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NONIDEAL MAGNETOHYDRODYNAMIC INSTABILITIES AND
TOROIDAL MAGNETIC CONFINEMENT

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NONIDEAL MAGNETOHYDRODYNAMIC INSTABILITIES AND TOROIDAL MAGNETIC
CONFINEMENT*

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ABSTRACT

The marked divergence of experimentally observed plasma instability phenomena from the predictions of ideal magnetohydrodynamics led in the early 1960s to the formulation of finite-resistivity stability theory. Beginning in the 1970s, advanced plasma diagnostics have served to establish a detailed correspondence between the predictions of the finite-resistivity theory and experimental plasma behavior -- particularly in the case of the resistive kink mode and the tokamak plasma. Nonlinear resistive-kink phenomena have been found to govern the transport of magnetic flux and plasma energy in the reversed-field pinch. The other predicted finite-resistivity instability modes have been more difficult to identify directly and their implications for toroidal magnetic confinement are still unresolved.

MASTER!

*This is the written version of the 1983 James Clerk Maxwell Address presented at the 1983 Meeting of the American Physical Society Plasma Physics Division in Los Angeles -- to be published in physics of Fluids

EP

1. EXPERIMENTAL MOTIVATION FOR THE DEVELOPMENT OF RESISTIVE MAGNETOHYDRODYNAMIC THEORY

The most obvious way to make a magnetic fusion reactor is to pass a strong current along a plasma column. The associated B_p (poloidal) magnetic field serves to shock-heat, compress, and confine the plasma. The simple "dynamic pinch"¹ was a principal topic of magnetic fusion research during the early 1950s; unfortunately it turned out to be unstable on the hydromagnetic time scale $\tau_H \sim 10^{-7}$ sec (for definition see Table I).

Ideal Magnetohydrodynamic (MHD) theory established its credentials as an experimental predictor by accounting for the observed instability phenomena of the dynamic pinch²: the sausage-like $m = 0$ and kinklike $m = 1$ modes (for definition see Table II). Theoretical attention then focused on the discovery of pinch configurations that might be stable within the rules of ideal MHD theory. In early 1956, Rosenbluth demonstrated the existence of a completely stable configuration (Fig. 1) based on the use of a trapped B_z (toroidal) magnetic field within the plasma column and an external conducting shell. The classified report containing the relevant Bessel functions and stability diagrams³ was regarded at the time as providing a likely near-term solution of the fusion reactor problem.

Experimental implementation of the "stabilized pinch" concept followed quickly, but the results proved to be unsatisfactory. The pinch was now found to be grossly stable on the dynamic time scale τ_H , but resistive interdiffusion of the equilibrium field led to the onset of terminal kink instability⁴

after only a modestly longer time interval, typically about $\tau_R/10 \sim 10^{-5}$ sec (for definition see Table I). Even more ominous, magnetic-field probes revealed that irreproducible fluctuations of substantial size were present already during the apparently stable phase of the discharge.

Refined magnetic probe measurements on the "stabilized pinch" configuration also showed that the radial profiles of plasma pressure and magnetic shear (Fig. 2) tended to violate the detailed prescriptions of ideal MHD stability theory.⁵ In Rosenbluth's original configuration, the plasma edge had been treated as of null spatial extent. In a follow-up paper⁶ published in 1958, he showed that the stabilization of a realistic, finite edge region would call for a moderate reversal of the B_z field outside the plasma column. (Accordingly, the stabilized pinch became known in later years as the "reversed-field pinch," or RFP.)

A major objective of the experimental program in the pinch area now became the realization of accurately shaped profiles of magnetic field and plasma pressure that would conform in detail with the prescriptions of ideal MHD theory. A more radical line of experimentation emerged in Livermore at this time⁷: The idea was to verify whether the ideal MHD theory really applied at all, on the basis of experiments in the so-called hard-core pinch configuration (Fig. 3). The hard-core pinch was recognized to be unsuitable as a toroidal reactor, but it had the advantage of offering an extremely simple test of ideal MHD theory: As long as $B_0(r)$ fell off more rapidly with radius than the vacuum field solution ($1/r$), the configuration had to be stable.

The significant -- and rather disturbing -- result of the Livermore experiments was that vigorous MHD instabilities persisted even in the supposedly stable case⁷ (Fig. 4). This conclusion was confirmed and refined in other hard-core pinch devices.^{8,9}

2. THE TEARING MODE

The prevailing wisdom around 1960 was that finite-resistivity corrections to the ideal MHD theory were probably to blame for the observed anomaly. It was easy to see that certain perturbations forbidden by ideal MHD theory would indeed serve to lower the magnetic energy: For example, a plasma-current layer is clearly unstable against tearing into parallel filaments. In the case of the Astron E layer, a rigorous analysis could be carried out for such a "tearing mode."¹⁰ Rebut developed a theory of neighboring equilibria¹¹ that accounted specifically for the threshold of instability observed in the hard-core pinch.⁸

The main reason why this type of mode was not yet taken seriously as an MHD instability was connected with its growth rate. The most plausible type of resistive tearing mode might be expected to grow on the resistive time scale $\omega = \tau_R^{-1}$ (see Table I). Such growth, however, would have been too feeble to explain the experimental observations, and also would not have been of long-term importance for fusion reactors -- where $\tau_R \sim 10^3$ - 10^4 sec. The critical question was, whether one could find a tearing mode that grew on an intermediate time scale

$$\tau_H^{-1} > \omega > \tau_R^{-1}$$

This problem was addressed by the authors of Ref. 12 during 1961-62. In its simplest form, the analysis uses the equations

$$\frac{\partial \bar{B}}{\partial t} = \nabla \times (\bar{v} \times \bar{B}) - \nabla \times \left[\frac{n}{4\pi} \nabla \times \bar{B} \right] \quad , \quad (1)$$

$$\nabla \times \rho \frac{d\bar{v}}{dt} = \nabla \times \left[\frac{1}{4\pi} (\nabla \times \bar{B}) \times \bar{B} \right] \quad , \quad (2)$$

$$\nabla \cdot \bar{v} = 0 \quad . \quad (3)$$

The addition of a magnetic field component along the direction of current flow has no physical effect: only the component $\bar{K} \cdot \bar{B}_0$ enters the relevant solutions of Eqs. (1)-(3). As illustrated in Fig. 5, we can therefore treat the tearing of an arbitrary current layer in terms of the reversed-field model of Fig. 6, where B_{x0} corresponds to $\bar{K} \cdot \bar{B}_0$. The simplifying assumption of incompressibility (3) turns out to be generally well justified in the case of a plane current-carrying layer.¹² The following discussion of tearing-mode growth is further simplified -- for the moment -- by excluding the effects of zero-order resistivity gradients and pressure gradients.

When Eqs. (1)-(3) are linearized, one obtains two coupled ordinary second-order differential equations, one for the perpendicular field perturbation B_{y1} (see Fig. 6) and one for the velocity perturbation v_{y1} . As soon as this seemingly difficult mathematical problem was put on the computer (Fig. 7), there resulted a happy insight: For large values of the magnetic

Reynolds number $S = \tau_R / \tau_H$, a narrow "singular layer" developed at the null of B_{x0} , where v_{y1} was large and the first derivative of B_{y1} underwent a sharp discontinuity. Outside this layer, a simple quasistatic, ideal-MHD, second-order equation for B_{y1} provided a good approximation to the full equations. The implication was that we could treat the whole problem analytically, finding the finite-resistivity solution near the singular surface $B_{x0} = 0$ by expansion in the coordinate y , and then matching it to the ideal MHD solution in the outer region. The critical parameter turned out to be the discontinuity across the singular surface of the logarithmic derivative $\Delta' = B'_{y1}/B_{y1}$. The quantity Δ' was found to depend only on the structure of the zero-order configuration B_{x0} and on the nature of the boundary conditions on B_{y1} -- such as the presence of a conducting shell. When Δ' was positive, as in Fig. 7, the tearing mode would grow.

To gain an insight into the rate of growth, it is helpful to rearrange the two second-order equations for B_{y1} and $\xi_{y1} = -iv_{y1} k/\omega$ so as to produce the following quadratic form¹³:

$$\begin{aligned} E &= V + && \text{(potential energy)} \\ &\omega \tau_R M + && \text{(decoupling energy)} \\ &\omega^2 \tau_H^2 K && \text{(kinetic energy)} \\ &= 0 \end{aligned} \tag{4}$$

where

$$V = \frac{a^2}{\langle B \rangle^2} \int dy \left[(B'_{y1})^2 + k^2 B_{y1}^2 - B''_{x0} B_{y1} \xi \right] , \tag{5}$$

$$M = \frac{1}{\langle B \rangle^2} \int dy \left[B_{y1} + \xi B_{x0} \right]^2, \quad (6)$$

$$K = a^2 \int dy \left[(\xi')^2 + k^2 \xi^2 \right] \quad (7)$$

The quantity V is the "potential" magnetic energy that can be liberated by the perturbation. Outside the singular layer, the simple ideal MHD relationship $B_{y1} = -\xi_{y1} B_{x0}$ could be used to transpose Eq. (5) into the form of the conventional energy principle. The novelty in the finite-resistivity case is that B_{y1} does not have to vanish on the singular surface. As a result, V can manage to make a negative contribution in Eq. (4) even in the case of the plane current layer, where there are no unstable ideal MHD modes at all. (Not surprisingly, V turns out¹³ to be proportional to $-\Delta'$.)

If B_{y1} fails to vanish at $B_{x0} = 0$, then the positive definite quantity M cannot vanish either. Since we are looking for modes that grow on a time scale $\omega \gg \tau_R^{-1}$, it follows from Eq. (4) that M must be made as small as possible. In other words, any departure from $B_{y1} = -\xi_{y1} B_{x0}$ must be restricted to some singular layer ϵ of small relative width ($M \sim \epsilon V$), inside which the maintenance of finite B_{y1} is unavoidable if V is to make its destabilizing contribution.

There is, of course, another way to keep M small, namely by letting ξ become very large, so as to preserve the relationship $B_{y1} = -\xi_{y1} B_{x0}$ as B_{x0} approaches its null. The penalty for this

tactic is that for $\xi_{y1} \sim \epsilon^{-1}$, $\xi'_{y1} \sim \epsilon^{-2}$, the positive definite quantity $K \sim \epsilon^{-4} M$ becomes very large. The best compromise is to make the M and K terms about equal. From Eq. (4) we then have

$$\omega \tau_R^M = \omega^2 \tau_H^2 K = -1/2 V \quad (8)$$

Using the previous estimates $K \sim \epsilon^{-4} M$ and $M \sim \epsilon V$, we obtain

$$\omega \tau_R^E \sim \omega^2 \tau_H^2 \epsilon^{-3} \sim 1 \quad (9)$$

The interesting conclusions are

$$\epsilon \sim (\omega \tau_R)^{-1} \quad (10)$$

$$\omega \sim (\tau_H^2 \tau_R^3)^{-1/5} \quad (11)$$

When the tearing-mode analysis is carried through more rigorously, the same basic scaling is obtained, along with particulars about the dependence of Eqs. (10) and (11) on the instability wavelength. These analytic predictions¹² have turned out to be in good agreement with subsequent computer work. The results of a latter-day computer run¹⁴ are shown in Fig. 8. [The figure, incidentally, covers the effect of a nonvanishing zero-order velocity ($v \neq 0$) -- i.e., a lack of zero-order resistive equilibrium -- which was neglected¹⁵ in Ref. 12.]

The first experimental identification of the tearing mode came from a reversed-field theta-pinch experiment.¹⁶ The photographic evidence and interpretation are shown in Figs. 9 and 10,

respectively. In the initial phase of the theta pinch (Fig. 10a), the configuration approximates an infinite reversed-axial-field cylinder. If there is sufficient radial distance between this cylinder and the outer conducting shell (the theta-pinch coil), one expects tearing instability: The fastest growing modes should have a wavelength several times greater than the radial thickness of the current layer, as in phase (b) of Fig. 10. Still longer wavelengths, however, have lower energy states, so that one would expect gradual merging of the initial short-wave modes as in (c) and (d). Because the actual theta-pinch configuration is of finite length, it must contract axially while thickening in the radial direction, until it reaches a true two-dimensional equilibrium (e). The degree of residual elongation depends on the closeness of the outer conducting shell. At this point, the lowest energy state has been reached: The stabilizing effect of the shell is automatically strong enough to suppress all tearing modes.

This simple interpretation of the experimental phenomena of Fig. 9 served to give valuable encouragement to tearing-mode theory. Subsequent studies of this topic have drawn attention to the importance of including the effects of radial acceleration (i.e., effective gravity) as a destabilizing term.¹⁷

The first detailed experimental confirmation of tearing-mode theory¹⁸ was obtained on the "triaxial pinch" (Fig. 11), a tubular dynamic sheet pinch with a z-directed, rather than θ -directed, plasma current. As illustrated in Fig. 12, the plasma tube tears into filaments, which contract like simple

dynamic pinches and then undergo secondary instabilities of the kink and sausage types. The filament growth rates and wave lengths were found to be in general agreement with the theory. The triaxial pinch geometry lends itself particularly well to the investigation of the tearing phenomenon, since an arbitrary axial magnetic field component can be superimposed. As has been mentioned in connection with Fig. 5, the presence of an axial field component should have no effect at all on the tearing mode. In the triaxial pinch experiments, the initial filamentation process was indeed found to be unaffected. As might also be expected, the secondary instabilities of the individual filaments were diminished by increasing the axial field component.

3. APPLICATION OF TEARING-MODE THEORY TO TOKAMAKS

During the latter part of the 1960s, the tokamak configuration (Fig. 13) began to exhibit outstandingly favorable stability and confinement properties.¹⁹ In normal operation, the tokamak discharge was found to be completely free of gross MHD disturbances over periods of many tens of milliseconds. Sensitive magnetic-probe measurements carried out at the plasma edge, however, revealed the persistence of low-level poloidal-field fluctuations,²⁰ particularly during the build-up phases of the plasma current and density. Some latter-day observations²¹ of this phenomenon -- now called "Mirnov oscillation" -- are shown in Fig. 14.

The Mirnov oscillation was found to have the following salient features: it comes in a range of discrete m -numbers (cf. Table II); time intervals having different m -numbers tend to be nonoverlapping and separated by distinctive negative voltage spikes (cf. Fig. 14); the observed pattern of field perturbation rotates at a frequency comparable to that of the electron diamagnetic drift. The magnetic perturbations were found to be essentially uniform in the direction along the magnetic field -- quite like the current filaments of Fig. 12.

The occurrence of particular m -numbers was found to be correlated in time with the value of the rotational transform i (cf. Fig. 13). Defining the "safety factor" $q(r) = 2\pi/i(r)$, one observes the approximate relationship $m \sim q(r_0)$, where the radius r_0 marks the edge of the main tokamak current channel²⁰ (Fig. 15). This result is consistent with the observation that the perturbations are aligned with the magnetic field: For $\vec{R} \cdot \vec{B} = 0$, we write $(m/r) B_p + (n/R) B_t = 0$, or $|m/n| = r B_t / R B_p = q(r)$. The modes shown in Figs. 14 and 15 correspond to $n = 1, m = 2, 3, 4, \dots$ (Note that in tokamak geometry the convention is to identify $B_\theta = B_p$ and $B_z = B_t$.)

By stretching ideal MHD stability theory a little, the Mirnov perturbations can be accounted for.²² The simplest tokamak model (Fig. 16) consists of a perfectly conducting inner plasma region $r \leq r_0$, with a uniform current density, surrounded by a perfectly conducting intermediate region $r_0 \leq r \leq r_c$, which does not carry any current, and an outer vacuum region $r_c \leq r \leq r_w$, which is bounded by a perfectly conducting wall.

Ideal MHD theory teaches that, for $q(r) < 1$, there is always an $m = 1$ instability, but if $q(r) > 1$ for all r , there can be no unstable mode unless it has a singular point $m/n = q(r_s)$ that falls into the vacuum region outside r_c . An additional condition for instability is that the singular radius r_s for a given mode must fall within a limited radial zone (where $\Delta' > 0$), as shown in Fig. 16a. To approximate the experimentally observed correlation between m and q for $n = 1$, the theoretical zones of instability are plotted against $q^{-1}(r_0)$, as in Fig. 16b. For the choice $x_c \equiv r_c/r_0 = 1$, there are nonoverlapping modes covering the entire range of q . For $x_c > 1$, one obtains stable gaps between modes. In the case of the low- m modes, substantial gaps can also be achieved by bringing the wall radius r_w close to the plasma. (The asterisked and double-asterisked cases in Fig. 16 correspond to $r_w/r_0 = 2.0$ and 1.33 , respectively.) The simple step-model of the tokamak current density is easily refined by rounding the sharp edge of $j(r)$ to various degrees (Fig. 17) -- in which case the narrow instability zones of the higher- m modes can be made to disappear entirely (as in the typical experimental case). The experimental observation that $n = 1$ modes predominate is closely related: For a given resonance $m/n = q(r_s)$, a higher n -value implies a higher m -value, which is generally more stable.

While this ideal MHD treatment seems rather satisfactory at first sight, it has some serious flaws. First of all, there is the formal objection that in the experiments the region outside the main plasma current channel is not really a vacuum region,

but rather a region of fairly good conductivity. A more serious problem is that in the experiments (cf. Fig. 18) there is generally a finite plasma current density $j(r)$ at the singular radius r_s of an unstable mode -- whereas in the ideal MHD theory one must either declare the region at r_s to be plasma, in which case there can be no instability in the first place, or vacuum, in which case $j(r_s)$ must vanish.

The finite-resistivity MHD stability theory of the tokamak^{22,23} uses essentially the same models and equations as the ideal MHD theory, but is able to treat the case $j(r_s) \neq 0$ as well. This point turns out to be crucial: As illustrated in Fig. 19, the stability of a particular mode is sensitive to the local derivatives of $q(r)$ at r_s ; these in turn are dominated by the derivatives of $j(r)$ at r_s . The general rule, proved in Ref. 23, is that for a given slope dq^{-1}/dr at r_s , an increase in the absolute magnitude of the quantity $\delta q^{-1}(r) = q^{-1}(r) - q^{-1}(r_s)$ (which measures the departure from $\mathbf{K} \cdot \mathbf{B} = 0$) is always stabilizing. Correspondingly, for a given global profile of $q^{-1}(r)$, a decrease in the local slope at r_s is always stabilizing for the local resonant mode $m/n = q(r_s)$. For all neighboring modes, of course, this flattening of q^{-1} at r_s implies a local steepening at their own singular points, and therefore a decrease in their own margins of stability.

These considerations have been applied to the optimization²⁴ of the tokamak current profiles shown in Fig. 20. The profile at the top is stable against all modes, without benefit of any outer conducting wall; it corresponds to the lowest permissible limiter

q value [$q(a) = 2.6$] for any completely stable profile. (The strategy of optimization is to minimize the slope of q at the most dangerous point, $q(r_s) = 2$, while preserving $\Delta' < 0$ for all other modes as well.) The bottom half of Fig. 20 shows an optimized profile with a conducting wall placed at $r_w = 1.2 a$, which is completely stable down to $q(a) = 1.7$.

The strategies for experimental stabilization of tokamaks that are drawn, respectively, from the ideal and resistive MHD theories are substantially different. Figure 16 encourages us to seek stability by virtue of high-conductivity plasma -- which, however, does not seem to work in practice -- the singular points of unstable modes being found to occur at essentially all radii. The only remaining remedy is to bring the conducting wall very close to the plasma surface: This does have a favorable effect, but is generally inconvenient to implement in experimental tokamak devices. The finite-resistivity theory, which is able to make use of $j(r_s) \neq 0$, tells us, first of all, to be careful not to cool the plasma on the $q(r) = 2$ surface (i.e., to avoid blowing in too much gas or impurities into that vicinity, and to compensate, if necessary, by local rf heating or by ramping up the plasma current so as to broaden the $j(r)$ profile by skin effect). This turns out to be relevant advice. Secondly, the nonlinear form of the resistive theory²⁵ tells us that there is a self-stabilizing mechanism at work: The magnetic islands associated with a given mode (Fig. 21) act to flatten the $q(r)$ profile at r_s , and therefore restrain that particular mode. Finally, a most important stabilizing technique arises from a realistic treatment of the $m = 1$, $n = 1$ mode.

In the ideal MHD theory, the $m = 1$ mode is particularly feared, since it cannot be suppressed by high plasma conductivity at r_s . (This traditional state of mind is reflected in giving the name "safety factor" to the quantity $q = 2\pi/l$ -- the measure of departure from the "Kruskal-Shafranov limit," where $q = 1$.) The theory also tells us, however, that the $m = 1$ mode is localized to the region between the magnetic axis and the singular point $q(r_s) = 1$, and this turns out to be a greatly mitigating factor. As shown in Fig. 22, the experimentally observed effect of the $m = 1$ mode is merely to flatten the $j(r)$ -profile (and T_e profile) periodically within the region where $q(r) \leq 1$. Outside, there is very little resultant heat transport -- indeed, one observes an exceptional degree of quiescence. The latter phenomenon is readily understood in terms of the comparison theorem of Fig. 19 and the analysis of Ref. 24: The effect of raising $q^{-1}(r)$ above 1 over an appreciable range in r is strongly stabilizing for the dangerous higher- m modes, which have their singular points at larger radii. As a paradoxical consequence of these considerations, the most satisfactory operating regime for an experimental tokamak is not the "high-safety" case $q(0) \gg 1$ but rather the "unsafe" case $q(0) \leq 1$.

While ideal MHD theory correctly predicts the general features of the linear $m = 1$, $n = 1$ mode, a full explanation of the experimentally observed "sawtooth" phenomenon of Fig. 22 calls for the finite-resistivity treatment. During the slow, rising part of the $T_e(0)$ sawtooth pattern, $j(0)$ is also rising in

time and $q(0)$ is dropping below 1. An $m = 1, n = 1$ field perturbation of the form shown in Fig. 23 then appears (see also Fig. 31). The $m = 1$ kink manifests itself near the crest of the sawtooth in Fig. 22 as a small growing oscillation, with a frequency corresponding to the electron diamagnetic drift. After a time interval that depends on the plasma resistivity, but is considerably smaller than τ_R , the quantity $q^{-1}(0)$ reaches a nonlinear threshold value where "catastrophic" magnetic reconnection takes place.²⁶ The essential idea is that the new helical magnetic axis straightens up and pushes out the old magnetic axis. Since helicity is conserved in the fast process, the q -value at the new axis is precisely unity, so that stability against the $m = 1$ mode is restored. By the same token, the trough of the $m = 1$ sawtooth is the most dangerous time for higher- m instability.

The nonlinear analysis of the higher- m modes²⁷ is much more complex than for $m = 1$, since the most interesting effects depend on the mixing of several modes with different pitches m/n . In that case, the two-dimensional treatment is insufficient and magnetic surfaces are not necessarily preserved. In Fig. 24, we see, first of all, a slow-growing $m = 2, n = 1$ mode in the mildly nonlinear "Rutherford regime."²⁸ (An ironical circumstance is that the fast linear growth rate of the tearing mode, which was initially so vital to the acceptance of resistive MHD theory, is hardly seen at all: the resistive kinks of the tokamak enter the Rutherford regime at very small amplitude.) Next, Fig. 24 illustrates a phenomenon that has already been discussed in the

context of Fig. 19: the growth of $m = 2$ islands flattens $q(r)$ around $q = 2$ and therefore steepens $q(r)$ at $q = 3/2$. As a result, the $m = 3, n = 2$ mode appears and flattens $q(r)$ around $q = 3/2$. Now the $m = 5, n = 3$ mode must grow in turn, and so forth. (This situation is somewhat analogous to a deteriorating economy, where each sector tries to shift the burden to another.²⁹) Soon the magnetic surfaces become ergodic, as in Fig. 25, and a catastrophic disruption occurs. The computer simulation is even able to approximate the "negative voltage spike" that is found to be characteristic of experimental tokamak disruptions. This higher- m reconnection process is much more damaging than the $m = 1$ sawtooth, since plasma energy is lost to the wall, impurities are ingested, and often the entire tokamak discharge is brought to an abrupt halt.¹⁹

Careful programming of the tokamak current and density profiles as functions of time can bring the discharge safely into its sawtooth regime, avoiding all major disruptions. Small disruptions, such as those shown in Fig. 14, tend to persist, and actually play a constructive role in relaxing the tokamak skin current. Tokamak transport simulations³⁰ predict that in large, hot tokamaks the skin current should be extremely pronounced unless the current-ramping is kept inconveniently slow (cf. Table I). When there is a strong skin current, the rotational transform becomes double-valued inside the plasma, as in Fig. 26, and even the ideal MHD theory then permits $m > 1$ instability.²² In the experiments, the consequence of skin current is the "double tearing" mode,²³ which accelerates current penetration by

a process of nonlinear reconnection. The overall energetics of this skin-relaxation process appear to be much more favorable than had been anticipated in Ref. 30.

Now that the predictive power of tearing-mode theory is fairly well established, some major resistive-MHD codes have been developed to interpret the experiments in further detail and assess the potential of improved tokamak geometries.³¹ For example, the "bean-shaped" tokamak of Fig. 27 is under extensive study and has been found to exhibit advantageous stability properties, even at high beta values.³²

4. APPLICATION OF TEARING-MODE THEORY TO REVERSED-FIELD PINCHES AND SPHEROMAKS

At the end of the 1960s, the reversed-field pinch (RFP) was eclipsed by the success of the tokamak. Only during the past five years has there been a notable resurgence of interest and accomplishment in the RFP area. The main stimuli of this resurgence have been the adaptation of tokamak experimental technology to the RFP,³³ and the growing understanding of the mechanism³⁴ whereby nonlinear MHD modes serve to maintain a long-term plasma confinement configuration³⁵ in spite of resistive diffusion (cf. Fig. 28). As a result of renewed RFP experimental activities, substantial increases of temperature and confinement time have already been achieved,^{33,35-37} and the observed scaling laws are favorable to continuing improvements at still higher plasma currents.

Following the discovery and experimental confirmation of resistive tearing modes, the theoretical stability of the RFP was reassessed.³⁸ As illustrated in Fig. 29, stable RFP configurations can still be found, even at substantial beta values. Tearing-mode stability merely requires that the magnetic field profiles be chosen with particular care and that the outer conducting shell be moved somewhat closer to the plasma surface than had been considered necessary in terms of the ideal MHD analysis. These findings of Ref. 38 may be considered to bring to a successful conclusion the long quest for definition of a theoretically stable pinch configuration that was undertaken in the early 1950s (cf. Sec. 1).

Many practical problems remain, however. The usefulness of the RFP configuration for long-time confinement of hot plasmas is threatened, first of all, by the tendency of ordinary resistive diffusion to drive the magnetic-field profiles away from the (rather narrow) range where complete stability can be achieved -- and in particular to eliminate the reversal of the toroidal field. RFP experiments indeed show a substantial level of MHD activity, but fortunately they have also been characterized by a remarkable reluctance to diffuse away from a field-reversed profile of the general form shown in Fig. 29: Some RFP experiments even demonstrate anomalous diffusion towards the desirable profile,³⁹ when starting from different initial conditions.

In this context, the particular merit of J.B. Taylor's analysis in Ref. 34 has been to provide a theoretical framework for interpreting the experimental observations, and a basis for

predicting favorable performance for high-temperature RFP's. Taylor's basic proposition is that resistive reconnections tend to conserve the helicity integral $K_h = \int \mathbf{A} \cdot \mathbf{B} \, dV$, which measures the total linkage between the toroidal and poloidal magnetic fluxes. Searching for the lowest energy state, subject to the constraint that K_h be conserved, he finds the solution $\mathbf{j} = \mu \mathbf{B}$, where μ is a constant. In other words, the lowest energy state is a force-free (null-beta) configuration of a particular type. Applying this model to the RFP, he is able to predict a certain relationship (Fig. 30) between the pinch ratio ϕ and the field-reversal index F .

The most exciting aspect of Taylor's theory is that it calls for a spontaneous reversal of the external toroidal field when the pinching becomes sufficiently strong -- as observed in the experiments.³⁹ A drawback of Taylor's treatment is that the null-beta case is not of much practical interest; however, there appears to be substantial continuity between null-beta and moderate-beta experiments (Fig. 30). Another drawback is that Taylor's model ($\mu = \text{constant}$) generally prevents $j(r)$ from vanishing at the plasma edge. This peculiarity can be buried by assuming a close-fitting conducting wall around the plasma,³⁴ but when this nonphysical treatment is modified by introducing a vacuum gap between the plasma and the wall, a new set of short-wave kink instabilities is introduced -- quite analogous to the high- m modes that afflict the step function tokamak model of Fig. 16. The point is that in this case the Taylor profile is no longer the true lowest-energy state, and one does better by using

the rounded $j(r)$ profiles of Fig. 29. These peculiarities, however, do not substantially diminish the heuristic value of the Taylor theory for understanding RFP phenomenology in general.

One would expect there to be some relationship between Taylor's energy principle and tearing-mode theory. This can be verified directly in a simple example: a cylindrical RFP obeying Taylor's rule -- the so-called Bessel-function model⁴⁰ of the RFP. The condition for onset of tearing modes is found to be identical with the condition calculated from Taylor's formalism for departure of the minimum energy state from axisymmetry. On the other hand, the detailed processes whereby the plasma is supposed to reach Taylor's distribution $\bar{j} = \mu \bar{B}$ are hard to relate to explicit resistive modes. Application of Taylor's formalism to tokamak stability is helpful in bringing out some of its peculiarities relative to ordinary resistive theory. In the tokamak case, Taylor's minimum-energy solution $\bar{j} = \mu \bar{B}$ corresponds very nearly to the uniform-current-density, constant- q distribution of Fig. 16, bounded by a perfectly conducting shell at $x = 1$. The resistive instability modes of principal concern in Sec. 3 (i.e., $n = 1$, $m = 2, 3, \dots$) are simply suppressed by the close-fitting shell, and so both types of analysis agree that the configuration should be stable. If we consider a different tokamak profile, like that in Fig. 17, where the current density is somewhat peaked rather than flat, with an associated q -profile where $q(0) < 1$ and $q(a) > 1$, then linear tearing-mode theory²³ predicts $n = 1$, $m = 1$ instability, and nonlinear theory^{25,27} describes a reconnection of magnetic field lines that serves to

level the $q(r)$ profile -- exactly the conclusions that would be drawn, for this case, on the basis of Taylor's formalism. More generally, however, ordinary resistive theory permits a wide range of profiles other than the constant- q solution to be perfectly stable, while Taylor's formalism would call for relaxation to a constant- q profile in every case.

The experimentally observed long-term persistence of the RFP profile in spite of resistive diffusion (Fig. 28) can be understood in general terms on the basis of Taylor's work,³⁴ but description of the details of the required "dynamo mechanism" remains a major challenge. As an introduction to this problem, it is helpful to review the sawtooth mechanism of the tokamak (cf. Section 3) which is, in fact, a dynamo of the desired type -- with the peculiarity that it can be treated explicitly by two-dimensional resistive MHD analysis.²⁵ As illustrated in Fig. 31a, if the rotational transform $q^{-1}(0)$ has risen above unity, then a resistive $n = 1$, $m = 1$ kink mode can restore the $q^{-1}(r)$ profile to $q^{-1}(0) = 1$. The process begins by the formation of a $m = 1$, $n = 1$ magnetic island on the surfaces where q^{-1} passes through unity. When the magnetic island has grown sufficiently large, nonlinear forces act to displace it towards smaller radii, crowding out the old straight magnetic axis and replacing it with the formerly helical axis of the magnetic island. In this process, helicity is conserved, while the total magnetic energy and poloidal flux are reduced. The analysis of the RFP case is more difficult: Reconnection phenomena can also be produced by two-dimensional computer studies,⁴¹ but the maintenance of the RFP field configuration clearly requires a more complicated three-dimensional dynamo.⁴²

If nonlinear MHD phenomena can be relied on to maintain a nearly stable toroidal magnetic field profile, while the external circuits need merely supply volt seconds to sustain the poloidal field, then a logical next step is to consider whether the entire RFP configuration can be operated in steady state. This approach has been shown to be workable in general terms,⁴³ but the detailed reconnection mechanisms remain to be identified.

In principle, this current-drive method could also be used for the tokamak. Ignoring, for the moment, the potential complications due to the various skin-current modes of Fig. 26, the tokamak geometry offers the possibility of an explicit two-dimensional illustration: The toroidal field of the tokamak is naturally steady-state, and the poloidal field could be maintained simply by driving the sawtooth dynamo backwards. The basic mechanism is illustrated in Fig. 31b, where the rotational transform $q^{-1}(0)$ is initially below unity. An $m = 1$, $n = 1$ magnetic island then forms on the inner $q^{-1} = 1$ surface, grows to nonlinear amplitude and finally replaces the old magnetic axis with the island axis. This process is the inverse of the sawtooth of Fig. 31a. To use the mechanism of Fig. 31b for current drive, a two-phase approach could be followed: first the helicity of the tokamak is slowly increased by rf-driving a skin current (or equivalently by cycling B_p and the limiter radius so as to shift the poloidal flux outward in r); when a centrally hollow $q^{-1}(r)$ profile has been established, one lets the skin current relax rapidly by reconnection, while conserving the helicity and increasing the total poloidal flux.

Discussion of explicit dynamo mechanisms for maintaining stable profiles or driving steady-state currents has the incidental merit of drawing attention to the probable drawbacks of such MHD activity from the point of view of high quality plasma confinement: The periodic replacement of the magnetic axis (which can be shown to be a necessary feature of any steady-state dynamo) is associated with correspondingly enhanced convection, just as in the case of the sawtooth of Fig. 22. Fortunately, one may expect that dynamo-related transport should decrease in larger, hotter plasmas, since the rate of field diffusion away from the stable profile appears to follow classical resistive scaling. This is the probable explanation of the strong improvement of confinement with rising T_e that has been characteristic of the RFP experiments.^{33,35-37} In a reactor, the required energy confinement time τ_E will be sufficiently shorter than τ_R , so that the coexistence of the MHD dynamo mechanism with a thermonuclear equilibrium burn is at least conceivable.

The recent rise of interest in the RFP has also contributed to the revival of the "spheromak" (Fig. 32), an RFP relative with null field reversal and very low aspect ratio.⁴⁴⁻⁴⁶ Surprisingly, this rudimentary confinement geometry appears to have favorable linear and nonlinear stability properties. The spheromak is a third member of the small class of potentially "Taylor-stable" tori, alongside the tokamak and the RFP. As illustrated in Fig. 33, the spheromak is also capable of utilizing an MHD dynamo mechanism to reoptimize its magnetic field profile during resistive diffusion.

5. THE RIPPLING MODE

In the finite-resistivity MHD analysis of Sec. 2, perturbations of the plasma resistivity itself were neglected. A more complete treatment¹² must allow for the perturbation of η due to plasma displacements along $\nabla\eta_0$. Under appropriate conditions, the resultant resistivity-gradient-driven "rippling mode" (Fig. 34) can be an important destabilizing mechanism.⁴⁷ At long wavelengths, kink modes of the tearing type are only weakly affected by inclusion of the resistivity-gradient effect, but at wavelengths below the critical value for tearing-mode stabilization, rippling instability is found to persist. The growth rates of all these modes follow the basic scaling derived for the tearing mode in Sec. 2, Eq. (11).

The rippling mode differs in character from the other MHD kinks in that it cannot be eliminated by "architectural" techniques, such as profile shaping, or nearby conducting shells. Fortunately, there is a microscopic mechanism at work that normally suppresses rippling modes: If the resistivity perturbation is due to a local perturbation in electron temperature resulting from convection along ∇T_e , then thermal conductivity along magnetic field lines should serve to suppress the instability, except at very low temperatures.¹² (The critical electron temperature T_c scales as $S^{3/10}n^{1/4}$ and is typically in the range 10-30 eV in present-day toroidal experiments.) On the other hand, if $\nabla\eta_0$ is due to a gradient in Z_{eff} , the rippling mode should grow and serve to flatten the $Z_{eff}(r)$ profile. P.H. Rutherford has pointed out that destabilizing gradients of

n_0 can also arise from the density dependence of neoclassical resistivity, from the use of nonthermal current carriers, and even from the logarithmic dependence of n on the Debye length, but these are generally weaker effects.

Since tokamaks and RFP's tend to have strong radial resistivity gradients, and since they exhibit kink modes that follow the prescriptions of resistive MHD theory in some detail, one might expect to find some experimental evidence of the rippling mode in those cases when it is clearly called for by the theory.⁴⁸ Tokamak regimes where gross MHD activity has been suppressed by architectural techniques should lend themselves particularly well to such a study.

The most obvious theoretical expectations associated with the rippling mode are shown schematically in Fig. 35. Most of the tokamak profile is characterized by $T_e > T_c$, so that the rippling-mode stability condition is simply $Z_{\text{eff}}(r) = \text{const.}$ Somewhere near the edge of the plasma, T_e must normally fall below T_c , in which case the stability condition becomes $Z_{\text{eff}}(r) = T_e^{3/2}(r)$. A pure hydrogenic plasma clearly cannot satisfy the latter condition.

Experimental tokamak plasmas never quite achieve $Z_{\text{eff}} = 1$, but the best cases come sufficiently close so that one would expect rippling instability in the region $T_e < T_c$. Unless a low integral value of q happens to fall into this narrow edge region, so that the corresponding kink mode is aggravated in the manner shown in Fig. 34, the effect on confinement in the main plasma may well be unimportant. (As indicated in Fig. 35, one might

take note that the use of a tokamak divertor has the effect of precluding the rippling mode by sustaining $T_e(r)$ above T_c on the inside of the separatrix, while suppressing $j(r)$ beyond the separatrix, where the field lines do not close -- but there are many other plausible explanations for the improvement of tokamak confinement in the "H-mode" of Ref. 49.)

In a tokamak with a strong gradient of Z_{eff} within the main plasma, the rippling mode should clearly become unstable, and one would then expect a serious deterioration of confinement. Again the most obvious impact would be in destabilizing the kink modes -- particularly the higher m -values. There is a good deal of circumstantial evidence that these considerations may really apply to experimental tokamak plasmas: Strong gradients of $Z_{eff}(r)$ are not normally seen inside the high-temperature region. Puffing of high- Z gas into the discharge can easily bring on a major disruption. The injection of low-velocity hydrogen pellets that stop short of the magnetic axis creates strongly marked interior steps in the profiles of $n(r)$, $T_e(r)$, and presumably $Z_{eff}(r)$, which are found to relax with surprising rapidity.⁵⁰ The latter observation suggests a critical test of rippling-mode theory: The injection of a series of pellets that match or mismatch the Z_{eff} of the unperturbed plasma to varying degrees.

The discussion of resistive MHD theory in Sec. 2 and in the present section has also neglected the effects of radiative cooling terms.⁵¹ Particularly in the treatment of stability at

the plasma edge, such effects may, however, turn out to be important. Their further investigation in the context of tokamak and RFP parameters would clearly be of interest.

6. THE RESISTIVE G MODE

Allowance for finite plasma pressure gradients introduces another important class of resistive instabilities. In toroidal confinement geometries, pressure gradients directed opposite to the radius of curvature R_c of the magnetic field tend to drive flute instabilities. In ideal MHD theory, these flutes can be stabilized by means of sufficiently strong magnetic shear (cf. Sec. 1, particularly Fig. 2). Finite resistivity greatly diminishes the effectiveness of shear stabilization. A simplified analysis of this phenomenon was carried out in Ref. 12 for a current layer in the xz-plane (cf. Sec. 2), with the destabilizing curvature simulated by a y-directed force of order $G = (a^2/4R_c) \partial B/\partial y$. For a flute mode having an unconstrained growth rate

$$\omega \propto \frac{G^{1/2}}{\tau_H} \quad (12)$$

the finite-resistivity treatment gives

$$\omega \propto (kL_s)^{2/3} \frac{G^{2/3}}{\tau_H^{2/3} \tau_R^{1/3}} \quad (13)$$

where k is the wave number of the flute and L_s is the shear length. The alarming feature is that the instability persists on a time scale that is only slightly slower than τ_H .

On the basis of the stability considerations emerging from tearing and rippling mode theory, a highly promising toroidal confinement configuration had been identified in the late 1950s: the toroidal analog⁵² of the linear hard-core pinch configuration of Fig. 3. To carry out toroidal confinement experiments with the stable type of magnetic field configuration shown in Fig. 4, a massive current-carrying ring was left transiently unsupported at the center of a hot-plasma tube. In this type of "levitron" device, plasmas were produced both by ohmic and electron cyclotron heating, in order to distinguish between current-driven and pressure-driven effects. Very low levels of MHD fluctuation were reached in the limit of low- β configurations in near-vacuum fields, but anomalous transport clearly persisted in all regimes. One plausible interpretation of these findings was, of course, in terms of the resistive G mode.

The demonstration of minimum-B stabilization⁵³ in 1961 had a powerful impact not only on mirror research but also on toroidal confinement concepts. For example, it is shown in Ref. 12 that the reversal of the sign of G (corresponding to the minimum-B case) not only eliminates the resistive G mode, but can even act to suppress the growth of unstable tearing and rippling modes. (See also Ref. 54 for a more advanced discussion.) Unfortunately, the nonexistence of closed minimum-B toroidal vacuum-field configurations was obvious from first principles,⁵⁵ but configurations with favorable average gradients of B were soon discovered.⁵⁶

The stability analysis for this case⁵⁷ gave the encouraging result that the flute instability would be transformed into a resistive "ballooning" mode (with enlarged amplitude in local regions of unfavorable magnetic curvature), which could only grow on the resistive time scale

$$\omega \propto (kL_c)^2 \frac{G}{\tau_R}, \quad (14)$$

where L_c is the connection length along magnetic field lines between regions of favorable and unfavorable curvature.

That the tokamak is a minimum-average-B configuration^{58,59} was pointed out in the early 1960s, but the superior experimental performance of the tokamak was not yet clearly evident at that time. The most plausible approach to the investigation of the basic principles of toroidal confinement was to eliminate all possible sources of instability, and preference was therefore given to the study of minimum-average-B vacuum-field configurations. On the basis of his tokamak analysis,⁵⁸ Yoshikawa suggested that the addition of a strong vertical magnetic field could also endow the ordinary levitron with powerful minimum-average-B properties. Subsequent experiments⁶⁰ with the Princeton Spherator and FM-1, and the Culham Levitron, using superconducting rings, did indeed show significantly improved confinement properties and indications of the resistive G mode -- though a unique identification was never made. Superior confinement properties had also been observed in magnetic multipole configurations,^{61,62} which use several internal rings to produce very strong minimum-average-B effects.⁶³ By the end

of the 1960s, a number of such experiments were flourishing, but since they were limited to fairly cold plasmas (typically 0.1-10 eV) and since internal-ring configurations are inherently implausible as steps towards a fusion reactor, they were readily displaced by the tokamak as the principal test bed for the study of toroidal confinement.

The ultimate importance of the resistive G mode in toroidal confinement remains unclear. In the case of the RFP and the spheromak, which have strongly unfavorable gradients of minimum-average-B, one would expect a serious limitation of confinement, unless non-MHD effects step in at high S-values and stabilize the mode.³⁸ Whether this stabilization will occur -- or indeed whether the resistive G mode is a problem in present experiments -- remains to be demonstrated. For the tokamak the theoretical outlook is more favorable, since only the slower-growing resistive ballooning mode is expected.⁶⁴ In particular, the bean-shaped tokamak of Fig. 27 tends to meet the architectural prejudices of Ref. 57 and offers good prospects for high-beta stability. Again, there is only circumstantial evidence⁶⁵ that unfavorable curvature is actually harming confinement in present-day tokamak experiments. The general observation that the tokamak edge plasma tends to flow across magnetic field preferentially on the large-R side does suggest that resistive G effects should be kept in mind.

7. CONCLUSION

During the past three decades, toroidal confinement research has advanced from relatively cold and highly unstable plasma conditions to the edge of the reactor-plasma regime. During the same period, the theory of MHD stability has been refined, and a substantial correspondence has been established between resistive MHD predictions and experimentally observed gross instability phenomena.

An appealing feature of this development has been the interaction among different branches of experimental fusion research in the development of a common physical understanding. The gross instabilities of various pinch configurations provided the first clues to resistive MHD theory. The substantial stability of the tokamak configuration lent itself particularly well to the identification of individual modes and the detailed verification of the theory. Improved theoretical understanding, in turn, has created new insight into the complex nonlinear phenomena of RFPs and spheromaks.

There is likely to be a good deal of opportunity for further constructive interaction during the next decade, as tokamaks, RFPs, and spheromaks all progress toward higher-S regimes, more refined diagnostics, and growing insight into the nature and utility of nonlinear resistive-MHD mechanisms.

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Table I. Resistive and Hydromagnetic Time Scales for Current Layers of Half-Width a_h .

	Typical Experiments			Illustrative Reactor Regime
	1950-60	1983	≥1986	
Resistive Time $\tau_R = \frac{4\pi a_h^2}{\langle \eta \rangle}$	$10^{-4}-10^{-3}$	1-10	10^2-10^3	10^3-10^4
Hydromagnetic Time* $\tau_H = \frac{a_h (4\pi \langle \mu \rangle)^{1/2}}{B}$	$10^{-7}-10^{-6}$	$10^{-8}-10^{-7}$	$10^{-8}-10^{-7}$	$10^{-8}-10^{-7}$
$\tau_{Hp} = \frac{B}{B_p} \tau_H$	$10^{-6}-10^{-5}$	$10^{-7}-10^{-6}$	$10^{-7}-10^{-6}$	$10^{-7}-10^{-6}$
Reynolds Number $S = \frac{\tau_R}{\tau_H}$	$\sim 10^3$	$\sim 10^8$	$\sim 10^{10}$	$\sim 10^{11}$
$S_p = \frac{B_p}{B} S$	$\sim 10^2$	$\sim 10^7$	$\sim 10^9$	$\sim 10^{10}$

*For tokamaks where $B_p \ll B$, the magnitude of τ_{Hp} is more directly significant than that of τ_H .

Table II. Illustrative Gross MHD Modes.

PINCH

$$|\bar{B}_1| \propto e^{i(m\theta + k_z z) + \omega t}$$

Principal Instability Modes:

• Sausage	$m = 0$	$k_z \sim 1/r$
• Strong Kinks	$m = 1$	$k_z \sim 1/r$
• Weak Kinks	$m \geq 2$	$k_z \leq m/r$

TOKAMAK

$$|\bar{B}_1| \propto e^{i(m\theta + n\phi) + \omega t}$$

Principal Instability Modes:

• Internal Kinks	$m = 1$	$n = 1$
• Strong External Kinks	$m = 2, 3, \dots$	$n = 1$
• Weak External Kinks	$m = 3, 4, 5, \dots$	$n = 2, 3, \dots$
-
-

FIGURE CAPTIONS

- Fig. 1 A longitudinal magnetic-field component can stabilize the short-wave kink mode (a) and sausage mode (b) of the simple pinch. An external shell is needed to stabilize the long-wave kink (c).
- Fig. 2 Magnetic probe measurements (a) on the Columbus S-4 device⁵ imply a pressure profile that violates the ideal MHD stability criterion. In the region $2.5 \text{ cm} < r < 5 \text{ cm}$, the pressure-gradient term exceeds the shear-stabilizing term (where μ is the pitch of the field lines).
- Fig. 3 The magnetic-field geometry of the conventional "stabilized pinch" can be inverted by means of a current-carrying center conductor. The resultant "hard-core pinch" configuration⁷ has a much broader range of ideal MHD stability.
- Fig. 4 Magnetic probe measurements on the hard-core pinch⁷ imply a pressure profile that is clearly ideal-MHD stable. The irreproducibility of the magnetic probe traces, however, demonstrates the persistence of gross instability.
- Fig. 5 The stability properties of a generalized sheet-pinch configuration coincide with those of the simple model of Fig. 6. [$\mathbf{k} \cdot \mathbf{B}_0$ corresponds to $B_{x0}(y)$.]
- Fig. 6 Magnetic-field structure of the tearing mode.
- Fig. 7 The magnetic-field and velocity perturbations of the tearing mode exhibit sharp discontinuities around the null of $B_{x0}(y)$.

- Fig. 8 The growth rate $p = \omega \tau_R$ of the tearing mode increases in proportion to $S^{2/5}$. The lines in the figure correspond to the simplified theoretical analysis.¹² The points were obtained by computer modeling.¹⁴ (The parameter μ measures the departure from initial ohmic equilibrium).
- Fig. 9 The evolution of a reversed-field theta pinch (of length L) is observed in streak photography (top) and instantaneous frames (bottom).¹⁶
- Fig. 10 The break up of the reversed-field theta pinch (a) of Fig. 10 is interpreted in terms of short-wave tearing-mode formation (b), followed by merging into longer wavelengths with lower total stored energy (c-e).
- Fig. 11 The Triax¹⁸ is a hard-core pinch with balanced internal and external B_0 -fields.
- Fig. 12 Instantaneous axial photographs of the Triax plasma of Fig. 11 (taken through a screen electrode), exhibit filamentation, followed by secondary instability of the filaments.
- Fig. 13 The tokamak configuration.
- Fig. 14 Tokamak magnetic-field perturbations of various mode numbers observed with magnetic probes during the initial current ramp-up.²⁰
- Fig. 15 The mode numbers of tokamak magnetic-field perturbations tend to be correlated with the instantaneous q value at the plasma edge.²¹

- Fig. 16 The simplest tokamak model has a uniform current density j , in the region $r/a = x \leq 1$. The shape of the rotational transform ι is shown in (a). As the absolute value of ι at $x=1$ is varied (b), the singular points $m/n = q(x)$ of various modes fall into the theoretically unstable ranges indicated in (a). The ranges of instability²³ can be narrowed by assuming perfect conductivity out to $x = x_0$.
- Fig. 17 When the tokamak model of Fig. 16 is replaced by a more realistic current profile, the high m numbers are stabilized.²³
- Fig. 18 Pinhole X-ray diagnostics on the PLT tokamak show that for $q(0) > 1$ the plasma core is quiescent, but the region around $q = 2$ exhibits a growing $n = 1$, $m = 2$ instability, leading to a "disruption."²⁶
- Fig. 19 A useful theorem for comparing the stability of various $q^{-1}(r)$ profiles²³ is applied here to the case of the $n = 1$, $m = 2$ mode. Variations in the q profile of type A are designed to increase $|q^{-1}(r) - q^{-1}(r_s)|$, while dq^{-1}/dr is held fixed at $r = r_s$, thus enhancing the magnetic-field-bending term $\mathbf{E} \cdot \mathbf{B}$ and improving stability. Conversely, variations of type B enhance the stability of the $m = 2$ mode by reducing dq^{-1}/dr at r_s . Note, however, that variation B will tend to destabilize neighboring modes ($q = 3/2, 5/2$, etc.) by increasing dq^{-1}/dr at their own resonant points, while variation A* - which is most effective at stabilizing $m = 2$ and all the higher m modes -- necessarily destabilizes the $m = 1$ mode.
- Fig. 20 Tokamak current-density profiles can be optimized²⁴ against resistive kink modes by adjusting details of $j(r)$ so that Δ' is negative for all m/n .

- Fig. 21 The magnetic island structure of the $n = 1$, $m = 2$ resistive kink mode.
- Fig. 22 Pinhole X-ray diagnostics on the PLT tokamak show that for $q(0) < 1$, there is a sawtoothlike excursion of central temperature, with $n = 1$, $m = 1$ disturbances at the crest of each sawtooth.²⁶ The exterior plasma region remains undisturbed.
- Fig. 23 The magnetic island structure of the $n = 1$, $m = 1$ resistive kink mode.
- Fig. 24 Computer simulation of tokamak resistive kink modes²⁴ shows the growth of the $n = 1$, $m = 2$ mode destabilizing first $n = 2$, $m = 3$ and then the $n = 3$, $m = 5$ mode.
- Fig. 25 The computer simulation of Fig. 24 demonstrates the destruction of magnetic flux surfaces as the various unstable modes begin to overlap.
- Fig. 26 A tokamak profile with a strong skin current J can have a double-valued rotational transform i . The stability diagram (cf. caption of Fig. 16) then exhibits a wide range of unstable modes.²³
- Fig. 27 Tokamak plasma of bean-shaped cross section can have favorable high-beta stability properties even for large R/a and small b/a , provided that $d/2a$ is not too small.³²

- Fig. 28 The internal toroidal magnetic field of the ZT-40 RFP maintains itself over many resistive diffusion times by means of a dynamo mechanism involving $m = 1$ tearing modes.³⁵ Note the increase of toroidal field after each disturbance, and the sawtoothlike loss of plasma temperature (X-ray signal) during the reconnection process.
- Fig. 29 The RFP profile can be made stable against resistive kink modes³⁸ by means of careful shaping and a fairly close-fitting conducting wall.
- Fig. 30 Experimentally observed RFP regimes tend to fall close to Taylor's $F-\theta$ curve for $\beta = 0$ plasmas. The field-reversal parameter F , is the ratio of the toroidal field B_t at the outer wall to B_t -average. The pinch ratio θ , is the ratio of the poloidal field at the wall to B_t -average.
- Fig. 31 In the sawtooth instability process (a) of the tokamak, a resistive $n = 1$, $m = 1$ kink mode serves to replace the original magnetic axis (characterized by $q(0) < 1$) the $q = 1$ axis of the $n = 1$, $m = 1$ magnetic island -- thus reducing the poloidal flux. In principle, the inverse process (b) could be utilized to help maintain the poloidal flux of the tokamak in steady state.
- Fig. 32 The spheromak magnetic-field configuration.⁴⁵
- Fig. 33 The toroidal magnetic field of the CTX spheromak at Los Alamos National Laboratory is maintained against resistive diffusion by a dynamo process that converts poloidal flux into toroidal flux.⁴⁶

- Fig. 34 The introduction of resistivity-gradient effects⁴⁷ extends the range of instability of the tearing mode to shorter wave lengths ($\alpha = ka > 1$).
- Fig. 35 Tokamak profiles of Z_{eff} and $T_e^{-3/2}$ are indicated schematically. For $T_e > T_c$, only the resistivity gradient due to Z_{eff} is destabilizing.
- Fig. 36 The spherator⁵³ is a toroidal vacuum-field minimum-average-B configuration similar in basic field structure to the hard-core pinch of Fig. 3, but with minimum-average-B properties analogous to those of the tokamak.

Figure 1

Figure 2

Figure 3

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Figure 4

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$B_{x0}(y)$

Figure 6

84A0075

Figure 7

Figure 8

Figure 9

Figure 10

Figure 11

(c)

(b)

(a)

Figure 12

81X0074

Figure 13

Figure 14

Figure 15

Figure 16

Figure 17

Figure 18

Figure 10

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Figure 20

Figure 21

Figure 22

Figure 23

Figure 24

ORNL-DWG 80-2812 FED

Figure 25

Figure 26

Figure 27

Figure 28

Figure 29

Figure 30

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Figure 31

Figure 32

RATIO OF TOROIDAL TO POLOIDAL FIELD

Figure 33

Figure 34

84A0072

Figure 35

SPHERATOR

TOKAMAK

Figure 36

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