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INTERACTION OF THE ATA BEAM WITH THE TM_{030} MODE
OF THE ACCELERATING CELLS*

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ABSTRACT

The interaction of the electron beam in the Advanced Test Accelerator with an azimuthally symmetric mode of the accelerating cells is investigated theoretically. The interaction possibly could cause modulation of the beam current at the resonant frequency of the mode. Values of the shunt impedance and Q value of the mode were obtained from previous measurement and analysis. Lagrangian hydrodynamics is employed and a WKB solution to the equation of motion is obtained. Results indicate that the interaction will not be a problem in the accelerator.

I. INTRODUCTION

Now that unstable transverse motion has all but been eliminated from ATA with the advent of laser guiding¹, one wonders if some sort of "longitudinal cousin" of beam breakup may arise from the interaction of the beam and the azimuthally symmetric modes of the accelerating cells. Although the shunt impedance of these modes was not measured by D. L. Bix^{2,3}, the modes were found and their Q values measured. In particular one mode, which Bix labeled

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the TM_{030} , had the highest Q value of all modes investigated in the undamped cell. The frequency of this mode was about 775 MHz, which is so close to the 830 MHz frequency of the troublesome TM_{130} mode that the two interactions might be hard to distinguish experimentally.

Using the theoretical model of the cell that was so useful in analyzing the TM_{1n0} modes⁴, we have determined that the TM_{030} has a shunt impedance of 60 Ω and a Q of five or six. With this information we perform the approximate calculation described in this report. We treat the longitudinal motion in the WKB approximation, and results indicate that interaction with the mode will not cause serious energy modulation or longitudinal bunching in ATA.

II. THE EQUATION OF MOTION

Our linear treatment employs Lagrangian variable ξ which is the axial displacement of an element of the beam from the unperturbed position. The first order current I_1 and the first order charge per unit length λ_1 are given by

$$\lambda_1 = -\lambda \partial \eta / \partial z \quad (2.1)$$

$$I_1 = \lambda \partial \eta / \partial t \quad , \quad (2.2)$$

where λ is the unperturbed charge density per unit length. The first order velocity v_1 is given by

$$v_1 = \dot{\eta}$$

$$= \frac{\partial \eta}{\partial t} + v \frac{\partial \eta}{\partial z} \quad (2.3)$$

with v the unperturbed axial velocity. We choose $\eta(z, t)$ of the form

$$\eta(z, t) = g(z) e^{i\omega\tau} \quad (2.4)$$

in which $\tau \equiv t - z/v$. In this section all first order quantities are assumed to have the factor $e^{i\omega\tau}$ appended.

Since the head of the beam will excite the TM_{0n0} modes, a linear calculation is not completely valid. However, excitation of the TM_{030} mode by a 10 kA beam with a 10 ns rise time is only about 6 kV, while the accelerating voltage on the cell is 250 kV, so we regard the head of the beam as being the perturbation at a given position z the time τ is the time measured from the time of passage of the beam head.

We note that the Lagrangian treatment does not include stabilizing effects such as a spread in time of transit of particles. All particles have the same total speed, but a spread in the amplitude of transverse motion yields a spread in transit time down the accelerator.

The complete equation for longitudinal motion is

$$\frac{d}{dt} (\gamma + \gamma_1) (v + \eta) = \frac{e}{m} [E_a + E_1(z)] \quad (2.5)$$

in which E_a is the applied accelerating electric field (taken as independent of z and t), E_1 is the first order electric field from excitation of the cell, and m is the rest mass of the electron. It is not difficult to show that

$$\gamma_1 = \gamma^3 \beta \eta / c \quad . \quad (2.6)$$

The equilibrium equation of motion is

$$\frac{d(\gamma v)}{dt} = \frac{e E_a}{m} \quad . \quad (2.7)$$

We neglect the variations of v with z and use the relation $1 + \gamma^2 \beta^2 = \gamma^2$ to obtain the first order equation of motion,

$$\frac{d}{dt} (\gamma^3 \dot{z}) = \frac{e E_1}{m} \quad . \quad (2.8)$$

With Eq. (2.4) inserted for $\dot{z}$, Eq. (2.8) becomes

$$\frac{d}{dz} (\gamma^3 \frac{dg}{dz}) = \frac{e E_1}{m v^2} \quad . \quad (2.9)$$

The mode of the cell has a shunt impedance Z , a quality factor Q , and resonance frequency ω_0 . The voltage V_1 induced on the cell by the first order current I_1 varying as $e^{i\omega t}$ is given by

$$V_1 = \frac{-i \omega \omega_0 Z I_1}{Q[\omega_0^2 - \omega^2 + i(\omega \omega_0 / Q)]} \quad . \quad (2.10)$$

We take the first order electric field E_1 to be

$$E_1 = V_1 / L \quad ,$$

with L the distance between accelerating cells. Employing Eqs. (2.2) and (2.4) and noting that $I = v\lambda$, we obtain

$$E_1 = \frac{\omega^2 \omega_0 \text{ZIG}(z)}{LvQ [\omega_0^2 - \omega^2 + i(\omega\omega_0/Q)]} . \quad (2.11)$$

Inserting this expression into Eq. (2.9), we obtain

$$\frac{d}{dz} [\gamma^3(z) g(z)] + k^2(\omega) g(z) = 0 , \quad (2.12)$$

with $k^2(\omega)$ given by

$$k^2(\omega) = \frac{-\omega^2 \omega_0 eZl}{mv^3 LQ [\omega_0^2 - \omega^2 + i(\omega\omega_0/Q)]} . \quad (2.13)$$

The WKB solution to Eq. (2.12) is

$$g(z) \propto \frac{1}{[\gamma(z)]^{3/4}} \exp \left\{ \pm i k \int_{z_0}^z \frac{dz'}{[\gamma(z')]^{3/2}} \right\} . \quad (2.14)$$

The validity of this solution is subject to the condition

$$k^2(\omega) \gg \frac{1}{3} \frac{d^2}{dz^2} \gamma^{9/4} . \quad (2.15)$$

As we shall see in Section III, the condition is not very well satisfied at the end of the machine.

III. ASYMPTOTIC BEHAVIOR

We define the variable u to be

$$u = \int_{z_0}^z \frac{dz'}{[\gamma(z')]^{3/2}} \quad (3.1)$$

To find the function $\eta(z, t)$ we must perform the Fourier inversion integral, namely

$$\eta(z, t) = \frac{1}{\gamma^{3/4}} \int_{-\infty - i\epsilon}^{\infty - i\epsilon} g_0(\omega) e^{i\omega\tau \pm ik(\omega)u} d\omega \quad (3.2)$$

The quantity g_0 is determined by the initial conditions at the beam head and does not concern us. We only wish to determine the maximum growth and the value $\tau(u)$ for which it occurs. The maximum amplitude occurs at the value of ω for which $\text{Im}k$ is maximum. If this value is $\omega = \omega_1$ then the maximum amplitude at any value of u is proportional to $\exp[\text{Im}k(\omega_1)u]$. The time τ at which the maximum occurs at any u is given by

$$\tau = u \frac{d[\text{Re}k(\omega)]}{d\omega} \bigg|_{\omega_1} \quad (3.3)$$

It remains to evaluate $\text{Im}k(\omega_1)$ and $\text{Re}k(\omega_1)$.

The frequency ω_1 will be near ω_0 , so in Eq. (2.13) we set $\omega^2 - \omega_0^2 = 2\omega_0 \delta$ and $\omega\omega_0 = \omega_0^2$ in the denominator and $\omega^2\omega_0 = \omega_0^3$ in the numerator of the right hand side. With their approximations, Eq. (2.13) becomes

$$k^2 = \frac{\omega_0^2}{2} \left(\frac{eZI}{\beta^2 mc^2} \right) \frac{1}{LvQ} \frac{1}{(\delta - i\alpha)} \quad (3.4)$$

in which $\alpha \equiv \omega_0/2Q$ and $\beta = v/c$. For the remainder of this calculation we set $\beta = 1$. A little algebra is required to solve Eq. (3.4) for the real and imaginary parts of k . We introduce k_0 by the definition

$$k_0^2 = \frac{\omega_0 eZI}{4LQ mc^2} \quad (3.5)$$

and obtain

$$\text{Im}k = \frac{k_0 \omega_0^{1/2}}{(\delta^2 + \alpha^2)^{1/4}} \left[1 - \frac{1}{(\delta^2 + \alpha^2)^{1/2}} \right]^{1/2} \quad (3.6)$$

$$\text{Re}k = k_0 \frac{\omega_0^{1/2}}{(\delta^2 + \alpha^2)^{1/4}} \left[1 + \frac{\delta}{(\delta^2 + \alpha^2)^{1/2}} \right]^{1/2} \quad (3.7)$$

The imaginary part of k is maximum for $\delta = -\alpha/\sqrt{3}$. At this value of δ we find

$$\text{Im}k(\omega_1) = 3^{3/4} k_0 (Q/2)^{1/2} \quad (3.8)$$

We note that the maximum of $\text{Im}k$ is independent of Q . A little more algebra is required to obtain

$$\left. \frac{d(\text{Re}k)}{d\omega} \right|_{\omega_1} = \frac{Q}{\omega_0} \text{Im}k(\omega_1) \quad (3.9)$$

Before putting numbers into Eqs. (3.8) and (3.9), let's examine the quantity u defined by Eq. (3.1). In ATA each accelerating cell imparts 250 keV to the electrons, and the change in γ is therefore about 0.5 at each cell. In our smooth approximation, we write

$$\gamma = \gamma_0 + \gamma' z \quad , \quad (3.10)$$

with

$$\begin{aligned} \gamma' &= d\gamma/dz \quad , \\ &= 1/2L \quad . \end{aligned}$$

Since $dz = d\gamma/\gamma'$, we may write u as

$$u = 2L \int_{\gamma_i}^{\gamma} \frac{d\gamma}{\gamma^{3/2}} \quad , \quad (3.11)$$

so that $u \propto L$. From Eqs. (3.5) and (3.6) we see that $\text{Im}k(\omega_1) \propto L^{-1/2}$, and $\text{Im}k(\omega_1) u \propto L^{1/2}$. In ATA the first 30 accelerating cells are spaced about 55 cm apart. The remaining 140 accelerating cells are placed about 27.5 cm apart. Most of the exponential growth occurs in the beginning of the accelerator where u is increasing rapidly. To obtain an upper limit of the growth as predicted by this calculation, we set $L = 55$ cm throughout the machine. For an initial value $\gamma_i = 6$ (corresponding to the injector energy) and a final value $\gamma_f = 90$ we find, at full energy,

$$u_f = 67 \text{ cm} \quad . \quad (3.12)$$

Values of the relevant parameters are given in Table I.

TABLE I. Parameters of ATA and the TM030 mode of the accelerating cells.

$$\begin{aligned}\omega_0 &= 2\pi \times 7.75 \times 10^8 \text{ s}^{-1} \\ Z &= 60 \Omega \\ I &= 10 \text{ kA (eZI} = 600 \text{ keV)} \\ mc^2 &= 511 \text{ keV} \\ c &= 3 \times 10^{10} \text{ cm/sec} \\ L &= 55 \text{ cm} \\ Q &= 5\end{aligned}$$

With these values we find $\text{Im}k(\omega_1) = 4.74 \times 10^{-2} \text{ cm}^{-1}$, and the final energy $\text{Im}k(\omega_1) u_f = 3.16$. This pessimistic calculation yields an optimistic result. Surely the damping mechanisms that have been left out of the calculation will suppress growth of only three e-folds in the entire accelerator.

The growth of three e-folds will occur if the time τ for which the maximum amplitude occurs is less than the duration of the pulse. Otherwise the growth during the pulse will be less than three e-folds. From Eqs. (3.3) and (3.9) we find that, at full energy $u = u_f$, the maximum amplitude occurs at 3.25 ns. We conclude that, at worst, some disruption may conceivably occur during the initial current rise.

As a final observation, we determine to what extent the condition (2.15) is satisfied at $\omega = \omega_1$. We find that $|k(\omega_1)|^2 = 1.5 \times 10^{-3} \text{ cm}^{-2}$, and at final energy,

$$\frac{1}{3} \frac{d^2}{dz^2} \gamma^{9/4} = 2.4 \times 10^{-4} , \quad (3.13)$$

so that the condition is satisfied by only about a factor of six. If the above calculation had predicted unacceptable growth (e.g., 10 or more e-folds), more work would be needed. But with the present result of three e-folds, the concern regarding this instability can safely be laid to rest.

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