DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

INTERACTING BOSON MODEL: SELECTED RECENT DEVELOPMENTS*

A. B. Balantekin[†]
Physics Division, Oak Ridge National Laboratory
Oak Ridge, TN 37831 U.S.A.

to be presented at

Second Conference on the Intersections Between
Particle and Nuclear Physics

Lake Louise, Canada
May 26-30, 1986

*Research sponsored by the Division of Nuclear Physics, U.S. Department of Energy under contract DE-AC05-84OR21400 with Martin Marietta Energy Systems, Inc.

[†]Eugene P. Wigner Fellow.

MASTER

By acceptance of this article, the publisher or recipient acknowledges the U.S. Government's right to retain a nonexclusive, royalty-free license in and to any copyright covering the article.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

INTERACTING BOSON MODEL: SELECTED RECENT DEVELOPMENTS*

A. B. Balantekin[†]

Physics Division, Oak Ridge National Laboratory
Oak Ridge, TN 37831 U.S.A.

ABSTRACT

The Interacting Boson Model is briefly reviewed. Recent applications of this model to the low-lying collective magnetic-dipole excitations and to the spectra of ¹⁹⁵Ir are described.

INTRODUCTION

Interacting Boson Model is an algebraic approach to even-even nuclei formulated in terms of bosonic variables with emphasis being placed on the symmetries associated with the quadrupole collectivity.¹ In this model, low-lying energy levels are generated as states of a system of N bosons carrying angular momentum L = 0 or L = 2. N is taken to be the number of correlated pairs of valence neutrons or pairs of valence protons. The pairs with L = 0 are similar to Cooper pairs in a gas of electrons. Introducing creation and annihilation operators for six types of bosons, b_i , $b_i^\dagger$, one obtains a bosonic realization of the U(6) algebra, $G_{ij} = b_i^\dagger b_j$. The one-boson terms of the Hamiltonian are written as a linear combination of the U(6) generators and the boson-boson interaction terms as a combination of quadratic products of these generators:

$$H = \sum_{ij} \epsilon_{ij} G_{ij} + \sum_{ijkl} u_{ijkl} G_{ij} G_{kl} \quad (1)$$

The coefficients in the above equation are free parameters to be fixed separately for each nucleus, subject to the restriction that the total angular momenta of the bosons is conserved. Eigenstates of the Hamiltonian are constructed by the application of boson creation operators on a vacuum N-times

$$b_{i_1}^\dagger \dots b_{i_N}^\dagger | 0 \rangle \quad (2)$$

i.e., one forms the completely symmetric representations of U(6). In general, such states are labeled by the boson number N and by the labels of all the representations of all possible subalgebras, included in the completely symmetric representation of U(6). Hence,

*Research sponsored by the Division of Nuclear Physics, U.S. Department of Energy under contract DE-AC05-84OR21400 with Martin Marietta Energy Systems, Inc.

[†]Eugene P. Wigner Fellow.

for most nuclei, the diagonalization of the Hamiltonian must be performed numerically. However, the popularity that the Interacting Boson Model has enjoyed in recent years is mostly facilitated by the existence of analytically solvable limits.

Analytical solutions are possible when the free parameters take special values such that the first term in Eq. (1) can be written as a combination of linear Casimir operators and the second term as a combination of quadratic Casimir operators of the algebra $U(6)$ and its subalgebras. In such cases the quantum numbers are given by the labels associated with the algebras in a particular chain starting with $U(6)$ and ending with $SO(3)$, and the Hamiltonian is said to possess a dynamical symmetry. This situation is reminiscent of the Gell-Mann-Okubo $SU(3)$ mass formula

$$E(I, I_3, Y) = E + a Y + b \left[(Y^2/4) - I(I+1) \right]. \quad (3)$$

which is associated with the group chain

$$SU(3) \supset SU(2) \times U(1) \supset SO(2) \times U(1). \quad (4)$$

There are three possible dynamical symmetries for the Interacting Boson Model Hamiltonian with the second group in the chain being $SU(5)$, $SU(3)$, or $SO(6)$.

The existence of dynamical symmetries of the Interacting Boson Model Hamiltonian implies the existence of analytical expressions not only for energy eigenvalues, but also for the eigenstates. Consequently, to establish whether or not a particular dynamical symmetry is physically realized, it is not sufficient to compare energy spectra. It is essential to test predictions for the electromagnetic transition rates, which are calculated using the wavefunctions. The dynamical symmetries of the Interacting Boson Model provide a simple framework to analyze and classify the experimental data and have been successfully used in this context during the last ten years.¹ In this talk, I highlight two of the recent developments in the applications of the Interacting Boson Model.

II. LOW-LYING COLLECTIVE MAGNETIC-DIPOLE EXCITATION MODE

There is a second version of the Interacting Boson Model, which is much richer in structure than the version discussed in the Introduction. If no distinction is made between bosons which are correlated pairs of neutrons and bosons which are correlated pairs of protons, then the wavefunction given in Eq. (2) contains only the completely symmetric representations of $U(6)$. However, if such a distinction is made, the states which are partially antisymmetrized between neutron and proton bosons should also be experimentally observable. This new version of the model is called the neutron-proton Interacting Boson Model.

Semiclassically, a deformed nucleus can be visualized as an ellipsoid inside of which protons and neutrons are uniformly distributed. When such a nucleus is placed in an external field, the proton ellipsoid translationally oscillates around the neutron

ellipsoid. The direction of the vibration can be either parallel or perpendicular to the major axis of the deformed nucleus. Consequently, a double-humped strength distribution can be experimentally observed. This is the well-known Goldhaber-Teller mode. For two separate ellipsoidal rigid bodies, which are slightly tilted away from each other, a different kind of oscillation is also possible where the two shapes wobble around the same axis² as depicted in Fig. 1. Such a motion would excite a $J^\pi=1^+$ state through the orbital current part of the M1 operator. One would expect that the energy of such a state is comparable to the energy of the E1 giant resonance (the Goldhaber-Teller mode).

Fig. 1. Classical picture of the collective 1^+ configuration

A collective magnetic-dipole excited 1^+ state has been discovered³ in electron scattering first over ^{156}Gd targets and later other nuclei in this region. The excitation energy in this mass region was found to be approximately 3 MeV. Recently (γ, γ') experiments on a series of Gd isotopes has confirmed⁴ the energies and more importantly the spin of these states. This excitation energy is much lower than one would expect for the wobble mode discussed in the previous paragraph, where the entire nuclear system participates. However, the mixed symmetry states of the neutron-proton Interacting Boson Model in the SU(3) limit give rise⁵ to a low-lying $K^\pi=1^+$ band, where the M1 matrix element connecting the

ground state and the bandhead is very large. Since in this model only the valence nucleons contribute to the wobble motion, as opposed to the entire nuclear system, the excitation energy would be relatively low. Furthermore, one can show⁶ that in the classical limit of the neutron-proton Interacting Boson Model Hamiltonian these 1^+ states can be regarded as small amplitude oscillations of the angle between the two symmetry axes of the deformed valence neutrons and protons. Consequently, one can interpret the experimentally observed 1^+ state in ^{156}Gd as a neutron-proton antisymmetric state in the Interacting Boson Model framework.

After the initial discovery in ^{156}Gd , the new collective isovector magnetic dipole mode has been observed in a number of nuclei.⁷ One should stress that this new mode is not an isolated example, but the prototype of an entire new class of states which occur at that

energy and have collective properties.⁸ Since these states are neutron-proton antisymmetric, their experimental study should yield a better understanding of the neutron proton forces in the collective models.

III. DESCRIPTION OF THE NUCLEUS ^{195}Ir IN THE FRAMEWORK OF $U(6/4)$ SUPERSYMMETRY

In order to study odd-even nuclei, where one of the nucleons is unpaired, in a framework similar to the Interacting Boson Model, fermionic degrees of freedom should be explicitly introduced. Such an extended model is the Interacting Boson-Fermion Model.⁹ If the angular momentum of the fermion has some specific value, and furthermore if the boson-boson and boson-fermion interactions have particular forms, one can also obtain analytical expressions for the energies and transition rates. An experimentally relevant example is provided when the bosonic core is described by the $O(6)$ dynamical symmetry of the Interacting Boson Model and when $j = 3/2$ for the unpaired fermion, since $j = 3/2$ is the angular momentum in the $SU(2)$ decomposition of the lowest-dimensional spinor representation of $O(6)$. In this scheme, bosonic states (i.e. those in the even-even nucleus) are transformed by the tensor representations of $O(6)$ and fermionic states (i.e. those in the neighboring odd-even nucleus) are transformed by the spinor representations, which are associated with the universal covering group $\text{Spin}(6)$. Such symmetries were termed as Bose-Fermi symmetries.¹⁰

In principle, the spectra of the even-even and the odd-even nuclei can be fitted separately. However, in certain cases the parameters appearing in the Hamiltonian and in the electromagnetic transition operators take the same value for both kinds of nuclei. In such cases, a supersymmetry is realized. Using the isomorphism $\text{Spin}(6) \sim SU(4)$, the scheme described in the previous paragraph can be embedded in the supergroup $SU(6/4)$. This supersymmetry has been observed¹¹ in the Os-Ir region where a number of nuclei are placed in the same supermultiplet (Fig. 2). There are other examples of approximate supersymmetries in nature, one of them being the equivalence of the slopes of the Regge trajectories for mesons and baryons. Indeed, the $U(N/M)$ type supergroups were first introduced in an effort¹² to explain this equivalence. The prominence of the nuclear supersymmetries, however, is that their well-defined predictions can be thoroughly tested in the laboratory. In the rest of this talk, I describe such a recent test.

There is a simpler description of the odd-even nuclei, called the weak coupling model. In this model the angular momentum of the fermion is directly coupled to the states $L = 0, 2, \dots$ in the even-even core and the resulting states are split by a $J(J+1)$ interaction. In the $\text{Spin}(6)$ scheme the $3/2$ member of the $3/2 \times 2$ multiplet is at an energy higher than that is expected from the simple-minded weak coupling picture. Since this state is generated by the quadrupole interaction between the bosons and the fermion, it does not belong to the same $\text{Spin}(6)$ representation as the ground state. Consequently, a precise measurement of the position of this state is requisite to

Fig. 2. Supersymmetric multiplets in the Os, Pt region.

establish the applicability of the Spin(6) Bose-Fermi symmetry.

Based on previous work in this region, ^{195}Ir is expected to be a good example of Spin(6) symmetry. The low-lying positive parity states and their electromagnetic de-excitations have recently been studied¹³ in the $^{193}\text{Ir}(2n, \gamma)^{195}\text{Ir}$ reaction. Using the observed E2 transition, the 3/2 state mentioned in the preceding paragraph has been located at 286 keV. In ^{195}Ir , the ground state is located in the $\sigma=13/2$ representation of Spin(6), and the 286-keV 3/2 state is the head of the $\sigma=11/2$ representation. Furthermore, using the parameters obtained for the nucleus ^{194}Os , one can predict the spectra of ^{195}Ir by invoking the U(6/4) Spin(6) supersymmetry. The resulting predictions are compared with the experimental level scheme in Fig. 3, taken from Cizewski, et al.¹³ The salient feature of this figure is the remarkable regularity in the repetition of the σ sequences, which is a unique signature of the supersymmetry scheme.

I would like to thank J. Cizewski for communicating her recent results prior to publication.

REFERENCES

1. For recent reviews see A. Arima and F. Iachello, *Adv. Nucl. Phys.* **13**, 139 (1984); A.E.L. Dieperink and G. Wenes, *Ann. Rev. Nucl. Part. Sci.* **35**, (1985).
2. N. Lo Iudice and F. Palumbo, *Phys. Rev. Lett.* **41**, 1532 (1978).
3. D. Bohle et al., *Phys. Lett.* **137B**, 27 (1984).
4. U.E.P. Berg et al., *Phys. Lett.* **149B**, 59 (1984).
5. F. Iachello, *Nucl. Phys.* **A358**, 89c (1981); A.E.L. Dieperink, *Prog. in Part. and Nucl. Phys.* **9**, 121 (1983).
6. A. B. Balantekin and B. R. Barrett, *Phys. Rev.* **C32**, 288 (1985); S. Pittel and J. Dukelsky, *ibid.* **335** (1985); R. Bijker, *ibid.*

-
- Figure 1 consists of two vertically stacked energy level diagrams for the nucleus ^{195}Ir . Both diagrams plot the excitation energy E_x in MeV on the y-axis, ranging from 0.00 to 1.00. The top diagram is labeled 'EXP' (Experimental) and the bottom diagram is labeled 'THY' (Theoretical). Both diagrams show a series of energy levels with spin-parity assignments (σ, τ) and transition labels.
- Experimental (EXP) Diagram:**
- Y-axis: E_x (MeV) from 0.00 to 1.00.
 - Levels and transitions:
 - Ground state: $\sigma = 1/2$ (at 0.00 MeV).
 - First excited state: $\sigma = 3/2$ (at ~0.15 MeV), with $\tau = 1/2$.
 - Second excited state: $\sigma = 1/2$ (at ~0.35 MeV), with $\tau = 3/2$.
 - Third excited state: $\sigma = 3/2$ (at ~0.55 MeV), with $\tau = 5/2$.
 - Fourth excited state: $\sigma = 1/2$ (at ~0.75 MeV), with $\tau = 3/2$.
 - Fifth excited state: $\sigma = 3/2$ (at ~0.95 MeV), with $\tau = 5/2$.
 - Transition labels: $3/2^+$, $1/2^+$, $5/2^+$, $3/2^-$, $5/2^-$.
 - Additional labels: $\Sigma = 6$, 195Ir , $1/2^+, 3/2^+, 5/2^+$, $1/2^-, 3/2^-, 5/2^-$, $3/2^+(6/2)^+$, $\tau = 7/2$.
- Theoretical (THY) Diagram:**
- Y-axis: E_x (MeV) from 0.00 to 1.00.
 - Levels and transitions:
 - Ground state: $\sigma = 1/2$ (at 0.00 MeV).
 - First excited state: $\sigma = 3/2$ (at ~0.15 MeV), with $\tau = 1/2$.
 - Second excited state: $\sigma = 1/2$ (at ~0.35 MeV), with $\tau = 3/2$.
 - Third excited state: $\sigma = 3/2$ (at ~0.55 MeV), with $\tau = 5/2$.
 - Fourth excited state: $\sigma = 1/2$ (at ~0.75 MeV), with $\tau = 3/2$.
 - Fifth excited state: $\sigma = 3/2$ (at ~0.95 MeV), with $\tau = 5/2$.
 - Transition labels: $3/2^+$, $1/2^+$, $5/2^+$, $3/2^-$, $5/2^-$.
 - Additional labels: $\Sigma = 6$, 195Ir , $U(6/4)$.

Fig. 3. Comparison between supersymmetry predictions and positive-parity states in ^{195}Ir (from Ref. 13).