MASTER

CONF-790318--8

ELECTROMAGNETIC INTERACTION OF COLLIDING BEAMS IN STORAGE RINGS *

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I. INTRODUCTION

When the beams cross each other in the intersection region of a storage ring, two distinct effects occur. A particle of one beam may undergo a fundamental interaction with a particle of the other beam. The average number of such events is equal to the product of the machine luminosity and the particle-particle total cross section. It is, of course, these particle interactions which the experimental physicist studies. In contrast to what happens for these fundamental interactions, a particle of one beam passing through the interaction region will always be influenced by the electric and magnetic fields produced by the entire other beam. Clearly, this classical electromagnetic interaction between the two beams will perturb the motion of the particles circulating in the machine. It is our purpose in this paper to present the basic aspects of this beam-particle force. We shall indicate how the effect of this electrical force is quite different in the case of continuous (unbunched) proton beams which cross each other at an angle (as in the ISR at CERN and in ISABELLE at Brookhaven), as compared to the case of electron-positron bunches which collide head on (as in the SPEAR II Ring and in PETRA and PEP).

II. UNBUNCHED INTERSECTING BEAMS

In Fig. (1) we show a particle trajectory crossing a continuous beam at an angle α_0 . We assume that the transverse (x,y) distribution of current (I) in the beam is Gaussian, that is,

$$\rho(x,y) = \frac{I}{2\pi\sigma_V\sigma_H} \exp \left[-\frac{x^2}{2\sigma_H^2} - \frac{y^2}{2\sigma_V^2} \right]. \quad (1)$$

It is clear from the figure that, during the passage of the particle from one side of the beam to the other side, it will sample a succession of gradient fields, with the result that its horizontal and vertical betatron tunes will be changed. For a central trajectory ($y=0$), these changes can be calculated by perturbation theory and are

$$\Delta\nu_V = \frac{\beta_V^* r_p I}{\sqrt{2\pi} e c \gamma \beta^2 \sigma_V^* \tan(\alpha_0/2)}, \quad (2)$$

*Work performed under the auspices of the U.S. Department of Energy
under Contract No. EY-76-C-02-0016.

Fig. 1. Particle Crossing a Continuous Beam.

and

$$\Delta v_H = 0, \quad (3)$$

where

α_0 = the horizontal total crossing angle (rad),

σ_V^* = Gaussian rms (vertical) size at crossing (m),

β_V^* = vertical amplitude function at crossing (m),

r_p = classical radius of proton (1.535×10^{-18} m),

(γ, β) = relativistic particle factors of colliding protons,

I = average current per beam (A),

$(\Delta v)_{V,H}$ = vertical or horizontal tune shift per crossing,

e = charge of proton (1.602×10^{-19} C), and

c = velocity of light (2.9979×10^8 ms $^{-1}$).

It is important to point out that in deriving these equations for the tune shift we have assumed that the cross-sectional size of the beam does not vary over the intersection region, and therefore that the betatron amplitude functions are constant.

A pertinent question to ask is: "What is the deflection of the particle trajectory as it crosses the continuous beam?". Because of the symmetrical way in which the particle approaches and leaves the center of the other beam, the net angular deflection in the horizontal plane is zero, similar to the tune shift, Eq. (3). Instead for the case of the vertical plane we have for the net change in slope,

$$\Delta\left(\frac{dy}{ds}\right) = \frac{2\pi r_p I}{ec\gamma\beta^2 \tan(\alpha_0/2)} \mathfrak{F}(y/\sqrt{2} \sigma_V^*) \quad (4)$$

where $\mathfrak{F}$ is the error function defined as

$$\mathfrak{F}(x) = \frac{2}{\sqrt{\pi}} \int_0^x dt e^{-t^2} . \quad (5)$$

In Fig. (2) we have sketched the variation of this angular deflection as a function of the displacement (y) of the particle path from the horizontal symmetry plane of the colliding beams.

Fig. 2. Angular deflection for a particle crossing an unbunched beam at an angle.

We note that the magnitude of the maximum deflection tends toward

$$(\Delta y')_{\max} = \frac{2\pi r_p I}{ec\gamma\beta^2 \tan(\alpha_0/2)} \quad (6)$$

for large vertical amplitudes of the incident particle. It is important to point out that since the linear term obtainable from Eq. (4) is proportional to the change in betatron tune expressed by Eq. (2), the magnitude of this value of $\Delta\nu$ has been conventionally designated as the beam-beam strength parameter. Thus, the change in tune per crossing, calculated from first order perturbation theory, serves as a measure of the strength of the nonlinear beam-particle interaction specified by the net angular deflection of an intersecting particle trajectory. This can best be seen by writing the net deflection in the form

$$\Delta\left(\frac{dy}{ds}\right) = (2\pi)^{3/2} |\Delta\nu_V| \frac{\sigma_V^*}{\beta_V^*} \mathfrak{F}\left(\frac{y}{\sqrt{2}\sigma_V^*}\right) . \quad (7)$$

III. HEAD-ON COLLISIONS OF BUNCHED BEAMS

For an electron traversing a bunch of positrons, such as occurs in e^+e^- storage rings, the situation is that depicted in Fig. (3).

Fig. 3. Particle Colliding head-on with a bunch.

A particle passing through the center of a bunch, whose density of charge is Gaussian in the transverse plane, will experience a betatron tune shift in both planes. These are calculated to be

$$\Delta\nu_V = \frac{C_V^* r_e N_B}{2\pi\gamma\beta^2 \sigma_V^* (\sigma_H^* + \sigma_V^*)}, \quad (8)$$

and

$$\Delta\nu_H = \frac{\beta_H^* r_e N_B}{2\pi\gamma\beta^2 \sigma_H^* (\sigma_H^* + \sigma_V^*)}. \quad (9)$$

We have used a notation such that $\sigma_{H,V}^*$ = rms horizontal or vertical size at the collision point (m), $\beta_{H,V}^*$ = betatron amplitude function over the collision region (m), N_B = number of particles per bunch, r_e = classical radius of an electron (2.818×10^{-15} m), and $\Delta\nu_{V,H}$ = vertical or horizontal tune shift per collision.

For head-on collisions the angular deflection of the particle path is not confined to the vertical plane as is the case for continuous beams crossing at an angle. When the bunch has a symmetrical Gaussian distribution of charge in the transverse plane, the net deflections of a colliding particle in the two planes are

$$\Delta\left(\frac{dx}{ds}\right) = \frac{-2N_B r_e x}{\gamma\beta^2 r^2} (1 - e^{-r^2/2\sigma^2}), \quad (10)$$

and

$$\Delta\left(\frac{dy}{ds}\right) = \frac{-2N_B r_e y}{\gamma\beta^2 r^2} (1 - e^{-r^2/2\sigma^2}), \quad (11)$$

where x and y are the horizontal and vertical offsets of the particle trajectory and r is the associated radial distance from the center line of the bunch. In Fig. (4) we show the variation of this net angular deflection in one of the planes.

Fig. 4. Angular deflection for a particle colliding head-on with a bunched beam.

In contrast to the unbunched case, the deflection after rising to a maximum value of about

$$(\Delta x')_{\max} \approx \frac{0.9 N_B r_e}{\gamma\beta^2 \sigma} \quad (12)$$

decreases to zero for particles passing through the outer fringes of a bunch.

Though the strength parameters, Eqs. (8) and (9), are applicable to unsymmetrical Gaussian bunches, the net deflections, Eqs. (10) and (11), only apply to cylindrically symmetrical beams, with an rms half-width of σ . It is therefore of interest to obtain the expression for the angular deflections for a Gaussian distribution with arbitrary aspect ratio, particularly since e^+e^- rings operate with a ratio of horizontal to vertical size at the collision point of about 20 to 1. To do this we start with the potential¹ produced by a two dimensional Gaussian distribution of charge having a total charge per unit length equal to λ_L . Thus we have

$$V(x,y) = \frac{\lambda_L}{4\pi\epsilon_0} \int_0^\infty \frac{1 - \exp\left[-\frac{x^2}{a^2+t} - \frac{y^2}{b^2+t}\right]}{(a^2+t)^{1/2} (b^2+t)^{1/2}} dt. \quad (13)$$

Differentiating this expression to obtain the electric field and using the fact that the particles are moving in opposite directions with essentially the speed of light, we find that the net angular deflection in the y and x planes are

$$\Delta\left(\frac{dy}{ds}\right) = \frac{-2N_B r_e y}{\gamma\beta^2} \int_0^\infty \frac{\exp\left[-\frac{x^2}{a^2+t} - \frac{y^2}{b^2+t}\right]}{(a^2+t)^{1/2} (b^2+t)^{3/2}} dt, \quad (14)$$

and

$$\Delta\left(\frac{dx}{ds}\right) = \frac{-2N_B r_e x}{\gamma\beta^2} \int_0^\infty \frac{\exp\left[-\frac{x^2}{a^2+t} - \frac{y^2}{b^2+t}\right]}{(a^2+t)^{3/2} (b^2+t)^{1/2}} dt \quad (15)$$

Again, N_B is the number of particles per bunch and r_e is the classical radius of the electron. If we consider the special case of the vertical angular deflection undergone by a particle when it is incident in the central vertical plane of a bunch [Eq. (14) with $x = 0$], then we derive

$$\begin{aligned} \Delta\left(\frac{dy}{ds}\right)_{x=0} &= \frac{-2\sqrt{\pi}N_B r_e}{\gamma\beta^2 (a^2 - b^2)^{1/2}} \left[\xi\left(\frac{a}{b} \frac{y}{(a^2 - b^2)^{1/2}}\right) \right. \\ &\quad \left. - \xi\left(\frac{y}{(a^2 - b^2)^{1/2}}\right) \right] \exp\left(\frac{y^2}{a^2 - b^2}\right), \end{aligned} \quad (16)$$

with ξ equal to the error function. When, in addition, the horizontal width of the bunch is considerably greater than its height ($a \gg b$), this result simplifies to

$$\Delta\left(\frac{dy}{ds}\right)_{x=0} = \frac{-2\sqrt{\pi}N_B r_e}{\gamma\beta^2 a} \left[\xi\left(\frac{y}{b}\right) - \xi\left(\frac{y}{a}\right) \right] \exp\left(\frac{y^2}{a^2}\right) \quad (17)$$

Under these conditions it is then possible to find the approximate value of the maximum deflection experienced by a particle passing through an unsymmetrical Gaussian bunch. Equation (17) yields for this value:

$$\Delta(y')_{\max} \approx \frac{\sqrt{2\pi} N_B r_e}{\gamma\beta^2 \sigma_H^*}, \quad (18)$$

into which we have inserted the rms half width at the collision point, $\sigma_H^* = a/\sqrt{2}$.

In general the integrals appearing in Eqs. (14) and (15) cannot

be written in closed form. However, with the stipulation that $b/a = \rho < 1$, they can be written in the following more convenient way:

$$\Delta\left(\frac{dy}{ds}\right) = \frac{-4N_B r_e y}{\gamma \beta^2 a^2 (1 - \rho^2)} \exp\left[-\frac{x^2 - y^2}{a^2 (1 - \rho^2)}\right] \times \int_0^1 \frac{dw}{w^2} \exp\left[\frac{1}{a^2 (1 - \rho^2)} \left(x^2 w^2 - \frac{y^2}{w^2}\right)\right], \quad (19)$$

and

$$\Delta\left(\frac{dx}{ds}\right) = \frac{-4N_B r_e x}{\gamma \beta^2 a^2 (1 - \rho^2)} \exp\left[-\frac{x^2 - y^2}{a^2 (1 - \rho^2)}\right] \times \int_0^1 dw \exp\left[\frac{1}{a^2 (1 - \rho^2)} \left(x^2 w^2 - \frac{y^2}{w^2}\right)\right]. \quad (20)$$

Imposing the condition $\rho \ll 1$ for wide beams, we now obtain

$$\Delta\left(\frac{dy}{ds}\right) = -4\pi |\Delta v_V| \frac{y}{\beta_V^*} \frac{\sigma_V^*}{\sigma_H^*} \exp\left[-\frac{x^2 - y^2}{a^2}\right] \int_0^1 \frac{dw}{w^2} \exp\left[\frac{1}{a^2} \left(x^2 w^2 - \frac{y^2}{w^2}\right)\right], \quad (21)$$

and

$$\Delta\left(\frac{dx}{ds}\right) = -4\pi |\Delta v_H| \frac{x}{\beta_H^*} \exp\left[-\frac{x^2 - y^2}{a^2}\right] \int_0^1 dw \exp\left[\frac{1}{a^2} \left(x^2 w^2 - \frac{y^2}{w^2}\right)\right]. \quad (22)$$

IV. BEAM-BEAM PARAMETERS OF (pp) AND (e^+e^-) MACHINES

Table I is a presentation of the parameters determining the luminosity and the beam-beam strength for the CERN ISR and the Brookhaven pp rings, ISABELLE. An attempt has been made to give a set of consistent values for these parameters which can be calculated from the listed equations. It may be of interest to mention that, for ISABELLE at 30 GeV, the beam-beam strength parameter and the maximum angular deflection are significantly higher than those at the ISR.

In Table II, we give an analogous list of parameters for the SPEAR II ring as well as for the PETRA and PEP machines. It is evident that e^+e^- rings operate with a beam-beam strength parameter which is larger by more than an order of magnitude than that for pp unbunched machines. In addition, these strengths are approximately equal in magnitude in the two transverse planes. The tabulation also indicates that the maximum deflection undergone by a particle in

traversing the other beam is three orders of magnitude greater for e^+e^- rings than the corresponding deflection in pp rings.

REFERENCES

1. B. W. Montague, CERN Rpt. 68-38 (1968).

Table IA. Beam characteristics of pp storage rings.

<u>pp Machines</u>		ISR CERN (p,p) $\equiv$ (1,2)	ISABELLE 30 x 30 GeV [Standard Ins.]	ISABELLE 400 x 400 GeV [Standard Ins.]
Energy	$E_1 = E_2 = E$	26.78 GeV	29.4 GeV	400. GeV
Particle Gamma	$\gamma = E/M_p$	28.49	31.3	426.3
Circum- ference	$2\pi R$	942.48 m	3833.8 m	3833.8 m
Current	$I_1 = I_2 = I$	28.6 A	8 A	8 A
Crossing Angle	α	14.77°	11.88 mrad	11.88 mrad
Number of Inter- sections		8	6	6
Betatron Tunes	$\nu_H \approx \nu_H$	8.9	22.6	22.6
Normalized Betatron Emittance (95% Phase Space)	$E_V = \frac{6\pi\sigma_V^2(\gamma\beta)}{\beta_V}$	$19.8\pi \times 10^{-6}$ radm	$15\pi \times 10^{-6}$ radm	$15\pi \times 10^{-6}$ radm
Beta Functions (Inter- section)	β_H^*	22 m	41.24	41.24
	β_V^*	14.25 m	7.5 m	7.5 m
Revolution Frequency	$f_o = c/2\pi R$	3.181×10^5 Hz	78.197×10^3 Hz	78.197×10^3 Hz
Number of Particles	$N = I/ef_o$	5.61×10^{14}	6.38×10^{14}	6.38×10^{14}
Classical Radius of Proton	r_p	1.535×10^{-18} m		

Table IB.

<u>p-p Machines</u>		ISR CERN	ISABELLE 30 x 30 GeV	ISABELLE 400 x 400 GeV
rms Verti- cal Size (Inter- section)	$\sigma_V^* = \sqrt{\frac{E_V \beta_V^*}{6\pi (\gamma \beta)}}$	1.278 mm	0.776 mm	0.21 mm
Effective Height	$h_e = 2\sqrt{\pi} \sigma_V^*$	4.53 mm	2.74 mm	0.743 mm
Angular Divergence (Inter- section)	$y'^* = \sqrt{6} \frac{\sigma_V^*}{\beta_V^*}$	2.2×10^{-4} rad	2.5×10^{-4} rad	0.68×10^{-4} rad
Luminosity	$L = \frac{I_1 I_2}{2\sqrt{\pi} e^2 c \sigma_V^* \tan \frac{\alpha}{2}}$	1.8×10^{31} cm ⁻² sec ⁻¹	5.4×10^{31} cm ⁻² sec ⁻¹	2.0×10^{32} cm ⁻² sec ⁻¹
Vertical Beam-Beam Strength Parameter Per inter- section	$\Delta v_V = \frac{-\beta_V^* r_p I}{\sqrt{2\pi} e c \gamma \beta^2 \sigma_V^* \tan \frac{\alpha}{2}}$	-1.1×10^{-3}	-5.6×10^{-3}	-1.5×10^{-3}
Horizontal Beam-Beam Tune Shift	Δv_H	0.	0.	0.
Beam-Beam Maximum Vertical Deflection	$(\Delta y')_{\max} = \frac{2\pi r_p I}{e c \gamma \beta^2 \tan \frac{\alpha}{2}}$	1.56×10^{-6} rad	9.2×10^{-6} rad	0.67×10^{-6} rad

Table IIA. Beam characteristics of e^+e^- storage rings.

e^+e^- Machines		SPEAR II (1,2) $\equiv (e^-, e^+)$	PEP (e^-, e^+)	PETRA (e^-, e^+)
Energy	$E_1 = E_2 = E$	3.71 GeV	15 GeV	15 GeV
Particle Gamma	$\gamma = E/m_e$	7260.2	29354	29354
Circum- ference	$2\pi R$	234.13 m	2200 m	2304 m
Magnetic Radius of Curvature	ρ	12.7 m	165.52 m	197.14 m
Number of Particles	$N_1 = N_2 = N$	1.75×10^{11}	2.52×10^{12}	3.84×10^{12}
Number of Bunches	n_B	1	3	4
Particles Per Bunch	$\frac{N}{n_B} = \frac{q}{e}$	1.75×10^{11}	8.4×10^{11}	9.6×10^{11}
Crossing Angle	α	0.	0.	0.
Betatron	ν_H	5.27	21.25	22.2
Tunes	ν_V	5.19	18.75	22.2
Beta Func- tion at Inter- section	β_H^*	1.197 m	2.8 m	3.0 m
	β_V^*	0.0998 m	0.11 m	0.15 m
ETA Func- tion at Inter- section	η^*	0.	0.49 m	
Revolution Frequency	$f_o = c/2\pi R$	1.22×10^6 Hz	1.36×10^5 Hz	1.3×10^5 Hz
Average Current	$I = e \left(\frac{q}{e} \right) n_B f_o$	35.9 mA	54.9 mA	80.0 mA
Classical Radius of Electron	r_e	2.818×10^{-15} m		

Table IIB.

e^+e^- Machines			SPR ⁺ R ₊ (e^-, e^+)	PEP ⁺ (e^-, e^+)	PETRA ⁺ (e^-, e^+)
Betatron Emittance	$\epsilon = \sigma^2/\beta$	ϵ_H	0.665×10^{-6} radm	0.138×10^{-6} radm	0.15×10^{-6} radm
		ϵ_V	0.902×10^{-8} radm	0.933×10^{-8} radm	0.75×10^{-8} radm
Coupling Coef- ficient	$K = \sqrt{\epsilon_V/\epsilon_H}$		0.12	0.26	0.22
Betatron rms Size (Inter- section)	$\sigma^2 = \beta \epsilon$	$(\sigma_H^*)_\beta$	0.892 mm	0.662 mm	0.67 mm
		σ_V^*	0.030 mm	0.032 mm	0.034 mm
Energy Spread	$\frac{\sigma_\epsilon}{E} = \left[\frac{3.84 \times 10^{-13} \gamma^2}{2\rho} \right]^{1/2}$		0.89×10^{-3}	1.0×10^{-3}	0.91×10^{-3}
Energy rms Size (Inter- section)	$\sigma_\epsilon^* = \eta^* \frac{\sigma_\epsilon}{E}$		0.	0.49 mm	
Total Horz. rms Size	$\sigma_H^2 = \sigma_\epsilon^{*2} + (\sigma_H^*)_\beta^2$		0.892 mm	0.79 mm	0.67 mm
(Energy Loss/Rest Mass) Per Turn Due To Syn. Radiation	$\left(\frac{\Delta W}{mc^2} \right)_{\text{turn}} = \frac{4\pi}{3} \frac{r_e}{\rho} \gamma^4 \beta^3$		2.582	52.94	44.45
Energy Loss (SR) Per Turn	ΔW		1.32 MeV	27.1 MeV	22.7 MeV
Total Po- wer Loss Per Beam	$(\Delta W)I$		47.4 kW	1.5 MW	1.8 MW
Average Radiated Power Per Particle	$P_\gamma = (\Delta W)f_0$		1.69×10^{12} eV/sec	3.68×10^{12} eV/sec	2.95×10^{12} eV/sec
Trans- verse Damping Time	$\tau_T = 2E/P_\gamma$		4.4×10^{-3} sec	8.1×10^{-3} sec	10×10^{-3} sec

Table IIC.

	e^+e^- Machines	SPEAR II	PEP	PETRA
Angular Divergence (Inter-section)	$y^* = \sqrt{6} \frac{\sigma_V^*}{\beta_V^*}$	7.6×10^{-4} rad	7.1×10^{-4} rad	5.5×10^{-4} rad
Luminosity Per Bunch Collision	$\mathcal{L}_{BC} = \frac{q_1 q_2}{e^2 4\pi \sigma_V^* \sigma_H^*}$	9.1×10^{24} cm ⁻²	2.2×10^{26} cm ⁻²	3.2×10^{26} cm ⁻²
Average Luminosity	$L = n_B^f \mathcal{L}_{BC}$	1.16×10^{31} cm ⁻² sec ⁻¹	9.1×10^{31} cm ⁻² sec ⁻¹	1.7×10^{32} cm ⁻² sec ⁻¹
Vertical Beam-Beam Strength Parameter	$\Delta v_V = \frac{\beta_V^* r_e (N/n_B)}{2\pi\gamma \beta^2 \sigma_V^* (\sigma_H^* + \sigma_V^*)}$	0.039	0.054	0.092
Horizontal Beam-Beam Strength Parameter	$\Delta v_H = \frac{\beta_H^* r_e (N/n_B)}{2\pi\gamma \beta^2 \sigma_H^* (\sigma_H^* + \sigma_V^*)}$	0.016	0.055	0.093
Approx. Max Particle Deflection Per Collision	$\sim \frac{\sqrt{2\pi} (N/n_B) r_e}{\sigma_H^* \gamma}$	2×10^{-4} rad	2.6×10^{-4} rad	3.5×10^{-4} rad