Monte Carlo Methods for Optimizing Torsatron Configurations

James A. Rome
Oak Ridge National Laboratory, Oak Ridge, TN 37830

It is desirable to optimize torsatron configurations to provide minimum transport losses. Neoclassical calculations of transport coefficients for torsatron configurations have been performed for simplified fields or under the assumption of conservation of the longitudinal adiabatic invariant, J. However, Fourier analysis of torsatron fields for real coil configurations shows that in general, about ten harmonics are required to represent the field. Furthermore, guiding center orbits computed in these fields do not conserve J. In fact, in present day torsatrons collisionless orbits may traverse the entire radial extent of the plasma. When collisions are included, it is therefore doubtful that the transport problem can be treated locally. When particles on these large orbits are lost from the plasma, a significant radial electric field is created which must be included in the analysis.

To treat these problems, Monte Carlo methods may be used to


DISCLAIMER

The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.
advantage because the full torsatron geometry and complicated guiding center orbits are correctly incorporated. Boozer\(^1\), has shown that the guiding center orbits can be elegantly derived from a Hamiltonian written in field line coordinates. In this coordinate system, the only required information about the torsatron geometry is the magnitude of the magnetic field. In addition to speeding up computation of the guiding center motion, field line coordinates annihilate the extraneous helical motion and thus permit an easy physical interpretation of orbits.

Collisions enter the orbit at each integration step by introducing pitch angle scattering according to the Monte Carlo scheme developed by Boozer\(^2\). Thermalization of superthermal ions and energy diffusion of thermal ions can also be introduced. However, instead of calculating a diffusion coefficient, we follow H. Wobig\(^3\) and calculate the particle confinement in steady state. Ions are started at a single energy (kept constant) and at a random location and pitch angle on a given flux surface. If an ion hits the wall, it is started again at its birth flux surface but with a different random location and pitch angle. The plasma density and temperature profiles are assumed constant, consistent with the constant energy model. We differ from Wobig in that a more detailed record is kept of the location and pitch angle of the ion throughout its orbit. This allows us to obtain a Green's function for the problem, \(f(\psi, \psi')\), the density of ions at \(\psi\) due to a source at \(\psi'\). Then, when the calculation is complete, the correct weighting function, \(h(\psi')\), is determined so that the assumed \(n(\psi)\) is given by \(n(\psi) = \int f(\psi, \psi') h(\psi') d\psi'\). Given \(h(\psi')\), any other moment of the
distribution function can be obtained. The internal \( \Psi \)-directed fluxes are obtained by accumulating the signed value of \( \Delta \Psi \) at each time step in each local \( \Psi \)-bin.

Typically, steady state is reached in a few 90° scattering times. For the cases of interest, this is long compared to the transit time of a circulating orbit and similar to the timescale for the complicated trapped particle orbits. Thus collisions help since they scatter ions from orbits that would be collisionlessly lost.

Fig. 1 shows the orbit of a 1-keV proton in an \( l = 2 \) torsatron field. In the polar plot, the radial coordinate is \( \Psi \), and the angular variable describes the poloidal position of the particle. When the ion hits the wall (\( \Psi = 1 \)), it is restarted at \( \Psi = 0.8 \); this is shown as a straight line on the plot. Fig. 2 shows \( f(\Psi,\Psi') \) for \( \Psi' = 0.8 \) as a function of \( \Psi \), and at various times (measured in 90° scattering times). The distribution function rapidly reaches steady state.

REFERENCES

Monte Carlo Run
06/25/82 02:13:01

\[ f(t, \psi) \]

Legend:
- \( t = 0.18 \)
- \( t = 0.38 \)
- \( t = 0.58 \)
- \( t = 0.78 \)
- \( t = 0.98 \)
- \( t = 1.18 \)
- \( t = 1.38 \)
- \( t = 1.58 \)
- \( t = 1.78 \)
- \( t = 1.98 \)