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Determination of Plasma Shape from Poloidal Field Measurements on ISX-B

D. W. Swain
S. Bates
G. H. Neilson
Y-K. M. Peng

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FIELD MEASUREMENTS ON ISX-B

D. W. Swain, S. Bates, G. H. Neilson, and Y-K. M. Peng

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ABSTRACT

The ISX-B tokamak has a poloidal coil system designed to produce circular, elliptical, and D-shaped plasmas. Plasma shape and low-order multipole moments of the plasma current distribution are determined from experimental measurements of B_Z , B_R , and/or ψ around the periphery of the vacuum chamber. The experimental arrangement and method of analysis of results, using a least squares method to fit the data points to a finite current filament model, are described in this report. Plasma shape results for circular and D-shaped plasmas with $b/a \leq 1.5$ and an analysis of the sensitivity of the technique to measurement errors are presented. The results indicate that this method gives accurate measurements of the plasma boundary and is relatively insensitive to errors.

1. INTRODUCTION

The ISX-B tokamak^{1,2} (see Fig. 1) is designed to contain circular, elliptical, and D-shaped plasmas, with elongations of $\epsilon \equiv b/a \lesssim 1.8$. Two neutral beam injectors can put up to 1.8 MW of beam power into the plasma, which creates a high beta plasma with unique properties. To assist in performing experiments on high beta equilibrium and stability, it was desired to make a diagnostic system that could provide information on plasma position, shape, and pressure as a function of time within 1 or 2 min after a shot.

This report describes a system that uses measurements of poloidal magnetic fields and poloidal flux to indicate plasma shape and position. Due to the requirement of rapid analysis, it appeared impractical to implement a full free-boundary MHD equilibrium code to do the analysis; therefore, a simpler model of the plasma current distribution has been developed. The continuous plasma current distribution is replaced in the model by a finite set of filamentary current elements (usually six) at fixed positions in the plasma volume. The currents in the filaments are determined by a linear least squares fit of the fields derived from them to the measured fields. As will be shown, this model gives results that are in excellent agreement with free-boundary MHD calculations. The data are analyzed on a PDP-10 computer in the time-sharing mode, and the program is capable of giving results in the required time. This technique is insensitive to small errors in measurement, and due to the redundancy of the measurements (i.e., many more measurements of ψ , B_R , or B_Z are made than are necessary to determine the filamentary currents), the analysis provides an estimate not only of the position and shape but also of the uncertainties in these quantities.

The outline of the technique and the setup of the experimental apparatus are described in Sect. 2, with typical results of the analysis given for several cases. Section 3 contains a sensitivity analysis of the method to errors of measurement or possible deficiencies in the theoretical computations of the fields (ISX-B has an iron core that makes the calculation of fields and flux considerably more difficult and uncertain than in an air-core tokamak). The results are summarized

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Fig. 1. Cross section of the ISX-B tokamak.

in Sect. 4. For completeness, details of the least squares fitting technique, error analysis, and calculations of iron-core effects are included in Appendixes A-C.

2. THE MEASUREMENT AND ANALYSIS TECHNIQUE

2.1 THE EXPERIMENTAL SETUP

The cross section of the tokamak in Fig. 2 shows the poloidal coils, a set of 36 magnetic field sensors, and a typical set of filaments used to model the plasma. The poloidal coils are divided into four sets: inner, outer, shaping, and radial coils. All coils in each set are connected in series; hence, the currents in all the coils in a given set are equal. Currents in each set are measured and stored digitally at 1-ms intervals during a shot (usually 100-200 ms long).

The plasma current and position (both vertical and horizontal) are feedback controlled. Relative changes in vertical position are computed from two magnetic probe measurements of B_R (one at the top and one at the bottom of the vacuum vessel). The vertical displacement signal is processed by an analog feedback circuit that sends an error signal to the radial field power supply. The radial field supply changes the current in the radial coils, creating a radial field that interacts with the plasma current to provide a vertical restoring force to center the plasma.

In a similar manner, the plasma current and horizontal position are held constant. However, the process is complicated by the unconventional poloidal coil system in ISX-B. The plasma current and the vertical field in the plasma can be written as

$$I_P = -N_1 I_1 - N_2 I_2 - N_3 I_3 \quad (1)$$

and

$$B_V = C_1 I_1 + C_2 I_2 + C_3 I_3, \quad (2)$$

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[I] = INNER POLOIDAL COILS
 [O] = OUTER POLOIDAL COILS
 [S] = SHAPING POLOIDAL COILS
 [R] = RADIAL POLOIDAL COILS

↑ = B_z SENSOR COIL
 → = B_R SENSOR COIL
 ● = PLASMA FILAMENTS

Fig. 2. ISX-B poloidal coils, magnetic sensors, and typical locations of plasma filaments.

where

- I_1 = current in the inner coil set,
- I_2 = current in the outer coil set,
- I_3 = current in the shaping coil set, and
- I_4 = current in the radial coil set.

Because the radial coil current directions are antisymmetric about $Z = 0$, I_4 does not affect either I_p or B_V (to lowest order). The current in the shaping coils (I_3) is preprogrammed to be a given function of time. A feedback system that senses both plasma current (I_p) and horizontal position (Δ_R) changes I_1 and I_2 [using Eqs. (1) and (2)] to maintain I_p and Δ_R at the desired values.

The magnetic loop/flux loop signals are processed by the system shown in Fig. 3. At present, we have data from ≤ 35 magnetic loops. The integrated signals from the sensors are digitized at 1-ms intervals throughout the shot and are stored, with current signals and other diagnostic data, in a PDP-10 computer where they can then be analyzed.

2.2 METHOD OF ANALYSIS

The problem is the following: given a set of coil currents I_1 - I_4 , the plasma current I_p , and a set of measurements of B_Z , B_R , and/or ψ at discrete points around the boundary, we wish to determine the position and shape of the outermost plasma flux surface and the moments of the plasma current distribution. We assume that the plasma is in equilibrium and that currents are not changing rapidly with time (i.e., a quasi-steady state). The problem is thus reduced to a magnetostatics problem; no information is required (or given) about variables such as plasma pressure, "internal inductance," etc. Further theoretical input (for example, a radial force balance equation) will allow some additional unknowns (such as " $\beta_p + \ell_1/2$ " of Shafranov) to be determined. However, a force balance equation for a high beta [$\beta_p \sim 0(A)$, where A = aspect ratio], noncircular plasma with an anisotropic pressure tensor due to massive neutral beam injection does not yet exist in a tractable form.

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Fig. 3. Data collection system.

The method of solution to the problem is outlined below:

(1) The plasma current distribution is modelled by a finite number of functions $f_i(R, Z)$ with undetermined coefficients α_i :

$$j_p(R, Z) = \sum_{i=1}^n \alpha_i f_i(R, Z) .$$

The functions f_i can take many forms. One of these forms is a multipole moment expansion in toroidal geometry. Another choice might be a "smooth" set of functions of R and Z that simulates a realistic current density distribution. A third form, which we have adopted here, is to simulate $j_p(R, Z)$ by a set of N filamentary currents located at predetermined points in the vacuum chamber; i.e.,

$$j_p(R, Z) = \sum_{i=1}^n \delta(R - R_i) \delta(Z - Z_i) I_{Pi} ,$$

where I_{Pi} = current flowing in the i th filament. The two reasons that this model is being used are (1) it is simple to calculate the fields at a sensor due to the filament currents and (2) the results from this choice, using six filaments, have been found *a posteriori* to give an excellent agreement with a free-boundary MHD equilibrium calculation. This point will be demonstrated in Sect. 3.

(2) The response matrix Q_{ij} is calculated where

$$y_i = \sum_{\text{coils}} Q_{ij}^C I_j + \sum_{\text{plasma}} Q_{ij}^P I_{Pj} \quad (i = 1 \text{ to } N_{\text{sensors}}) . \quad (3)$$

y_i is the field quantity (either B_Z , B_R , or ψ) measured by the i th sensor. This equation is the basic one in the magnetostatics computation. We implicitly assume that the fields measured by the sensors are due to known currents (I_1 - I_4) in the external poloidal coil circuits and to a set of currents simulating the plasma (I_{P1} - I_{PN}) of unknown magnitudes

but prescribed locations. (In Sect. 3 we show that the calculated boundary is relatively insensitive to the locations chosen, as long as they are not too close to the boundary.) All of the geometry of the problem, including coil locations and configurations, the effect of the iron core, aspect ratio, etc., are contained in the calculation of the Q matrix. For an air-core machine, Q is readily calculable as combinations of elliptic integrals. In an iron-core machine, the presence of the core modifies the calculation of the fields. The model used to calculate the iron-core contribution is discussed in Appendix C.

(3) The unknown currents (I_{P1} - I_{PN}) are determined by a weighted least squares fit; that is,

$$\chi^2 = \sum_{\text{sensors}} \frac{(y_i - \hat{y}_i)^2}{\sigma_i^2} \quad (4)$$

is minimized where y_i = the fields calculated using Eq. (3), $\hat{y}_i$ = measured field at sensor i, and σ_i = estimated error associated with the measurement at sensor i.

(4) The values of I_{P1} - I_{PN} obtained from step (3) are used to find the location on the limiter at which the calculated value of ψ is a minimum. This is the plasma edge flux value. The convention assumed here is that ψ is minimum at the magnetic axis of the plasma and increases as the plasma edge is approached. The curve defined implicitly by $\psi(R,Z) = \psi_{\min}$ is traced to determine the location and shape of the outermost plasma flux surface.

(5) Approximate values for the moments of the plasma current distribution are determined from the magnitudes and locations of I_{P1} - I_{PN} . In particular, the centroid of the current distribution

$$R_I = \sum R_j I_{Pj} / \sum I_{Pj} \quad (5)$$

and

$$Z_I = \sum Z_j I_{Pj} / \sum I_{Pj}$$

can be calculated.

This procedure determines the plasma position and low-order current moments. However, moments of the current distribution (even if many are known) do not allow the unambiguous determination of $j_p(R,Z)$ because there is no unique unfolding of $j_p(R,Z)$ from the moments.

There are two important variations of the method that can be used to provide a self-consistency check on the calculations. First, all currents (including coil currents) can be calculated from the sensor measurements and the coil currents compared with their measured values. Second, the plasma current ($I_p = \sum I_{Pj}$) can be constrained to equal the measured value of I_p and the results compared with the unconstrained calculation. These variations, when compared with the original calculations, provide a simple way of checking to see that the results are sensible. (The results are discussed in more detail in Appendix A.)

2.3 EXAMPLES

Figure 4 shows results for an ohmically heated discharge. Figure 4a shows the plasma horizontal shift $\Delta_H(t)$ calculated by the analog position control circuit (solid curve) and the plasma position determined by the method of analysis described above at four times during the discharge. The plasma position was deliberately moved inward 4 cm starting at $t = 80$ ms; plasma current remained constant at $I_p = 118$ kA.

Figures 4b-4e show the results of the plasma shape calculations at each time. The calculated outermost flux surface is shown by the solid curve. The plasma is resting against the inner limiter (at $R = 0.66$ m) for all the times and fills an area considerably smaller than the area available. The elongation for this case (low β , no current in the shaping coils) is $\epsilon = b/a \approx 1.2$. The circles shown inside the flux contours are the locations of the six filamentary currents used to model the plasma current.

PLASMA SHAPE AT 4 TIMES IN THE SHOT

Fig. 4. Data from a typical shot. Top: Plasma position (analog calculation) and density. Plasma is shifted in 4 cm starting at $t = 80$ ms. Bottom: Calculated plasma edge at four times. Dots are locations of filaments used in the calculation.

Figure 5 is a plot of $B_Z(s)$ and $B_R(s)$ at $t = 120$ ms in the same shot. The quantity s is the normalized arc length around the rectangle on which the sensors are located. The agreement between measured and calculated values is good; the rms average error is $<5\%$ of the rms average field.

Measurements from a shot with active plasma shaping are shown in Figs. 6 and 7. In this experiment the plasma was started up and allowed to come to an equilibrium without any shaping. At $t = 50$ ms, the current in the shaping coils was turned on, changing the elongation from $\epsilon \cong 1.2$ to $\epsilon = 1.4$. The comparison between measured and calculated $B_Z(s)$ and $B_R(s)$ is shown in Fig. 7. There is a substantial change in both measured and calculated $B_R(s)$ when the shaping coils are energized. As in the previous example, agreement between the measured and calculated values is very good.

3. SENSITIVITY TO ERRORS

The results of the previous section indicate that the technique of simulating the plasma by six filamentary currents can yield good agreement between measured and calculated fields. In this section we attempt to evaluate the reliability with which the method will compute an accurate outermost flux surface. In particular, we will consider four questions:

- (1) How well does the calculation using six current filaments compare with a full free-boundary MHD equilibrium calculation?
- (2) How sensitive is the computed plasma boundary to the locations of the filamentary currents?
- (3) How sensitive is the computed boundary to errors in measuring coil currents and magnetic fields?
- (4) How reliably can the fields due to known currents be calculated, and do the measured and calculated fields agree in cases when all currents are known (is the model for the iron core satisfactory)?

Fig. 5. B_z and B_R vs normalized arc length(s) around the vessel at $t = 120$ ms. (a) Diagram illustrating definition of s , (b) comparison of measured and calculated B_z vs s , and (c) comparison of measured and calculated B_R vs s .

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Fig. 6. Calculation of plasma edge for a shot with active shaping.

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Fig. 7. Comparison of measured and fitted values of $B_R(s)$ and $B_Z(s)$ for the two times shown in Fig. 6.

3.1 COMPARISON WITH MHD EQUILIBRIA

A free-boundary MHD equilibrium code that models the ISX-B coil/plasma system³ was modified to provide as output the information needed as input to the fitting program (namely, coil currents and locations and sensor measurements and locations). These data were then analyzed using the fitting program. Figure 8 is a comparison between the plasma boundary computed from the full MHD equilibrium code (solid curve) and the fitting program for two cases. The results indicate that the filamentary model is capable of calculating the plasma edge in very close agreement with the MHD equilibrium calculation.

The speed and ease of the filamentary model permit analysis of data between shots, with little need for active operator interaction. The main disadvantage of this fitting method is that although it provides accurate magnetostatics information (i.e., plasma shape, current moments), it does not give any information about the plasma pressure profile or the flux surfaces inside the plasma.

3.2 SENSITIVITY OF COMPUTED BOUNDARY TO FILAMENT LOCATIONS

Figure 9 shows the plasma boundary computed using four different locations for the filaments. Each computation used the same values of coil currents and measured magnetic fields. In the first case shown, the filament locations are too close to the plasma boundary because local bulges in the boundary can be seen. However, the results from the other three cases agree with each other very well. Table 1 lists the computed filamentary currents, plasma boundary parameters, and current centroid locations. The boundary is fairly insensitive to the filament locations because the least squares algorithm accounts for a change in location by a redistribution of current in the filaments.

3.3 SENSITIVITY TO ERRORS IN COIL CURRENTS

Figure 10 shows the plasma position calculated with the current in the inner coil varied. In the case shown, the outer coil current I_2 was measured to be 11.0 kA and the measured inner coil current I_1 was

Fig. 8. Comparison of the plasma edge calculated using a free-boundary equilibrium code and the filamentary model. (a) No shaping, $b/a \approx 1.1$ and (b) active shaping, $b/a \approx 1.6$.

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SHOT 10426 $t = 120$ msec

Fig. 9. Effect of varying filament locations on the calculated plasma edge.

Table 1. Comparison of plasma boundary calculation for different filament locations (see Fig. 9)

Filament location and current	Case			
	a	b	c	d
R_1	105	105	100	100
Z_1	0	0	0	0
I_1	-19.9	-2.3	-15.5	-4.1
R_2	98	98	95	100
Z_2	20	13	15	15
I_2	-15.5	-28.4	-23.5	-14.0
R_3	82	82	85	90
Z_3	20	13	15	15
I_3	-10.6	-10.5	-3.4	-8.9
R_4	75	75	80	85
Z_4	0	0	0	0
I_4	-22.6	-12.3	-24.1	-42.1
R_5	82	82	85	90
Z_5	-20	-13	-15	-15
I_5	-18.2	-19.9	-15.8	-22.8
R_6	98	98	95	100
Z_6	-20	-13	-15	-15
I_6	-10.3	-23.6	-14.7	-5.2
<u>Plasma edge</u>				
R	90.4	90.2	90.3	90.4
Z	-0.8	-0.8	-0.8	-1.3
b/a	1.42	1.40	1.43	1.43
δ	-0.20	-0.18	-0.17	-0.15
<u>Current</u>				
R_j	89.3	90.2	90.1	90.4
Z_j	-0.5	-0.6	-0.6	-0.8

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Fig. 10. Effect of varying the inner coil current (I_1) on the calculated plasma edge.

5.0 kA. The corresponding plasma edge is shown by the solid curve. The other curves indicate the edge computed with all magnetic sensor signals the same, but with I_1 changed by ± 1.0 kA ($\pm 20\%$). The plasma center changes in both cases, but not dramatically. The estimated error in coil current measurements is $< 5\%$, so these results indicate that errors in the inner coil current of the size expected will not cause a significant error in the calculated plasma shape. Similar effects are also observed for errors in the outer coil current.

The squared error [χ^2 in Eq. (4)] in the three cases shown in Fig. 10 changes significantly. Table 2 lists χ^2 for the three cases. Also shown is a case in which the outer coil current is changed.

3.4 SENSITIVITY TO ERRORS IN FIELD CALCULATION

For the least squares fit, we assume that the Q matrix can be calculated exactly, given the locations of currents and sensors. If there is an error in the field calculation, then the entire analysis can yield inaccurate results. The only way to check on the accuracy of the field calculation is to compare it with experiment in which the location and amplitudes of all currents are known (i.e., no plasma current). The ideal procedure would be to put a known current through each coil set individually, with all other coil currents zero, and then compare measured and calculated values. However, this is not practical in ISX-B because it is an iron-core device. Because driving only one coil set quickly saturates the core, and the fields with a saturated versus unsaturated core are quite different, the comparison of fields with only one coil set energized is inappropriate to determine the effect of the iron core under normal conditions. The method chosen is to drive only one coil set with the leads from another set shorted. The current in the energized coils induces a current of opposite sign in the shorted set such that the total number of ampere turns linking the core is ≈ 0 .

Figures 11 and 12 show a comparison between measured and calculated values of B_z and B_R for a case in which the outer coil set was energized and the inner set was shorted. These measurements yield an estimated error of the measured fields

Table 2. Effect of changes in coil currents on weighted errors

$$\chi^2 = \sum_i (y_i - \hat{y}_i)^2 / \sigma_i^2$$

I_1	I_2	χ^2	
5.0	11.0	9.0	measured currents
6.0	11.0	18.0	I_1 changed
4.0	11.0	22.0	I_1 changed
5.0	10.0	37.0	I_2 changed

Fig. 11. Measured vs calculated B_Z for a "shorted" shot in which all currents are known.

Fig. 12. Measured vs calculated B_R for a "shorted" shot.

$$\sigma_i^2 = B_{\text{abs}}^2 + f^2 B_i^2 ,$$

where the absolute error $B_{\text{abs}} \approx 20$ G and the fractional error $f \approx 0.05$. The absolute error can arise from slight misalignments of the sensor coils with the relative error coming from errors in coil sensitivities, integrator calibrations, and ADC conversions. In a field with a B_z component of 400 G (a typical value), a 3° misalignment of a B_R sensor will cause a 20-G absolute error. It is possible that there are also some errors in the calculation of the fields. However, these cannot be distinguished from the expected random errors observed.

The sensors can also pick up changes in the toroidal field if the probes are misaligned toroidally. In practice, the toroidal field is held constant within 1% during a shot, so the errors caused by this effect are small (<10 G for a 3° misalignment).

Figure 13 is a plot of the plasma edge determined including and not including the effects of the iron core. The difference corresponds to a change in plasma position of <2 cm. This indicates that slight errors in calculating the iron contribution are not important in determining the plasma position and shape for the iron core in ISX-B. Unfortunately, this statement cannot be generalized to other iron-core tokamaks, although it may be true in many cases. A more detailed discussion of the way in which the iron-core effects are calculated is given in Appendix C.

The differences between the measured and calculated fields can provide an estimate of the uncertainty with which the plasma boundary was calculated, when subjected to the error analysis described in Appendix B. Figure 14 shows the results of this error analysis, assuming known coil currents. The solid curve is the $\epsilon = 1.4$ boundary shown in Fig. 6; the dashed lines are ± 1 standard deviation of the boundary (see Appendix B).

4. CONCLUSIONS

The method described uses a linear least squares fitting technique to determine the plasma shape and current moments. Several filamentary

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Fig. 13. Effect of the iron core on the calculated plasma edge.

Fig. 14. Calculated boundary (solid curve) ± 1 standard deviation (dashed curves).

currents at prescribed locations (inside the plasma) and unknown magnitudes (determined from the least squares fitting) are utilized to model the continuous plasma current distribution. Investigation of the accuracy of this method indicates the following:

1. Plasma location and shape appear to be accurately computed, barring an unknown systematic error.
2. Calculations of the plasma edge using the six-filament model are in excellent agreement with free-boundary MHD equilibrium calculations and can be done using much less computer time. However, pressure profile information cannot be obtained.
3. The calculated boundary is insensitive to (a) location of plasma filaments, (b) measurement errors in coil currents or fields, and (c) errors in the iron-core model.

In cases where eddy currents in the vacuum vessel, etc., are significant, due to rapidly changing coil or plasma currents, this approach could also be applied if the eddy currents can be properly modelled.

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The contributions of many people have made the data system and analysis technique possible. We would like to thank Russ Overbey for his help in setting up the computer data acquisition system, Tom Tucker for his help in calculating the iron-core effects on the magnetic fields, and Dennis Strickler for his work in implementing and running the ISX-B free-boundary MHD equilibrium code.

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APPENDIX A

Weighted Least Squares Fit with Constraint

At a specified time, we measure a set of M fields $\hat{y}_j$ ($j = 1 - M$); we use the word "field" to refer to the value of B_Z , B_R , or ψ at a given point and a specified time. In general, the y_i will contain a mixture of B_Z , B_R , and ψ measurements. We assume that the vector of measurements Y (of length M) is related to a set of N other parameters x_i ($i = 1, N$) by the relation

$$Y = QX, \quad (A1)$$

where Q is an $M \times N$ matrix that is assumed to be known. In our case the x_i 's are currents, but they could just as well be multipole moment amplitudes, etc. If we have an estimate of the error of each value $\hat{y}_i$, then we can weigh each measurement by a weight w_i [$w_i = (\sigma_i^2)^{-1}$] and do a least squares fit to the data to find X , i.e., minimize

$$\sum_{i=1}^M w_i (\hat{y}_i - y_i)^2. \quad (A2)$$

We may also want to impose an additional constraint on X of the form

$$A^T X = B, \quad (A3)$$

where A is a vector of length N and B is a scalar. In our case this constraint can be used to make the sum of the x_i 's equal to the measured plasma current.

We use the technique of Lagrange multipliers and minimize

$$\epsilon^2 = \sum_{i=1}^M w_i \left(\hat{y}_i - \sum_{j=1}^N q_{ij} x_j \right)^2 - 2\lambda \left(B - \sum_{j=1}^N a_j x_j \right). \quad (A4)$$

Equation (A4) is written as if we do not know any currents in the problem. If we are not solving for the primary currents (i.e., the measured currents in the poloidal coils), then we must break the currents vector X into the currents we know and those we are trying to calculate:

$$\begin{aligned} \epsilon^2 = & \sum_{j=1}^M w_i \left[\left(\hat{y}_i - \sum_{j=1}^P q_{ij} x_j \right) - \sum_{j=P+1}^N q_{ij} x_j \right]^2 \\ & - 2\lambda \left[\left(B - \sum_{j=1}^P q_{ij} x_j \right) - \sum_{j=P+1}^N q_{ij} x_j \right]. \quad (A5) \end{aligned}$$

The currents x_i ($i = 1 - P$) are known, and x_i ($i = P + 1 - N$) are to be solved for. Taking the variation of Eq. (A5) yields $N-P$ equations of the form

$$\sum_{i=1}^M w_i \sum_{j=P+1}^N q_{ij} q_{ik} x_j + \lambda a_k = \sum_{i=1}^M w_i \hat{y}_{Li} q_{ik}, \quad (A6)$$

where $\hat{y}_{Li} = \hat{y}_i - \sum_{j=1}^P q_{ij} x_j$. Define the quantities

$$t_k = \sum_{i=1}^M w_i \hat{y}_{Li} q_{ik},$$

$$S_{jk} = \sum_{i=1}^M w_i q_{ij} q_{ik}.$$

Then Eq. (A6) becomes

$$\sum_{j=P+1}^N S_{jk} x_j + \lambda a_k = t_k \quad (A7)$$

for $k = P + 1 - N$, which must be satisfied with the constraint equation

$$\sum_{j=P+1}^N a_j x_j = B - \sum_{j=1}^P a_j x_j . \quad (A8)$$

If we partition Q , A , and X into primary and secondary components,

$$Q = \begin{bmatrix} j=1,P & j=P+1,N \\ q_{ij}^P & q_{ij}^S \end{bmatrix}, \quad X = \begin{bmatrix} x_i^P \\ - \\ x_i^S \end{bmatrix}, \quad A = \begin{bmatrix} a_i^P \\ - \\ a_i^S \end{bmatrix},$$

then the preceding equations become in matrix notation

$$\hat{Y}_L = \hat{Y} - Q^P X^P \quad (A9)$$

$$T = Q^{ST} \hat{W} \hat{Y}_L \quad (w_{ij} = w_i \delta_{ij}) \quad (A10)$$

$$S X^S + \lambda A^S = T \quad (A11)$$

$$A^{ST} X^S = B - A^{PT} X^P \equiv B^S \quad (B^S = B - A^{PT} X^P) . \quad (A12)$$

Let X_{unc}^S be the unconstrained solution to Eq. (A11); i.e.,

$$X_{unc}^S = S^{-1} T . \quad (A13)$$

The vector X_{unc}^S can be solved by standard matrix inversion methods. Now, explicitly eliminating λ from Eq. (A11) and solving for X^S in terms of X_{unc}^S yield

$$\lambda = \frac{A^{ST} X_{unc}^S - B^S}{a_S}, \quad (A14)$$

where $a_S = A^{ST} S^{-1} A^S$ and

$$X^S = X_{unc}^S - \lambda S^{-1} A^S. \quad (A15)$$

Note that λ is proportional to the amount that the unconstrained solution fails to satisfy the constraint equation (in our case, $a_S \lambda$ = net ampere-turn error of the unconstrained solutions).

The procedure used to find X^S is (1) solve Eq. (A13) to get X_{unc}^S , (2) evaluate λ using Eq. (A14), and (3) find X^S from Eq. (A15). This gives both the unconstrained and constrained solutions, which allows them to be compared. In most cases, λ is small (i.e., the unconstrained solution comes close to satisfying the constraint condition). A large value of λ can be used as a signal that there may be an error in either the measurements or the calculation.

This procedure is used whether solving for all the currents (in which case $P = 0$) or only the secondary currents. Solving for the secondaries is somewhat faster because a 6×6 matrix must be inverted instead of a 10×10 matrix; both are done in a short time on a PDP-10 computer.

APPENDIX B

Error Analysis

Definitions and Formulas

For a vector $X = (x_1 \dots x_n)$, define the covariance matrix

$$\text{Cov}(X)_{ij} \equiv \tilde{X}_{ij} = \langle (x_i - \langle x_i \rangle)(x_j - \langle x_j \rangle) \rangle \quad (\text{B1})$$

where the $\langle \rangle$ denotes an average over many measurements.⁴ The diagonal terms of the $\tilde{X}$ matrix are the variances of the x vector; the (symmetric) off-diagonal terms are the covariances.

For a matrix equation of the form

$$Y = AX,$$

where $\text{Cov}(X) = \tilde{X}$ is known, the covariance matrix for Y is given by

$$\tilde{Y} = A\tilde{X}A^T. \quad (\text{B2})$$

If $Y = X + Z$, then

$$\tilde{Y} = \tilde{X} + \tilde{Z} \quad (\text{B3})$$

if, and only if, X and Z are uncorrelated [i.e., $\langle (x_i - \langle x_i \rangle)(z_j - \langle z_j \rangle) \rangle = 0$ for all i and j].

Assumptions

We assume that errors in the primary current measurements X^P are uncorrelated:

$$\tilde{X}_{ij}^P = \delta_{ij} f_i^2, \quad (\text{B4})$$

where f_i = expected rms error in current x_i^P . We also assume that the errors in $\hat{Y}$ are uncorrelated:

$$\tilde{y}_{ij} = \delta_{ij} e_j^2. \quad (B5)$$

Calculation of Covariances

Use Eqs. (B1)-(B5) to do an error analysis of Eqs. (A9)-(A15). The $\sim$ symbol implies a covariance matrix,

$$\tilde{Y}_L = \text{Cov}(\hat{Y} - Q^P X^P) = \tilde{Y} - Q^P \tilde{X}^P Q^{PT}, \quad (B6)$$

which, using Eqs. (B4) and (B5), yields

$$\tilde{y}_{Lij} = \delta_{ij} e_j^2 - \sum_{k=1}^P q_{ik}^P f_k^2 q_{jk}^P \quad (B7)$$

and

$$\tilde{T} = \text{Cov}(Q^{ST} W \hat{Y}_L) = Q^{ST} W \tilde{Y}_L W Q^S \quad (B8)$$

and gives

$$\tilde{t}_{ij} = \sum_{k=1}^M \left[q_{ki}^S w_k^2 e_k^2 q_{kj}^S - \sum_{\ell=1}^M q_{ki}^S w_k w_\ell q_{\ell j}^S \left(\sum_{m=1}^P q_{km}^P f_m^2 q_{\ell m}^P \right) \right]. \quad (B9)$$

Then the covariance of the unconstrained solution is

$$\tilde{X}_{\text{unc}}^S = S^{-1} \tilde{T} S^{-1}, \quad (B10)$$

because $S^{-1T} = S^{-1}$.

Now if we substitute the expression for λ in Eq. (A14) into Eq. (A15) and regroup, we get

$$X^S = V X_{\text{unc}}^S + W, \quad (\text{B11})$$

where $V = I - S^{-1} A^S A^{ST} / a_S$, $W = S^{-1} A^S B^S / a_S$, and I = identity matrix. Because all quantities in W are constants, $\tilde{W} = 0$, so

$$\tilde{X}^S = V \tilde{X}_{\text{unc}}^S V^T. \quad (\text{B12})$$

Equations (B6)-(B12) are suitable for calculating the covariance matrix of the currents that have been solved. Because of the close proximity of adjacent plasma filaments, the covariance between adjacent currents I_{pj} and I_{pj+1} is large (that is, the calculated fields at the sensor positions are relatively insensitive to increases in the current in one filament in conjunction with a corresponding decrease in an adjacent filament). However, we are not interested in $\tilde{X}^S$ but in errors in the computed plasma shape. To determine this, we compute the variance of ψ at each point on the boundary.

For a point (R, Z) , we can write

$$\psi(R, Z) = G^T X = G^{PT} X^P + G^{ST} X^S, \quad (\text{B13})$$

where the elements of the G vector, $g_i(R, Z)$, are Green's functions relating ψ to currents. For (R, Z) located at the sensor position i , $g_j(R_i, Z_i) = q_{ij}$. The variance in ψ is then given by

$$\tilde{\psi} = (\delta\psi)^2 = G^T \tilde{X} G. \quad (\text{B14})$$

The error in the plasma boundary at a point is given by

$$\Delta \vec{r}_b = \pm \frac{(\vec{\nabla} \psi)(\delta \psi)}{|\nabla \psi|^2},$$

which can easily be evaluated.

In a similar manner, the variance in the plasma current (for the unconstrained solution) can be easily calculated:

$$I_P = \sum I_{Pj} = A^{ST} X_{unc}^S, \quad (B15)$$

and

$$\tilde{I}_P = (\delta I_P)^2 = A^{ST} \tilde{X}_{unc}^S A^S. \quad (B16)$$

If the constrained solution is used, $\tilde{I}_P = 0$ because I_P is constrained to be a fixed value in this case.

APPENDIX C

Calculation of Iron-Core Effects

ISX-B has an iron core with a single return leg. An accurate analysis would require a three-dimensional iron core with air gaps, anisotropic iron permeability, and permeability changing with field. However, a much simpler model has been used with free parameters that can be adjusted to give good fits *a posteriori* to measured experimental data (as in Sect 3).

The model is illustrated in Fig. 15 and is from work done by Peng and Tucker.⁵ The iron core and return leg are simulated by an infinite solid iron cylindrical core, with an annular iron cylinder modelling the return leg. Variables in the model are the inner core radius r_1 , the radius of the annulus r_2 , and the iron permeability μ . The thickness of the annulus is specified by requiring that the iron area of the core and the annulus be equal.

The fields due to the presence of the iron region between the core and the annulus can be solved for analytically in terms of cylinder functions. Because the evaluation of these expressions is time-consuming, in practice, the iron contribution is computed by interpolation of a Green's function. For example,

$$B_z^{\text{Fe}}(r,z) = G_z^{\text{Fe}}(r,z - z_c, r_c) I_c ,$$

where G_z^{Fe} is the iron contribution to the field at a point (r,z) due to a unit current at a point (r_c, z_c) and I_c is the current. The total field is given by

$$B_z^{\text{total}}(r,z) = B_z^{\text{vac}}(r,z) + B_z^{\text{Fe}}(r,z) ,$$

where $B_z^{\text{vac}}(r,z)$ is computed by the usual elliptic integral expression. For our case, $|B_z^{\text{Fe}}| \leq 0.3 |B_z^{\text{vac}}|$. The function $G_z^{\text{Fe}}(r,z,r_c)$ does not change rapidly as a function of the arguments, so a spacing between

Fig. 15. Model used to calculate the iron-core effect.

points of 0.10 m is used. Even with the large spacing between points, the error made in interpolation is $\leq 2\%$ of the total calculated field.

Other analytically tractable two-dimensional models have been analyzed. It is not difficult to obtain a reasonable agreement between almost any model and measurements of B_Z and B_R after the measurements have been made. However, the prediction of field perturbations by an iron core, even with a complicated three-dimensional iron model, is still not reliable.

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73. Claude Mercier, Service du Theorie des Plasmas, Centre d'Études Nucléaires, Fontenay-aux-Roses (Seine), France

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