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TITLE: COLLECTIVE, STOCHASTIC AND NONEQUILIBRIUM BEHAVIOR OF HIGHLY EXCITED HADRONIC MATTER

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COLLECTIVE, STOCHASTIC AND NONEQUILIBRIUM BEHAVIOR OF HIGHLY EXCITED HADRONIC MATTER

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We discuss selected problems concerning the dynamic and stochastic behavior of highly excited matter, particularly the QCD plasma. For the latter we consider the equation of state, kinetics, quasiparticles, flow properties and possible chaos and turbulence. The promise of phase space distribution functions for covariant transport and kinetic theory is stressed. The possibility and implications of a stochastic bag are spelled out. A simplified space-time model of hadronic collisions is pursued, with applications to A-A collisions and other matters. The domain wall between hadronic and plasma phase is of potential importance: its thickness and relation to surface tension is noticed. Finally we review the recently developed stochastic cell model of multiparticle distributions and KNO scaling. This topic leads to the notion that fractal dimensions are involved in a rather general dynamical context. We speculate that various scaling phenomena are independent of the full dynamical structure, depending only on a general stochastic framework having to do with simple maps and strange attractors.

1. INTRODUCTION

The purpose of this lecture is to present speculations and formulate problems concerning everyday-life problems of QCD, lying neither in the limit of infrared slavery nor asymptotic freedom. We are concerned with finding simplicity in the many-particle dynamics both in multiparticle production, and in the internal excitations and collective (flow) behavior of excited QCD matter, which we shall call for simplicity the QCD plasma, even though many phases are conceivable.

Many important questions remain to be clarified for the extended system. These include:

(1) What are the quasi-particles of the plasma?

(2) What are the fluctuations in the plasma? In particular, can there be regimes in which chaotic or turbulent behavior are manifest?

(3) What are the transport properties (heat conductivity, viscosity, conductivity tensor, and generalized dielectric constant)?

(4) What are the rate processes when the particle species are out of equilibrium?

Many of the foregoing problems are formidable in their own right. When we consider the possible formation of sparks of plasma in a hadronic collision, we encounter still more complex issues. How indeed, can we get "on" the phase diagram, beginning with hadronic collisions? Beginning with a hadron of unknown structure in quarks and glue, a highly virtual state of unknown dynamics may precede a complex kinetic phase before reaching the safety of the phase diagram. Even then, the hydrodynamic expansion of the localized plasmoid, possible latent heat created at phase transitions and entropy generation precede the little understood process of hadronization.

Other speakers have focussed on recent progress in hydrodynamics and lattice calculations of thermodynamics and phase behavior of extended QCD matter. Our purpose is to outline problems and sometimes to suggest solutions for additional dynamical behavior (expected or conceivable) exhibited by such systems. We shall stress the nonequilibrium and stochastic behavior which are inevitable and moreover remarkably interesting. I am convinced that simple structures pervade these out-of-equilibrium problems and point to some examples of phenomena which already exhibit simplicity in the absence of LTE (i.e., local thermodynamic equilibrium). Simple structures inhabit highly excited extended matter in ways described by simple stochastic field ensembles and by what we shall call fractal dynamics. We conjecture that for many empirical features, especially those involving scaling or self-similar properties, there is a skeletal underlying dynamics of simple low-dimensional maps which exists within the very complex dynamics of the full system. By identifying these underlying structures, we not only describe data but strongly constrain the behavior of the full system. The results of fractal dynamics, whose language is that of maps (usually containing strange attractors with nonintegral Hausdorff dimension), can be expected to be simple and potent.

If the foregoing claims seem too wild, recall that ten years ago, during the tyranny of lns physics, scaling, parton models, etc., the key ideas of the new dogma as expressed at this meeting (quark matter, hydrodynamics for hadronic physics) were widely despised or ignored. Moreover QCD had just been proposed, and only now is the crucial role of glue becoming appreciated by a community long washed in the lore of quarks, quark bremsstrahlung, "flat" rapidity plateaus, and so on. In my polemic[1] "Heretical Models of Particle Production" the phenomenological success of a modernized Fermi-Landau statistical hydrodynamical model (SHM) was posed as a counter to the prevailing orthodoxy. Even then, with ISR data, the (true) rapidity distribution was not flat, but Gaussian; the multiplicity was not lns but more like the square root of the available c.m. energy as predicted (both coefficient and power) by SHM. The scaling violations predicted at small x were in agreement with data as well as the increase of dN/dy at 90° with increasing energy. To be sure, the Fermi-Landau[2,3] picture involves dubious assumptions, particularly that of instant thermalization within the Lorentz-contracted initial volume. Lately a new view of the initial condition has been proposed by Bjorken[4] and developed by the Helsinki school[5] in connection with A-A collisions. Although usage for hadron-hadron collisions is not yet authorized by the authorities, some careful analysis along these lines is very much in order. Remember that L.T.E. is not a prerequisite to the use of collective variables.

In the 1960's the advent of S-matrix theory (bootstrap and the j-plane) created a generation unacquainted with the substantial SHM results of Soviet and Japanese theorists. By 1973 quark models had made many inroads, though not so much to high energy collision dynamics or the properties of extended matter. As proof I can mention the oblivion of my pre-QCD paper:[6] "Quarkium, a Bizarre Fermi Liquid." The gluons were not SU_3^C, just U(1). Even so, one could speculate about quark stars and various phases, for example, the low density Wigner lattice, the medium density ferromagnetic phase due to the strong exchange force, quasi-particles such as plasmons, zero sound and possibly superconductivity in the hadron condensate leading to ESP-like phase information in the leptons (now implementable with GUTS).

Despite the current infatuation with QCD matter (erroneously called quark matter in the conference title) and despite the willingness to embrace hydrodynamic techniques for orientation to the evolution problems expected in A-A collisions, it is clear that the key problem of this new field is the proper description of nonequilibrium behavior of hadronic matter by a relativistic and quantum mechanical kinetic theory. If equilibration is occurring in some region of space time, we can learn about it from the kinetic theory. Hydrodynamic equations result from taking various moments of the equations of motion of the reduced distribution functions. Even in the absence of LTE, formal hydrodynamic structures exist and could provide useful information.

Since 1974 I have been working out a covariant quantum transport theory[7] in collaboration with F. Zachariasen. This theory is a straightforward field-theoretic extension of the phase space representation of quantum mechanics put forward[8] by Wigner in 1932. Recently we published a summary[9] of our work on scattering theory in this framework, emphasizing the nonrelativistic N-body problem which had not been adequately studied. The formalism needs extensive development to become a useful prediction tool for realistic problems in A-A collisions. We feel that the investment of effort is worthwhile, however, in view of the flexibility and intuitive content of the formalism.

2. DYNAMICAL PROPERTIES OF THE QCD PLASMA.

Our discussion centers on three topics: (1) Quasi-particles (thermodynamics, effective interactions, kinetics); (2) Equation of State; (3) Possible Chaos.

2.1. Quasi-Particles.

Most prior studies of the QCD plasma have been bound to a perturbative or lattice computational framework. The perturbation series converges badly and does not (and is not expected to) reveal information on the quasi-particles of the medium. Lattice calculations have their own strengths and drawbacks, sharing with pertubative calculations the defect of exhibiting numerical answers instead of representing physical quantities in terms of low-lying excitations of the system.

By QCD plasma[10] we shall mean the deconfined naive vacuum with the basic "potential" being coulombic. We stress this only because of the persistence of papers imagining that a confining potential persists in the unconfined phase. Further, our reference system will have (except for occasional special remarks) massless quarks. The mass scales inhabiting the full system are then the QCD reference mass Λ and the temperature T. Although some calculations pretend to be in the $T \gg \Lambda$ limit, realistic constraints based on conceivably attainable temperatures dictate that almost all length/mass scales are rather comparable for contemporary applications, e.g., $T \sim \Lambda \sim 200$ MeV, or sometimes, simply $m_\pi \sim 140$ MeV. Similarly the plasma frequency, and the thermal quark mass are expected to be of this magnitude. Hence no asymptotic simplifications should be expected for the description of twentieth century A-A collisions. Nevertheless, experience with condensed matter physics is reassuring in that a quasi particle description is often useful and even semiquantitation outside idealized limits.

Although an enormous number of possible phases can be contemplated, we shall focus on the "obvious" excitations which may be expected in a thermal plasma without controllable c-number sources. The gluons will acquire mass, as will the quark. A plasmon is expected and in addition, a hydrodynamic phonon[11] with a velocity of $1/\sqrt{3}$. Careful calculations of damping need to be done to see whether these quasi particles are really approximate normal modes of the system, and if so in what range of Λ,T. The change in spectrum due to turning on the interactions is indicated in Fig. 1. The shape of the dispersion curve $\omega(p)$, though important for kinetic rates, is not known at present.

The effect of screening, as seen in the electric and magnetic gluon mass, is the analogue of the usual plasma excitation[13] in an electron gas (or electron-ion gas). Difficult gauge-invariance problems have caused much controversy concerning the glue mass, particularly for the transverse (magnetic) degrees of freedom. Following Applequist[12] we shall adopt

$$M_G \text{ (electric)} \sim gT$$
$$M_G \text{ (magnetic)} \sim g^2T \qquad (2.1)$$
$$M_Q \sim gT$$

For accessible energy parameters g ~ 1 and these masses are all of order T. Yet another excitation not visible in single particle propagators is the color singlet phonon (sound wave). This spinless excitation will exist when collisions are sufficiently frequent as to achieve local equilibrium, yet not so rapid as to overdamp the mode. In contrast to the electron-ion plasma, the color difference is such that the plasmon and phonon will not be distinct roots of the same eigenvalue equation.[13]

FIGURE 1
The dispersion curves for the QCD plasma are shown qualitatively. The noninteracting massless spectrum ($\omega = p$) is shown in (1a). When interactions are turned on, the spectrum changes to (1b). The dotted line disappears while massive gluons and quarks appear, along with a phonon with $\omega = 1/\sqrt{3}\, p$, $p \to 0$. The dashed lines indicate our ignorance of the true dispersion for larger p.

From this discussion we anticipate quasi-particles whose low-momentum energies are roughly

		color
$\omega_{G(E)}$	$\simeq gT$	$\underline{8}$
$\omega_{G(M)}$	$\simeq g^2T$	$\underline{8}$
ω_Q	$\simeq gT$	$\underline{3},\bar{\underline{3}}$
ω_{ph}	$\simeq p/\sqrt{3}$	1

(2.2)

Owing to screening the long-range static coulomb potential becomes

$$V(r) \simeq \frac{g^2}{4\pi r} + g^2 \frac{e^{-gTr}}{r} \qquad (2.3)$$

So that a short-range residual force (energy $\simeq g^2 m_\pi/e \sim 20$ MeV) acts between the colored plasma quasi-particles.

Having suggested a set of low-lying excitations, we can attempt to imitate procedures in condensed matter physics. First of all we should be able to describe the thermodynamics of the plasma in terms of the contributions of the quasi-particles. Secondly the original Hamiltonian should be expressible in terms of the quasi-particle coordinates in a way that gives not only the effective interaction but also the vertices describing transitions among various quasi-particle configurations.

First consider the most primitive thermodynamic function, the energy density ε. In the case of free massless glue and quarks, we know that the black-body formula holds

$$\varepsilon = \frac{\pi^2}{30} N_d T^4 \tag{2.4}$$

where N_d is the active number of degrees of freedom, which for an equilibrated plasma of N_f flavors of quarks is

$$N_d = 8 \times 2 + \frac{7}{8} \times 2 \times 2 \times 3 \times N_f \tag{2.5}$$

Note that N_f = 2, 3 gives N_d = 37, 47.5, enormously greater than the value N_d = 3 available to Fermi and Landau when only three types of pions were available.

Remarkably, lattice calculations[14] have shown that the "free" Stefan-Boltzmann law (2.4) is reached at nonasymptotic values of T. Does this really imply that the excitations are the massless free quarks and gluons expected in the limit $T \to \infty$. We believe not, and suggest that as the excitation spectrum shifts (Fig. 1) when the interactions are turned on, a certain compensation occurs. First note that when the gluon mass becomes finite, two effects occur. First of all there are three rather than two polarization modes. Secondly since $M_G \sim T$, no matter how high T becomes, the "Boltzmann" factor, rather Bose-Einstein factor, suppresses the contribution to thermodynamic quantities.

$$\frac{1}{e^{E/T}-1} \lesssim \frac{1}{e^{g}-1} \tag{2.6}$$

Overall the glue contribution to ε decreases, as does that of the newly massive quark. At the same time, however, the easily excited phonon mode compensates to a large extent. In fact we easily find

$$\varepsilon_{ph} \simeq \frac{3\sqrt{3}}{16} \varepsilon_{\text{free glue}} \tag{2.7}$$

Even though the phonon is a color singlet, its dispersion curve is so soft as to contribute $\simeq$1/3 of that of <u>massless</u> glue (whose contribution is suppressed by a comparable amount).

Although the thermodynamic functions themselves do not seem to be sensitive to the quasi-particle spectrum, it is clear that controllable probes should be able to detect the quasi-particles. Unfortunately controllable probes do not seem to exist for A-A collisions. In a metal we are accustomed to detect plasmons by measuring the energy loss spectrum of fast electrons. Phonon dispersion curves can be precisely determined by scattering thermal neutrons from crystals. The controlled color probe (e.g., a fast quark or gluon) seems unobtainable. Hard collisions will produce such probes, but when smeared over the broad-band beam which the collision elements represent, it is hard to imagine seeing a recognizable signal. Color singlet density excitations of phonon shock modes should create a high $p_\perp$ signal, but again other explanations for such a signal could easily be invented. Hence this system of plasmons, phonons and nearly free quark excitations interacting by short-range screened interactions, seems difficult to verify by the excitation processes expected in the A-A collision process.

Note, however, that the equilibrium state involves an active set of fluctuations (in color, due to plasma oscillations, and in color singlet density oscillation due to the phonons). The anticipated photon/lepton pair signal may be sensitive to these fluctuations. New calculations taking into account these source fluctuations should be made.

As far as we know, no serious investigation has been made of the effective Hamiltonian for quasi-particle interactions. In one way these interactions may be simpler than in the analogous electron-ion plasma. In the latter the phonon oscillation creates a charge density to which (charged) quasi-free electrons couple. In the present case the phonon is color neutral, so that the colored quasi-quarks and quasi-gluons do not interact except in pairs.

Next consider the kinetics of the plasma in the context of traditional rate equations. The quasi-particle vertices (Fig. 2) are the basic ingredients. A new feature concerns the possibility of on-shell three particle reactions forbidden for the original spectrum but allowed by the downward dispersion of the quasi-particle spectrum as exhibited in Fig. 3a. If the interactions convert the spectrum to convex, one can imagine the heat bath to catalyze processes previously forbidden in mass sh ll kinetics. These processes can compete with or even dominate the usual calculations based on cross-section kinetics.

Although some useful rates have been estimated[15] by kinetic equations, much remains to be done. The estimate of the time dependent populations of various species resulting from differing initial conditions may not seem very exciting, yet is extremely important in planning and interpreting experimental data involving A-A collisions. We can imagine several significant relaxation times. Below we shall argue that in some cases the initial configuration can be pure glue, out of thermal equilibrium. This glue very likely can equilibrate before enough quark pairs have been created to give true thermodynamic equilibrium. Further the production of strange quark pairs may be delayed due to the larger strange quark mass. Finally the value of the baryon chemical potential can strongly bias the populations reached at a given time from a specific initial condition.

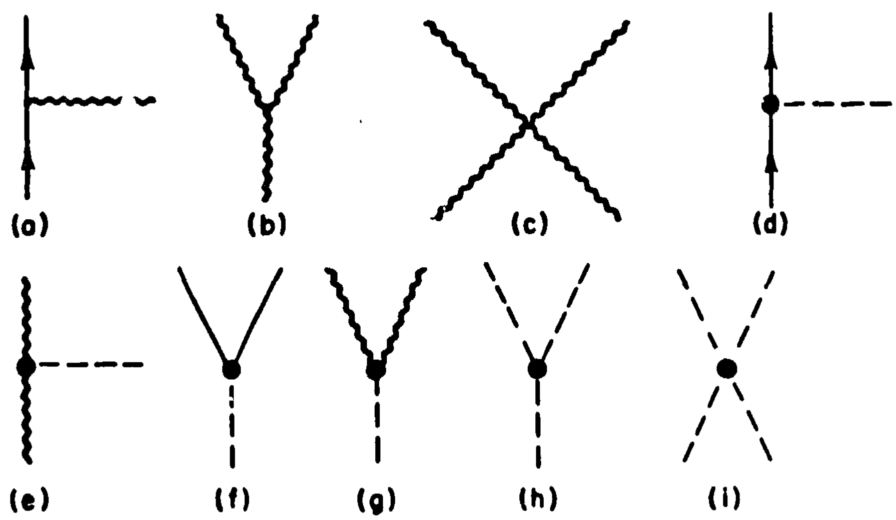

FIGURE 2
Quasiparticle Feynman graphs are shown for the QCD plasma. The solid line represents the (quasi) quark, the wiggly lines represent various massive gluons and the dashed line is the phonon mode.

FIGURE 3
Convex and concave dispersion curves dictate whether real three-particle transitions can occur. If the dispersion shown in (3a) occurs, for example, we can have energy transport via real thru-phonon transitions.

The traditional rate equations for a spatially uniform system assume random initial phases so that the rate dN_i/dt for a specified momentum p_i, spin S_i and color C_i is calculated from the "golden rule"

$$\frac{dN_i}{dt} = \sum_j \{[\text{production rates } j \text{ creating } i] - [\text{destruction rates } j\text{, removing } i]\} \quad (2.8)$$

with a typical term looking like

$$2\pi \sum_{1,2,2'} \delta(E_1 + E_2 - E_{1'} - E_{2'})|M|^2 N_1 N_2 (N_2' + 1)(N' + 1) \quad (2.9)$$

for creation of 1′. Here the occupation nunbers are bosonic, with particle 1′ selected. In QCD $|M|^2$ is infrared divergent and needs to be cut off at $Q^2 \simeq \Lambda^2$ or T^2. This seems to be the least of the problems involved.

Rather than fill pages with conceptually trivial yet unsolved equations, we simply point out once again the practical importance of some theoretical rate estimates to get a better feeling for the population chemistry of the QCD plasma.

2.2. Is the Plasma Chaotic?

Recent investigations of the time dependence of spatially uniform classical Yang-Mills theories by the Yerevan school[16-19] has led to the fascinating conclusion that such systems are inherently chaotic and possess strong mixing properties. It is further claimed that the full system should inherit this behavior. Indeed the highly excited QCD plasma of significant spatial extent would seem quite close to the assumed condition for the validity of the foregoing calculation.

We note that in classical mechanics, the chaotic behavior is in fact fully deterministic. We would surmise several things should chaotic behavior be indeed the rule. First of all the conventional quasi-particle picture might become irrelevant. Kinetic-diffusive transport would be replaced by highly efficient convective turbulent transport. Furthermore, perturbative space-time arguments, on which inside-outside cascades and the like are based, may have to be replaced by a totally different physical picture. However, the final physical picture may be simpler than the one presented nowadays. Also note that if the excitations are energized in a nucleus-nucleus collision, considerable angular momentum is likely to be concentrated in vorticle eddies, which could decay as coherent blasts of vector particles such as ρ, ω, ϕ (see Fig. 4). Signals of this sort seem new to the limited repertoire currently discussed and hence merit serious attention.

2.3. Finite Plasma Excitations-Plasmoids.

Suppose that in some manner one has created a finite volume of plasma phase (Fig. 5a) within the normal vacuum. Possible methods for accomplishing this will be described in the next section. The structure and evolution of this plasmoid is very complex. Here we only mention some points which do not seem to have attracted much notice in current literature, with exception of Ref. 20.

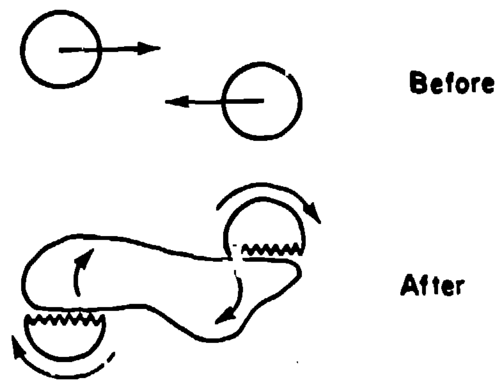

FIGURE 4
This figure is intended to suggest the possibly enormous angular momentum involved in a high energy nucleus-nucleus collision. One possible decay channel is through a coherent burst of polarized vector mesons.

Since the domain of excitation is small, it seems unlikely that the phase boundary is a mathematical surface. Instead it has a thickness δ, which is apparently unknown and if true to form is also of order $1/M_\pi$. In this case it seems fair to ask the colonels of the lattice to estimate this quantity.

From Fig. (5b) we note that the average density in the shell between the two phases is about $\frac{1}{2}B$. Since the energy in the shell is $\frac{1}{2}B \times 4\pi R^2\delta$, the change in energy of the shell when we change R is

$$dW_{shell} = (\tfrac{1}{2}B\delta)\, dA \equiv \sigma dA \tag{2.10}$$

where dA is the area change and σ the usual surface tension: here we see

$$\sigma \simeq \tfrac{1}{2}B\delta \tag{2.11}$$

The bulk change in volume energy due to the bag constant alone is

$$dW_{bulk} = BdV \tag{2.12}$$

so
$$\frac{dW_{shell}}{dW_{bulk}} = \frac{\delta}{R} \tag{2.13}$$

with $A = 4\pi R^2$, $V = 4\pi R^3/3$.

The energy in the surface is not necessarily negligible if $\delta \sim 1$ f. Physically it seems useful to distinguish the energy in the surface shell from the volume. For example, when plasma blobs encounter each other, it is advantageous for them to fuse, eliminating the positive surface energy. (At the same time a small heating should occur.) If δ is substantial, then many discontinuous effects may get smoothed out. In summary it seems worthwhile to calculate δ. Other problems may be solved in the process.

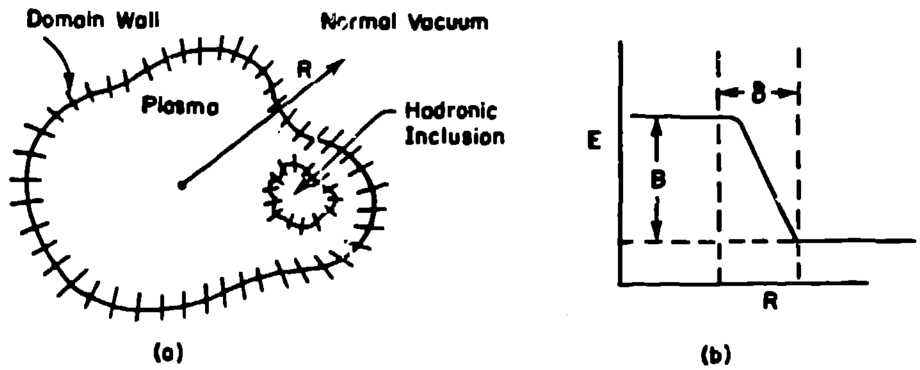

FIGURE 5
An excited region of space-time containing QCD plasma is shown. The phase boundary is not a mathematical surface but involves a domain wall of thickness δ and associated surface tension.

A particularly important consequence of surface tension is the suppression of potential Rayleigh-Taylor instabilities in the later phases of expansion when the acceleration $\ddot{R}$ may be negative. Usually one would expect short wavelength fluctuations to develop exponential instabilities under suitable conditions, thereby providing a mechanism for converting internal thermal energy into the kinetic energy of soft jets. But the vastly increased surface energy in the jet makes the latter quite unlikely. At the same time we realize that the aforementioned surface tension prefers spherical objects. Otherwise the irregular geometrical structures of excited QCD plasma might become nearly indescribable except in event-by-event fractal terms.

2.4. The Trace Theorem and the Equation of State.[11,21]

The ideal relativistic equation of state

$$p = \frac{1}{3}\,\varepsilon \tag{2.14}$$

with p and ε defined to be Lorentz scalars, is much used. The adiabatic sound velocity c_0, given by

$$c_0 = \left(\frac{dp}{d\varepsilon}\right)^{1/2} \tag{2.15}$$

is in this case $1/\sqrt{3}$. A connection with the classical energy-momentum tensor

$$T_{\mu\nu} = (\varepsilon + p)\, u_\mu u_\nu - p g_{\mu\nu} \tag{2.16}$$

is given by the trace

$$T^\mu_\mu = \varepsilon - 3p \tag{2.17}$$

The vanishing of the trace (massless quanta) can then be associated with the validity of (2.14).

In field theory the trace of the energy momentum tensor is expressed in[21] terms of various operators in the theory. In Ref. 11 it is argued that in the QCD plasma (2.14) can be a reasonable approximation even in the presence of interactions.

3. GLUONS, QUARKS AND HADRONIC COLLISIONS

3.1. Proton Structure, Fluctuations and Proton-Proton Collisions

Despite much work, little is known about the detailed structure of nucleons. An extensive phenomenology of valence quarks has evolved but is inadequate to describe hadronic collision reactions. What is needed is something like a Fock space wave function of the proton. Such information is not available at present. Indeed, deep inelastic leptonic probes excite only the quarks, which carry about one-half the energy-momentum inside the proton. Formerly, in the quark era, it was imagined that the remaining half might reside in a wee sea of $q\bar{q}$ pairs. Lately it has become evident that the missing half must be due to the (confining) glue; indeed, indirect measurements of the gluon structure functions have been presented. Computation of hard jets[22] based on such information have given quantitative confirmation of perturbative QCD with the quark-gluon distributions inferred from leptonic probes. It appears that as the energy becomes still higher, gluon collisions will become the predominant reaction mechanism.

The thrust of the present section is to present arguments that beginning at still lower energies (i.e., Fermilab on up) the preexisting gluon cloud is principally responsible for the degradation of initial kinetic energy into soft hadrons. Although many of these arguments were recently published,[23] we shall repeat the main points since they are at variance with prevailing opinion. To be clear, we put our position as follows:

(1) Quark bremsstrahlung is an effect of secondary importance.

(2) Color separation and dielectric breakdown (i.e., strings) are of minor importance for multihadron production in hadron-hadron collisions.

(3) The collision of the preexisting confining gluon clouds is the principal vehicle for transforming kinetic energy to soft multiparticle final states.

The qualitative framework described here is an extension of the picture proposed in 1974 by Pokorski and Van Hove.[24]

Our conceptual framework can be expressed as a series of questions:

(a) What are the quanta comprising the proton and how are they distributed?

(b) How do these degrees of freedom share the available energy-momentum? What are the fluctuations?

(c) When two protons collide, what is the differential behavior of the initial quarks and gluons?

(d) Immediately following the collision, what is the nature of the excitations (diffractive and central, e.g.).

(e) What are the space-time evolution and hadronization properties of the distinguishable subsystems?

To first approximation the proton is composed of three valence quarks escorted by a cloud of confining glue. Although the energy is fixed, there could be interesting fluctuations in the amount shared at a given time between the gluon and quark subsystems. As a speculative digression, let us ask the following question: Is the "bag" stochastic? We ask this question because of the example of chaotic behavior found in the uniform classical Yang-Mills theories as described in Refs. 16-19. Supposing that the effect of confinement does not destroy the chaotic fluctuations induced by the glue self-couplings, we imagine the glue energy to be distributed over a large number of virtual modes. In that case the valence quarks can be imagined to lie in an effective heat bath. Even though the (classical) motion is deterministic, there will exist a coarse-grained entropy and temperature. In this manner one might be able to justify otherwise dubious attributions of temperature to an energy eigenstate, here the proton. It has been noticed that structure functions do resemble thermal distributions[25] but no prior ideas have been produced to justify such a picture. In a nucleus, the effect of binding will be to decrease the effective temperature and also to change the effective size of the nucleons. (Hagedorn and Rafelski[26] have shown how the EMC effect can be fit by a 20% increase in nucleon radius in a thermal model of the hadron.)

Fluctuations in the energy share of hadronic constituents are experimentally testable, extremely important and probably already gathering dust on data tapes. What is needed is event-by event analysis of the asymmetry of two identified leading particles, with momenta P_R and P_L. Besides the invariant mass of the residual system ("fireball," call it)

$$(P_R - P_L)^2 = M^2 \tag{3.1}$$

we need the rapidity distribution Y of the fireball.

To put the possibilities in a vivid light, we give a (simplified) but instructive history of a p-p collision in the Pokorski-Van Hove model. We begin with the observation that there is an ordering of cross sections[27] in QCD Born terms

$$\sigma_{GG} \gg \sigma_{Gq} \gg \sigma_{qq} \tag{3.2}$$

This certainly overstates the case; what is really certain, however, is the ratio of color weights in the final state, i.e.,

$$\begin{aligned} \frac{\sigma_{GG}}{\sigma_{Gq}} &\sim \frac{8\times 8}{8\times 3} = \frac{8}{3} \\ \frac{\sigma_{Gq}}{\sigma_{qq}} &= \frac{8\times 3}{3\times 3} = \frac{8}{3} \end{aligned} \tag{3.3}$$

Eq. (3.2) may then be regarded as the "large 8/3 approximation."

Before giving cautionary remarks, consider the intuitive space-time picture given in Fig. 6 for a nearly head-on collision. Because of (3.2) the glue is stripped from the valence quarks which then reconstitue themselves as diffractively excited objects in a characteristic time γ/Λ. The bulk (nondiffractive, nonjet) hadronization is then due to the degradation of the gluon energy of motion into soft modes.

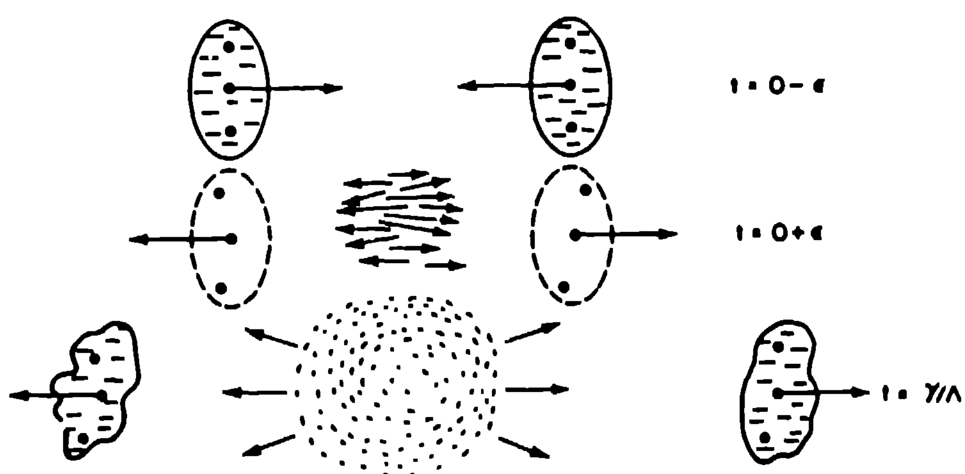

FIGURE 6
This cartoon shows stages in a proton-proton collision. (The dashed lines denote virtual glue.) At $t = 0 + \varepsilon$ the confining glue has been stripped leaving bare quarks escaping the region wherein the gluon clouds are streaming through each other. At a later time $t \simeq \gamma/\Lambda$ the bare quarks have reconstituted themselves while the degraded gluon cloud is evolving towards hadronization into many quanta.

(1) It is completely natural in this picture that the leading particles should have about ½ the total initial energy. (An important topic not addressed here is the apparent increase in this fraction ongoing from ISR to $\bar{p}p$ collider energies.)

(2) In this model the collision of the preexisting (confining) glue clouds dominates. Hadrons are transparent to the valence quarks, with radiation of glue (or photons) from quarks playing a minor role.

(3) Diffractive excitation can be color coherent. Although $\underline{8}$, $\underline{10}$ (and higher dimensional) color configuration are possible, with strings attached to the "fireball," color tubes are not compulsory (in contrast to e^+e^- annihilation) and could be negligible.

(4) Universality of hadronic multiplicities and their distributions is natural due to the passive role of the valence quarks in this model.

(5) Nondiffractive soft hadronization depends only on the "available" energy left behind by the leading particles

$$W_{had} = fW \tag{3.4}$$

Failure to take account this elementary fact has led to a multiplicity literature full of confusion.

(6) As mentioned above, f will differ event by event. <u>It is extremely important to know P(f) in order to learn more about the structure of the proton and internal fluctuations of the proton.</u>

Further discussion of this model can be found in Ref. 35. At present the picture is mainly qualitative and requires much work to be promoted from model to theory.

3.2. What Happens When the Gluon Clouds Collide?

Accepting the notion that the principal event in a typical hadronic collision is the collision of the glue clouds, we are faced with the problem of describing the dynamical evolution of this system. In the past the most common approaches belong to two extreme limits:

(a) Thermalization in the Lorentz-contracted Fermi-Landau volume (strong interaction limit),

(b) perturbative, now leading log calculations of quasi-soft processes (weak coupling limit).

There is a third point of view, which can be traced back to Heisenberg,[28] and which we think has merit, as discussed below:

(c) Nonlinear field couplings transform kinetic energy to internal energy in a manner similar to fluid turbulence.

In our view, elements of each of these viewpoints are necessary in a correct picture. The conceptual problem is to figure out when to emphasize one view or the other. To discuss this we consider space-time to be dissected by hypersurfaces [the simplest[29] are hyperboloids $\tau^2 = t^2 - \underline{x}^2 = \tau_0^2$; a familiar one in the Landau model is the hadronization surface defined by $T(t,\underline{x}) = M_\pi$].

We speak of a sequence of stages, which may depend on momenta, as well as other variables.

(a) Initially, the overlap of virtual glue will not be describable by on-shell kinetic theory (e.g., with cross-section determined rates). Instead one envisions the interpenetration of two streaming glue clouds, dominated by low-momentum transfer amplitudes. In a perturbative description, one expects a web of branching (three-point) vertices degrading the initial momenta into a large number of soft gluons. During this phase, it may be that few quarks and antiquarks are created.

(b) At a certain stage, mass shell kinetic theory becomes appropriate. Since the system is highly nonuniform and expanding, numerical predictions will not be easy. (This is where the covariant phase space distributions advertised in the introduction could be useful.) Plasma computer simulation techniques may be of use in this regard. Although the system may be far from equilibrium, the number of variables necessary for its description is drastically reduced (for example to one- and two-particle distribution functions, complicated by the existence of lots of quantum numbers). <u>Hydrodynamic behavior may exist without thermodynamic equilibrium</u>.

(c) If the system does not separate, experience shows that the kinetic system will establish a local equilibrium. It is at this point that many people get cold feet with regard to attainment of LTE in <u>any</u> stage of a hadronic collision. The larger space-time arena offered in A-A collisions seems safer. We have an open mind on this issue, believing that experiment will provide the answer. Note that several kinds of equilibrium can be contemplated. First of all the gluons could thermally equilibrate on a time scale such that equilibrium has not yet been established with $q\bar{q}$ pairs. Also the strange quark pairs are expected to appear later as a result of mass suppression.

(d) After the foregoing evolution through the infinite dimensional phase space, we can hope (locally) to have arrived "on" the phase diagram of equilibrium statistical mechanics. In that case we can use hydrodynamics supplemented by microscopic transport coefficients. Remember, however, that the <u>internal</u> motions due to the quasi-particle spectrum is occurring and may be very important for observable signals. In addition, chaotic turbulent behavior of the highly excited system (having significant vorticity) seems likely.

(e) Entropy and Hadronization. Contemporary QCD hadronization predictions are based on a combination of largely empirical fits and prayer. Therefore, it would seem useful to modernize one of Landau's especially ingenious ideas, namely to emphasize the privileged role of the entropy in the hydrodynamic evolution in his model.[3] Recall that although the numbers of species (and even their type) change in the evolution, the entropy four current is conserved apart from latent heats at first order phase transitions and irreversible effects (hopefully calculable). In Landau's theory there is a hypersurface of separation on which the final hadronic populations are just becoming free, though in equilibrium. In this circumstance the number of quanta (expanded now in hadron coordinates) is $\frac{1}{4}S_f$ where $S_{final} \equiv S_f$ is the same as $S_{initial} \equiv S_i$ defined on whatever hypersurface equilibration sets in.

It would seem extremely useful, therefore, to try to extend the entropy concept to the field theoretic domain. As shown by Boltzmann, the entropy is very useful in kinetic theory regardless of questions of local equilibrium. For example, for probabilities p_i, we have the prototype formula

$$S = - \sum_i p_i \,\ell n\, p_i \tag{3.5}$$

and in quantum theory we can use $-\mathrm{Tr}(\rho \,\ell n\, \rho)$ with ρ the density matrix.

Next consider two estimates of entropy production derived from the hydrodynamical model but corresponding to two different initial conditions. First we consider the original Fermi-Landau boundary condition, i.e., instant thermalization in the Lorentz contracted volume. Using free thermodynamic functions, we found[35]

$$S_f = 7.4\, N_d^{\frac{1}{4}} f^{\frac{1}{4}} W_{had}^{\frac{1}{2}} \tag{3.6}$$

where f was defined in Eq. (3.4).

which translates to a charged multiplicity

$$N_{ch} \simeq 2.46\, f^{1/4} W_{had}^{1/2} = 2.46\, W_{had}^{3/4}/W^{1/4} \tag{3.7}$$

Using the average f of 0.4-0.5, we get $N_{ch} \simeq 2\, W_{had}^{\frac{1}{2}}$, which is a rather spectular prediction, good to within 10%. However, the $f^{\frac{1}{4}}$ dependence allows one to check on the separate W, W_{had} dependence. The data[30] of Basile et al. in fact conflict with the $f^{\frac{1}{4}}$. Although recalibrations for distinct ISR energies may be problematic, we are left with the likely failure of the initial geometry but a lingering puzzle regarding the otherwise good agreement.

Continuing doubts concerning the assumptions underlying the Fermi-Landau initial condition have led to the formulation of an alternative one, based on Bjorken's "inside-outside" cascade model.[29] Here the early stages are declared intrinsically quantal, with hydrodynamic evolution occurring within a two-dimensional (positive time) hyperboloid $t^2 > \tau_0^2$. The equations of motion are as before but with an injection of energy momentum on the hyperboloid $t = \tau_0$ according to[31-32]

$$\partial^\mu T_{\mu\nu} = \varepsilon_0(y)\, u^\nu \delta(t - \tau_0) \tag{3.8}$$

The variables are $\tau^2 = t^2 - x^2$, $y = \frac{1}{2} \ln (t + x)/(t - x)$; $\varepsilon_0(y)$ is an input energy density as a function of fluid rapidity y. τ_0 is imagined to be of order 1 f and $\varepsilon_0(y)$ is estimated from observed hadronic rapidity densities.

Having waited until a later (cooler epoch) to begin the hydrodynamical evolution, it is of interest to verify that the correct entropy (hence multiplicity) is predicted by this approach. Projecting Eq. (3.8) on the four-velocity u^ν, we find the entropy flow

$$\frac{u^\nu \partial^\mu T_{\mu\nu}}{T} = \partial^\mu s_\mu = \frac{\varepsilon_0(y)}{T} \delta(t - \tau_0) \tag{3.9}$$

integration gives for the entropy production

$$S = \frac{\varepsilon_0 \tau_0}{2T_0} \cosh^{-1} \frac{t_{had}}{\tau_0} \cong \frac{\varepsilon_0(y)\ \tau_0}{2T_0} \ln (2t_{had}/\tau_0) \tag{3.10}$$

where $t_{had} \simeq \tau_0\ \gamma$ is the typical hadronization time. $\varepsilon_0(y)$ has been estimated[4] to be ($A_\perp$ is the colliding area and $m_\perp$ the transverse mass)

$$\varepsilon_0(y) \simeq \frac{\langle m_\perp \rangle}{\tau_0 A_\perp} \frac{dN}{dy} \tag{3.11}$$

or

$$\varepsilon_0(y) \simeq \left[\frac{1}{1.6\tau_0 A_\perp} \left(\frac{dN}{dy}\right)\right]^{1+c_0^2} \tag{3.12}$$

by Bjorken[4] and Gyulassy and Matsui.[33] For $c_0^2 = 1/3$ the distinction is not important here so we use (3.11). The multiplicity then becomes (removing $A_\perp$ to get to the uniform two-dimensional case)

$$N \simeq \frac{\langle m_\perp \rangle}{4T_0} \cdot \frac{dN}{dy} \cdot \ln \frac{W}{m_p} \tag{3.13}$$

dN is well fit by $0.5 \ln W/m_p$ from ISR to collider energy. The multiplicity then varies as

$$N \simeq K \ln^2 s \tag{3.14}$$

with s measured in GeV^2, and $K \simeq 0.25$. A reasonable experimental value is $K \sim 0.72$. Considering uncertainties, (3.14) is in reasonable accord with data, given all the uncertainties.

(f) Threshold for Disappearance of Cascades in A-A Collisions. Consider an idealized A-A collision along the lines of the Pokorski-Van Hove model. If the energy is high enough, the stripped valence quarks cannot reconstitute their glue clouds in time to encounter another strong stripping (although 3/8 is not comfortably close to zero for successive collisions!) Question--what is the threshold for the suppression of exponential cascading, or more colloquially the onset of transparency? (At this meeting we have seen the reputation of transparency clouded by Busza's new data.)[34] To answer this we compute the energy for which the dressing distance equals the length of traversed nuclear matter. Using $R = r_0 A^{1/3}$ ($r_0 \cong 1.4$ f) we get

$$\frac{2R}{\gamma} \simeq \frac{\gamma}{\Lambda} \quad (3.15)$$

Since $\gamma = E/m_p$ for a nucleon we find

$$E_{c.m.} \gtrsim 2A^{1/6} \text{ GeV} \quad (3.16)$$

We note that Eq. (3.15) has been derived in a different context by Gyulassy.[34] The coefficient is crude but the $A^{1/6}$ dependence is characteristic. For pA collisions the coefficient decreases by $\sqrt{2}$.

Perhaps this prediction has already been disproved. Regardless of its merit, as long as one speaks of "transparency" the question posed is interesting.

4. THE STOCHASTIC CELL MODEL OF MULTIPLICITY DISTRIBUTIONS; FRACTAL DYNAMICS

Everyone agrees on one thing--there will be lots of particles produced in relativistic A-A collisions. The number produced in accelerator pp or $\bar{p}p$ collisions is impressive enough, with an average of 12, 29 charged particles at 63 GeV, 540 GeV c.m. energies. What is more, the multiplicity distribution has a long n tail with significant population even at $z = n/\bar{n} \sim 3$. For a 100 on 100 GeV per particle collider, the nucleon-nucleon average charged multiplicity will be about 20. Scaling this by your favorite A dependence creates a horde of particles which give detection problems, but also inspires the hope of creating in the laboratory dense concentrations of hadronic matter.

Every popular model for total multiplicities is model dependent to varying degree. Most common are

$$\begin{aligned}
&\text{lns} && \bar{n}_{ch} = A + B\,\ell ns + C(\ell ns)^2 + D(\ell ns)^3 + \ldots \\
&\text{SHM} && \bar{n}_{ch} = KS^p \qquad p \simeq \tfrac{1}{4}\;,\; K \simeq 2 \qquad\qquad (4.1)\\
&\text{QCD} && \bar{n}_{ch} = a + b \exp\left[c \sqrt{\ell n\,(s/d)}\,\right]
\end{aligned}$$

There are various unresolved problems, even for data analysis. The first question is: what is the correct s to use in 4.1? In Sec. III we argued that for bulk hadronization at high energy we should remove two units of charge for leading particles and (1 - f) W for the energy residing in the leading particles. Therefore in (4.1) we suggest

$$\bar{n}_{ch} \to \bar{n}_{ch} - 2$$
$$s^{1/2} \to fW \tag{4.2}$$

(Still, f should vary event by event, a fact not properly accounted for in 4.1.) Once this correction is made each of formulas (4.1) is numerically satisfactory through collider energies.[36] [Note that the rescaling of $s^{\frac{1}{2}}$ in (4.2) rearranges terms in the log expansion of 4.1.]

Different information should be contained in the multiplicity distributions $P_n = \sigma_n/\sigma_{in}$ where σ_n and σ_{in} are the n-prong and inelastic cross sections, respectively. The shapes of P_n vs. n are rather different for hadron production in hadron-hadron, lepton hadron and e^+e^- collisions. Yet all seem to obey the scaling property

$$\bar{n}P_n \sim \psi(n/\bar{n}) \tag{4.3}$$

known as Koba-Nielsen-Olesen scaling[37] (KNO).

Many dynamical/geometrical models have been brought forth to explain (4.3), which was originally explained by Feynman scaling. The latter works less well than (4.3), an observation which has recently inspired considerable interest.

Recently we noticed[38-39] that a quantitative description of hadron-hadron multiplicity distributions followed from a model-independent stochastic theory borrowed from quantum optics. This stochastic cell model has two basic assumptions:

(1) The emitting system can be partitioned into k independent sources;

(2) A simple statistical distribution is assigned to the emitting fields. For hadron-hadron collisions one assumes Gaussian random variables for cells of equal intrinsic strength.

In this case the counting distribution is the negative binomial or generalized Bose-Einstein

$$P_n^k = \frac{(n + k - 1)!}{n!(k - 1!} \; \frac{(\bar{n}/k)^n}{(1 + \bar{n}/k)^{n+k}} \tag{4.4}$$

and the asymptotic form is very simple:

$$\bar{n}P_n^k \sim \psi_k(z) = \frac{k^k}{(k - 1)!} \; z^{k-1}e^{-kz} \tag{4.5}$$

Fig. 7 shows an example of fit; consult Refs. 38-40 for more detailed information and also the modified density matrix needed to describe the narrow e^+e^- hadron distributions. Here we only note that the number of cells (which can be an average and not necessarily an integer) lies between 3 and 4 as we go from Fermilab to $\bar{p}p$ collider energy.

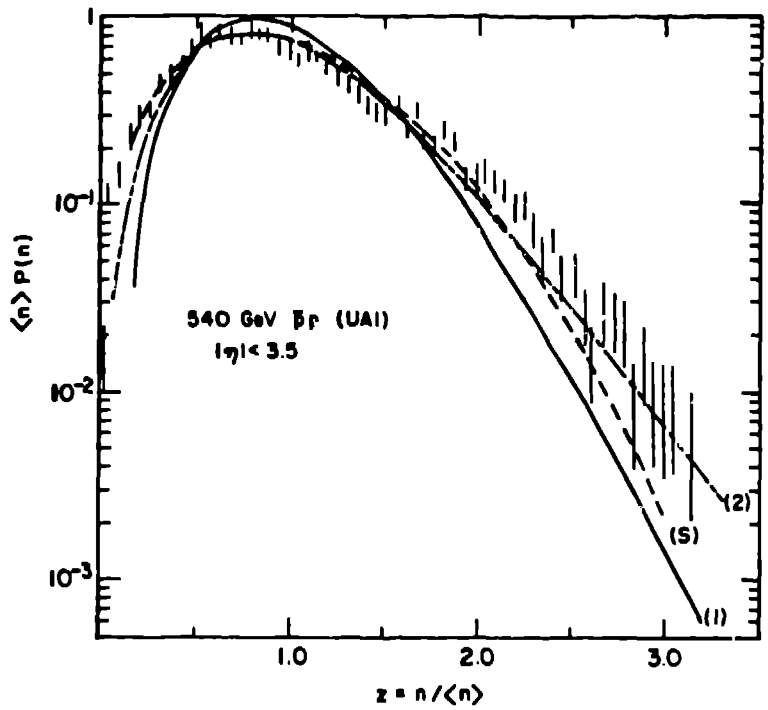

FIGURE 7
The KNO plot for UA1 collider data is shown for the original Slattery fit (Fermilab energy) and formula (4.5) for k = 4 [labelled (2)] and k = 6 [labelled (1)]. We have used n_{ch} = 20 corresponding to the indicated pseudo rapidity interval $|\eta| < 3.5$. Data sources are given in Ref. 38.

A slow increase of k with increasing $\bar{n}$ will show up as a sharpening of the KNO plot (i.e., a systematic but slow violation of energy scaling). A particular prediction of this will be given below, for Desertron pp energies (40 TeV in c.m.) and for A-A collisions. Except for an ever-present large n tail, the central multiplicity may become Poisson in character with increasing A. Hence the A dependence of the KNO plot snould provide a sharp test of theories of the multiplicity distributions.

In the stochastic cell model, the main question is to interpret its one parameter--the number k of abstract cells. Traditionally one would think of a small number of emitting fireballs or (super) clusters. In our opinion such an approach is not an explanation but only provides an overly simplistic name to the emitting object. We believe that the cells possess a dual role: that of labeling independent ergodic cells in phase space (having topological structures, very likely) and also characterizing simple dynamical maps which contain the essence of the full problem.

We recently suggested[41] that three very different phenomena--hadronic multiplicity distributions, galaxy counts in clusters and turbulence--share a common fractal dimension (2.6). These systems are essentially self-similar, dissipative and proceed through high excitation. This result provides a strong hint towards an eventual description of highly excited but perhaps non-equilibrium QCD matter.

For dealing with such problems we replace the infinite number of degrees of freedom homomorphically with a low-dimensional map (this is where the hard work lies!). Although the simplest one-dimensional cascade (basically the Cantor set) is very suggestive of the observed universality, generally we will expect a more structured map, with an associated strange attractor of nonintegral fractal dimension. For such sets continuity is the exception and beautiful fractal structures (composed by self-similar insertions) as discussed by Mandelbrot[42] probably exist in the complex many-body dynamical systems, imposing themselves on prominent features of the data as described above. Finally it seems likely that many of these results can be obtained without the necessity of thermal equilibrium.

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