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EXPLORATION FOR SUBSURFACE MOLTEN OR QUASI-MOLTEN
BODIES USING RHEOLOGICAL METHODS

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ELASTOMECHANICAL METHODS IN THE EXPLORATION
OF THE VALLES CALDERA, NEW MEXICO

Final Report

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ABSTRACT

Variations in barometric pressure and tidal force fields generate small amplitude elastomechanical response signals that can be observed at the ground surface as displacement, strain and tilt signals. The amplitudes involved are quite small, but the signals may in many cases be observed with the help of modern instruments. The bubble box tilt meter is a particularly suitable instrument, and ground tilt signals are therefore the preferred field observable. An advantage of the elastomechanical signals is that they are sensitive to the rheological properties of the subsurface formations and ground tilt field data may thus contain signatures of shallow magma bodies that are useful in detecting and delineating such structures.

The Valles caldera in the Jemez Mountains of New Mexico is an interesting case of an active caldera that is a very attractive object for a comprehensive ground tilt field program to map both elastic and rheic discontinuities. Based on a rather comprehensive theoretical development that is outlined in five separate Essays, estimates of the amplitudes of some of the possible ground tilt signals can be obtained. Based on the results, the strongest signals of the order of 100-200 nanoradians would appear to result from the purely elastic response of the caldera fill to a barometric forcing. Moreover, assuming a magmatic underplate below the Valles, barometric forcing may lead to a flexure of the upper crust that could generate tilt signals of a similar magnitude. Present data material is insufficient to allow a useful estimate to be made of the possible tilt signals due to the rheic properties of a Valles pluton that may be in the state of partial fusion. However,

the models investigated indicate that such signals could be of a significant magnitude and provide useful information on the physical state of the Valles structure.

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(I) INTRODUCTION

(1) The following is a report to the Lawrence Livermore Laboratory of the University of California, Livermore, CA, on work carried out under Subcontract # 4077001 between the Regents of the University of California and Oregon State University. This work was carried out during the project period of 8th June to 8th October 1981. The principal investigator is Gunnar Bodvarsson, Professor of Geophysics and Mathematics at Oregon State University.

(2) Project background.

Variations in barometric pressure and tidal force fields generate small amplitude elastomechanical field signals in the upper crust. The barometric fields are associated with the passage of atmospheric maxima/minima that may involve pressure transients of the order of a few kPa (kilopascal) over periods of 10^4 to 10^6 s. The tidal strain fields are strictly periodic with amplitudes of a few 10^{-8} m/m and principal periods of approximately 4.3×10^4 and 8.6×10^4 s. In both cases the rate of the resulting strain is so small that the processes are subinertial, that is, purely elastostatic or rheostatic (creeping) where inertia forces are negligible.

The resulting deformation of crustal structures depends on local elastic/rheic parameters and many types of discontinuities in the formation properties result in local anomalies in the stress, strain and ground tilt response fields. Although the amplitudes involved are generally quite small, the response fields are in many practically important situations observable with modern instrumentation and thereby

provide for an interesting exploration technique that has the potential of supplementing or complementing other field methods.

The elastomechanical or strain/tilt sensing methods have two advantages over conventional field methods that tend to open up new dimensions in geophysical exploration. First, the strain/tilt fields are directly sensitive to the rheological or flow properties of the formations and can consequently provide data that can not be obtained by conventional methods. This is of particular interest with regard to the detection of magma bodies since conventional exploration methods are quite insensitive to such structures. The elastomechanical techniques are consequently of particular interest in geothermology where magmatic heat sources are of basic importance. Second, in the case of purely elastic deformations due to given forcing, the fields are directly sensitive to the elastic moduli. The seismic methods, on the other hand, measure the velocities of seismic waves that depend on the square root of a modulus/density ratio.

Recent experience (Bodvarsson and Axelsson, 1980) indicates that ground strain fields are considerably more contaminated by noise than ground tilt fields. At this juncture, tilt measurements with bubble box instruments (Kinemetrics, Inc) are therefore the preferred field observable.

During the summer of 1979 the Oregon State/Portland State Universities group conducted a tidal ground tilt study at the Krafla volcanic/geothermal area in Iceland (Axelsson, 1980). The work was successful in the sense that a significant tilt field anomaly was detected and is being interpreted as the result of the presence of a local subsurface magma body. During the same year, a brief study at the Newberry caldera in Oregon in-

licated a radial tidal tilt field anomaly at the ring fracture of this structure (Bodvarsson and Axelsson, 1980). Observations of barometric tilts have frequently been reported in the literature (see for example, Tomaschek, 1953 and 1959).

The work on the following report has been carried out to investigate the applicability of the elastomechanical methods for the exploration and investigation of the Valles caldera structure in the Jemez Mountains of New Mexico. In more detail, the technical scope of the work is divided into 4 tasks:

- 1) Investigate basic theory and establish quantitative relations for rheological behavior.
- 2) Estimate probable magnitude of response functions for ranges of body volume and degree of partial melt.
- 3) Investigate the applicability of the rheidity sensing methods and recommend specific applications.
- 4) Research required instrumental resolution.

The above topics (1) to (4) are being addressed in the following report. To attain the final results, a considerable amount of theoretical modeling was carried out on various relevant situations. We have concluded that it is convenient to separate the theoretical work from the main report and describe this part of our effort in five Essays that follow the report below.

(II) THE VALLES CALDERA

(3) Structure.

Bailey et. al. (1976) emphasize that in the western United States, only three calderas are known to be large enough and young enough to possibly still contain residual magma chambers, that is, the Valles NM (≈ 1.1 My old), the Long Valley Ca (≈ 0.7 My), and Yellowstone WY (≈ 0.6 My). The Valles sketched in Fig. 1 has been in the focus of interest for some time. Smith et. al. (1961, 1970) have carried out a detailed geological mapping of the caldera structure. Moreover, Smith and Bailey (1968) have discussed the evolution of the Valles as one of the most interesting cases of a resurgent cauldron. A thermal evolution model of the Valles has been developed by Kolstad and McGetchin (1978) and recently a number of papers have appeared on the Baca geothermal field that is located within the Valles (see for example, Dondanville, 1978; Atkinson and Gulati, 1979).

With regard to the development to follow it is of interest to list two additional literature sources that are relevant in the present context. First, the reports on the Geothermal Investigations of the U.S. Geological Survey in Long Valley CA 1972-73 published in the Journal of Geophysical Research, vol. 81, 1976. Second, the paper by Oftedahl (1978) on the development and structure of the Permian cauldrons of the Oslo rift zone in Norway. Since the deep interior of many of the Oslo cauldrons has been exposed by erosion, Oftedahl's observations are rather unique and fundamental to our concepts in cauldron mechanism.

For a brief overview of the available material, we include the following three sketches from the paper by Smith and Bailey (1968):

VALLES CALDERA, NEW MEXICO

Figure 1

Figure 2. Geological map of the Valles Caldera by Smith and Bailey (1968).

Fig. (2), a generalized geological map of the Valles, Fig. (3) a physiographic sketch and Fig. (4) a generalized cross section of a resurgent cauldron of the Valles type.

The Valles is transected by a northeasterly trending graben that is 2.5 to 4 km wide and is clearly visible in Figs. (2) and (3). Quite apparently, this graben, that we will refer to as the Redondo graben provides the structural control for the Redondo Creek or the Baca geothermal field. (see Fig. 1). This is a liquid dominated system with a base temperature in excess of 260°C (Dondanville, 1978) and a maximum observed temperature of 330°C .

(4) Temperature profile.

Important borehole temperature data have been obtained on the LASL Hot-Dry-Rock experiment at Fenton Hill. The temperature profile for borehole GT-2 shown in Fig. 5 is furnished by Kolstad and McGetchin (1978). A temperature of 197°C at a depth of 3 km is indicated. Very recent data from a new hole EE-2 indicate 320°C at 4.5 km. Although a straight downward extrapolation is quite uncertain, the observational data at hand would indicate 800°C , approximately the temperature of fusion of silicic magma, at the depth of 10 km.

Kolstad and McGetchin (1978) have used the available material to work out plausible temperature profiles for the Valles structure. Based on the simplified model structures shown in Fig. (6) and assuming that the pluton was formed instantaneously one My ago they have computed temperature histories for various pluton dimensions and ages. Their results for a pluton of $h = 20$ km, $r = 8$ km and an age of one My are shown in Fig. (7). A better fit with the Fenton Hill Borehole data is obtained by taking $r = 12$ km.

Figure 3. Physiographic map of the Valles caldera from Smith and Bailey. (1968)

Figure 4. Sketch of the terminal stage of a resurgent cauldron by Smith and Bailey. (1968)

Figure 5. Geothermal gradient, lower Precambrian section, Borehole GT-2 at Fenton Hill.

Figure 6. Basic model for the Valles structure used in the temperature field computations of Kolstad and McGetchin. (1978)

Figure 7. Computed isotherms for a pluton of 16 km diameter, 20 km height and one My age from Kolstad and McGetchin. (1978)

(5) Magma profile.

The very considerable amount of eruptive material that has been delivered by the Valles during its active history indicates the presence of a large supply of subsurface magma. Smith and Shaw (1976) estimate the volume of the Valles magma chamber at a few 10^3 km^3 . As indicated by the youngest domes at the ring fracture system, magma has been flowing out of the chamber as recently as 0.05 to 0.1 My ago. Moreover, the Baca geothermal system is no doubt magma

related and would appear to indicate the presence of magma at depths of a few km below the Redondo graben. The presence of numerous CO_2 - H_2S gas seeps at and around the Baca field reinforces this inference. The indications are that there is a magma subpluton protruding up into the caldera floor below the Redondo graben.

Recently, seismic field work by Sanford et. al. (1977) and magnetotelluric field work by Hermance and Pedersen (1980) have indicated the presence of rather extensive intracrustal thin sill-type magma layers at depths of roughly 20 km below parts of the Rio Grande rift zone. It would appear reasonable to speculate that a layer of this type may form a magmatic underplate (see Lachenbruch, 1979/80) below the Valles. Because of the thermal anomaly at the Valles, it is rather likely that the depth to the magma layer may there be substantially less than 20 km and it may protrude up into the Valles pluton. This situation is indicated by the temperature profiles quoted in section (4) above.

It is to be noted that Beblo and Bjornsson (1980) have found evidence for similar underplating below the rift zone of Iceland. Since there is no doubt that such structures would be entirely mafic, the similarity to the situation below the Rio Grande rift could be interpreted as indicating a similar chemistry there. Since mafic magmas have a much lower viscosity than the more felsic type, this observation is of relevance for the present work.

(III) MATERIALS OF THE VALLES STRUCTURE

(6) Elastic parameters.

Little published material is available on the seismic/elastic parameters of the Valles structure. For the present purpose, we will therefore rely on seismic material from a similar structure, the Long Valley caldera, CA that is displayed in Fig. 8 and 9. It is to be expected that the Valles and Long Valley calderas have similar seismic profiles.

(7) Composition of the magmas.

The nature of the Valles subsurface magmas is uncertain. Obviously, the great volume of silicic eruptives indicates a very large volume of silicic magma. Nevertheless, the material may be the product of differentiation and there may be (see (5) above) a large volume of more mafic low-viscosity magma deeper in the reservoir/underplate system. In this context, we will underline the fact that occasionally large volumes of silicic eruptives are poured out by extremely mafic volcanic systems such as the rift zone of Iceland. The eruption of the Askja caldera in Northern Iceland in 1875 is a case at hand (see McBirney and Williams, 1978).

Obviously, the rheology of magmas is complex and available data are sparse. Moreover, it is doubtful whether laboratory measurements on refused lava samples yield results that are relevant to magmas in situ. In the present context, we are mainly interested in the rheology at very low stresses, that is, whether magmas in situ behave Newtonian down to ultrasmall stresses. Work by Hulme (1974) indicates a small but finite Bingham type yield strength of lavas. It is, on

Figure 8. Map showing Long Valley caldera floor and 10 mGal gravity contours. Caldera faults are closely associated with caldera floor outline (see Bailey et al., 1975).

Figure 9. Cross section showing P wave velocity structure under profile BB'. Symbols are the same as in Figure 8.

the other hand, to be underlined that chemical differentiation of subsurface magmas is an obvious relatively rapid process occurring in all major magma chambers. Perhaps the most interesting field case is the observation of Thorarinsson (1954) that the composition (55 to 65% SiO_2) of the initial lava in each eruption of the Hekla in Iceland is a function of the time of repose (50 to 100 years) between eruption. It would appear that such differentiation indicates that convection types of flow can take place at very low local stress in the magma. Our conclusion is therefore that the question of a Bingham type yield strength remains unsettled.

(8) Magma rheidity.

Since we have no data on the viscosity of the Valles magmas, it is of interest to consider the rheidity problem from a more general point of view. Below, we will present a number of relevant data.

McBirney and Williams (1978) furnish the following two graphs portraying laboratory data on the viscosity of magmas. The graph in Fig. 10 shows the viscosity as a function of lava composition and temperature. The graph in Fig. 11 indicates the influence of H_2O content on the viscosity of granitic liquids at various temperatures. The following table summarizes some relevant data.

Table (I)

Table I. Effects of H_2O on the viscosities of various common igneous melts. Data from Scarfe (1973) and Murase (1962).

Composition	T(°C)	Viscosity dry, Pas.	Viscosity with H_2O
Granite	785°	10^{11}	10^5 (5% H_2O)
Andesite	1,150°	10^3	$10^{2.5}$ (4% H_2O)
Tholeiitic basalt	1,150°	$10^{2.2}$	10^2 (4% H_2O)
Olivine basalt	1,250°	10	10 (4% H_2O)

Figure 10. Viscosity of some common igneous rocks as a function of temperature. 1 is a rhyolite that was glassy or liquid through the entire range of temperature, 2 is andesite, 3 a tholeiitic basalt, and 4 an alkali basalt. (From Murase and McBirney 1973.)

Figure 11. The effect of H_2O on the viscosity of granitic liquids at various temperatures. (From Murase, 1962.)

We will supplement the above data by the following observations that are based on field evidence.

The spine of Mont Pelee in 1903. The volcanic spines that are conspicuous structures in the West Indies, probably represent cases of the most viscous lavas ever erupted at the earth's surface. It is therefore of some interest to estimate the viscosity in such extreme cases.

The spine of Mont Pelee was slowly thrust out of the earth during a period of 6 months and reached a final height of 230 m. The diameter at the base was about 100 m. It would appear that the buoyancy of the magma was the driving force. Let D be the diameter of the spine, w be the average upward velocity, η be the viscosity and ∇p be the available pressure gradient. Assuming Poiseuille flow, (Lamb, 1932), we have then the relation

$$\eta = (D^2/32w)\nabla p \quad (1)$$

On the basis of available data, we conclude that $D = 100$ m and $w = 1.4 \times 10^{-5}$ m/s are fairly good values and estimate the average upward buoyancy pressure gradient at $\nabla p = 5 \times 10^3$ Pa/m. The estimate on the basis of equation (1) is then of the order of $\eta \approx 10^{11}$ Pas. This is the value given in Table I for dry granitic magma and probably represents an extreme.

Valles post-caldera lava domes. A discontinuous ring of rhyolite pyroclastic cones, domes and flows was formed in 18 separate eruptions at 10 major vent areas by ring-fracture volcanism in the later stages of the Valles structure. These domes are conspicuous features in the Valles landscape. Fig. 12 shows the topography of the large dome located in the East of the Valles caldera floor.

Figure 12. Topographic map of the Cerro del Medio, a large magma dome on the eastern slope of the Valles.

Since these domes represent the most recent volcanism in the Valles cycle possible 0.05 to 0.1 My ago, it is of interest to make an attempt at estimating the viscosity of the rhyolitic magma that forms the domes. It is likely that the dome lava gives an indication of the properties of the upper parts of the Valles pluton magma. Unfortunately, the task of estimating the viscosity of the initial dome magma is highly underdeterminate.

A brief theory of the mechanism of lava flow is given in Essay (I) that is attached to this report. The final equation (1.17) represents a non-linear parabolic partial differential equation for the form of a

thin sheet lava flowing over a surface of little relief. Clearly, since this is an evolutionary equation, time is an independent variable, and viscosity can thus not be estimated unless data on mass flows or flow velocities are at hand. Unfortunately, no information is available on the evolutionary aspects. Moreover, it is to be realized that the form displayed in Fig. 12 is the final form upon solidification. Nevertheless, there are good reasons to conclude that the fossil form is representative of the final flowing stage. Moreover, based on historical records it would appear that such domes are formed within periods of one to 100 years. Accepting these premises, we proceed as follows.

Resorting to an analysis of a unidirectional flow ease over a plane surface on the basis of the development in Eassay (I), we obtain the vertically averaged horizontal flow velocity from equation (1.16)

$$v_a = -(g/3\eta)h^2\nabla h \quad (2)$$

where h is the lava thickness, ∇h the profile gradient and η the viscosity. Thus

$$\eta = -(g/3v_a)h^2\nabla h \quad (3)$$

We estimate that the dome in Fig. 12 was formed during a period of 10 years such that the average outward velocity was about $v_a = 10^{-5}$ m/s. Moreover the average thickness $h = 200$ m and the average gradient $\nabla h = 0.1$. On the basis of equation (3) this leads to an estimate of $\eta \sim 10^9$ Pas. Hence, we conclude that the value may be in the range 10^8 to 10^{10} Pas depending on the age.

(IV) CALDERA MECHANISM AND STRUCTURE

In the previous section, we have briefly reviewed the available data material on the Valles with particular emphasis on data that is relevant to a field investigation of the structure by elastomechanical methods. The present section will elaborate on the picture that has emerged by discussing briefly some of the geological/geophysical processes that appear to be at work in calderas of this type. We feel that although somewhat fragmentary the discussion to follow is of interest for the overall evaluation of the prospective effectivity of the elastomechanical methods and will be helpful in providing concepts and ideas to form a basis for the interpretation of field data.

(9) Caldera dimensions and the ring fracture system.

It is well known that the diameter of most calderas is in the range 10 to 20 km. The Valles is at the upper margin of the range. Quite obviously, the prevalence for this rather narrow range is significant and must be related to some mechanical or thermal aspects of caldera evolution. In particular, the size of the diameter may be governed by stress/strength relations in the cap rock or by the dimensions of the pluton that depend primarily on thermal processes.

Collapsed caldera floors are bounded by ring fracture and dike systems that are clearly visible in many eroded fossil calderas. Oftedahl (1978) reports that many of the Oslo cauldrons have well developed ring dikes that may be 30 m to 1000 m wide. Obviously, the ring dikes are significant magma channels that in the case of the Valles have provided the conduits for the rhyolitic magma forming the rather recent domes located along the ring fracture system.

(10) Collapse because of shear failure.

We will now consider the caldera collapse from the point of view of the strength of the roof over a large magma pluton. There is strong evidence that the caldera floor sinks because of the loss of support resulting either from the withdrawal of magma by subsurface displacement or because of eruption at the surface. For an elementary view of the situation consider the sketch in Fig. 13 of a circular caldera floor block of radius R and thickness h . Let g be the acceleration of gravity and ρ be the density of the rock. We assume that the liquid support is suddenly withdrawn leaving the block to be supported only by vertical shear stresses along the circumference. Obviously, the stress is then

$$\sigma = \pi R^2 h \rho g / 2\pi R h = R \rho g / 2 \quad (4)$$

The block will sink if this stress exceeds the shear strength of the rock. It is interesting that the stress depends only on the density and the radius. Inserting a common values of $\rho = 2500 \text{ kg/m}^3$ and $R = 8 \text{ km}$ we obtain $\sigma = 10 \text{ MPa}$ (megapascal) or 10^3 bar . This is a reasonable value for the averaged or bulk strength of the rock in situ (see Clark, 1966) and provides some understanding of the range of caldera radii in nature. The most important implication of this model is that the caldera diameter may mainly be a measure of the shear strength of upper crustal rocks and that it is likely that the magma chamber has considerable larger dimensions than the caldera block.

Figure 13. The circular caldera block.

(11) Age and evolution of the Valles pluton.

The analysis of Kolstad and McGetchin (1978) referred to in the previous section is based on the assumption of a thermal initial condition that is represented by the temperature field due to a pluton of given dimensions that was emplaced instantaneously one My ago. This is a rather drastic assumption that would appear to neglect various evolutionary aspects of the pluton emplacement process and also reduce the depth of diffusion of the temperature field in the crustal/upper mantle layers adjacent to the pluton system. The assumption reflects the point of view that the Valles pluton is a single anomaly in an otherwise thermally relatively normal environment. As a matter of fact, Kolstad and McGetchin (1978) assume a background vertical temperature gradient of only 25°C/km that would correspond to a normal heat flow.

For our conceptions about the Valles structure and its environment, it is of importance to reconsider the above assumption from a general point of view.

We have already noted that geological field evidence indicates that volcanism in the Jemez Mountains started about 10 My ago. The above assumption of a pluton age of only one My would therefore appear

to be rather specific. As a matter of fact, referring to estimates of the rate of magma/rock heat transfer processes in Essay (II), there are no valid arguments against considering the case that the pluton was emplaced considerably earlier, sometime during the past 10 My. As of now, the temperature field in the upper approximately 10 km around the pluton could then have reached a quasi-steady state, and in this case, we would interpret the temperature profile in Figure 5 as indicating about 800° at the depth of 10 km. This is the temperature of fusion of granitic magma and may therefore represent the upper part of a magma underplate below the Valles that may extend for tens of km out from the structure. This picture would be supported by the seismic field results of Sanford et.al. (1977) and magnetotelluric results of Hermance and Pedersen (1980).

The presence of the Baca geothermal field with a base temperature in excess of 300°C (see section (3) above) would appear to require a magmatic heat source at rather shallow depths. Our conjecture is that the thermal activity is supported by a magmatic subpluton that protrudes up under the Redondo graben. In summing up the presently available information on the magma profile of the Valles we arrive at the conjectured cross section displayed in Figure 14 below that is an alternative to the model of Kolstad and McGetchin (1978).

Figure 14. Revised model of the Valles structure.

(V) GROUND TILT RESPONSES TO BAROMETRIC AND TIDAL FORCING

(12) The various types of responses.

It is now possible to turn to the matter of principal concern in this report, that is, the estimating of the magnitude of conjectured elastomechanical/rheic responses of the Valles and its supporting structures to barometric and tidal loading. The principal purpose of this effort is to probe whether a ground tilt program at the Valles would be warranted and sufficiently likely to yield useful results. It is necessary to underline the conjectural nature of the estimates. The effects to be considered are regarded only as likely responses. Nature may not cooperate and the reality can turn out to be different.

For convenience, the development of all relevant and useful quantitative relations has been relayed to Essays (I) to (V) that follow below. By referring to the theoretical development, we can here concentrate on the matter of main interest, that is, the estimating of ground tilt amplitudes.

There are two main forcing fields for elastomechanical signals, that is, barometric load at the surface and tidal stress fields that are mainly horizontal. To indicate the magnitude of possible barometric loads, we refer to a graph furnished by Panofske et al. (1975, page 306) showing barometric pressure amplitudes. Based on this data, we can conclude that barometric transients with peak to peak amplitudes of about 3 kPa are sufficiently common to be regarded as forcing fields for ground tilts. The time scales involved are of the order of 10^5 to 10^6 s.

The tidal strain field is obtained by standard formulas (see, for example, Tomaschek, 1957). Basic periods are about 12 h and 24 h but

the amplitudes vary considerably in space and time. The fields will have to be computed separately for individual regions. For the present purpose it is sufficient to state that the volume strain at middle latitudes is of the order of a few 10^{-8} and the associated stress amplitudes are a few 10^2 to one kPa.

Variations in the elastic/rheic properties generate anomalous local responses to the forcing fields. Five basic causes of local ground tilt anomalies at the Valles can be envisioned, viz.,

- (i) Low elastic moduli of the caldera fill.
- (ii) Temperature related reduction of the solid rock elastic moduli of the upper crust in and around the caldera.
- (iii) Rheic responses due to a partially liquid pluton and/or subpluton.
- (iv) Rheic responses due to the conjectured Valles underplate.
- (v) Transient signals due to geothermal processes within the Baca system.

Clearly, the effects are more or less superposed, but we will discuss them separately.

(i) The caldera fill. Due to the lack of seismic data on the Valles, we will adopt elastic parameter estimates based on the Long Valley data referred to in section (6) above. This data material indicates that the Young's modulus E of the caldera fill may vary from 0.5 to 4.0×10^{10} Pa. We adopt an average of 2×10^{10} Pa corresponding to the Lamé parameter $\mu = 0.8 \times 10^{10}$ Pa. Except for the 1.5 km thick sedimentary cover that also forms the lower part of the caldera fill (see Figure 6), the surrounding granitic shield probably has a value of $E = 8 \times 10^{10}$ Pa.

Since tilts are generated by relative displacements, we can ignore the effects of the sedimentary cover. Moreover, since the upper caldera fill is considerably softer than the surrounding granite, the relative vertical displacement of the caldera fill can be approximately estimated on the basis of an effective caldera fill modulus of $E = (8/(8-2))2 \times 10^{10} = 2.7 \times 10^{10}$ Pa corresponding to $\mu = 1.06 \times 10^{10}$ Pa. Because of many approximations and uncertainties, we will round off this value to simply $\mu = 10^{10}$ Pa.

The caldera forms approximately a circular inclusion with an average radius of $R = 7.5 \times 10^3$ m and an effective thickness of about 1.5×10^3 m. Using the results for circular inclusions obtained by the constrained displacement approximation in conjunction with an effective modulus of rigidity of $\mu = 10^{10}$ Pa, we can obtain a good approximation of the caldera fill response to barometric load. The vertical displacement is given by equation (3.55) and the radial tilt by (3.57). Since there are some numerical problems with the convergence of the series in (3.57), it appears more convenient to calculate the displacement on the basis of (3.55) and estimate the tilt by a graphical procedure. An appropriate correction will then have to be made because of the one-sided errors of the CDA.

The calculations based on (3.55) are quite simple. Using the above dimensions, parameters and summing up the first 5 terms of the eigenfunction series, we obtain the vertical (relative) displacement and tilt responses of the caldera fill to the barometric load of 3 kPa as shown in Figure 15. The graph is smoothed for numerical errors due to slow convergence of the eigenfunction series in (3.55).

Figure 15. Estimated vertical displacement at the caldera rim due to a barometric load of 3 kPa.

The results of the CDA computations show that the tilt is concentrated in a rather narrow boundary layer of less than 2 km thickness. Moreover, this average tilt within 1 km of the boundary is approximately equal to 140 nr , that is approximately the maximum displacement divided by the thickness of the responding layer.

Comparable estimates for the solid earth tidal forcing are more difficult to obtain. The impressed strain is then horizontal and hence in the plane of the caldera fill plate. However, to obtain the order of magnitude of the tidal tilt response, we can proceed along similar lines as above. As before, we assume that the caldera fill has considerably lower elastic moduli than the country rock and we can then neglect its influence on the deformation of the surrounding rock. The caldera is simply a hole in the upper crust. Moreover, it is quite reasonable to assume that the condition prevails to greater depths such that the elastic stress/strain field in the uppermost few km is approximately two-dimensional. According to Jaeger and Cook (1976, p 253) the radial strain amplitude across the hole in the direction of the tidal stress is then about $2s/E_c$ where s is the stress amplitude in the country rock and E_c

the Young's modulus of the same rock. Assuming a Poisson's ratio of $1/4$ this longitudinal strain is associated with a vertical strain of $s/2E_c$ that results in a lowering of the surface of the central sections of the caldera fill of $sh/2E_c$ where h is the effective thickness of the caldera fill. Relying on the results above, we conclude then that the associated boundary layer tilt estimate will be $s/2E_c$. The tidal stress amplitude is of the order of one kPa and we can take that our estimate for the average radial tilt over about a one km wide boundary layer amounts to 6 nr. This is a very small tilt that would be very difficult to observe.

(ii) Temperature related reduction of solid rock elastic moduli.

The Valles structure is characterized by abnormally high temperatures in the upper crust. The data from the Fenton Hill boreholes of the LASL-HDR program indicate a linearly increasing temperature to about 800°C at the depth of 10 km. Laboratory and field experiments (Clark, 1966) indicate a relative isobaric reduction of the elastic moduli with increasing temperature of the order of 2 to $4 \times 10^{-4}/^{\circ}\text{C}$ for temperatures in the range 25°C to 300°C . Since we are here comparing formations within the thermal halo of the Valles to the country rock, pressure effects do not alter the relative reduction of the moduli.

The downward and outward extension of the Valles temperature field is quite uncertain, and it is therefore difficult to arrive at a numerical estimate of the possible magnitude of the present effect. We will therefore offer only the following simple argument. Our concern here is mainly the effect on the solid rock above the conjectured Valles underplate. The roof of the underplate may be 10 km thick and the average temperature anomaly in the roof of the order of 250°C that

would imply an average reduction of the elastic moduli by 10 %, that is, Young's modulus would be reduced from 8×10^{10} Pa to 7.2×10^{10} Pa. Comparing this reduction to the above case of the caldera fill, where the modulus was reduced from 8×10^{10} Pa to 2×10^{10} Pa over a depth range of 1.5 km, we find that the total vertical displacement under barometric load would be 1/4 of the caldera fill value. Moreover, since the thickness of the yielding zone is now about 7 times the thickness of the caldera fill, the total tilt effect should be about $1/28$ of the case of the caldera fill, that is an average of about $140/28 = 5$ nr in a boundary layer of 10 km thickness along the boundary of the thermal halo. This effect would not be observable, and the temperature effect on the solid rock does therefore not appear to be of interest.

(iii) Rheic responses due to a partially liquid pluton and/or sub-pluton. This type of response is of particular interest and the experimental detection the effect would constitute the principal purpose of a ground tilt program at the Valles. Unfortunately, the rheic processes are very uncertain and the basis for presenting useful ground tilt estimates for these effects is questionable. The material presented in section (6) of Essay (III) does provide some semi-quantitative insight into the possible magnitude of the response but is really insufficient for a quantitative development. It is, however, clear that any relative displacement of the Redondo graben master faults due to a liquid subpluton could lead to considerable local ground tilt anomalies along the faults. As to possible effects of a liquid or semi-liquid pluton below the whole caldera floor the following reasoning might be offered.

Assuming that the Valles pluton system has the form conjectured in

Figure 14 and the pluton is composed of Newtonian magma of sufficiently low viscosity, we can estimate the amount of caldera floor flexure and tilt that would result from a barometric surface load. Neglecting all pressure diffusion effects, we can assume that the floor can be modeled as a circular Hookean plate of radius R , thickness h , Young's modulus E , resting on a low viscosity Newtonian liquid and clamped along the edge. Using standard results of elastic plate theory (see section (2) of Essay (III) and Love, 1927) the deflection of the plate $u(r)$ under a uniform pressure load p is

$$u = (p/64\xi)(R^2 - r^2)^2 \quad (5)$$

where r is the radial coordinate, ξ the stiffness

$$\xi = Eh^3/12(1-\sigma^2) \quad (6)$$

and σ the Poisson's ratio that can be taken to be 0.25. The radial tilt of the plate is

$$\partial_r u = -(p/16\xi)r(R^2 - r^2) \quad (7)$$

that has a maximum at $r = R/3^{1/2}$ amounting to

$$(\partial_r u)_{\max} = -(0.024p/\xi)(R/h)^3 \quad (8)$$

Assuming an average $E = 4 \times 10^{10}$ Pa and thus $\xi = 4 \times 10^9 h^3$, and, moreover, $p = 3 \times 10^3$ Pa we find

$$(\partial_r u)_{\max} = 2 \times 10^{-8} (R/h)^3 = 20(R/h)^3 \text{ nr} \quad (9)$$

In the present case values of $R = 7.5$ km and $h = 5$ km may be reasonable guesses and we find then that the maximum is about 80 nr that would be measured at a radial distance of $r = 4.3$ km. Obviously, the estimate of h is quite uncertain and the result furnishes only a possible order of magnitude. However, an effect of this magnitude is quite interesting.

(iv) Rheic responses due to the conjectured Valles underplate.

Referring to section (5) above, there are indications that the Valles structure is placed on a magmatic underplate that may possibly protrude up to the depth of 10 km and extend radially to a distance of several tens of km. The underplate is rather likely to be mafic and may therefore have a fairly low viscosity, that is, perhaps no more than 10^6 to 10^8 Pas and probably lower. Very little is known about the downward extension of the liquid phase state. The magnetotelluric results of Hermance and Pedersen (1980) for other sections of the Rio Grande rift appear to indicate that the thickness there may be only 1 km in the case of pure liquid phase but several km if the plate is composed of partial melt. It would appear reasonable to expect a greater extension of melts or partial melts below such active structures as the Valles. As a matter of fact, referring to the situation at the Yellowstone system (see Eaton et al., 1975), the partial melt may extend down for tens or even a few 10^2 of km. It is therefore quite conceivable that the Valles rests on a basement of partial melts of relatively low viscosity that extends into the upper mantle. In this situation, it is of interest to investigate the upper limit of the basement viscosity that

would lead to observable tilt effects.

For this purpose, we consider a very simple model, viz., an infinite half-space consisting of a solid Hookean crust of thickness h and Young's modulus E resting on a Newtonian viscous basement of viscosity η . We load this model with a transient pressure load of amplitude p that is harmonic in space with a wave-number k . Let the time scale involved be t_0 and the question arising is then the magnitude of the maximum basement viscosity that would not greatly affect the bending amplitude of the Hookean layer. The viscosity bound is easily estimated on the basis of equation (3.87) of section (5) of Essay (III). The second term in the exponent is the critical decay factor. In the present case we simply set

$$\exp(-\xi k^3 t/\eta) \leq b \quad (10)$$

where b is an appropriate upper bound. Assuming, for example, about 85% decay would imply $b = 1/e^2 = 0.14$ and hence on the basis of (10),

$$\eta < \xi k^3 t_0 / 2 \quad (11)$$

or, in more detail

$$\eta < [(2\pi)^3 E / 12(1-\sigma^2)] (h/L)^3 t_0 \quad (12)$$

where L is the wave-length of the forcing pressure field. Assuming for the Valles $h = 10$ km, a characteristic wave-length of $L = 50$ km, a characteristic barometric load time $t_0 = 10^4$ s in conjunction with

$E = 8 \times 10^{10}$ Pa and $\sigma = 0.25$ leads to the estimate $\eta \leq 1.4 \times 10^{14}$ Pas. Comparing this result with the viscosity values quoted for magmas in section (8) above, we conclude that the viscosity bound is really very high and any partial melt is fairly likely to have a viscosity below this bound.

Assuming this scenario at the Valles, we can resort to the above clamped plate model to estimate tilt responses to a barometric load. Following the same procedure and assuming the same load in conjunction with $E = 8 \times 10^{10}$ Pa, $R = 25$ km and $h = 10$ km, we arrive at a tilt amplitude of 150 nr that would be observed around a radial distance of 14 km from the center of the Valles. Obviously, any Bingham type yield strength and inertia forces in the basement have been neglected in the above procedure.

Estimates of tidally induced tilts due to the same type of Newtonian basement are even less certain. Assuming, for example, that the tides cause a pressure amplitude of 500 kPa in the basement would induce a similar flexure of the Valles crustal plate with an amplitude of 1/6 of the above value that is about 25 nr.

The detection of such crustal flexure tilts would be of considerable interest and constitutes a major objective of any ground tilt program at the Valles.

(v) Transient signals due to geothermal processes within the Baca system. The geothermal activity at the Baca may lead to two types of elastomechanical signals. First, high-temperature geothermal systems are characterized by multi-phase related pressure fluctuations of various types, amplitudes and frequencies, that may constitute sources for elastomechanical signals. Moreover, the high-temperature situation

affects the local elastic moduli and may thereby lead to localized anomalous responses to barometric and tidal forcing fields. We have no sound basis for estimating the magnitude of the amplitudes involved in both types of processes.

To provide a quick overview, the above estimates are listed in Table (II) below.

(13) Proposed field program.

To monitor and record the conjectured ground tilt signals, it would appear reasonable to employ 4 recording Kinematics bubble box tilt meters at selected locations in and around the Valles caldera. Initially, the four meters should be placed in an array as indicated in Figure (16) below.

Figure 16. Initial 4 tiltmeter sites.

Overview of Conjectured Tilt Signals at the Valles Caldera

	Responding structural elements (see Figure 14)	Estimated response amplitude Unit: nrad (10^{-9} rad)		Remarks
		Forcing field		
		Atmospheric Pressure based on $\Delta p = 3\text{kPa}$	Solid earth tidal stress	
(i)	Caldera formations.	100 - 200 Purely elastic response of formations, maximum tilt amplitude to be observed over a strip of 1 to 2 km width along the interior of ring fracture.	4 - 10	Should be a fairly realistic estimate of the purely elastic component.
(ii)	High temperature halo.	Possibly small additional response due to lower elastic moduli within the high temperature region.		<u>Conjectured processes</u> No quantitative estimates possible because of lack of data on structure and formation parameters.
(iii)	Liquid or partially liquid pluton or sub-pluton.	Additional response predominantly due to the rheidity and compressibility of the magma. Maximum amplitude should be observable along the interior of the ring fault and along the master faults of the Redondo graben.		
(iv)	Liquid or partially liquid pluton base and mafic underplate.	~ 150 Additional response due to elastic flexure of upper crust covering the liquid underplate. Maximum amplitude to be observed over the exterior slopes of the caldera rim.	~ 25	<u>Conjectured model</u> Underplate diameter ~ 50 km Magma assumed to be Newtonian.
(v)	Baca geothermal system.	Additional local response due to reduced elastic moduli in the geothermal system. Short period transients possibly due to two-phase related pressure transients.

Figure 17. Subsequent 4 tiltmeter sites.

Following a 2 to 3 month monitoring period, two of the tilt meters should be moved to sites along the Redondo graben as indicated in Figure (17) and be employed there for a period of 2 months at each site.

At this juncture, it would appear that the continuation of the field program will have to be designed on the basis of the results obtained at the initial 8 field sites. However, we expect that upon the completion of this phase additional monitoring along the ring fracture system, both within and outside the caldera, and along the Redondo graben will be advisable.

Considerable care will have to be exercised in selecting the field sites and one will in this respect have to rely on experience in both the Western U.S. and in Iceland. One or two recording ground thermometers should be employed at each site and a recording high resolution barometer should be operated at a convenient site at the Valles.

Suitable smoothing and signal extraction methods should be employed in the processing of the data obtained and particular care taken to suppress the local thermoelastic noise at the field station.

An overall tilt resolution of 15 to 30 nr can probably be achieved in the field work.

Although this report is mostly concerned with elastomechanical techniques, it is of some interest to point out that other types of unconventional exploration techniques may be of interest in conjunction with the concepts set forth here. Since the subsurface temperature field is of particular interest, we point out that a ground level geomagnetic survey based on magnetic field fluctuation spectra to detect the depth to the Curie temperature would be of interest. There is now an enhanced interest in such techniques. An improved theoretical basis for these techniques is presented in section (1) of Essay (III).

ESSAYS

- (I) Lava Flows and Forms
- (II) Effusion Rates, Emplacement and Heat Transfer
- (III) Elastomechanical, Viscous Flow and Viscoelastic Fields
- (IV) Elastohydromechanical Fields
- (V) Remarks on Tidal Forcing

ESSAY (I): LAVA FLOWS AND FORMS

As a result of the high viscosity, the mode of most lava flows can be taken to be that of a viscous creep where inertia forces can be neglected. This is a tremendous simplification but there are other aspects that greatly complicate lava dynamics. Some are listed below.

(1) The rheological properties of lavas are apparently complex and only poorly known. According to Hulme (1974) there are indications that magmas may behave as Bingham liquids, that is, possess a yield strength. At stresses above the Bingham limit, magmas probably behave as Newtonian liquids, but the viscosity is highly temperature dependent.

(2) As a consequence of the temperature dependent viscosity the heat balance of lavas is of great importance for the rheology. In particular, it is to be noted that lavas generally form a solid crust and a solid bottom layer that reduce the effective height of the liquid section.

(3) Lava flow have a free surface and their motion is therefore subject to a non-linear surface condition.

(4) The mechanical processes in the fronts of lava flows differ from the situation in the liquid interior. The solidified crust piles up at the front and forms a type of barrier or wall that has to be partially pushed ahead of the flow or under the advancing lava. This condition remains to be quantified.

(5) Since the front is moving, the front condition is inherently non-linear regardless of the physical relations under (4) above.

(6) In general, lavas move on an uneven ground and the shape of the ground surface will thus enter into consideration.

(7) The productivity of the eruptive sources feeding lava flows is generally highly variable and unpredictable.

(8) Very few detailed field data are available on lava flows in motion. On the other hand, a great number of frozen or fossil lava flows are available for observation in various regions of the world. While the fossil lava data are useful, it is to be emphasized that the form and dimensions of such formations represent lavas in the final solidified state and may therefore not directly apply to the case of advancing flows that are still liquid. In the following, we will attempt at quantifying the simplest type of lava flow by presenting a theory of the subinertial free surface flow of a thin sheet of a homogeneous Newtonian liquid.

The Thin Newtonian Sheet. Consider a thin layer of a homogeneous incompressible Newtonian liquid flowing over an almost horizontal ground as indicated in Figure (1.1). The variations in the altitude of the ground and all slopes are assumed to be small.

Figure 1.1. Model of a lava flow.

We place a coordinate system with the origin in a horizontal plane surface Σ below ground level and with the z -axis vertically up. Let $S = (x,y)$ be a point on Σ , $f(S)$ be the ground altitude and $h(S)$ be the thickness of the fluid over the point S . Moreover, let ρ be the density of the fluid, μ and ν be the absolute and kinematic viscosities, respectively, $p(P,t)$ be the pressure and $\bar{u}(P,t) = (u,v,w)$ be the velocity in the fluid at the field point $P = (S,z)$ and at time t .

Since the fluid is assumed to be Newtonian, the motion is governed by the Navier-Stokes equation

$$\rho \frac{D\bar{u}}{Dt} = -\nabla p + \mu \nabla^2 \bar{u} - \bar{g}\rho, \quad (1.1)$$

where the material derivative

$$\frac{D}{Dt} = \partial_t + u\partial_x + v\partial_y + w\partial_z \quad (1.2)$$

and $\bar{g}$ is the acceleration of gravity. The condition of incompressibility is

$$\nabla \cdot \bar{u} = q \quad (1.3)$$

where $q(P,t)$ is the volume source density. The pressure at the free surface Ω is constant and can be assumed zero. The free surface boundary condition is therefore

$$\left. \frac{Dp}{Dt} \right|_{p=0} = 0 \quad p \text{ on } \Omega \quad (1.4)$$

The condition at the bottom of the fluid layer is that the flow velocity be parallel to the ground surface Γ . In the other words

$$\bar{n} \cdot \bar{u} = 0 \quad p \text{ on } \Gamma \quad (1.5)$$

when $\bar{n}$ is the outward normal to Γ . Finally, an initial condition may have to be adjoined to the above conditions.

The mathematical problem of solving equation (1.1) taking (1.2) to (1.5) into account is immense and beyond present capabilities. For our purpose we will make the following quite drastic but, nevertheless, plausible simplifications (a) and (b) which will reduce the above problem to a more manageable form.

(a) In view of the very slow flow, we can assume creep conditions where the mass forces can be neglected as compared with the viscous and gravitational forces. The term on the left of equation (1.1) will thus be neglected.

(b) Since the vertical velocity w in a creeping thin almost horizontal layer is small as compared with the horizontal components, we will assume that

$$|w| \ll |u| \approx |v| \quad (1.6)$$

and neglect w everywhere except in the conservation of volume condition (1.3).

(c) Moreover, the thin layer creep also implies that

$$\begin{aligned} |\partial_{xx} u| \approx |\partial_{yy} u| \ll |\partial_{zz} u| \\ |\partial_{xx} v| \approx |\partial_{yy} v| \ll |\partial_{zz} v| \end{aligned} \quad (1.7)$$

Let $\bar{u}(S) = (u, v)$ now be the horizontal velocity vector and $-\nabla_2^2$ the 2-dimensional Laplacian. The viscous term on the right of (1.1) then takes the form

$$\nabla_2^2 \bar{u} = (\partial_{zz} u, \partial_{zz} v). \quad (1.8)$$

(d) As a further consequence of the thin layer creep, the vertical viscous forces can be neglected as compared with the gravitational forces. We will thus assume for the fluid pressure in the layer that

$$p \approx g\rho(h - z) \quad (1.9)$$

The pressure gradient thus takes the form

$$\nabla p = (g\rho \partial_x h, g\rho \partial_y h, -g\rho). \quad (1.10)$$

As a consequence of the above approximations, the horizontal component of equation (1.1) reads now

$$0 = -g\rho \nabla_2 h + \mu \partial_{zz} \bar{u}, \quad (1.11)$$

where ∇_2 is the horizontal gradient. The vertical component simply reduces to the equation for the hydrostatic pressure in the layer.

Since the first term on the right of (1.11) is independent of z , the equation is easily integrated with respect to z . The boundary conditions require that the fluid velocity vanish at the bottom of

the layer and that there be no viscous force at the top. Hence, the conditions are

$$z = f, \quad \bar{u} = 0 \quad (1.12)$$

$$z = h, \quad \partial_z \bar{u} = 0$$

and the integration of (1.11) thus results in

$$\bar{u} = (g/2\nu)(z-f)(z+f-2h)\nabla_2 h \quad (1.13)$$

Equation (1.3) can now be used to eliminate the 2-dimensional velocity $\bar{u}(S)$. We integrate (1.3) with respect to z from $z = f$ to $z = h$ and apply the boundary conditions

$$z = f \quad w = 0 \quad (1.14)$$

$$z = h \quad w = -\partial_t h$$

Hence, interchanging the vertical integration and the horizontal differentiation, we obtain

$$\nabla_2 \cdot \left[\int_f^h \bar{u} dz \right] + \partial_t h = Q \quad (1.15)$$

where Q is the vertical integrated source density. The integral is obtained from (1.13) resulting in

$$\int_f^h \bar{u} dz = -(g/3\nu)(h - f)^3 \nabla_2 h \quad . \quad (1.16)$$

The final equation is obtained by inserting (1.16) into (1.15), viz.,

$$(g/3\nu) \nabla_2 \cdot [(h-f)^3 \nabla_2 h] = \partial_t h + Q. \quad (1.17)$$

This is the result of our approximations and represents the equation the creeping thin Newtonian fluid layer of the layer is the only remaining unknown.

To solve this equation, we need data on the kinematic viscosity ν , the bottom form $f(S)$ the source density $Q(S)$ and the front boundary condition that has yet to be formulated. As already stated the front condition is enigmatic, but to obtain some semi-quantitative results, we propose to proceed as follows.

The simplest approach, and perhaps the only viable one, consists in regarding the front velocity $\bar{v}$ measured in the direction of the front normal as a purely empirical function of the front height h_0 and the lava viscosity. The front condition based on the conservation of lava volume and equation (16) reads then

$$-(g/3\nu)(h_0 - f)^3 \nabla_2 h = (h_0 - f) \bar{v} \quad (1.18)$$

or

$$-(g/3\nu)(h_0 - f)^2 \nabla_2 h = \bar{v} \quad (1.19)$$

where the values of $(h_0 - f)$ and $\nabla_2 h$ are to be taken at the front.

As of now, there appear no possibilities of obtaining meaningful estimates of the front velocity on the basis of the mechanical conditions at the front.

Because of the non-linearities in $h(S)$, equation (1.17) with (1.19) can only be solved by numerical methods.

ESSAY (II): EFFUSION RATES, EMPLACEMENT AND HEAT TRANSFER.(1) Effusion rates.

Little material is available on the long-term average regional rates of lava effusion. While a few data are available on the great mafic plateaus, only one case of continental silicic volcanism, that is, the Oregon-Washington Cascades, has been studied from this point of view (McBirney, 1978). We will briefly review this material.

Mafic volcanism. The great flood basalt plateaus which represent the most volumous lava formations on the surface of the Earth were built up relatively slowly by a great number of individual eruptive events. According to Kuno (1969) the Columbia River plateau ($220,000 \text{ km}^2$) in North America and the Deccan plateau ($518,000 \text{ km}^2$) in India, which both have an average thickness of about 1 km, were produced during periods of 9 and 15 million years, respectively. The average rate of accumulation has therefore been of the order of 70 to 100 m/Myears. Since the sensible heat content of basaltic magma is of the order of $1.6 \times 10^6 \text{ J/kg}$, and its density around $3 \times 10^3 \text{ kg/m}^3$, this rate when averaged over the plateau area corresponds to an outward heat transport of 10 to 13 mW/m^2 . This is approximately 1/5 of the normal outward rate of conduction of heat through the Earth's surface. Although conspicuous structures, the formation of the large mafic plateaus did not represent a significant increase in the overall local outward transport of terrestrial heat.

Although no reliable data are available on the thickness of the main parts of the Brito-Arctic basalt province, the rate of accumulation does not appear to have been substantially different from the above cases.

The basalt plateau of Iceland ($103,000 \text{ km}^2$) represents perhaps the most prominent part of the Brito-Arctic province. For the present purpose, a vertical rate of accumulation can be defined on the basis of the seismic results of Palmason (1971) that show that the thickness of the series having the physical characteristics of flood basalts varies from 2 to 9 km. Using an average thickness of 4 km and assuming that the series has been built up since early Miocene, the average rate of accumulation is found to be 200 m/Myeas, that is, about the double of the above figure for the Columbia River and Deccan plateau.

It is interesting to note that during historic time individual eruptions in Iceland have produced from 0.1 to 10 km^3 of lava with an average well below one km^3 . The total number of eruptions required for the formation of the visible part of the flood basalt plateau of Iceland is therefore of the order of 10^6 . Similar figures can be expected in the case of the Columbia River and Deccan plateaus. All three basalt plateaus mentioned above were produced mainly by fissure eruptions. However, a number of significant central vents are known in Iceland.

Since the volcanism of Iceland has a definite axial rift-zone character it is of importance to note that during historic times the average effusion of sensible lava heat measured per unit length rift axis has been about 23kW/m axis. (Bodvarsson, 1981).

There is a number of smaller basalt plateaus which, on the other hand, have been produced mainly by central vents. These include the Manchuria-Korea plateau ($22,000 \text{ km}^2$) listed by Kuno (1960), the Snake River plateau in North America (Kuno, 1969), the Patagonian plateau ($60,000 \text{ km}^2$) in South America (Tyrrell, 1932) and the Trans-Mexican volcanic belt described by Mooser and Maldonado-Koerdell (1967).

The central vent character of the Mexican plateau is especially pronounced in the state of Michoacan where the area of Quaternary activity covers about $30,000 \text{ km}^2$. There is a very large number of cinder cones of various sizes scattered throughout the area. The reason for the different eruptive mechanisms of the smaller plateaus is not known.

Felsic volcanism. The Oregon-Washington Cascades represent an axial volcanic system of a predominantly andesitic character. Episodic activity has persisted throughout the Cenozoic. McBirney (1978) concludes that averaged over a period of 20 Myears, the effusive output has been about 10^{-5} km^3 per year and km axis length. This corresponds to an averaged heat output of one kW/m. Obviously, this activity is considerably below the case of Iceland. No output data are available for the Rio Grande rift but a similarity to the Oregon-Washington Cascades would appear likely.

(2) Emplacement.

In the present context, we are interested in the mode and rate of emplacement of magmatic intrusions, in particular, in the case of large intrusions of the Valles pluton type. Observations have been made on intrusive processes but no quantitative data are available on rates.

The mechanism of emplacement of discordant magmatic bodies such as plutons, stocks and volcanic necks presents a number of interesting problems in thermal geophysics. In contrast to dikes and sills that clearly have been formed by a displacement of the country rock, the more irregular bodies have apparently been emplaced largely by a replacement

of the rock. Available field evidence (see for example Frankel, (1967)) indicates that the process most frequently involves stoping, that is, a progressive piecemeal thermal stress fracturing of the country rock by the invading magma and a subsequent removal of the fragments by sinking in the liquid. There is little evidence that a direct fusion of the country rock at the magma/rock interface is an important factor in the emplacement of upper crustal intrusions. The thorough investigation of McBirney (1959) of the factors governing the emplacement of shallow volcanic necks in sedimentary rock in Arizona and New Mexico presents a particularly interesting field case. He concludes that thermal stoping has been the dominant emplacement process and that convective motion in the invading magma has removed the fragments from the rock/magma interface.

Fracturing induced by stresses of thermoelastic origin is likely to proceed only in relatively brittle rock. Since rocks become more ductile with increasing confining pressure and temperature, the tendency for thermal cracking is likely to decrease with increasing depth and may disappear below a certain critical depth that may be of the order of 5 to 10 km. Direct fusion of the country rock by the invading magma, which apparently is of little importance in the case of shallow intrusions, may become the dominant mode of emplacement at greater depths.

It is evident that the emplacement, whether involving thermal fracturing or fusion, requires a continuous supply of heat to the areas of contact between the magma and the country rock. The case of pure fusion is rather clear. Heat has to be brought to the rock/magma interface to supply the latent heat of fusion and also to maintain the temperature of fusion at the surface that is advancing into the solid phase. The situation in the case of thermal fracturing is more obscure. It is

fairly evident that a certain temperature range has to be maintained at the interface and it would appear likely that a sufficiently steep temperature gradient into the solid phase has to be present to promote fracturing. Unfortunately, we have no plausible methods of quantitatively estimating these particular requirements. Nevertheless, in the following section, a few relations will be set forth that help to throw some light on the more quantitative aspects.

(3) Heat transfer.

Heat balance at a moving interface. Consider a homogeneous and isotropic solid of density ρ , heat conductivity k , capacitivity c and diffusivity a . A plane of uniform temperature T_0 moves at a constant velocity v through the solid. The temperature of the solid at a great distance in front of the interface is zero. We are interested in the steady state temperature distribution in front of the interface. The case is one-dimensional and for the present purpose, we can put the coordinate origin $x = 0$ in the moving plane. The heat transport equation for the temperature field $T(x)$ is then

$$aD^2T + vDT = 0 \quad (2.1)$$

where $D = d/dx$. The solution satisfying the boundary conditions is

$$T = T_0 \exp(-vx/a), \quad (2.2)$$

and the conduction heat loss per unit area from the interface to the

solid in front is

$$H_c = -kDT|_{x=0} = \rho c v T_0 \quad (2.3)$$

If melting takes place at the interface, the total heat required to maintain a steady state situation is

$$H_m = \rho v (cT_0 + L) \quad (2.4)$$

where L is the latent heat of fusion and T_0 is then the differential between the temperature of fusion and the temperature of the rock away from the interface.

Magma/rock heat transfer in parallel flow. To investigate the transfer of heat from the magma to the rock, we consider the simple case of a Newtonian viscous flow parallel to the solid surface. The transfer of heat from the liquid to the surface under these circumstances has been studied extensively and we can here quote the following result based on boundary layer theory (Schlichting, 1968). The transfer per unit surface area as arranged over h units length from the origin is given by

$$H_o = 0.7(k\Delta T/h)P^{1/3}Re^{1/2} \quad (2.5)$$

where k is the heat conductivity of the liquid, T the temperature differential between the unaffected liquid and the (uniform) surface temperature, P is the Prandtl number and Re the Reynold number, that is,

$$P = \nu/a, \quad Re = wh/\nu \quad (2.6)$$

where ν is the kinematic viscosity, a the diffusivity and w the velocity of the liquid.

Vertical heat transport by liquid convection. We will consider a simplified model of laminar flow convection in a vertical pipe of constant diameter. The flow section consists of an upflow core of radius R and an outer downflow annulus. Let ν be the kinematic viscosity of the liquid and p be the driving pressure gradient in the upflow core. Based on laminar flow theory (Lamb, 1932) the total mass upflow in the core is then

$$M = (\pi R^4 / 8\nu) \nabla P \quad (2.7)$$

A similar relation although slightly more complex holds for the downflow annulus. In the present case, the total driving buoyancy gradient is provided by the differences in density between the upflow/downflow columns. Let α be the thermal expansivity of the liquid and there be an approximately uniform temperature differential T_d between the columns. In essence, this is equivalent to assuming that the heat is being taken up at the bottom of the circulation and withdrawn at the top. No cooling along the pipe wall is taken into consideration. These restriction will be elaborated on below. The total available pressure gradient is then $\alpha \rho T_d$ where ρ is the density of the liquid. This gradient drives the total flow circulation and since the downflow annulus has two bounding surfaces, we assume that for application in (2.7),

$$\nabla P = \alpha g \rho T_d / 3 \quad (2.8)$$

The mass flow up the core is then

$$M = \pi \alpha g \rho c T_d R^4 / 24 \nu \quad (2.9)$$

Under the circumstances assumed the total rate of heat transport by the section is then

$$Q = \pi \alpha g \rho c T_d^2 R^4 / 24 \nu \quad (2.10)$$

where c is the heat capacitivity of the liquid. Introducing the pipe radius R_0 that for a approximately balanced flow has to be $R_0 = 1.6 R$, we rewrite (2.10) as

$$Q = \pi \alpha g \rho c T_d^2 R_0^4 / 160 \nu \quad (2.11)$$

and obtain then the heat transport per unit pipe cross section

$$H_c = \alpha g \rho c T_d^2 R_0^2 / 160 \nu \quad (2.11)$$

This result is an ultrasimple estimate in the sense that no method of estimating the temperature differential T_d is provided. This quantity depends on the conditions at the heat supply/withdrawal ends. In applying (2.11) for order of magnitude estimates, we will simply assume plausible values for T_d .

Numerical examples. Using plausible parameter values, that is, $\rho = 3 \times 10^3 \text{ kg/m}^3$, $c = 10^3 \text{ J/kg, } ^\circ\text{C}$, $L = 4 \times 10^5 \text{ J/kg}$ and assuming, for example, $T_0 = 500^\circ\text{C}$, we find that equation (2.4) reduces to $H_m = 2.7 \times 10^9 v \text{ W/m}^2$. Considering a very slowly progressing melting interface with, for example, $v = 10 \text{ km/Myers}$ that might correspond to the upward migration of a pluton, we find then that approximately $H_m = 1 \text{ W/m}^2$. This is a small value but, nevertheless, 10 times the normal outward conduction heat in the earth's crust.

Proceeding to equation (2.5) and assuming a very viscous magma with $\eta = 3 \times 10^9 \text{ Pas}$, that is, $\nu = 10^6 \text{ m}^2/\text{s}$, we find that with $a = 10^{-6} \text{ m}^2/\text{s}$, $P = 10^{12}$. Moreover, let $k = 3 \text{ W/m, } ^\circ\text{C}$, $\Delta T = 100^\circ\text{C}$ and consider the case of $h = 10^3 \text{ m}$ and the velocity $w = 10^{-4} \text{ m/s}$. We find then that $\text{Re} = 10^{-7}$ and hence $H_0 = 0.7 \text{ W/m}^2$. This value is of the same order as H_m above and indicates that the two processes are energetically compatible at the conditions assumed.

Turning to the convective heat transport as given by equation (2.11), we can take that $\alpha = 2 \times 10^{-5}/^\circ\text{C}$ and assume again a very viscous magma of $\nu = 10^6 \text{ m}^2/\text{s}$. Inserting a small $T_d = 10^\circ\text{C}$ and $R_0 = 10^3 \text{ m}$, we find, nevertheless, approximately $H_c = 400 \text{ W/m}^2$. In spite of the adverse conditions assumed, this is a large value showing that convective heat transport in large structures containing Newtonian magma is an effective process.

Comparing the heat flow values obtained, it is quite evident that the heat exchange at the magma/rock interphase constitutes the principal impedance to the moving magma front. This conclusion applies to both types of migration mechanisms, that is, stoping or fusion, but the impedance would appear to be more critical in the case of fusion. The rate at which a magma front can proceed therefore depends critically on the conditions

at the magma/rock front. It is quite likely that the magma/rock and even more the invading magma/rock melt density contrasts are factors of dominating importance.

On the other hand, our results appear to indicate, that the magma/rock heat transfer is sufficiently efficient to support an upward migration rate of the order of 3×10^{-10} m/s = 10 km/Myeas. This result is relevant to the Valles pluton.

ESSAY (III): ELASTOMECHANICAL, VISCOUS FLOW AND VISCOELASTIC FIELDS.

In the following, we will consider a number of Hookean elastostatic and elastorheic fields and models that are of interest in the present context. Since such fields mostly are transforms of simple potential fields, it is of interest to review a number of concepts that appear in geophysical potential theory.

(1) Simple potential fields

Geophysical exploration is either source or parameter oriented. the first case is rather simple and we will therefore turn first to this case.

Gravitics and magnetics are typical source oriented cases where the exploration effort is directed towards the detection and delineation of source bodies. The potential field $u(P)$, where $P = (x, y, z)$, satisfies the simple potential equation

$$\Pi u = f \quad (3.1)$$

where $\Pi = -\nabla^2$ and $f(P)$ is the source field density that has to be adjusted with regard to physical dimensions. The solution of (3.1) is

$$u(P) = (1/4\pi) \int_{sp} (f(Q)/r_{PQ}) dv_Q, \quad (3.2)$$

where $Q = (x', y', z')$, $dv_Q = dx' dy' dz'$ and

$$r_{PQ}^2 = (x-x')^2 + (y-y')^2 + (z-z')^2, \quad (3.3)$$

The force field is then

$$\vec{g}(P) = -\nabla u = (1/4\pi) \int (f(Q) \vec{r}_{PQ} / r_{PQ}^3) dr_Q, \quad (3.4)$$

In source oriented exploration, $\vec{g}$ is the primary observable and $f(Q)$ is the unknown target.

Exploration on the basis of electrical D.C. conduction currents is the primary example of parameter oriented work. Let $\sigma(P)$ be the formation conductivity and the corresponding equation for the electrical potential $u(P)$ is then for isotropic materials

$$\Pi(\sigma) = -\nabla \cdot (\sigma \nabla u) = f. \quad (3.5)$$

With a variable $\sigma(P)$ this equation has no closed form solution corresponding to (3.2) above.

The primary observable is now the potential $u(P)$, or rather potential differentials $[u(P_1) - u(P_2)]$, the source field density $f(P)$ is usually known and the exploration target is the conductivity parameter $\sigma(P)$. Invariably, the observations on the potential are restricted to the earth's surface and there is therefore a high degree of indeterminacy in the problem setting.

Comparing equations (3.5) and (3.1) it becomes evident that in the general setting the source and parameter oriented cases are of a very different nature. However, in the very restricted case of homogeneous materials, equation (3.5) reduces to (3.1) and there is then a complete analogy. In such cases, the fundamental problem setting

is that of equation (3.1) and the solutions (3.3) and (3.4).

We will now emphasize 3 aspects of simple potential field mechanics, that are relevant in the present context but have, as of now, received insufficient attention, viz., (1) implications of gradients as observables, (2) a commutation theorem and (3) application of statistical fluctuations for exploration purposes.

Gradients as observables. The primary potential fields such as the gravitational and magnetic potentials are practically non-observable. Moreover, the absolute value of the electrical potential field is an impractical observable. In all such cases, we therefore prefer to observe field gradients such as the gravitational force or the difference in electrical potential between adjacent stations. Although this procedure is practically convenient in the electrical case, the field gradients depend critically on local formation parameters and have therefore to be interpreted with a considerable degree of caution. Because of fluctuation in the parameters of natural formations, a large number of observational data may have to be collected to obtain representative average values for the parameters.

The commutation relation Field gradients represent linear operations on the primary field. Consider now a linear operator L acting on potential fields that commutes with the potential operator such that

$$\Pi L - L \Pi = 0 \quad (3.6)$$

Operating with L on both sides of equation (3.1) we obtain then

$$\Pi(Lu) = Lf, \quad (3.7)$$

and hence

$$\Pi v = Lf \quad \text{if} \quad v = Lu, \quad (3.8)$$

In other words $v(P)$ is the potential function generated by the source density Lf . Any properly defined and localized field gradient can thus be interpreted as the potential function due to a transformed source density Lf . This interpretation is often of practical value and is helpful to understand information content of observational data.

Application of source density fluctuations. As emphasized above, natural formations are invariably inhomogeneous and exhibit fluctuations in the intrinsic parameters such as the density, magnetization and electrical conductivity. Quite often two formations exhibit the same average properties but have different fluctuation characteristics. They can then often be distinguished on the basis of the fluctuation statistics. A technique based on magnetic source density fluctuation to determine the depth to the Curie temperature in the earth's crust has recently been developed by Gudmundsson (1967) and Spector (1968). This subject is of some interest in the case of the Valles. The development by Spector (1968) that is now being adopted by a number of exploration groups is, however, based on assumptions that are rather irrelevant and below we will therefore recast the theory into a more straight forward setting.

Fourier transform formulation of potential field data. We will commence with case of the gravity field.

Using the notation of above the force of gravity is

$$\vec{g}(P) = -\gamma \int_{sp} (\vec{r}_{PQ}/r_{PQ}^3) \rho(Q) dv_Q, \quad (3.9)$$

and the vertical component is

$$g_z(P) = -\gamma \int [(z-z')/r_{PQ}^3] \rho(Q) dv_Q = \gamma \int [(z'-z)/r_{PQ}^3] \rho(Q) dv_Q \quad (3.10)$$

Taking the Fourier transform with respect to (x,y) with $\vec{k} = (k_1, k_2)$ we get

$$\hat{g}_z(\vec{k}, z) = 2\pi\gamma \int \exp[-k|z'-z|] \hat{\rho}(\vec{k}, z') dv_k dz' \quad (3.11)$$

Taking the point of observation at $z = -h$ and $z' \geq 0$ we have in the case of a maximum source depth H

$$F_h(\vec{k}) = g_z(\vec{k}, -h) = 2\pi\gamma \exp(-kh) \int_0^H \exp(-kz') \hat{\rho}(\vec{k}, z') dz' \quad (3.12)$$

and if $\hat{\rho}(\vec{k}, z') = A(\vec{k})$ is constant on vertical lines

$$F_h(\vec{k}) = 2\pi\gamma \exp(-kh) A(\vec{k}) (1/k) [1 - \exp(-kH)] \quad (3.13)$$

or

$$F_h(\vec{k}) = (2\pi\gamma/k) A(\vec{k}) [\exp(-kh) - \exp(-k(h+H))] \quad (3.14)$$

Depending on the definition of $|F_k(\vec{k})|^2$ and $|A(\vec{k})|^2$, we have then the energy or energy density spectrum of F_h

$$|F_h(\vec{k})|^2 = (2\pi\gamma/k)^2 |A(\vec{k})|^2 [\exp(-kh) - \exp(-k(h+H))]^2 \quad (3.15)$$

The case of magnetic force fields follows easily from the development above. Since all oblique force/source fields can be transformed by convolution type operations to the vertical, it is most convenient to express our results in terms of the vertical case. The vertical magnetic force field amplitude due to a vertical homogeneous magnetization density m follows then by an operation by $-(\mu_0 m / 4\pi\gamma) \partial_z$, on ((3.11) above (μ_0 = permeability of the vacuum). Assuming vertical source density homogeneity the results corresponding to (3.14) and 3.15) are then

$$M_h(\vec{k}) = (\mu_0 m / 2) B(\vec{k}) [\exp(-kh) - \exp(-k(h+H))] \quad (3.16)$$

where $B(k)$ is the FT amplitude of the source density. Moreover

$$|M_h(k)|^2 = (\mu_0 m / 2)^2 |B(k)|^2 [\exp(-kh) - \exp(-k(h+H))]^2 \quad (3.17)$$

It is of interest to note that by measuring $M_h(\vec{k})$ at a known $z = -h$ we obtain data on $B(\vec{k})$ that may characterize the underlying formation. Moreover, the maximum of $|M_h|^2$ given by (3.17) will depend on $|B|^2$. Observational data on $M_h(\vec{k})$ of sufficient quality will very frequently allow the determination of H which is the main task in Curie depth investigations.

(2) Elastomechanical fields

Consider a Hookean solid with the Lamé parameters λ and μ and the displacement vector $\vec{u} = (u_x, u_y, u_z)(P)$. The impressed body volume force density is $\vec{f}(P)$. Limiting ourselves to only vertically heterogeneous elastic media, we have then the basic equation for infinitesimal displacements (see Menahem and Singh, 1981)

$$\mu \nabla^2 \vec{u} + (\lambda + \mu) \nabla \nabla \cdot \vec{u} + \vec{z} (d\lambda/dz) \nabla \cdot \vec{u} + (d\mu/dz) (2\partial_z \vec{u} + \vec{z} \times \nabla \times \vec{u}) + \vec{f} = 0, \quad (3.18)$$

where $\vec{z}$ is the vertical unit vector. In homogeneous media the equations simplify to the standard form

$$\mu \nabla^2 \vec{u} + (\lambda + \mu) \nabla \nabla \cdot \vec{u} + \vec{f} = 0, \quad (3.19)$$

Let the volume strain $\nabla \cdot \vec{u} = e$ and operate with $\nabla \cdot$ on (3.19) resulting in

$$(\lambda + 2\mu) \nabla e = \nabla \cdot \vec{f} \quad (3.20)$$

Hence, in homogeneous media, the volume strain satisfies a simple potential equation of the type (3.1). Equations (3.19) and (3.20) indicate the close relation between the elastomechanical fields and simple potential fields.

The elastomechanical observables are the displacement vector $\vec{u}$, linear strain and tilt components, that is, the various derivatives of $\vec{u}$ with respect to the coordinates and the volume strain e . As a matter of course, the latter quantity is a sum of three linear strain components, but in proper experimental situations it can be observed independently. Because of the importance of the volume strain we will below elaborate on the experimental techniques involved. Two models of particular interest for which solutions of (3.18), (3.19) and/or (3.20) have been obtained will be described briefly.

The homogeneous/isotropic Hookean half-space. Consider a half-space with the boundary surface Σ and place the (x,y) plane of the coordinate system in Σ with the vertical z -axis positive into the half-space. Let a pure pressure $p(S)$, where $S = (x,y)$, act on Σ . Assuming no body force, viz., $\vec{F} = 0$, the resulting displacement is obtained on the basis of the well known Bussinesq solution (Love, 1927) for the half-space. It is convenient to express the solution in form of the Green's function response to $p = \delta(S)$, which expressed in radial coordinates is

$$u_{rg} = (1/4\pi\mu)[(rz/R^3)-(1-2\sigma)r/R(z+R)], \quad (3.21)$$

$$u_{zg} = (1/4\pi\mu)[(2/R)(1-\sigma)+z^2/R^3], \quad (3.22)$$

where σ is the Poisson's ratio and $R^2 = (r^2+z^2)$. The subscript g refers to the Green's function response. In the present context, we are mainly interested in the downward displacement of the Σ plane and obtain these on the basis of (3.22) the vertical displacement component at $z = 0$ for a pressure load of $p(S)$ that

$$u_{z0}(S) = [(1-\sigma)/2\pi\mu] \int_{\Sigma} (1/r_{ST}) p(T) da_T, \quad (3.23)$$

where T is again the surfact point of integration, r_{ST} the distance between S and T and da_T is the surface element at T . Equation (3.23) is a convolution and a 2-dimensional Fourier transform in Σ will therefore convert it into a simple product of the transforms of $(1/r_{pT})$ and

$p(T)$. Since $F(1/r_{pT}) = (1/k)$ and let $\hat{p}$ and $\hat{u}_{zo}$ be the transforms of p and u_{zo} respectively, we find that equation (3.23) transforms into the simple form

$$\hat{u}_{zo}(K) = [(1-\sigma)/\mu](1/k)\hat{p}(K), \quad (3.24)$$

where K is the field point in transform space. Hence,

$$\hat{p} = [\mu/(1-\sigma)]k\hat{w} \quad (3.25)$$

and the inverse transform is

$$p(S) = [\mu/(1-\sigma)]\Pi_2^{1/2}w(S) \quad (3.26)$$

where $\Pi_2^{1/2} = (-\nabla^2/2)^{1/2}$ is a surface differential operator of a fractional order.

For later convenience, we rewrite equation (3.24) in terms of the Lamé parameters

$$\hat{u}_{zo}(K) = (2\mu+\lambda)\hat{p}(K)/2k\mu(\lambda+\mu), \quad (3.27)$$

The elastic plate. The homogenous Hookean plate of thickness h and Young's modulus E is another model of interest. Let the surface of the plate coincide with the Σ plane introduced above and be acted on by the pure pressure $p(S)$. The plate is assumed to have finite horizontal dimensions and be supported in a prescribed manner at the

boundary. Standard thin elastic plate theory gives then the basic equation for the vertical displacement u (subscript omitted)

$$\xi \pi_2^2 u = p \quad (3.28)$$

where

$$\xi = Eh^3/12(1-\sigma^2) \quad (3.29)$$

is the stiffness of the plate. The boundary conditions have to be adjoined to (3.28). The 2-dimensional Fourier transform of (3.28) is

$$\xi k^4 \hat{u} = \hat{p}$$

The constrained displacement approximation. Three-dimensional elastomechanical field problems in inhomogeneous media are generally quite involved and solutions difficult to obtain. We are therefore interested in suitable approximation methods that help to simplify the mathematical procedure and provide results that are sufficiently close to the exact solutions. Such methods are particularly useful in situations where the elastic parameters are incompletely known and the effort expended for exact solution would therefore not be warranted.

The constrained displacement approximation (CDA), to be described below, is a useful method under such circumstances. We will focus our attention on Hookean half-spaces that are loaded by normal surface force fields and test the method on this case. First, we simplify (3.24) by

using Poisson's relation $\lambda = \mu$ and obtain then

$$\hat{u}_{z0} = 0.75\hat{p}/k_{\mu} \quad (3.31)$$

On the CDA we assume that the pure pressure loading on the half-space results mainly in a vertical displacement such that in the case of one-dimensional loading we can approximate the displacement vector by

$$\vec{u} = (0, 0, u(P)) \quad (3.32)$$

where $P = (x, z)$ is the general field point and we have omitted the subscript on $u_z(P)$. On the CDA, the basic equations then reduce to

$$\mu \partial_{xx} u + (\lambda + 2\mu) \partial_{zz} u = 0 \quad (3.33)$$

which with $\lambda = \mu$ becomes

$$\partial_{xx} u + 3\partial_{zz} u = 0 \quad (3.34)$$

This equation is easily solved with the help of a Fourier transformation with respect to x and we find that

$$\hat{u}_0 = 0.58\hat{p}/k_{\mu} \quad (3.35)$$

Relative to the exact solution (3.31) this is an underestimate by almost 25%. It is noted that the two solutions deviate only by a constant factor and the approximation is therefore independent of the wave-number.

In the present context where there is great uncertainty with regard to underlying models and even the rock parameters, we tend to regard this approximation as satisfactory, in particular, since we have a measure of the degree of approximation and can then make some useful corrections.

Application of the CDA to the case of a caldera model. We will now apply the CDA to a specific case of a Hookean elastic half-space with a flat boundary surface Σ . The space is isotropic but inhomogeneous in the sense that it contains a flat inclusion of thickness h located just below Σ . A cross section through the model is shown in Fig. (3.1). The inclusion covers a horizontal area A , has the boundary contour B and its horizontal dimensions are made larger than h . We are interested in the vertical displacement of Σ in response to a (fluid) pressure field p acting on Σ . To further simplify the case, we assume that the inclusion has much lower elastic moduli than the half-space. In estimating the relative deformation of the inclusion, we can then assume that the half-space is completely rigid. The CDA is particularly suited to estimate the tilt of the surface in this case.

Figure 3.1. Caldera fill model.

To apply the CDA, we assume that the vertical displacement vector $u(P)$ as expressed by (3.32) above when $P = (x, y, z)$ is a general field point. Assuming again Poisson's relation $\lambda = \mu$, we obtain then the equation

$$\nabla^2 u + 3\partial_{zz} u = 0, \quad (3.36)$$

where $\nabla^2 = \partial_{xx} + \partial_{yy}$. The pressure boundary condition on the surface Σ is

$$p = -3\mu \partial_z u|_{z=0}, \quad (3.37)$$

and

$$u = 0 \quad \text{for } P \text{ on } C \quad (3.38)$$

The equations above are easily solved by a standard eigenfunction technique. We assume that

$$u = \sum_j \phi_j [b_j \exp(-a_j z) + c_j \exp(a_j z)], \quad (3.39)$$

where $\phi_j(x, y)$ are the normalized eigenfunctions satisfying

$$-\nabla^2 \phi_j = \lambda_j \phi_j, \quad (3.40)$$

the λ_j 's are eigenvalues and because of (3.38) the boundary conditions on ϕ_j are

$$\phi_j = 0 \text{ for } (x,y) \text{ on } C \quad (3.41)$$

Moreover, because of (3.37), we have the condition at $z = 0$

$$p = - 3\mu \sum_j \phi_j a_j [c_j - b_j], \quad (3.42)$$

and since the surrounding half-space is rigid the condition at $z = h$ is $u = 0$, that is

$$\sum_j \phi_j [b_j \exp(-a_j h) + c_j \exp(a_j h)] = 0, \quad (3.43)$$

Inserting u as expressed by (3.39) into (3.36) and using (3.40) gives the relation

$$a_j = (\lambda_j/3)^{1/2} \quad (3.44)$$

Moreover, because of the orthonormality of the ϕ_j 's, we obtain from (3.42) that

$$\int_A p \phi_j da = f_j = - 3\mu a_j (c_j - b_j), \quad (3.45)$$

where da is the element of A and f_j is a notation for the integral on the left. Similarly, from (3.43) we obtain

$$b_j \exp(-a_j h) + c_j \exp(a_j h) = 0, \quad (3.46)$$

Since p is assumed to be known and the ϕ_j 's are determined by the form of A , the f_j 's are known and the set of equations (3.45) and (3.46) determine the expansion coefficients b_j and c_j . Solving, we obtain

$$b_j = (f_j/3\mu a_j)[1 + \exp(-2a_j h)]^{-1}, \quad (3.47)$$

$$c_j = - [f_j \exp(-2a_j h)/3\mu a_j][1 + \exp(-2a_j h)]^{-1}, \quad (3.48)$$

Inserting in (3.39) the final result emerges

$$u = (1/3\mu) \sum_j (f_j \phi_j / a_j) [1 + \exp(-2a_j h)]^{-1} [\exp(-a_j z) - \exp(-a_j (2h - z))] \quad (3.49)$$

The displacement of the boundary Σ is obtained by taking $z = 0$, that is

$$u_{z=0} = (1/\mu 3^{1/2}) \sum_j f_j F_j \phi_j / \lambda_j^{1/2} \quad (3.50)$$

where

$$F_j = [1 - \exp(-2a_j h)]/[1 + \exp(-2a_j h)], \quad (3.51)$$

is the reflection factor. Equation (3.50) represents our final result from which we can obtain the tilts of the surface.

The case of a penny-shaped inclusion of radius R where $R \gg h$ is of particular interest. Following standard procedure for elementary potential fields in cylindrical coordinates, we find then (see, for example, Carslaw and Jaeger, 1959) that with the origin at the center of the inclusion and r

the radial coordinate,

$$\phi_j(r) = J_0(\alpha_j r) / \pi^{1/2} R J_1(\alpha_j R), \quad (3.52)$$

where α_j is the j 'th root of $J_0(\alpha R) = 0$ and

$$\lambda_j = \alpha_j^2, \quad a_j = \alpha_j / 3^{1/2} \quad (3.53)$$

In the case that the pressure is constant $p = p_0$ over A , we have

$$f_j = 2\pi^{1/2} p_0 / \alpha_j \quad (3.54)$$

and hence equation (3.50) takes the form

$$u_{z=0} = (2p_0 / \mu R 3^{1/2}) \sum_j F_j J_0(\alpha_j r) / \alpha_j^2 J_1(\alpha_j R), \quad (3.55)$$

and the radial derivative

$$\partial_r u \Big|_{z=0} = -(2p_0 / \mu R 3^{1/2}) \sum_j F_j J_1(\alpha_j r) / \alpha_j J_1(\alpha_j R), \quad (3.56)$$

The tilt at the boundary $r = R$ is then

$$\partial_r u \Big|_{\substack{z=0 \\ r=R}} = -(2p_0 / \mu R 3^{1/2}) \sum_j (F_j / \alpha_j), \quad (3.57)$$

The convergence of this expression is, however, in question. The point $r = R$ may be a singularity in the tilt field and we may have to disregard this point and compute tilts for $r < R$ on the basis of (3.56).

Improved approximation. In essence, the CDA procedure, as described above, is based on the neglecting of two displacement components and thereby replacing the volume strain $\nabla \cdot \vec{u}$ by $\partial_z u$ where u is the z - component of $\vec{u}$. Clearly, even at the model conditions assumed, the two components will generally not vanish, although they may be sufficiently small for the CDA to give satisfactory results. This situation does evidently leave room for improvement on the basis of a perturbation approach. Having obtained the first approximation to $u(P)$, the derivative $\partial_z u$ can be inserted in the first equations of the vector set in (3.30) and solutions for the two displacement components can be obtained by standard potential theoretical procedures. The two functions can then be applied to improve $\nabla \cdot \vec{u}$ in the third equation of (3.30). A new solution for $u(P)$ can then be obtained that will represent a second order approximation. The procedure can then be repeated. Since there is a considerable algebra involved, we will refrain from discussing details of the perturbation approach.

(3) Newtonian viscous half-spaces

To study the deformation of viscous bodies in response to external force fields, it is of some conceptual interest to consider the case of subinertial viscous creep in a homogeneous incompressible Newtonian liquid half-space that is deformed by an impressed pressure field at the boundary surface Σ . In the following, we will confine our attention to the simple case where Σ remains approximately plane under the impressed pressure field $p(S)$ where S are points on Σ .

We place the (x,y) plane of our coordinate system in Σ with the z -axis vertically down into the half-space. Let $P = (x,y,z)$ be the general field point, $\vec{v} = (v_x, v_y, v_z)(P)$ the liquid velocity vector, $p(P)$ the pressure field and η be the absolute viscosity of the liquid. Under the conditions assumed,

the basic dynamic and kinematic equations for the flow fields are

$$\eta \Pi \vec{v} = -\nabla p, \quad (3.58)$$

$$\nabla \cdot \vec{v} = 0 \quad (3.59)$$

where $\Pi = -\nabla^2$. Operating with $\nabla \cdot$ on (3.58) results then in the scalar potential equation for the pressure field

$$\Pi p = 0 \quad (3.60)$$

Using standard Green's function technique, this equation can readily be solved for p and the three components of $\vec{v}$ can then be obtained by a similar procedure on the basis of (3.58). However, to determine $\vec{v}$, appropriate boundary conditions at Σ have to be prescribed.

In the present context, we will concentrate on the case of a harmonic pressure load on Σ that is taken to be a free surface such that all tangential forces vanish, or in other words, the Neumann type conditions

$$\partial_z v_x = \partial_z v_y = 0, \text{ for } z = 0, \quad (3.61)$$

prevail at Σ . The surface load is conveniently expressed by

$$p(S) = A \exp(ik_1 x + ik_2 y) \quad (3.62)$$

and the solution of (3.59) is then simply

$$p(P) = A \exp(ik_1 x + ik_2 y - kz), \quad (3.63)$$

where

$$k^2 = k_1^2 + k_2^2, \quad (3.64)$$

To obtain v_x and v_y , the following equations

$$\eta \Pi v_x = -ik_1 A \exp(ik_1 x + ik_2 y - kz), \quad (3.65)$$

$$\eta \Pi v_y = -ik_2 A \exp(ik_1 x + ik_2 y - kz), \quad (3.66)$$

have to be solved with the conditions (3.61). Omitting a little algebra we obtain with the help of the Neumann type of Green's function

$$v_x = -(iAk_1/\eta k^2)(kz + 1) \exp(ik_1 x + ik_2 y - kz), \quad (3.67)$$

$$v_y = -(iAk_2/\eta k^2)(kz + 1) \exp(ik_1 x + ik_2 y - kz), \quad (3.68)$$

and to obtain v_z it is convenient to solve (3.60) directly by

$$v_z = \int_z^\infty (\partial_x u + \partial_y v) dz', \quad (3.69)$$

Inserting from (3.67) and (3.68)

$$v_z = (A/\eta) \int_z^\infty (kz' + 1) \exp(ik_1 x + ik_2 y - kz') dz' \quad (3.70)$$

and hence by a simple integration

$$v_z = (A/k\eta)(kz + 2)\exp(ik_1x + ik_2y - kz). \quad (3.71)$$

Therefore, in the one-dimensional case with a wave-length L , we obtain the amplitude relation at $z = 0$

$$v_{z0} = 2A/\eta k = \hat{p}L/\pi\eta \quad (3.72)$$

where $\hat{p}$ is the pressure amplitude at Σ .

Another case of interest is obtained when the boundary conditions (3.61) are replaced by the Dirichlet type zero-velocity conditions at Σ ,

$$v_x = v_y = 0 \quad \text{for } z = 0 \quad (3.73)$$

The corresponding solutions are obtained by the same procedure as above except that a Dirichlet type of Green's function has to be applied in solving for v_x and v_y . We obtain then instead of (3.71)

$$v_z = (A/\eta k)(zk+1)\exp(ik_1x+ik_2y-kz), \quad (3.74)$$

and thus at $z = 0$

$$v_{z0} = A/\eta k = \hat{p}L/2\pi\eta \quad (3.75)$$

or inverted into coordinate space

$$\eta \Pi_2^{1/2} v_{z0} = A \quad (3.76)$$

These values are just the half of the previous values given by (3.72).

Since time is not an explicit variable in this section, $\hat{p}$ and thereby also v_{z0} can be any slowly varying functions of time that are consistent with the assumption of creep.

It is of interest to point out the similarity between the displacement relation (3.24) and the velocity relation (3.75).

(4) The viscoelastic half-space

The generalization of the above results to a viscoelastic half-space is straight forward. For this purpose, we have to derive a combination of the two basic sets of equations (3.19) and (3.58), (3.59). This is most easily achieved by introducing viscous terms in (3.19) by replacing the Lamé parameters λ and μ by two operator type parameters $(\lambda + \lambda' \partial_t)$ and $(\mu + \mu' \partial_t)$ where ∂_t is the partial with respect to time and λ' and μ' are viscosity type parameters that can be adjusted to particular material situations. It must be emphasized that this approach is somewhat specific and that more general operators can be applied. However, there are good reasons to expect that our approach will be adequate for the present purpose. Equation (3.19) is then modified to

$$(\mu + \mu' \partial_t) \nabla^2 \vec{u} + (\lambda + \lambda' \partial_t + \mu + \mu' \partial_t) \nabla \nabla \cdot \vec{u} + \vec{f} = 0, \quad (3.77)$$

where $\vec{u}$ and $\vec{f}$ are now to be taken as functions of time. In the present case our interest focuses on obtaining an appropriate impulse response of the model governed by (3.77). By assuming all functions causal and resorting to the Laplace transformed version with s as the transform

of time t the parameters in (3.77) are then simply $(\lambda+s\lambda')$ and $(\mu+s\mu')$ and our specific viscoelastic case is then transformed into the Hookean case. The solution obtained for the Hookean half-space can therefore be applied to the viscoelastic case by a reinterpretation of the parameters. For the present purpose it is convenient to work in the (k_1, k_2, z) Fourier transform space and thus focus the attention on the viscoelastic version of equation (3.24) or more explicitly (3.27).

To arrive at the impulse response for $\hat{u}_z(k)$, the forcing pressure Fourier transform $\hat{p}(K)$ is to be taken to represent the Laplace transform of an impulse and the parameters have to be repaced as indicated above. The new version of (3.27) is then following some rearrangement

$$\tilde{u}_{z1}(K, s) = (1/k) \tilde{p}(K) [(A/(1+as)(1+bs)) + (Bs/(1+as)(1+bs))], \quad (3.78)$$

where

$$A = (\lambda + 2\mu)/2\mu(\lambda + \mu), \quad B = (\lambda' + 2\mu')/2\mu(\lambda + \mu) \quad (3.79)$$

$$a = (\lambda' + \mu')/(\lambda + \mu), \quad b = \mu'/\mu$$

The Laplace inversion for the impulse response is now straight forward and yields

$$\begin{aligned} \hat{u}_{z1}(K, t) = (1/k) \hat{p}(K) [& (A/(a-b))(\exp(-t/a) - \exp(-t/b)) \\ & + (B/(a-b))(-1/a \exp(-t/a) + 1/b \exp(-t/b))] \end{aligned} \quad (3.80)$$

Hence, for a given pressure load $\hat{p}(K)f(t)$ where $f(t)$ is a causal function of time, the displacement amplitude is

$$\hat{u}_z(K,t) = \int_0^t \hat{u}_{z1}(K,t-\tau)f(\tau)d\tau \quad (3.81)$$

This is the final result on the vertical displacement of the Σ plane due to a given surface pressure load acting on the viscoelastic half-space.

(5) Composite half-spaces

Composite models provide an alternative method of combining elastic and rheic properties. The most important elementary model of this type is represented by the homogeneous Hookean elastic plate placed on the top of a viscous or elastoviscous half-space. We will consider three such cases. It is convenient to place the Σ plane at the top of the elastic plate, and since we are mainly interested in the vertical displacement due to a pure pressure load on Σ , it is convenient to use $u(S)$ without subscript to denote this displacement component. Since we are interested in wavelengths L that are of the order of a few tens of kilometers, we can assume that the thickness of the plate $h \ll L$. Moreover, it can be assumed that h remains constant upon deformation and that the displacement u applies both to the Σ plane and the lower plane of the plate. Finally, we will depend on the same type of representation as above, that is, derive an impulse response of u to a pressure load $p(S)$ in the 2-dimensional Fourier transform representation. As before we are only interested in slow displacements, or creep, in other words, where inertia forces can be neglected. There is no contradiction in introducing an impulse response for creeping motion, if it is applied only to sufficiently slowly varying functions of time. Based on the

results above, the development is now straight forward.

The elastic plate on a Newtonian viscous half-space. Let p_1 be the pressure distribution at the bottom of the plate (see Fig. 3.2) Using the notation of section (2) above the equation for creeping motion of the plate is then

$$\xi \Pi_2^2 u = p - p_1 - \rho g u \quad (3.82)$$

where ρ is the density of the plate, g the acceleration of gravity. The third term on the right of (3.82) has been introduced to account for the hydrostatic forces. We assume that the plate and the lower half-space have the same density. The equation applying to the top plane of the lower half-space is (see (3.76))

$$\eta \Pi_2^{1/2} \partial_t u = p_1, \quad (3.83)$$

Eliminating p_1 results in

$$(\rho g + \xi \Pi_2^2) u + \eta \Pi_2^{1/2} \partial_t u = p, \quad (3.84)$$

and taking the 2-dimensional Fourier transform in the Σ plane in conjunction with assuming that the transform of $p(S,t)$ is $\hat{p}(K)\delta(t)$ and $D = d/dt$ results in the equation for the amplitude $\hat{u}(K,t)$

$$(\rho g + \xi k^4)u + \eta k D\hat{u} = \hat{p}\delta(t), \quad (3.85)$$

or

$$D\hat{u} + [(\rho g/\eta k) + (\xi/\eta)k^3]\hat{u} = (\hat{p}/\eta k)\delta(t), \quad (3.86)$$

The solution for the impulse response is elementary

$$\hat{u} = (\hat{p}/\eta k)\exp[-((\rho g/\eta k) + (\xi/\eta)k^3)t] \quad (3.87)$$

which holds for $L \gg h$ and $t > 0$.

Figure 3.2. Elastic plate on a viscous half-space.

The elastic plate on a thin Newtonian viscous slab resting on a rigid half-space. In continuation of the development above it is of interest to replace the viscous half-space by a viscous slab of relatively small thickness H such that the liquid motion can be taken to be predominantly horizontal. This could be the case of a viscous underplate. The above development has then to be modified in the following way.

Let $\vec{q}$ be the vertically integrated horizontal mass flow vector in the viscous slab that has the kinematic viscosity ν and p_1 be the flow pressure that is assumed to be constant over the slab thickness H . The model is sketched in Fig. (3.3). On the basis of standard Newtonian viscous flow theory, we have then

$$\vec{q} = -(H^3/12\nu)\nabla_2 p_1 \quad (3.88)$$

and the conservation of liquid mass requires

$$\nabla_2 \cdot \vec{q} = \rho \partial_t u \quad (3.89)$$

The equation for the plate is the same as (3.82). Eliminating $\vec{q}$ and p_1 results after some algebra in

$$D\hat{u} + (H^3/12\eta)(\xi k^6 + \rho g k^2)\hat{u} = (k^2 H^3 \hat{p}/12\eta)\delta(t) \quad (3.90)$$

The impulse response is then

$$\hat{u} = (k^2 H^3 \hat{p}/12\eta) \exp[-(H^3/12\eta)(\xi k^6 + \rho g k^2)t] \quad (3.91)$$

This equation holds for $L \gg h$, $L \gg H$ and $t > 0$.

Figure 3.3. Elastic plate above a viscous slab.

(6) Elastic effects around liquid spaces.

The elastic effects of liquid inclusions such as magma bodies, plutons and underplates are of primary concern in the present context. Unfortunately, the problems of a quantitative theory of such phenomena are very complex and the usefulness of a major theoretical development is in considerable doubt because of the sparsity data on the rheology and other physical properties of the liquid magma. As emphasized above, the viscosity of magma depends critically on temperature, chemistry and content of volatiles such as water. The question of a Bingham type yield strength remains open. Moreover, it is quite obvious that the compressibility will also be very dependent on the water/gas content. The volatiles may either be totally dissolved in the liquid or occupy separate spaces, in particular, at the top of magma bodies.

In view of the combined complexity and uncertainty, we will limit our present remarks to a brief discussion of one of the main phenomena involved, viz., the pressure redistribution at liquid spaces. The density of the liquid will be taken to be the same as that of the surrounding solid material such that density effects can be neglected. As a matter of fact, the density of magma is generally not significantly different from the country rock. The same pressure effect will also take place at gaseous inclusions, but the density effect will be there.

Consider now an elastic half-space with a liquid inclusion as sketched in Fig. (3.4) below

Figure 3.4. The liquid inclusion.

To obtain a maximum degree of simplicity, we assume that the liquid space is approximately cylindrical with vertical walls and a flat bottom that is at a depth d from the surface of the half-space. This model could represent a pluton without a caldera-block type solid cover that, however, has been omitted because it is of little relevance to the elastic effects. Let the liquid be low-viscosity Newtonian, the surrounding solid homogeneous/isotropic Hookean and assume that the surface of the half-space is subject to a unit-step type increase in pressure of amplitude p . We are interested in the relative displacement of the liquid surface in response to the pressure load. Obviously, only displacement within the surface slab of thickness d is of interest.

Let the coordinate system be placed in the surface with the z -axis vertically down, u_s be the downward displacement of the solid surface relative to the depth level d and σ_h be the horizontal stress in the surface slab both quantities measured away from the inclusion. Using the notation above an ultrasimple exercise in elasticity shows that

$$u_s = pd/(\lambda + 2\mu), \quad \sigma_h = p\nu/(1-\nu), \quad (3.92)$$

where ν is the Poisson's ratio of the solid. The liquid, on the other hand, transmits the total surface pressure to the walls of the inclusion and the liquid therefore exerts an overpressure on the solid

$$\Delta p = p - (p\nu/(1-\nu)) = p(1-2\nu)/(1-\nu), \quad (3.93)$$

Assuming Poisson's relation $\lambda = \mu$ implying $\nu = 0.25$, we find that

$\sigma_h = p/3$ and $\Delta p = 2p/3$. The overpressure will expand the volume of the inclusion V and assuming that the elastance of the volume is E , we find that the volume increment is

$$\Delta V_1 = 2Ep/3 \quad (3.94)$$

Moreover, the reduction in liquid volume due to compressibility is

$$\Delta V_2 = -cpV \quad (3.95)$$

where c is the compressibility of the liquid. Assuming that the area of the inclusion is A , the displacement of the liquid surface relative to the solid surface is

$$u_1 = ((2E/3)+cV)(p/A) - u_s$$

or

$$u_1 = [((2E/3)+cV)(1/A)-(d/3\mu)]p \quad (3.96)$$

To obtain a useful order of magnitude estimate, we observe that the elastance E is not too shape dependent and that we can use results for spherical volumes of radius R (Love, 1927) where (still assuming $\lambda = \mu$),

$$E = \pi R^3/\mu \quad (3.97)$$

In this case it is reasonable to take that $A = \pi R^2$ such that $E/A = R/\mu$ that we equate approximately to $d/2\mu$. For the cylinder $V/A = d$ and inserting these values in (3.96) reveals that the displacement of the

liquid surface due to the volume elastance is approximately equal to the displacement of the solid surface represented by the last term in the brackets of (3.96). The relative displacement of the liquid surface is thus precominantly due to the liquid compressibility and amounts to

$$u_l = pcd \quad (3.98)$$

Clearly, this result furnishes only an order magnitude estimate.

Another case of liquid pressure effects of interest results from the tidal deformation of liquid underplates. For order of magnitude estimates of displacements an analog procedure can be helpful.

Consider a liquid underplate placed at some depth in the crust. To remain within a simple context, we envision this to be a horizontal circular magma space of radius R and thickness h where $R \gg h$. The principal tidal strain in the region is horizontal generating a regional volume strain e_t . Assuming that the underplate is dilated by this amount implies a liquid pressure reduction of $\Delta p = e_t/c$ where c is again the liquid compressibility. However, the liquid pressure effect results in a relative axial contraction of the underplate and the total Δp will be smaller than estimated above. To estimate the contraction Δh along the symmetry axis of the underplate, we apply a result of Sneddon (1946) for penny-shaped cavities of radius R implying that at a value of $\mu = 2 \times 10^{10}$ Pa, the increase of internal pressure by Δp increases the axial width by $3 \times 10^{-11} R \Delta p$. Taking Poisson contraction into account results then in our estimate of

$$u_l = 2 \times 10^{-11} R e_t / c \quad (3.99)$$

ESSAY (IV): ELASTOHYDROMECHANICAL FIELDS.

Natural formations generally contain fluids in pores, fractures and intercrystalline spaces. Liquid water is the most prominent of such interstitial fluids. Most igneous formations have a very low porosity and the interaction of the interstitial water with the elastic fields can then for most practical purposes be neglected. The situation in more fractured formations and clastic sediments is, however, different and the hydroelastic interactions, as we shall name them can then not be neglected. Moreover, as already emphasized above, the observation of fracture/pore fluid pressure is an integral part of elastomechanical field work. We will therefore devote some space to the basic set of elastohydromechanical equations that can be assumed to govern such processes to a satisfactory approximation.

(1) Basic equations

Consider a wet Hookean formation that can be taken to be homogeneous and isotropic on a scale much larger than the dimensions of the fluid carrying fractures and pores. We will use the same notation as above for the elastic fields and parameters, with the exception that p is now to denote the pressure of the interstitial fluid. Obviously, the Lamé constants now refer to the wet formation.

The first question to be encountered involves the effect of the interstitial fluid on the stress/strain fields, that is, the modification of equation (3.19) to encompass the hydroelastic effects. We will here follow a procedure of Biot (1947) where the forces due to the interstitial fluid pressure are taken to be represented by a simple body force density equal to $-\theta \nabla p$ where θ is a dimensionless parameter that is small

compared to unity. Equation (3.19) thus modify to

$$\mu \nabla^2 \vec{u} + (\lambda + \mu) \nabla \nabla \cdot \vec{u} - \theta \nabla p + \vec{f} = 0 \quad (4.1)$$

This equation has to be adjoined by an equation for the fluid transport inside the solid. Here, we will follow a procedure of Bodvarsson (1970) and introduce the equation for the fluid pressure

$$\partial_t p + a \nabla \nabla p = -(\delta/s) \partial_t \nabla \cdot \vec{u}, \quad (4.2)$$

where $a = C/\rho s$ is the formation hydraulic diffusivity, C the hydraulic conductivity, ρ the density of the fluid, s the hydraulic capacitivity and δ is the matrix coefficient introduced by Bodvarsson (1970). Equations (4.1) and (4.2) with appropriate boundary conditions constitute the basic equations that are assumed to approximately govern the elastohydromechanical fields of interest in the present context.

The simplified and approximate character of (4.1) and (4.2) is, however, to be noted. Without further elaboration, we will state that (4.1) is likely to hold mainly for formations of low fluid content as is the case for igneous formations. Moreover, the parameters θ and δ are to be taken to be purely empirical and are therefore to be adjusted to individual situations.

As may emerge a little more clearly, the pressure term in (4.1) has been introduced to mainly account for the pressure/elastic interaction whereas the forces on the matrix due to the flow resistance are being neglected. This approach is likely to be permissible in the case

of formations of low porosity and fluid content. The introduction of the friction terms would greatly complexify the equations without much enhancing the validity for the present purpose.

There are no formal difficulties in including formation rheic effects in (4.1) along the same lines as in section (3) above. For this purpose, we will replace the Lamé parameters in (4.1) by operators such that

$$(\mu + \mu' \partial_t) \nabla_2 \vec{u} + (\lambda + \lambda' \partial_t + \mu + \mu' \partial_t) \nabla \nabla \cdot \vec{u} - \theta \nabla p + \vec{f} = 0, \quad (4.3)$$

where the notation is as in section (3). Equation (4.3) thus includes elastic, rheic and hydroelastic effects.

Obviously, equation (4.1) to (4.3) are quite involved and more general types of solution are not derived too easily. However, as already emphasized, the parameter θ is generally less than unity and this permits a first order decoupling of the equations, that can then be solved by a perturbation approach. As a matter of fact, we are generally only interested in first order effects. The procedure is then to neglect the pressure term in (4.1) or (4.2) and solve for the first order displacement field $\vec{u}$. This field is then introduced as the source term in (4.2) that can then be solved uniquely. The matrix parameter δ is somewhat uncertain but is in most cases probably close to unity. The perturbation method outlined allows us to solve the elastohydro-mechanical fields with the help of the development in earlier sections.

(2) A specific hydroelastic model

It is instructive to consider a simple specific hydroelastic model that has the advantage of allowing a specific estimate of the

parameter θ . This is the case of the vertically fractured elastic half-space that is loaded by a uniform pressure p_0 on the Σ plane. The vertical fractures are assumed to be continuous from the Σ plane down and be fluid saturated. The overall aporosity ϕ due to the fractures is assumed to be uniform and small compared to unity. We will now derive the basic equations for this model that is sketched in Fig. (4.1).

Figure 4.1. A specific hydroelastic model.

Let σ_1 and σ_2 be the vertical, respectively horizontal principal stresses in the solid material. Similarly, let ϵ_1 and ϵ_2 be the corresponding principal strains. Moreover, let p be the fluid pressure in the fractures. Due to the horizontal homogeneity, all quantities are functions of depth and time only. Using the notation above, we have then the modified Hooke's law for the solid material

$$\sigma_1 = (\lambda + 2\mu)\epsilon_1 + 2\lambda\epsilon_2 \quad (4.4)$$

$$\sigma_2 = -p = \lambda\epsilon_1 + 2(\lambda + \mu)\epsilon_2 \quad (4.5)$$

Moreover, let q be the vertical fluid mass flow, ρ the density and c_f the compressibility of the fluid. The conservation of fluid mass requires that

$$(1/\phi)\partial_t \varepsilon_2 - (1/\rho)\partial_z q = c_f \partial_t p \quad (4.6)$$

Equation (4.5) gives

$$\varepsilon_2 = -(p + \lambda \varepsilon_1)/2(\lambda + \mu), \quad (4.7)$$

and the fluid flow can be assumed to be governed by Darcy's law

$$q = -C \partial_z p \quad (4.8)$$

when C is the fluid conductivity. Let u be the vertical displacement and hence $\varepsilon_1 = \partial_z u$. Inserting these quantities in (4.6) yields the equation for the fluid pressure

$$[1 + (1/2c_f(\lambda + \mu))]\partial_t p - (C/\rho c_f)\partial_{zz} p = -[\lambda/2c_f\phi(\lambda + \mu)]\partial_{tz} u \quad (4.9)$$

and (4.4) modifies to

$$\sigma_1 = [\lambda + 2\mu - (\lambda^2/(\lambda + \mu))]\varepsilon_1 - [\lambda/(\lambda + \mu)]p \quad (4.10)$$

If f is a vertical volume force, we obtain then the elastic equation

$$[\lambda + 2\mu - (\lambda^2/(\lambda + \mu))]\partial_{zz} u - [\lambda/(\lambda + \mu)]\partial_z p + f = 0 \quad (4.11)$$

Equation (4.11) corresponds to one component of (4.1) and (4.9) is the present version of (4.2).

The above equations can now be simplified by assuming Poisson's relation $\lambda = \mu$ and introducing the diffusivity $a = C/\rho c_f$. Moreover, the factor $(1/2c_f(\lambda+\mu)) \ll 1$ and hence (4.9) simplifies finally to

$$\partial_t p - a \partial_{zz} p = -(1/4c_f) \partial_{tz} u \quad (4.12)$$

and (4.11) is

$$2.5\mu \partial_{zz} u - (1/2) \partial_z p + f = 0 \quad (4.13)$$

Hence the parameter $\theta = 1/2$.

(3) Displacement/interstitial fluid pressure relations

Observations of interstitial fluid pressure and surface displacement/tilt are complementary and the relation between these observables are of considerable interest. In the following, we will discuss this topic in a little more detail in the case of a homogeneous/isotropic permeable slab of uniform thickness h that can be assumed to be governed by equations (4.1) and (4.2). The slab is assumed to be liquid-saturated, placed on an impermeable fixed basement and for convenience we assume that the thickness h is large compared with the hydraulic skin depth at the time frequencies of main interest. The surface load is assumed to be barometric of the unit step type in time and uniform in space with an amplitude $b(t)$. Notation is as in previous sections. Before turning to the basic equations, a few remarks have to be made as to the boundary conditions for the fluid pressure at the upper surface that will coincide with

the ground water level. Several types of situations are possible.

In the case when the ground water level coincides with the free surface, the boundary pressure on the interstitial fluid would be the same as on the solid matrix, that is,

$$p = b(t), \quad z = 0. \quad (4.14)$$

However, in most terrain, in the SW of the U.S., in particular, the ground water table is quite low and there is a thick dry surface layer that is saturated by air.

Since the ratio of air/water compressibility is of the order of 10^4 and the corresponding viscosity ratio is roughly 1/25, the (compressibility x viscosity) ratio is 400 and hence, assuming the same permeability, the pressure diffusivity in the wet formation will be 400 times as large as that of the dry layer. In fact, the diffusivity of the dry layer will in general be so small that barometric pressure transients will penetrate only a few tens of meters into the layer. A thick dry surface layer will therefore be largely isobaric with regard to pore/fracture pressure during normal barometric loading at the surface.

In this situation, the barometric load transmitted down to the wet formation results in an increased pore/fracture pressure that forces the interstitial water upward. Assuming small amplitudes, we have then because of the constant pressure a boundary condition of the mixed type at the ground water level (Bodvarsson, 1977), that is

$$(g/\phi)\partial_t p - C\partial_z p = 0 \quad (4.15)$$

where g is the acceleration of gravity.

We can now express the basic equation for this one-dimensional case of (4.1) and (4.2),

$$(\lambda + 2\mu)\partial_{zz}u - \theta\partial_z p = 0, \quad (4.16)$$

and

$$\partial_t p - a\partial_{zz}p = -(\delta/s)\partial_{tz}u, \quad (4.17)$$

where p can now be taken to be the pressure in excess of hydrostatic. Moreover, in the present case, we assume causal initial conditions and taking that the barometric load is of the unit-step type of amplitude b , the boundary conditions for the displacement are

$$-(\lambda + 2\mu)\partial_z u \Big|_{z=0} = bU(t), \quad (4.18)$$

where $U(t)$ is the unit-step function, and at the base of the slab

$$u = 0, \quad z = h \quad (4.19)$$

There are no formal problems in solving the above equations with either condition (4.14) or (4.15). However, since we are here only considering the general character of the hydroelastic effects, we will resort to discussing the simplest case that results when a low porosity wet formation is overlain by a high porosity dry layer such that the condition at the ground water level placed at $z = 0$ degenerates into the simple condition

$$p = 0, \quad z = 0 \quad (4.20)$$

Proceeding with solving for this specific case, we assume that the porosity is sufficiently low that the above set of equations can be decoupled and solved by the perturbation procedure referred to above.

The first order term u_1 is then obtained by

$$(\lambda+2\mu)\partial_{zz}u_1 = 0, \quad (4.21)$$

that yields

$$u_1 = Az+B \quad (4.22)$$

where A and B are constants for $t>0$. On the basis of the boundary conditions follows then that

$$u_1 = (b/(\lambda+2\mu))(h-z)U(t), \quad (4.23)$$

Inserting in (4.15) results in

$$\partial_t p_1 - a\partial_{zz}p_1 = (\delta b/s(\lambda+2\mu))\delta(t), \quad (4.24)$$

and because of the large thickness h this equation has the approximate solution

$$p_1 = (\delta b/s(\lambda+2\mu))\text{erf}(z/2(at)^{1/2}) \quad (4.25)$$

On the basis of (4.16), a second order displacement is obtained from

$$(\lambda+2\mu)\partial_z u_2 - \theta p_1 = A \quad (4.26)$$

where A is again a constant. We obtain

$$u_2 = (\theta/(\lambda+2\mu)) \int_0^z p_1 dz + Az \quad (4.27)$$

and since $u_2 = 0$ at $z = h$

$$A = -(\theta/(\lambda+2\mu)h) \int_0^h p_1 dz \quad (4.28)$$

which results then in

$$u_2 = -(\theta\delta b/s(\lambda+2\mu)^2)[h-z-2(at)^{1/2}(\text{ierfc}(h/2(at)^{1/2})-\text{ierfc}(z/2(at)^{1/2}))] \quad (4.29)$$

where ierfc is the integral-erfc function. The displacement at $z = 0$, that is the quantity of main interest is then to the second order

$$u|_{z=0} = (bh/(\lambda+2\mu))[1-(\theta\delta/s(\lambda+2\mu))(1-F^{-1}(\text{ierfc}(F)-0.56))] \quad (4.30)$$

where $F = h/2(at)^{1/2}$ is the Fourier number. The extreme values of the second order approximation are then

$$\begin{aligned} t = 0+, \quad u|_{z=0} &= (bh/(\lambda+2\mu))[1-(\theta\delta/s(\lambda+2\mu))] \\ t \rightarrow \infty \quad u|_{z=0} &= (bh/(\lambda+2\mu)) \end{aligned} \quad (4.31)$$

This can be portrayed in the graph in Fig. (4.2) below

Figure 4.2. Displacement response of a porous fluid saturated solid.

Hence, the solid behaves as a firmoviscous or Kelvin-Vogt solid.

The first order pore/fracture pressure as given by equation (4.25) above is a monotonously decreasing function of time with the initial amplitude $(\delta b/s(\lambda+2\mu))$.

(4) Further development

Although the material in the previous section presents the hydro-elastic processes in a very simple setting, many of the essential features emerge quite clearly. In particular, liquid saturated low-porosity formations tend to exhibit a Kelvin-Vogt type of behavior. There are no difficulties in extending our results to a more general setting. The general solution for p_1 in the one-dimensional case with the mixed type of boundary condition given by (4.15) is given by Carslaw and Jaeger (1959) and can be easily applied to generalize the above development. However, the algebra becomes more involved and it would appear that further work along these lines would not be warranted at this juncture. Obviously, the same applies to the study of more general three-dimensional

models. There are no principal difficulties but the algebra is very involved. As of now, the main interest centers on obtaining field data.

It is, nevertheless, appropriate to indicate the procedure of applying the constrained displacement approximation (CDA) of section (3) in Essay (III) to the hydroelastic setting. In essence, we would then reduce the displacement vector equation (4.1) to a one-dimensional form and insert only one displacement component in the divergence on the right of the pressure equation (4.2). However, it will depend on the geometry and the diffusion parameters whether any reduction of the number of independent variables in (4.2) can be made. This will have to be considered in the case of each individual model.

(5) Monitoring of interstitial fluid pressure

The interstitial fluid pressure has been included as a scalar p in the above equations implying that this quantity can be taken to be a localized point-function. Although there are no formal difficulties involved, it must, nevertheless, be underlined that because of observational port effects, the observation of strictly local values of p presents some difficulty.

The monitoring of p is generally carried out by placing pressure transducers in boreholes that extend to sufficient depths and serve as observation ports. Two difficulties arise under these circumstances. First, although the holes can be cased over most of their depth, most holes have an open section of a finite length and this tends to preclude the observation of strictly local values of the pressure. The values obtained are more likely to be averages over the open section. Second, all boreholes have a finite capacitance such that it is impossible to observe instantaneous values of the pressure. Again, only weighted

averages over certain periods of time can be observed. The capacitance is particularly large in the case of open holes with a free liquid surface.

The capacitive effects of observation ports have been discussed briefly in a paper by Bodvarsson (1979). Although the paper addresses the problem mainly from the point of view of well interference testing, the results have relevance to a more general setting involving fluid pressure monitoring in boreholes. This paper is attached as Appendix (1).

(6) Exploratory relevance of interstitial fluid pressure data

The practical importance of the fluid pressure data in elastomechanical exploration and testing rests primarily in the pressure/volume-strain relation given by equation (4.2) above. Let H be the diffusion operator on the left of (4.2) and e the volume strain such that the equation can be expressed as

$$Hp = -(\delta/s)\partial_t e, \quad (4.32)$$

and its solution then

$$p = -(\delta/s)H^{-1}\partial_t e, \quad (4.33)$$

In the case of uniform strain, or very low fluid diffusivity, and at distances substantially larger than hydraulic skin depths from boundaries equation (4.33) reduces to the simple relation

$$p = -(\delta/s)e \quad (4.34)$$

implying that the fluid pressure is proportional to the volume strain. Thus, provided the pressure data can be processed for observation port effects and the material parameter (δ/s) is known, the data give a direct information on the local value of the strain. Herein lies the main importance of the pressure observations.

ESSAY (V): REMARKS ON TIDAL FORCING.

The development in previous essays has been designed mainly to apply to the case of barometric forcing at the ground surface. In other words, the forcing field is vertical and generates mainly a vertical displacement. The situation with tidal forcing is different in that the forcing stress field has then to be taken to be horizontal such that local displacements and tilts observed at the surface and reduced for global effects are primarily due to Poisson contraction and local inhomogeneity effects under the horizontal strain. In general, the theory of such effects tends to be more involved than the case of barometric forcing.

In the case of the Valles caldera, a tidal effect of interest involves the response of the underlying pluton to the horizontal tidal stress field. A method for estimating the tidal effect on a spherical viscoelastic body has been derived by Hanson (1981) and will therefore not be elaborated on here.

Another tidally induced phenomenon concerns the hydroelastic effects in interstitial fault gauge fluids that are of direct interest for the observation of interstitial fluid pressure and formation volume strain. We will elaborate briefly on this subject.

From a more general point of view, the mechanics of tidally induced hydroelastic processes in fault zones is a specific case of a more general problem that can be stated as follows. Given an infinite impermeable elastic space that includes a region B of an elastic fluid saturated

porous/fractured material. The region B has a finite fluid conductivity such that the interstitial fluid flows in response to impressed pressure gradients. The entire elastic space is deformed by an impressed slowly oscillating stress field. In the general case, the elastic parameters of region B differ from the surrounding space and the region and its neighborhood will therefore be deformed relative to the space. The interstitial fluid moves in response to the impressed stress resulting in a time-dependent modification of the fluid pressure fluid that again acts on the elastic matrix of region B and the surroundings. We are here confronted with a case of interacting elastic and fluid pressure fields. The basic equations are (4.1) and (4.2) with appropriate boundary conditions.

Since the present report is mainly concerned with the more general physical implications of the hydroelastic effects, we will resort to briefly considering the following simplified model of a fault zone system. Consider a porous/permeable fluid saturated slab of thickness $2h$ that is embedded between two symmetric elastic half-spaces. The model is sketched in Figure (5.1). The slab is Hookean in a generalized sense but can be inhomogeneous on a scale that is large compared to the thickness $2h$. Moreover the thickness can vary on a similar scale. The inhomogeneities are, however, restricted such that there are a well defined average thickness $2h_0$ and average material properties. The flow of the interstitial fluid is assumed to be governed by Darcy's law. Both half-spaces are homogeneous Hookean with a Poisson ratio 0.25 and a modulus of rigidity μ_c .

The entire system is acted on by an oscillating force field that is perpendicular to the plane of the slab and generates a principal

stress of σ_0 in both half-spaces. Since the slab is inhomogeneous, the stress varies in its neighborhood but takes on the constant asymptotic value σ_0 away from the slab. Obviously, the slab will breathe under the stress and the inhomogeneities will then lead to local oscillating movements of the interstitial fluid.

Figure 5.1. Model of a porous slab.

As of now, no assumptions have been made with regard to the longitudinal dimensions of the slab. For the present purpose where the interest centers on the hydroelastic effects, it is most convenient to take that the slab is of infinite dimension. Without a significant loss of generality this eliminates boundary conditions that tend to complicate the analytical development. Because of the symmetry of the model, we have then only to consider one half of it, that is, the half-slab of thickness h welded to the adjacent half-space. The outer surface of this truncated model has to remain flat under the imposed stress. We place a rectangular coordinate system with the (x,y) plane in this surface and the z -axis vertically into the half-space.

On the truncated model the breathing of the slab leads to a displacement of the slab-half-space interface in the direction of the z-axis that conveniently is expressed $(u_0 + u)$ where u_0 is an average value that is uniform over the entire slab and $u(x,y)$ is a perturbation due to local inhomogeneities in the slab material. This displacement $(u_0 + u)$ results in a linear strain of $(u_0 + u)/h$ parallel to the z-axis and it is again convenient to separate this quantity into two components such that

$$(u_0 + u)/h = w_0 + w \quad (5.1)$$

where w_0 is the averaged component corresponding to u_0 . Associated with the strain is a resulting stress component parallel to the z-axis that we likewise write $(\sigma_0 + \sigma)$ and a pressure of the interstitial fluid of $(p_0 + p)$ where σ_0 and p_0 are again uniform over the entire slab.

Since the material of the slab is Hookean, there is a linear relation between the strain, stress and the fluid pressure that can be expressed

$$\sigma_0 + \sigma = (k(u_0 + u)/h) - \zeta(p_0 + p) \quad (5.2)$$

where k is a specific elastic modulus, a variant of Young's modulus that depends on the material, pore geometry and the scale of the inhomogeneities. The second parameter ζ is, on the other hand, a pure number that is considerably less than unity. We adjust the parameters such that

$$\sigma_0 = (ku_0/h) - \zeta p_0 \quad (5.3)$$

and hence

$$\sigma = (ku/h) - \zeta p \quad (5.4)$$

Because of the scale of the inhomogeneities, the stress in (5.2) and the interstitial fluid pressure are approximately constant across the slab and the equation for the pressure then reduces to a field equation in two independent variables (x,y)

$$\rho h s \partial_t (p_0 + p) - \nabla_2 \cdot (ch \nabla_2 p) = \delta \rho \partial_t (u_0 + u), \quad (5.5)$$

where $\nabla_2 = (\partial_x, \partial_y)$. Because p_0 is uniform over the slab this term drops out of the second term on the left of (5.5). The expression on the right represents the effects of the volume strain where the dimensionless parameter δ , that has values of the order of unity, has been introduced to relate the volume strain to the linear strain and to account for the effects of the pressure reactance.

Finally, the displacement parallel to the z -axis, the resulting stress in the slab and the fluid pressure are related by the displacement/stress equation for the surface of the half-space. To avoid a singularity, the relation includes only the perturbation components such that

$$(\phi p - \sigma) = Hw \quad (5.6)$$

where $H = (\mu_c/1.5)\pi_2^{1/2}$ is the operator defined by equation (3.58) in essay III and ϕ is a dimensionless parameter that is about equal to the area porosity and has been introduced to account for the effects of the

fluid pressure on the half-space. The addition of (5.6) completes the system of equations.

Although the underlying model has been simplified considerably, and the resulting equations are linear, the solution of the set (5.3) to (5.6) poses a non-trivial problem. The combination of the diffusion and the half-space operator H is at the center of the difficulty. This underlines the complexity of the general tidal deformation problem we are considering in this section.

To look into the physics of the above model, we observe that when all parameters are constant, equation (5.5) reduces to the time integrated form

$$h s p_0 = \delta w_0 \quad (5.7)$$

and the perturbation components vanish. In the inhomogeneous case where the integrated capacitivity hs varies along the slab, and there are no external fluid sources, p_0 is the source of p . Turning to a conventional perturbation case where all parameter/component perturbations are small compared to the zero order set, we obtain the following situation for the first order components. Assuming that c is constant but the capacitivity $hs = h_0 s_0 + C$ where C is a perturbation, we take that now

$$h_0 s_0 p_0 = \delta w_0 \quad (5.8)$$

and the first order form of (5.5) then reduces to

$$\rho h_0 s_0 \partial_t p + c h_0 \pi_2 p = \delta \rho \partial_t w - \rho C \partial_t p_0, \quad (5.9)$$

where $\pi_2 = (-\nabla^2)^{1/2}$.

The displacement w can be eliminated with the help of (5.4) combined with (5.6) and it is convenient to introduce the diffusivity $a = c/\rho s_0$. Equation (5.9) can be expressed in a final form

$$[1 - \delta(h_0 s_0 H + s_0 k)^{-1}(\phi + \zeta)]\partial_t p + a\pi_2 p = -(C/h_0 s_0)\partial_t p_0 \quad (5.10)$$

where the term on the right is now a source term for the p -field.

To draw essential physical conclusions on the basis of (5.10) it is of main interest to consider the magnitude of the operator term in the brackets of the first term on the left. Since the operator H has a positive norm, the term $(h_0 s_0 H + k s_0)$ is bounded from below by $k s_0$. In the case of rock formations of interest in the present context, the capacity s_0 is generally of the order of a few 10^{-11}Pa^{-1} . Moreover, the parameter k is of the same magnitude as Young's modulus that generally is a few 10^{10}Pa . Hence, $k s_0$ will generally be of the order unity plus, that is, in the range 2 to 3. Moreover, δ is of the order of unity and based on the definition of ϕ and ζ , the sum of these parameters is likely to be no larger than 0.2 to 0.5. Hence, the norm of the entire operator term in the brackets of the first term on the left of (5.10) is in most cases below 0.2 to 0.4 and the entire brackets therefore of the order of unity. In most practical cases, equation (5.10) therefore is a typical diffusion equation and the related phenomena therefore dominated by the hydraulic skin-depth. This parameter gives a measure of the range of pressure diffusion under oscillatory stress. Most low-to-medium fluid conductivity cases are at tidal frequencies associated with skin-depths of the order of no more than 100 m (see Bodvarsson, 1970). We can there-

fore conclude that hydroelastic phenomena are not likely to greatly affect the tidal breathing of fault zones with a moderately conductive gouge. The phenomena are, however, of definite interest in the case of higher conductivities involving permeabilities well in excess of 0.1 Darcy. Skin depths can then amount to several 10^2 m and are likely to substantially modify the fault breathing. In particular, the portion of the fault zone within a distance of one skin depth from the surface will be affected. The various types of tidal borehole tests discussed above provide important tools of obtaining experimental data on both stress and strain fields and around fault zones of this type.

These phenomena are of particular interest with regard to interstitial fluid pressure processes in the Redondo graben fault zones (see Section (3) of the report).

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Appendix (I)

CAPACITIVE PERTURBATIONS IN WELL INTERFERENCE TESTING *Introduction

Conventional well interference testing is applied to obtain observational data on reservoir parameters such as fluid conductivity, fluid diffusivity, and structural inhomogeneities or boundaries. Test results are usually interpreted on the basis of forward type curve-matching methods (Ramey, 1970).

Field procedures are generally based on the use of standard size wells for both injection and response monitoring. The pressure sensors are placed into the wells that serve as observational ports. Obviously, the monitoring wells constitute capacitive inhomogeneities that can perturb the reservoir flow field and thereby distort the pressure readings. In particular, the capacitance of wellbores with two-phase fluids, gas caps or even a free fluid surface is relatively large and the perturbation can then be substantial. Quite erroneous test results may be obtained in such situations. Moreover, analog perturbations can result from the presence of inactive high-capacitance wells and other reservoir "soft spots" in the neighborhood of the test wells. For example, geothermal systems that appear liquid-dominated may actually include local spots with two-phase pore fluids that have a higher compressibility than the pure liquid. In particular, such soft-spots are likely to develop in regions with temperatures close to boiling and/or high gas content liquids.

As a matter of course, the capacitive effects are well known and are in the petroleum industry usually referred to as wellbore storage effects. A considerable literature exists, mainly relating to such effects in single-

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well pressure-buildup or drawdown testing (see, for example, Ramey, 1970; Earlougher and Ramey, 1973; Raghavan, 1976; Chen and Brigham, 1978; Miller, 1979). For further references, we refer to the monograph by Earlougher (1977). A number of aspects relating specifically to interference testing have been discussed by Prats and Scott (1975), Jargon (1976) and Sandal et al. (1979).

In passing, it is of interest to remark that sensor capacitance is a matter of extremely general relevance. For example, capacitive effects interfere with the measurement of time-varying temperatures. Just as we refer to temperature gauges as thermometers, we will here apply the term pressometers for the pressure monitoring devices. In interference testing, the pressometer consists of the entire monitoring well setup.

The purpose of the present short note is to discuss the capacitive effects from a rather general point of view and, in particular, to derive some basic expressions to enable us to correct for pressometer and soft-spot capacitance. The approach will be based on the assumption of a forward type interpretational procedure. In other words, the development is based on definite field models that lead to a well-posed problem setting. By varying the model parameters, the solutions yield the type-curves that are used to interpret field data. Data interpretation on the basis of so-called inverse procedures is usually not feasible and would lead to a practically impossible problem setting.

Notation and basic equations are as given in the paper by Bodvarsson (1978).

An ultrasimple model

In the case of slowly varying fields, well pressometers can quite often be lumped into an equivalent admittance (conductance) A and an equi-

valent capacitance C . That is to say, that given the ambient time-varying pressure field to be measured $p(t)$ and the pressometer reading $p_m(t)$, the mass flow into the meter system satisfies the following equations

$$q = A(p - p_m), \quad q = \rho C D p_m \quad (1)$$

where $D = d/dt$ and ρ is the density of the fluid. Let $t_0 = \rho C/A$ be the pressometer relaxation or response time and the above equations can then be combined into one equation.

$$p_m + t_0 D p_m = p \quad (2)$$

including only the parameter t_0 . It is convenient to introduce the correction pressure p_1 that has to be added to p_m to obtain the ambient field p such that $p = (p_m + p_1)$. We have then the relation

$$p_1 = t_0 D p_m \quad (3)$$

Depending on a number of circumstances such as the flow and phase situation in the well, the parameters A , C and t_0 can be taken to be constant over a limited range of pressure. Equation (2) is then a simple differential equation in p_m and the relation (3) involves only a single differentiation.

In interference testing, we can usually take that pressure monitoring proceeds from an equilibrium situation where $p_m = p$ and because of linearity, we can then put $p_m(0) = p(0) = 0$, that is, the system is causal. In operational form, the solution of (1) is then

$$p_m = (1 + t_0 D)^{-1} p \quad (4)$$

which yields a Taylor series in the operator $t_0 D$.

An injection/monitoring well pair in a distributed reservoir model

Consider now a more general situation involving a porous and permeable fluid-saturated reservoir model with a given fixed boundary Σ where prescribed conditions are to be applied. Let $c = \beta/\nu$ be the fluid conductivity operator where β is the formation permeability operator and ν the kinematic viscosity of the fluid. The density of the fluid is ρ , the wet formation capacitivity or storage coefficient is s and hence the fluid diffusivity $a = c/\rho s$. Let $P = (x,y,z)$ be the general field point.

Two wells have been drilled into the reservoir, one for injection purposes and the other is to be applied as a pressometer that is assumed to have a known capacitance C taken to be constant within the pressure range of interest. Moreover, the distance between the two wells is taken to be very large compared with the dimensions of the formation/well contact openings such that for mathematical convenience the injection zone, as seen from the monitoring well, can be lumped into a point Q . This situation is easily generalized to the case of a distributed source.

The problem setting is now the following. The system is initially in equilibrium and starting at $t = 0$ the mass flow $f(t)$ is being injected into the reservoir at Q . In the case of fluid withdrawal the sign of $f(t)$ is negative. The fluid injection raises the reservoir pressure leading to the reading $p_m(t)$ at the pressometer well. We are interested in the pressure $p(P,t)$ at the monitoring well as unperturbed by the pressometer capacitance and will therefore derive the correction pressure p_1 such that at the pressometer $p = p_m + p_1$. As compared with the simple situation in the preceding section, the present case is more complex in that the correction pressure is a field function $p_1(P,t)$ that has to be derived by the integration of a diffusion type PDE.

To derive the pressure field equations, we simplify the topology of the reservoir by neglecting the conductivity perturbation of the pressometer and replace it by an equivalent capacity function $su(P)$ such that the space integral over this function is equal to C . Obviously, the function $u(P)$ is localized in that it is zero everywhere except at the pressometer well. Moreover, let $\Pi(c)$ be the generalized Laplacian that in the case of a homogenous and isotropic medium simplifies to $\Pi(c) = -c\nabla^2$.

In this setting with the pressometer well included, the total pressure field $(p-p_1)$ satisfies the equation

$$\rho s(1+u)\partial_t(p-p_1) + \Pi(c)(p-p_1) = f\delta(P-Q), \quad (5)$$

where $\delta(P-Q)$ is the spatial delta-function centered at Q . The pressure field p unperturbed by the pressometer capacitance satisfies

$$\rho s\partial_t p + \Pi(c)p = f\delta(P-Q), \quad (6)$$

and hence the correction field p_1 satisfies

$$\rho s\partial_t p_1 + \Pi(c)p_1 = \rho s u \partial_t (p-p_1), \quad (7)$$

To cope with this equation, we observe that the pressure within the pressometer is constant and equal to the observed pressure p_m . Since p_m represents the total field there, we take that $p_m = (p-p_1)$ over the support of $u(P)$ and equation (7) can thus be expressed

$$\rho s\partial_t p_1 + \Pi(c)p_1 = \rho s u D p_m, \quad (8)$$

where $D = d/dt$ and the expression on the right is now a known function in space and time. Moreover, it is quite obvious that $(p-p_1)$ and p should satisfy the same boundary conditions on Σ and p_1 therefore satisfies homo-

geneous conditions there. It follows that equation (8) can be integrated to obtain the perturbation pressure p_1 . Let the diffusion operator on the left of (7) be expressed

$$H = \rho s \partial_t + \Pi(c), \quad (9)$$

and H^{-1} be its inverse at the conditions specified. The solution of (8) is thus formally

$$p_1(P, t) = \rho H^{-1}(s u D p_m), \quad (10)$$

This equation takes now the place of equation (3) which holds only in the much simplified lumped case of constant A . In fact, in the case of fields varying so slowly that the time-derivative on the right of (7) can be neglected, equation (8) reduces to a potential equation that can be integrated to yield an expression for p_1 . For the field pressure at the pressometer, this integral then reduces to the same form as (3).

It is of importance to remark that to correct the pressometer reading $p_m(t)$, the pressure $p_1(t)$ has to be evaluated at the source of this field. The result is therefore strongly dependent on the selection of a correct local model for the pressometer system. We will refrain from entering into a detailed discussion of the integral (10), and limit our remarks to perhaps the simplest case relevant in the present context. We assume a homogeneous and isotropic reservoir of such an extent that as viewed from the pressometer, it can be taken to be infinite. Moreover, the pressometer system consists of, or is equivalent to, a spherical cavity of radius R . To investigate the response of a system of this type, we turn to a spectral type of analysis and investigate the integral (10) when $p_m = p_0 \exp(i\omega t)$ and ω is the circular frequency. These assumptions simplify the

procedure very considerably. Omitting details, we obtain for the open cavity model the amplitude of the correction pressure p_1 at the cavity

$$p_1 = i\omega\rho C p_0 / A_s [(1+i)(R/d) + 1] , \quad (11)$$

where C is the capacitance of the pressometer, $A_s = 4\pi cR$ its static admittance and $d = (2c/\rho s\omega)^{\frac{1}{2}}$ is the skin depth of the pressure field at the frequency ω . This expression is useful in that it gives the amplitude of the spectral components of p_1 . The dominant physical factor is the ratio R/d . Expression (11) is analog to (3) above.

Soft spots

As already stated, even small reservoir soft spots, that are sufficiently close to the pressometer, can perturb the pressure readings. For example, there may be an inactive well with a free liquid surface in the close vicinity of the monitoring well. Considering such a case that has a known capacitance K , we are interested in some estimates of the resulting perturbation of $p_m(t)$. Here, it is appropriate to make the simplifying assumption that multiple pressure field scattering can be neglected. In other words, the secondary fields due to the pressometer and the soft spot do not scatter and their perturbation of the primary field therefore additive. Hence, to obtain the perturbation due to the soft spot at the pressometer we can neglect the capacitance of the pressometer.

Under these circumstances, it is possible to proceed in much the same way as above. Let $p_2(p,t)$ be the perturbation pressure field due to the soft-spot. The field has then to satisfy an equation of the same type as (7) above. There is, however, the important difference in that there is no monitoring device in the soft spot and the pressure there is now unknown. The integration expressed by (10) can therefore not be performed. There

are nevertheless, the ameliorating circumstances that we are only interested in the value of p_2 at the pressometer. As viewed from this distance, details of the model of the soft-spot are less important and we can in many practically relevant cases resort to a perturbation approach where the pressure field at the soft-spot is taken to be equal to the primary field at this position. On this basis the integration prescribed by (10) can be performed by equating p_m with the primary field and we have then an estimate of p_2 that can be evaluated at the pressometer well.

A variety of situations are of practical interest, but we will refrain from a detailed discussion. A simple investigation on standard size wells shows that in inference tests, the action radius of inactive wells is relatively small. Beyond distances of the order of a very few hundred meters, open inactive wells can for the most practical purposes be neglected.

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