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COO-3084/77

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Void Nucleation in Spheroidized Steels
During Tensile Deformation

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MASTER

U.S. Department of Energy
Contract EY-76-S-02-3084
Technical Report No. 77

April 1980

E(11-1) 3084-77

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Abstract of
Void Nucleation in Spheroidized Steels
During Tensile Deformation

by

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Brown University, June 1980

An investigation was conducted to determine the effects of various mechanical and material parameters on void formation at cementite particles in axisymmetric tensile specimens of spheroidized plain carbon steels.

Desired microstructures for each of three steel types were obtained by controlled tempering of quenched rods, resulting in various particle size distributions. Cylindrical tensile specimens were then strained to prescribed reductions in neck area corresponding to deformation levels near fracture.

Observations of void morphology with respect to various microstructural features were made using optical and scanning electron microscopy. While it was noted that non-equiaxed, irregularly shaped particles often failed by internal fracture, spheroidal particles were observed to form voids by decohesion of the particle-matrix interface in the vicinity of the particle poles. Large particles were favored sites for void nucleation as were particles situated on ferrite grain boundaries.

The area fraction of voids, f_v , and the number of voids per unit area of transverse cross section, n_A , were measured as functions of deformation in the neck of each specimen.

Normal stress requirements and energy balance considerations were incorporated into a criterion for stable void nucleation. Three dimensional solutions for the energy release rate associated with polar void cap formation were developed. Contributions to the stored elastic energy in the vicinity of the particles due to applied loading and local plastic incompatibilities were considered and models were evaluated for the present alloy systems in accordance with the proposed nucleation criterion.

The models predict enhanced void nucleation at larger particles and at particles situated on ferrite grain boundaries, consistent with observations. It was determined that small changes in the work required to form void surfaces can drastically alter the level of deformation necessary for stable void formation.

Effects of non-spherical particle geometries and particle interactions were considered in relation to particle fracture and interfacial decohesion at closely spaced particles.

Good correlation between the computed and measured dependence of η_A on deformation was obtained, providing support for the energy analysis used to predict void nucleation.

Acknowledgments

I extend to Professor Joseph Gurland my sincerest gratitude for his many contributions to this work. His guidance, patience, encouragement, and friendship were most appreciated.

I wish also to thank Mr. Herbert Stanton whose assistance and expertise in the laboratory were vital to the experimental phase of this investigation. The technical assistance of Mr. Joseph Tracey and Mr. Robert Dean was also appreciated.

I wish to thank Ms. Stephanie Moore and Mr. F.A. Fisher for their contributions to the typing of the manuscript.

This work was supported by the U.S. Energy Research and Development Agency under contract EY-76-S-02-3084.

Table of Contents

	Page
1. Introduction	1
2. Literature Review	3
2.1 Experimental observation of voids	3
2.2 Role of voids in current fracture theories	9
2.3 Local deformation at inclusions	12
2.4 Void nucleation criteria	14
2.5 Void nucleation models	15
2.5.1 Particle cracking	15
2.5.2 Particle-matrix decohesion	21
2.6 Axisymmetric tensile deformation	33
2.6.1 Tensile deformation in spheroidized steels	34
2.6.2 Stress and strain in necked tensile specimens	35
3. Experimental Procedure	39
3.1 Materials	39
3.2 Heat treatments	39
3.3 Mechanical testing	40
3.4 Metallography	42
4. Results and Analysis	44
4.1 General microstructural features	44
4.1.1 Undeformed specimens	44
4.1.2 Deformed specimens	44
4.2 Microstructural parameters	47

4.3	Spatial distribution of particles	48
4.4	Particle size distribution	49
4.5	Tensile test data	51
4.6	Relation of neck geometry to local deformation state	52
4.7	Quantitative void measurements	53
4.8	Effect of void growth	55
4.9	Comments on void growth and relation to fracture	58
4.10	Effects of microstructure on ductility	61
5.	Analytical Models for Void Nucleation	64
5.1	Introduction	64
5.2	Model A: Spherical particle in a homogeneous isotropic matrix	64
5.2.1	Introduction	64
5.2.2	Elastic energy release based on void cap geometry	65
5.2.3	Elastic energy sources	72
5.2.4	Evaluation of local unrelaxed strains, ϵ_p^*	76
5.2.5	Computed energy difference curves	77
5.2.6	Normal interfacial stress criterion	78
5.2.7	Discussion of model A	79
5.3	Model B: Spherical particles on grain boundaries	81
5.3.1	Introduction	81
5.3.2	Details of plastic zone	82
5.3.3	Results of model B	89
5.3.4	Discussion of model B	90
5.3.5	Comparison with experiment	91

5.4 Effects of particle shape and particle interactions	95
5.4.1 Particle shape	95
5.4.2 Particle interactions	97
6. Summary and Conclusions	100
6.1 Void formation in spheroidized steels	100
6.2 Void nucleation criteria and analytical models	101
6.3 Recommendations for further study	104
Appendix A	106
Appendix B	115
References	119
Tables	125
Figures	135

I. Introduction

While the strengths of many alloy systems have been enhanced by the presence of precipitates or dispersed particles, it has been observed that these same inhomogeneities can act as sites for the initiation of voids during plastic deformation. It is apparent that characterization of the processes by which these voids form in relation to the many parameters which govern the deformation of a particular system is important if the events which lead to the ultimate failure of a material are to be understood and possibly controlled.

For this purpose a systematic study of void nucleation at cementite particles in spheroidized steels during tensile deformation was conducted in an attempt to characterize the nature of the phenomenon in terms of (1) the mode of void formation, (2) location with respect to microstructural features (e.g. particles, grain boundaries), (3) void geometry and (4) material deformation.

Consistent with these results, a criterion for void nucleation based on an analysis of the local stresses and elastic energy storage at spherical particles in a plastically flowing matrix was developed.

In the next chapter, a number of observations relating to void nucleation in a variety of systems are reviewed and current models for void formation are evaluated so as to familiarize the reader with various concepts relating to the subject. The experimental details of the present investigation are presented in Chapter 3 and the results and corresponding analyses appear in Chapter 4. The analytical models for void nuclea-

tion are developed in Chapter 5 and comparisons between experiment and theory are made.

Although mechanical testing and void characterization were performed on a single class of materials, it is felt that, due to their comprehensive nature, the analytical models which have been proposed may be applied to a large number of alloy systems. Thus, it is hoped that this investigation will serve as an important contribution to our fundamental understanding of the initiation of failure in engineering materials.

2. Literature Review

2.1 Experimental observations of voids.

Studies dealing with the deformation of materials containing dispersions of hard second phases or inclusions have shown that the nucleation of microcracks or voids is associated with these particles when the material is subjected to various types of deformation. These voids appear either as cracks in the particles or as failures of the particle-matrix interfacial bonding. The actual void morphology depends upon the interrelation of various microstructural parameters as well as the local deformation state. In very high purity materials, the absence of particles acting as void sites requires that other mechanisms of void nucleation be operative. Bauer and Wilsdorf [1] made in situ TEM studies of high purity silver and stainless steel ribbons during deformation. Many voids were observed to nucleate ahead of propagating cracks in the absence of material discontinuities but it was observed that these areas were characterized by high dislocation densities. Gardner et al. [2] investigated beryllium and iron single crystals in a similar manner and determined that cracks initiated at dislocation structures which had evolved into cell boundaries.

During high temperature creep deformation, voids have been observed to form at grain boundary triple points [3] or at particles located on grain boundaries [4] as a result of high stresses due to incompatibilities associated with grain boundary sliding. Void formation may be associated

with diffusion controlled processes in which grain boundaries act as sinks and sources for vacancies with directional diffusion occurring as a result of the local stresses. [5]

Although voids are not always associated with particles, as indicated in the above discussion, void formation in commercial grade materials deformed at ambient temperatures is attributed to the presence of inhomogeneities in the form of dispersed inclusions or second phases. Early investigations by Puttick [6], Rogers [7], and Crussard [8] were some of the first studies to confirm the role of inclusions as sites for nucleation of voids. Since then a number of researchers have dealt with the deformation of particle-containing materials, leading to a limited understanding of the effects of various parameters on void initiation and ductile failure. The term "limited" is used because of the lack of good quantitative data on void formation and because of contradictory observations and conclusions which have resulted from these studies. In the past the effect of particles on ductile fracture has been viewed either qualitatively or with the total fracture process in mind including void growth and coalescence to failure. [9, 10] While these studies discuss the effect of particles on overall ductility (e.g. strain to fracture) they say little about the factors which control the initiation of voids.

In the few studies which have been conducted to investigate void formation, those variables which have received the most attention are (1) particle size and shape, (2) effect of state of stress, and (3) effect of plastic strain. Other variables which have been considered

are (4) testing temperature and (5) alloying effects as related to changes in interfacial surface energies and matrix workhardening. Some of the results from these studies will now be reviewed.

The cracking of inclusions is related to both brittle and ductile fracture initiation. The microcracks act either as Griffith-type flaws from which macroscopic brittle crack propagation begins,^[11] or the cracks grow plastically under the influence of the imposed stress and strain resulting in eventual coalescence and ensuing ductile rupture.

Several authors^[14, 22] have noted that large particles tend to crack or decohere at earlier stages of deformation than small ones, although others have found little correlation^[12] or even an opposite effect.^[13] Gangulee and Gurland^[14] studied the fracture of silicon particles embedded in an aluminum matrix and found that the probability of a particle being broken at a given stress level was proportional to the square root of the particle diameter. Cox and Low^[15] found that TiCN inclusions in 4340 steels tended to break up when the matrix underwent plastic straining with larger ones cracking at lower strains. Brock^[16] noted the fracture of large inclusions in a number of aluminum alloys but only very few voids at small inclusions, resulting from particle-matrix decohesion, even at very large plastic strains. A number of investigators have observed carbide cracking in steels. Lindley et al.^[17] found that particle aspect ratio was the controlling geometrical factor for fracture of grain boundary carbides. Liu and Gurland^[18] noted that large spheroids cracked preferentially in their work on spheroidized steels. Butcher^[19] looked at the cracking of copper oxide inclu-

sions in copper and discovered that for constant aspect ratio, larger inclusions broke at lower strains.

The cracking of inclusions may be a stress controlled phenomenon or the result of local plastic straining, or a combination of both. In their study of silicon particles in aluminum Gangulee and Gurland [14] found that the probability of particle cracking was proportional to the applied stress. However, Butcher's work [20] on MnS particles in free cutting steel showed that the number of broken particles was more nearly a function of applied strain than applied stress. Liu and Gurland [18] determined that the distribution of cracked particles was uniform throughout the fracture cross section of necked tensile specimens of spheroidized steels. Since the maximum stress component decreases from the center to the edge of the specimen, it was concluded that crack initiation is based upon shear flow and not tensile stress. Barnby [21] showed that rod-shaped chromium carbides in stainless steel crack as a result of concentrated local stresses at the particles due to impingement of matrix slip lines. He concluded that an applied matrix strain is responsible for creating a critical stress concentration at the particles leading to internal fracture.

While particle cracking is commonly observed in plate and rod-shaped inclusions (having aspect ratios greater than one) and also in particles with complex shapes (spheroid agglomerates in steel), interfacial decohesion of the particle-matrix interface is frequently associated with particles having more equiaxed geometries or weak interfacial bonding.

Particle-matrix debonding often occurs during the latter stages of tensile deformation, after necking instability, and in some materials it appears to be suppressed almost until the fracture event. [16]

As in the case of particle cracking, decohesion of the interface seems to be favored at larger particles. Palmer and Smith [22] attribute this effect to the relaxation of interfacial stresses by matrix cross slip which can occur at smaller particles in FCC materials. They based this reasoning on the experimentally determined particle size dependence of void formation, which diminished with decreasing temperatures of deformation in internally oxidized copper. This, however, does not explain the size dependence for systems containing large particles where cross slip does not play a significant role in the relaxation of stresses about particles. Argon et al. [12, 23] proposed that for spheroidized carbides in 1045 steel the apparent tendency for voids to form at large particles was attributable to the interactions between particles of greater than average size having less than average spacings. Goldenberg et al. [24] observed such an effect in their studies of void nucleation in spheroidized steels. On the other hand Easterling et al. [13] noted an inverse relationship between particle size and void nucleation which they suggested is a result of the functional relationship between the unrelaxed strains about the particles and the particle size.

The work required to create void surfaces at the interface of an inclusion and the matrix will certainly have an effect on void nucleation. It has been shown [13, 25, 26] that alloying changes in some systems can

significantly alter the work of adhesion (i.e. the energy required to form the free surfaces). By adding cobalt to nickel containing 2.5 volume percent alumina, Hancock et al.^[26] found that the interfacial energy between the alumina and the matrix was substantially lowered resulting in less void formation and increased ductility compared to the same alloy without cobalt. Easterling et al.^[13] made similar studies of austenitic and ferritic irons which were alloyed with a variety of transition elements. They found that large increases in the strain to nucleate cavities could be obtained due to large changes in the work of adhesion at the interface of alumina particles as a result of alloying differences. Cialone and Asaro^[27] measured an increase in void nucleation at Fe_3C particles in 1045 steel due to hydrogen charging, a result which may be due to an increase in the particle-matrix interfacial energy.

Attempts to determine the effects of applied stress and plastic strain on the nucleation of voids have also been made. Argon and Im^[12] maintain that cavities form at the interface of particles in spheroidized steels, copper-chromium alloys, and maraging steels when the combined stresses on the interface due to the drag of the plastically flowing matrix and the local hydrostatic tension attain some critical value. This study represents one of the few quantitative investigations which relates experimental measurements to a model for void nucleation. LeRoy^[28] also studied voids in spheroidized steels and concluded that a critical

value of plastic strain was required for cavity formation at particle interfaces. Easterling et al. [13] also determined critical strain values for creation of voids and then modified the energy criterion of Brown and Stobbs [29] to obtain a model which was compatible with their experimental findings.

Some of the above observations of particle cracking and decohesion have been related to various models for void formation. The salient features of these models will be discussed in section 2.5.

2.2 Role of voids in current fracture theories

The creation of micro-flaws in a material undergoing deformation represents the first step in a series of processes leading to ultimate failure. A better understanding of the mechanics of ductile fracture requires further study of the micromechanisms which operate during the early stages of void initiation. Current fracture models attempt to relate microstructure to the critical conditions for crack initiation and growth, and increased knowledge of the role that microstructure plays in void initiation can only help to characterize the fracture process in engineering materials.

Since particles and inclusions are instrumental in void nucleation processes, it is worthwhile to consider some theories which relate the presence of inhomogeneities in polycrystalline materials to various fracture criteria. Ritchie, Knott and Rice (RKR) [30] have proposed a model for cleavage fracture in which they developed a relationship between critical tensile stress, σ_c , and fracture toughness, K_{IC} (a material

property which is a measure of the critical elastic stress intensity factor required for crack extension). The model postulates that slip-induced cleavage will proceed when the maximum principal stress ahead of a crack tip is equal to, or exceeds, the fracture stress, σ_c , over some "microstructurally significant" distance, χ , which depends on material morphology. In steels, cleavage fracture is generally associated with cracking of grain boundary carbides, possibly by the mechanism proposed by Smith.^[11] The propagation of these microcracks into the ferrite matrix may be controlled by a Griffith-type criterion which is dependent on the local stress field ahead of the main crack. This field falls off rapidly with distance from the crack tip and it is argued that for unstable crack propagation to occur the local stress must be great enough to not only nucleate a carbide microcrack, but also to extend it through a ferrite grain to the next void site. Thus, a choice for χ of two grain diameters is proposed over which σ_c must be attained to insure unstable crack growth. Good agreement between experimental results and predicted values for K_{IC} have been obtained based on the RKR model.^[30, 31, 32]

A model for ductile failure initiation at the tip of a crack has been proposed by Rice and Johnson^[33] which suggests that a critical value of the crack tip opening displacement, CTOD, is required for an initially spherical void, located at some distance from the original crack tip, to grow and coalesce with the blunting crack. This is essentially a strain controlled fracture model which assumes voids are

originally present in the region ahead of the crack. In fact, however, some amount of CTOD (strain) is required for the formation of voids at existing homogeneities in front of the crack.^[34] Thus, some void nucleation criterion which incorporates elements of the microstructure as well as the local deformation fields must be applied to determine the CTOD required to create voids. The CTOD then needed for void growth and coalescence will depend upon the spacing between void nucleating sites. The total CTOD or strain required for crack growth will be that needed for nucleation of voids plus that to link the voids with the crack tip, both of which are dependent on the microstructure. Costin^[34] and Green and Knott^[35] have both used this model to successfully explain ductile fracture initiation in steels.

The models for cleavage initiation and ductile crack growth just presented both allow for, and indeed require, the introduction of microstructural features as part of a fracture criterion. It is apparent that macroscopic fracture initiation is actually controlled by microscopic processes and that the initiating event is the formation of flaws (cracks or voids) on the microstructural level. The development of void nucleation models incorporating microstructural details and local deformation conditions thus presents itself as an important step in the refining of fracture theories.

2.3 Local deformation at inclusions.

Particle fracture and interfacial decohesion result as a consequence of the local state of deformation which exists in the vicinity of void nucleating inhomogeneities. The values of the stresses and strains in and about particles of various geometries have been calculated for specific loading conditions and material characteristics. Both continuum and micromechanistic models have been postulated.

While the problem of an elastic inclusion in an elastically deformed matrix has been treated extensively for the cases of a two-dimensional circular particle [36, 37] and three-dimensional ellipsoid geometries, [38, 39] these solutions are not very useful for describing the stresses and strains which contribute to void nucleation at elastic inclusions embedded in a plastically flowing matrix. Of course, the inhomogeneity effect resulting from elastic incompatibility must be considered, but it is usually overshadowed by the stress and strain concentrations which result from unaccommodated plastic flow, especially for alloys whose constituents have comparable elastic constants.

The two-dimensional problem of a circular elastic inclusion in an elastic-plastic matrix subject to pure transverse shear, has been solved by Orr and Brown [40] and Huang [41] using finite difference techniques. Argon [42] discusses the details of these models which predict maximum interfacial stresses at 17° and 12° to the direction of principal stress, respectively. Argon, Im, and Safoglu [23] produced a finite element

solution for the case of an elastic non-hardening plastic continuum flowing in shear about a rigid circular inclusion for which the maximum interfacial stresses were found to be 1.5 times the remote shear stress in the direction defined by the principal tensile axis. They also concluded that a pure hydrostatic stress, σ_T , could be added directly to this solution to obtain results for general states of loading.

Such two-dimensional models may only be used to approximate the stress conditions about real spheroidal particles during plastic deformation. Ashby [43] proposed a dislocation model to describe the large interfacial stresses which are produced at spherical particles in a plastic matrix due to the back stresses from prismatic dislocation loops which are produced at the interface to accommodate the misfit between particle and matrix. Brown and Stobbs [29] developed a micromechanistic model for the back stresses based on primary slip and subsequent relaxation at silica particles in copper and Chang and Asaro [44] extended this model to the case of multiple slip. Both models employed the methods of Eshelby [45] to compute the elastic strains in and about the particles resulting in maximum interfacial stress values along the principal tensile stress direction and uniform stresses inside the particles. Shibata and Ono [46] extended these solutions to the case of ellipsoidal (as opposed to spherical) inclusions.

The stress concentrations in and about elongated, or irregularly shaped, inclusions may be much larger than for spherical inhomogeneities as predicted by the fiber loading model of Kelly and Davies [47] and the

experimental analyses of Edmonds and Beevers.^[48] Micromechanistic solutions for these geometries are not yet available due to limited knowledge of the local deformation processes which operate at such particles.

The quasi-continuum description of Chang and Asaro^[44] is regarded by this author to be the most realistic and useful model for calculating the deformation at spherical cementite particles in steel. The void nucleation models in chapter 5 are based on their results.

2.4 Void nucleation criteria

Existing models for void nucleation are generally grouped into one of three categories based upon either an energy criterion, a local stress criterion or a local strain criterion. Some models contain elements from more than one category resulting in a more complex criterion.

Those models based on energy arguments propose that cracks or voids will form when the available stored elastic energy in the vicinity of a particle or inclusion is equal to or greater than the energy required to create the void surfaces. This criterion is considered to be a necessary, but not sufficient, condition for hole nucleation.

The local stress criterion states that voids will form when the normal stress inside inclusions or at the inclusion-matrix interfaces attains some critical value which exceeds the fracture strength of the particle or the bonding strength of the interface.

Void nucleation models based on local strain criteria deal with the large strain concentrations which can develop at non-deforming particles

in a plastically flowing matrix. These regions of concentrated strain may be associated with dislocation pile-ups or with more uniformly distributed displacement discontinuities at the particle-matrix interfaces. In either case, high local stresses may result leading to cavity formation.

Several existing models for void nucleation will be discussed in an effort to evaluate the main features in light of the present work.

2.5 Void nucleation models.

The three types of void nucleation criteria introduced in section 2.4 refer to the particular physical quantity - stress, strain, or elastic energy - which governs the conditions favoring nucleation. Confusion may result from the fact that a critical local stress criterion may be formulated in terms of a critical value of applied plastic strain or a critical energy criterion may be represented by a critical applied stress value. The interrelationships between stored elastic energy, local stress, local strain, applied stress, and applied strain may give rise to a number of nucleation formulae which are all based on the same criterion. This will become apparent as the following discussions are developed.

2.5.1 Particle cracking.

Models for the cracking of hard, brittle particles during plastic deformation generally follow one of two approaches.

- 1) The micromechanistic models [49, 50] propose that cracks are nucleated in particles when high stresses at the ends of impinging slip bands

reach the fracture strength of the particle and the associated energy of the pile-up is sufficient to produce the free surfaces of the crack. 2) Continuum models [47, 51] assume that the large stress concentrations at the tips of pile-ups are relieved by local yielding and that for particles which can be viewed optically, slip is essentially uniform in the vicinity of the particles. The load transfer from plastic matrix to elastic particle then becomes the controlling factor for crack nucleation.

Stroh [50] was responsible for one of the earliest models which provided a criterion for crack nucleation at the tip of a pile-up of dislocations. He proceeded by considering the energy associated with introducing a crack of length c and unit thickness in a body subjected to a tensile stress, σ , given by

$$E_c = - \frac{\pi}{8} (1-\nu) \frac{\sigma^2 c^2}{\mu} + 2\Gamma c \quad (2.1)$$

where Γ is the free surface energy of the material. The stability condition for crack nucleation was taken to be

$$E_c < 0$$

or

$$\frac{16 \Gamma \mu}{\pi (1-\nu)} < \sigma^{*2} c \quad (2.2)$$

where σ^* is the required tensile stress for a stable system. It was assumed that Γ is related to the shear modulus, μ , as

$$\Gamma = a b \mu \quad (2.3)$$

where b is the Burgers vector and a is a constant estimated to be .006.

Stroh determined the maximum normal stress, σ_m , ahead of a pileup to be

$$\sigma_m = 2 \left(\frac{L_0}{3r} \right)^{1/2} \tau_0 \quad (2.4)$$

where L_0 is the pileup length, τ_0 is the applied stress on the pileup, and r is the radial distance ahead of the tip of the pileup.

The pileup length was determined to be

$$L_0 = \frac{\mu b n}{\pi(1-\nu)\tau_0} \quad (2.5)$$

where n is the number of dislocations. Combining equations 2.2-2.5 leads to the final form for the energy stability condition

$$\tau_0 > \frac{0.72\mu}{n} \quad (2.6)$$

For carbides in steel a value of n on the order of 100 would be required to produce cracking for typical values of τ_0 .

Smith [49] treated the same problem by replacing the discrete dislocation distribution of the pileup with a continuous distribution function. The form of the distribution was determined from equilibrium requirements assuming that an effective shear stress, τ_0 , acts on the dislocations external to the crack while a shear stress, τ_c , acts on the crack. While the energy of the pileup-crack system was not determined, the stationary values for the energy were obtained. For $\tau_c = 0$ (traction free crack)

$$\tau_0 > \left(\frac{2\Gamma\mu}{\pi(1-\nu)L_0} \right)^{1/2} \quad (2.7)$$

is the criterion for stable crack nucleation. Substituting the values for Γ and L_0 used by Stroh into this equation yields

$$\tau_0 > \frac{0.12\mu}{n} \quad (2.8)$$

indicating that Smith's model predicts nucleation of cracks for shorter pileups at equivalent levels of applied stress.

Smith and Barnby [52] were able to refine this model by assuming the barrier which creates the pileup to have finite thickness, d , as in the case of a real particle. This results in an associated reduction in load carrying capacity as the crack is formed. The modified form of equation 2.7 is

$$\tau_o > \left(\frac{2\gamma\mu}{\pi(1-\nu)L_o} \right)^{1/2} \left(\frac{2d}{L_o + 2d} \right)^{1/2} \quad (2.9)$$

For $d \ll L_o$ this reduces to

$$\tau_o > \left(\frac{2\gamma\mu}{\pi(1-\nu)L_o} \right)^{1/2} \left(\frac{2d}{L_o} \right)^{1/2} \quad (2.10)$$

and a reduction in the stress required for crack stability may result for large L_o .

Evidence for the presence of dislocation pileups in particle-containing systems is lacking. Local plastic flow may result in blunting of the pileups producing dislocation tangles at the particles. [53] For extensive local flow, the models of Stroh and Smith are no longer valid.

McClintock [51] postulated that the mechanics of fracture at inclusions may be studied in terms of continuum plasticity with emphasis on the state of plastic strain. He found that for rigid particles in a perfectly plastic matrix, infinite strain concentrations can exist near interfaces and that stress concentrations of about 2 can be obtained in circular (cylindrical) inclusions.

Kelly and Davies [47] considered stress equilibrium at discontinuous fibers in a plastically flowing continuum with shear flow stress τ_o . For a cylindrical fiber aligned along the z axis, force equilibrium

was used to arrive at an expression for the axial stresses in the fiber:

$$\sigma_{zz} = \frac{2\tau_o z}{r_o} + g, \quad 0 < |z| < \frac{l}{2} \quad (2.11)$$

where l is the length of the fiber and g is a constant. For cylindrical fibers in which a stress $\sigma_o = 2\tau_o$ is transmitted across the end surfaces, the total normal stress in the axial direction will be

$$\sigma_{zz} = \sigma_o + \frac{2\tau_o}{r_o} \left(\frac{l}{2} - |z| \right) \quad (2.12)$$

For a critical value of the aspect ratio of the fiber, $(l/2r_o)_c$, σ_{zz} at the center of the fiber ($z=0$) will be equal to the fracture strength of the fiber

$$\sigma_{zz} = \sigma_f = \sigma_o + 2\tau_o \left(\frac{l}{2r_o} \right)_c. \quad (2.13)$$

This model predicts that the tensile stresses in a particle with large aspect ratio are limited only by yielding in the particle or by internal fracture, depending on the relative values of yield stress and fracture strength.

Lindley et al. [47] have studied cracking of carbide plates in steel and found a propensity for cracking near the midpoints of the plates indicating agreement with equation 2.12.

Argon [42] has determined solutions for elastic-plastic deformation about hard particles. Limiting solutions for non-hardening and linearly hardening materials were used to obtain bounds on the solutions for real materials after Rhee and McClintock. [54] This method was applied to a

rod of length l and radius r_0 aligned in the tensile direction in a linearly hardening matrix. Using asymptotic solutions for large values of l/r_0 results in the following expression for the stresses across the ends of the rod

$$\sigma_{zz} = \sigma_0 \left(\frac{1 + \frac{l}{r_0}}{1 + \frac{E_m}{E_p} \frac{l}{r_0}} \right) \quad (2.14)$$

where E_p is the Young's modulus of the rod and E_m is the linear hardening rate of the matrix. For rigid inclusions

$$\sigma_{zz} = \sigma_0 \left(\frac{l}{r_0} + 1 \right) \quad (2.15)$$

This stress component is nearly constant along the length of the particle and is approximately equal to the average stress in the fiber according to the non-hardening case of equation 2.12.

The actual solution for the stresses in a fiber are assumed to be bounded by these two solutions so that in typical strain hardening materials containing particles with large aspect ratios, the distribution of axial normal stress is not expected to be uniform along the length of the particle; however, the stress concentration at the midpoint will be less than that predicted by the non-hardening case. This conclusion is still compatible with the results in [17].

Of course the stresses in real particles will be influenced by particle shape with possible large stress concentrations in particles with complex geometries. Also, pre-existing flaws (interface facets, internal boundaries) may enhance cracking.

Because of the apparent uniformity of plastic flow at inclusions in steel, it is felt that the continuum models best describe the stress intensification in carbide particles.

2.5.2 Particle-matrix decohesion

Several models have been proposed which attempt to characterize the nucleation of interfacial voids in terms of some critical value of one or more parameters of a given system. Ashby [43] has proposed a model for interfacial void nucleation which is based upon a critical local stress criterion. The normal stresses at a spherical inclusion were determined from a model for secondary slip at small particles, resulting in back stresses on the particles which can lead to cavitation at the interface.

He assumed that an imposed shear strain, γ_p , due to slip on a primary system is accommodated by the punching of prismatic interstitial and vacancy loops from the particle interface along axes which are 45° to the direction of primary glide. The resulting strain on the secondary system is γ_s , and if continuity is maintained at the particle-matrix interface, the elastic strains in the particle and the interfacial stresses may be computed in terms of γ_p and γ_s .

Ashby then showed that when these interfacial stresses attain some critical value at which prismatic punching of secondary dislocations occurs, the strain on the secondary system is very nearly equal to $\gamma_p/2$. Since the prismatic loops must account for these strains, the magnitude

of the maximum displacements at the interface of a particle of radius r_0 are

$$u = \frac{\gamma_p r_0}{2} . \quad (2.16)$$

If there are n secondary loops, each of Burgers vector b , to account for these displacements, then

$$n = \frac{\gamma_p r_0}{2b} . \quad (2.17)$$

Ashby assumed that cavity nucleation occurs when the back stresses on the interface due to a stack of dislocations of length k are equal to some critical value, σ_c . He argued that since close to a dislocation loop the stresses fall off as $\frac{1}{R}$, while the distant field decays as $1/R^3$, the interfacial stresses will arise mainly from the closest loop which will be at an approximate distance of $R = \frac{k}{n}$ from the particle. Using equation 2.17 then gives

$$\sigma \sim \frac{1}{R} = \pm \frac{\gamma_p r_0}{2kb} . \quad (2.18)$$

This relation predicts that the maximum interfacial tensile stress will be linearly related to the applied plastic shear strain and the particle size, indicating that large particles will nucleate cavities before smaller ones.

This model assumes an idealized arrangement of ordered secondary slip which may apply to small particles as shown by the experimental evidence of Humphrey's and Martin.^[55] However, this model cannot account for plastic accommodation about large particles where multiple slip occurs and where alternate relaxation mechanisms operate. It is also not valid

to conclude that the stresses increase linearly with particle size since the relationship between σ_0 and k is not known and in fact it might be argued that r_0/k is virtually independent of particle size, thus eliminating r_0 from equation 2.18.

As stated by Ashby the stress required to punch out each successive loop will increase slightly due to the fact that the previous loops must be forced outward. Since n is proportional to r_0 , there will be some resultant effect on the interfacial stress due to particle size. As the strains increase and the outer loops begin to interact with primary glide dislocations or with prismatic loops from neighboring particles, k will essentially become fixed and the stresses will then rise more rapidly as a function of n . Linear dependence of σ on r_0 is then valid. If k is taken to be one half of the nearest neighbor spacing between particles, λ , then according to Brown and Embury [56]

$$\frac{k}{r_0} = \frac{1}{4} \left(\sqrt{\frac{2\pi}{3c}} - \sqrt{\frac{8}{3}} \right) \quad (2.19)$$

for spherical particles of volume fraction c . This ratio is a function only of volume fraction so that the size dependence in Ashby's cavitation model is basically the same as that postulated by Argon et al. [24] concerning particle interactions.

This model, proposed by Argon, Im and Safoglu, [23] also involves a critical local stress criterion which governs interfacial cavity formation at hard particles in a plastically flowing matrix. They determined that the necessary elastic energy required for formation of the resulting

free surfaces is always available for particles greater than about $200 \text{ \AA}$ in diameter upon reaching the interfacial strength. Thus it is concluded that void formation occurs spontaneously when the critical interfacial stress is attained for most particles.

The particle was treated as a rigid cylinder embedded in an elastic-plastic continuum subjected to pure shear deformation. Limiting expressions for the stress concentration at the interface were arrived at by obtaining finite element solutions for the case of a non-hardening matrix and then comparing them with the known solutions for a linearly hardening matrix having zero yield stress. Rhee and McClintock [54] used this principle to determine strain concentrations in inhomogeneous deformation fields. From such an analysis it was found that the maximum normal interfacial tensile stress, σ_n , can be bounded such that

$$\frac{3}{2} \tau_0 \leq \sigma_n \leq 2\tau_0 \quad (2.20)$$

where τ_0 is the flow stress in shear. For some other distant deformation field other than pure shear containing a hydrostatic stress component, σ_T , the interfacial stress is given by superimposing the hydrostatic stress on the solution for pure shear giving

$$\frac{3}{2} \tau_0 + \sigma_T \leq \sigma_n \leq 2\tau_0 + \sigma_T \quad (2.21)$$

and since the limits are rather close it is assumed that

$$\sigma_n \approx Y(\bar{\epsilon}_p) + \sigma_T \quad (2.22)$$

where $Y(\bar{\epsilon}_p)$ is the flow stress in tension and is a function of the local homogeneous plastic strain. Thus for a state of simple tension the maxi-

imum interfacial stress will be approximately $\sigma_n = \frac{3}{2} Y(\bar{\epsilon}_p)$. If geometrical instability in the form of necking occurs σ_n will increase due to increasing σ_T . It is assumed that when σ_n reaches some critical value, σ_c , an interfacial void will nucleate, yet this criterion should be considered as a lower bound since σ_c must be attained over some finite area of the interface, while σ_n in equation 2.22 corresponds to the maximum interfacial stress.

A critical stress criterion for cavity nucleation based on dislocation models has also been proposed by Argon and his coworkers. After Ashby [43] it is assumed that as plastic incompatibilities mount between the flowing matrix and a rigid spherical particle due to a shear strain γ_p , a plastic accommodation zone is produced as dislocation loops are punched out from the interface. These dislocations move away from the inclusion against the influence of an effective shear drag, τ_s , forming a cylindrical region of radius $r_0/\sqrt{2}$ where r_0 is the particle radius. For very small inclusions ($r_0 \leq 100 \text{ \AA}$) it is assumed that no plastic relaxation occurs internally in the cylinder. Force equilibrium then leads to an equation for the normal stress, σ_n , at the interface of the particle and the cylinder given by

$$\sigma_n = \tau_0 \left(1 + \sqrt{\frac{6\sqrt{6}}{m}} \gamma_p / \gamma_0 \right) \quad (2.23)$$

where τ_0 is the yield stress in shear of the polycrystal ($\tau_0 = \frac{m\tau_s}{\sqrt{3}}$, m being the Taylor factor) and γ_0 is the corresponding yield strain. At a critical value of stress, σ_c , corresponding to a critical value of

plastic strain, γ_p^c , interfacial decohesion can occur:

$$\gamma_p^c = \frac{m\gamma_0}{6\sqrt{6}} \left(\frac{\sigma_c}{\tau_0} - 1 \right)^2. \quad (2.24)$$

According to this model, for interfacial strengths much larger than the matrix flow stress τ_0 , the applied strain required to nucleate cavities will be proportional to the square of the bonding strength. This could explain why some observers [25, 26] have noted large changes in strain to cavitation for rather small alloying changes which affect particle bonding strength.

For particles with r_0 greater than 100 Å it is assumed that plastic relaxation can occur within the previously elastic glide cylinder. Although the same equilibrium equation must be solved, the stress-strain relation is no longer linear. Instead, power law hardening is assumed with

$$\frac{Y(\bar{\epsilon}_p)}{Y_0} = \left(\frac{\bar{\epsilon}_p}{\bar{\epsilon}_0} \right)^n \quad (2.25)$$

where $\bar{\epsilon}_0$ and Y_0 are the tensile yield strain and stress respectively.

The total interfacial stress is determined now to be

$$\sigma_n = \tau_0 \left[\left(\frac{\gamma_p}{\gamma_0} \right)^n + \sqrt{3} \left(\frac{\sqrt{6}(m+1)}{m} \right) \frac{\gamma_p}{\gamma_0} \right]^{\frac{n}{1+n}}. \quad (2.26)$$

For a given plastic strain γ_p , the corresponding interfacial stress is much less than when no plastic relaxation occurs. This result predicts that small particles should nucleate cavities after less plastic strain than larger particles provided the energy condition is satisfied, which

is contrary to most observations. Instead, Argon attributes the observed size dependence for void nucleation to interactions between inclusions of larger than average size with smaller than average spacings (i.e. regions of higher than average volume fraction). An approximate solution, which predicts enhanced interfacial stresses for smaller values of particle spacing, was obtained for the case of overlapping plastic zones of adjacent particles due to large local volume fraction or large plastic strains.

The stress controlled nucleation criteria of Argon et al. depend upon the models which are used to compute local stresses at inclusions. The continuum model is a two-dimensional treatment whereas experiments^[12] were performed on materials containing spheroidal inclusions. Generalization of this model to three dimensions may not be valid.

The dislocation models are based on rather idealized modes of plastic accommodation which, however, allow for the incorporation of important parameters including local volume fraction and applied plastic strain. An important result is that absolute particle size is not a factor in these nucleation models aside from the minimum size requirement for energetically stable voids. From energy considerations it will be shown in chapter 5 that this requirement is not generally satisfied, even at high levels of deformation, and that a dependence on absolute particle size exists.

An energy based criterion for interfacial void nucleation was presented by Tanaka, Mori, and Nakamura^[57]. They treated the case of a hard, elastic, spherical particle in a plastically flowing matrix under

uniaxial tension. The results of the Eshelby transformation problem [45] were used to determine the total energy in and around the particle before (E_T^B) and after (E_T^A) void formation, including appropriate surface energy terms. Void nucleation is assumed to occur when

$$E_T^A < E_T^B. \quad (2.27)$$

The values of the applied matrix strains were chosen as the transformation strains and thus plastic relaxation in the vicinity of the particle was neglected. Also, the particle and matrix were assumed to have different elastic constants and therefore the internal stresses and strains were determined by the equivalent inclusion method. [58] The total energy before and after surface decohesion was determined by summing the following contributions: (1) E_1 , the elastic stored energy resulting from internal stresses caused by plastic incompatibility, (2) E_2 , the energy of interaction between the applied field and the internal fields, (3) E_3 , the interaction energy of the elastic inclusion with the applied stress, (4) E_4 , the energy associated with the entire specimen volume due to the applied loading in the absence of the inclusion, and (5) E_5 , the surface energy associated with relevant interfaces.

Void nucleation was assumed to occur as decohesion of spherical void caps at the particle poles. For these conditions, the inequality 2.27 was found to lead to a criterion which is formulated in terms of a critical tensile strain

$$\epsilon_p^c \geq \beta r_o^{-1/2} \quad (2.28)$$

where β is a constant involving the elastic constants of the matrix and inclusion and the free surface energies of the matrix, inclusion, and interface respectively.

The surface energy term, E_s , was computed assuming decohesion of the total inclusion surface. The consequence of this is an overestimation of the critical strain to nucleation since the other energy quantities were determined assuming only partial decohesion. Equation 2.28 favors void formation at larger particles with strain to decohesion being inversely proportional to the square root of particle size, corresponding to a number of qualitative observations. However, for typical levels of deformation, this criterion is virtually always satisfied for particles greater than about 200 Å, which agrees with Argon's analysis.

Because plastic relaxation was not accounted for in this model, the energy associated with the internal stresses is much larger than the energy terms associated with the applied fields. For this reason there is only a minor effect of applied loading on reducing the critical strain to stable void formation. As relaxation occurs the internal stresses are reduced and the effect of the applied loading will be enhanced. Also, a corresponding increase in the critical strain to nucleation will occur. The effect will produce a large increase in the size of particles which can form energetically stable voids and equation 2.28 will be applicable to materials with a wide range of particle sizes.

An energy criterion for void nucleation was developed by Brown and Stobbs [29] based on a model for work hardening in FCC materials contain-

ing hard spherical particles. Included in their model was a detailed description of the dislocation processes leading to plastic relaxation. The resulting secondary dislocation structures about the particles produce two effects. First, local hardening about the particles strengthens the overall material by inhibiting primary slip and second, these dislocation tangles impede further plastic relaxation creating large back stresses on the particles. It is these unaccommodated strains and stresses which give rise to elastic energy storage in the vicinity of the particles. Brown and Stobbs equated this energy to the surface work, W , required for total interfacial decohesion to determine the critical shear strain for void formation and determined that

$$\gamma_p^* \geq \sqrt{\frac{3W}{\mu r_0}} \quad (2.29)$$

gives the condition for energy stability. In this expression γ_p^* represents the uniform unrelaxed strains about the particles which were equated to the transformation strains in the Eshelby solution [58] for the purpose of computing elastic energy storage. In order to relate this result to measured deformation, an approximate relationship between the unrelaxed strains, γ_p^* , and the applied strain, γ_p , was developed based on details of the plastic zones about the particles. It was determined that

$$\gamma_p^* = \left(\frac{4b}{r_0}\right)^{5/8} \left(\frac{\alpha}{\pi}\right)^{2/8} \gamma_p^{3/8} \quad (2.30)$$

where b is the Burgers vector of the secondary dislocations and α is a constant, generally assigned a value between $\frac{1}{5}$ and $\frac{1}{3}$. This equation

indicates that relaxation is more complete at larger particles. The energy criterion in equation 2.29 then becomes

$$\gamma_p \geq \frac{\pi}{2} \left(\frac{3W}{4b\mu} \right)^{4/3} r_o^{1/3} \quad (2.31)$$

indicating that the critical strain for decohesion increases as r_o to the one-third power which is contrary to a number of studies cited in section 2.1.

While this criterion considers the effects of plastic relaxation at particles, it fails to account for the elastic stored energy associated with the applied loads. It will be shown in chapter 5 that this contribution cannot be ignored.

In real systems, total interfacial decohesion is not observed so that both the elastic energy released and the surface work required upon void formation must be altered if a model of the type used by Brown and Stobbs is to be employed. Such considerations were made by Goods and Brown. [59]

The models which have been discussed, pertaining to void nucleation based on energy requirements, merely detail the conditions for energy stability of the final state of the system without considering whether such a configuration can actually be attained. Sundström [60] has developed an energy criterion for void initiation based upon an elastic continuum analysis involving finite element calculations for a two-dimensional circular particle embedded in an elastic matrix. While the model does not apply to real, plastically deformed materials with spheroidal

particles, it does propose the concept of a critical minimum stable void size and an accompanying activation energy requirement. A compliance method was used in which the energy release rate, $\dot{G}$, was determined for a number of crack angles, ϕ , defined in three dimensions in figure 5.1. For a constant value of the work of adhesion of the interface, W , the total integrated energy difference

$$\Delta E = \int_{\phi=0}^{\phi=\phi_0} (W - \dot{G}) dA \quad (2.32)$$

was determined as a function of ϕ_0 for a fixed particle size, r_0 , and various values of applied tensile stress. The results of this model indicate that below a critical value of the applied stress, σ_c , stable interfacial crack formation is not energetically favorable for any value of ϕ_0 . Also, for an applied stress greater than σ_c , voids with subtended angles less than a critical value, ϕ_c , are not stable. Corresponding to this critical angle is a critical value of the energy difference function, ΔE_c , defined by $\frac{d\Delta E}{d\phi} = 0$. ΔE_c acts as an activation energy barrier to void nucleation.

The critical stress required for stable interfacial void formation was found to be a function of $\sqrt{W/r_0}$, which indicates that larger particles with weak interfaces will fail at lower applied stresses. Such behavior is in agreement with experimental observations.

Each of the models presented in this section contains elements which must be considered in the development of a comprehensive nucleation criterion, and the important aspects of each one are summarized in

Table 2.1. As indicated throughout this review, various deficiencies associated with the concepts, details, or applicability of each model exist. It is the purpose of the present study to develop a nucleation criterion which avoids these deficiencies and which can adequately describe the phenomenon of void formation in a variety of systems.

2.6 Axisymmetric tensile deformation

In order to compare analytical models for void nucleation to quantitative experimental results, the deformation in an experimental system must be measured in terms of the parameters which are used in the models. For real systems it is not possible to determine the conditions which exist at each material point because of local variations in microstructure. However, statistically valid average quantities may be obtained if the continuum solutions for the deformation have slowly varying values over microstructural distances. The conditions which exist in the neck of a tensile specimen are ideal for measuring effects of deformation on void nucleation. Both analytical and numerical solutions for the stresses and strains in a homogeneous material are available for selected positions in the necked region. Widely varying states of deformation exist yet changes in the deformation are small over distances which are large compared to relevant microstructural spacings.

Since the present study deals with void nucleation in spheroidized steels, as measured in deformed tensile specimens, a brief review of the general features of the deformation in these materials during tensile testing will be presented.

2.6.1 Tensile deformation of spheroidized steels

The tensile deformation of spheroidized steels from initial elastic straining to final fracture is described by Anand [61] in conjunction with the load-elongation curve of figure 2.1. The specimen is deformed elastically from A to B where pre-yield microstrain causes a deviation from linearity of the stress-strain curve (B to C). This initial plastic deformation is distributed homogeneously throughout the specimen. At the upper yield stress (C) microplastic heterogeneities known as Lüders fronts are nucleated and these regions of localized plastic flow propagate along the length of the specimen at an applied stress which is lower than the stress required to nucleate them (D). This lower yield stress remains fairly constant during propagation of the Lüders bands (D to E) and drops slightly (E to F) as the last undeformed section is rapidly reduced in area. After Lüders deformation, the entire specimen is forced to deform at the applied strain rate which causes a jump in the stress (F to G). Homogeneous plastic deformation proceeds and the matrix material hardens. Another form of localization occurs (H) due to geometrical instability which results in the formation of a neck. Beyond this point strains are no longer distributed uniformly along the axis of the specimen or across an arbitrary section perpendicular to the axis. The state of deformation in the necked region is discussed further in section 2.6.2.

Continued deformation produces localized strains in the neck and a drop in the load associated with deformation (H to I) is due to the continuous reduction in area of the neck.

In many materials, this latter stage of deformation results in nucleation and growth of microvoids. At some level of deformation (I) the conditions are such that these imperfections coalesce to form the nucleus for the radial crack which is associated with the final fracture of the specimen. In a conventional test, the elastic energy released by the specimen and the testing machine upon formation of the crack will cause unstable growth and fast fracture of the specimen will occur (I).

Bluhm and Morrissey [62] designed a rigid testing device with which they were able to control the elongation of the specimen and the growth of the radial crack. This portion of the deformation is associated with a load drop (I to J) corresponding to the rapid reduction in load carrying area as the crack grows.

2.6.2 Stress and strain in necked tensile specimens

Necking represents a critical stage in the deformation of a tensile specimen since a triaxial state of stress is created which increases with the severity of the neck. Bridgman [63] has provided an analysis of the stresses on the minimum cross section of a necked specimen. By solving the equations for stress equilibrium and assuming that the strains are uniform throughout the section (i.e. the von Mises yield function remains constant), the stresses σ_{zz} , σ_{rr} , $\sigma_{\theta\theta}$ are expressible in terms of the flow stress Y_0 , the minimum neck radius, a , and the neck contour radius, R (figure 3.1). Experimental proof of the uniformity of strains

was obtained for reductions in area up to 92%. The analytical expressions are

$$\begin{aligned}\sigma_{rr} = \sigma_{\theta\theta} &= Y_0 \ln \left(\frac{a^2 + 2aR - \rho^2}{2aR} \right) \\ \sigma_{zz} &= Y_0 \ln \left(1 + \frac{a^2 + 2aR - \rho^2}{2aR} \right)\end{aligned}\tag{2.33}$$

where ρ is the radial position.

It can be seen that this system of stresses is given by a uniform flow stress Y_0 and a superimposed triaxial field which decreases from the central axis to the specimen surface. Argon et al. [64] have extended this analysis to calculate the stresses and strains along the axis of a tensile specimen as a function of distance, z , from the minimum cross-section. The normal component of stress along the axis is

$$\sigma_{zz} = Y_0 \left[1 + \ln \left(1 + \frac{a}{2R} \right) - 2 \ln \left(1 + \frac{z^2}{a(a+2R)} \right) \right]$$

and (2.34)

$$\sigma_{rr} = \sigma_{\theta\theta} = Y_0 \left[\ln \left(1 + \frac{a}{2R} \right) - 2 \ln \left(1 + \frac{z^2}{a(a+2R)} \right) \right].$$

The use of a constant Mises yield function will give rise to some error in this expression since the strains are not constant along the axis and the yield function is indeed a function of the strains, $Y_0 = Y(\bar{\epsilon}_p)$. It should be noted that for large enough z , σ_{zz} can become compressive; however, for very large z values in the uniformly deformed region outside the neck, σ_{zz} must approach $Y_0(\bar{\epsilon}_p)$. Eq. (2.34) does not account for this boundary condition.

Several numerical analyses of the stress and strain distributions in uniaxial tensile specimens have been conducted. Needleman [65] used the following constitutive law to model plastic flow:

$$\bar{\epsilon}_p = \epsilon_o \left[\left(\frac{\bar{\sigma}}{Y_o} \right)^n - \frac{\bar{\sigma}}{Y_o} \right] \quad (2.35)$$

where $\epsilon_o = Y_o/E$ is the yield strain of the material, $\bar{\sigma}$ and $\bar{\epsilon}_p$ are equivalent stress and equivalent plastic strain respectively. Experimentally obtained stress-strain curves were used to determine ϵ_o , Y_o and n . Finite element techniques were employed to compute the shape of the necked region and the stresses and strains therein on the basis of J_2 flow theory.

Chen [66] used a Ramberg-Osgood hardening law for equivalent plastic strain of the form

$$\bar{\epsilon}_p = \frac{3}{7} \epsilon_o \left[\left(\frac{\bar{\sigma}}{Y_o} \right)^n - 1 \right] \quad (2.36)$$

to carry out similar calculations.

Norris et al. [67] obtained a uniaxial flow stress curve from a detailed photographic history of a specimen of A-533 steel in tension. They developed an empirical flow law of the form

$$\bar{\sigma} = \frac{c_1 + c_2 \bar{\epsilon}_p + c_3 \bar{\epsilon}_p^2 + c_4 \bar{\epsilon}_p^3}{1 + c_5 \bar{\epsilon}_p + c_6 \bar{\epsilon}_p^2} \quad (2.37)$$

with c_1 through c_6 being numerical constants. A dynamic finite-difference solution was obtained which is valid for large area reductions.

All three of the numerical studies give comparable results and all predict larger triaxial stresses than the analytical solutions of Bridgman. The model of Norris et al. also predicts non-uniform strains across the

plane of the neck, contrary to the assumptions of Bridgman. The numerical models all account for compression zones on the axis away from the neck indicating that the surface of the specimen must carry the tensile loads for these cross sections. The axial stresses again approach $Y_0(\bar{\epsilon}_p)$ at large distances from the neck. The results of Needleman and Norris et al. are used in chapters 4 and 5.

3. Experimental Procedure

3.1 Materials

Two carbon steels with .17 (type B) and .44 (type W) weight percent carbon respectively were used for the quantitative studies of this investigation. The supplier, Bethlehem Steel Corporation, provided the chemical compositions and size specifications of the as received material given in table 3.1.

A third steel of 1.05 (type H) weight percent carbon was used for supplementary studies and the chemical analysis provided by Walter M. Saunders, Inc., Providence, Rhode Island, is also given in table 3.1.

3.2 Heat treatments

The purpose of the heat treating procedure was to obtain uniformly distributed spheroidized cementite particles in a matrix of α iron. The as received materials were subjected to the following sequence of heat treating operations.

a) Type B and Type W Specimens

Rod sections of approximately 3.25 in. (.0826 m) in length were austenitized at 50°C above the A_3 temperature for 1.5 hours. The austenitized specimens were then rapidly quenched in ice water in order to obtain fine bainitic or martensitic structures. The quenched rods were then tempered at 700°C for 1) 1 hour 2) 24 hours and 3) 120 hours in order to obtain different particle size distributions in specimens of the same type.

b) Type H Specimens

Following the standard austenitization treatment at 50°C above the A_3 temperature for 1.5 hours, the specimens were quenched in oil as a precaution against cracking which sometimes accompanied ice water quenching in these high carbon steels. Tempering at 700°C for 72 hours produced the desired spheroidized structures.

3.3 Mechanical testing

The heat treated rods were machined into standard tensile specimens of 1.00 in. (.0254 m) gage length and 0.25 in. (.0064 m) diameter. All testing was done at room temperature. Load-elongation curves in tension were obtained for each specimen using a mechanical Instron testing machine. Deformation rates corresponded to a crosshead speed of .05 in/min. (2.12×10^{-5} m/s). After removing the strain gage, the specimens were fractured and the reduction in area (RA) of the minimum cross section was determined.

Another series of tests was then performed in which each specimen was deformed to a prescribed RA which resulted in severe necking but which was conservatively less than the RA corresponding to fracture. For type B specimens the minimum gage diameter measured 0.13 in. (.0033 m) and for type W specimens it measured 0.16 in. (.0041 m) corresponding to reductions in area of 73% and 59% respectively. Deformed and undeformed specimen geometries are shown in figure 3.1.

A third series of tests was performed in which the radial crack associated with the ultimate failure of axisymmetric tensile specimens was

subjected to controlled growth. This procedure was accomplished by two different methods.

a) Hydraulic Instron Testing Machine

The specimens were predeformed until a visible neck was obtained. A strain gage was then positioned across the neck and the test was resumed in the strain control mode. The deformation rate was determined by imposing a ramp generating function which produced a controlled rate of extension of the gage arms of .005 in/min. (2.11×10^{-6} m/sec). The test was stopped immediately when an abrupt load drop indicated the presence of an internal crack.

b) Rigid Testing Apparatus (RTA)

A rigid testing device was constructed (figure 3.2) by modification of the design used by Bluhm and Morrissey.^[62] Pre-necked specimens were deformed in the apparatus which was fitted to the mechanical Instron machine. Specimen loads were recorded by an internal load cell and specimen elongation was determined by the method described in [62]. Again, an abrupt change in the load on the specimen signaled the presence of an internal crack and the test was halted.

While both methods were able to effect stable radial crack growth in type B specimens, neither was able to prevent unstable failure in W type specimens. Since the precision with which the load versus elongation could be determined was better for method (a), this procedure produced specimens containing smaller internal cracks which proved more useful for study of specimen fracture.

3.4 Metallography

Metallographic observations were made on both undeformed and deformed specimens using both optical and electron microscopy. All observations of undeformed specimens were made on transverse sections while observations of deformed specimens were made on both longitudinal and transverse sections as represented in figure 3.3.

Longitudinal sections were prepared so as to contain the central tensile axis. This was done by constantly monitoring the specimen dimensions throughout the grinding and polishing operations. The transverse sections were prepared by first inscribing circumferential rings at predetermined positions along the length of the deformed section using a lathe. A special holder was designed which was used to fix the specimen in the proper orientation and which also allowed for constant monitoring of the orientation and position of the particular transverse surface relative to the specimen axis during polishing. For each specimen, a series of transverse sections was prepared corresponding to successively smaller axial distances from the minimum cross section. Each new section was obtained by grinding to the next premarked position and thus the previous sections were destroyed. Therefore all the data required from a given section had to be obtained before preparation of the succeeding one. Each section was carefully polished and etched after preliminary use of various grades of abrasive papers. In order to insure that extraneous metal was not masking the true nature of the surface and existing void population, a scheme of repeated polishing, etching and ultrasonic cleaning was employed which resulted in reproduceable microstruc-

tures. This scheme consisted of 1) polishing the surface with .05 μ alumina in water 2) ultrasonic cleaning for 1-2 minutes 3) etching with a 4% picral solution followed by a rapid etch with a 3% nital solution 4) ultrasonic cleaning for 1-2 minutes. This process was repeated four times. The nital etching procedure, used to reveal grain boundary structures, was carefully controlled because overetching can cause extensive surface damage in highly deformed regions.

The microstructural parameters were determined in both deformed and undeformed specimens. For the deformed specimens the areal density of voids, η_A , and the volume fraction of voids, f_v , were obtained from transverse sections by standard metallographic techniques performed on scanning electron micrographs taken at a magnification of 2000 times. For each section, η_A and f_v were determined corresponding to radial positions of ρ/a equal to 0, .5, and 1.0 where ρ is the radial coordinate and a is the value of ρ at the specimen surface for a given section.

The B type specimens which were deformed to produce an internal crack were sectioned longitudinally as well as transversely. Qualitative observations of the nature of the crack path, in relation to the void structures and other pertinent microstructural features, were made.

4. Results and Analysis

4.1 General microstructural features

4.1.1 Undeformed specimens

The described heat treatments produced uniform distributions of cementite particles in a ferrite matrix. These particles were generally connected by a network of ferrite grain boundaries and subgrain boundaries (figure 4.1). An increase in average particle size was accompanied by an increase in the mean grain intercept length. Roughly 70% of the particles were associated with visible boundaries for each specimen, although local variations of this measurement were large. The non-boundary particles were smaller and more spherical than the overall population while the boundary particles tended to be larger and less equiaxed with maximum extent in the plane of the boundary. It is important to realize that while on a two dimensional micrograph these elongated particles may appear as prolate spheroids with major axes parallel to the grain boundary trace, they are actually oblate spheroids with minor axes perpendicular to the plane of the boundary.

4.1.2 Deformed specimens

a) Longitudinal sections

In the regions of maximum deformation near the center of the neck, the microstructure was characterized by grains elongated in the tensile direction. This observation was most apparent in the B-type steels which were deformed to maximum strains of 1.4 (figure 4.2). Due to the large amounts of plastic deformation and associated high dislocation

densities of the ferrite matrix, nital etching of grain boundaries was only partially effective. Overetching with nital produced an irregular surface which was no longer representative of the true microstructure. This hampered attempts to characterize the nature of the void distribution with respect to the microstructure. However, observations of micrographs like those in figure 4.3 allow one to conclude that grain boundaries play an important role in the nucleation of voids for these materials. Because of the high deformation levels associated with void nucleation and the associated grain elongation, the ellipsoidal boundary particles tended to align in the tensile direction and the voids which nucleated at these particles were associated with these boundaries.

As indicated by the micrographs in figure 4.4 there was a definite particle size effect on the nucleation of voids. Interfacial decohesion was associated with larger than average particles. There were very few instances of voids at particles of less than average size.

In the type H steels, the primary mode of void formation was particle fracture as indicated in figure 4.5. These polished and etched longitudinal sections were prepared from necked tensile specimens deformed to a RA of 35%. A subsequent heat tinting treatment consisting of 20-30 seconds at 600°C in air caused a thin oxide layer to form on the cementite particles. The resulting shading of the particles made it easier to differentiate them from the matrix during optical metallographic inspection. It is clear that cracking was favored at large, irregular

particles with aspect ratios greater than one in the tensile direction. In the type B and type W steels having smaller average particle sizes with more regular shapes and smaller aspect ratios, interfacial decohesion was the prevalent failure mode for void nucleation as shown in figures 4.2 and 4.3. These observations may be explained by the models proposed in chapter 5.

b) Transverse sections

SEM micrographs of deformed transverse sections (figure 4.6) differed from those of the undeformed sections in two respects. First, the deformed sections were populated with voids and second, while the average radial deformation appeared isotropic, the deformation of individual grains was not homogeneous as indicated by the sometimes curved and swirling nature of the flow in individual grains as seen in figure 4.6a.

It is observed that the voids tended to have elliptical cross sections similar to those of the particles, as might be expected since the particles were nucleation sites for these voids. Figure 4.6a provides evidence of the importance of grain boundary particles to the nucleation process since it is apparent that the voids are associated with the traces of the boundaries.

A third type of metallographic section was prepared corresponding to the geometry of figure 3.3c. Special care was required in order to retain the corner of the intersection between the longitudinal and transverse faces. The results of this type of specimen preparation

are shown in figure 4.7b, with the specimen orientation corresponding to figure 4.7a. The voids were generally cylindrical in nature having elliptical cross sections perpendicular to the tensile direction.

The highly elongated nature of the voids (figures 4.3 and 4.4) on grain boundaries aligned in the tensile direction suggests that perhaps void growth is enhanced by boundary separation. The planar nature of the boundaries, however, would be expected to produce lenticular void cross sections when viewed on transverse planes. It might be argued that the voids assume an elliptical geometry due to the radial stresses in the neck of the specimen, but there are no such stresses at the surface of the specimen ($\rho = a$) where the voids are still elliptical. It seems then that the nucleated void shape and subsequent plastic growth of the voids are the factors governing the ultimate void size and shape.

4.2 Microstructural parameters

The quantitative analysis of both deformed and undeformed structures was performed on transverse sections. The reasons for this will be made clear in section 4.7. Standard techniques of quantitative microscopy [68] were used to determine the following quantities for each specimen.

- P_P - Fraction of test points on a plane corresponding to cementite particles.
- $(N_A)_P$ - Number of particle intercepts per unit area of test plane.
- $(N_L)_P$ - Number of particle intercepts per unit length of random test line.
- $(N_L)_g$ - Number of grain boundary intercepts per unit length of random test line.

These quantities were used to obtain the following parameters:

- f_p - Particle volume fraction ($f_p = P_p$).
- r_o - Mean particle radius assuming spherical particles ($r_o = .75 f_p / (N_L)_p$).
- $(N_v)_p$ - Number of particles per unit volume
($(N_v)_p = (N_A)_p / 2r_o$)
- λ_p - Mean free path between particles
($\lambda_p = (1-f_p) / (N_L)_p$)
- D - Mean grain intercept length ($D = \frac{1}{(N_L)_g}$)

These parameters were evaluated for each specimen in both the deformed and undeformed conditions with the deformed state corresponding to the central plane of the neck. D was not determined for the deformed specimens due to difficulties in etching of the boundaries. The results appear in table 4.1. Differences in r_o and λ_p between the deformed and undeformed microstructures did not appear to be significant. For perfectly equiaxed spheres these quantities should be independent of the deformation. Any significant change of r_o or λ_p due to the deformation must be associated with the rotations of non-equiaxed particles and their subsequent alignment with the tensile axis. (See appendix A).

4.3. Spatial distribution of particles

The spatial randomness of particles in both deformed and undeformed specimens was considered using the methods of Ashby and Ebeling.^[69]

The experimentally determined probability that a square cell or a planar section will contain ω particles is given by

$$P(\omega) = \frac{\text{Number of such cells containing } \omega \text{ particles}}{\text{Total number of cells}} \quad (4.1)$$

The calculated Poisson probability function for random distributions is given by

$$P(\omega) = \frac{\mu^\omega}{\omega!} \exp(-\mu) \quad (4.2)$$

where μ is the average value of ω over all cells counted. The cell size was chosen so that $\mu \sim 1$. A representative comparison of $P(\omega)$ versus ω for deformed and undeformed structures is shown in figure 4.8 for specimen B3. Both the theoretical and measured curves are drawn. This result is representative of the other specimens and is important when considering deformation processes and the effects upon nucleation of voids. The fact that the distribution of particles remains more or less random, even at very large strains, means that a rearrangement of particles (clustering, stringing, etc.) is not responsible for affecting void formation at high deformation levels. Also, since $(N_A)_p$ and $(N_V)_p$ remain constant as discussed in appendix A, any accompanying void nucleation and growth should not be related to geometrical changes in the particle distribution.

4.4 Particle size distributions

The effect of particle size on void nucleation is an important factor in many of the current models for nucleation, including the present

one, and consequently particle size distribution curves were obtained for each B and W type specimen. These curves were computed using the methods of Underwood [68] which incorporate the measured quantities $(N_A)_p$, $(N_L)_p$ and $(N_V)_p$.

A log-normal distribution function is assumed as is common for these types of structures involving dispersions of a second phase. [70]

$$f(d) = \frac{1}{\sqrt{2\pi} \ln \sigma_g} \exp \left[-\frac{1}{2} \left(\frac{\ln d - \ln d_g}{\ln \sigma_g} \right)^2 \right] \quad (4.3)$$

where $d = 2r_o$, d_g is related to the average particle size and σ_g is related to the width of the distribution. The i th moment of the distribution is given by

$$\mu_i(d) = \exp \left[i \ln d_g + \frac{1}{2} i^2 \ln^2 \sigma_g \right] \quad (4.4)$$

so that

$$\begin{aligned} \mu_1(d) &= \frac{1}{(N_V)_p} \sum_j [(N_V)_p]_j \cdot d_j = \exp(\ln d_g + .5 \ln^2 \sigma_g) \\ \mu_2(d) &= \frac{1}{(N_V)_p} \sum_j [(N_V)_p]_j \cdot d_j^2 = \exp(2 \ln d_g + 2 \ln^2 \sigma_g) \\ \mu_3(d) &= \frac{1}{(N_V)_p} \sum_j [(N_V)_p]_j \cdot d_j^3 = \exp(3 \ln d_g + 4.5 \ln^2 \sigma_g) \end{aligned} \quad (4.5)$$

For spherical particles

$$\begin{aligned} (N_A)_p &= (N_V)_p \mu_1(d) \\ (N_L)_p &= \frac{\pi}{2} (N_V)_p \mu_2(d) \\ f_p &= \frac{\pi}{6} (N_V)_p \mu_3(d) \end{aligned} \quad (4.6)$$

Algebraic manipulation leads to the following relations for σ_g and d_g :

$$\begin{aligned}\ln^2 \sigma_g &= 2 \ln \frac{(N_A)_p}{(N_V)_p} - \ln \frac{2(N_L)_p}{\pi(N_V)_p} \\ \ln d_g &= 2 \ln \frac{(N_A)_p}{(N_V)_p} - \frac{1}{2} \ln \frac{2(N_L)_p}{\pi(N_V)_p}\end{aligned}\quad (4.7)$$

These relations define the log-normal distributions of particle sizes plotted in figure 4.9 for specimens B1, B2, B3 and W1. The results for W2 and W3 were almost coincident with B2 and B3 and are not shown. These graphs will be used later to relate proposed nucleation criteria to the particular size distribution for each specimen.

4.5 Tensile test data

From the load-elongation curves (figure 4.10) and measurements of RA at the minimum neck sections, the following quantities were obtained:

- σ_y^L - Lower yield stress
- ϵ_{TS} - Strain at necking
- σ_{TS} - Ultimate tensile stress
- $\bar{\sigma}_{min}$ - Average axial stress on the minimum section for interrupted tests
- $\bar{\sigma}_f$ - Average axial stress on minimum section at fracture
- $\bar{\epsilon}_f$ - Strain to fracture ($\bar{\epsilon}_f = \ln \frac{A_0}{A_f}$)

These quantities appear in table 4.2 for B and W type specimens.

Two other parameters which were required in order to determine the stresses and strains in the neck of each specimen were the minimum section radius, a , and the radius of curvature of the neck contour, R . These are defined in figure 3.1 and were determined for each specimen deformed to a prescribed strain (section 3.3) by magnifying the contour using a comparator, tracing the contour, and measuring a and R from the tracing. The value of a/R for all B type specimens was 0.95 and for all W type specimens it was 0.70. These values correspond well to the previous data of Bridgman [63] for steels.

4.6 Relation of neck geometry to local deformation state

In order to relate experimental data on nucleation and growth of voids to a given model, the details of the local state of the deformation must be available. Analytical and numerical solutions of the stresses and strains in the neck of a tensile specimen were presented in section 2.6.2. These were evaluated at selected positions in the deformed specimens for the conditions corresponding to measured values of a/a_0 and a/R . The flow stress Y_0 in equations 2.33 and 2.34 was taken to be a function of equivalent strain ($Y_0 = Y(\bar{\epsilon}_p)$) and was assumed to satisfy the Holloman type equation (2.25) where Y_0 now represents the yield stress and ϵ_0 the corresponding yield strain obtained by extending the curves of figure 4.10 back to the elastic loading line. Y_0 , ϵ_0 and n were adjusted to give a best fit to the recorded deformation curves. These values appear in table 4.3 for each specimen. The term ϵ actually represents the total strain but this is very nearly equal to the plastic strain $\bar{\epsilon}_p$ for large deformations.

The numerical results of Needleman [64, 65] were extrapolated to estimate the values of σ_{zz} and $\bar{\epsilon}_p$ for the current specimen geometries. This was somewhat approximate because of the length of the curve which had to be extrapolated in order to apply the numerical solutions to such high levels of strain. The numerical and analytical (equation 2.34) values for σ_{zz} along the specimen axis are plotted in figure 4.11 for specimens B2 and W2 and while the absolute values do not coincide exactly there is reasonable agreement over the range of computed values. It is felt that the numerical solutions give better results at large values of z while the analytical solutions are more accurate for $z=0$. The numerical values of σ_{zz} at $z=0$ were thus matched to the corresponding analytical values and the rest of the numerical curve was adjusted proportionately to obtain the values of σ_{zz} , $Y(\bar{\epsilon}_p)$ and σ_H used in the forthcoming analysis (table 4.4). $Y(\bar{\epsilon}_p)$ and σ_H are plotted for each steel in figure 4.12 as functions of z/a_0 .

The analytical values for σ_{zz} and σ_H corresponding to the plane of the minimum cross section were computed using equation 2.33. $Y(\bar{\epsilon}_p)$ was assumed constant on this plane. Figure 4.13 shows the axial stress component, σ_{zz} , as a function of ρ/a , corresponding to the values in table 4.5.

4.7 Quantitative void measurements

The areal density of voids, n_A , measured as number of voids per unit area of transverse section and the volume fraction of voids, f_v , are

plotted as functions of z (with $\rho=0$) and of ρ (with $z=0$) for each specimen in figures 4.14-4.17. These quantitative measurements were made on transverse (as opposed to longitudinal) sections for the following reasons:

1) Metallographic preparation of longitudinal sections posed various difficulties in terms of obtaining reproducibility of void counts and void volume fraction. The large strain gradients along the lengths of such sections produced differences in the properties of the etched surface and the scatter in the data was large. The much smaller strain gradients on transverse sections made it possible to prepare the entire surface uniformly. The tedious procedure of polishing each successive section proved worthwhile in terms of reproducibility of results.

2) A certain amount of ambiguity existed when attempting to define the contour of a void on longitudinal sections. It was often difficult to distinguish very shallow voids from otherwise highly deformed matrix surface. The void contours on transverse sections were well delineated due to the depth of the voids perpendicular to the plane of the sections.

3) The use of η_A as a measure of void profusion had some subtle disadvantages regardless of whether longitudinal or transverse sections were considered. η_A was not a true measure of void nucleation since it also contained information about void growth perpendicular to the surface on which the measurements were made. Such information was not readily attainable, but certain estimates of axial void growth were used to interpret the significance of η_A as measured on transverse sections. This

will be made clear in section 4.8.

The results show that a threshold level of $\eta_A \sim .5 \times 10^{10} \text{ m}^{-2}$ extended beyond the confines of the neck (figure 4.14). This may be attributed to early cavity formation at MnS inclusions or cracking of large, irregular particles as noted by Argon and Im [12] and Cialone and Asaro. [27] Other general features of these curves are the gradual rise in void density followed by a rapid rate of void density increase and an apparent saturation in this rate as the minimum section is approached. This observation may at first seem perplexing since only 10% to 40% of the particles had associated voids on the minimum cross section as determined from longitudinal sections. However, the proposed nucleation models in chapter 5 explain these results on the basis of a particle size effect.

In figure 4.15 there is a decrease of 30% to 50% in the value of η_A from the central axis ($\rho=0$) to the surface of the specimen at $\rho=a$ for the specimens tested. Assuming that the strains across the section remain uniform throughout the deformation, this effect must be related to the variation of σ_H on the minimum cross section. If $\bar{\epsilon}_p$ is not constant with respect to ρ on the plane $z=0$ as predicted by Norris et al. [67], then changes in η_A will be related to the effects of both σ_H and $\bar{\epsilon}_p$ on void nucleation and growth. The models of chapter 5 can also be used to explain the results in figure 4.15.

4.8 Effect of void growth

Since η_A contains information relating to both the nucleation and

growth of voids, it does not provide a valid representation of the distribution of voids. In principle, the number of voids per unit volume, η_V , can be determined from η_A if the mean void dimension, $\bar{H}$, normal to the surface can be obtained. Due to the complicated void geometries and void shape changes during deformation, $\bar{H}$, can not be accurately determined from measurements made on longitudinal sections. The following analysis provides a scheme by which η_A can be separated into a nucleation term η_A^n and a growth term η_A^g so that the effects of growth can be eliminated from the data.

An increment in the deformation of a void containing material will result in an increment in the measured value of η_A which can be represented as the sum of a nucleation term and a growth term. In order to evaluate the effect of growth it is helpful to transform the curves in figure 4.15 to functions of equivalent plastic strain $\bar{\epsilon}_p$. Figure 4.18 shows the relationship between $\bar{\epsilon}_p$ and z/a_0 and the corresponding contours of the B and W type specimens are indicated by the dashed lines. The solid curves were obtained by extrapolation of the numerical results [65] for $\bar{\epsilon}_p$ along the axis of a tensile specimen matched to the measured strain at $z=0$. The resulting relationship between η_A and $\bar{\epsilon}_p$ is shown in figure 4.19.

For a volume void distribution density defined by

$$\eta_V = \sum_i (\eta_V)_i \quad (4.8)$$

and

$$\bar{H} = \frac{\sum_i (\eta_V)_i H_i}{\sum_i (\eta_V)_i} \quad (4.9)$$

where i is used to label an increment in the size distribution of voids. The expression relating areal void density to volume void density

$$\eta_A = \eta_V \cdot \bar{H} \quad (4.10)$$

may be written as

$$\eta_A = \sum_i (\eta_V)_i \cdot H_i. \quad (4.11)$$

Considering differential quantities

$$\begin{aligned} \dot{\eta}_A &= \sum_i (\dot{\eta}_V)_i \cdot H_i + \sum_i (\eta_V)_i \cdot \dot{H}_i \\ &= \dot{\eta}_A^n + \dot{\eta}_A^g \end{aligned} \quad (4.12)$$

where the superscripts n and g refer to nucleation and growth respectively.

Without knowing the values of the H_i , or the exact void geometries, certain assumptions concerning $\dot{H}_i$ may be made from existing models for void growth. For the case of spherical voids, Rice and Tracey [71] determined that

$$\dot{H}_i = M(\sigma_T/Y(\bar{\epsilon}_p)) H_i \dot{\bar{\epsilon}}_p \quad (4.13)$$

where M is a function of the ratio of the hydrostatic stress, σ_T , to the flow stress, $Y(\bar{\epsilon}_p)$, and $\dot{\bar{\epsilon}}_p$ is the remote strain rate in the direction of H_i . For small values of $\sigma_T/Y(\bar{\epsilon}_p)$, M is only slightly affected by σ_T . Based on computed values of $\sigma_T = \frac{1}{3} \sigma_{22} + \frac{2}{3} \sigma_H$ in the neck of the deformed specimens (having a maximum value of .73 in B type specimens) it was maintained that M could be treated as some constant average value.

Equations 4.12 and 4.13 lead to the expression

$$\dot{\eta}_A^n = \dot{\eta}_A - M \eta_A \dot{\bar{\epsilon}}_p \quad (4.14)$$

The total contribution of η_A^n to the measured quantity η_A at a strain of $\bar{\epsilon}_p^0$ can be determined by integration of equation 4.14 over the strain interval from $\bar{\epsilon}_p = 0$ to $\bar{\epsilon}_p = \bar{\epsilon}_p^0$ giving

$$\eta_A^n = \eta_A(\bar{\epsilon}_p^0) - \eta_A(0) - M \int_0^{\bar{\epsilon}_p^0} \eta_A d\bar{\epsilon} \quad (4.15)$$

which can be evaluated numerically from the form of η_A in figure 4.19.

The results for each specimen are plotted in figure 4.20 for $M = 1$ and $M = 2$ corresponding to void elongation rates equal to that of and twice

that of the matrix respectively. In his work on spheroidized steels

LeRoy [28] determined that M has an average value of 1.4. The models

of Rice and Tracey [71] upon which equation 4.13 is based predict

values of M between 1.7 and 2.3 over the extent of the necked regions.

Since the voids in spheroidized steels are not spherical and are probably

constrained somewhat by the particles from which they form, it is felt

that the selected values for M give reasonable limits to the growth

term.

4.9 Comments on void growth and relation to fracture

The average void diameter, $\bar{d}_v$, on transverse sections was determined as a function of distance from the minimum cross section along the axis (figure 4.21). This was done by dividing the void volume

fraction (i.e. area fraction) at a given point (figure 4.16) by the corresponding number density of voids (figure 4.14) to arrive at the average void cross sectional area for each specimen, from which $\bar{d}_v$ was obtained. These results are consistent with the predicted particle size dependence on void nucleation (chapter 5), since as the deformation increases, smaller particles are capable of forming voids. Even without this effect a general decrease in lateral void area is expected since models for void growth [71] suggest that lateral void contraction occurs for values of $\sigma_T/Y(\bar{\epsilon}_p)$ less than about 1. The point here is that the failure of these materials should not be viewed in terms of lateral void growth and coalescence, but rather as a result of longitudinal void growth and continuous nucleation, producing a net decrease in the mean free path between voids on transverse sections.

The interrupted fracture tests which were performed on the B type specimens contributed a number of qualitative details about the ultimate failure of these materials. Referring to the micrograph in figure 4.22 the elongated void structure in specimen B3 is clearly visible and the absence of lateral void growth is apparent. There is some void coalescence near the outer edge of the crack but this may actually be part of the crack itself. The incidence of longitudinal void coalescence is much greater.

Enhanced void growth along grain boundaries has been related to the large defect content of the boundaries in terms of dislocation coalescence. [72] Fractographic studies of low carbon steels by Onodera [73]

showed that grain boundaries play an important role in expanding small voids during the latter stages of deformation. Yet the arguments of section 4.1.2 suggest that enhanced boundary void growth does not occur in these materials.

Because the crack was allowed to open appreciably, it was difficult to associate the actual fracture path with particular microstructural features. However, figure 4.23 shows a series of micrographs of specimen B1 where the crack remained partially closed. A rare phenomenon was observed corresponding to points A and B in figure 4.23a which appears to be a bifurcation of the main crack. These regions, along with those portions of the main crack labelled C and D were characterized by a failure of the material along grain boundaries which were highlighted in SEM micrographs as seen in figures 4.23b and 4.23c. It appears that the material was sheared along these segments which act as paths between large void structures. Whether these voids were formed as a result of the shearing, or whether they served as initiation sites from which the crack propagated, is not clear. In any case, the grain boundaries played an important role in the failure of this specimen.

In similar tests on a .17% C pearlitic steel, Clausen [74] also observed intervoid failure along boundaries and ductile grain boundary fracture was investigated in Al-4.5 Cu alloys by Leupp and Epprecht. [75]

While the grain boundary failure mode of the present investigation was associated with large strains and high ductility, it may provide

some insight into the apparent transition from fibrous, ductile failure to brittle, intergranular fracture when ferrous alloys are subjected to embrittling conditions prior to impact testing. Indeed, intergranular failure may be common to both fracture modes as indicated in this discussion. The notion of ductile versus brittle boundary behavior was treated by Rice [76] with reference to the embrittling effects of hydrogen on materials and the relative energy requirements for brittle crack extension and ductile crack blunting.

4.10 Effects of microstructure on ductility

A number of experimental studies [9, 77, 78] have shown that the strain to fracture in tensile specimens containing a dispersed second phase is very nearly independent of particle size (as is confirmed in table 4.2), but depends only upon the volume fraction of second phase. This is not a universal material characteristic, however, as indicated by the work of Wilcox and Clauer, [79] and the reasons for this are clear if one considers separately the effects of a number of variables on ductility.

1) For equal volume fractions, the number of spherical particles (void nucleating sites) per unit volume is proportional to $1/r_0^3$, which favors a large value of $\tilde{\epsilon}_f$ for distributions of larger particle sizes.

2) At a given level of deformation, void nucleation is favored at larger particles based on experimental observation as well as the results of chapter 5. This effect favors greater ductility for specimens with smaller particle size distributions.

3) For specimens with equal volume fractions of particles which have been deformed to the same strain, the severity of the stress state increases with decreasing average particle size, producing an expected decrease in ductility.

4) For the current material systems being investigated, a smaller average particle size is associated with a smaller grain size for a given particle volume fraction. The resulting increase in grain boundary surface may enhance the boundary failure mode of figure 4.23 and thus decrease ductility.

5) Assuming that nearly all of the voids lie on boundaries and that the resultant weakening of the boundaries enhances failure, equal volume fractions of voids will not produce equal amounts of grain boundary damage in specimens with differing void (particle) sizes. For two steels of equal void volume fraction having uniform spherical voids of radii r_S and r_L , corresponding to small and large void sizes respectively, the ratio of the grain boundary areas occupied by voids will be

$$\frac{A_S^B}{A_L^B} = \frac{r_L}{r_S} \quad (4.16)$$

which favors ductility in specimens with large particles for constant total grain boundary area.

6) Countering the result in (5) is the increase in total boundary area per unit volume for smaller particle distributions if it is assumed that the effect discussed in (4) is not significant.

7) For the conditions in (5), if δ_S and δ_L are the planar nearest neighbor distances of spherical voids located on the boundaries, then it follows that

$$\frac{r_S/\delta_S}{r_L/\delta_L} = \frac{r_L}{r_S} \quad (4.17)$$

which indicates that void interactions will be more prevalent for distributions of small voids (assuming constant f_v) leading to a decrease in ductility.

Rosenfield [80] and Rogers [81] have both reviewed the effects of material variables on ductility but neither was able to make conclusive statements about the effects of particle size on strain to fracture. It is clear that the overall effect of particle size and grain size on ductility is a result of the total contribution from a number of factors. Apparently, on the basis of experimental results, the net effect on the current systems of spheroidized steels is small and ductility can be related to particle volume fraction. Predictions of $\bar{\epsilon}_f$ can not be made unless the contribution from each particular variable is thoroughly understood and quantitatively analyzed.

5. Analytical Models for Void Nucleation

5.1 Introduction

A comprehensive treatment of void nucleation at elastic inhomogeneities in a plastically flowing matrix must include the following considerations: (1) particle size, (2) particle shape, (3) particle spacing in relation to the interactions between neighboring particles, (4) surface energy quantities, (5) applied loading including the triaxiality associated with specimen geometry, (6) local deformation fields due to the incompatibility between particle and matrix, (7) critical debonding stress of the interface and (8) energy stability of the particle-void system. In addition, for the materials used in this investigation, the effect of particle location with respect to grain boundaries must be included. While the proposed models attempt to incorporate each of the factors listed above, the complexities associated with non-spherical particle geometries and interactions between closely spaced particles are avoided during the initial development of the theory. These effects are treated semi-quantitatively in section 5.4.

5.2 Model A: Spherical particle in a homogeneous isotropic matrix

5.2.1 Introduction

This model proposes that void nucleation will occur when the normal interfacial stress, σ_n , at the particle-matrix boundary attains a critical value, σ_c , over the cap region subtended by a critical angle, ϕ_c , as shown in Figure 5.1. The significance of ϕ_c arises from consideration of the conditions for stable void nucleation with respect to the total energy of the system where ϕ_c represents the minimum angle for which a stable void

exists. Contrary to the conclusions of Tanaka et al.^[57] and Argon and Im^[12], who determined that the energy criterion for void formation is satisfied upon yielding for particles having radii greater than about 100 Å, the present model proposes a specific scheme by which elastic energy is released upon introduction of an interfacial crack, leading to the conclusion that energy requirements for stable void formation are satisfied only at large particles subjected to high levels of deformation associated with the conditions in the neck of a tensile specimen for the alloy systems described in Chapter 4.

5.2.2 Elastic energy release based on void cap geometry

The first step in the analysis is to determine the elastic energy release due to the introduction of a spherical cap-shaped crack of angle ϕ_0 at an inclusion of radius r_0 subjected to a combined state of elastic loading consisting of a uniaxial tension upon which is superimposed a state of purely hydrostatic stress as shown in figure 5.1. Although 3-dimensional crack solutions corresponding to this geometry are not available, the appropriate crack tip stress intensity factors and corresponding expressions for energy release rate may be approximated by considering known solutions to related problems with 2 and 3-dimensional crack geometries.

The particle and matrix are assumed to have equivalent elastic constants (a reasonable approximation for cementite in α -iron^[82]). Thus the elastic problem reduces to that of a crack in a homogeneous body. Figure 5.2 shows six types of crack configurations. Exact solutions are available for the first four.^[83] The flat crack of figure 5.2a has a stress intensity factor $K_I^a = \sigma\sqrt{\pi a}$ where a is the half-crack length. The corresponding

value of K_I for the three-dimensional penny crack of figure 5.2b is $K_I^b = \frac{2}{\pi} \sigma \sqrt{\pi a}$. With this result in mind, the two-dimensional tunnel crack configurations of figures 5.2c and 5.2d are examined as possible approximations to the three-dimensional cases of a spherical cap in figures 5.2e and 5.2f. For the crack configurations of figures 5.2c and 5.2d

$$\begin{aligned}
 K_I^c &= \sigma \sqrt{r_0} \sin \phi \left[\frac{\cos \frac{\phi}{2} (2 - 3 \sin^4 \frac{\phi}{2} - 2 \sin^2 \frac{\phi}{2})}{2(1 + \sin^2 \frac{\phi}{2})} \right] \\
 K_{II}^c &= \sigma \sqrt{r_0} \sin \phi \left[\frac{\sin \frac{\phi}{2} (2 - 3 \sin^4 \frac{\phi}{2})}{2(1 + \sin^2 \frac{\phi}{2})} \right] \\
 K_I &= \sigma_H \sqrt{r_0} \sin \phi \left[\frac{\cos \frac{\phi}{2}}{1 + \sin^2 \frac{\phi}{2}} \right] \\
 K_{II}^\alpha &= \sigma_H \sqrt{r_0} \sin \phi \left[\frac{\sin \frac{\phi}{2}}{1 + \sin^2 \frac{\phi}{2}} \right]
 \end{aligned} \tag{5.1}$$

Existing limiting solutions for the three-dimensional spherical cap at $\phi \rightarrow 0$ and $\phi \rightarrow \pi$ can be used to approximate the actual values of stress intensity and energy release rate over the entire range $0 \leq \phi \leq \pi$. At the lower limit where $\phi = 0$, K_I^c and K_{II}^d reduce to the value of K_I^a for a flat crack. Likewise, the solutions for the spherical cap must reduce to K_I^b so as a first approximation

$$\begin{aligned}
 K_I^e &= \frac{2}{\pi} K_I^c \\
 K_{II}^e &= \frac{2}{\pi} K_{II}^c \\
 K_I^f &= \frac{2}{\pi} K_I^d \\
 K_{II}^f &= \frac{2}{\pi} K_{II}^d
 \end{aligned} \tag{5.2}$$

The plane strain energy release rate, $\mathcal{G}$, is given by

$$\mathcal{G} = \frac{1 - \nu^2}{E} (K_I^2 + K_{II}^2) = \frac{(1 - \nu)(K_I^2 + K_{II}^2)}{2\mu} \quad (5.3)$$

and is assumed to apply in the case of 3-dimensional crack extension. The total energy released by formation of a crack subtending the angle ϕ_0 due to an applied load, τ , will be

$$\Delta E_\sigma = \int_0^{\phi_0} \mathcal{G} dA = 2\pi r_0^2 \int_0^{\phi_0} \mathcal{G} \sin\phi d\phi \quad (5.4)$$

Thus to evaluate the validity of equation 5.2 the energy released according to equation 5.4 was calculated for $\phi_0 = \pi$ (i.e., total decohesion) and compared to the known result for the energy change associated with the introduction of a spherical hole in an infinite elastic medium subject to prescribed loading. These solutions were determined using the methods of Eshelby [45] for solving the perturbation of a uniform applied stress field by a spherical cavity. Before the void is introduced the total energy in the system, E_T , will just be that due to the applied stresses (i.e., $E = E_A$). When the hole is present, the elastic energy associated with the local fields of the void (E_V), and the interaction energy between these local fields and the applied field, (E_i), must be included, so that

$$E_T = E_A + E_V + E_i \quad (5.5)$$

The energy change resulting from introduction of the void will be

$$E_\sigma = -(E_V + E_i) \quad (5.6)$$

In order to solve this problem, the transformation strains ϵ_{ij}^T must be determined which, when used to solve the inhomogeneity problem, give

stresses σ_{ij}^I in the transformed region that disappear as a result of the applied stresses σ_{ij}^A , producing a stress free region which may be replaced by a void.

For the case of uniaxial tension $\sigma_{33}^A = \sigma$, all other $\sigma_{ij}^A = 0$, and the transformation strains are determined to be

$$\begin{aligned}\epsilon_{11}^T &= \epsilon_{22}^T = \frac{\sigma}{6\mu} \left[\frac{3(1-\nu)}{2(1+\nu)} - \frac{15(1-\nu)}{7-5\nu} \right] \\ \epsilon_{33}^T &= \frac{\sigma}{6\mu} \left[\frac{3(1-\nu)}{2(1+\nu)} + \frac{30(1-\nu)}{7-5\nu} \right],\end{aligned}\quad (5.7)$$

The elastic energy associated with the transformation operation is

$$E_V = -\frac{1}{2} \int_V \sigma_{ij}^I \epsilon_{ij}^T dV \quad (5.8)$$

where the integral is taken over the volume of the transformed region. The energy of interaction with the applied field will be

$$E_i = -\int \sigma_{ij}^A \epsilon_{ij}^T dV. \quad (5.9)$$

The total energy change from equation 5.6 becomes

$$\begin{aligned}E_\sigma &= \frac{1}{2} \int \sigma \epsilon_{33}^T dV \\ &= \frac{V\sigma^2}{\mu} \left[\frac{1-\nu}{8(1+\nu)} + \frac{5(1-\nu)}{2(7-5\nu)} \right] \\ &= .385 \frac{V\sigma^2}{\mu} \quad \text{for } \nu = .3\end{aligned}\quad (5.10)$$

A similar calculation for purely hydrostatic loading ($\sigma_{11}^A = \sigma_{22}^A = \sigma_{33}^A = \sigma_H$) gives

$$\epsilon_{11}^T = \epsilon_{22}^T = \epsilon_{33}^T = \frac{3}{4} \frac{\sigma_H}{\mu} \frac{(1-\nu)}{(1+\nu)} \quad (5.11)$$

and

$$E_{\sigma} = .606 \frac{V\sigma_H^2}{\mu} \quad (5.12)$$

The corresponding results for the energy release as calculated from equations 5.1—5.4 are

$$E_{\sigma} = .200 \frac{V\sigma^2}{\mu} \quad (5.13)$$

for uniaxial loading and

$$E_{\sigma} = .508 \frac{V\sigma_H^2}{\mu} \quad (5.14)$$

for hydrostatic loading. It can be seen that while the case of purely hydrostatic loading gives reasonable agreement with the theoretical value, the computed result for uniaxial tension is 48% too low. For this reason a correction factor of $(1 + \frac{1}{2} \sin \phi)$ is applied to K_I^e and K_{II}^e in equation 5.2. This factor was chosen on the basis that it provides the correct behavior at $\phi = 0$ and the proper magnitude at $\phi = \pi$, and as a simple geometrical factor it contains the proper symmetry with respect to ϕ . Of course the actual problem lies in the fact that the two-dimensional angular dependence is not adequate for the three-dimensional case, although with the proposed modifications of the two-dimensional quantities reasonable approximations have been obtained. Using

$$\begin{aligned} K_I^e &= \frac{2}{\pi} (1 + \frac{1}{2} \sin \phi) K_I^c \\ K_{II}^e &= \frac{2}{\pi} (1 + \frac{1}{2} \sin \phi) K_{II}^c \end{aligned} \quad (5.15)$$

in equation 5.4 yields

$$E_{\sigma} = .380 \frac{V\sigma^2}{\nu} \quad (5.16)$$

which compares quite favorably with equation 5.10. The final forms for the approximations of the stress intensity factors corresponding to the conditions of figures 5.2e and 5.2f are

$$\begin{aligned} K_I^e &= \frac{2}{\pi} (1 + \frac{1}{2} \sin \phi) \sigma \sqrt{\pi r_0} \sin \phi \left[\frac{\cos \frac{\phi}{2} (2 - 3 \sin^4 \frac{\phi}{2} - 2 \sin^2 \frac{\phi}{2})}{2(1 + \sin^2 \frac{\phi}{2})} \right] \\ K_{II}^e &= \frac{2}{\pi} (1 + \frac{1}{2} \sin \phi) \sigma \sqrt{\pi r_0} \sin \phi \left[\frac{\sin \frac{\phi}{2} (2 - 3 \sin^4 \frac{\phi}{2})}{2(1 + \sin^2 \frac{\phi}{2})} \right] \\ K_I^f &= \frac{2}{\pi} \sigma_A \sqrt{\pi r_0} \sin \phi \left[\frac{2 \cos \frac{\phi}{2}}{2(1 + \sin^2 \frac{\phi}{2})} \right] \\ K_{II}^f &= \frac{2}{\pi} \sigma_A \sqrt{\pi r_0} \sin \phi \left[\frac{2 \sin \frac{\phi}{2}}{2(1 + \sin^2 \frac{\phi}{2})} \right] \end{aligned} \quad (5.17)$$

As justification for these formulae the case of combined loading will now be considered with the condition that $\sigma_H = Q\sigma$ where Q is a measure of the triaxiality produced in the neck of a uniaxial tensile specimen. A uniaxial stress state corresponds to $Q = 0$ and purely hydrostatic loading is approached as $Q \rightarrow \infty$. Superposition gives

$$\begin{aligned} K_I &= \frac{2}{\pi} \sigma \sqrt{\pi r_0} \sin \phi \left[\frac{\cos \frac{\phi}{2} (2Q + [1 + \frac{1}{2} \sin \phi] [2 - 3 \sin^4 \frac{\phi}{2} - 2 \sin^2 \frac{\phi}{2}])}{2(1 + \sin^2 \frac{\phi}{2})} \right] \\ K_{II} &= \frac{2}{\pi} \sigma \sqrt{\pi r_0} \sin \phi \left[\frac{\sin \frac{\phi}{2} (2Q + [1 + \frac{1}{2} \sin \phi] [2 - 3 \sin^4 \frac{\phi}{2}])}{2(1 + \sin^2 \frac{\phi}{2})} \right] \end{aligned} \quad (5.18)$$

and the corresponding energy release rate is

$$\begin{aligned} \mathcal{G} = \frac{(1 - \nu)\sigma^2 r_0 \sin\phi}{2\pi\mu} & \left[4Q^2 + 4Q\left(1 + \frac{1}{2}\sin\phi\right)\left(2 - 2\sin^2\frac{\phi}{2} - \sin^4\frac{\phi}{2}\right) \right. \\ & \left. + \left(1 + \frac{1}{2}\sin\phi\right)^2\left(4 - 8\sin^2\frac{\phi}{2} + 8\sin^6\frac{\phi}{2} - 3\sin^8\frac{\phi}{2}\right) \right] / (1 + \sin\frac{\phi}{2})^2 \end{aligned} \quad (5.19)$$

Equation 5.4 has been integrated numerically from 0 to π (i.e., total decohesion) using this expression for $\mathcal{G}$ for various values of Q . The results for E_G vs. Q are plotted in figure 5.3.

The corresponding Eshelby solution for combined loading involves the transformation strains

$$\begin{aligned} \epsilon_{11}^T = \epsilon_{22}^T &= \frac{\sigma}{6\mu(1 + \nu)} \left[\frac{-15(1 - \nu^2)}{7 - 5\nu} + \frac{3(1 - \nu)(3Q - 2\nu - 6\nu Q + 1)}{2(1 - 2\nu)} \right] \\ \epsilon_{33}^T &= \frac{\sigma}{6\mu(1 + \nu)} \left[\frac{30(1 - \nu^2)}{7 - 5\nu} - \frac{3(1 - \nu)(3Q - 2\nu - 6\nu Q + 1)}{2(1 - 2\nu)} \right] \end{aligned} \quad (5.20)$$

Substitution into equation 5.10 yields

$$E_G = \frac{V\sigma^2}{\mu} \left[\frac{1}{8}(1 + 3Q)^2 \left(\frac{1 - \nu}{1 + \nu} \right) + \frac{5}{2} \left(\frac{1 - \nu}{7 - 5\nu} \right) \right] \quad (5.21)$$

This result is also shown in figure 5.3. Agreement between the computed energy release for total decohesion and the theoretical Eshelby result for the energy released upon introduction of a spherical cavity in an elastic body is reasonably good. It is concluded that $\mathcal{G}$ in equation 5.19 is an adequate approximation to the actual value.

The relevance that this result has to the present investigation hinges on the validity of using linear elastic fracture mechanics to solve this particular problem involving large amounts of plastic deformation. Such a treatment is justified if one considers that the present analysis deals

only with the introduction of a crack and not with the conditions for growth of a pre-existing crack. Thus plastic deformation processes, which are important in ductile crack growth, are assumed not to be associated with energy dissipation during crack nucleation although they certainly will dictate the local conditions in the vicinity of a particle and ultimately the final nucleation event. Crack formation can then be related to elastic energy release while subsequent crack growth will be controlled by plastic hole growth and ductile crack extension.

5.2.3 Elastic energy sources

It remains to determine the various contributions to the elastic energy release due to the applied loading and the local deformation fields associated with the particles. There are three contributions to the total energy available for surface formation when a crack nucleates at the pole of an isolated particle in a plastically flowing matrix: (1) E_G , the elastic energy associated with the applied loading which was introduced in the last section, (2) E_p , the stored energy resulting from the unrelaxed plastic strains which are present due to plastic incompatibilities between the elastically deforming particle and the plastically deforming matrix, and (3) E_i , the energy of interaction between these internal fields and the applied field.

Chang and Asaro ^[44] have provided an extensive analysis of the internal stresses associated with plastic incompatibility at spherical particles of cementite in steel. Their results may be used to compute the corresponding elastic energy storage. The overall specimen is assumed to undergo uniaxial tensile deformation with symmetric multiple slip in the matrix giving rise

to an equivalent plastic strain $\bar{\epsilon}_p$. Plastic incompatibilities and the associated stresses at the particles can be relaxed to a large extent by local secondary slip at the particle interfaces (figure 5.4). Complete plastic relaxation is not achieved and residual strain incompatibilities exist giving a local equivalent strain of ϵ_p^* . The uniform nature of slip in these materials leads to uniform transformation strains of ϵ_{ij}^T which may be applied to Eshelby's results. [45]

$$\begin{aligned}\epsilon_{11}^T &= \epsilon_{22}^T = \frac{\epsilon_p^*}{2} \\ \epsilon_{33}^T &= -\epsilon_p^*\end{aligned}\quad (5.22)$$

The problem of an elastic spherical inclusion in a plastic matrix undergoing plastic relaxation has thus been reduced to a purely elastic formulation. The resulting elastic stresses and strains in the particle and matrix have been computed [44] yielding uniform stresses σ_{ij}^I in the particle of the form

$$\begin{aligned}\sigma_{11}^I &= \sigma_{22}^I = -\alpha\mu\epsilon_p^* \\ \sigma_{33}^I &= 2\alpha\mu\epsilon_p^* \\ \sigma_{ij} &= 0, \quad i \neq j\end{aligned}\quad \text{where } \alpha = \frac{7 - 5\nu}{15(1 - \nu)}.\quad (5.23)$$

The total elastic energy due to the internal fields is

$$E_p = -\int_V \sigma_{ij}^I \epsilon_{ij}^T dV = \frac{3}{2}V\alpha\mu\epsilon_p^{*2}\quad (5.24)$$

where V is the volume of the inclusion. The interaction energy between the field of the inclusion and the applied field is

$$E_i = - \int_V \sigma_{ij}^A \epsilon_{ij}^T dV. \quad (5.25)$$

σ_{ij}^A represents the applied stress field and is given by

$$\begin{aligned} \sigma_{11}^A &= \sigma_{22}^A = Q\sigma \\ \sigma_{33}^A &= (1 + Q)\sigma \end{aligned} \quad (5.26)$$

so that

$$E_i = \sigma \epsilon_p^* V. \quad (5.27)$$

Thus, the formation of a cap crack is accompanied by energy release from the three sources E_σ , E_p and E_i . The release of energy due to the applied loading has been discussed in terms of the approximate value of $\mathcal{J}$ and will have a maximum value of E_σ corresponding to total decohesion at the particle-matrix interface. It is assumed that E_p and E_i can also be released upon crack formation (i.e., the incompatibility is relieved as the constraints of the interface are removed). It is convenient to also compute these quantities using $\mathcal{J}$ based on appropriate choices for Q . The internal stresses in the particle due to plastic incompatibilities given in equation 5.23 are of the form of equation 5.26 where $\sigma = 3\alpha\mu\epsilon_p^*$ and $Q = -\frac{1}{3}$. The stresses in the matrix are not uniform and do not fit the form of equation 5.26; however, because the total energy storage due to the inhomogeneity is a function only of the particle stresses, the transformation strains, and the particle volume, it is reasonable to conclude that $Q = -\frac{1}{3}$ is an appropriate value to use in $\mathcal{J} = \mathcal{J}(Q)$. The value of E_i in equation 5.27 is a function of the applied stress field but contains no dependence on σ_H due to the form of the transformation strains. Hence,

$Q = 0$ will be used in ~~$\mathcal{H}$~~ for determining the release rate of the interaction energy.

The nucleation of a cap-shaped crack will therefore involve an amount of combined energy release, ΔE_T , consisting of the three contributions described above (ΔE_i , ΔE_p , ΔE_i). Each term will be of the form

$$\Delta E = E\Theta \quad (5.28)$$

where Θ represents the normalized angular dependence which controls the release of E . It has the form

$$\Theta = \frac{\int_0^\phi \theta(\phi, Q) d\phi}{\int_0^\pi \theta(\phi, Q) d\phi} \quad (5.29)$$

where $\theta = \mathcal{H} \sin\phi$ in equation 5.4. The expression for the combined energy release becomes

$$\Delta E_T = E_\sigma \Theta_\sigma(\phi, Q) + E_p \Theta_p(\phi, Q = -\frac{1}{3}) + E_i \Theta_i(\phi, Q = 0) \quad (5.30)$$

and from equations 5.21, 5.24, and 5.27

$$\begin{aligned} \Delta E_T = & \frac{4\pi\sigma^2 r_0^3}{3\mu} \left[\frac{1}{8}(1 + 3Q)^2 \left(\frac{1 - \nu}{1 + \nu} \right) + \frac{5}{2} \left(\frac{1 - \nu}{7 - 5\nu} \right) \right] \Theta_\sigma \\ & + 2\pi\alpha\mu\epsilon_p^{*2} r_0^3 \Theta_p + \frac{4}{3}\pi\sigma\epsilon_p^* r_0^3 \Theta_i \end{aligned} \quad (5.31)$$

The angular quantity Θ has been determined numerically using equations 5.4 and 5.19 for various values of Q (Table 5.1).

A function Z is now introduced such that

$$Z = \Delta W - \Delta E_T \quad (5.32)$$

where ΔW is the energy associated with crack surface formation and is directly proportional to the cap area.

$$\Delta W = 2\pi r_0^2 (1 - \cos\phi_0) \Gamma_W \quad (5.33)$$

with

$$\Gamma_W = \Gamma_I + \Gamma_M - \Gamma_{IM} \quad (5.34)$$

where Γ_I , Γ_M , and Γ_{IM} are the free surface energies of the particle, matrix, and interface respectively. An evaluation of these quantities for the iron-carbon system is presented in Appendix B. For a given value of crack size, ϕ_0 , a positive value of Z indicates that stable crack formation is not favorable. Under the proper conditions, which are detailed in the schematic form of figure 5.5, an energetically stable system may exist. When $\frac{dZ}{d\phi} = 0$ a critical state exists for which an increase in the crack size can lower the energy of the particle-void system. A stable configuration is not obtained until the energy difference function Z is less than or equal to zero. These points will be discussed further in section 5.2.7.

5.2.4 Evaluation of the local unrelaxed strains, ϵ_p^*

Before computing Z for various values of particle size, crack angle, and deformation, it is necessary to determine the form of ϵ_p^* in terms of known macroscopic quantities. Chang and Asaro [44] have obtained good agreement between measured and predicted values of the internal stresses in spheroidized steels. The predicted values were based on relations between ϵ_p^* and $\bar{\epsilon}_p$ determined through modification of the work of Brown and Stobbs. [29] The results of their analysis may be expressed as

$$\epsilon_p^* = K(r_0) \bar{\epsilon}_p^m \quad (5.35)$$

Taking $m = .3$ and $K(r_0) = 2.6 \times 10^{-7} r_0^{-.7}$ provides a good fit to numerical solutions over a large range of $\bar{\epsilon}_p$ and r_0 .

As discussed in section 4.6 the steels used in this study have flow curves which can be fitted to a power hardening law of the form in equation 2.25. The relationship between ϵ_p^* and $\bar{\epsilon}_p$ can then be transformed to

$$\epsilon_p^* = 2.6 \times 10^{-7} r^{-.7} \left(\frac{\epsilon_0^n Y(\bar{\epsilon}_p)}{Y_0} \right)^{\frac{.3}{n}} \quad (5.36)$$

The values of ϵ_0 , Y_0 and n are given for each specimen in table 4.3.

5.2.5 Computed energy difference curves

The normalized energy difference function $\frac{Z}{r_0^2}$ can now be computed for a particular steel for specific values of ϕ_0 , r_0 , $Y(\bar{\epsilon}_p)$ and Q from equations 5.31, 5.32 and 5.36 and from the data in table 5.1. The physical constants have been assigned the following values (see Appendix B):

$$\Gamma_I = \frac{3J}{m^2}$$

$$\Gamma_M = \frac{4J}{m^2}$$

$$\Gamma_{IM} = 1.3 \frac{J}{m^2}$$

$$\mu = 7.95 \times 10^4 \frac{MN}{m^2}$$

$$\nu = .3$$

$\frac{Z}{r_0^2}$ has been evaluated for two values of $Y(\bar{\epsilon}_p)$ and Q for each steel corresponding to the conditions at first nucleation of voids and to the final state at the center of the neck. These results are plotted as a function of ϕ_0 for steels B1 and W1 in figures 5.6 and 5.7. These curves indicate that the energy criterion is not satisfied for a wide range of particle sizes even at large deformation levels for isolated particles.

5.2.6 Normal interfacial stress criterion

While Z is a measure of the energy stability of the particle-void system, it is required additionally for void nucleation that the normal stresses, σ_n , at the interface satisfy the critical condition that σ_n be greater than or equal to the critical bonding stress of the interface, σ_c , over the area of the nucleated crack. Thus, a double criterion is proposed

1. $\sigma_n \geq \sigma_c$ over the area defined by ϕ_0
2. $\phi_0 \geq \phi_c$

which defines the conditions for void nucleation.

The normal stresses at the particle surface due to back stresses associated with unaccommodated plastic misfit have been determined [44] as

$$\sigma_n^I = \alpha u (3n_3^2 - 1) \epsilon_p^* \quad (5.37)$$

where $n_3 = \cos\phi$ is the axial component of the unit normal to the interface.

Adding to this the normal component of the applied stresses

$$\sigma_n^A = Y(\bar{\epsilon}_p) (Q + n_3^2) \quad (5.38)$$

and using equation 5.36, the total normal component of stress on the particle becomes

$$\sigma_n^T = 1.08 \times 10^4 (3\cos^2\phi - 1) \left[\frac{\epsilon_0^n Y(\bar{\epsilon}_p)}{Y_0} \right]^{\frac{3}{n}} + Y(\bar{\epsilon}_p) (Q + \cos^2\phi) \quad (5.39)$$

This expression has been evaluated for the same conditions used to compute

$\frac{Z}{r_0^2}$. The results are plotted in figure 5.8 for specimen W1 corresponding to the most severe interfacial stress conditions of the six specimen types.

5.2.7 Discussion of model A

The present model predicts that cavity formation at spherical, isolated particles is energetically favorable only at very large deformation levels and at larger particles in a given size distribution for the conditions corresponding to room temperature tensile deformation of the spheroidized steels tested. The mechanism by which voids form may be viewed in terms of a standard nucleation and growth process in which a crack nucleus of angle ϕ_0 is formed by the action of large normal interfacial stresses. Referring to the schematic in figure 5.5, if ϕ_0 is less than some critical value, ϕ_c , corresponding to an activation energy $(\frac{Z}{r_0^2})_c$, the nucleated crack embryo will be unstable resulting in a totally bonded interface. If a crack embryo larger than or equal to ϕ_c nucleates, the energy of the system can be lowered by continued growth of the crack and subsequent energy release. The growth, however, will be controlled by the plastic work required to advance the crack in the presence of a highly deformed matrix. Immediate blunting of the stable crack will limit the amount of free surface formation and the flow of dislocations into the cavity will be responsible for the ultimate void size and shape as proposed by Broek.^[84] Thus, curves of the type in figure 5.5 are valid only to the point when nucleation is stable. Further growth is not controlled by the elastic model and it may be assumed that an increase in ϕ above the nucleation value of ϕ_c will be negligible due to the large plastic work term associated with further crack growth.

The normal tensile stresses plotted in figures 5.8 are quite large (up to $2800 \frac{\text{MN}}{\text{m}^2}$) at the particle poles but become small or even compressive for $\phi_0 \geq \frac{\pi}{3}$. For this reason it is unlikely that voids can nucleate or grow along the entire interface. Observations support this contention.

The effects of Γ_W and σ_H on the proposed energy criterion have been investigated in terms of the present model. Figure 5.9 shows $\frac{Z}{r_0^2}$ curves for steel B1 subject to maximum deformation at the center of the neck with $Y(\bar{\epsilon}_p) = 640 \frac{\text{MN}}{\text{m}^2}$ and various values of $\frac{\Gamma_W}{\Gamma_0}$ for the two cases corresponding to $Q = .4$ and $Q = 0$, where $\Gamma_W^0 = 5.7 \frac{\text{J}}{\text{m}^2}$ is the value of Γ_W determined in Appendix B. While σ_H has a rather minor effect on the nucleation of voids, Γ_W can drastically alter the level of deformation required to form stable voids.

The presence of σ_H due to specimen geometry is overshadowed by the local conditions occurring near the particles at large deformation levels, yet a definite shift occurs in the energy difference curves which tends to favor nucleation at larger σ_H for constant $Y(\bar{\epsilon}_p)$.

The literature values of Γ_W corresponding to a number of systems [12, 59, 60] have been estimated to be $.04 \sim 2 \frac{\text{J}}{\text{m}^2}$. It is unlikely that the value of Γ_W for the present system is so low since, even at 1450°C , $\Gamma_W = 1.98 \frac{\text{J}}{\text{m}^2}$ for BCC iron. [85] Of course the actual value of Γ_W will be altered by testing temperature and also by alloying and/or impurity elements. Large values of Γ_W can explain, in part, the large deformations required for void nucleation at Fe_3C particles in spheroidized steels.

Model A is limited in its application to the present study for the following reasons. While it does predict a low incidence of void nucleation at isolated particles contained in a homogeneous matrix it does not account for 1) the fact that voids are most often associated with particles which reside on ferrite grain boundaries and 2) the often observed decohesion at two neighboring particles closely spaced along boundaries in the tensile direction, followed by coalescence of the voids in this direction. The

present model for isolated particles within the grain bulk has been modified in an attempt to explain the observed phenomena. These are discussed in the following sections.

5.3 Model B: Spherical particles on grain boundaries

5.3.1 Introduction

The results of model A support the present experimental observations of spheroidized steels that void nucleation is a rare event at particles which are not situated on ferrite grain boundaries. In order to account for the enhanced void initiation at boundary carbides, the details of model A have been modified to include the effects of the boundary on the local deformation in the vicinity of a spherical particle.

During the early stages of deformation, pronounced hardening occurs near the grain boundaries producing inhomogeneous deformation within individual grains of a polycrystal as discussed by Thompson.^[86] Ashby^[87] introduced the concepts of "statistically" and "geometrically" stored dislocations to explain the hardening of non-homogeneous materials. The statistically stored dislocation density, ρ^S , can be associated with single crystal deformation and results from the random encounters and interactions of dislocations in the crystal. The geometrically stored dislocation density, ρ^G , represents the dislocations which are produced in order to maintain material continuity by the accommodation of plastic incompatibilities. The quantity ρ^G dominates the hardening at grain boundaries during the initial stages of deformation. An increment in dislocation density $d\rho^G$ is necessarily proportional to $\frac{1}{\lambda_g} \frac{d\gamma_p}{b}$ where $d\gamma_p$ is the plastic strain increment, b is the Burgers vector and λ_g is the mean slip distance.

While such a relation leads to a Hall-Petch parabolic stress-strain equation

$$\tau = \tau_0 + K \left(\frac{\gamma_p}{D} \right)^{1/2} \quad (5.40)$$

when λ_g is taken as the mean grain size, D , the range of applicability is limited to small strains. In actuality $\rho^g = \rho^g(\gamma_p)$ and $\lambda_g = \lambda_g(\gamma_p)$, the explicit forms being unknown. Physically one may argue that for large deformations the geometrical slip distance is reduced and ρ^g becomes increasingly localized in the grain boundary region. Thompson [86] has found that such a configuration can adequately explain the observed hardening in a variety of polycrystalline systems. The presence of rigid grain boundary and subgrain boundary particles will naturally complicate the compatibility requirements in the vicinity of the particle-boundary intersection. Anand [61] considered the roles of particles and boundaries in his work hardening theories for spheroidized steels. No distinction was made, however, between the internal stress contribution arising from the plastic strain incompatibilities for particles on the boundaries and particles inside the boundaries. On the basis of internal stress measurements, Chang and Asaro [44] did suggest that plastic accommodation at particles could be assisted by the presence of subgrain boundaries, but did not discuss the effect of high angle grain boundaries. In his experimental studies of ductile fracture of Al-Mn alloys, Melander [88] accounted for differences in spacing parameters associated with boundary particles but did not consider the accompanying changes in the local deformation.

5.3.2 Details of plastic zone

The determination of E_p in model A was based on the work of Chang and

Asaro [44] and evolved from consideration of the plastic zone about spherical particles in a matrix undergoing multiple slip. The model for plastic relaxation proposed by Brown and Stobbs [29] for particles embedded in a FCC matrix experiencing slip on a single primary system was modified by Chang and Asaro to account for a spherically symmetric plastic zone associated with uniform relaxation at cementite particles in a BCC ferrite matrix. The following relations and assumptions involving the local dislocation distributions about the particles were employed in their analysis:

1. As noted, the plastic zone about the particle is considered to be symmetric with volume

$$V_p = \frac{4}{3}\pi(R^3 - r_0^3) \quad (5.41)$$

where R represents the extent of the plastic zone as measured from the particle center.

2. The primary Orowan loops about the particles which are converted to prismatic loop dipoles of approximate radius r_0 result in a local secondary dislocation density

$$\rho_L = \frac{hnr_0}{V_p} \quad (5.42)$$

where n is the number of relaxed Orowan loops and $h \sim 8$. [29]

3. The number of relaxed loops is proportional to the relaxed plastic strain and is given by Ashby [43] as

$$n = \frac{2r_0 m(\epsilon_p - \epsilon_p^*)}{b} \quad (5.43)$$

where m is the Taylor factor having a value of 2 for the case of multiple slip.

4. The local flow stress in the plastic zone, τ_ℓ , is related to the local dislocation density ρ_ℓ by (see [89])

$$\tau_\ell = c\mu b\sqrt{\rho_\ell} \quad (5.44)$$

where c is taken to be .25.

5. The extent of the plastic zone, R , is determined by a Frank-Read type criterion where the stresses at the periphery of the zone due to the particle are equated to the stress required to expand dislocation loop segments of approximate length r_0

$$\frac{3}{2} \alpha \mu \epsilon_p^* \left(\frac{r_0^3}{R^3} \right) = \frac{\mu b}{r_0} \quad (5.45)$$

The above conditions lead to a cubic equation in ϵ_p^* relating ϵ_p^* , ϵ_p and r_0 from which the form of equation 5.36 was determined. Further modifications will now be proposed in order to account for the presence of grain boundaries.

It must be realized that details of the dislocation structures which form the plastic zone are unknown and that only general characteristics can be attributed to the zone configuration. The following features are included in the present analysis:

1. The presence of the grain boundary produces a non-symmetric plastic zone of extent B along the boundary and extent R perpendicular to the boundary resulting in an ellipsoidal annulus of volume

$$V_p = \frac{4}{3}\pi(B^2R - r_0^3) \quad (5.46)$$

as depicted in figure 5.10.

2. The concept of a primary Orowan loop of constant Burgers vector b about the particle is no longer valid due to discontinuity of slip at the

boundary. Primary glide dislocations will result in half loops and an accompanying boundary dislocation density ρ^g will arise in order to maintain compatibility at the boundary. An approximation for ρ^g is made on the basis of physical arguments for the case when no particles are present. As mentioned, an increment in the geometrical dislocation density $d\rho^g$ required to accommodate slip at a boundary may be related to the relaxed plastic strain increment at the boundary

$$d\rho^g = \frac{A n (d\bar{\epsilon}_p - d\epsilon_g^*)}{\lambda_g b} \quad (5.47)$$

where ϵ_g^* is analogous to ϵ_p^* and represents the unrelaxed strains at the boundaries. This quantity is much less than $\bar{\epsilon}_p$ and may be neglected. The parameter A is a quantity which relates the number and arrangement of accommodating boundary dislocations to the number of primary glide dislocations impinging on the boundary. For an extremely low angle boundary, slip is nearly continuous and A will have a value close to 1. Larger misorientations between grains will require the operation of a larger number of slip systems in order to maintain continuity of the boundary thus increasing the value of A. Also, while λ_g is often taken as the mean grain intercept length, D, at low strains, for strains greater than $\bar{\epsilon}_p \sim .2$ the results of Thompson [86] and Zankl [90] indicate that the geometrical slip distance of the grain boundary generated dislocations may be approximated as

$$\lambda_g = \sqrt{D} \quad (5.48)$$

where D is given in microns. Hence, the geometrical dislocation density will be localized in the vicinity of the boundary and the superscript g

will subsequently refer to quantities associated with the grain boundaries. The reduction of slip distance of the boundary generated dislocations will also produce an increase in A as more dislocations will be necessary in order to produce the required displacements at the boundary. The net effect of the boundary then is to produce a high localized dislocation density and corresponding local hardening of the boundaries:

$$\rho^g = \frac{2m\bar{\epsilon}_p}{b\sqrt{D} \times 10^{-6}} \quad (\text{m}^{-2}) \quad (5.49)$$

where A has conservatively been assigned the value 2. In determining this expression from equation 5.47, A and λ_g were assumed to be independent of strain and were assigned appropriate average values.

3. The dislocation density ρ_L associated with the relaxation of plastic incompatibilities between the particle and the matrix will again result from the conversion of primary loops (or semi-loops) to prismatic dipoles. While this process can be envisioned in regions away from the boundary along the dimension defined by R (figure 5.10), more complex processes must operate in the vicinity of the boundary. Relaxation must occur independently in the adjacent grains maintaining compatibility at the particle interface as well as the grain boundary. Without knowing the actual processes which occur, it is conservatively assumed that, in order to accommodate the same misfit between inclusion and matrix as when no boundary is present, an equivalent dislocation density will result (i.e.,

$$\rho_L = \frac{8nr_0}{V_p} \text{ with } n = \frac{2m\bar{\epsilon}_p r_0}{b}).$$

4. In the vicinity of the grain boundary-particle junction along the dimension defined by B (figure 5.10), the total dislocation density, ρ^B ,

is taken to be the sum of the boundary density, ρ^B , and the density of the secondary zone of the particle ρ_L ,

$$\rho^B = \frac{2m\bar{\epsilon}_p}{b\sqrt{D} \times 10^{-6}} + \frac{hnr_0}{V_p} \quad (5.50)$$

By assuming linear additivity, interactions between ρ^B and ρ_L have been neglected.

Away from the boundary, along the dimension R , the effect of ρ^B will be greatly reduced and

$$\rho^R = \rho_L = \frac{hnr_0}{V_p} \quad (5.51)$$

For subsequent computations an average local dislocation density, $\bar{\rho}$, must be employed. This was determined by taking the average over the three directions defined by x_1, x_2, x_3 in figure 5.10 giving

$$\bar{\rho} = \frac{4m\bar{\epsilon}_p}{3b\sqrt{D} \times 10^{-6}} + \frac{hnr_0}{V_p} \quad (5.52)$$

5. The Frank-Read criterion discussed earlier is again used to determine the plastic zone size about the particle. Along the dimension defined by R , this condition will be equivalent to the case when no boundary is present.

$$\frac{3}{2}\alpha\mu\epsilon_p^* \left(\frac{r_0^3}{R^3} \right) = \frac{\mu b}{r_0} \quad (5.53)$$

The extent of the zone along the boundary, B , will be more restricted due to the complexities of the dislocation structures. Relaxation mechanisms which occur in this region will involve dislocation segments which are, on the average, smaller than those away from the boundary. For the deter-

mination of β this average segment length will be designated as βr_0 , where β is less than 1, giving

$$\frac{3}{2} \alpha \mu \epsilon_p^* \left(\frac{r_0^3}{\beta^3} \right) = \frac{\mu b}{\beta r_0} \quad (5.54)$$

For the purpose of evaluating these conditions, β was chosen as .75, which leads to an expression for the plastic zone volume of the form

$$V_p = \frac{4}{3} \pi r_0^3 \left[\left(\frac{3}{2} \right)^{\frac{1}{3}} \left(\frac{9}{8} \right)^{\frac{2}{3}} \frac{\alpha}{\beta^2} \epsilon_p^* r_0 - 1 \right] \quad (5.55)$$

which is about 15% smaller at large strain levels than in the case for no boundary.

6. The relationship between ϵ_p^* and $\bar{\epsilon}_p$ is now determined by equating the maximum interfacial shear stress, as determined by Chang and Asaro [44], to the local flow stress giving the condition for relaxation at the interface

$$\frac{3}{2} \alpha \mu \epsilon_p^* = c \mu \sqrt{\bar{\rho}} \quad (5.56)$$

To assume that the relationship between ϵ_p^* and $\bar{\epsilon}_p$ can be expressed by the same physical model over the entire range of deformation being considered is certainly questionable. The agreement between computed and experimental values of internal stress obtained by Chang and Asaro [44] was verified only up to 5% plastic strain. The measured values, which were based on the amount of permanent softening determined during cyclic testing of spheroidized steels, were found to saturate at strains greater than 5%. Since these experimental quantities correspond to average, far field values, they can not be related directly to the local conditions at the particles. Measured saturation of the internal stresses, which occurs rather abruptly at strains

of 5%, may result from a screening of the particle stresses by the dislocation structures which develop about the particles. [91]

There is no obvious reason why the relaxation mechanisms at the particles should be altered or why the present model cannot be employed at strains much greater than 5%. Until the micro-mechanistic details of plastic flow in dispersion hardened materials are better characterized, the present model for local deformation is proposed as a reasonable approximation.

5.3.3 Results of model B

Equations 5.52, 5.55 and 5.56 lead to the following relationship:

$$\left(9\pi\alpha^2 \epsilon_p^{*2} - \frac{32\pi c^2 b \bar{\epsilon}_p}{3\sqrt{D} \times 10^{-6}} \right) \left(1.24 \frac{\alpha}{b} \epsilon_p^* r_0 - 1 \right) r_0 - 12c^2 b h \bar{\epsilon}_p = 0. \quad (5.57)$$

The unrelaxed strains ϵ_p^* were determined numerically for selected values of $\bar{\epsilon}_p$, r_0 , and D and the results are plotted in figure 5.11. An expression of the form of equation 5.36 is not obtainable from these results over the necessary range of r_0 and $\bar{\epsilon}_p$. Instead ϵ_p^* must be determined graphically.

The normalized energy difference quantity (equation 5.32) is then expressed as

$$\begin{aligned} \frac{Z}{r_0^2} = & 35.8(1 - \cos \phi_0) - [5.27 \times 10^{-11} Y(\bar{\epsilon}_p) r_0 (6.73 \times 10^{-2} (1 + 3Q)^2 + .318) \Theta_\sigma \\ & + 2.62 \times 10^{11} \epsilon_p^{*2} r_0 \Theta_p + 4.19 Y(\bar{\epsilon}_p) \epsilon_p^* r_0 \Theta_1] \end{aligned} \quad (5.58)$$

This quantity is plotted in figures 5.12 and 5.13 as a function of cap angle ϕ_0 for specimens B1 and W1 for several particle sizes corresponding to the conditions of maximum deformation in the neck. The results for the

case when no boundary is present are also plotted for comparison. It is noted that the presence of grain boundaries has a significant effect on void nucleation for larger particles but that the effect is insignificant for small particles ($r_0 \leq .25\mu$).

5.3.4 Discussion of model B

While the magnitude of the effect of the grain boundaries on void nucleation is subject to the values chosen for A , λ_g , and β , the model does provide results which explain the experimental observations of enhanced void formation at particles situated on grain boundaries. This effect is due to the constraints on the plastic zone size of the particle and the increased dislocation density associated with plastic strain accommodation at the boundaries. A larger value of the unrelaxed plastic strains at the particle results and the corresponding increase in elastic energy storage is then available to satisfy the energy criterion at an earlier stage of the deformation.

It must be recalled that the model system of a perfectly spherical particle either on or off the grain boundary is a highly idealized situation. Relaxation mechanisms at actual particles may be affected by shape and orientation and as a result the unrelaxed strains about the particles will not be uniform. In such cases non-uniform stresses can exist inside particles as predicted by fiber loading shear lag models. Also local concentrations of elastic energy may alter the requirements for interfacial void formation based on the proposed energy criteria. Thus it is felt that the present nucleation model results in an upper bound since it applies to spherical particles.

Void nucleation at Fe_3C particles in spheroidized steels begins at high levels of deformation. The effect of the corresponding matrix strain on the plastic zone shape has been neglected. Since the plastic zone is a region which hardens rapidly with respect to the surrounding matrix, the zone shape will be controlled by the local deformation at the particle and not by the overall strain. Thus the present methods used to determine V_p are justified.

So far the effects of particle shape and particle interactions have been neglected. Semi-quantitative models which incorporate these effects into the energy criterion are discussed in section 5.4.

5.3.5 Comparison with experiment

In order to compare the results of the analytical model with experimentally measured quantities, a standard criterion must be chosen which defines whether a particle will or will not nucleate a void. Since the interfacial normal stresses on the particle are quite small for angles greater than $\frac{\pi}{4}$, it is assumed that voids larger than this will not nucleate and so the condition that $\frac{Z}{r_0^2} = 0$ for $\phi_0 = \frac{\pi}{4}$ is used to define void stability.

This criterion was employed to determine the relationship between r_0^c and $\bar{\epsilon}_p$ which was computed from equation 5.58 and is plotted in figure 5.14 for specimens B1, B2, W1, W2 where r_0^c represents the critical particle size for stable void nucleation as defined above. While r_0^c is represented as a function of $\bar{\epsilon}_p$, it is actually a function of the total deformation state. It can be seen from figure 5.10 that with increasing deformation each succeeding increment in the deformation corresponds to smaller increments in

nucleating particle size. That is, satisfaction of energy criterion becomes increasingly difficult for small particles.

An increment in particle size may be related to an increment in the percentage of particles with voids for a given distribution by considering figure 4.19 from which the area increment under the curve may be computed for a given specimen. This was done for successive increments in the deformation along the axis of each specimen and the total calculated percentage of particles with voids is plotted as a function of $\bar{\epsilon}_p$ in figure 5.15 for $\Gamma_W = 5.7 \frac{J}{m^2}$ and $\Gamma_W = 4.0 \frac{J}{m^2}$.

The figure indicates that a 30% decrease in Γ_W can produce significant changes in the number of particles with voids for a given level of deformation. While there is little effect on the shape of the curves relative to the level of strain, $\bar{\epsilon}_p$, the percentage of particles with voids is 2 to 5 times larger at the point of maximum deformation for the lower value of Γ_W .

This result is important in light of the deleterious effects which embrittling agents can have on mechanical properties of materials. The work of Cialone and Asaro [27] on hydrogen embrittlement in plain carbon steels showed that while hydrogen did produce large losses in ductility, the failure mode could not be distinguished from that of non-hydrogenated specimens. The processes leading to fracture in equiaxed tensile specimens appeared to merely be accelerated at a given level of deformation. This observation is in line with the present result if it is assumed that the presence of hydrogen does indeed lower the work of adhesion of the carbide-ferrite interface, thus enhancing the nucleation of voids at lower deformation levels.

If it is assumed that a one to one correspondence exists between voids and particles, then the percentage of failed particles can be directly related to the number of voids per unit volume $\{(N_V)_V = (N_V)_P\}$. Since the experimental results are given in terms of the number density of voids per unit area, some estimate of the average void length parallel to the tensile axis is required. This was accomplished by estimating average void lengths, $\bar{H}_0$, from measurements made on longitudinal sections at the center of the neck. Assuming the same growth rate parameter, M , used to compute nucleation areal void densities in section 4.8, the average void length at any point along the axis was estimated to be

$$\bar{H} = \bar{H}_0 - M\bar{H}_0 \int_{\bar{\epsilon}_P^0}^{\bar{\epsilon}_P^{\max}} d\bar{\epsilon}_P \quad (5.59)$$

where $\bar{\epsilon}_P^{\max}$ is the equivalent plastic strain at $z = 0$ and $\bar{\epsilon}_P^0$ is the strain at the point of interest. Using equation 4.10, the value of η_A may be computed from knowledge of η_V and equation 5.59. The results are plotted in figures 5.16 and 5.17 for B1, B2, W1, W2 showing both experimental and computed curves of areal void density versus plastic strain with $M = 1$ and $M = 2$. The measured values of $\bar{H}_0$ were $.47\mu$, $.57\mu$, $.26\mu$ and $.31\mu$ for the respective specimens.

The qualitative agreement between experiment and theory is very encouraging. General features of the curves correspond including position with respect to the strain axis. The absence of quantitative agreement may be due to several details involved in the analysis.

1) the one to one correspondence between voids and particles is not realistic, especially in specimens having small λ_P where it was often observed

that one void was associated with two or more particles. This may explain the large computed values of η_A for specimen W1.

2) Approximately 70% of the particles lie on grain boundaries so that the nucleation model should only be applied to these particles, yet the particle size increments were related to the overall particle size distribution function.

3) The determination of $\bar{H}_0$ from longitudinal sections was highly approximate due to the non-uniformity of void shape and the associated difficulties of relating planar measurements of longitudinal void dimensions to the actual value of $\bar{H}_0$.

4) The particles are not ideally spherical, nor are they isolated, and the effects of particle shape and particle interactions will increase the measured values of η_A over those for spherical, widely spaced particles.

A similar comparison was made between theory and experiment for η_A as a function of ρ on the plane $z = 0$. The results appearing in figure 5.18 for specimens B1, B2, W1 and W2 were computed assuming that the only variable change affecting the nucleation condition is the value of Q as determined by equation 2.33. A decrease in Q from $\rho = 0$ to $\rho = a$ is responsible for a decrease in η_A , but the experimental results indicate a larger dependence of η_A on ρ . The computed curves were scaled to match the experimental curves at $\rho = 0$ for the sake of comparison. As mentioned previously, Norris et al.^[67] predicted that the strain on the minimum neck section is not constant but decreases from the center to the edge of the specimen. Incorporation of their results into the model used to compute η_A produced closer agreement between experiment and theory (figure 5.19).

The comparisons made in figures 5.16-5.19 indicate that the present energy based nucleation criterion is able to predict the qualitative behavior of the relationship between void nucleation and state of deformation in a necked tensile specimen. The form of the computed curves is due to the particle size dependence on nucleation, resulting from the proposed model by which elastic energy is released when a void cap is formed.

5.4 Effects of particle shape and particle interactions

5.4.1 Particle shape

As noted in section 4.1.2, particle cracking was the dominant mode of void formation in the type H steels as opposed to interfacial decohesion in types B and W. The fractured particles were most often characterized by large, irregular, non-equiaxed geometries. The fracture plane was normal to the applied tensile direction and in some cases may have been associated with the boundary between contiguous particles.

For non-spherical particles, a nucleation model based upon uniform stress relaxation at the interface is no longer valid. This is exemplified by the extreme case of long fibers discussed in terms of shear lag models in section 2.6.1. Such models implicitly assume that complete relaxation occurs at the ends of a fiber while no relaxation occurs along the length of the fiber, producing a state of stress in the fiber which increases linearly with distance from the ends. In real systems there must be some relaxation at the fiber-matrix interface but it need not be uniform along the length of the fiber. Similarly, for ellipsoidal particles, relaxation may not be uniform and the elastic fields in and around such inhomogeneities may not be computed on the basis of the Eshelby solution for which uniform

transformation strains are assumed. The resulting elastic fields inside the particles will no longer be uniform and a stress build-up from the ends to the center of the ellipsoidal region will exist, the form of which will depend on the details of the relaxation at the particle-matrix interface.

An approximate criterion for particle fracture, which is based on the energy stability model already proposed for interfacial decohesion, has been developed. The total maximum axial stress on the particle cross section is due to the applied stress, σ_{zz}^A , and the internal stresses, σ_{zz}^S , which arise as a result of back stresses and load transfer. If it is assumed that the fracture plane in the particle is circular with radius a_0 , then the energy released by the crack can be computed from the form of K_I for a penny shaped crack. The energy difference function, Z , of equation 5.32 may be evaluated to give

$$Z = \Gamma_W \pi a_0^2 - \frac{4}{3} \frac{(1 - \nu)}{\mu} \sigma^2 a_0^3 \quad (5.60)$$

where $\sigma = \sigma_{zz}^A + \sigma_{zz}^S$

The condition that $Z \leq 0$ when the particle plane is completely fractured is used to define a stable crack. Using the parameters which were employed to compute Z for the case of decohesion, Γ_W for particle fracture is $6 \frac{J}{m^2}$ and will not likely be altered by alloying changes which may affect the work of adhesion of the particle-matrix interface.

For spherical particles, σ_{zz}^S is equal to the axial component of the back stresses and σ is defined above. For this case the condition that $Z = 0$ was used to compute the critical value of a_0 for which a stable crack could

exist when the material was subjected to the maximum deformation state at the center of the neck. For each specimen this value falls just below the solid lines of figure 5.14. These results indicate that the incidence of particle cracking versus interfacial decohesion is extremely sensitive to the various surface energy quantities of the system. For low values of interfacial work of adhesion, decohesion will be favorable while for higher values particle cracking is predicted. For elongated or irregular particles, the possibility of higher internal stresses due to load transfer associated with non-uniform relaxation may increase σ_{22}^S to a value which favors particle fracture. Cracking along a properly oriented boundary between contiguous particles may also be favored due to a smaller value of Γ_W associated with this plane.

5.4.2 Particle interactions

Observations by Cialone and Asaro [27] as well as those of the present study indicate that many voids are associated with more than one particle and it seems reasonable to expect that enhanced void formation can result from the interactions of neighboring particles. The stress intensification at the particle interface due to impingement of plastic zones of closely spaced particles has been addressed by Argon et al. [23]. A similar idea could be applied to the present models for nucleation of voids in which constraints on the plastic zones about particles are introduced by nearest neighbors. However, certain details of the deformation make such a treatment appear to be invalid. First, if it is assumed that the deformation and subsequent relaxation mechanisms are unaltered by the fact that particles are closely spaced, it must also be assumed that the spacing between these

particles will change in accordance with the remote strain. If so, it can be shown that for particles closely spaced along the tensile axis as a result of grain boundary alignment, the particle spacing will increase at a faster rate than the plastic zones of neighboring particles can expand. On the other hand, if it is realistically assumed that the deformation is indeed affected by the presence of neighboring particles, then some estimates about the nature of the deformation and the subsequent relaxation at the interfaces must be made. For this purpose, it is useful to consider the model depicted in figure 5.20 in which two cylindrical particles represent the platens of a brazed joint and the shaded matrix region represents the filler metal of the joint. The unshaded matrix region is allowed to deform as it would about isolated particles, but the ligament is constrained causing an increase of the local tensile flow stress in the region. A continuum solution for the normal stress, P , required to produce plastic flow at a particular radial position in the ligament has been obtained [92] having the form

$$P = Y_0 \exp \left[\frac{r_0 - r}{l} \right] \quad (5.61)$$

where Y_0 represents the unconstrained matrix flow stress and l , r , and r_0 are defined in figure 5.20. This solution applies to perfect bonding at the interfaces.

Extending this model to the system of neighboring spherical particles shown in figure 5.21, force equilibrium dictates that

$$2\mu P(2\pi r dr) = -dQ[2\pi r(l + 2r_0(1 - \cos\theta))]. \quad (5.62)$$

For $\mu = .5$ (perfect bonding) and imposing the condition that $dP = dQ$ as obtained from the yield condition, equation 5.62 may be expressed as

$$\ln P = -\frac{1}{2} \int \frac{\cos \theta d\theta}{A - \cos \theta} \quad (5.63)$$

where $A = \frac{\ell}{2r_0} + 1$. Integration yields

$$P = K \exp \left\{ -\frac{1}{2} \left[-\theta_0 + \frac{2A}{\sqrt{A^2 - 1}} \tan^{-1} \left(\sqrt{\frac{A+1}{A-1}} \tan \frac{\theta_0}{2} \right) \right] \right\} \quad (5.64)$$

The condition that $P = Y_0$ at $\theta_0 = \frac{\pi}{2}$ is used to eliminate the constant K , so the expression for the flow stress in the ligament becomes

$$P = Y_0 \exp \left\{ -\frac{1}{2} \left[\frac{\pi}{2} - \theta_0 + \frac{2A}{\sqrt{A^2 - 1}} \left(\tan^{-1} \left(\sqrt{\frac{A+1}{A-1}} \tan \frac{\theta_0}{2} \right) - \tan^{-1} \sqrt{\frac{A+1}{A-1}} \right) \right] \right\} \quad (5.65)$$

Figure 5.22 shows the dependence of the flow stress on the radial position in the ligament for several values of $\frac{\ell}{r_0}$.

From these results it is seen that plastic flow in the ligament may be severely restricted for closely spaced particles. Under these conditions the particle-ligament-particle system may be approximated as a single elongated elastic region within a plastically flowing matrix. Load transfer mechanisms may operate causing large stresses in the ligament and at the ligament-particle interfaces. For sufficient constraint the triaxial stress may become great enough to satisfy the energy criterion for stable void formation at the interfaces at deformation levels significantly below those at which void nucleation at isolated particles would be favorable.

6. Summary and Conclusions

6.1 Void formation in spheroidized steels

Before a realistic model for void nucleation at second phase particles can be formulated, various details of the void morphology must be obtained by experimental investigation. Failure to incorporate these details into a corresponding analytical treatment may likely lead to inaccurate nucleation criteria for a particular system. For example, a number of models have assumed total decohesion of the particle-matrix interface, while observations generally show that only partial decohesion occurs. Thus, a number of observations and conclusions relating to the nature of the voids in spheroidized steels have been made, providing the basis for the proposed nucleation models in this work. In summary it was observed that:

1. Voids occur at isolated spheroidal cementite particles by partial decohesion of the interface in the vicinity of one of the particle poles defined by the tensile axis.
2. Voids are generally associated with particles of greater than average size. They rarely form at very small, isolated particles, even for the severe state of deformation which exists in the neck of a tensile specimen.
3. Particles situated on ferrite grain boundaries are favored sites for the nucleation of voids.
4. Voids often form by decohesion of the interfaces of particles which are closely spaced along the tensile axis.

5. Appreciable interfacial decohesion begins at high levels of deformation corresponding to $\bar{\epsilon}_p = .6$ in W type steels and $\bar{\epsilon}_p = .75$ in B type steels.
6. The maximum gradient in the void nucleation profile occurs at strains of $\bar{\epsilon}_p = .8$ and $\bar{\epsilon}_p = 1.15$ for W and B type specimens respectively.
7. While only 10% to 40% of the particles have associated voids at the center of the specimen neck as determined from longitudinal sections, an apparent decrease in the rate of void nucleation with deformation occurs at higher deformations.
8. Voids elongate in the tensile direction but maintain elliptical cross-sections, indicating that plastic hole growth, and not ferrite grain boundary separation, dictates the final void geometry.
9. Non-equiaxed or irregularly shaped cementite particles are often subject to internal fracture (figure 4.5). The resulting cracks tend to be oriented normal to the tensile direction and may sometimes be associated with boundaries between contiguous particles.
10. Voids are more easily resolved on transverse cross sections than on longitudinal sections and a corresponding improvement in the reproducibility of data is obtained.
11. Areal void density, n_A , by itself, is not an accurate measure of void nucleation, regardless of whether longitudinal or transverse sections are examined, due to the effects of void growth.

6.2 Void nucleation criteria and analytical models

A comprehensive treatment of void nucleation at spherical particles

has been developed based on experimental observations of spheroidized steels. The following aspects of material deformation have been incorporated into the proposed models:

1. Applied loading including the effects of specimen geometry.
2. Misfit between elastic particle and plastically deforming matrix.
3. Subsequent relaxation at the particle-matrix interface.
4. Effects of grain boundaries on local deformation.

A double criterion for void nucleation has been proposed based on a combined critical normal stress requirement and an elastic energy requirement:

1. $\sigma_n > \sigma_c$ over the entire interfacial area defined by ϕ_o .
- and
2. $\Delta E > \Delta W$ which defines a critical angle, ϕ_c , such that
- $$\phi_o > \phi_c.$$

Based on these considerations, models for void nucleation have been formulated and evaluated, leading to the following conclusions:

1. The energy criterion is not generally satisfied even for large deformations.
2. The factors which favor void nucleation are:
 - i) Large particle size.
 - ii) Low work of adhesion, Γ_w .
 - iii) High flow stress, $Y(\bar{\epsilon}_p)$.
 - iv) High triaxiality, Q .

- v) Large plastic strain, $\bar{\epsilon}_p$.
- vi) Small grain size, D .
- vii) Location of particles on grain boundaries.

3. Computed void distribution curves based on the proposed energy criterion correlate well with experimental results. Relative magnitudes of void number density for different specimens as well as the functional dependence upon deformation are in agreement.

4. Small changes in Γ_w can significantly alter the conditions for stable void formation. A decrease in Γ_w from 5.7 J/m^2 to 4.0 J/m^2 can increase the percentage of void nucleating particles by as much as a factor of 5 in the center of the neck.

5. The particle size dependence predicted by the models is supported by the experimental determination of average transverse void diameter, $\bar{d}_v$.

6. The energy criterion can be extended to the case of particle cracking. Whether voids form by interfacial decohesion or internal fracture of the particles depends upon particle shape in relation to load transfer, and the relative values of Γ_F and Γ_C .

7. Constraints on plastic flow in the matrix between closely spaced particles can lead to an enhanced triaxial state of stress, locally. The corresponding increase in elastic energy storage favors void nucleation at lower levels of deformation.

8. The energy criterion of equation 5.32 can be used to determine conditions for void nucleation during axisymmetric tensile deformation

in any alloy system containing spherical particles. However, the relationship between ϵ_p^* and $\bar{\epsilon}_p$ is dependent upon the model chosen to describe the local deformation at the particles.

6.3 Recommendations for further study

The results obtained in this work suggest possible avenues for future experimental and analytical investigations relating to void formation.

A. Experimental

1. Determination of the effects of alloy additions on the surface chemistry of particle-matrix interfaces, possibly by Auger spectroscopic studies of fracture surfaces.
2. Quantification of the effects of grain size on void nucleation in specimens having equivalent particle size distributions.
3. Study of void nucleation in tensile specimens subjected to hydrostatic pressure in order to determine the effects of Q .
4. Alteration of specimen geometry to determine whether the proposed models can predict void density profiles for arbitrary states of stress and deformation gradients.
5. Further interrupted fracture testing in order to better characterize the process of gross material failure in relation to voids and grain boundaries.

B. Analytical

1. Quantitative analyses of the effects of particle interactions on void nucleation.

2. Detailed development of models to describe local deformation at particles situated on grain boundaries.

3. Quantitative analysis of the relationship between the back stresses arising from plastic accommodation models [44] and load transfer arising from continuum models [47], incorporating descriptions of plastic relaxation at non-spherical particles.

Appendix A

The Effect of Alloy Deformation on the Average Spacing
Parameters of Non-Deforming Particles*

Introduction

Recent work [93, 94] on geometrical contributions to deformation and fracture mechanisms in particle strengthened alloys has led to an examination of the effect of plastic deformation on the average particle spacing parameters. Consideration of questions such as whether the planar particle density is a function of strain in the neck of a tensile specimen, or whether the directional length distribution of the mean free path is anisotropic in uniaxially deformed specimens are relevant to the understanding of the role of particles and inclusions in deformation and fracture. The following discussion of these and other problems is based on the principles of stereology as presented by Underwood.^[68]

Theory

The sample cube of volume $V_T = \ell^3$ (figure A.1a) contains particles of constituent β embedded in a homogeneous plastic matrix, α . The poly-dispersed system of β particles is characterized by the following average spatial dispersion parameters^[68]:

Average volume fraction

$$(V_v)_\beta = \bar{V}_\beta N_{v\beta} \quad (A.1)$$

*Submitted for publication to Metallurgical Transaction by J. Fisher and J. Gurland, Division of Engineering, Brown University.

Average areal fraction

$$(A_A)_\beta = \bar{A}_\beta N_{A\beta} \quad (A.2)$$

Average number of intersection of particles per unit test area

$$N_{A\beta} = \bar{H}_\beta N_v \quad (A.3)$$

Matrix mean free path between non-contiguous particles

$$\lambda_\alpha = \bar{L}_{3\beta} \frac{1 - (V_v)_\beta}{(V_v)_\beta} \quad (A.4)$$

where $N_{v\beta}$ is the average number of β particles per unit volume of the sample cube, $\bar{V}_\beta$ is the average particle volume, $\bar{A}_\beta$ is the mean particle intercept area, $\bar{H}_\beta$ is the particles' mean projected height or mean caliper diameter, and $\bar{L}_{3\beta}$ is the mean intercept length of the particles. It is noted that the average dispersion parameters $(V_v)_\beta$, $(A_A)_\beta$, $N_{A\beta}$ and λ_α are each a function of $N_{v\beta}$, the average number of particles per unit volume, and one or more of the mean particle size parameters $\bar{V}_\beta$, $\bar{A}_\beta$, $\bar{H}_\beta$ and $\bar{L}_{3\beta}$. The average dispersion parameters are measured on statistically random planar test sections, usually by a systematic point count for $(A_A)_\beta$, or a count of intersections of particles for $N_{A\beta}$, or by an intercept count along test lines for λ_α . [68]

The sample cube of α and β material is now deformed at constant volume to a rectangular parallelepiped (figure A.1b) such that $\ell_x \neq \ell_y \neq \ell_z$. The following discussion will present the effect of the deformation on the particle dispersion parameters.

Equiaxed non-deforming particles of any spatial arrangement

Neither the average number of particles per unit volume, $N_{v\beta}$, nor the average particle size parameters $\bar{V}_\beta$, $\bar{A}_\beta$, $\bar{H}_\beta$ and $\bar{L}_{3\beta}$ are changed by the

deformation. Consequently, according to equations A.1-A.4, the particle dispersion parameters $\{V_v\}_\beta$, $\{A_A\}_\beta$, $N_{A\beta}$ and λ_α remain invariant throughout the deformation. Also, since neither $N_{v\beta}$ nor $\bar{V}_\beta$, $\bar{A}_\beta$, $\bar{H}_\beta$ and $\bar{L}_\beta$ are functions of directionality, the particle dispersion parameters are independent of the orientation of the test planes and test lines and

$$(A_A)_{\beta xy} = (A_A)_{\beta yz} = (A_A)_{\beta xz} = (A_A)_\beta \text{ random} \quad (A.5)$$

$$N_{A\beta xy} = N_{A\beta yz} = N_{A\beta xz} = N_{A\beta} \text{ random} \quad (A.6)$$

$$\lambda_{\alpha x} = \lambda_{\alpha y} = \lambda_{\alpha z} = \lambda_\alpha \text{ random} \quad (A.7)$$

where the subscripts x, y, z refer to test planes or test lines parallel to the axes or the planes defined by the coordinate system x, y, z of figure A.1. These conclusions apply to any type of spatial distribution of equiaxed rigid particles in either deformed or non-deformed materials, regardless of whether the spatial distribution is random or non-random, or whether it has directional character or not, or whether the particle size is uniform or non-uniform. In the determination of average spacing parameters a single test (or sampling) plane is sufficient for random particle distributions, but in the case of directional sampling of non-random, and possibly anisotropic, spatial distributions of particles it is required that the number and spacing of the directional and parallel test planes and test lines be such that the measurements provide statistically valid results.

b) Non-deforming particles of non-equiaxed shape.

1) Initially of random orientation. Non-equiaxed or non-spherical particles are subject to rotation during plastic deformation of the aggregate

with resulting preferential orientation of their largest axes parallel to the direction of the largest tensile strain. The spatial dispersion parameters affected by the rotation are $N_{A\beta}$ and λ_{α} , because the lengths $\bar{H}_{\beta}$ and $\bar{L}_{3\beta}$ acquire directional character. For instance, if, as a result of preferred particle orientation in the deformed state, $\bar{H}_{\beta z} > \bar{H}_{\beta x}$ and $\bar{L}_{3\beta z} > \bar{L}_{3\beta x}$, then $N_{A\beta xy} > N_{A\beta yz}$ according to equation 3.A and $\lambda_{\alpha y} < \lambda_{\alpha z}$ according to equation 4.A. While $N_{A\beta}$ and λ_{α} are now directionally dependent, their average values established over all orientations of test planes and test lines remain equal to the average values (on any plane or in any direction) in the undeformed state.

The area fraction $(A_{A\beta})$ remains invariant during the deformation and is equal in all directions. This is seen convincingly if equation A.1-A.3 are combined to give:

$$(A_A)_{\beta} = \bar{A}_{\beta} N_{A\beta} = \bar{A}_{\beta} \cdot \bar{H}_{\beta} \cdot N_{v\beta} = \bar{V}_{\beta} N_{v\beta} = (V_v)_{\beta} \quad (A.8)$$

Because neither $\bar{V}_{\beta}$ nor $N_{v\beta}$ is a function of deformation or direction, equation A.8 holds independently of the direction of measurement or of deformation history, subject only to the requirement of statistical validity.

ii) Initially of parallel orientation. For the case of elongated particles or discontinuous fibers aligned with their major axes parallel to the direction of uniaxial deformation, it is seen from equation A.1-A.4, and with reference to the geometry of figure A.1b, that $N_{A\beta xy} > N_{A\beta yz}$, $N_{A\beta xy} > N_{A\beta xz}$, $\lambda_{\alpha x} < \lambda_{\alpha z}$, in the undeformed state. Ideally, if the particles or fibers do not rotate during the uniaxial deformation, the average spacing parameters in each direction remain invariant during the deformation,

provided that the directional size parameters remain unchanged, i.e. $\bar{H}_{\beta z}$ and $\bar{L}_{3\beta x}$, for instance.

Experiments

For the purpose of experimental verification, the microstructural spacing parameters related to the dispersion of Fe_3C particles were measured in several spheroidized tool steels, with nominally 1.05% C, in the "as received" and deformed conditions. The deformation consisted of either uniaxial tension, uniaxial compression or rolling.

The area fraction of cementite, $(A_A)_\beta$, and the mean free path of ferrite, λ_α , were measured on the planes and in the direction indicated by the subscripts x, y, z, where z is the direction of the primary deformation. The particle orientation factor (O.F.) is established by the ratio of the numbers of particle intercepts (N_x , N_y , N_z) with test lines respectively perpendicular and parallel to the deformation direction. The orientation factor is a measure of the average aspect ratio of the planar particle intercepts in the direction of deformation.

Within the limits of the experimental precision, the data of figures A.2 and A.3 confirm the theoretical conclusions, namely that 1) in spite of considerable scatter, the area fraction measurements are not in disagreement with the expected constancy of area fractions during deformation; 2) the mean free path increases in the direction of elongation and decreases in the direction of compression in correlation with changes of the orientation factor indicating rotation of nonequiaxed particles (figure A.2 and figure A.3, rolled rod), and 3) the mean free path is approximately invariant during deformation in the steel with approximately constant orientation

factor (figure A.3, rolled bar). The particles in this steel also appeared to be more equiaxed than those in the steels with pronounced particle rotation.

Discussion

The average microstructural measures of particle concentration and spacing remain unchanged by deformation if the particles are equiaxed. The most apparent effects of deformation are due to the rotation of non-equiaxed particles. In practice, in many alloys the most visible effects of deformation are particle stringing, due, for instance, to geometrical alignment of particles on grain boundaries, or particle fibering, due to particle deformation and fracture. If the particles are equiaxed, the average dispersion parameters $(A_A)_\beta$, $N_{\beta A}$ and λ_β do not reflect observable structural anisotropy, because they are independent of the orientation of test planes and test lines. A more complete characterization of the microstructure requires additional information about the directional variation of the distribution of spacing parameters, such as the frequency distribution of the intercept length components of the mean free path in various directions, or the frequency distribution of the number of particles in directionally oriented test cells. An application of the analysis of the heterogeneity parameters of fibered or layered microstructures, relevant to mechanical properties, was developed by Underwood and Starke [95], and the effect of deformation on the dispersion morphology of deformable particles was described by Durham. [96]

A spacing parameter which merits further consideration is the nearest neighbor distance which, for instance, is of importance when considering

particle interactions. The average nearest neighbor distances in a volume ($0.554 N_V^{-1/3}$) or on a plane ($0.500 N_A^{-1/2}$) as calculated for random distributions of point particles [68], do not change as a result of deformation as long as the distributions remain random, even though the distance vector between a specific pair of point particles does change in length and direction in accordance with the deformation of the specimen. Also, since the average number density of the intersections of equiaxed particles on test planes remains constant throughout the deformation, any planar measures of average interparticle distance based on N_{AB} (such as $N_{AB}^{-1/2}$) also remain invariant. If the particle placement is idealized by associating it with an imaginary cubic grid, as is often done in particle strengthening theories, the grid point separation would change in accordance with the imposed deformation, but the statistical average planar particle intercept density, N_{AB} , measured on randomly spaced parallel test planes, would remain constant with deformation and change of test plane orientation, for equiaxed particles, even if the particles were regularly and exactly placed at the grid points. As far as particle spacing in real particle systems is concerned, any inference based only on the deformation of an imaginary grid could be grossly misleading.

The above considerations are particularly relevant to the analysis of ductile fracture processes involving particle-nucleated void growth in the necked portion of tensile specimens. In modeling the fracture process it is often assumed that either the particle spacing or the void spacing or both are a geometrical function of strain [51, 94, 97, 98] (for instance [97], $\lambda_a(\epsilon) = \lambda_a(0) \exp(-\frac{\epsilon}{2})$, where ϵ is the longitudinal strain and λ_a is the

mean free path on a transverse planar section). The assumption derives from theoretical models with parallel cylindrical voids of infinite length [51, 98], but it has been shown here that it does not generally apply to measures of the average spacing if the particles or holes are appreciably smaller than the specimen or neck length. In the neck, as well as elsewhere in the specimen, the average planar number density of equiaxed particles and the mean free path remain constant with strain and orientation. The planar number density of void intercepts (and their average center-to-center spacing) on a cross-sectional plane is invariant with deformation for longitudinal voids ($\bar{H}_2 \ll \lambda_2$) parallel to the tensile specimen axis, unless the volume number density, N_V , or the longitudinal caliper length, $\bar{H}_2$, of the voids increases with strain, by nucleation of new voids or plastic growth of existing voids, respectively.

In general, the proper evaluation of the role of particles in deformation and fracture requires careful consideration of the spacing parameters and of their presumed dependence on strain.

Conclusions

1. The average spacing parameters $(A_A)_\beta$, N_{AB} and λ_α of non-deforming equiaxed particles are invariant with respect to plastic deformation.
2. In the case of non-deforming, non-equiaxed particles, the average spacing parameters N_{AB} and λ_α may acquire directional character by rotation of the particles into a preferred orientation during deformation of the alloy.
3. The average directional spacing parameters $(A_A)_{\beta z}$, N_{ABxy} , and $\lambda_{\alpha x}$ of non-deforming and non-rotating, non-equiaxed particles are invariant during uniaxial deformation.

Acknowledgements

The experimental data were obtained by J. Anastasio, undergraduate research assistant. The work was supported by the U.S. Department of Energy under contract EY-76-S-02-3084 with Brown University.

Appendix B

In order to evaluate ΔW in equation 5.33, values for the free surface energies of ferrite, cementite, and the ferrite-cementite interface (Γ_F , Γ_C , Γ_{FC}) must be determined. The availability of experimental data relating to these quantities is limited and the imprecision of testing methods casts uncertainty upon the reliability of these data. Furthermore, surface energy values of solid-vapor interfaces are generally performed at high temperatures, near the melting point, and do not apply to corresponding temperatures at ambient temperatures. It is therefore apparent that only approximate values of Γ_F , Γ_C and Γ_{FC} can be used in the computation of ΔW .

The relationship between surface tension and surface free energy was considered by Shuttleworth [99] who equated the increase in the Helmholtz surface free energy, F_A , to the work done against surface stresses, σ , in expanding the area, A , of an interface. For σ equal to the surface tension, Γ , the relationship is

$$\Gamma = F_A + A \left(\frac{dF_A}{dA} \right) \quad (B.1)$$

In the case of liquids, or solids at high temperature, the number of atoms per unit area of surface remains fairly constant and $\frac{dF_A}{dA} \sim 0$. The surface energy can then be equated to the surface tension. For decreased atomic mobility in solids at lower temperatures, $A \frac{dF_A}{dA}$ can have magnitudes comparable to F_A and Shuttleworth predicted that some

inert gas crystals and alkali halides have negative surface tensions at absolute zero. It is assumed that high temperature zero creep tests are generally conducted under conditions corresponding to high atomic mobility so that $F_A = \Gamma$.

The analysis by Swalin [100] based on surface excess quantities at the interface between two phases, α and β , leads to a relationship of the form

$$d\Gamma = -S_A dT - \sum_i C_i d\mu_i \quad (B.2)$$

where S_A is the excess surface entropy, C_i is the surface concentration of the i^{th} component of the system and μ_i is the corresponding chemical potential. At the solid-vapor interface (sv) of a single phase, assuming $d\mu_i = 0$ and $\frac{dF_A}{dA} = 0$, this leads to

$$F_A = \Gamma^{sv} = \Gamma_0^{sv} - S_A (T_0 - T) \quad (B.3)$$

where Γ_0^{sv} and T_0 correspond to reference values of surface tension and temperature determined by experiment near the melting point of the solid. Thus, F_A may be estimated as a function of temperature if the behavior of S_A is known, where the relationship

$$-S_A = \frac{dF_A}{dT} = \frac{d\Gamma^{sv}}{dT} \quad (B.5)$$

was used to compute surface entropy from high temperature data.

The results of a number of investigations based on zero creep measurements are presented in Table B.1 along with values of F_A^{sv} for

iron at 25°C computed from equation B.3. On the basis of these results Γ_F is assigned an approximate value of 4 J/m².

The surface free energy of the ferrite-cementite interface was determined by Kramer et al.^[104] by calorimetry measurements in which they obtained the interfacial surface enthalpy, $H_A = \left(\frac{\partial H}{\partial A}\right)_T$, associated with the formation of pearlite from austenite in eutectoid steels at 723°C. By varying the thickness of the lamellar spacing of the pearlite at the constant transformation temperature, they obtained a value for H_A of 1.4 J/m². Their suggested value of $S_A = .00067 \text{ J/m}^2 - ^\circ\text{C}$ was used in the relation

$$F_A = H_A - S_A T \quad (\text{B.5})$$

to obtain a value for Γ_{FC} of about 1.2 J/m² at 25°C.

In Birnfield's^[105] study of the growth kinetics of cementite spheroids on grain boundaries, he made dihedral angle measurements in order to obtain a relationship between the surface energy of the ferrite grain boundary and the ferrite-cementite interface. His data has been analyzed to give $\Gamma_{FC} = .266 \text{ J/m}^2$ at 600°C and $S_A = - .0019 \text{ J/m}^2 - ^\circ\text{C}$. Equation B.3 then predicts that $\Gamma_{FC} = 1.3 \text{ J/m}^2$ at 25°C. An approximate value of 1.3 J/m² was chosen to represent Γ_{FC} in the evaluation of ΔW .

Experimental values of the surface energy of free cementite are not available and were therefore approximated by Anatov's rule.^[106]

$$\Gamma_C = |\Gamma_F - \Gamma_{FC}|$$

Using the previous results for Γ_F and Γ_{FC} , this expression predicts a value of 2.7 J/m^2 for Γ_C at 25°C . Because of the high uncertainty associated with this value due to the unreliability of equation B.7, a single digit value of 3 J/m^2 is assigned to Γ_C .

It should be noted that the thermodynamic relationships used in this analysis are based on planar interfaces. As reported by Hirth,^[107] for a spherical particle with radius r ,

$$\Gamma = \Gamma_0 \left(1 - \frac{2\delta}{r}\right) \quad (\text{B.7})$$

where Γ_0 now refers to the value of the interfacial energy for a surface of zero curvature and δ is of the order of atomic dimensions. It can be concluded that Γ will not be significantly altered by radius of curvature for cementite particles in spheroidized steel.

The values $\Gamma_F = 4 \text{ J/m}^2$, $\Gamma_C = 3 \text{ J/m}^2$ and $\Gamma_{FC} = 1.3 \text{ J/m}^2$ refer to averages with respect to crystal orientation and thus, the effects of anisotropy on surface energy have not been considered.

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Table 2.1

Investigator	Failure Mode	Criterion	Basis
Stroh [50]	Particle Cracking	Critical Shear Stress $\tau_0 > \frac{.72\mu}{n}$	Energy
Smith [49]	Particle Cracking	Critical Shear Stress $\tau_0 > \frac{.12\mu}{n}$	Energy
Smith and Barnby [52]	Particle Cracking	Critical Shear Stress $\tau_0 > \frac{.12\mu}{n} \left(\frac{2d}{L_0} \right)^{\frac{1}{2}}$	Energy
Kelly and Davies [47]	Particle Cracking	Critical Normal Stress $\sigma_0 \left(1 + \frac{L}{2\tau_0} \right) > \sigma_f$	Load transfer from plastic strain incompatibility
Ashby [43]	Interfacial Decohesion	Critical Normal Stress $\text{constant} \times \frac{\gamma_p r_0}{2kb} > \sigma_f$	Back stresses from plastic strain incompatibility
Argon et al. [23]	Interfacial Decohesion	Critical Normal Stress $\tau_0 \left[1 + \sqrt{\frac{6\sqrt{6}}{m}} \frac{\gamma_p}{\gamma_0} \right] > \sigma_f$ or $\tau_0 \left[\left(\frac{\gamma_p}{\gamma_0} \right)^n + \sqrt{3} \left(\frac{6\sqrt{6}(m+1)}{m} \frac{\gamma_p}{\gamma_0} \right)^{\frac{n}{1+n}} \right] > \sigma_f$	Applied strain (Dislocation model, No relaxation) Applied strain (Dislocation model, With relaxation)

(table 2.1, cont.)

Investigator	Failure Mode	Criterion	Basis
Tanaka et al. [57]	Interfacial Decohesion	Critical Plastic Strain $\epsilon_p > \beta r_0^{-1/2}$	Energy
Brown and Stobbs [29]	Interfacial Decohesion	Critical Plastic Shear Strain $\gamma_p > \frac{\pi}{a^2} \left(\frac{3W}{4b\mu} \right)^{1/3} \left(\frac{r_0}{4b} \right)^{1/3}$	Energy
Sundström [60]	Interfacial Decohesion	$\frac{d\Delta E}{d\phi} = 0, \Delta E = \int_0^{\phi_0} (W - \frac{2}{3}) dA$	Energy

Table 3.1

Type	Stock Diameter (meters)	C	Mn	Si	Al	P	S
B	.0020	0.17	0.52	0.006	—	0.007	0.024
W	.0019	0.44	0.69	0.15	—	0.007	0.024
H	.0016	1.05	0.34	0.24	0.008	0.012	0.011

Table 4.1

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Specimen	f_p		%C		r_0	λ_p	D	$(N_A)_p$	$(N_V)_p$
	meas.	actual	meas.	actual	microns	microns	microns	meter ⁻² $\times 10^{10}$	meter ⁻³ $\times 10^{10}$
B1	.029	.026	.19	.17	.21	9.4	4.0	32.3	76.9
B2	.024	.026	.16	.17	.42	22.7	7.1	7.6	7.4
B3	.024	.026	.16	.17	.52	28.5	10.9	5.2	4.1
W1	.066	.067	.44	.44	.14	2.7	1.9	152.8	509.0
W2	.070	.067	.46	.44	.51	9.1	2.2	18.9	18.9
W3	.066	.067	.44	.44	.57	10.8	3.4	14.5	12.3
H	.169	.160	1.11	1.05	.69	4.5	—	—	—

128

MAXIMUM DEFORMATION
TRANSVERSE SECTIONS

Specimen	f_p		%C		r_0	λ_p	$(N_A)_p$	$(N_V)_p$
	meas.	actual	meas.	actual	microns	microns	meter ⁻² $\times 10^{10}$	meter ⁻³ $\times 10^{10}$
B1	.021	.026	.14	.17	.14	9.0	51.2	182.8
B2	.023	.026	.15	.17	.48	25.4	7.1	7.4
B3	.022	.026	.14	.17	.48	28.6	6.2	6.5
W1	.067	.067	.45	.44	.14	2.5	102.5	732.1
W2	.068	.067	.45	.44	.42	7.6	28.8	34.3
W3	.066	.067	.44	.44	.49	11.2	15.5	15.8

Table 4.2

Tensile Test Data

Specimen	$\sigma_y^L \left(\frac{\text{MN}}{\text{m}^2} \right)$	ϵ_{TS}	$\sigma_{\text{TS}} \left(\frac{\text{MN}}{\text{m}^2} \right)$	$\bar{\sigma}_{\text{MIN}} \left(\frac{\text{MN}}{\text{m}^2} \right)$	$\bar{\sigma}_f \left(\frac{\text{MN}}{\text{m}^2} \right)$	$\bar{\epsilon}_f$
B1	269	.186	447	853	881	1.35
B2	250	.212	424	844	870	1.42
B3	243	.226	420	827	827	1.42
W1	517	.128	710	1,157	1,339	1.12
W2	424	.172	588	977	1,111	1.10
W3	393	.182	552	925	1,062	1.10

Table 4.3

Holloman Constants

Specimen	ϵ_0	$\sigma_0 \left(\frac{\text{MN}}{\text{m}^2} \right)$	n
B1	.001	175	.180
B2	.0005	123	.205
B3	.0005	116	.210
W1	.0022	407	.135
W2	.0018	262	.175
W3	.0015	228	.185

Table 4.4

Stresses in Necked Specimens

$$\left(\frac{\text{MN}}{\text{m}^2}\right)$$

	B1			B2			B3			W1			W2			W3		
$\frac{z}{a_0}$	σ_{zz}	$Y(\bar{\epsilon}_p)$	σ_H	σ_{zz}	$Y(\bar{\epsilon}_p)$	σ_H	σ_{zz}	$Y(\bar{\epsilon}_p)$	σ_H	σ_{zz}	$Y(\bar{\epsilon}_p)$	σ_H	σ_{zz}	$Y(\bar{\epsilon}_p)$	σ_H	σ_{zz}	$Y(\bar{\epsilon}_p)$	σ_H
0.0	884	637	247	856	618	238	841	606	235	1190	915	275	1010	776	234	945	743	223
0.1	866	630	236	839	610	229	823	598	225	1160	910	250	982	770	212	919	737	201
0.2	822	624	198	801	603	198	787	592	195	1120	905	215	953	765	188	890	732	174
0.3	774	616	157	759	596	163	746	584	162	1060	899	161	907	758	149	850	725	138
0.4	720	610	110	713	588	125	700	576	124	980	894	86	850	753	97	798	720	86
0.5	671	601	70	670	578	92	658	566	92	915	883	32	792	740	52	746	707	43
0.6	607	589	20	613	562	51	602	550	52	826	870	-44	727	727	0	686	693	-8
0.7	526	572	-46	542	546	-4	532	534	-2	732	856	-124	650	711	-61	617	677	-60
0.8	439	553	-114	466	526	-60	457	514	-57	627	837	-210	566	691	-125	540	658	-130
1.0	214	510	-296	266	479	-213	262	497	-205	368	797	-429	362	649	-287	354	615	-288
1.2	73	487	-414	144	454	-310	142	442	-300	176	771	-595	210	621	-411	217	587	-408

Table 4.5

Stresses on Minimum Cross Section

$$\left(\frac{\text{MN}}{\text{m}^2} \right)$$

$\frac{\rho}{a}$	B1			B2			B3			W1			W2			W3		
	σ_{zz}	$Y(\bar{\epsilon}_p)$	σ_H	σ_{zz}	$Y(\bar{\epsilon}_p)$	σ_H	σ_{zz}	$Y(\bar{\epsilon}_p)$	σ_H	σ_{zz}	$Y(\bar{\epsilon}_p)$	σ_H	σ_{zz}	$Y(\bar{\epsilon}_p)$	σ_H	σ_{zz}	$Y(\bar{\epsilon}_p)$	σ_H
0.0	885	637	248	858	618	240	842	606	236	1190	915	275	1009	776	233	966	743	223
0.1	883		246	856		238	840		234	1187		272	1007		231	964		221
0.2	876		239	850		232	834		228	1180		265	1001		225	958		215
0.3	866		229	840		222	824		218	1168		253	991		215	948		205
0.4	851		214	825		207	809		203	1151		236	976		200	934		191
0.5	831		194	806		188	791		185	1128		213	957		181	916		173
0.6	806		169	782		164	767		161	1100		185	933		157	893		150
0.7	775		138	752		134	737		131	1065		150	903		127	865		122
0.8	738		101	716		98	702		96	1024		109	878		92	831		88
0.9	692		55	671		53	658		52	974		59	826		50	791		48
1.0	637		0	618		0	606		0	915		0	776		0	743		0

Table 5.1

$$\theta = \frac{\int_0^{\phi} \theta(\phi, Q) d\phi}{\int_0^{\pi} \theta(\phi, Q) d\phi}$$

ϕ	Q	$-\frac{1}{3}$	0	.1	.2	.3	.4	∞
$\frac{\pi}{100}$		1.47×10^{-5}	1.86×10^{-5}	1.89×10^{-5}	1.90×10^{-5}	1.89×10^{-5}	1.88×10^{-5}	1.36×10^{-5}
$\frac{\pi}{16}$		4.13×10^{-3}	4.90×10^{-3}	4.98×10^{-3}	4.97×10^{-3}	4.92×10^{-3}	4.87×10^{-3}	3.25×10^{-3}
$\frac{\pi}{10}$		1.79×10^{-2}	2.09×10^{-2}	2.09×10^{-2}	2.08×10^{-2}	2.05×10^{-2}	2.03×10^{-2}	1.29×10^{-2}
$\frac{\pi}{8}$		2.57×10^{-2}	4.10×10^{-2}	4.10×10^{-2}	4.06×10^{-2}	3.99×10^{-2}	3.94×10^{-2}	2.46×10^{-2}
$\frac{\pi}{6}$		8.45×10^{-2}	9.56×10^{-2}	9.48×10^{-2}	9.41×10^{-2}	9.23×10^{-2}	9.08×10^{-2}	5.50×10^{-2}
$\frac{\pi}{4}$		2.52×10^{-1}	2.82×10^{-1}	2.80×10^{-1}	2.77×10^{-1}	2.71×10^{-1}	2.68×10^{-1}	1.59×10^{-1}
$\frac{\pi}{3}$		4.60×10^{-1}	5.22×10^{-1}	5.24×10^{-1}	5.16×10^{-1}	5.08×10^{-1}	5.01×10^{-1}	3.09×10^{-1}
π		1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table B.1

Temperature	$F_A^{SV} \Big _T \frac{J}{m^2}$	$S_A \frac{J}{m^2 - C^\circ}$	$F_A^{SV} \Big _{25^\circ C} \frac{J}{m^2}$	Ref.
1450°C	1.95	$\frac{0}{.0009}$	3.23	Murr [85]
1534°C	2.08	.0015	4.83	Jones and Leak [101]
1410°C	2.32	.0011	3.84	Hondros [102]
1500°C	1.85	.0013	3.77	Price et al. [103]

$$\Gamma_F \sim 4 \frac{J}{m^2}$$

Fig. 2.1. Representative load-elongation curve for plain carbon steels.

Fig. 3.1. Geometries of deformed and undeformed axisymmetric tensile specimens.

Fig. 3.2. Rigid Testing Apparatus (RTA) used to control radial crack growth in B type specimens.

Fig. 3.3. Types of sections which were prepared for metallographic examination to determine void morphology.

(a)

(b)

Fig. 4.1. Scanning electron micrographs of undeformed specimens.
(a) .17% carbon, specimen B2 (b) .44% carbon, specimen W2.

(a)

(b)

Fig. 4.2. Scanning electron micrographs of longitudinal sections showing grain elongation and particle alignment for (a) specimen W2 with $\bar{\epsilon}_p = .6$ and (b) specimen B1 with $\bar{\epsilon}_p = 1.3$. Tensile axis is vertical.

Fig. 4.3. Scanning electron micrograph of a longitudinal section taken from the center of the neck of specimen B2 showing the association between voids and particles which are situated on grain boundaries. While there is extensive longitudinal void growth, lateral growth is minimal. Tensile axis is vertical.

(a)

(b)

(c)

Fig. 4.4. Scanning electron micrographs of longitudinal sections showing that large particles are favored sites for void nucleation for a given level of deformation. (a) specimen W1 (b) specimen B1 (c) specimen B3. Tensile axis is vertical.

(a)

(b)

Fig. 4.5. Optical micrographs of longitudinal, heat tinted sections from type H specimen. Particle cracking is the dominant mode of void formation with preferential failure of large, non-equiaxed particles. (a) $\bar{\epsilon}_p = .25$ (b) $\bar{\epsilon}_p = .4$. Tensile axis is verticle.

(a)

(b)

Fig. 4.6. Scanning electron micrographs of transverse surfaces from the minimum neck section of (a) specimen B1 and (b) specimen W2 showing elliptical void cross section.

(a)

(b)

Fig. 4.7. Scanning electron micrographs of corner section (fig. 3.3c) taken from specimen B1. (a) low magnification showing specimen orientation. (b) high magnification showing void structures.

Fig. 4.8. Experimental and theoretical spatial distribution curves for particles in specimen B3 corresponding to deformed and undeformed regions. Measurements were made on transverse sections.

Fig. 4.9. Computed particle size distribution curves for spheroidized carbides based on log-normal distribution functions.

Fig. 4.10. Measured load-elongation curves for B and W type specimens based on a one inch gage length.

Fig. 4.11. Axial stress as a function of axial distance from the minimum neck section computed for specimens B2 and W2. These results were obtained from extrapolation of numerical results by Needleman [65] and from analytical solutions proposed by Argon [64].

Fig. 4.12. Tensile flow stress and superimposed hydrostatic stress along the axis of necked specimens based on solutions by Needleman [65] and Argon [64].

Fig. 4.13. Axial stress as a function of radial position on the plane of minimum cross section computed from the solution of Bridgman [63].

Fig. 4.14. Experimental curves showing the areal density of voids as a function of distance from the minimum neck section along the specimen axis.

Fig. 4.15. Experimental curves showing the areal density of voids as a function of radial position on the plane of minimum cross section.

Fig. 4.16. Void volume fraction as a function of equivalent plastic strain along the specimen axis.

Fig. 4.17. Void volume fraction as a function of radial position on the plane of minimum cross section.

Fig. 4.18. Relationship between equivalent plastic strain and axial position based on the numerical results of Needleman [65]. Measured neck contours for B and W type specimens are also shown.

Fig. 4.19. Areal void density curves of figure 4.14 replotted versus equivalent plastic strain along the specimen axis.

Fig. 4.20. Areal void density as a function of axial strain corrected for void growth. These curves represent the contribution to η_A from void nucleation only.

Fig. 4.21. Average transverse void diameter as a function of axial position.

Fig. 4.22. Arrested radial crack in specimen B3 showing voids elongated in tensile direction. The absence of lateral void coalescence is apparent. Tensile axis is vertical.

(a)

(b)

(c)

Fig. 4.23. (a) Scanning electron micrograph of arrested radial crack in specimen B1 where crack sections A-D are characterized by grain boundary failure. (b) and (c) Higher magnifications of crack section at C showing grain boundary failure and crack path. Tensile axis is verticle.

Fig. 5.1. Spherical particle with void cap and accompanying stress state in the neck of a tensile specimen.

Fig. 5.2. (a - d) Relevant crack configurations and associated stress intensity factors used to approximate corresponding solutions for the 3-dimensional crack geometries in (e) and (f).

Fig. 5.3. Computed elastic energy release upon formation of a spherical void in an elastic body as a function of Q . Reasonable agreement is obtained between the present energy release method (i.e. equation 5.4 with $\phi_0 = \pi$) and the corresponding Eshelby solution.

Fig. 5.4. Schematic representation of local relaxation of plastic incompatibilities between particle and matrix resulting in local elastic stresses and strains.

Fig. 5.5. Schematic representation of normalized energy difference curves versus crack angle showing factors which affect nucleation.

Fig. 5.6. Computed normalized energy difference curves versus crack angle for several particle sizes corresponding to the state of deformation at first observed void nucleation in specimens B1 and W1.

Fig. 5.7. Computed normalized energy difference curves versus crack angle for several particle sizes corresponding to the maximum state of deformation at the center of the neck in specimens B1 and W1.

Fig. 5.8. Total normal interfacial stress as a function of polar angle and particle size in specimen W1.

Fig. 5.9. Computed energy difference curves versus crack angle in specimen B1 corresponding to a particle size of $r_0 = .5\mu\text{m}$ subject to the state of maximum deformation in the neck of the specimen. The effects of Γ_W and Q are shown where $\Gamma_W^0 = 5.7 \frac{\text{J}}{\text{m}^2}$.

Fig. 5.10. Plastic zone configuration for spherical particle situated on a grain boundary.

Fig. 5.11. Relationship between unrelaxed plastic strain and applied strain as a function of particle size and grain size.

Fig. 5.12. Energy difference curves for particles on and off boundaries computed for the conditions in figure 5.7 for specimen B1.

Fig. 5.13. Energy difference curves for particles on and off boundaries computed for the conditions in figure 5.7 for specimen W1.

Fig. 5.14. Critical particle radius required for nucleation as a function of deformation for two values of Γ_W .

Fig. 5.15. Percentage of particles with voids versus deformation based on the results in figures 4.9 and 5.14, where a one-to-one correspondence between particles and voids has been assumed.

Fig. 5.16. Comparison of computed and measured areal void densities along the specimen axis with $M = 1$.

Fig. 5.17. Comparison of computed and measured areal void densities along the specimen axis with $M = 2$.

Fig. 5.18. Comparison of computed and measured areal void densities on the minimum neck section assuming uniform $\bar{\epsilon}_p$. Curves are scaled to match at $\frac{\rho}{a} = 0$.

Fig. 5.19. Comparison of computed and measured areal void densities on the minimum neck section assuming $\bar{\epsilon}_p$ varies from the center to the edge of the specimen according to the results of Norris et al. [67]. Curves are scaled to match at $\frac{\rho}{a} = 0$.

$$P = \gamma_0 \exp \left[\frac{r_0 - r}{l} \right]$$

Fig. 5.20. Cylindrical geometry used to compute tensile flow stress in the ligament between two closely spaced particles (equation 5.61).

Fig. 5.22. Ratio of constrained ligament flow stress to unconstrained matrix yield stress as a function of radial position for several ratios of particle size to particle spacing.

Figure A.1

Fig. A.2 Change of areal fraction $(A_A)_{\beta}$, mean free path λ_q , and orientation factor O.F., in tension and compression.

Fig. A.3 Change of areal fraction $(A_A)_{\beta}$, mean free path λ_{α} , and orientation factor O.F., with reduction of thickness and elongation in rolling.