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MASTER

FINITE ELEMENT ANALYSIS OF THERMAL CONVECTION IN DEEP OCEAN SEDIMENTS*

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INTRODUCTION

The need to develop a method for the permanent disposal of radioactive waste material has led to consideration of a number of geologic formations as possible repository sites. One of the more attractive disposal options currently under investigation involves the use of the deep ocean sediments located in oceanic mid-plate, mid-gyre regions (Bishop and Hollister 1973, 1974). Current proposals for seabed disposal methods (Talbert 1977, 1978) envision the waste material to be suitably encapsulated and then buried in the sediment layer. Of obvious importance to the study and engineering of such a disposal scheme is the determination of the temperature and fluid flow fields existing in the sediment layer and the perturbation of these fields due to the implantation of localized heat sources.

The fluid mechanical and heat transfer process occurring in oceanic sediments may be characterized as free (or natural) convection in a porous material. In the case of an undisturbed sediment layer, the driving force for the natural circulation of pore water comes from the geothermal heat flux. Current theories for heat flow from the sea floor suggest the possibility of large scale hydrothermal circulation in the oceanic crust (see e.g., Ribando, et al. 1976) which is in turn coupled with a convection process in the overlying sediment layer (Anderson 1980, Anderson, et al. 1979). The introduction of a local heat source, such as a waste canister, into a saturated sediment layer would by itself initiate a convection process due to buoyancy forces.

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Since the mathematical description of natural convection in a porous medium is of sufficient complexity to preclude the use of most analytic methods of analysis, approximate numerical procedures are often employed. In the following sections, a particular type of numerical method is described that has proved useful in the solution of a variety of porous flow problems. However, rather than concentrate on the details of the numerical algorithm the main emphasis of the presentation will be on the types of problems and results that are encountered in the areas of oceanic heat flow and seabed waste disposal.

PROBLEM FORMULATION

Though the motivation for the present work is derived from the problem of seabed waste disposal, the formulation of the boundary value problem has been kept as general as possible to permit extension to other problems of engineering interest. To properly define the class of porous flow problems under investigation, the following assumptions have been made:

- (a) two-dimensional geometries,
- (b) rigid porous matrix,
- (c) single phase, incompressible, Newtonian fluid,
- (d) laminar, Darcy flow with inertia effects neglected,
- (e) Boussinesq approximation, and
- (f) fluid and matrix are in thermal equilibrium.

Under these restrictions, the equations describing heat and mass transfer in a porous material may be expressed in Cartesian coordinates as:

Continuity:

$$\frac{\partial \langle v_i \rangle}{\partial x_i} = 0 \quad (1)$$

Darcy's Law (Momentum):

$$\langle v_i \rangle = - \frac{k_{ij}}{\mu} \left(\frac{\partial \langle P \rangle^*}{\partial x_j} + \rho_f g \frac{\partial z}{\partial x_j} \right) \quad (2)$$

Energy:

$$(\rho C)_e \frac{\partial T}{\partial t} + \rho_f C_f \langle v_i \rangle \frac{\partial T}{\partial x_i} = \frac{\partial}{\partial x_i} \left[(\lambda_{e_{ij}} - \phi E_{ij}) \frac{\partial T}{\partial x_j} \right] + Q \quad (3)$$

Local Continuity

where $(\rho C)_e = \phi \rho_{f_o} C_f + (1 - \phi) \rho_s C_s$

$$\lambda_{e_{ij}} = \phi \lambda_f + (1 - \phi) \lambda_{s_{ij}}$$

$$\rho_f = \rho_{f_o} [1 - \beta(T - T_o)] \quad (4)$$

In equations 1 through 4, v_i are the Darcy velocity components in the x_i coordinate direction, P is the pressure, T is the temperature, and t the time. Also, ρ is the density, C the heat capacity, μ the viscosity, β the coefficient of volumetric expansion, λ_{ij} the thermal conductivity tensor, ϕ the porosity, k_{ij} the permeability tensor, E_{ij} the thermal dispersion tensor, and Q the volumetric heat generation. A bulk volume average is denoted by $\langle \rangle$ while a pore volume average is indicated by $\langle \rangle^*$ (Note: $\langle \rangle = \langle \rangle^* \phi$). Finally, the subscripts e, f, s, and o refer to effective, fluid, solid (matrix), and reference quantities, respectively.

The boundary value problem described by equations 1 through 3 is completed by the specification of an appropriate set of initial and boundary conditions for the primary dependent variables, v_i and T . Though equations 1 through 3 form a valid description of general heat and mass transfer in porous materials, the equations are somewhat inconvenient in terms of developing a numerical solution procedure. The major difficulty stems from the fact that equation 2 must be solved subject to the (incompressibility) constraint condition given by equation 1. Such systems are notoriously difficult, as is evidenced by the vast literature on methods for solving the incompressible, Navier-Stokes equations. In the present case, most of the difficulty may be avoided by considering the combination of equations 1 and 2 to yield the following equation for the pressure

$$\frac{\partial}{\partial x_i} \left(- \frac{k_{ij}}{\mu} \frac{\partial P^*}{\partial x_j} \right) = \frac{\partial}{\partial x_i} \left(\frac{k_{ij}}{\mu} \rho_f g \frac{\partial z}{\partial x_j} \right) \quad (5)$$

Equations 5 and 3 form an appropriate system for the numerical computation of the pressure and temperature, with the velocity field being recovered from equation 2 as required. Also, it should be noted that boundary and initial conditions for this system must now be specified in terms of the pressure rather than the fluid velocity.

NUMERICAL METHOD

The numerical procedure employed to solve the boundary value problem outlined in the previous section is a Galerkin form of the finite element method (FEM). As the FEM is by now a well established and documented method (e.g., Zienkiewicz 1971, Huebner 1975, Gray, et al. 1976), a detailed derivation of the equations for the present problem will not be undertaken here. Rather, the discussion will be limited to a brief outline of the most important features of the numerical procedure. Details of the method may be found in Gartling and Hickox (1979).

Application of the Galerkin procedure to the partial differential equation system given by equations 5, 3, and 2 produces a corresponding system of discretized equations of the following form

$$\underline{\underline{K}}(\underline{\underline{T}})\underline{\underline{P}} = \underline{\underline{F}}(\underline{\underline{T}}, t) \quad (6)$$

$$\underline{\underline{N}}\dot{\underline{\underline{T}}} + \underline{\underline{D}}(\underline{\underline{u}})\underline{\underline{T}} + \underline{\underline{L}}(\underline{\underline{T}}, \underline{\underline{u}})\underline{\underline{T}} = \underline{\underline{G}}(\underline{\underline{T}}, t) \quad (7)$$

$$\underline{\underline{u}} = \underline{\underline{H}}(\underline{\underline{T}})\underline{\underline{P}} + \underline{\underline{B}}(\underline{\underline{T}})\underline{\underline{T}} \quad (8)$$

Note that $\underline{\underline{\sim}}$ indicates a matrix and $\underline{\sim}$ a vector. Also, $\underline{\underline{P}}$, $\underline{\underline{T}}$, and $\underline{\underline{u}}$ represent vectors of nodal point pressures, temperatures and velocities; the remaining matrices and vectors correspond to the appropriate differential operators or boundary conditions found in equations 5, 3, and 2. The functional dependence of each operator is shown in parenthesis.

Solution algorithms for the above system of equations must obviously reflect the degree to which the system is coupled for a particular problem (e.g., highly coupled for natural convection; decoupled for constant property forced convection) and whether the problem is of a steady-state or transient nature. Basically, the procedures adopted for equations 6 through 8 parallel the methods described by Gartling (1977a) for convection problems involving the Navier-Stokes equations. That is, for coupled steady-state problems, the energy and pressure equations are solved in a cyclic manner (beginning with the energy equation) with the coefficient matrices updated whenever new values of the dependent variables become available. In the case of decoupled or isothermal flows, the pressure equation is solved first, followed by a direct solution of the energy equation as required. Transient flows are processed by the same equation sequence but with a modified Crank-Nicholson time stepping procedure used in place of the iteration cycle. Further details of the solution algorithms may be found in Gartling and Hickox (1979).

The numerical procedures outlined in the previous paragraphs have been incorporated in a user-oriented computer code called MARIAH. The code structure and features of MARIAH are based on its predecessor, NACHOS (Gartling 1977b) a finite element code

for incompressible flows described by the Navier-Stokes equations. In the following section, a number of porous flow problems associated with flows in ocean sediments will be described. All of the computations cited were performed using the MARIAH code, which should provide some insight into the utility of the proposed finite element procedure.

CONVECTION IN DEEP OCEAN SEDIMENTS

Two different problems of convection in ocean sediments are considered in the present section; the first analysis deals with naturally occurring hydrothermal circulation while the second illustrates a typical analysis for a waste disposal application.

Sealing of a Basaltic Layer

As stated previously, there is some evidence to suggest that natural hydrothermal circulation occurs in the oceanic crust due to the geothermal heat flux. In an idealized sense, the problem of heat flow in a crustal layer may be viewed as the problem of a saturated porous layer of horizontally infinite extent that is heated from below. Natural circulation is known to occur in such a geometry when the Rayleigh number ($Ra = k\rho_0 g \beta q L^2 / \mu a_e \lambda_e$, q = heat flux, L = depth of layer) for the layer exceeds a threshold value. Ribando, et al. (1976) and others have analyzed such a problem in the context of oceanic heat flow using a variety of numerical methods. In all of these cases, the upper boundary of the crustal layer was assumed to be either completely permeable or completely impermeable--the bounding cases for the physical situation. The ocean crust is, in fact, subjected to a gradual sealing process due to the continual deposition of relatively impermeable sediments on the sea floor. The gradual buildup of the sediment layer serves to eventually isolate the convection process in the basaltic crust from interacting with the ocean environment.

The question naturally arises as to what conditions must be met such that the crustal layer may be considered to be sealed. Anderson (1980) has employed a simple analytic model to propose that convective transport through the sediment is no longer important when

$$\frac{k_{sed}}{h} \ll a k_{bas} \quad (9)$$

In equation 9, k_{sed} and k_{bas} are the sediment and basalt permeabilities, h is the thickness of the sediment layer, and a ($= \pi/D$) is the horizontal wave number for the convection cell that is assumed to be present in the oceanic crust. In an effort to investigate the validity of equation 9, a finite element model of the sediment/basalt system was constructed for use with the MARIAH code.

Finite Element Model

The problem was idealized as shown in figure 1 where the assumed boundary conditions and geometric parameters are also identified. The first task in attempting to validate Anderson's hypothesis was the determination of the appropriate wave number for the convection cell. Having made the assumption that the flow is two-dimensional and periodic in the horizontal coordinate, it was felt that the size of the computational domain should reflect the aspect ratio of the convective motion for the Rayleigh number in question. Adopting the philosophy described by Combarrous and Bories (1975), the preferred aspect ratio was taken to be the one which proved to be the thermodynamically most efficient in transferring energy between the lower and upper surfaces. For the present case, where a constant heat flux is specified along the lower boundary, the measure of efficiency was taken to be the mean temperature difference between the upper and lower surfaces. The most efficient convection process minimized this temperature difference.

Figure 1. Schematic of Basalt/Sediment Problem.

A brief parameter study was undertaken in which the aspect ratio ($AR = D/L$) of the computational domain was varied (i.e., height, L was fixed at 2900 m, width, D was varied) between 0.6 and 4.0 for a fixed Rayleigh number (Ra is based on the heat flux and height, L , of the basalt layer). For all cases, the ratio of sediment to basalt permeabilities was 10^{-3} , which corresponds to a nearly impermeable upper boundary. Shown in figure 2 are a series of steady-state streamline plots corresponding to various

AR = 0.60

AR = 1.0

AR = 2.0

AR = 4.0

Figure 2. Streamlines for Flow in the Basalt Layer
Various Aspect Ratios, $Ra = 190$.

AR and a $Ra = 190$. The evolution from a single cell solution at small AR to a multiple cell flow at higher AR is quite evident. The bias of the cellular motion toward one corner of the domain is a result of the variation of the fluid viscosity with temperature. Computations were made for the mean bottom boundary temperature

$$T_{\text{mean}} = \frac{1}{L} \int_0^L T(s) ds$$

for each case shown in figure 2 and these values are plotted versus AR in figure 3. The ordinate in figure 3 is in fact the ratio of the temperature difference across the layer when natural convection is present to the temperature difference that would exist if conduction was the only heat transfer mechanism. For a given Rayleigh number (i.e., given heat flux), figure 3 clearly shows that there is an aspect ratio for which the mean temperature of the lower surface is a minimum. Also, it may be observed that as the Rayleigh number is increased, the aspect ratio for the preferred cell decreases. These results are consistent with the cases described by Combarnous and Bories (1975) in which the lower boundary of the porous layer is maintained as an isothermal surface as opposed to the constant flux condition specified here. For the isothermal boundary case, the mean Nusselt number (heat transfer coefficient) is maximized to find the preferred aspect ratio.

From figure 3, it was determined that for a Rayleigh number of 190 ($L = 2900$ m, $q = \text{geothermal flux} = 0.220 \text{ W/m}^2$) a suitable aspect ratio would be in the range $0.8 \leq AR \leq 1.0$. Therefore, having chosen the $AR = 0.8$ cell as the basic geometry, a series of computations were made to investigate the appropriateness of the inequality in equation 9. Three of the four parameters in equation 9 were held fixed ($a = \pi/0.8L$ m, $k_{\text{bas}} = 1.0 \times 10^{-14} \text{ m}^2$, $h = 100$ m); the sediment permeability was varied over three orders of magnitude ($1.0 \times 10^{-17} \text{ m}^2 \leq k_{\text{sed}} \leq 1.0 \times 10^{-14} \text{ m}^2$). Streamline plots for several of these computations are shown in figure 4. As anticipated, the figure clearly shows that increasing the sediment permeability results in an increase in the volume of fluid that enters and leaves the basalt layer.

For each of the cases treated, a check was made to test the validity of equation 9. In general, Anderson's hypothesis was proved appropriate though the form of equation 9 makes a quantitative statement about sediment sealing somewhat difficult. In an attempt to make equation 9 a more precise statement, the graph in figure 5 was constructed. The missing quantity in Anderson's analysis is a parameter to measure the effectiveness of the sediment in preventing fluid flow into and out of the basalt layer. An obvious choice for this parameter is the volume inflow (or outflow) crossing the sediment/ocean interface. Shown in figure 5 is a plot of the computed volume flow as a

Figure 3. Normalized Mean Temperature Difference Versus Cell Aspect Ratio.

function of sediment permeability. The volume flow is normalized with respect to the volume flow that occurs when $k_{sed} = k_{bas}$. If a definition of sealing can be established, as say, the point at which the volume flow through the sediment is reduced by a given percentage, then equation 9 can be quantified. As an example, let the basalt layer be considered sealed when the volume inflow has been reduced by 90% due to an increase in sediment flow resistance. Though the data in figure 5 is somewhat limited, this criteria corresponds to the condition

$$k_{sed}/k_{bas} \leq 0.10 \quad (10)$$

That is, when the inequality in equation 10 is satisfied, then the basalt layer is considered sealed.

Though the above result must be considered a preliminary attempt to describe and quantify the process of sediment sealing, it has demonstrated the utility of a finite element model in verifying and extending a theory for a complex physical process.

Cylindrical Heat Source in Ocean Sediment

As an example of the type of problem encountered in the seabed waste disposal program, the situation sketched in figure 6 was considered. The illustrated problem is considered to be generic in nature and is more fully discussed by McVey, et al. (1979).

A cylindrical heat source (waste canister) is emplaced at a

Figure 4. Streamlines for Basalt/Sediment Problem, Various Sediment Permeabilities, $Ra = 190$.

specified depth in a layer of ocean sediment. The boundary and initial conditions for the problem are shown in the figure. The radial and vertical extent of the computational domain was chosen such that the effects of the "far field" boundary conditions were minimized over the time interval of interest. The pore fluid properties were assumed to be those of pure water; sediment properties were obtained from the work of Silva and Calnan (1979). The heat source had an initial power output of 1.5 kW which varied with time according to the thermal decay curve for typical high level nuclear waste.

The free convection process induced by the cylindrical heat source was analyzed using the MARIAH program. Shown in figure 7 are plots of typical isotherms near the source for times of 1 and 10 years after canister emplacement. The symmetry of the temperature field about the horizontal mid-plane of the canister indicates that the heat transfer process in the sediment is conduction dominated. This result is to be expected since the Rayleigh number for this problem is $\sim 10^{-3}$, a value for which

Figure 5. Normalized Volume Flow Rate Versus Sediment Permeability, $AR = 0.8$, $Ra = 190$.

Figure 6. Schematic for Waste Canister Buried in Ocean Sediment.

convective energy transport is expected to be negligibly small (Hickox and Watts 1979).

Even though the motion of the pore fluid is not strong enough to alter the thermal field, fluid motion is present and of some importance in determining the movement of nuclides that may be released from the canister. Shown in figure 8 are plots of the stream function for times corresponding to the thermal results in figure 7. The pore fluid is seen to develop a weak cellular (toroidal) motion due to the local heating by the waste canister. The preferential growth of the cell in the radial direction is a result of the anisotropic permeability of the sediment ($k_r = 10 k_z$). The maximum fluid velocity (~ 0.02 m/yr) occurs at the canister surface approximately 2 years after emplacement. As the heat source decays, the fluid velocity also decreases though the size of the convection cell continues to expand outward from the canister and eventually penetrates the sediment surface.

Results such as those described above have proved to be of great value in the feasibility and engineering studies for the waste disposal program. Canister sizing and thermal power loading are areas that have directly benefited from such analyses. In addition, studies in thermochemistry and ion migration (McVey, et al. 1979) are dependent on the detailed thermal and velocity fields computed by the numerical procedure outlined here.

SUMMARY

The present paper has attempted to present the motivation for a class of porous flow problems as well as a method of solution. The general boundary value problem of heat and mass transfer in a porous media was described. Based on a Galerkin form of the finite element method, a numerical procedure was developed that was suitable for both steady-state and transient problems of convective heat transfer. The utility of the proposed scheme was illustrated through the study of two problems involving flow in oceanic sediments. Through the diversity of the studies undertaken, it is apparent that the finite element procedure proposed here provides a versatile and powerful tool for the solution of engineering problems as well as the study of fundamental physical phenomena.

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Figure 7. Isotherms Near the Waste Canister, 1 and 10 Years After Emplacement.

Figure 8. Streamlines Near the Waste Canister, 1 and 10 Years After Emplacement

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