/V) USED

ψ PHASE ANGLE ~ DEGREES MEASURED FROM β (SEE FIGURE A.12)

Figure A.13. Nondimensional Modulation Schedule for α , V , or U/V .

For the blade $C_L = \frac{dC_L}{d\alpha} \alpha$, where C_L is the lift

coefficient, $\frac{dC_L}{d\alpha}$ is the slope of the lift coefficient versus angle of attack curve in the linear range, and α is the angle of attack. Thus C_L follows the same schedule as the angle of attack.

The spinning cylinder lift coefficient was found in wind tunnel tests to be a function of U/V , where V is actually the relative wind to the cylinder, or the wind tunnel speed. On the track, the ideal relative speed is $VR = W \sqrt{1 + \lambda^2 - 2\lambda \sin\psi}$ so that actually $C_L = C_L(U/VR) = C_L(U/[W \sqrt{1 + \lambda^2 - 2\lambda \sin\psi}])$. That is, if constant U is used, C_L will be variable around the track. If it is desired to maintain constant C_L , it will be necessary to be continually changing U as the cylinder progresses around the track. The gain in useful power may offset the power expended to regulate C_L . If U is held constant at a value which produces near maximum power over the upper half of the orbit, the lower half of the orbit will go into super circulation and operate at the maximum C_L . Under these conditions, more net power may result since none is spent in regulating C_L . As W or λ decreases, U , and hence the cylinder spin, can be greatly reduced which reduces the power expended in rpm reversal. The control system must sense conditions and automatically adjust.

The lift coefficient for the spinning cylinder is not a linear function of the speed ratio U/V . Thus, if U/V is modulated parabolically with ψ , during lift reversal, C_L will not have a parabolic variation with ψ .

SECTION A-6

IDEAL BLADE VECTOR DIAGRAM

An ideal blade vector diagram, which shows the relative velocities and aerodynamic forces acting on the blade at an arbitrary phase angle ψ , is presented in Figure A.14. The view is looking along the axis of rotation out the positive ψ axis. The circle is the path around which the blade moves in a counterclockwise direction at constant angular velocity $\Omega = 2\pi n$. R is the radius of the path and n is the number of revolutions per unit time. The peripheral speed is $V = \Omega R = 2\pi n R$ and is tangent to the path. The wind speed is W and is in the direction of the negative X axis. The velocities are shown relative to the blade which may be thought of as being instantaneously at rest. VR is the resultant aerodynamic velocity which produces the aerodynamic forces of lift and drag, where $L = C_{L2} \frac{1}{2} \rho S V R^2$ is the lift and $D = C_{D2} \frac{1}{2} \rho S V R^2$ is the drag. The phase angle, ψ , is less than the eccentricity angle, β , so this represents conditions for a lifting/thrusting rotor so that the angle of attack, α , is positive. The resultant velocity, VR , is at the angle ϕ relative to the X axis, and the drag, D , acts parallel and in the direction of VR . The lift, L , acts at right angles to VR in the direction shown, since α is positive. If α were negative, the lift would act in the opposite direction which it would do for a windmill, and β would be negative and in the upper quadrant.

The lift and drag when added vectorially give the single resultant aerodynamic force which is resolved into components FX along the X axis, FZ along the Z axis, FR along the radius, and FQ normal to the radius. FX is the local contribution to the rotor thrust, FZ is the local contribution to the rotor lift, FR is the local aerodynamic force along the radius which with the inertia force designs the radial blade support arms in tension and compression, and FQ is the local torque force which is positive as shown and would require power to turn the rotor. The power is $P = FQ.V$. Algebraic expressions were developed in the section on Ideal Blade Theory for the X and Z components of velocity, and FX ,

Figure A.14. Ideal Blade Element Vector Diagram of Cyclogiro.

FZ , FR , and FQ . When these are integrated around the orbit the rotor lift, thrust, torque, and power are expressed as time average values as $\overline{FX}$, $\overline{FZ}$, $\overline{FQ}$, and $P = \overline{FQ} \cdot V$. The blade orientation relative to X axis is given by $\chi = \phi - \alpha$, where χ is the so-called "blade rocking angle." When χ is computed, a cam could be cut which would automatically orient the blade. Kirsten showed that the "swinging-sliding block" mechanism would produce the ideal angle ϕ for all values of λ , and with a servo motor to follow this motion would drive the blade properly for all λ except $\lambda = 1$. An eccentric cam contoured to the blade modulation schedule could subtract α from ψ to obtain χ . The axis of the cam would be oriented at the angle β to control the amount of thrust and lift developed. Differential α control would provide a rolling moment about the x axis so that ailerons would not be needed. Differential β control would produce a yawing angle about the z axis so that a rudder would not be needed. Step inputs to α would provide direct lift control so that extreme maneuverability of the vehicle could be achieved. When β is negative, the device autorotates, extracts power from the wind stream and becomes a windmill.

SECTION A-7
REAL BLADE VECTOR DIAGRAM

When the rotor develops a net thrust and lift, $\overline{FX}$ and $\overline{FZ}$, the momentum theorem requires that a velocity increment be given to the fluid in the opposite direction to the force generated. Let these increments be WX and WZ . Figure A.15 is a Real Blade Vector Diagram where WX and WZ have been added as shown with all else the same. The resultant velocity has been changed in magnitude and rotated to the direction ϕ_r . This in turn rotates and changes the magnitude of L and D . Compare with Figure A.3. Note that FX , FZ , FR , and FQ , have all been changed in magnitude. The thrust force and radial force, FX and FR , have been reduced in magnitude, and the rotor lift and torque force, FZ and FQ , have been increased in magnitude. Thus, it will take more power to produce less thrust. Since FX and FZ will vary all around the orbit, WX and WZ will vary all around the orbit, and the use of constant values assumed by many investigators is not valid. When acting as a windmill, FX and FR are increased in magnitude, and FQ and FZ reduced. In both cases, the presence of the velocity increments WX and WZ reduces the desired performance of the rotor. The central problem in cyclogiro theory is to be able to predict WX and WZ at every point on the orbit. This led to the vortex theory.

Figure A.15. Real Blade Element Vector Diagram of Cyclogiro.

SECTION A-8

CIRCULATION AT THE BLADE AS A FUNCTION OF PHASE ANGLE

Kutta and Joukowski showed independently that a circulation was required to produce a lift force and formulated the Kutta-Joukowski law as: $L' = \rho V \Gamma$. For a uniform finite span wing, this becomes $L = \rho V \Gamma b = C_L \frac{\rho S}{2} V^2$, from which $\Gamma = C_L S V / 2b$. This circulation, Γ , is simulated mathematically by a bound vortex in the wing. For a blade traversing the orbit the circulation becomes:

$$\Gamma(\psi) = \frac{C_L S}{2b} W \sqrt{1 + \lambda^2 - 2\lambda \sin \psi}, \text{ and is}$$

seen to be a function of ψ , the phase angle. The maximum value occurs when $\psi = -\pi/2$ and the minimum when $\psi = \pi/2$. These values are:

$$\Gamma_{\max} = \frac{C_L S}{2b} \lambda (1 + \lambda) \text{ and } \Gamma_{\min} = \frac{C_L S}{2} |1 - \lambda| \text{ in magnitude.}$$

It is convenient to nondimensionalize $\Gamma(\psi)$ by dividing by $\Gamma_{\max}$ to remove the effect of C_L , W , and geometry. The result is:

$$\frac{\Gamma(\psi)}{\Gamma_{\max}} = \frac{\sqrt{1 + \lambda^2 - 2\lambda \sin \psi}}{(1 + \lambda)}$$

This function gives the nondimensionalized circulation as a function of phase angle for constant lift coefficient as the blade traverses the orbit. It is plotted in Figure A.3 for the value of $\lambda = 2$ for the ideal blade. It is seen that $\Gamma(\psi)/\Gamma_{\max}$ has considerable change around the orbit solely due to the changing resultant velocity. The only way that the bound vortex can change and satisfy the Helmholtz Vortex laws is to shed vorticity

to form a wake which must form closed paths and be connected to the bound vortex. The shed vorticity is shed spanwise parallel to the bound vortex. The shed vorticity is shed spanwise parallel to the trailing edge of the blade, bends forward at each tip and joined to the bound vortex to form a rectangular ring. Its strength is the differential, $d\Gamma$. As the blade traverses the orbit, a continuously varying sheet would be formed. If the sheet were rigid as Prandtl and Goldstein assumed, the sheet shed from each blade would form a cycloidal surface. The vorticity shed in the sheet would have opposite rotation to the bound vorticity.

The nondimensional rate of shedding with respect to phase angle is easily found to be:

$$\frac{1}{\Gamma_{\max}} \frac{d\Gamma(\psi)}{d\psi} = \frac{-\lambda \cos \psi}{(1 + \lambda) \sqrt{1 + \lambda^2 - 2\lambda \sin \psi}}$$

The differential strength of vorticity shed is

$$d\Gamma = \frac{d\Gamma(\psi)}{d\psi} d\psi$$

and the exact strength of vorticity shed at periodic intervals is $d\Gamma = \Gamma(\psi + d\psi) - \Gamma(\psi)$. This will be used later.

The nondimensional rate of vortex shedding at $\lambda = 2$ is plotted in Figure A.2. It is seen that it is zero at two points on the circle - $\frac{\pi}{2}$ and $\frac{\pi}{2}$. From $-\frac{\pi}{2}$ to $\frac{\pi}{2}$, $\frac{1}{\Gamma_{\max}} \frac{d\Gamma(\psi)}{d\psi} < 0$, and from $\frac{\pi}{2}$ to $\frac{3\pi}{2}$, $\frac{1}{\Gamma_{\max}} \frac{d\Gamma(\psi)}{d\psi}$ is positive.

The reason is that from $+\frac{\pi}{2}$ to $\frac{3\pi}{2}$ the velocity is increasing. It is seen that there are two maxima. It is easy to show that for $\lambda < 1$ the maxima occurs at $\sin \psi = \lambda$, and for $\lambda > 1$, it occurs when $\sin \psi = 1/\lambda$. These points are marked in Figure A.2 at the tick marks, and for $\lambda = 2$ occurs at $\psi = \pi/3 = 30^\circ$ and $\psi = 5\pi/6 = 150^\circ$. It is also seen that $30^\circ \leq \psi \leq 150^\circ$ that the rate of shedding changes much more rapidly than for $-\frac{\pi}{2} \leq \psi \leq \pi/3$ and

$5\pi/6 \leq \psi \leq 3\pi/2$. Figure A.4 shows that the local strength of vorticity shed in a rigid cycloidal sheet is extremely variable with regions of positive and negative vorticity when the lift coefficient is constant. It also shows that the part of the cycloid above the x axis has less shed vorticity than the part of the cycloidal surface below the x axis or for positive Z. Since the area under the curve is proportional to the area between the axis and the curve, this can be seen by comparing the area between $\psi = -\pi/2$ to $\psi = 0$ with the area from $\psi = 0$ to $\psi = \frac{\pi}{2}$. Thus, the shedding is more concentrated below the x axis. This fact will be utilized later.

If the blade were flipped instantaneously at the flip point, the lift coefficient would go from C_L to $-C_L$ or from $-C_L$ to C_L . The vorticity shed at blade flip would be twice the local value in Figure A.1 at $\psi = \beta$ and $\psi = \beta + \pi$. Naturally, the local value can be anything between the minimum and the maximum depending upon the actual value of β which is between $-\pi/2$ and $\pi/2$. The lift coefficient reversal changes the sign of the circulation shed. If $\beta = -\pi/2$, all the shed vorticity is the same rotation such that the sum of twice the minimum and all the vorticity shed with phase angle is equal to twice the maximum. This is actually true for all values of β , but the value shed at blade flip must be taken into account. When an acceleration-deceleration ramp is used instead of an instantaneous blade flip it is still true. Instantaneous blade flip would create discontinuities in the circulation distribution which would mean highly concentrated vorticity. An acceleration-deceleration ramp would result in a continuous circulation distribution, but two regions of very strong local vorticity. If the vortex sheet were rigid, this would only be a mathematical problem in evaluating integrals used in computing induced velocities. Discontinuities would cause improper integrals which would require special care. Actually, the discontinuity is a singular point. The acceleration-deceleration ramp removes this discontinuity and provides a continuous function.

Mathematically, the shed vorticity is continuous and would form a continuous sheet of zero thickness if a real fluid viscosity is present and forms a boundary layer on the surface of the body. Joukowski showed that a cusp at the trailing edge created a singular point which would be removed by adding a proper amount of circulation. Von Karman⁶⁶ extended this to a finite trailing edge angle. Prandtl⁶⁷ examined the viscous flow near the sharp trailing edge experimentally, and made approximate calculations to try to show the formation of a vortex at the trailing edge, called the starting vortex, as the trailing edge singularity in potential flow was removed physically. Associated with this viscous action is the formation of the boundary layer on the surface of the airfoil. This boundary layer is continuously generating vorticity which tends to accumulate at the trailing edge. The rate of accumulation is different on the upper and lower surface due to the velocity distribution. Because of this, a critical accumulation is reached alternately on the upper and lower surface which is periodically shed. This means two things: the bound vortex strength fluctuates about a mean value, and the vortex sheet is not shed continuously, but in discrete finite increments of alternating sign. Careful wind tunnel measurements and flow visualization techniques have verified these deductions for steady stream flow. The von Karman⁶⁸ vortex sheet behind cylinders is the classic example. When these concepts are applied to the varying circulation and shedding of the cyclogiro, it may also be deduced that the vortex sheet shed is not continuous, but is actually a series of discrete finite vortices shed periodically. The frequency and strength of these discrete vortices is unknown, but provides physical arguments for using a discrete vortex wake array to model the real wake. Such a model will be discussed next.

SECTION A-9

SEMIRIGID WAKE STRUCTURE

Associated with each differential segment of a vortex filament is an induced velocity field which extends to infinity, and which is given by the Biot-Savart Law. The total induced velocity at any point is obtained by integrating the contribution of each element over all the vortex filaments in the field. Prandtl showed that when the total induced velocity is combined vectorially with the free stream velocity, an induced angle of attack results which changes the angle of attack, and hence lift coefficient, which an airfoil in the resultant flow produces. This influences the circulation about the airfoil and the vorticity shed in the flow. Differentio-integral equations can be written which must be solved to obtain a solution. These equations may be time dependent. Prandtl assumed a steady flow with a flat trailing rigid vortex sheet for an isolated wing with a continuous distribution of vorticity in the sheet. Prandtl found that an elliptic spanwise loading for the bound vorticity satisfied the integrals for lift and drag of a wing and resulted in the minimum drag for the restrictions imposed.

It is possible to write similar equations for the cyclogiro if the wake is assumed to be rigid cycloidal surfaces, but the equations are time dependent. At one position of the rotor, the equations are time independent, but the equations cannot be solved in closed form. It is possible to compute the induced velocity at any point in the field by assuming finite filaments in the spanwise and streamwise direction, and approximating the rigid cycloidal surface by a mesh of finite length segments to form polygonal arcs. Numerical integration is then performed adding the increments of induced velocity at a point vectorially. Spanwise distribution can be handled by using a finite number of control points across the span. Convergence is achieved by iteration. To set up the problem, the shape of the wake and the vortex strength of each segment in the network must be known. The shape can be assumed to be cycloid for the wake surfaces, and the strength can be computed from the bound vortex circulation

distribution with phase angle, including lift coefficient reversal and acceleration-deceleration ramp effects. This is accomplished by using finite increments of phase angle. The vorticity shed at each increment of phase angle becomes $\Delta\Gamma = \Gamma(\psi + d\psi) - \Gamma(\psi)$. The spanwise loading can be approximated by using finite increments in the bound vorticity which approximates the elliptical loading or the Shrenk loading to estimate conditions. Iteration would produce the final loading. The experimental evidence that the vorticity shed from a wing is finite periodic increments rather than continuous variation tends to support the assumption of finite element analysis.

In principle, a finite system of simultaneous equations could be written to find a solution which would be solved by matrix techniques. The number of equations would be enormous, and inverting the matrix would probably saturate the largest computer known, and, if it did not, would require so much computation time that it would not be feasible. An alternate approach would be to start the rotor impulsively and compute the vorticity shed and track its location in the wake. A free vortex moves at the local velocity in the flow which is the vector sum of the freestream velocity and the induced velocity at a point due to all the finite segments in the flow. This includes all nonsteady effects, and would generate an approximation to the true wake. Initially, the computation time is quite small, but as the number of vortex segments in the flow increases the computing time increases as $n^2 + n$ where n is the number of segments in the flow. In this approach, it would be necessary to continue the wake generation until the induced velocity at points on the orbit approached a limit. This requires enormous core memory or mass storage and computing time for the simple case of uniform loading.

Since the velocity in the wake gradually decreases from free stream value far upstream to the value computed from the momentum theorem, the wake does not translate at uniform velocity, but widens as the velocity decreases to satisfy continuity. The rigid wake ignores this effect, while the wake generation technique includes this effect.

It was decided to construct a semirigid wake to investigate the effect of streamwise reduction of the wake velocity. It was assumed that the velocity passing through the rotor decreased linearly from freestream velocity $1.5R$ upstream to the final wake velocity $1.5R$ downstream from the center of the rotor. Wake widening was neglected since these computations were made on a small programmable calculator. The shed vorticity was assumed to move at the local velocity from the point of shedding to $1.5R$ downstream after which it moved at constant velocity. In the varying velocity region, the wake would distort, while after $1.5R$ downstream it would be rigid. Induced velocity effects of the wake on itself were ignored since the wake was assumed semirigid. The effects of lift reversal and acceleration-deceleration ramp of $\Delta = 15^\circ$ with varying angles of attack were included. The shed vorticity was assumed to originate from the trailing edge of the blade as the blade was modulated around the orbit always oriented at the ideal value of ϕ . Since this was for the windmill state, β was assumed to be $-\pi/2$. Three blades were used to show multiblade effects. The rotor was rotated in increments of 5° , and $\lambda = 2$ was used. A uniform loading was assumed so the wake was essentially two dimensional. For comparison purposes, ideal wakes were constructed with a digital computer and plotted by Calcomp. These were for $\lambda = 0.5, 1$, and 2 and blade angle of attack $-\alpha 0 + 12^\circ$ and constant.

Figures A.16a and A.16b illustrate the ideal wake for $\lambda = 2.0, 1.0$, and 0.5 for $\alpha = 0^\circ$. It is seen that these are respectively prolate cycloids, cycloids, and curtate cycloids. These are rigid wakes and translate at the freestream speed. Note that the intersections of the surface create cells which would make any potential multivalued and multiple connected regions. Figure A.17a and A.17b illustrate the ideal wake for $\lambda = 2.0, 1.0$, and 0.5 for $\alpha = +12^\circ$. These are cycloidal like surfaces, but since the wake emanates from the trailing edge, the resulting shapes are distorted from cycloids. It must be expected that blade modulation will distort the surface generated from a cycloidal motion of the pivot. Each arc for each blade is similar, but translated from the previous blade by a fixed distance. The intersection again produce cells.

Figure A.16a. Illustration of the Ideal Wake for $\lambda = 2.0, 1.0$, and 0.5 for $\alpha = 0^\circ$,

IDEAL WAKE - PATH OF BOUND VORTEX

$\lambda = 1.0$
CYCLOID

$\lambda = 0.5$
CURTATE
CYCLOID

Figure A.16b. Illustration of the Ideal Wake for $\lambda = 2.0, 1.0$, and 0.5 for $\alpha = 0^\circ$,

Figure A.17a. Illustration of the Ideal Wake for $\lambda = 2.0$, 1.0 , and 0.5 for $\alpha = +12^\circ$.

Figure A.17b. Illustration of the Ideal Wake for $\lambda = 2.0, 1.0$, and 0.5 for $\alpha = +12^\circ$.

Figure A.18 is the semirigid wake with discrete vortex shedding every 5° of rotation for $\lambda = 2$ with blade modulation, $\Delta = 15^\circ$, and $\alpha = 12^\circ$. This wake is nearly a prolate cycloid with distortion evidence near the blade flip points of $\pm \frac{\pi}{2}$. Each vortex shed is indicated by a small arrow headed circular arc which shows the direction of the vortex rotation. The vortex strength is roughly proportional to the size of the symbol. The bound vortices are also shown, and the bottom blade is instantaneously at zero angle of attack. The line joining each symbol represents the tip joining vortex. Note that at each flip point there are strong concentrated vortices shed as the angle of attack is reversed, and the upper and lower concentrated vortices occur at nearly the same value of x . In between the concentrated vortices is the distributed vorticity which is very weak, the largest is only 1.5 percent of the concentrated vorticity which would occur with instantaneous blade flip and occurs when $\psi = 30^\circ$, and occurs approximately one-third between the x axis and the bottom of the rotor. There is more distributed vorticity located in the lower one-third of the wake than in the upper two-thirds of the wake, and is more closely spaced.

The influence of the slowing down of the wake can be inferred from the width and spacing of the lower loops. The end of slowing down region is the vertical dashed line. Downstream of that the wake velocity is two-thirds of free-stream. The wake should be $\frac{\sqrt{3}}{3}$ wider to satisfy continuity but this was omitted in this simple calculation. It is seen that the lower loops compress and become narrower as the wake slows down. One loop is in both regimes. The last two are the same shape since that region is rigid and translates at a uniform velocity.

It is noticed that the concentrated vorticity on the upper and lower half occur at near the same value of x . The concentrated vorticity on the lower half is one-third the concentrated vorticity on the upper half, and the sum of the distributed vorticity is two-thirds of the upper and of opposite rotation for this value of $\lambda = 2$. Each segment on the wake is

Figure A.18. Semi-Rigid Wake Array with Distributed Vorticity Shed Every 5° of Rotation.

inducing velocity on every other part of the flow. Since the wake is rigid except for wake slowing down, the wake does not feel this effect. If the wake were free, this induced effect would cause the wake to distort continuously. The concentrated vorticity would cause the distributed vorticity to rotate around the concentrated vorticity in spiral like path and gradually coalesce into two concentrated vorticity of equal strength but opposite rotation. The upper half of the wake would wrap around the upper concentrated vorticity and being of opposite sign reduces the strength to two-thirds the initial value. The lower loops of the wake would wrap around the lower concentrated vorticity and being of the same sign would add to the strength. The net result is that two concentrated vortices of equal strength would form. The final strength of these two concentrated vortices would be the mean of the concentrated vorticity shed at each blade flip for the particular value of β . This mean strength occurs due to the roll up and the algebraic sign of the distributed vorticity. To satisfy the Helmholtz vortex laws a line of vortices must join at the tips, and also join each pair of concentrated vortices. The line joining the concentrated pair would have vanishing strength as the roll-up continued. The final roll-up would occur far downstream but calculations from generating the wake shows that it starts very rapidly and by one turn of the rotor roll-up, is well developed. Thus, there is a near-field far-field structure with an intermediate region which transitions from one to the other. Spanwise load distribution would concentrate tip joining vortices near each blade tip. This concentration would tend to hasten ring formation which would rapidly distort the original rectangular ring patterns to elongated ovals and perhaps finally to toroidal rings. This is speculation based on the trailing vortex system of wings and some experimental evidence of the far wake of propellers.

The far wake structure has been deduced earlier from studies of vortex motion and used to develop the "Primitive Vortex Theory of the Cyclogiro." It was used to expedite the initial research

on the giromill, and tests have indicated its usefulness. It was the starting point for continued work. The exact theory of wake generation was developed, and verified that this structure existed in the far field. The primitive theory was then modified to include wake expansion and better modeling of the near field. This has resulted in improved convergence but some exceptions still occur. This theory is presently being modified to include better near field modeling at the expense of longer computing time. This will use cycloidal like wake near the rotor combined with the far field structure. Wake widening is included.

SECTION A.10

WAKE MODEL FOR IMPROVED PRIMITIVE THEORY

The primitive theory used a series of rectangular vortex rings for the uniformly loaded blade, starting in the immediate vicinity of the rotor. The rings were of equal strength and equal spacing and translated as the rotor rotated at a velocity equal to the free stream velocity plus the mean induced velocity computed at the capture area. No wake expansion was considered. The induced velocity at the capture area was computed by the momentum theorem from the forces on the blade. The x component of velocity on the orbit was assumed to be the induced velocity at the capture area corrected by a sine function of ψ to reduce the upstream side and increase the downstream side to allow for velocity decrease through the rotor. This corrected the angle of attack, and iteration performed until the circulation on the orbit and the mean induced velocity computed from the momentum theorem converged. The ring spacing was then determined and the induced velocity on the capture area computed from the wake and the bound vortex system, was iterated until it converged to the value computed from the momentum theorem by adjusting the strength of the vorticity in the wake. The wake was then translated as the rotor rotated and the performance computed. Subroutines were used to compute various steps in the calculation.

The exact wake generation showed that the wake widened in the spanwise and lateral direction, that the far field structure was generated quite close to the rotor, but that near the rotor the cycloidal structure still existed. It was decided to incorporate as many of these features into a modified primitive theory as possible, but to keep the computing time as short as possible.

Figure A.19 is a sketch obtained from calculated results for an eight-bladed rotor which incorporates most of these features. The circle is the rotor orbit and the bound vortices are small circles on the orbit. The wake boundaries are the curved lines

Figure A.19. Wake Structure Assumed for the Improved Primitive Vortex Theory.

extending downstream from the capture area, and the rings of concentrated vorticity are indicated by circles on the boundary and diagonal lines joining pairs of concentrated vorticity on the boundaries. The shape of the boundaries is determined by an empirical fit to the wake expansion based on continuity and the final velocity in the wake computed from the momentum theorem. The sidewash due to the rotor lift force has been included and tends to cause the wake to deflect to the side opposite the direction of the rotor lift force. Spanwise widening is also incorporated, but is symmetrical. As the rotor rotates the wake is translated downstream.

The bound vortex system and the tip vortices joining to the wake to satisfy the Helmholtz Vortex laws presented a problem. If the cycloidal surfaces were incorporated, many vortex segments would be needed to simulate these surfaces, and the computing time, especially with translation, would be excessive. Examination of Figure A.19 suggested a solution which only partially satisfied the Helmholtz vortex laws, but which required very little computing time. This was to join the bound vortex to the free concentrated vortex which was shed when the bound vortex was at the flip point by a straight line vortex segment of strength equal to the local bound vortex strength, and to add a vortex to the free concentrated vortex to form a closed ring. The strength of this semibound ring varied continuously as the rotor rotated and vanished at the lower flip point. The front bound vortices were joined to the upper half of the wake, and the lower bound vortices to the lower half of the wake. This was superior to the way the bound vortices had been treated in the primitive theory which did not connect them to the wake. This model has the defect that the joining vortices are constant strength and the first few free concentrated vortices are too strong. This induces too high velocity on the orbit and tends to degrade the performance. However, it greatly reduces the computing time and seems to give satisfactory results. The program was run to check the McDonnell-Douglas tests of three-bladed giromill and the agreement was excellent.

The section pertaining to the Madaras system follows.

SECTION A.11

APPLICATION TO THE MADARAS ROTOR PLANT

The vortex computer program was used to compute the performance of a series of Madaras plants to determine effects of mutual interference between rotors as a function of the design variables. The variables investigated were: track radius, R ; wind speed, V_ω ; tip speed ratio, λ ; cylinder rotation speed ratio to relative wind speed, U/V , rotational speed, U ; ramp angle, Δ ; eccentricity, β ; and number of cylinders, NB . The cylinder chosen had a projected area of 2000 square feet, an aspect ratio of 6, with an end cap ratio, e/d , of 2. This gave a cylinder diameter of 5.56 m and a height of 33.4 m. The values of R investigated were $R = 457$ m, 610 m, 762 m, 915 m, and 1196 m. The wind speed (V_ω) was varied from 4.5 m/s to 13.5 m/s in increments of 1.1 m/s. The tip speed ratio, λ , was varied from 0.8 to 2.5 to find the peak power output of the cylinders. U/V was varied from 2 to 6 in increments of 0.5 to determine an optimum. U was varied from 46 to 61 m/s in increments of 7.6 m/s. Ramp angle, Δ , was checked at 15° , 30° , and 60° ; however, most data was computed only at 30° and 15° . Ramp angle is the angle during which the cylinder rotation is accelerated from full stop to full speed, and vice versa. The eccentricity angle, β , was checked at -85° , -87.5° , and -90° ; but -85° was used for most calculations.

The number of cylinders was varied from 1 to 18 in increments of 1. The wake length was arbitrarily set as being 5 radii long. In computing, points were selected on the track as near to 1° apart as possible. The program used as an input the number of points on the orbit, $N2$, which is an integer. $N2 = \text{largest integer in the ratio } [(\frac{360}{NB})] \times NB + 1$. If NB is an integral multiple of 360° , the points are 1° apart. This is a characteristic of the program and must be followed exactly or spurious results occur. The number of points on the capture area can be any number equal to or less than 201, but must be integers. The number of cylinders must be integers, but may be odd or even. The wake length is expressed in terms of the number of rotor diameters (NR). The wake

extends both upstream and downstream. It is characterized by an input parameter of NR which must be integer. The data was computed for NR = 5, but NR = 10 would have been preferred. NR = 5 was used to reduce the computing time with only a small loss of accuracy.

Preliminary runs for a 6-cylinder plant were run for $4.5 \leq V_{\omega} \leq 13.5$ m/s on R = 457 m radius track with $0.1 \leq \lambda \leq 1.5$ and $\Delta\lambda = 0.1$ and $\Delta = 15^\circ$, with $2. \leq U/V \leq 6$. with $\Delta U/V = 0.5$ at $\beta = -85^\circ$ to find the value of $\lambda = 1.1$ and $U/V = 5.5$. The value of U/V was modulated to maintain U/V at a constant value for all values of orbit angle except when reversing the rotation. The computations showed that the rotor was self-starting in the wind, and generated about 1.1 MW per rotor at peak power. We also learned that at each wind speed, peak power occurred at a track speed of $V = 10.5$ m/s.

A computation of the power required to accelerate and decelerate the rotors with a ramp angle of $\Delta = 15^\circ$ showed that very large electric motors would be required. Therefore, the ramp angle, was increased to 30° and the data recomputed. Only a small loss in gross power generated resulted, but the power to spin the rotors in acceleration-deceleration was reduced to approximately 1/4 that required for $\Delta = 15^\circ$.

Operation at constant U/V meant that a costly cylinder rpm control sensing and regulation system would be required, and that cylinder spinning power would be variable around the track, since V is not the track speed but the resultant aerodynamic velocity. Since the electric motors would operate more efficiently at constant rpm and the control system would be simplified, it was decided to operate at constant peripheral speed, U.

The program was run again with 6 cylinders on a 457 m track with $4.5 \leq W \leq 13.5$ m/s, $\Delta W = 1.1$ m/s; $0.1 \leq \lambda \leq 1.5$ with $\Delta\lambda = 0.1$; and $\Delta = 30^\circ$, and $\beta = -85^\circ$. Examination of detailed data showed that as V_{ω} and λ decreased, the cylinder approached super circulation over part or all of the orbit. If U were reduced proportionally to V_{ω}

as V_ω reduced, supercirculation would be avoided and a small gain in power would result. More importantly, the acceleration-deceleration power reduces as U is decreased by the cube of the speed ratio, and an increase in the net power would result. The program was not written to accomplish this except by changing control cards. This procedure would be too time consuming, so it was elected to perform all the runs at a constant U . The program could be altered later to include this option as time did not permit altering the program or changing U during one run by control cards.

A conference was held to outline the range of variables to be investigated as minimum objectives. Since an excessive number of computer runs would be required if all combinations were studied, it was decided that certain variables would be investigated to try to find an optimum, and then held constant while the next variable was optimized, and then held constant. This procedure would be continued until all variables had been investigated. Although the number of runs required was reduced greatly, we recognized that a true optimum would not be reached.

Starting with the 457 m radius track, the number of cylinders was increased from 1 to 8 in increments of 1. It was found for $0.8 \leq \lambda \leq 1.3$ that the power generated increased linearly with NB up to 6, and then began to depart from a linear increase with power increasing more slowly as the number of cylinders increased. Thus, there is a maximum in power generated as NB increases. A maximum has not been reached, but cylinder mutual interference effects were beginning to be significant as the number of cylinders increased beyond 18. Above $\lambda = 1.3$, the loss in power per rotor increased markedly as NB increased, with maximums occurring at different values of λ as NB was increased. This indicated that there was a minimum spacing between cylinders below which cylinder mutual interference was important. This spacing occurred when $R_1 \sin(180^\circ/NB_1) = R_2 \sin(180^\circ/NB_2)$, with $R_1 = 457$ m and $NB_1 = 6$. Thus, as R_2 is increased, NB_2 could be computed for a given R_2 . It would be necessary to investigate $NB > NB_2$, when $R_2 > R_1$, since

below NB_2 the cylinder mutual interference would be negligible. This would reduce the computation needed to investigate larger radii track.

The time available to accelerate and decelerate the cylinders in rotation increased directly as R for fixed Δ . This has a significant effect on the power required for deceleration-acceleration, and since this power is a significant portion of the gross power generated, anything which can reduce this power has potential pay-off. Increasing R appeared attractive, so R was increased from 457 m to 1220 m in steps of 152 m. At each value of R_2 , several values of number of cylinders, NB , were chosen and the performance was computed.

The peak power per rotor at each value of V_ω was plotted and labeled for each value of NB versus the Radius, R . Selected values of NB were computed at each value of R so that trend lines could be established. At fixed V_ω and NB , the peak power increased monotonically with increasing R , and asymptotically approaches a limit. The rate of increase decreased as R increased. Physically, this meant that as R increased with fixed rotor spacing, the blade mutual interference effects decreased until only a few adjacent cylinders affected one another, and those on the far side of the orbit had vanishing effect. As NB increased beyond that for critical spacing, the peak power per rotor decreased indicating an increase in mutual blade interference. As NB was increased, the trend lines of peak power per rotor were essentially parallel but below one another.

It was obvious that increasing R was beneficial. However, the track length increases directly with R increase, and if track costs are found to be a significant portion of the cost, the larger radii may not be an attractive trade-off since the beneficial effects of increase in power with increase in R would be offset more by the increase in cost.

It was decided to investigate the influence of larger numbers of cylinders on the performance at $R = 457$ m. Data was then run for $R = 457$ m with $NB = 9, 10, 11, 12, 13, 14, 16, 18$. As NB increased, λ for peak power at each wind speed decreased, so for $NB = 14, 16, 18$ it was necessary to cover the range of $0.6 \leq \lambda \leq 1.5$, with $\Delta \lambda = 0.1$. It was found the power at each wind speed neared a maximum as NB approached 18. Although a maximum was not defined there was only a marginal increase in power for $NB = 18$ over that for $N = 16$, so no larger values of NB were used. In addition computing time would increase prohibitively.

The power required to decelerate-accelerate the rotor was found to be critical at $R = 457$ m. The simplest method to reduce this would be to increase Δ . However, an increase of Δ to 60° resulted in a serious loss in power generated. It appears that $\Delta = 30^\circ$ is the largest Δ that can be tolerated. Decreasing U is as effective as increasing Δ . An examination of detail data at $U = 54$ m/s, shows that at the lower wind speeds, the cylinder is in supercirculation, and U can be greatly reduced. At $V_\omega = 13.5$ m/s, the cylinder is in supercirculation on a small part of the orbit, since λ for peak power is about unity. A decrease in U could be tolerated with almost no effect on the power generated, but if decreased too much the power would be reduced. The supercirculation phenomenon will be discussed in more detail later.

A single cylinder was run on the $R = 457$ m track since this run enabled a comparison with a single cylinder operating in undisturbed flow. The single cylinder experiences no inter-cylinder mutual interference, but has self-induced interference with the tip joining vortex and the wake. These effects are usually omitted in a undisturbed flow analysis, but are actually present.

The power generated by a single cylinder on a 457 m radius with $U = 54$ m/s and $\Delta = 30^\circ$ is shown in Figure A.20 for 5 different values of V_ω as a function of λ . It is to be noted that as V_ω increases, the value of λ corresponding to maximum or peak power decreases. It is interesting that the track speed at which the maximum occurs is approximately 10.9 m/s for all values of V_ω .

Figure A.20. Gross Power Output versus λ for Various Wind Speeds, One Rotor Spinning at 183 rpm, 915 m Diameter Track.

This feature is a desirable feature for synchronous generators. Also of interest is the fact that each value of V_ω , the track speed V_t at which the power developed becomes zero is approximately 26 m/s. This shows that the system is self limiting, that it cannot run away, and that an essentially constant optimum track speed exists for all wind speeds. When the effect of track resistance, motor power losses, and generator power losses are included, the maximum track speed the rotor will reach will be always less than this limiting speed based only on rotor characteristics. The reason the speed for peak power and the limiting speed are essentially constant speeds can be traced to the fact that U had been held constant.

The wind tunnel tests reported in Section 3 showed that C_L and C_D of a spinning cylinder are a function of (U/V) , where U is the cylinder peripheral velocity, and V is the relative aerodynamic speed. When the rotating cylinder is moved on a circle in a uniform flow, ideal theory neglecting induced velocities show that V_ω is actually $VR = VW \sqrt{1 + \lambda^2 - 2\lambda \sin\psi}$, and (U/V) actually is (U/VR) . The tests also showed for $A = 6$ and $e/d = 2$, that when $(U/VR) = (U/VR)_{crit} = 6$, C_L and C_D reach their maximum values; and when $(U/V) > (U/VR)_{crit}$ the C_L and C_D are essentially constant. The reason for this is that at $(U/VR)_{crit}$, supercirculation occurs on the cylinder.

Supercirculation is a term used to indicate that the stagnation point in the flow around a body has moved off the body and occurs in the fluid near the body. This is illustrated in Figure A.21. Figure A.21a illustrates the streamlines associated with potential flow around a circular cylinder without circulation. Figure A.21b illustrates the real fluid taken from photographic techniques used to visualize the flow as reported in Reference 62. When a vortex is placed at the center of the cylinder to add circulation, the streamline pattern changes, as in Figure A.21c and A.21d. The stagnation points occur where the streamlines come normal to the cylinder at the points S. For real flow, stagnation point occurs on the upstream side of the cylinder, but a wake exists on the

Figure A.21. Ideal and Real Flow Around a Circular Cylinder for Various Circulation from 0 to a Supercritical Value.

downstream side. The wake reduces the lift magnitude for a given ratio of U/V so that the lift generated is less than predicted by theory. Figures A.21e and A.21f illustrate the critical value of circulation which causes the two mathematical stagnation points to coalesce into a single stagnation point at the bottom of the cylinder.

If the circulation is greater than the critical value the stagnation point moves out into the fluid, and the cylinder is surrounded by a layer of fluid which rotates around the cylinder inside the closed loop. Effectively this has changed the circular cylinder into a teardrop cylinder. Mathematically, S is a singular point, and since this occurs out in the fluid, some researchers believe this could not occur in reality. This belief is not true, because at S the velocity is zero, and as shown in Figure A.21h, this situation in a real fluid was photographed by Prandtl. This situation occurs on a rotating cylinder because of viscosity. When the circulation is strong enough to be greater than the critical value, it is referred to as supercirculation. It can be created by rotation on a circular cylinder, or by blowing jets on airfoils. It does exist.

Viscosity is not powerful enough to increase the fluid viscosity sufficiently to continue to cause the lift coefficient to increase after some value of U/V . For the cylinder traveling at a track speed V_t , $(U/VR)_{crit} = U/[V_\omega \sqrt{1 + \lambda^2 - 2\lambda \sin\psi}]$ where $\lambda = \frac{V_t}{V_\omega}$. It is seen that $(U/VR)_{crit}$ or supercirculation can occur at points on the orbit depending upon U , W , λ , and ψ . Of particular interest is when $\psi = -\pi/2$ since this corresponds to the maximum value of VR which makes (U/VR) a minimum. Then, $(U/VR)_{crit} = \frac{U}{V_\omega(1 + \lambda)}$. If U is a constant, either V_ω or λ can be solved for, since $V_\omega = U/[(1 + \lambda)(U/VR)_{crit}]$, and $\lambda_{crit} = U/[V_\omega(U/VR)_{crit}] - 1$. It is seen that as V_ω increases, the value of λ at which supercirculation occurs at the upper flip point decreases. The significance of λ_{crit} is that for each value of V_ω for $\lambda < \lambda_{crit}$, the cylinder has supercirculation over the whole orbit, and C_L and C_D are essentially constant at the maximum values around the whole orbit. When $\lambda > \lambda_{crit}$,

part of the upper orbit (region of advancing cylinder) has $(U/V_R) < (U/V_R)_{crit}$, and that portion of the orbit has C_L and C_D less than the maximum, but variable with ψ . The value of ψ at which $(U/V_R)_{crit}$ is reached on the orbit can be shown to be $\psi_{crit} = \sin^{-1} \left\{ \frac{1 + \lambda^2 - [(U/W)/U/VR]^2}{2\lambda} \right\}$. The minimum value of C_L and C_D on the orbit occurs at $\psi = -\pi/2$ and increases to the maximum value at ψ_{crit} . If U is a constant = 53 m/s, and $(U/V_R)_{crit} = 6$, then $\psi_{crit} = 0$ for $V_\omega = 8.9$ m/s. Thus, for all $V_\omega < 8.9$ m/s the cylinder is in supercirculation for the complete orbit. This means until $\lambda = \lambda_{crit}$, C_L and C_D are constant at the maximum values. As λ increases beyond λ_{crit} the part of the upper orbit for which supercirculation does not exist increases and C_L and C_D on this part of the orbit decrease with increasing λ . C_L and C_D are a minimum at $\psi = -\pi/2$ and gradually increase until $\psi = \psi_{crit}$ after which C_L and C_D become constant. Since the upper part of the orbit produces the most power, this means that the rate of increase of total power with increasing λ decreases below that which would occur if C_L and C_D were constant.

The circulation is $\Gamma = \frac{C_L S V_R}{2 b}$ so that it is proportional to the product $CL \cdot V_R$. But, $V_R = V_\omega \sqrt{1 + \lambda^2 - 2\lambda \sin\psi}$, so that V_R increases with increase of λ . The vorticity in the wake is proportional to Γ and the Γ density in the wake increases with λ since there are more closely packed surfaces or vortex rings as λ increases. The induced velocities are proportional to Γ and the number of rings in the wake. Thus, as λ increases the induced velocities increase in magnitude, and these reduce the magnitude of V_R . The net effect is that the power generated increases with increase in λ until the induced velocities become significant, and then the power generated decreases. At some value of λ for the given value of V_ω , the peak power is generated. For λ greater than this value, the power generated decreases. The effect of constant U is to reduce the effective C_L as λ increases, and this

reduces the power generated from that which would be generated if C_L were constant in magnitude. This causes the peak power to occur at a constant V and a more rapid decrease in power generated once the peak has been passed. The mechanism is complex and the results are not easily predictable, but come from detailed calculation.

The shape of the power generated versus λ curves can be more readily understood. Since the power generated is proportional to the cube of the wind speed, increasing wind speed should have a powerful effect on the peak power generated. For example, an 8.9 m/s wind speed should generate 30% of the power developed at 13.4 m/s at the same value of λ . Thus the peak power for 1 cylinder at $\lambda = 1.2$ would be expected to equal 0.33 MW at 8.9 m/s, since a wind of 13.4 m/s generates 1.11 MW. However, the computed power at 8.9 m/s at $\lambda = 1.2$ is 0.39 MW which is greater than would be expected. The reason for this is that at 13.4 m/s the ideal value of ψ_{crit} is 1.6° . Consequently, the effective lift coefficient at 13.4 m/s is less than the effective lift coefficient at 8.9 m/s since for winds of 13.4 m/s, less of the orbit has supercirculation than for 8.9 m/s winds. This reduction in effective lift coefficient causes the power to increase less rapidly than the cube of the wind speed. Therefore, the more rapid decrease of power above peak power as wind speed increases is associated with the reduction in effective C_L as λ increases due to less of the orbit being in supercirculation. Based on these no loss computations, if constant U/V could be used rather than constant U more power could be generated at all λ and W , than for constant U . These two types of operation will be studied to determine which yields the most net power per dollar.

As mentioned before, as the number of cylinders is increased, mutual cylinder interference effects begin to have importance when $NB > 6$. The reason for this is that the spacing between the cylinders is $h = 2R \sin(180^\circ/NB)$, where h is the spacing. The induced velocity in two dimensions is $V_\omega = \Gamma/(2\pi r)$, where r is the radius from the bound vortex to the point in question, since the spacing, h , reduced with increasing number of

cylinders. Thus the induced velocity on neighboring cylinders increases as the number of cylinders is increased. The total induced velocity felt by a cylinder is the vector sum of the induced velocity associated with each cylinder on the orbit and its associated semi-bound vortex ring. This sum increases in magnitude, and its direction changes as the number of cylinders increases. Those cylinders nearest any given cylinder have the major effect, as the radius r is less. If each cylinder had the same lift coefficient and velocity, the direction of the mutual induced velocity would be radially outward on the upstream side of the orbit, and radially inward on the downstream side of the orbit. Because of the variation in velocity and lift coefficient around the orbit, the direction is not radial, but may be reasonably close to radial. It will still be outward on the upstream side and inward on the downstream side for a windmill and opposite for a propulsive device. This mutual blade interference has profound effects on the shape of the power generated versus λ curves.

Figure A.22 is a series of power generated versus λ curves for a series of wind speeds for a 14 rotor plant having 33.4 m high by 5.6 m diameter rotors. The cylinders operate on a circular track having a 457 m radius at a rotation speed, U , of 53.4 m/s. Note that only C_D losses of the cylinder are included in this data. It is seen that the value of λ for peak power increases as wind speed increases. This means that the track speed for peak power is no longer constant, but increases as wind speed increases. The reason for this is that the mutual interference becomes less as wind speed increases. This means that synchronous generators geared to the car wheels would not generate constant frequency. If dc current or an induction generator were used, this would not be important. In addition, the decrease of power beyond peak power is much more rapid at the lower wind speeds. As wind speed increases, this effect is less pronounced, since the mutual induced velocities are a smaller fraction of the total velocity, and do not effect Γ and V_R as much. The peak power per rotor is less than that for a single cylinder as would be expected, since

Figure A.22. Average Gross Power Output from a Madaras Plant of Fourteen Rotating Cylinders versus λ for Various Wind Speeds. Fourteen Rotating Cylinders, Aspect Ratio = 6, $e/d = 2$, Cylinder Area = 186m^2 , Diameter = 5.6m, Track Radius = 457m, Cylinder Rotating at 183 rpm.

the magnitude and direction of the resultant velocity at each point on the orbit has been changed.

Figure A.23 presents the data when the plant size has been increased to 16 of the same sized cylinder and track radius. The character in the curves is the same as for 14 cylinders. The total power has increased while the power per rotor has decreased. The value of λ for peak power has reduced as well as the limiting value of λ . The primary advantage of 16 cylinders over 14 is that the problem of connecting the rotors in an endless train is easier, but the plant cost is higher since more cylinders must be used.

Figure A.24 presents the data when the number of cylinders has been increased to 18. The peak power has been increased only slightly while the power generated per rotor has decreased again. All the comments for the 16 cylinder rotor apply. This appears to be about the maximum power which can be generated if the trends continue. No further increase in number of cylinders has been run since the gain in total power is marginal, and computing time increases rapidly with NB and was prohibitive. The power per rotor is only 0.7407, than that for a single rotor cylinder at 13.4 m/s. This plant produces nearly the maximum power.

THE EFFECT OF RAMP ANGLE ON THE POWER PRODUCED

The ramp angle (the angular distance during which the cylinder is spun-up) was initially chosen to be 15° , but spin-up power was excessive. The ramp angle was then increased to 30° as calculations showed this may be acceptable, especially at the larger track radii. When the radius was held to 1500 feet to reduce the track cost, the number of cylinders was increased and the data computed. It was found that at the higher wind speeds, the spin-up power was again becoming a large fraction of the total power generated. Calculations showed that a 60° ramp angle would be desired to reduce the spin-up power, so the ramp angle was increased to 60° and the data computed at near the peak power at $\lambda = 1.2$ for the 16 cylinder rotor. The results were disastrous. The power generated was only about two-thirds of that generated

Figure A.23. Average Gross Power Output from Madaras Plant versus λ for Various Wind Speeds. Sixteen Rotating Cylinders, Aspect Ratio = 6, $e/d = 2$, Cylinder Area = 186m^2 , Diameter = 5.6m, Track Radius = 457m, Cylinder Rotating at 183 rpm.

Figure A.24. Average Gross Power Output from Madaras Plant versus λ for Various Wind Speeds, Eighteen Rotating Cylinders, Aspect Ratio = 6, $e/d = 2$, Cylinder Area = 186m^2 , Diameter = 5.6m , Track Radius = 457m , Cylinder Rotating at 183 rpm.

at the same value of λ when the ramp angle was 30° . This result was anticipated as the upper half of the orbit produces the major contribution to the power, and one-third of this had been at reduced C_L due to the large ramp angle.

It may be possible to optimize ramp angle power loss with spin-up power loss by varying the ramp angle over a wide range in small increments. This has not been done due to the press of time and the long computing times for large number of blades. There will probably be a different optimum condition at each number of rotors and wind speed. This would require large amounts of time and funds for computer analysis, and is recommended if the Madaras system is funded for future research.

APPENDIX B

COST ESTIMATE DETAILS

APPENDIX B
COST ESTIMATE DETAILS

B.1 PLANT COST

A. LAND

To be
Considered
Separately

B. SITE PREPARATION AND EARTHWORK

Clear Site, 70 Ac @ \$750.00	\$ 52,500	
Layout Work	52,000	
Fencing, 11,500 lf @ \$10.00	<u>115,000</u>	
TOTAL SITE PREPARATION AND EARTHWORK		<u>\$ 219,500</u>

C. SITE BUILDINGS AND VIADUCT

Assembly Building, 100' x 160'	\$2,013,400	
See attached sheet		
Control Building, 30' x 50'	75,000	
1,500 sf @ \$50.00		
Access under Track:		
Excavation, 5,500 cy @ \$2.60	143,000	
Concrete, 1,015 cy @ \$137.00	139,000	
Roadway, 295 sy @ \$15.25	<u>4,500</u>	
TOTAL SITE BUILDINGS AND VIADUCT		<u>\$2,374,900</u>

D. UTILITIES, ROADWAYS, TRACKS AND TROLLEYS

Utilities:

Perforated Drains, 61,000 lf		
@ \$6.85	\$417,850	
Catch Basins, 160 @ \$75.00	140,000	
Corr. Metal Pipe 24" dia.		
16,000 lf @ \$15.00	240,000	
Corr. Metal Pipe 12" dia.		
10,000 lf @ \$7.50	75,000	
Corr. Metal Pipe 8" dia.		
6,000 lf @ \$4.35	26,100	
Excavation & Backfill,		
24,000 cy @ \$3.60	<u>86,400</u>	985,350

Roadways:

Concrete, 34,700 cy @ \$15.25	\$529,200	
Gravel, 20,315 cy @ \$8.00	<u>162,500</u>	691,700

Tracks:

Subgrade		
Compaction, 170,000 sy		
@ \$.20	\$ 34,000	
Testing	50,000	
Cuts & Fills, 143,000 cy		
@ \$2.60	<u>371,800</u>	

Borrow Fill, 160,000 cy		
@ \$8.00	1,280,000	
Fine Grade, 170,000 sy		
@ \$.30	<u>51,000</u>	1,786,800
Roadbed		
Concrete, 10,500 cy		
@ \$52.25	\$ 548,700	
Reinforcing Steel, 945 tons		
@ \$660.00	623,700	
Joints, 41,000 lf @ \$6.00	24,600	
Finish & Cure, 243,700 sf		
@ \$2.25	<u>61,000</u>	1,258,000
Steel Track		
Bolts, 15,235 pc @ \$30.00	\$ 457,050	
Track Plates, 4,200 tons		
@ \$660	3,150,000	
Grout, 317 cy @ \$1485.00	<u>470,800</u>	4,077,850
Trolleys		
Poles, 380 pc @ \$1100.00	383,800	
Joists, 230 ton @ \$700.00	161,000	
Support Angles, 115 ton		
@ \$1200.00	138,000	
Collector Track, 13,180 lf		
@ \$40.00	527,200	
Trolley Brackets, 28 pc		
@ \$1200.00	33,600	
Trolley Shoes, 84 pc		
@ \$570.00	47,800	
Pole Excavation, 240 cy		
@ \$100.00	24,000	
Pole Foundation Concrete,		
240 cy @ \$45.00	10,800	
Pole Anchor Bolts, 1,520 pc		
@ \$25	<u>38,000</u>	1,321,280
Power Collection System		
1000 amp Aerial Cable,		
6,000 lf @ \$69.00	414,000	
1000 amp Taps on Trolley	6,100	
Poles, 150 pc @ \$275.00	<u>41,250</u>	461,350
Wind Sensors and Poles		
Wind Sensors, 24 pc @ \$650.00	15,600	
Sensor Poles, 24 pc @ \$275.00	6,600	
Underground Cables, 20,000 lf		
@ \$2.25	<u>45,000</u>	<u>67,200</u>
TOTAL UTILITIES, ROADWAYS, TRACKS AND TROLLEYS		\$10,649,530

E. ROTOR AND CAR

Wheel and Suspension System:

(see attached sheet) 28 pc @ \$220,565 \$6,175,820

(includes generator and speed increaser)

Couplings and Train Assembly

Cables, 25,000 lb @ \$5.00 125,000

Cable Sockets, 56 pc @ \$1000.00 56,000

Cable Pins, 56 pc @ \$100.00 5,600

Positioning of Cars on Track 50,000

Rental of Prime Mover 83,000

Car Frame and Housing

(see attached sheet) 28 pc @ \$167,230 4,682,440

(includes spin motor and speed increaser)

Rotor Tower

(see attached sheet) 28 pc @ \$94,960 2,658,880

(includes bearings, shafting, lube system)

Balance Rotor 28 pc @ 2500.00 70,000

Generator Controls & Wiring:

Generator Control Center 36,500.00

Generator Wiring 1,230.00

Spin Motor Wiring 600.00

Lube Pump Wiring 1,270.00

Car Lighting 920.00

Telemetry and Controls 37,500.00

Grounding 2,500.00

Car Power Wiring 1,050.00

28 pc x 81,570.00 \$2,283,960

TOTAL ROTOR AND CAR

\$18,850,420

F. POWER AND CONTROL SYSTEMS

Power Factor Correction \$ 29,000

2000 amp Disconnects 96,000

Control, Computers, Displays 292,050

TOTAL POWER AND CONTROL SYSTEMS

\$ 417,050

B.2 SUMMARY

DIRECT COSTS

• Construction

•Site Preparation \$ 219,500

•Buildings & Viaduct 2,374,900

•Utilities

•Roads 10,649,530

•Tracks & Trolleys

Total Construction

13,243,930

• Mechanical

\$19,523,650

•29 Rotor Cars (includes 1 spare) 417,050

Total Mechanical

19,940,700

TOTAL DIRECT COSTS (no land)

\$33,184,630

• Land Net Acreage = 70 acres

INDIRECT COSTS

• Land	none
• General Consturction Indirect Cost (24 months)	
• Insurance on Labor, Workmen's Comp., FICA, Unemployment: 16% x 3,600,000	\$ 576,000
• Office Expense: 3% x 13,243,930	397,320
• Field Personnel:	
Project Mgr. 80,000	
Field Engr. 200,000	
Supervision 56,000	
Timekeeper 40,000	380,000
• Overhead 5% x 13,243,930 (direct)	662,200
• Profit 3% x 13,243,930 (direct)	397,320
• Temp. Heat & Electric	110,000
• Total General Construction Cost	<u>\$ 2,522,840</u>
∴ Indirect Construction Cost = 19.05% of Direct Construction Cost	
• Mechanical Construction Indirect Cost (36 months)	
• Insurance on Labor 18% x 890,000	\$ 160,000
• Office Expense 4% x 19,940,700	797,630
• Field Personnel	
Project Mgr. 165,000	
Field Engr. 150,000	315,000
• Temp. Heat & Electric	160,000
• Small Tools	100,000
• Overhead & Profit 15% x 19,940,700	2,991,110
• Total Mechanical Construction Cost	<u>\$4,523,740</u>
∴ Indirect Mechanical Cost = 22.69% of Direct Mechanical Cost	

TOTAL PROJECT COSTS

LAND	To be considered Separately
GENERAL CONSTRUCTION	
• Direct	\$13,243,930
• Indirect	<u>2,522,840</u>
• Total Construction	\$15,766,770
MECHANICAL CONSTRUCTION	
• Direct	\$19,940,700
• Indirect	<u>4,523,740</u>
• Total Mechanical	24,464,440
CONSTRUCTION FINANCING	
11.7% Total Direct (\$32,511,400)	<u>3,882,600</u>
TOTAL COST	<u>\$44,113,810</u>

B.2 ANNUAL CHARGES (less Land Cost)

Mortgage Payments	\$39,420,060 @ 8.75% per annum	\$3,449,260
Depreciation	\$39,420,060 @ 30 years	1,314,000
Taxes: Real Estate		221,000
Income		<u>300,000</u>

TOTAL ANNUAL CHARGES 5,284,260

Total Annual Charges = 12.2% Plant Cost (excluding Land)

OPERATING COSTS

Wages

Secy. & Billing Clerk	13,000	
Station Manager	50,000	
Outside Labor	50,000	
Electricians (2)	25,000	
Millwrights (2)	25,000	
Labor Insurance & Taxes	<u>30,000</u>	
		233,000

Utilities

Heat & Lights:		
Control Building	3,000	
Assembly Building	<u>8,000</u>	11,000

Expendable Parts

Bearing (616)	60 @ 3500	\$21,000	
Wheels (224)	8 @ 3000	24,000	
Reducers (140)	4 @ 15000	60,000	
Lube Oil Pumps (56)	2 @ 1500	3,000	
Couplings (392)	16 @ 450	7,200	
DC Brushes (56)	56 @ 600	33,600	
Contractors (504)	504 @ 80	40,320	
Track Shoes (84)	84 @ 300	25,200	
Collector Rails (13,180)	1200 @ 40	<u>48,000</u>	262,300

Insurance	250,000
Miscellaneous Items	<u>200,000</u>

TOTAL OPERATING COSTS \$956,300

∴ Total Operating Cost = 2.2% of Plant Cost (excluding Land)

B.3 DETAILED COMPONENT BREAKDOWNS

ROTOR CAR WHEEL AND SUSPENSION SYSTEM (four trucks)

Structural Frames	4 pc @	4,000.00	\$16,000
Wheel Bearings	16 pc @	1850.00	29,600
Wheels	8 pc @	3000.00	24,000
Bolts & Nuts	64 pc @	4.00	255
Axles	8 pc @	450.00	3,600
Keys & Set Screws	8 pc @	30.00	240
Retaining Rings	8 pc @	80.00	640
Sprockets	8 pc @	200.00	1,600
Chain	125 lf @	10.00	1,250
Falk Coupling 160T10	4 pc @	820.00	3,280
Keys & Set Screws	8 pc @	20.00	160
Support Pins 12" dia.	4 pc @	600.00	2,400
Retainer Rings	8 pc @	120.00	960
Falk Speed increaser	4 pc @	12,500.00	50,000
Falk Coupling 120T10	4 pc @	270.00	1,080
Keys & Set Screws	8 pc @	10.00	80
Indusction Generator 250 kW	4 pc @	15,000.00	60,000
Bolts & Nuts	40 pc @	3.00	120
Camrol Guides	8 pc @	150.00	1,200
Bolts & Nuts	8 pc @	12.00	100
Support Pin & Bearings	4 pc @	4500.00	18,000
Assembly labor on site			<u>6,000</u>

TOTAL \$220,565

ROTOR CAR FRAME AND HOUSING

Main Girders	70,000 lb @	\$.35	\$24,500
Gusset Plates	2,500 lb @	\$.35	875
Steel Tubes	10,600 lb @	\$.35	3,710
Field Welding 3/4"	250 lf @	7.70	1,925
Field Welding 3/8"	125 lf @	3.25	600
Bolts & Nuts	400 pc @	2.50	1,000
Gaskets	1,300 lf @	2.00	2,600
Structural Tubes and 1/8" Plate Skin	52,000 lb @	\$.45	23,400
DC Motor & Controls	1 pc @	78,000	78,000
Reducer	1 pc @	19,000	19,000
Coupling 140T10	1 pc @	475.00	475
Coupling 130T10	1 pc @	350.00	350
Car Floor Tubing	10,000 lb @	\$.35	3,500
Car Floor Grating	1,150 sf @	1.75	2,015
Bolts & Nuts	10 pc @	8.00	80
Set Screws & Keys	4 pc @	10.00	40
Concrete Ballast	23.5 cy @	15.25	360
Field Assembly Labor			<u>4,800</u>

TOTAL \$167,230

ROTOR CAR TOWER

Steel Tube 72" dia.	22,000 lb @ \$.75	\$16,500
Cone Base	58,000 lb @ \$.35	20,300
Gusset Plates	10,000 lb @ \$.35	16,100
Bearings	6 pc @ 2350.00	14,100
Couplings	4 pc @ 450.00	1,800
Key & Set Screws	10 pc @ 40.00	400
Bearing Support Upper	5,400 lb @ \$.30	1,620
Ladder	102 lf @ 30.00	3,060
Bearing Support Lower	300 lb @ \$.45	135
8" Shafting	99 lf	11,200
Lube Oil System	2 pc @ 2500.00	5,000
Bolts & Nuts	24 pc @ 7.00	170
Shaft Bearing Supports	1,500 lb @ \$.45	675
Oil Seals	55 lf	1,400
Oil Retainers	lump sum	2,500
Field Labor		<u>12,600</u>

TOTAL \$94,960

ROTOR AND END CAPS

Aluminum Trusses	10,300 lb @ 1.40	14,470
Truss Assembly		12,000
Corrugated Alum. Skin	3,170 sf @ 1.45	4,600
Screws	10,000 pc @ \$.02	2,000
Cover Plates	1,000 sf @ 1.60	1,600
Ring Gear & Retainer	1 pc @ 12,000.00	12,000
Ring Gear Supports	800 lb @ 2.40	1,920
Lower Bearing Retainer	1 pc @ 10,000.00	10,000
Upper Bearing	1 pc @ 10,000.00	10,000
Lower Bearing	1 pc @ 8,000.00	8,000
Upper Bearing Retainer	1 pc @ 10,000.00	10,000
Erection Labor		<u>8,400</u>

TOTAL \$94,990

APPENDIX C

ROTOR STRUCTURAL WEIGHT AND INERTIA SCALING EQUATIONS

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ROTOR STRUCTURAL WEIGHT AND INERTIA SCALING EQUATIONS

The procedure for scaling rotor weight and mass moment of inertia for various geometric changes in rotor and end cap size was a two step process.

First, a detailed reference structural design was developed for the following conditions.

- Design geometry described in Paragraph 4.1.
- Design wind and operating conditions described in Paragraph 4.1.
- Design life, environmental, and system criteria described in Paragraph 4.1.
- Design loads described in Paragraph 4.2 and all subparagraphs thereunder.
- Structural configuration as described in Paragraph 4.3 and all subparagraphs thereunder.

Second, scaling equations based on variations from the reference geometry and operation conditions were generated. These equations, presented below, were then programmed in our Madaras System Performance Simulation Program in order to properly account for the effects of rotor geometry and spin condition variations on system performance.

C.1 CYLINDER WEIGHT

The weight of cylinders having a constant projected area of 2000 ft² is

$$W_{cyl} = \left[\frac{r_{cyl}}{11.1} \times \left(\frac{RPM}{186} \right)^{1.75} \right] W_{REF} \quad \text{where,}$$

W_{cyl} = weight of cylinder in lbs

r_{cyl} = radius of cylinder in feet

RPM = maximum rotor spin rate in rev. per minute

W_{REF} = weight of cylidner having aspect raio of
4.0 = 18,889 lbs.

C.2 CYLINDER WEIGHT PER UNIT HEIGHT

In order to be able to predict the weight of a cylinder having an arbitrary projected area it is desirable to develop an expression for cylinder weight per unit of height.

For cylinders having a 2000 ft² projected area,

$$A = 2 \pi r_{cyl} h = 2000$$

$$h = \frac{1000}{r_{cyl}}$$

$$\therefore W'_{cyl} = \frac{W_{cyl}}{h} = \frac{W_{cyl} r_{cyl}}{1000} = W_{cyl} r_{cyl} \times 10^{-3} \text{ lb/ft}$$

where h = cylinder height in feet.

C.3 CAP WEIGHT

The weight of the cap is given by

$$W_{cap} = \left[\left(\frac{r_{cap}}{16.65} \right)^2 \times \left(\frac{RPM}{186} \right)^{1.40} \right] W_{REF}^{CAP}, \text{ where}$$

r_{cap} = radius of cap in feet

W_{REF}^{CAP} = weight of cap for cylinder having aspect ratio of 4.0 and cap radius to cylinder radius ratio of 1.5
= 3127 lbs

C.4 MASS MOMENT OF INERTIA

The mass moment of inertia of the rotor with respect to the geometrical axis is expressed as

$$I_{rotor} = I_{cyl} + I_{cap}$$

$$I_{cyl} = m_{cyl} (0.84 r_{cyl})^2$$

$$I_{\text{cap}} = I_1 + I_2$$

$$I_1 = \left[\frac{r_{\text{cyl}}^3}{4r_{\text{cap}}} \right] m_{\text{cap}}$$

$$I_2 = m_{\text{cap}} \left(1 - \frac{r_{\text{cyl}}}{r_{\text{cap}}} \right) \left[\frac{2}{3} r_{\text{cyl}} + \frac{1}{3} r_{\text{cap}} \right]^2$$

where,

I = mass moment of inertia in slug-ft²

m = mass in slugs.

C.5 MINIMUM WEIGHT CUT-OFF

The equations presented above are based on perturbations about the previously defined Maderas design point. The accuracy of these expressions for large deviations from this point have not been tested. However, it is obvious that there will exist some minimum weight for both the cap and the cylinder. This weight will be influenced by minimum gage requirements for material handling, damage resistance, and structural stability.

A minimum weight of 9,500 lbs for cylinders having a projected area of 2000 ft² is recommended. The minimum weight for other cylinder sizes is given by

$$w_{\text{cyl}}^{\text{min}} = \left(\frac{A_{\text{proj}}^{\text{cyl}}}{2000} \right) \times 9500 \quad \text{where}$$

$A_{\text{proj}}^{\text{cyl}}$ - projected area of cylinder.

A minimum weight for the rotor cap has not been defined. The cap weight calculated using the equations of Section III will be used for all cylinder sizes and geometries.

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