FLUCTUATIONS, TURBULENCE AND TRANSPORTS IN THE PRESENCE OF DRIFT WAVES

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ABSTRACT

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Turbulence spectrum and plasma diffusion have been studied by theoretical analysis as well as by numerical simulations using a drift wave model in which electrons are assumed to follow the Boltzmann distribution, while ions are assumed to move two-dimensionally in the plane perpendicular to the magnetic field. For a plasma near local thermal equilibrium, it is found that while the ion density diffusion is negligibly small, the energy diffusion is much larger and is comparable to the test particle diffusion. In the presence of strong turbulence, $\delta n \sim n_0$, fluctuation energy is found to cascade toward long wavelength modes which accompanies the density diffusion. The resultant turbulence spectrum is isotropic in two dimensions perpendicular to the magnetic field. However, the particle diffusion coefficient remains small. When the ion density gradient is maintained by freezing the electron background density, it is found that the turbulence spectrum cascades toward smaller wavenumbers only in the direction

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perpendicular to the density gradient. As a result an anisotropic turbulence spectrum is formed indicating an appearance of zonal flow in this direction.

I. INTRODUCTION

Drift wave instabilities and the associated anomalous diffusion of plasmas have been an important subject in both plasma kinetic theory and laboratory experiments in conjunction with the research of controlled fusion.¹ A great deal of work has been carried out on linear and nonlinear properties of such instabilities.² Determination of turbulence spectrum and the associated diffusion of particles and energy are important aspects of nonlinear theory.

Here we consider a drift wave model in which the electrons are assumed to obey the Boltzmann distribution

$$n_e/n_0 = e\phi/T_e, \quad (1)$$

where n_e and ϕ are electron density and potential fluctuations and n_0 and T_e are average density and electron temperature, respectively. Such a model is justified for an electrostatic turbulence in which the magnitude of the ion vorticity $|\nabla \times \mathbf{v}|$ is larger than or comparable to the drift wave frequency³ which is a situation commonly observed in tokamak plasmas for the fluctuations whose wavelengths are comparable to ion gyroradius.⁴ Full dynamics is employed for the motion of ions in a two-dimensional plane normal to magnetic field. It is well known⁵ that the assumption (1) results in the quasilinear particle flux of electrons

$$\Gamma_x^e = \langle e n_e E_y / B \rangle, \quad (2)$$

to vanish since the flux is proportional to the linear phase difference between n_e and ϕ .

In the presence of strong turbulence, however, quasilinear approximation may be invalid and strong coupling among different modes plays an essential role in determining turbulence spectrum and transport. This is especially so in laboratory experiments in which a steady state turbulence is maintained through the balance of the particle and energy inputs in the anomalous diffusion.

The approximation (1) saves computing time in numerical simulations since the electron dynamics are totally ignored in this model.⁶ Determination of average electron density in this model is left unspecified and we shall employ high and low mobility electron models. In the

high mobility model, the average electron density is adjusted freely to follow the ion average density. In the *low mobility model*, electron average density is frozen to the initial value. To understand the turbulent diffusion, we first consider test particle diffusion for ions in the presence of thermal fluctuations in the limit of a vanishing density gradient. It is shown that the test particle diffusion is only slightly smaller than the convective cell diffusion. When both finite density and temperature gradients are introduced keeping the low level of fluctuations, test particle diffusion decreases considerably due to the appearance of real frequency associated with the diamagnetic drift. This is in contrast to convective cells in which density and temperature gradients do not produce the real part of the frequency. The density diffusion is even less than the reduced test particle diffusion in the case of thermal fluctuation. It is found that the temperature diffusion is much larger than the density diffusion and is comparable to the test particle diffusion.

In the presence of strong turbulence, $\delta n \sim n_0$, density diffusion takes place on the same time scale as that of the inverse cascade through mode-coupling⁷ in a high electron mobility model. The resultant fluctuation spectrum is almost isotropic in a two-dimensional plane normal to magnetic field. When the density diffusion is prohibited by means of the low mobility electron model, energy transfer toward long wavelength modes takes place primarily in the direction perpendicular to density gradient resulting in the formation of anisotropic fluctuation spectrum as predicted by Hasegawa, McLennan and Kodama.⁸

In Section II, diffusion and spectrum in thermal plasma is discussed. Inhomogeneity is introduced in Section III for a plasma near local thermal equilibrium. Very large density fluctuations are considered in Section IV. Discussions and remarks are given in Section V.

II. HOMOGENEOUS PLASMA

Let us first consider test particle diffusion in a thermal plasma in the limit of small density gradient. Because the ion motions are treated as two-dimensional there exists no normal mode with a finite frequency. Here the test particle diffusion coefficient is calculated in a following manner.

$$D = (c^2/B^2) \sum_k \langle E_k^2 \rangle \tau_k, \quad (3)$$

where $\langle E_k^2 \rangle$ is the electric field fluctuations associated with drift waves and τ_k is the correlation time between particles and field. $\langle E_k^2 \rangle$ may be found using fluctuation-dissipation theorem;⁹

$$\frac{\langle E_k^2 \rangle}{8\pi} = \frac{T}{2} \frac{k^2 \lambda_e^2}{1 + k^2 \rho_i^2}, \quad (4)$$

for $k^2 \ll \lambda_e^{-2}$, ρ_i^{-2} where λ_e and ρ_i are electron Debye length and the ion gyroradius at the electron temperature. Since there is no linear mechanism for the correlation damping, it is reasonable to assume $\tau_k = (k^2 D)^{-1}$ in which particle diffusion will determine the correlation. We then find from (3) and (4)

$$D \approx \frac{c}{B} \frac{\Omega_i}{\omega_{pi}} (T)^{1/2}, \quad (5)$$

where the k integration is cut off at $k_{\max} = \rho_i^{-1}$. Ω_i and ω_{pi} are ion gyro and plasma frequencies. This diffusion is independent of magnetic field and is similar to convective cell diffusion for $\omega_{pi} > \Omega_i$,¹⁰

$$D = \frac{c}{B} \frac{\Omega_i}{\omega_{pi}} T^{1/2} (\ln k_{\max}/k_{\min})^{1/2}. \quad (6)$$

The only difference between (5) and (6) is the long wavelength divergence, $(\ln k_{\max}/k_{\min})^{1/2}$, for the convective cells.

Numerical simulations are carried out to check Eqs. (4) and (5) using a two-dimensional ion model in which the electrons are assumed to have a Boltzmann distribution.⁶ Typically 128×128 ions are used on a 64×64 grid. Figure 1 shows the test ion diffusion coefficients for $\omega_{pi} = 2\Omega_i$ and $\omega_{pi} = \Omega_i$. We find the measured diffusion is very close to Eq. (5) and is insensitive to magnetic field strength. Time averaged electric field fluctuations are shown in Fig. 2 for these runs. For long wavelength, $k^2 \ll \lambda_e^{-2}$, ρ_i^{-2} , the measurements agree very well with (4) but for short wavelength, the spectrum decreases rapidly with k due to the shielding and finite-gyroradius effects neglected in (4). In addition the use of finite size particles in the simulations

also reduced the amplitude of short wavelengths.

It is interesting and instructive to compare this diffusion with convective cell diffusion as shown in Figs. 3 and 4. Simulation parameters are the same as before ($\omega_p = \Omega_e$) and both electrons and ions are treated as discrete particles. From Fig. 3, diffusion is found $D_x = 2.6 \times 10^{-1} \Delta^2 \omega_p$, which is more than a factor of two larger than the former case in which the electrons are in Boltzmann distribution, here Δ is the mesh size for computation.¹⁰ The convective cell diffusion is in good agreement with the theory.

Fluctuation spectrum shown in Fig. 4 agrees well with theoretical thermal spectrum¹⁰ which indicates the monotonic decay with increasing wavenumbers. This is in sharp contrast to the former fluctuations which is peaked at a wavenumber $k \approx \rho_i^{-1}$. This is one of the reasons that convective cells generally give rise to diffusion much larger than drift waves. While these simulations are carried out near thermal equilibrium, characteristics of the spectra remain more or less the same in a non-uniform plasma so long as the fluctuation amplitude remains small.

III. INHOMOGENEOUS PLASMA

We now study plasma transport in an inhomogeneous plasma with density and temperature gradients. Here fluctuation level is still assumed in near local thermal equilibrium and their amplitude is therefore small. Test particle diffusion as well as the diffusion of density and temperature gradients are also measured. Initially, density and temperature for the ions are both assumed

$$n_i, T_i \sim \exp(-x^2/\delta_{n,T}^2) \quad (7)$$

where $\delta_{n,T}$ is typically ten grid cells. Simulation parameters are the same as for the homogeneous case. Since the local measurement of diffusion coefficient is difficult because of the scattering of the measurements, average diffusion coefficient for density and temperature is measured using the Fourier coefficient for the density and temperature.¹¹ High mobility electron model was used in the simulations shown in this section.

Figure 5 shows the time history of the dominant Fourier coefficient for density and tempera-

ture. We observe that the density diffusion is essentially zero while the temperature diffusion is about $D = 0.02 \Delta^2 \omega_{pi}$ which is almost identical to the test particle diffusion to be shown later. Quasilinear particle flux of the ions given by (2) remains very small since the phase difference between the ion density fluctuations n_i and the potential is small because the linear damping rate for the fluctuations is small. Particle flux associated with the nonlinear polarization drift¹²

$$\Gamma'_x(x) = \frac{c^2 n_0}{\Omega_i B^2} \langle E_y \frac{\partial E_x}{\partial x} \rangle_y, \quad (8)$$

where the bracket means the averaging over y -direction, is also very small. This is because the potential may be approximated by a superposition of sinusoidal drift waves for small amplitude fluctuations given by

$$\phi(x, y, t) = \sum_k \phi_k(x) \sin(\omega_k t - k_y y + \alpha_k), \quad (9)$$

where α_k is the phase of the k -th mode and the frequency ω_k is given by¹

$$\omega_k = \frac{k_y T_e}{m_i \Omega_i} \frac{dn}{dx} \frac{\exp(-k^2 \rho_i^2) I_0(k^2 \rho_i^2)}{2 - \exp(-k^2 \rho_i^2) I_0(k^2 \rho_i^2)}. \quad (10)$$

Then it is straightforward to show that the flux given by (8) vanishes identically. In order to produce net particle flux given by (8), it is essential, therefore, that the nonlinear frequency shift is comparable to or larger than the linear frequency given by ω_k . The amplitude for density fluctuations for this to occur has been studied in detail¹³ using a high electron mobility model and it was found that for long wavelength modes, $k\rho_i \ll 1$, nonlinear mode coupling is so small that even for relatively large density fluctuations, the shift of the frequency from the linear frequency remains small resulting in the very small net particle flux.

A large energy transfer can exist even in the absence of density diffusion because energy can be transferred by replacing particles with each other when a finite temperature gradient is present.

To make this point more precise, let us consider the ion heat flow equation neglecting collisions and heat flux tensor.

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \frac{T}{n} \nabla \cdot (n \mathbf{v}) . \quad (11)$$

The absence of density diffusion implies that the effect of polarization drift is negligible, because $E \times B$ drift with the Boltzman density distribution does not contribute to the density diffusion. Let us, hence, take only the $E \times B$ drift here, where upon Eq. (11) reads

$$\frac{\partial T}{\partial t} - \frac{\nabla \phi \times \hat{z}}{B_0} \cdot \nabla T = 0 . \quad (12)$$

Assuming a periodic variation in the y direction, we take average with respect to the y variations

$$\frac{\partial \bar{T}}{\partial t} = \frac{1}{B_0} \frac{\partial}{\partial x} \left[\frac{\partial \phi}{\partial y} \bar{T} \right] , \quad (13)$$

where the bar shows the average over the y direction and the fluctuation of the temperature $\tilde{T}$ is given from Eq. (12) by

$$\frac{d\tilde{T}}{dt} = \frac{1}{B_0} \frac{\partial \phi}{\partial y} \frac{\partial \tilde{T}}{\partial x} . \quad (14)$$

Here d/dt indicates the derivative along test particle trajectories.

Integrating Eq. (14) along the trajectory and substituting $\tilde{T}$ in Eq. (13), we obtain

$$\frac{\partial \bar{T}}{\partial t} = \frac{1}{B_0^2} \frac{\partial}{\partial x} \left[\frac{\partial \phi}{\partial y} \int' \frac{\partial \phi(r')}{\partial y} \frac{\partial \bar{T}(r')}{\partial x} dt' \right] . \quad (15)$$

If we now take the ensemble average by assuming that the diffusion time is much longer than the correlation time of the fluctuating electric field, we have

$$\frac{\partial \bar{T}}{\partial t} = \frac{\partial}{\partial x} (D_T \frac{\partial \bar{T}}{\partial x}) , \quad (16)$$

where

$$D_T = \frac{1}{2} \frac{\partial}{\partial t} \langle \Delta x^2 \rangle , \quad (17)$$

with

$$\Delta x = - \frac{c}{B_0} \int' \frac{\partial \phi(t'')}{\partial y} dt'.$$

This verifies the identity of the test particle and the temperature diffusion coefficient when the density diffusion is absent.

Figure 6 shows the test particle diffusion for the inhomogeneous case. Note the diffusion coefficient is the same to the heat diffusion coefficient. Note also that the test particle diffusion in an inhomogeneous plasma is smaller than that for a homogeneous plasma by a factor of five or so even if the fluctuation spectrum of the electric field shown in Fig. 7 remains the same for both cases. The smaller test particle diffusion originates from the existence of oscillating waves in an inhomogeneous plasma. In the presence of waves with frequency ω_k , Eq. (3) is reduced to

$$D = \frac{c^2}{B^2} \sum_k \langle E_k^2 \rangle \frac{\gamma_k}{\omega_k^2}, \quad (18)$$

where γ_k is the rate in which the correlation of the mode k is lost. In the absence of linear damping mechanism, γ_k may be given by nonlinear mode-coupling rate of Hasegawa-Mima equation.³

$$\begin{aligned} & \frac{\partial}{\partial t} (1+k^2 \rho_s^2) \frac{e\phi_k}{T_e} + i\omega_k \frac{e\phi_k}{T_e} \\ & = \frac{\Omega_e}{2} \sum_{k=k'+k''} \rho_s^4 (k' \times k'') \cdot \hat{z} (k''^2 - k'^2) \frac{e\phi_{k'}}{T_e} \frac{e\phi_{k''}}{T_e} \end{aligned} \quad (19)$$

where ω_k^* is the drift wave frequency given by $-k \times \hat{z} \cdot \nabla (\ln n_0) \rho_s^2 \Omega_e$. Numerical evaluation of the nonlinear term in (19) reveals $\gamma_k \ll \omega_k$ for $k\rho_s \ll 1$ while $\gamma_k \sim \omega_k$ near $k\rho_s \sim 1$ and the estimate from (10) is not too far from the simulation results.¹³

We note here that a reduction of diffusion coefficient in an inhomogeneous plasma also takes place for convective cell induced diffusion. In this case the amplitude of convective cells is reduced in an inhomogeneous plasma compared with the amplitude for a homogeneous plasma. This is because the more energy is associated with the ion flute modes than the energy in the convective cells in an inhomogeneous plasma.¹³

IV. LARGE AMPLITUDE DRIFT FLUCTUATIONS

We have seen that the particle diffusion associated with the thermal fluctuations of drift waves in the absence of quasilinear flux is very small and can be neglected in practice. When large amplitude fluctuations are present, we may expect the enhanced diffusion due not only to quasilinear flux but also to nonlinear polarization flux, (8), since the strongly interacting large amplitude waves are no longer coherent. To understand the nature of large amplitude drift wave turbulence, let us first review the theoretical understanding of the turbulence property of Eq. (19). The mode coupling equation (19) was derived assuming two-dimensional cold ions, electrons with Boltzmann distribution and quasi-neutrality. Hence the density perturbation δn_k is related to ϕ_k through

$$\delta n_k = \frac{e \phi_k}{T_e} n_0. \quad (20)$$

We see from Eq. (19) that the nonlinear term dominates over the linear term ω_k^* if

$$k^4 \rho_i^4 |e \phi_k / T_e| > |\omega_k^* / \Omega_i|. \quad (21)$$

Since $\omega_k^* / \Omega_i \approx 10^{-2}$, if $e \phi_k / T_e \approx 0(1)$, at $k \rho_i = 1$ as in the case of simulation to be presented later, the nonlinear term dominates if $k^3 \rho_i^3 \geq 10^{-2}$, or $k \rho_i \geq 0.22$. In this range of k , one would expect a development of inertia range turbulence spectrum of the two-dimensional hydrodynamic type⁷ where the energy spectrum $W_k = e^2 (1 + k^2 \rho_i^2) |\phi_k|^2$ obeys power law of k^{-4} at $k > k_i$ and of $k^{-8/3}$ at $k < k_i$ where k_i is the source (unstable) wave number.

For a wave number regime smaller than $0.22 \rho_i^{-1}$, the linear term dominates hence the spectrum develops according to the prediction of weak turbulent theory, that is, the turbulence energy cascades into lower frequency regime hence to smaller value of k_y , where y is the direction perpendicular to the density gradient.⁸ When the density diffusion takes place simultaneously to the energy cascade in k space, the situation becomes more complicated because ω_k^* decreases as the density gradient decreases, thus the transition wavenumber from strong to weak turbulence also decreases.

In an extreme case in which the diffusion takes place much faster than the spectrum cascade, ω_k^* becomes zero hence the spectrum becomes isotropic for the entire wavenumber domain and obeys the power law spectrum.

in the other extreme case in which the particle diffusion is much slower, the spectrum becomes anisotropic below $0.22 \rho_i^{-1}$. The spectrum is expected to condensate at the smallest k_y and at $k_x \approx 0.22 \rho_i^{-1}$. This means that the turbulence energy condensates to a zonal flow of plasma particles in the y direction.⁸

In order to numerically study the diffusion in the presence of large amplitude fluctuations, ions are loaded initially so as to form a square wave in the y -direction with its wavelength equal to the fourth mode in the system. The initial amplitude is taken $\delta n/n_0 = 1$ and 0.5.

Figure 9 shows the fluctuation spectrum at the initial time and at around $t \approx 5000 \omega_{pi}^{-1}$. Initially harmonics of the fourth modes are excited. At $t = 5000 \omega_{pi}^{-1}$, the spectrum for $k\rho_i \leq 1$ indicates the inverse cascade property peaking toward smaller k 's which is quite different from the small amplitude fluctuations. The shape of the electric field spectrum $|E_k|^2 = k^2 |\phi_k|^2$ appears reasonably close to theoretically predicted k_i^{-2} dependence for $k\rho_i \leq 1$.⁸ The spectrum for $k\rho_i \geq 1$ reaches quickly the shape for thermal equilibrium despite the presence of large amplitude fluctuations.

Initial inhomogeneous plasma density has spread out almost uniformly by $t \approx 2000 \omega_{pi}^{-1}$ as shown in Fig. 9 and the spectrum is almost isotropic with respect to k_y and k_x . Clearly for very large density fluctuations, $\delta n \sim n_0$, the nonlinear term in (19) exceeds the linear frequency and the flux given by (8) is finite.

Test particle diffusion and the density diffusion is shown in Figs. 10 and 11. We again observe the test particle diffusion is much larger than the density diffusion. These diffusion coefficients are rather modest however, regardless of the presence of very large amplitude fluctuations.

When the initial density perturbations are reduced to $\delta n/n_0 = 0.5$, diffusion for density and test particles become smaller as shown in Fig. 10 and 11. Test particle diffusion in this case is

smaller than the convective cell diffusion in thermal equilibrium as shown in Fig. 3. In these calculations, electrons are assumed to diffuse together with the ions on the average and no ambipolar field is set up.

It is interesting to study the fluctuation spectrum when the initial density profile is maintained throughout the simulations. This is done by freezing the electron density to the initial ion profile thereby setting up the ambipolar electric field to hold the ion density to its initial value (low mobility electron model).

Figure 12 (a), (b) show the contour plots for the electrostatic potential at $t = 2000 \omega_{pi}^{-1}$ and $5000 \omega_{pi}^{-1}$ for the low mobility model with the initial density fluctuation $\delta n \approx n_0$. It is clearly seen that more or less isotropic spectrum at an earlier time has spread only in the homogeneous y-direction to the longest mode available. Spectra with respect to k_x and k_y are shown in Figs. 13 (a) and 13 (b). The spectrum with respect to k_x is peaked at $k_x \rho_s = 0.8$ at an early time due to the initial excitation of such modes. By the end of the run, fluctuation energy becomes monotonic and peaks at the longest wavelength of the system $k_x \rho_s = 0.2$. The shape of the spectrum $|E_k|^2 = k^2 |\phi_k|^2$ is reasonably close to the theoretically predicted k^{-2} .

The spectrum with respect to k_y indicates quite a different behavior and in particular no sign is present for the energy to migrate toward the longest mode available and the resultant spectrum remains anisotropic with respect to k_x and k_y . The cascade with smaller k_y is not allowed because the density is frozen in its initial profile. The appearance of zonal flow in the y direction is apparent because constant ϕ contour is the stream line contour in agreement with the earlier prediction.⁸ Recent experimental study of fluctuations in a tokamak¹⁵ reveals a significant longer density correlation in the azimuthal direction as compared with the radial direction supporting the evidence of an existence of zonal flows. Since the zonal flow in the azimuthal direction tends to inhibit the particle diffusion in the radial direction, the appearance of such a zonal flow may be closely related with the observations in the laboratory experiments in which negligible diffusion was observed in spite of the presence of large density fluctuations.^{16,17}

Ion density profile maintains the initial smooth Gaussian profile and no diffusion was observed due to the ambipolar electric field set up by the immobile electrons.

V. CONCLUDING REMARKS

Using a drift wave model in which electrons are assumed to obey the Boltzmann distribution while retaining the full dynamics of ions, diffusion of particle and energy and the fluctuation spectrum are studied for a thermal plasma and a turbulent plasma. It was found the particle diffusion in this model remains small even in the presence of large amplitude density fluctuations. This is attributed to the fact that the nonlinear particle flux given by (8) remains small. It appears to us therefore that proper treatment of quasilinear particle flux and coupling to vortex (convective cell) modes are more important in determining anomalous particle diffusion. Ion heat flux is found large and is comparable to the test particle diffusion coefficient given by (18).

In the presence of very large density fluctuations, $\delta n \sim n$, it was found that the fluctuation spectrum shows the inverse cascade process transferring the energy to the longest mode available. The inverse-cascade process accompanies particle diffusion which destroys the original density profile leading to a uniform plasma. When the density diffusion is not allowed by freezing the electron density profile, it was found the cascade takes place only in the direction perpendicular to the density gradient resulting in the anisotropic fluctuation spectrum with respect to k_y and k_z and in the formation of zonal flow.⁸ It should be emphasized that a very large density fluctuation is required in order for the fluctuation energy to be transferred into the long wavelength modes, $k\rho_s \ll 1$, in the present model. This is consistent with earlier conclusion¹³ that the nonlinear mode-coupling term is very small in (19) for long wavelength modes, $k\rho_s < 1$.

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FIGURE CAPTIONS

- Fig. 1 Test particle diffusion associated with drift wave fluctuation in the limit of small density gradient. 64×64 grid and 128×128 ions are used in the simulations.
- Fig. 2 Time-averaged fluctuation spectrum for drift fluctuations near thermal equilibrium. Note $E_k^2 \sim k^2$ for long wavelength modes, $k\rho_i \leq 1$.
- Fig. 3 Test particle diffusion associated with convective cells. 64×64 grid and 128×128 ions and electrons are used with $m_i = m_e$.
- Fig. 4 Time-averaged fluctuation spectrum for convective cells near thermal equilibrium. E_k^2 is constant with k for $k\rho_i \leq 1$.
- Fig. 5 Diffusion of particle density and temperature in a nonuniform plasma near local thermal equilibrium. Note the temperature diffusion is much larger than the density diffusion.
- Fig. 6 Test particle diffusion coefficient in an inhomogeneous plasma. It is comparable to ion temperature diffusion coefficient.
- Fig. 7 Time-averaged fluctuation spectrum for drift waves in a non-uniform plasma. Note the spectrum is same to Fig. 2 for a uniform plasma.
- Fig. 8 Fluctuation spectrum in a strongly turbulent plasma at $t = 5000 \omega_{pi}^{-1}$. Initial perturbation at $k\rho_i = 0.8, 1.6, 2.4$ are transferred to the longer waves in the system. Spectrum is almost isotropic with respect to k_x and k_y due to the flattening of the original non-uniform density.
- Fig. 9 Time-development of the density profile in a turbulent plasma with $\delta n/n_0 \approx 1$ initially. Anomalous diffusion flattens the initial non-uniformity after $t = 2000 \omega_{pi}^{-1}$.
- Fig. 10 Test particles diffusion in a turbulent plasma with the initial density perturbation of $\delta n/n_0 = 1$ and 0.5 . High mobility electron model is used for these simulations.

Fig. 11 Density diffusion coefficients in a turbulent plasma.

Fig. 12 Electrostatic potential contours at $t = 2000 \omega_{pi}^{-1}$ and $5000 \omega_{pi}^{-1}$ in a plasma in the initial density perturbation $\delta n/n_0 = 1$. Electron density profile is frozen to its initial profile. The spectrum more or less isotropic initially evolves into large structures in the y -direction, a formation of a zonal flow.

Fig. 13 Fluctuation spectrum with respect to k_y and k_x in a turbulent plasma with fixed electron density. Note the spectrum with respect to k_y becomes monotonic at $t = 4500 \omega_{pi}^{-1}$ roughly scaling as k_y^{-2} while the spectrum with respect to k_x is peaked at $k_x \rho_s = 0.4$.

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