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**Finite Element
Analysis of Arc
Welding**

MASTER

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January 1980

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FINITE ELEMENT ANALYSIS OF ARC WELDING

E. Friedman

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FINITE ELEMENT ANALYSIS OF ARC WELDING

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Analytical models employing numerical simulation techniques have been developed to predict results of the gas tungsten-arc welding process. These results include temperature transients, weld bead dimensions, transient and residual stresses, and distortions.

Heat input from the welding arc to the workpiece and the resulting heat transfer mechanisms in the weldment are characterized analytically for both stationary and moving arcs. Implementation of the welding heat transfer models into two-dimensional finite element formulations is discussed. Included is an accurate, efficient procedure to account for phase change/latent heat effects. Results of weld thermal analyses are presented to illustrate the sensitivity of the details of the heat input distribution. The effects of electromagnetically-induced arc pressure and weld puddle surface tension on weld penetration and bead shape are assessed. A temperature-dependent elastoplastic material characterization, with properties near the melting point extrapolated from material data at lower temperatures, is employed for weld mechanical behavior. Although it would be desirable to have material properties data available at very high temperature levels, the extrapolated properties are used in conjunction with output from welding thermal analyses for calculations of residual stresses, plastic strains, and distortions. Evaluation of the residual plastic strain distribution for a bead-on-plate weld suggests a possible means of assessing high temperature cracking susceptibility. An analysis of temperatures and distortions induced by closure welding of a toroidal seal illustrates the complex geometries that can be treated.

FINITE ELEMENT ANALYSIS OF ARC WELDING

E. Friedman

I. INTRODUCTION

The welding of metal structures provides a means of joining together a number of components. A major objective is to minimize the weld-induced impairment of the properties of those components. Considerable work has been done on developing effective welding techniques for a large number of metals and alloys. Efforts have also been made to identify the various problems that result from welding processes, determine the strength of welded joints and structures subject to specified loading conditions, and establish guidelines and criteria for most effective joint design. The information accumulated in these areas over the years has been overwhelmingly experimental.

Attempts have been made to establish empirical approaches toward understanding the complex behavior of materials due to welding. However, comparatively little effort has been expended in developing and applying analytical models to explain and predict the transient thermal response and the transient and residual mechanical response. Papers published by the Welding Research

Council deal with many of the analysis methods that do exist. These include surveys on heat flow due to welding [1] and on welding-induced stresses and distortions [2]. A recent ASME symposium on numerical modeling of manufacturing processes [3] included several papers that are representative of the state-of-the-art welding analysis.

One of the more commonly used welding processes is gas tungsten-arc welding (GTAW). In this process, pieces of material are joined by heat produced in an electric arc maintained between a nonconsumable (tungsten) electrode and the workpiece. A blanket of gas is used to shield the work from the atmosphere. The passage of high current through the gas ionizes it, generating the heat necessary to melt weldment material. Solidification of the molten metal fuses the pieces.

At the core of the GTAW process is the heat supplied by the arc to the workpiece and then transferred within the workpiece to produce melting. This heat transfer process, or thermal cycle, has many consequences. Complex metallurgical changes take place in the fusion zone where metal is melted and solidified. In adjacent "heat-affected" regions, material is heated to temperatures that are below the melting point, yet are sufficiently high to produce changes in microstructure and mechanical properties. Nonuniform heating and cooling generate plastic deformation and residual stresses that often must be reduced by post-weld treatments. In addition, shrinkage of weld metal and heat-affected zone material induces permanent distortion of the weldment. These phenomena occur to some degree during any arc welding procedure. Various weld defects may also be produced as a result of the thermal cycle. Some of these are lack of fusion, lack of penetration, porosity, hot tearing, and cold cracking [4].

Analytical treatments of temperature transients during welding have most often modeled the effective thermal energy supplied by the heat source as a point source or a line source, depending on the geometry of the weld. Early work done in this area was concerned with the quasistationary transient temperature solution (steady-state with respect to a moving coordinate system) resulting from a line source of heat traveling in a straight path at a constant speed through an infinite plate. The classical line source solution was determined independently by Boulton and Lance-Martin [5] and Rosenthal and Schmerber [6].

From this, Tall [7] developed a numerical model to calculate the longitudinal stresses (stresses in the direction of the weld line) at a section of the plate normal to the weld line. Plastic flow near the weld line was included in the calculation, and mechanical properties - though not thermal properties - were taken to be temperature-dependent. Masubuchi, Simmons and Monroe [8] used Tall's method of analysis to write a digital computer program to calculate temperatures and stresses in welded plates.

Analytical studies of welding stresses and distortions for various weldment configurations are quite complex. Factors considered include transient multi-dimensional temperature distributions, effects of weldment fixturing and surrounding structure on heat sink and constraint characteristics, and realistic models of inelastic material behavior in the vicinity of the weld line. This suggests the use of numerical methods of analysis - in particular, the widely-adaptable finite element method - for predicting transient and residual stresses and distortions.

A determination of the complete thermomechanical response due to welding would require a full three-dimensional, inelastic analysis of mechanical behavior accompanied by computation of the three-dimensional transient temperature distribution. Computer run times would be very large and the resulting costs perhaps prohibitive. Therefore, for weldments under quasistationary thermal conditions, it is reasonable to develop numerical techniques for planar analysis in sections normal to the direction of welding.

The work of Hibbitt and Marcal [9] represented an initial step in the development of numerical analysis techniques to simultaneously account for many welding parameters and factors that previously had been considered piecemeal or not at all. Hibbitt and Marcal designed a finite element transient thermal analysis model to simulate the gas metal-arc welding process, which employs consumable electrodes. This model was used to generate the thermal input for a finite element stress analysis.

The weld thermomechanical model described in this report also utilizes the finite element method to model both the thermal and the mechanical behavior.

Although the analysis techniques are applied to gas tungsten-arc welding, suitable modification of the thermal model would enable the method to be employed for other welding processes as well.

Descriptions of analytical models of heat transfer from the arc to the workpiece and of the weldment thermal cycle are presented first. Included are (1) a summary of the means of including phase change/latent heat effects in the finite element formulation, (2) some representative results of parameter studies that have been carried out for both stationary and moving arcs, and (3) a discussion of the effects of electromagnetically induced arc pressure and weld puddle motion on weld penetration. This is followed by a description of the finite element model formulated to calculate stresses and distortions. Results are presented for transient and residual stresses, distortions, and plastic strains calculated for bead-on-plate welds. Results are also presented for temperatures and distortions resulting from the closure welding of a flexible, toroidal seal.

II. THERMAL ENERGY APPLIED TO WORKPIECE

In the gas tungsten-arc welding process, energy is transferred to the workpiece mainly by four mechanisms [10]:

1. Kinetic energy of the electrons comprising the arc current
2. Heat of condensation of the electrons (work function) penetrating the solid work surface
3. Radiation from the arc
4. Thermal conduction from the arc plasma to the workpiece.

To avoid the extremely complex details of energy transfer from the arc, the magnitude of heat input to the workpiece is expressed simply as the product of the arc power and some efficiency factor η . The arc power is the product of the electric potential E times the arc current I . The arc efficiency is a rather nebulous factor intended to account for all power losses. A determination of the heat input magnitude $Q = \eta EI$ can be made either by correlating computed temperatures with thermocouple readings from test welds, or by making calorimeter measurements for a specified set of welding conditions. Minor changes in welding process variables - e.g., shielding gas, electrode configuration, arc gap, minor element additions to the arc - will affect the arc

efficiency. Numerical simulation of energy transfer in the welding arc, accompanied by supporting test data, helps to explain how these variables perturb the welding process [11].

III. ANALYSIS OF WELDING TEMPERATURES

A. Source Solutions for Simulating Welding Temperatures

Prior to the utilization of sophisticated numerical analysis techniques to simulate the welding thermal cycle, weld-induced temperatures were calculated for bead-on-plate welds by idealizing the heat from the welding arc to be either a point source or a line source of heat. Solutions for this class of problems follow [12]:

Consider a plate of infinite extent bounded by the planes $z = 0$ and $z = h$. Heat is input at a point moving at uniform speed v on the surface $z = 0$ so that at any time t , the coordinates of the point source are given by $x = vt$, $y = 0$, $z = 0$. Neglecting end effects resulting from initiation or termination of the heat source or from the finite length of the weld plate, the resulting temperature distribution is stationary with respect to a moving coordinate system whose origin coincides with the point of application of the heat source. This class of temperature response is termed "quasistationary." Introducing the coordinate transformation $\xi = x - vt$ into the general form of the heat conduction equation yields:

$$\kappa \nabla_1^2 T + v \frac{\partial T}{\partial \xi} = 0 \quad , \quad (1)$$

where $\nabla_1^2 = \partial^2/\partial \xi^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ in a Cartesian system. For this equation to be valid, both the diffusivity κ and the conductivity k are constant. Three basic closed-form solutions to this equation for plates of various degrees of thickness are now presented. In each case, the surfaces $z = 0$ and $z = h$ are assumed adiabatic, and no account is taken of latent heat effects due to melting and solidification that take place in actual welds.

1. Very Thick Plate

Heat is conducted through the plate without hindrance until the insulating effect of the adiabatic surface $z = h$ is felt. For a sufficiently thick plate,

however, the temperature rise at $z = h$ is so small that the surface effect is negligible. For computational purposes, the solution for a plate of infinite thickness (i.e., a semi-infinite body) is applicable. With the strength of the heat source given by Q , the temperature solution T is [13]:

$$T(\xi, y, z) = T_0 + \frac{Q}{2\pi kh} \left[e^{-\frac{v}{2k}(R+\xi)} / (R/h) \right], \quad (2)$$

where T_0 is the reference temperature and $R = (\xi^2 + y^2 + z^2)^{1/2}$.

2. Moderately Thick Plate

In this case, the temperature rise at $z = h$ is large enough to necessitate the inclusion of the surface effect. The point source solution for a semi-infinite space results in non-zero heat flow at $z = h$. An image source applied at $\xi = 0, y = 0, z = 2h$ insures that $z = h$ is adiabatic. There is, however, non-zero heat flow at $z = 0$ due to the image source. Another image source applied at $\xi = 0, y = 0, z = -2h$ is now required to satisfy conditions at $z = 0$. Carrying this along ad infinitum, an infinite distribution of image sources is applied at points $\xi = 0, y = 0, z = \pm 2nh, n = 1, 2, \dots$. The superposition of the very thick plate solution and the image solutions yields the temperature solution satisfying the adiabatic conditions at $z = 0$ and $z = h$ [13]:

$$T(\xi, y, z) = T_0 + \frac{Q}{2\pi kh} e^{-\frac{v\xi}{2k}} \sum_{n=-\infty}^{\infty} \left[e^{-\frac{vR_n}{2k}} / (R_n/h) \right], \quad (3)$$

where $R_n = [\xi^2 + y^2 + (z + 2nh)^2]^{1/2}$. Note that the thicker the plate, the more rapidly the series solution converges.

3. Thin Plate

The solution for the moderately thick plate would apply to this case as well, but convergence would be extremely slow. As an alternative, the heat source of strength Q , rather than being applied at a point on the surface $z = 0$,

is applied as a uniform line source of heat through the thickness. The temperature distribution is then uniform through the thickness as well, and is given by [5,6]:

$$T(\xi, y) = T_0 + \frac{Q}{2\pi kh} e^{-\frac{v\xi}{2\kappa}} K_0\left(\frac{vr}{2\kappa}\right), \quad (4)$$

where $r = (\xi^2 + y^2)^{1/2}$, and K_0 is the modified Bessel function of the second kind and zero order.

Offshoots of these three fundamental cases have also been developed. These include: (1) consideration of plates of finite width, in which case temperatures due to image sources are superposed on the infinite plate solutions to satisfy the necessary adiabatic edge conditions [14]; (2) infinite plates with temperature-varying conductivity but constant diffusivity [15]; (3) line sources of heat traveling on a circular path [16]; and (4) certain non-stationary conditions which simulate initiation or termination of the heat source [17].

Several limitations exist with respect to employing these solutions to simulate the welding thermal cycle. The more severe of these pertain to the calculation of temperatures in the weld metal and heat-affected zones, where changes in microstructure take place. In these regions finite distribution of the heat input (as opposed to energy deposited in an infinitesimally narrow band of material), and consideration of energy absorbed and released due to melting and solidification, respectively, have a profound influence on temperature distribution, heating and cooling rates, and weld metal and heat-affected zone dimensions. While it is possible to superpose source solutions to simulate the response to a finite distribution of heat input, numerical evaluation of temperatures would be difficult. Moderately thick plates of finite width, which generally would be of most interest, would be especially difficult to evaluate. Temperatures computed from a source solution may be of some use outside the heat-affected zone, however, even though thermal properties are most often assumed to be constant and heat losses to the surroundings by convection and radiation are usually neglected. Hence, if one is careful, it may be possible

to use line source solutions for thin plates to calculate residual stresses in welded structures built up from thin, flat plates [7]. These solutions, however, are not sufficiently reliable to simulate the thermal response in the region near the heat source.

B. Analytical Model for Welding Thermal Cycle

In addition to providing more reliable temperature solutions than the point and line source techniques, numerical solutions make it possible to obtain weld thermal cycles for weldment configurations other than flat plates. The model described here follows that given by Friedman [18].

1. Quasistationary Thermal Conditions

Calculation of the transient temperature distribution is based on attainment of quasistationary conditions, as described previously. Consider the planar weldment illustrated in Figure 1. The temperature at any point is expressed functionally as:

$$T(x_1, x_2, x_3, t) = T(x_1, x_2, x_3 - vt) \quad (5)$$

Thus, given the transient temperature distribution at any one section of the weldment normal to the weld direction (for example, $x_3 = 0$), the temperature at any other section is determined by an appropriate shift in the time scale as follows:

$$T(x_1, x_2, x_3, t) = T(x_1, x_2, 0, t - x_3/v) \quad (6)$$

The problem is therefore reduced to finding the two-dimensional unsteady temperature field at a section normal to the weld line. A planar analysis may be used for this purpose when the weld speed, relative to a characteristic thermal diffusion rate for the material, is sufficiently high that the amount of heat conducted ahead of the welding electrode is small relative to the total heat input. In this case, the net heat flow across any infinitesimally thin slice of the weldment normal to the weld line is assumed negligible relative to heat diffused within the slice itself, and the term $(\partial/\partial x_3)[k(T)(\partial T/\partial x_3)]$ is neglected in the heat conduction equation. Two-dimensional heat conduction at the section $x_3 = 0$ is thus treated. (For a welding heat source moving on a

circular path at constant speed in an axisymmetric weldment, and effects resulting from initiation and termination of the heat source are neglected, and heat conduction in a radial slice is treated disregarding net heat flow in the circumferential direction.)

2. Characterization of Heat Input

The most critical input data required for welding thermal analysis are the parameters necessary to describe the heat input to the weldment from the arc. Both the magnitude and distribution of heat input will influence: (1) the dimensions of the weld metal and heat-affected zones, (2) the cooling rates, (3) the peak temperature distribution, and (4) the temperature gradients necessary to calculate stresses and distortions. Following the approach of Pavelic, et al. [19], the heat from the arc is, at a given point in time, assumed to be deposited on the weldment surface as a radially symmetric normal distribution function. Letting r be the distance from the center of the heat source, the heat flux q acting on the surface is given by

$$q(r) = \frac{3Q}{\pi \bar{r}^2} \exp[-3(r/\bar{r})^2] \quad , \quad (7)$$

where $\bar{r}$ is a characteristic radial dimensional distribution parameter that defines the region in which 95 percent of the heat flux is deposited. If $t = 0$ is defined as the time at which the center of a moving heat source passes over the section $x_3 = 0$, the heat flux distribution on the surface of the weldment at section $x_3 = 0$ is given by

$$q(x_1, t) = \frac{3Q}{\pi \bar{r}^2} \exp[-3(x_1/\bar{r})^2] \exp[-3(vt/\bar{r})^2] \quad (8)$$

The three heat input parameters characterizing a moving arc weld - Q , $\bar{r}$ and v - are embodied in this formula. A heat input of Q/v per unit length of weld is supplied by the moving arc. For a stationary arc, a total amount of heat Qt^* , where t^* is the duration of heating, is supplied. The inverse of the speed of a moving arc is therefore akin to the duration of a stationary arc. On those surfaces not subject to direct heating by the arc, heat is assumed lost to the surroundings by combined radiation and natural convection.

3. Finite Element Method Applied to Welding Temperatures

The finite element method of analysis for transient heat conduction [20,21] has in recent years become a powerful tool for thermal analysis of complex shapes. The finite element discretization for transient heat conduction analysis entails the reduction of the transient heat conduction equation from its general partial differential form to a set of simultaneous ordinary differential equations. These equations contain temperatures and time derivatives of temperatures at a discrete number of points, or nodes. A finite difference scheme - the Crank-Nicholson method [20] in this case - is used to integrate these equations. This results in a set of simultaneous algebraic equations which must be solved at each of a number of time steps. Nonlinearities resulting from temperature-dependent thermal properties and boundary coefficients (radiation, natural convection) are treated by having these quantities re-evaluated at each time step, at the known temperatures at the beginning of the step. The material and boundary coefficients are therefore made to be piecewise-constant functions of temperature. Except for those regions undergoing phase change, the analysis procedure is linear and noniterative in each time step.

The finite element used for the welding thermal analysis is the two-dimensional, biquadratic, isoparametric element [21]. The finite element formulation for this analysis has been incorporated into the heat conduction portion of the BESTRAN finite element system [22].

C. Procedure for Including Phase Change/Latent Heat Effects

The method employed for treating melting and solidification effects in weldments considers phase change in alloy systems, in which phase transformation takes place over a finite temperature range [23,24]. The procedure described to include latent heat effects in numerical solution techniques separates the latent heat from the specific heat. This avoids inaccuracies inherent in procedures which treat the latent heat as an apparent increase in the specific heat. The method is equally applicable to finite difference and finite element transient heat conduction computer codes, although it has been included only in the BESTRAN finite element system [22].

Analytical methods developed to solve heat conduction problems with freezing or melting may be divided into two broad classifications. The first approach is to consider the phase change front as a boundary whose motion is initially unknown, and across which the flow of heat is discontinuous. By specifying continuity of temperature and the proper energy balance across the solid/liquid interface, equations are formulated to solve for the location of this interface as a function of time. Boundary value problems formulated in this manner are all outgrowths of the Stefan problem - the class of boundary value problems of the parabolic type with boundaries moving at initially unknown rates. They are strictly applicable to phase change in pure metals.

The present treatment, however, is addressed toward the solution of phase change problems in alloys, in which latent heat liberated or absorbed due to phase transformation is effected over a finite temperature range. For this class of problems, a well-defined phase change front is not specified; rather, the latent heat is considered as an increase in the heat capacity of the material in the phase change range. This complexity virtually requires the employment of numerical methods of heat conduction analysis.

A common means of incorporating phase change into numerical solution methods is to treat the latent heat as an increase in the specific heat between the solidus and liquidus temperatures. This was done, for example, by Hibbitt and Marcal [9] for welding thermal analysis. Comini, et al[25] defined the increased latent heat in terms of an averaging of enthalpy gradients. This is designed to avoid the possibility of missing peak values of heat capacity in the phase change range.

The phase change treatment presently considered recognizes that a very large increase in heat capacity over a relatively small temperature range requires that, for a given time increment, the proper magnitude of latent heat involved in the transformation is dependent upon temperature both at the beginning and at the end of the time step. An iterative method is thus employed to directly account for the latent heat. Although the method is iterative, convergence is usually quite rapid. Time increments larger than those required for the noniterative increased specific heat procedure may generally be used, even though the inaccuracies inherent in a procedure which does not account for the temperature at the end of the increment are avoided.

For a given transformation, phase change takes place over a range of temperatures bounded by T_S and T_L . For melting or freezing, T_S and T_L represent the solidus and liquidus temperatures, respectively, while for solid state transformations, they define the applicable temperature range. Consider a unit mass of material, initially at temperature $\bar{T}$, being heated to a temperature T . Part or all of this temperature rise may take place in the phase change range. The heat absorbed by the material during this temperature rise is the enthalpy change ΔH , given by:

$$\Delta H = \int_{\bar{T}}^T \left\{ c(T) + \left[\frac{\ell}{T_L - T_S} - c(T) \right] \left[h(T - T_S) - h(T - T_L) \right] \right\} dT, \quad (9)$$

where c and ℓ are the specific heat and latent heat of the material, respectively. Note that in this form, ℓ represents the total thermal energy required to raise the temperature of the material from T_S to T_L . In Equation (9), h is the Heaviside step function, whose value is unity or zero depending on whether its argument is positive or negative. The heat absorbed during the temperature rise (or liberated during a temperature drop) is illustrated in Figure 2(a) as the enthalpy change. It is assumed in Equation (9) that enthalpy varies linearly with temperature during phase change.

In the absence of phase change, the variation of specific heat with temperature is, for practical applications, sufficiently small that this material property can be idealized to be piecewise constant for the temperature range considered. That is, the specific heat is taken to be constant over each time step in the transient problem, and its value is computed based on the known temperature at the beginning of the step. Although it is possible to iterate on the specific heat so that it is evaluated, say, at the middle of the time step, this time-consuming procedure is seldom warranted.

Consider now the presence of phase change. For those computational techniques in which the latent heat is made equivalent to an apparent increase in the specific heat, Figure 2(b) would then represent a plot of specific heat against temperature. Thus during a temperature rise in an increment in which phase change takes place, a constant value of specific heat is used for the entire time step. Whether this specific heat is based on the known temperature

$\bar{T}$ existing at the beginning of the time step, the temperature T at the end of the increment, or some temperature that lies between $\bar{T}$ and T (which can be obtained by iterative means), it is clear that the specific heat employed has either a relatively low value based upon its evaluation at a temperature outside the phase change range, or a high value evaluated from a temperature within the range. Either is inaccurate since, as can be seen from Figure 2, the heat absorbed depends on some weighted average of the two, the assigned weights depending on both the temperatures at the beginning and at the end of the time step. The errors inherent in employing a uniform specific heat throughout a time step containing a phase change can be minimized by using progressively smaller time steps; however, this may prove to be a highly inefficient means of accurately accounting for latent heat effects.

1. Phase Change Coefficients

The phase change computational technique explicitly separates the latent heat from the specific heat, thus eliminating the source of error. The specific heat is now a relatively smoothly varying function of temperature. It is approximated as a piecewise constant function, so that $c(T) \approx c(\bar{T})$ in a time increment, and Equation (9) becomes

$$\Delta H = c(\bar{T})(T - \bar{T}) + \left[\frac{\ell}{T_L - T_S} - c(\bar{T}) \right] \int_{\bar{T}}^T \left[h(T - T_S) - h(T - T_L) \right] dT \quad (10)$$

The integral in this equation supplies the weight of the contribution of latent heat to the enthalpy change. Its value depends on the relative magnitudes of T_S , T_L , $\bar{T}$ and T . If $\bar{T}$ falls within the phase change range, $c(\bar{T})$ is evaluated as the specific heat of either the liquid (for melting) or the solid (for freezing).

Equation (10) is expressed in the form:

$$\Delta H = c(\bar{T})(T - \bar{T}) + \left[AT - (B_1 T_S + B_2 T_L + B_3 \bar{T}) \right] \left[\frac{\ell}{T_L - T_S} - c(\bar{T}) \right], \quad (11)$$

where the four phase change coefficients A , B_1 , B_2 , B_3 are given in Table 1.

Table 1. Phase Change Coefficients

Range of $\bar{T}$	Range of T	A	B_1	B_2	B_3
$\bar{T} \leq T_S$	$T \leq T_S$	0	0	0	0
	$T_S < T < T_L$	+1	+1	0	0
	$T_L \leq T$	0	+1	-1	0
$T_S < \bar{T} < T_L$	$T \leq T_S$	0	-1	0	+1
	$T_S < T < T_L$	+1	0	0	+1
	$T_L \leq T$	0	0	-1	+1
$T_L \leq \bar{T}$	$T \leq T_S$	0	-1	+1	0
	$T_S < T < T_L$	+1	0	+1	0
	$T_L \leq T$	0	0	0	0

This non-linearity must be treated by an iterative procedure.

2. Iterative Procedure

The finite element idealization yields a set of simultaneous equations for each element at each time step. These are designated the thermal stiffness equations. They relate the "thermal forces" at the n nodes of the element to the nodal temperatures at the end of the time step, and to the distributed "thermal loads" acting on the element. In matrix notation,

$$\{F\} = [K^E] \{T\} + \{F^E\} \quad (12)$$

$\{F\}$ is the nodal force vector, $\{T\}$ the nodal temperature vector, $[K^E]$ the thermal stiffness matrix, and $\{F^E\}$ the vector of nodal forces required to balance the distributed thermal loads. If there were no phase change to be considered, the elements of $[K^E]$ and $\{F^E\}$ would all be known. With latent heat effects, they now depend on temperature, so that

$$\{F\} = [K^E(T)] \{T\} + \{F^E(T)\} \quad (13)$$

The iterative process employed merely evaluates $[K^E]$ and $\{F^E\}$ for iterate i as functions of temperature solutions computed for iterate $i-1$. (Acceleration parameters may also be introduced to speed convergence.) Initially $i=1$. An educated guess is made for the temperature distribution to evaluate $[K_1^E]$ and $\{F_1^E\}$. In general, then,

$$\{F_i\} = [K_i^E] \{T_i\} + \{F_i^E\}, \quad i = 1, 2, \dots, \quad (14)$$

where $[K_i^E] = [K^E(T_{i-1})]$ and $\{F_i^E\} = \{F^E(T_{i-1})\}$. T_0 (corresponding to $i = 1$) is the guess temperature, which may be found, for example, by linear extrapolation of the temperature solutions for the two previous time increments.

The manner in which the latent heat terms of Equation (11) are included in $[K^E(T)]$ and $\{F^E(T)\}$ is described by Friedman [24]. The linking of and subsequent solution for an assembly of finite elements may then proceed in one of the usual ways as discussed, for example, by Zienciewicz [26].

D. Thermal Response of Stationary Arc Welds

The finite element method for transient heat conduction analysis has been employed to calculate temperatures and weld bead dimensions in 6.35 mm (0.25 in) thick, 50.8 mm (2 in) square NiCrFe Alloy 600 flat plates subject to a stationary GTAW heat source [27]. Tests were run to provide data with which the computed thermal cycle can be correlated. The experimental arrangement is shown in Figure 3, and is described in detail in the paper by Glickstein, et al. [28].

The geometry of the analytical model is that of a section of the plate normal to its plane and passing through its center. Heat flow in regions removed from the plate edges is assumed axisymmetric about the center of the heat source, which is coincident with the center of the plate. Heat flow in the square plate is thus simulated by conduction in a 50.8 mm (2 in) diameter circular plate. This assumption was validated by tracings taken from thermocouples located at the undersurface of the plate at the same distance from the weld centerline, but at different sections passing through the center. The finite element mesh employed for the weldment cross-section is shown in Figure 4. (Only half the section is shown due to symmetry about the centerline.) A relatively fine mesh is used in the vicinity of the centerline because of the very high temperature gradients developed in this region.

Weld-induced temperature calculations are carried out using the heat input distribution given by Equation (7). Computed temperature transients are in good agreement with thermocouple data in weldment regions removed from the weld metal and heat-affected zones. Typical results are shown in Figure 5 for arc current $I = 91$ A, arc potential $E = 7.7$ V, and duration of welding of 10 s. With an assumed arc efficiency of 0.76, the heat input $Q = 530$ W. The assumed heat input distribution parameter $\bar{r}$, is 3.81 mm (0.15 in).

While the predicted transient temperature response is satisfactory at the thermocouple locations shown in Figure 3, the calculated shape of the weld bead (defined by the furthest extent of the melting isotherm) is not in good agreement with bead shapes observed from metallographic sections. Although calculated depths of penetration of the weld bead compare favorably with measured values, the predicted width of the fusion zone is much less than the measured width. This discrepancy is attributed to the use of a high thermal conductivity value to simulate heat transfer in the weld puddle. Improved analytical models are required to better account for convective heat transfer in the puddle due to electromagnetically-induced circulation, and to ascertain the effects of changing puddle shape due to arc jet forces. The latter phenomenon will be discussed subsequently.

The thermal response of a weldment depends essentially on the heat input from the arc, the heat transfer characteristics of the weldment, and the weldment geometry. With respect to heat input from the arc to the workpiece, the magnitude of the heat input is obviously of prime importance. In addition to its effects on the temperature histories throughout the weldment and the weld bead penetration characteristics, the heat input magnitude relative to the thickness of the weldment determines whether a "thick" plate or a "thin" plate type of thermal response is obtained. The type of response, in turn, determines the nature of both the growth of the weld puddle as heat is added and the temperature distribution. The distribution of heat input from the arc, though having a relatively small effect on the thermal response in regions removed from the weld metal and heat-affected zones, has a significant influence on the geometry of the weld puddle.

A study has been performed [27] to investigate the growth of the weld puddle and the influence of the details of the heat input distribution on weld bead penetration and shape. One of the more interesting phenomena examined pertains to the effect of the undersurface of the weldment on heat retention near the weld centerline and its effect on penetration. For relatively low magnitudes of heat input, or arc current, such that the puddle is shallow relative to the weld thickness, heat is conducted with little resistance in all directions (see Section III.A). The puddle grows as if the plate were extremely thick. As current is increased and weld penetration is about 20 percent of the plate thickness, the resistance of the undersurface comes into play, accelerating penetration. At a depth of penetration on the order of 60 percent of the thickness, the insulating effect of the undersurface predominates. A large amount of heat is accumulated in the high-temperature, solid region between the puddle and the undersurface. Very slight increases in arc current produce rapidly increasing puddle depths until a full penetration weld is attained. The thickness effects are illustrated in Figure 6.

The dimensions of a weld bead depend on both the magnitude and the distribution of heat input from the arc. The sensitivity to the distribution of the heat input is depicted in Figure 7, which shows the variation of weld bead dimensions with the characteristic distribution parameter $\bar{r}$, for a number of durations of heating at a fixed magnitude of heat input. The different localized heat flow patterns that result from different distributions of heat input have a marked influence on both the size and shape of the weld metal zone.

Additional results of the analysis and test program for heat transfer in stationary arc welds may be found in the paper by Friedman and Glickstein [27]. These typical examples of the results of analytical modeling of the welding thermal cycle illustrate the potential of welding thermal analysis for determining the interrelationship of heat input magnitude, distribution, and duration (or welding speed for moving arc welds) as they influence penetration, weld metal zone shape, and temperature transients. The versatility of the finite element method also permits complex weldment shapes to be treated and changes in weld joint geometry to be assessed. In particular, welding analysis in conjunction with limited testing can be used to calculate the combination of

thermal and geometric parameters that will produce an optimum weld configuration for a given joint. Further study on the physics of the welding arc, however, is required to understand and quantify such weld parameter effects as electrode shape, arc gap, shielding gas, and trace elements on the magnitude (or arc efficiency) and distribution of the heat input.

E. Effects of Arc Pressure and Puddle Surface Tension on Weld Bead Configuration

Implicit in the stationary arc weld thermal analysis procedure is the assumption that the molten weld puddle maintains a fixed shape as heat from the welding arc is being applied. However, gravitational forces, arc pressure [10], and fluid flow induced by magnetic and convective forces in the weld puddle cause local distortion of the puddle. The magnitude of this local distortion is most pronounced during the root pass of a full penetration weld and results in the weld underbead upon solidification.

In an extension of the stationary arc weld analysis [27] the molten puddle is considered to be acted upon simultaneously by distributions of heat input from the arc, pressure from the arc, and gravitational forces in the puddle [29]. The effects of magnetic forces and convective motion are not addressed in this treatment. The arc pressure and gravitational forces result in the top surface of the puddle being depressed below its original level prior to melting. If full penetration is achieved, the same forces cause the bottom surface of the puddle to be depressed. Surface tension at interfaces between the liquid puddle and the gaseous atmosphere serves to resist puddle distortion. For full penetration welds, the surface tension supports the molten material and prevents burn-through.

1. Analytical Model of Puddle Distortion

The analytical model of weld puddle distortion considers a stationary arc heating the surface of a flat plate. This implies that the distributions of both heat and pressure acting on the weldment surface are axisymmetric. The model is based on the weld puddle deforming under the action of pressure from the welding arc and gravitational forces in the puddle only at temperatures exceeding the melting point. For purposes of calculating these local deformations (as opposed to distortion resulting, for example, from shrinkage strains, which will be discussed in Section IV), material at temperature less than the

liquidus is taken to be rigid. At any given time during which heat is being applied by the stationary heat source, a given volume of the weldment material may be in the molten state. This volume may or may not fully penetrate the thickness of the weldment. Nevertheless, its shape is axisymmetric with respect to the center of the axisymmetric heat source. The molten puddle is acted upon by axisymmetric distributions of thermal energy, arc pressure, and gravitational forces, while surface tension at the interface of the weld puddle with the surrounding atmosphere supports the molten material.

The new features added to the previous heat transfer model [27] are (1) simulation of pressure from the welding arc and gravitational forces in the puddle, (2) distortion of the puddle due to these forces, (3) resistance to puddle shape change by surface tension at puddle interfaces with atmosphere, and (4) heat transfer in the weldment as the puddle continually changes shape.

The present analysis procedure deviates from the previous one when weldment material melts. At any time step during which the temperature at one or more nodes of the finite element mesh exceeds the specified liquidus temperature, the motion of these nodes due to arc pressure and gravitational forces is determined. The nodes at temperatures less than the melting point are completely restrained from any motion. As illustrated in Figure 8, the calculated nodal displacements are employed to update the undistorted mesh geometry. The resulting distorted mesh is used for temperature calculations in the next time step. An outline of the required calculations is as follows:

1. Calculate nodal temperatures in time increment.
2. If all nodal temperatures are less than the melting point (liquidus), return to Step 1 for next time increment.
3. If any nodal temperature exceeds the melting point, calculate displacements of nodes in puddle.
4. Add displacements to undeformed grid coordinates to obtain distorted mesh geometry.
5. Return to Step 1 for next time increment using the distorted geometry.
6. Continue until calculations are completed for the time period of interest or until solution becomes unstable. (This occurs subsequent to full penetration when the undersurface sag becomes very large, indicating the onset of burn-through.)

In general, calculations during an increment of time involve determinations of temperatures throughout the weldment, and of distortion in that part of the weldment in which temperatures exceed the melting point. The BESTRAN finite element system [22] is used to make these computations. The output of the temperature calculations is employed to determine what region of the weldment is in the molten state and, therefore, subject to distortion. The results of the distortion calculations are utilized to calculate the distorted finite element mesh required for the next set of temperature calculations.

The heat input from the welding arc is applied as the radially symmetric normal distribution function given by Equation (7). The heat supplied by the arc is conducted through the weldment in its distorted configuration. Under the action of arc pressure and gravitational forces, the heated surface of the weld puddle in the vicinity of the weld centerline is depressed below its original elevation, as shown in Figure 8. This depression may significantly affect the penetration characteristics of the weld. Since all calculations are carried out in increments of time, the weld puddle distortion progresses in discrete steps, rather than continuously.

2. Characterization of Arc Pressure

Puddle distortions are computed by specifying mechanical properties of the weldment to simulate the behavior of a nearly incompressible liquid with negligible shear resistance. The pressure applied by the arc to the surface of the weldment is taken to be distributed as a radially symmetric normal distribution function, such that

$$p(r) = p_0 \exp [-3(r/r_f)^2] \quad , \quad (15)$$

where p is the pressure acting on the surface, p_0 is the maximum pressure at the weld centerline, r is the distance from the weld centerline, and r_f is a dimensionless parameter defining the region in which 95 percent of the electromagnetically-induced arc force is applied. p_0 and r_f completely define the pressure distribution from the stationary arc. This is analogous to the steady distribution of heat applied to the weldment (Equation (7)).

3. Simulation of Surface Tension

Distortion of molten material under the action of arc pressure and gravitational forces is inhibited by the surface tension existing at the interface between puddle and atmosphere. At a given time, there may exist one or two of these interfaces, depending upon whether partial or full penetration has been attained.

Surface tension is simulated in the finite element model by a membrane with uniform isotropic tension equal to the surface tension N . Consider a circular membrane of radius b supported around its edge and subject to a normal pressure $q(r)$, which may vary radially. The normal deflection w of the membrane satisfies the equilibrium equation [30]:

$$(1/r) (d/dr) (r dw/dr) = -q/N, \quad (16)$$

provided $d^2w/dr^2 \ll 1$.

Integrating this equation with the conditions $(dw/dr)|_{r=0}=0$ and $w(b) = 0$, the following is obtained for the deflection:

$$w(r) = \ln(b/r) \int_0^b (q/N) \rho d\rho - \int_r^b (q/N) \ln(\rho/r) \rho d\rho. \quad (17)$$

This equation is used to establish a relationship between the restraining forces of the membrane and the membrane deflections at a discrete number n of locations (corresponding to finite element nodes) on the puddle surface. The deflection $w_i = w(r_i)$ at a location defined by $r = r_i$, due to a ring load P_j applied at a distance r_j from the center of the membrane, is found by letting the pressure distribution $q(r) = [P_j/(2\pi r_j)] \delta(r-r_j)$, where δ is the Dirac-delta function, and $i, j = 1, 2, \dots, n$ ($r_{i+1} > r_i$). Substituting this expression into Equation (17), the following linear flexibility equation is obtained:

$$w_i = C_{ij} P_j, \quad (18)$$

where

$$C_{ij} = \ln(b/r_j) \quad j \leq i \\ = \ln(b/r_i) \quad j > i. \quad (19)$$

Equation (18) is generalized in matrix notation by:

$$\{w\} = [C] \{P\} \quad , \quad (20)$$

where the elements of the symmetric flexibility matrix $[C]$ are given by Equation (19).

The inverse of the flexibility matrix is the membrane stiffness matrix $[K]$. The constraint effect of puddle surface tension is incorporated into the finite element model as an equivalent elastic foundation with stiffness $[K]$.

4. Results of Parametric Studies

In order to assess the effects of arc pressure, gravitational forces, and surface tension on weld penetration, weld bead dimensions, and puddle distortion, all computations were made for a heat input of fixed magnitude and distribution applied to the center of a 50.8 mm (2 in) diameter, 6.35 mm (0.25 in) thick circular Alloy 600 plate. The thermal input parameters (see Equation (7)) used are $Q = 1000 \text{ W}$, $\bar{r} = 3.81 \text{ mm}$ (0.15 in). The magnitude of heat input is equivalent to an arc current of 125 A, potential of 10 V, and arc efficiency of 0.8.

The effects of arc pressure and gravity forces on penetration and weld bead dimensions are illustrated by determining the growth of the weld puddle with time neglecting puddle distortion (i.e., neglecting arc pressure and gravity forces or assuming very large surface tension). The results are then compared with those obtained using representative puddle distortion parameters. For this purpose, a reasonable estimate of the maximum arc pressure is 1000 Pa (0.15 psi) [10,31]. The arc pressure distribution parameter r_p is taken to be 3.81 mm (0.15 in) (this is consistent with the limited number of measurements reported in the literature), and a surface tension of 1 N/m (0.0057 lb/in) is used.

The results plotted in Figure 9 show that the distortion of the puddle contributes significantly to the penetration characteristics of the weld - especially when the depth of penetration approaches the weldment thickness. When the effects of puddle distortion due to arc pressure and gravitational forces are included, full penetration occurs after 32 s of heating; neglecting puddle distortion, full penetration is not yet attained at 40 s.

The maximum arc pressure resulting from electromagnetic forces is proportional to the square of the current [10]. It is thus of interest to determine the effects of arc pressure on the growth and penetration of the weld puddle. (Although an increase in current produces increases in both heat input and arc pressure, only the latter is considered here. Effects of heat input on penetration and weld bead shape are illustrated in Figures 6 and 7.)

The results are presented in Figure 10, in which the growth of the puddle dimensions with time are plotted for a number of values of the maximum arc pressure, and in Figure 11, which shows how the time to full penetration is influenced by the magnitude of the arc pressure. Figure 10 also demonstrates that arc pressure significantly influences weld penetration when the puddle penetrates to approximately 60 percent of the weldment thickness. For these penetrations, the depression of the heated surface of the puddle is of sufficient magnitude that the rate of further penetration depends strongly on the degree to which the surface is depressed. The effect on bead width (at the heated (top) surface) is less pronounced, since the influence of puddle depression on the radial transfer of heat in the weldment is less than on the heat transfer through the thickness.

A study was also performed to illustrate the influence of surface tension on puddle, or weld bead, dimensions. Calculations were made for values of surface tension $N = 1, 0.5, \text{ and } 0.1 \text{ N/m}$ ($0.0057, 0.0029, \text{ and } 0.0006 \text{ lb/in}$) to illustrate the sensitivity of this physical property. Figure 12 shows the effect of surface tension on penetration when the puddle depth exceeds about 60 percent of the weldment thickness. Puddle width at the heated surface is relatively insensitive to changes in surface tension. For the case of $N = 0.1 \text{ N/m}$ (0.0006 lb/in), the restraint forces introduced by the surface tension are so small that the puddle distorts severely, resulting in an ill-behaved finite element solution. At the other extreme, values of surface tension only slightly above 1 N/m (0.0057 lb/in) produce a restraint effect sufficiently large that the limiting case of no distortion is approached.

Further efforts are required to measure and control pressure in the welding arc and to investigate surface tension in molten metals. For example, a determination of how minor elements affect surface tension [32] would be a

major step in controlling both puddle distortion and weld penetration. In addition, analytical and experimental work on fluid motion and on convective heat transfer in the puddle needs to be pursued.

F. Thermal Analysis of Moving Arc Welds

The heat flow pattern resulting from a moving arc weld is three-dimensional; temperature gradients exist through the thickness of the weldment, as well as in directions parallel and transverse to the welding direction. The added complexities arising from the nonlinear phenomena previously discussed require that approximations and idealizations be made to formulate an analytical model that is computationally manageable while maximizing the usefulness of the results obtained.

Analytical models have been constructed for conditions in which the welding arc is moving at constant speed v on a regular path. End effects resulting from initiation or termination of the weld have received little consideration. The spacial variable x_3 coincident with the welding direction (see Figure 1) is combined with time t to form a new variable $\tau = t - x_3/v$ (see Equation (6)). Assuming thermomechanical uncoupling, the three-dimensional, transient heat conduction equation is transformed to:

$$\frac{\partial}{\partial x_1} \left(k \frac{\partial T}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left(k \frac{\partial T}{\partial x_2} \right) + \frac{1}{v^2} \frac{\partial}{\partial \tau} \left(k \frac{\partial T}{\partial \tau} \right) - \rho c \frac{\partial T}{\partial \tau} = 0 \quad (21)$$

Although written for variable thermal properties, Equation (21) is equivalent to Equation (1) with $\tau = -\xi/v$. With this formulation, the time-dependent solution at any section normal to the welding direction (for example, $x_3 = 0$) is expressed as a function of the variable τ , and may therefore be used directly to evaluate temperatures at any other section.

As discussed in Section III.A.3, analytical treatments of the "quasistationary" welding problem for thin weldments have often been considered by assuming the thermal energy supplied by the welding heat source to be deposited as a line source. For this approach, the second term of Equation (21) is neglected, and temperature transients are calculated independent of the thickness direction. An alternative procedure is to retain the second term and revert back to the form of the quasistationary heat conduction equation given by Equation (1). Incorporation of this equation into a finite element

formulation produces a set of equations akin to a steady-state formulation in terms of the three spacial variables ξ , y and z . However, the thermal stiffness matrix $[K^E]$ given in Equation (12) is unsymmetric.

Another alternative to the line source approach is to retain the second term of Equation (21) in order to calculate heat flow in the thickness (x_2) direction, and neglect the third term. This is justified when the weld speed is sufficiently high relative to a characteristic diffusion rate. A transient, two-dimensional heat conduction formulation for any section ($x_3 = 0$) normal to the welding direction remains. With $t = \tau + x_3/v$, the complete, three-dimensional solution - neglecting end effects - is computed. This procedure is amenable for direct incorporation into existing two-dimensional finite difference or finite element heat conduction computer programs, and has been implemented in the BESTRAN [22] finite element system. The heat input distribution applied to the weldment at $x_3 = 0$ is given by Equation (8).

The restriction of high weld speed can be relaxed. To account for net heat flow in the welding direction, the third term of Equation (21) may be approximated and included in the formulation as an equivalent internal heat generation term. The two-dimensional character of the formulation is thus retained.

An extensive analysis and test program for various thickness NiCrFe Alloy 600 bead-on-plate welds has recently been completed [33]. Typical of the information obtained is a set of correlations developed between analytically predicted weld metal zone dimensions and dimensions measured from test weld sections. For example, calculations were made for 6.35 mm (0.25 in) thick plates subject to heating from a gas tungsten-arc source at currents ranging from 70 to 140 A, and weld speeds of 2.067 and 4.133 mm/s (4.88 and 9.76 in/min). Heat input magnitudes were specified to give good agreement between computed and measured temperatures. (This is one of the methods previously mentioned to determine the actual heat input magnitude and, therefore, the arc efficiency.) Arc efficiencies of approximately 0.80 were obtained. Since the distribution of heat input, characterized by the parameter $\bar{r}$, cannot be ascertained experimentally, and the weld metal zone dimensions are sensitive to $\bar{r}$, calculated and measured values of both weld penetration and weld bead width were correlated in order to estimate the distribution parameter for a

specified set of welding conditions. Values of $\bar{r}$ were varied until satisfactory agreement between calculated and measured weld bead dimensions were obtained. The results, shown in Figure 13, imply more diffuse distributions of heat input for higher arc currents and lower weld speeds.

IV. ANALYSIS OF WELDING STRESSES AND DISTORTIONS

A. Analytical Model for Weld Stresses and Distortions

1. Material Behavior Characterization

Material subject to the welding thermal cycle is postulated to behave mechanically as an initially isotropic, elastoplastic, strain hardening continuum, such that a component of total strain ϵ_{ij} , at an instant of time t , is given by [34]:

$$\epsilon_{ij}(t) = \epsilon_{ij}^{EL}(t) + \epsilon_{ij}^{PL}(t) + \epsilon_{ij}^{TH}(t) \quad , \quad (22)$$

where

$$\epsilon_{ij}^{EL}(t) = A_{ijkl}(T)\sigma_{kl}(t) \quad (23)$$

$$\epsilon_{ij}^{PL}(t) = \int_{t_0}^t \dot{\epsilon}_{ij}^{PL}(t) dt \quad (24)$$

$$\epsilon_{ij}^{TH}(t) = \epsilon^{TH}(T)\delta_{ij} \quad . \quad (25)$$

Equations (23), (24), and (25) represent components of elastic, plastic, and thermal strains, respectively. A_{ijkl} and σ_{kl} represent components of the isotropic, temperature-dependent tensor of elastic moduli and of stress, respectively. $\dot{\epsilon}_{ij}^{PL}$ is the rate of plastic deformation, ϵ^{TH} is the free linear thermal expansion, and δ_{ij} is the Kronecker delta. Although creep effects could be included in Equation (22), rate-dependency is neglected in the present treatment.

Applying incremental plasticity theory, the plastic strain rates are replaced by plastic strain increments, such that the total plastic strain is accumulated in a finite number of increments; that is,

$$\epsilon_{ij}^{PL}(t) = \sum_{n=1}^{N(t)} (\Delta\epsilon_{ij}^{PL})_n \quad , \quad (26)$$

where the end of the Nth increment is associated with time t . Therefore, with $\Delta\epsilon_{ij}^{PL}(t)$ representing the change of the plastic strain component in the increment bounded by $t-\Delta t$ and t , Equation (26) becomes

$$\epsilon_{ij}^{PL}(t) = \epsilon_{ij}^{PL}(t-\Delta t) + \Delta\epsilon_{ij}^{PL}(t) \quad , \quad (27)$$

where $\epsilon_{ij}^{PL}(t-\Delta t)$ is the plastic strain accumulated in the first $N-1$ increments. Consider yielding to be governed by the Mises criterion and plastic flow by the Prandtl-Reuss law. Then, as for example derived by Hill [35], the increment of plastic strain may be expressed in the form

$$\Delta\epsilon_{ij}^{PL} = \frac{3S_{ij}}{2\bar{\sigma}} \Delta\bar{\epsilon}^{PL} \quad , \quad (28)$$

where $S_{ij} = \sigma_{ij} - 1/3 \sigma_{kk} \delta_{ij}$ is a component of deviatoric stress, $\bar{\sigma} = (\frac{3}{2} S_{ij} S_{ij})^{1/2}$ is the generalized stress, and $\Delta\bar{\epsilon}^{PL}$ is an increment of the generalized plastic strain. A conventional power law relation is postulated to characterize the generalized plastic strain increment, such that

$$\Delta\bar{\epsilon}^{PL}(t) = \text{maximum} [(\bar{\sigma}(t)/K)^n - \bar{\epsilon}^{PL}(t-\Delta t); 0] \quad , \quad (29)$$

where K and n are parameters that characterize the yield strength and the strain hardening capacity of the material. They are obtained from the uniaxial stress-strain curve and, in general, are temperature-dependent. Defining the yield strength σ_y as the stress level at which the plastic strain in a uniaxial, isothermal test is 0.2 percent, K may be evaluated as follows:

$$K(T) = \sigma_y(T)(0.002)^{-1/n(T)} \quad . \quad (30)$$

The requirement that $\Delta\bar{\epsilon}^{PL}$ be either positive or zero is the means by which continued plastic loading, accompanied by isotropic expansion of the yield surface, and elastic unloading from the yield surface are accommodated in the formulation [34]. The generalized plastic strain is accumulated by adding, at each load step, the incremental value obtained from Equation (29) to the previously accumulated plastic strain.

2. Relief of Plastic Strain at High Temperatures

During the welding process, material in close proximity to the center line of the weld experiences loading to a plastic state of stress prior to melting. The plastic deformation accumulated during this time is completely relieved when melting takes place. At temperatures below the melting point T_M , the plastic strain is reduced by annealing processes, whose effects are significant above the recrystallization temperature. It is not the intent of the present work to dwell upon the establishment of precise formulae to describe the effects of recrystallization-induced relief of plastic deformation. Instead, the following factor is used to simulate the reduction of plastic strain at high temperatures [36]:

$$\begin{aligned} f(T) &= \exp [\alpha(T-T_M)/T] & T \leq T_M \\ &= 1 & T > T_M \end{aligned} \quad (31)$$

where T and T_M are the absolute temperature and melting temperature, respectively, and α is a parameter proportional to an activation energy for the process.

3. Finite Element Method Applied to Welding Stresses and Distortions

The incremental plasticity relations used in the welding analysis are incorporated into a small strain, small displacement finite element representation by linearizing the stress-strain equation in each increment of loading and using the tangent stiffness method to evaluate the material stiffness [26]. The linearized element stiffness matrices are obtained in BESTRAN [22] by using a Newton-Raphson type of iteration procedure in each increment [26]. The nonlinear stress-strain law is linearized about some prescribed value of stress (equivalent to using the first two terms of a Taylor series expansion about this value). Finite element calculations are performed, new values of stress calculated, and the process repeated until suitable convergence criteria are met. Since the transient and residual stress and distortion patterns that are generated during welding are based on the build-up of irreversible deformations, the accuracy of the accumulated plastic strain is of paramount importance. Convergence is thus based on the cumulative plastic strain, and the convergence criterion is set equal to 10^{-5} . In order to facilitate convergence, suitable acceleration parameters are employed in the iterative process.

Based on the discussion of the thermal model, it has been assumed that, for sufficiently high weld speeds, a two-dimensional transient analysis of temperatures at a section normal to the direction of welding is adequate for calculation of the welding thermal cycle under quasistationary conditions. The results of the temperature calculations are then used as input for the analysis of stresses and distortions.

A moving weld not subject to extremely rigid constraints will, due to non-uniform metal shrinkage, distort in directions both parallel and transverse to the welding direction. The result is a complex three-dimensional deformation pattern which, considering the nonlinear material characterization, would be impractical to analyze. To alleviate this, attempts have been made [2,3,7,9,37] to reduce the dimensionality of analytical models. Solutions to one- or two-dimensional elastoplastic problems, though certainly not complete, yield useful data on distortions and stresses.

A two-dimensional mechanical model has been formulated [18] which is consistent with the gas tungsten-arc welding thermal model. It uses the quasi-stationary temperature solution obtained from the thermal analysis as input. The computational technique was developed for plane sections normal to the direction of welding, as shown for example in Figure 1. Through-the-thickness temperature variations and weldment geometries other than those of flat plates are accommodated. Motion in the welding direction is completely restrained (plane strain conditions), preventing characterization of the longitudinal distortion mode observed in unrestrained bead-on-plate welds, yet allowing the calculation of longitudinal and transverse stresses and transverse bending distortions in the neighborhood of the weld metal and heat-affected zones.

The plane strain restriction may be relaxed by introducing a small number of out-of-plane degrees of freedom to calculate longitudinal distortions while maintaining the two-dimensional character of the analytical model. In the present treatment, however, the mechanical response at a given section normal to the welding direction is determined only by the temperature transients computed for that section. Calculation of distortion ahead of the welding arc is not taken into account.

A biquadratic, isoparametric element that is similar to the element employed in the thermal analysis is used in the stress and distortion analysis, although the finite element mesh configurations need not be the same for the two sets of calculations.

4. Extrapolation of Material Properties at High Temperatures

Mechanical properties are not usually known at temperature levels near the melting point. For computational purposes, the three mechanical properties of interest - elastic modulus, Poisson's ratio, yield strength - are extrapolated linearly to temperature levels that exceed the highest temperatures at which these properties are known. The elastic modulus is extrapolated linearly to the solidus temperature, and decreases to a negligible value at the liquidus temperature (melting point) and in the molten region. Poisson's ratio is also extrapolated linearly to the solidus, while at and above the liquidus, this property is such that a finite bulk modulus is maintained for the weld puddle. The yield strength is extrapolated linearly to some higher temperature level. (The magnitude of yield strength at this temperature is taken arbitrarily as one-tenth the value existing at the highest temperature for which data exist.) It is then decreased to an arbitrarily low value at and above the solidus temperature.

B. Stress and Distortion Analysis of a Bead-on-Plate Weld

A thermomechanical welding analysis was carried out [18] for a 2.54 mm (0.1 in) thick Alloy 600 weldment subject to no external geometric constraints in the transverse plane. Thus the weldment is completely free to expand and bend in the transverse direction; however, due to the plane strain conditions imposed in the welding direction, motion is completely restrained in the longitudinal direction.

The welding parameters chosen for this analysis are as follows: heat input $Q = 703$ W, characteristic radius of heat input distribution $\bar{r} = 5.08$ mm (0.2 in), weld speed $v = 2.12$ mm/s (5.0 in/min). These values were selected to illustrate the thermomechanical response of a full penetration weld with a weld metal cross-section that varies in width through the weldment thickness.

The thermal energy from the welding arc is distributed over a finite portion of the heated surface of the weldment, resulting in a variation through the weldment thickness of the thermal response in the vicinity of the weld line.

(In contrast, a line source of heat yields no thickness variations whatsoever.) A variation of shrinkage is produced through the thickness during cool-down. This causes the characteristic bending distortions, which are accompanied by a nonuniform accumulation of plastic strain in the weld metal and heat-affected zones.

The unconstrained motion of the weldment in the plane normal to the weld line produces transient and residual stress patterns that are dominated by the longitudinal stress. Longitudinal stress distributions at a number of points in time, as well as the residual stress distribution at final cool-down, are plotted in Figure 14. Stress histories at various locations are plotted in Figure 15. It is of interest to note some of the characteristics of the transient stress distribution.

Upon initial heat-up (prior to the time at which the center of the electrode passes over the section $x_3 = 0$ being analyzed), the localization of severe temperature gradients in the immediate vicinity of the weld line produces compressive yielding in this region. As more energy is supplied by the arc and temperatures increase, the yield strength quickly decreases until, at the melting point, it is negligible. During the time period prior to solidification of the weld metal, all material outside the puddle is in compression with the region immediately adjacent to the molten zone in a plastic state of stress. Since temperatures in this region are extremely nonuniform, the yield stresses vary from zero at the melting point to about 260 MPa (37.7 ksi), which is only slightly lower than the room temperature yield strength (291 MPa (42.2 ksi)). Plastic deformation in molten material has been completely relieved.

Upon solidification, the fusion zone material yields initially in compression at rather low stress levels. Upon further cooling, yield strength is increased and unloading from the yield surface proceeds elastically until the material yields in tension. In the weld metal and heat-affected zones, unloading and reversed yielding occur over a very narrow stress range. Hence, as cool-down proceeds, that portion of the weldment that is in tension longitudinally grows steadily. By final cool-down, the residual longitudinal stresses, which are appreciable only in the region within about 50 mm (2 in) of the weld centerline, are completely tensile. This is due to the plane strain restrictions placed on the analysis. Though relaxation of this constraint would yield

residual stresses in compression outside this region, it is important to note that the tensile stresses in the highly stressed region of interest (within about 25 mm (1 in) of the centerline) are governed primarily by plastic behavior and should change little.

The residual stress distribution is such that there is little variation of stress through the thickness. The residual stresses are essentially uniform at 390 MPa (56.6 ksi) in the weld metal and heat-affected zones and then decrease uniformly to the room temperature yield strength at a location 23 mm (0.9 in) from the weld centerline. In the elastic region beyond this location, the stresses drop off quite rapidly. The residual stresses in the plastic region exceed the room temperature yield strength because of material strain hardening. In this process, the initial short-time compressive yielding produces an expansion of the yield surface followed, during cool-down, by yielding in tension at a stress level higher than the monotonic yield strength. This type of behavior is consistent with the residual stresses measured for bead-on-plate welds by Nagaraja Rao and Tall [38]. It was noted in this study that residual stresses in (or in the close vicinity of) the weld metal zone are usually about 50 percent above the yield strength of the base metal. The results presented here, therefore, appear to be in good qualitative agreement with these test data.

Plastic deformations are confined to a localized region as illustrated in Figure 16, in which transient and residual generalized plastic strain distributions are plotted. Consistent with the plastic zone associated with the residual stress distribution, plastic strains greater than 0.2 percent are generated within 23 mm (0.9 in) of the weld centerline. The highest plastic strain levels exist in the weld metal and heat-affected zones throughout the entire welding cycle, with the exception of that time period in which the weld metal is molten and the plastic strain nonexistent. Beyond the heat-affected zone the plastic strain decreases quite rapidly.

The plots of plastic strain against time at various locations (Figure 17) illustrate the manner in which plastic deformation is accumulated in the weld. In the weld metal zone, plastic strain is accumulated rapidly as the material yields in compression. At the melting point, the plastic strain, which has reached a level of 2.3 percent, is completely relieved. Upon solidification,

the plastic strain very rapidly reaches a level in close proximity to that existing just prior to melting. As cooling continues and yielding in tension progresses, the plastic strain steadily increases to its final value of 4.3 percent. In the heat-affected zone, plastic strain is accumulated during heat-up in much the same way as in the weld metal zone, but not quite as rapidly. At the very high temperature levels just below the solidus, the accumulated plastic strain is partially relieved. As the material cools, it yields in tension and further plastic strain is accumulated. In regions removed from the weld metal and heat-affected zones, there is no relief of plastic strain, and elastic withdrawal from a plastic state of compressive stress is characterized by no accumulation of plastic strain until yielding in tension commences.

The generalized plastic strain, which characterizes permanent deformation, may be used as an indicator of cumulative "damage" in the weld metal and heat-affected zones during the welding process. Contours of the accumulated generalized plastic strain at final cool-down are plotted in Figure 18 and are superposed on curves defining the farthest extent of the liquidus and solidus isotherms (the former defines the weld metal zone). Considerable variation of plastic strain through the thickness of the weld is indicated in the fusion and heat-affected zones. The maximum residual plastic strain exists at the underside of the weldment at the weld metal/heat-affected zone interface and reaches a level of 4.5 percent. Maximum damage can thus be expected to occur in this vicinity, which agrees qualitatively with conclusions based on plastic strain distributions calculated by Hibbitt and Marcal [9]. This suggests the possibility of coupling the calculated plastic strain with ductility measurements (as carried out for example by Yeniscavich [39]) to assess the susceptibility of weld material cracking at elevated temperatures during specified weld thermal cycles.

V. ANALYSIS OF COMPLEX TOROIDAL SEAL CLOSURE WELD

The thermomechanical models for analysis of moving arc welds have been applied to a weldment with a complex geometric configuration [40]. A flexible, toroidal pressure boundary is fabricated by applying circular weld beads to join two circular NiCrFe Alloy 600 membranes and form a seal designed to contain system pressure and to withstand motions of the structures to which the membranes are attached. A representative seal weld configuration was chosen to illustrate

implementation of the analytical method and to present some typical results. Temperatures and distortions were calculated for the first, or fusion, pass of the multipass welding process, and compared with measured values.

The finite element mesh configurations used for the seal weld temperature and distortion analyses are shown in Figure 19. The thermal analysis mesh is characterized by relatively fine grid spacing in the weld metal and heat-affected zones, as well as in surrounding base metal in which temperature gradients are quite severe relative to those in membrane regions somewhat removed from the heat-affected zone. Rather coarse grid spacing is employed in the regions with low temperature gradients. The mesh employed for the distortion analysis, on the other hand, is more uniform because displacement gradients are high throughout the seal and the entire weldment must be treated as an elastoplastic continuum.

Temperatures, weld bead dimensions, and transient and residual distortions were calculated for a GTAW heat source moving at 2 mm/s (4.75 in/min). The arc current and potential are 74 A and 9 V, respectively. Transient temperature plots at two locations on the undersurface of the weldment in the vicinity of the weld metal/heat-affected zone interface are shown in Figure 20. Temperatures obtained from thermocouple traces are also plotted in Figure 20. Although there is good agreement between calculated and measured temperatures, it should be noted that cooling rates calculated subsequent to solidification are somewhat high. The bulk of the permanent seal distortion occurs as a result of the first pass closure weld thermal cycle. The first pass residual distortion for the representative seal configuration is shown in Figure 21. The seal distortion pattern arises from two interrelated phenomena: (1) the overall inward displacement of both membranes due to the inelastic response of the toroidal seal to the weld thermal cycle, and (2) shrinkage in the weld metal and heat-affected zones to draw the two membranes closer to each other.

This example serves to illustrate use of the finite element method to treat complex weldment geometries - that is, configurations other than flat plates. The two-dimensional analysis procedures (in this case, axisymmetric analyses of temperature and distortion in a radial slice of the weldment) fall somewhat short of three-dimensional techniques necessary to completely determine the thermomechanical response. Nevertheless, they provide temperatures that

are in reasonably good agreement with thermocouple data, and calculated weld bead dimensions that are within 10 percent of measured dimensions [40]. The computed distortions, on the other hand, are in poor agreement with measured distortions during the heating, phase change and short-time cooling periods, due to the assumptions implied in the two-dimensional axisymmetric analysis approach. Residual distortions, however, correlate well with the test data. The effects of the heat capacity and the flexibility of the attachment structures on temperatures and distortions have been accounted for in the analysis. Details of this analysis and test program may be found in the paper by Friedman and Sattinger [40].

VI. SUMMARY

The incorporation of analytical models of the gas tungsten-arc welding process into finite element computer programs provides a valuable tool for determining the welding thermal cycle, weld bead shape, and penetration characteristics, as well as for evaluating the stresses and distortions generated as a result of the temperature transients. The analysis procedures are applicable to planar or axisymmetric welds with arbitrary cross-sectional geometries, under quasistationary conditions.

The method used for determining temperatures features an iteration procedure to accurately account for the latent heat absorbed during melting and liberated during solidification of the weld. By simulating the heat input from the arc to the workpiece by a normal distribution function, temperature transients, weld bead dimensions, and cooling rates are evaluated as functions of both the magnitude and distribution of heat input, weldment geometry, and weld speed (or duration of heating for stationary arcs). Modeling of the welding thermal cycle is a prerequisite to analytical treatments of metallurgical changes in weld metal and heat-affected zone material, residual stresses and distortions, and weld defects. A quasistationary formulation for moving welds enables temperatures to be calculated using a two-dimensional heat conduction computer program. The present limitation of high welding speed can, however, be relaxed without altering the two-dimensional framework of the procedure.

Effects of arc pressure and weld puddle surface tension on weld penetration and bead configuration can be evaluated by extending the thermal analysis model for stationary arcs to include effects of weld puddle distortion. The influence of these two phenomena on heat transfer in the weld puddle is significant, and development of means to measure and control arc pressure and surface tension would be major steps in controlling weld penetration and weld bead configuration.

The results of welding thermal analyses are used as input to finite element analyses of distortion and stress. For the sake of reasonable computer time and storage requirements, a two-dimensional treatment of the mechanical response in planes normal to the welding direction has been developed. Material representation characterized by a temperature-dependent, elastoplastic, strain hardening continuum, and a set of assumed properties at temperatures near and above the melting point, enables transient and residual stresses and distortions to be evaluated. A procedure to use these results to assess weldment damage developed in the weld metal and heat-affected zones is suggested. An example of the application of the welding analysis techniques to a geometrically complex weldment illustrates the potential of these procedures for assessing structural weld joint configurations.

Development of the weld thermomechanical model has also served to identify other areas of work that would yield a more thorough understanding of the welding process, in addition to providing more reliable input data to the finite element analyses. Among these are determinations of material properties at very high temperatures, studies of heat transfer mechanisms in the weld puddle, and investigations of the physics of the welding arc to determine the means by which thermal energy is transferred from the arc to the workpiece.

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Figure 1. Planar Weldment Configuration and Coordinates

Figure 2. Changes in (a) Enthalpy and (b) Heat Capacity for Material Undergoing Phase Change

(a)

(b)

Figure 3. Experimental Arrangement for (a) Holding the Weld Plate and (b) Thermocouple Locations for Stationary Arc Weld Test (1 mm = 0.0394 in.)

Figure 4. Finite Element Mesh Used for Weld Thermal Analyses (1 mm = 0.0394 in.)

Figure 5. Comparison Between Calculated and Experimental Transient Temperature Response at Four Thermocouple Locations. $I = 91$ A, $E = 7.7$ V, Duration of Heating = 10 s, $Q = 530$ W, $\bar{r} = 3.81$ mm (1 mm = 0.0394 in., $^{\circ}\text{F} = 1.8^{\circ}\text{C} + 32$)

Figure 6. Effects of Heat Input Magnitude and Weldment Thickness on Penetration for Stationary Arc Weld.
 Duration of Heating = 9.5 s, Heat Input
 Distribution Parameter $\bar{r} = 3.81$ mm (1 mm = 0.0394 in.)

Figure 7. Weld Metal Zone Dimensions for Different Durations of Heating t^* , and Heat Input Distribution Parameters $\bar{r}$. Stationary Arc Weld Heat Input $Q = 1060 \text{ W}$ ($1 \text{ mm} = 0.0394 \text{ in.}$)

Figure 8. Weld Puddle Distortion as Depicted by Deformed Finite Element Mesh (1 mm = 0.0394 in.)

Figure 9: Effect of Puddle Distortion on Puddle Dimensions.
 Maximum Arc Pressure = 1000 Pa (0.15 psi),
 Surface Tension = 1 N/m (0.0057 lb/in.) (1 mm = 0.0394 in.)

Figure 10. Effect of Arc Pressure on Puddle Dimensions.
 Maximum Arc Pressure = p_0 , Surface Tension = 1 N/m
 (0.0057 lb/in.), ($1000 \text{ Pa} = 0.15 \text{ psi}$, $1 \text{ mm} = 0.0394 \text{ in.}$)

Figure 11. Effect of Arc Pressure on Time to Full Penetration.
Surface Tension = 1 N/m (0.0057 lb/in.) (1000 Pa = 0.15 psi)

Figure 12. Effect of Surface Tension on Puddle Dimensions.
 Surface Tension = N , Maximum Arc Pressure = 1000 Pa (0.15 psi).
 (1 N/m = 0.0057 lb/in., 1 mm = 0.0394 in.)

Figure 13. Effects of Arc Current and Weld Speed on Distribution of Heat Input.
Weldment Thickness = 6.35 mm (0.25 in.)
(1 mm/s = 2.36 in./min, 1 mm = 0.0394 in.)

Figure 14. Transient and Residual Stress Distributions.
 (1 MPa = 145 psi, 1 mm = 0.0394 in.)

Figure 15. Longitudinal Stress vs. Time at Various Distances (x_1) From Weld Centerline.
 (1 MPa = 145 psi, 1 mm = 0.0394 in.)

Figure 16. Transient and Residual Generalized Plastic Strain Distributions (1 mm = 0.0394 in.)

Figure 17. Generalized Plastic Strain vs. Time at Various Distances (x_1) From Weld Centerline.
(1 mm = 0.0394 in.)

Figure 18. Contours of Residual Generalized Plastic Strain

Figure 19. Finite Element Grids for Toroidal Seal; (a) Temperature Analysis,
(b) Distortion Analysis

Figure 20. Calculated and Measured Temperatures at Weld Metal/Heat-Affected Zone Interfaces of Toroidal Seal Weld

Figure 21. First Pass Toroidal Seal Distortion.
Magnification = 10X