Simulation of Atmospheric Turbulence

**W. C. Cliff
D. L. Hall**

January 1980

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SUMMARY

This paper develops a method for constructing an artificial temporal sequence of velocity values for simulating atmospheric turbulence. The method develops the conditional probability density distribution of velocity given that the previous value of velocity is known. A value is randomly selected from the conditional probability density which then serves as a known value for the next conditional probability density function. Continuing this procedure provides the desired simulated sequence of velocity values.

Based upon this simulation method a program is developed for the Hewlett-Packard HP-29C^(a) hand-held calculator for simulating the longitudinal component of atmospheric turbulence during neutral or strong wind conditions. The program takes 88 steps, and the only required inputs are the mean wind speed, the height above grade to be simulated, the local surface roughness, and the time desired between successive velocity values. The computer code may be easily modified for use on other comparable hand-held calculators and can obviously be modified for use on normal computing facilities.

(a) A specific product is used for example only; its use does not constitute an endorsement of the product.

TABLE OF CONTENTS

ACKNOWLEDGMENTS		iii
SUMMARY		v
LIST OF FIGURES		ix
LIST OF TABLES		ix
1.0 INTRODUCTION		1
2.0 CONCLUSIONS		3
3.0 APPROACH		5
4.0 SIMULATION OF THE LONGITUDINAL COMPONENT OF ATMOSPHERIC TURBULENCE		7
5.0 CALCULATOR PROGRAM FOR SIMULATING THE LONGITUDINAL COMPONENT OF ATMOSPHERIC TURBULENCE		11
6.0 FREQUENCY DISTRIBUTION AND CORRELATION OBTAINED FROM A SAMPLE SIMULATION		17
REFERENCES		21

LIST OF FIGURES

6.1	Frequency Distributions from Example Simulation and Theory, Normal Distribution	19
6.2	Autocorrelation Functions for Example Simulation and Theory (Negative Exponential)	20

LIST OF TABLES

5.1	Program Inputs	11
5.2	Code for Simulation of Longitudinal Component of Turbulence for the HP-29C Hand Calculator	12
5.3	Storage Register Contents	15
6.1	Frequency Distributions from Simulation and Theory	18
6.2	Autocorrelation Functions Derived from Simulated Wind History and Theory	19

1.0 INTRODUCTION

A knowledge of the interaction of turbulence with man-made structures such as wind turbines, airplanes, submarines, buildings, bridges, and power lines is essential to predict the structures' behavior and to provide the necessary criteria for adequate design. Since it is virtually impossible to provide measured records of turbulence for all conditions that a structure will encounter, methods to simulate turbulence under various conditions have been developed.^(1,2,3) Fichtl et al.⁽²⁾ provides excellent coverage of current turbulence simulation concepts and references.

The method developed in this paper uses the autocorrelation function and the conditional probability distribution of velocity given the preceding observation to simulate a time sequence of wind velocities. Most previous techniques have used the spectrum of turbulence for simulating velocity histories by appropriately filtering white noise to obtain the desired spectral characteristics. These methods require a greater number of steps than does the method presented in this paper.

For the special case of the longitudinal component of atmospheric turbulence a simple simulation program is developed that can be programmed on many available hand-held calculators. The simulation assumes that the turbulence is normally distributed and has the classical negative exponential autocorrelation. A negative exponential autocorrelation function has the property that if sample points separated by time τ have a correlation value of $\rho(\tau)$, then the correlation between points separated by time $n\tau$ is $(\rho(\tau))^n$. A specific program that provides the simulation will be presented after development of the method.

It should be noted that this method does not depend on the assumptions of normality or on the assumption of a negative exponential autocorrelation, and can be used to simulate histories with other, more complex, autocorrelation structures and with other density functions.

2.0 CONCLUSIONS

A program for simulating atmospheric turbulence whose probability density function is Gaussian is developed for use on the Hewlett-Packard HP-29C hand calculator. This program takes 88 steps and requires as input the mean wind speed, the height at which the simulation is to be made, the local surface roughness, and the time between sequential velocity samples. The simulation method may be used to generate a single velocity history at a point. The simulation method may also be used to generate a velocity history at another location coincident in time with the wind history at a given location (which may have been generated by the simulation method itself).

3.0 APPROACH

The basic approach to providing a method for simulating turbulence is to develop the conditional probability distribution of velocity, given that other velocities have been observed. Once the conditional distribution function is established, a value is selected from this distribution. This random value then becomes the desired simulated velocity, which in turn is used in selecting the next velocity value.

The conditional probability for the velocity at time $t + \tau$, given other velocities observed at previous times, is expressed as

$$P(V(t + \tau)|V(t), V(t - \tau) \dots, V(t - n\tau)) = \frac{P(V(t + \tau), V(t) \dots, V(t - n\tau))}{P(V(t), \dots V(t - n\tau))} . \quad (1)$$

The conditional density function of $V(t + \tau)$ given previous velocities, is given by

$$f(V(t + \tau)|V(t), \dots V(t - n\tau)) = \frac{f(V(t + \tau), V(t), \dots, V(t - n\tau))}{f(V(t), V(t - \tau) \dots V(t - n\tau))} . \quad (2)$$

Thus, to establish the conditional density function one is required to know the full joint density function and the joint density function of the given variables. Once these are established, the simulation of turbulence is initiated by assuming (or obtaining by other means) nominal values for $V(t)$, $V(t - \tau)$... $V(t - n\tau)$ and randomly selecting a value of $V(t + \tau)$ from the conditional density function for $V(t + \tau)$.

A suggested method for simulating turbulence in both space and time is to first simulate in time a given location and then use that time history for simulating the time history at a location spatially separated from the first location.

4.0 SIMULATION OF THE LONGITUDINAL COMPONENT OF ATMOSPHERIC TURBULENCE

The longitudinal component of atmospheric turbulence may be simulated using the conditional probability method by using the standard assumptions that the turbulence is normally distributed and that the autocorrelation has the form of a negative exponential. Since the autocorrelation is assumed to have the form of a negative exponential, $V(t + \tau)$ depends only on the previous velocity $V(t)$ and on no other velocities.⁽⁴⁾ Thus, only $f(V(t + \tau)|V(t))$ need to be considered.

The conditional probability density function of the longitudinal velocity at time $t + \tau$, $V(t + \tau)$, given the velocity at time t , $V(t)$, becomes

$$f(V(t + \tau)|V(t)) = \frac{1}{\sigma\sqrt{1 - \rho^2}} \phi\left(\frac{V(t + \tau) - \bar{V} - \rho(V(t) - \bar{V})}{\sigma\sqrt{1 - \rho^2}}\right) \quad (3)$$

where $\phi(X) = \frac{1}{\sqrt{2\pi}} e^{-\frac{X^2}{2}}$

ρ = correlation coefficient between $V(t + \tau)$ and $V(t)$

$\bar{V}$ = mean velocity

σ = the standard deviation of velocity (rms).

All quantities except $V(t + \tau)$ in equation 3 may be solved for as follows:

$\bar{V}$ = mean wind velocity at the height at which the simulation is to be performed. This value is selected as desired ($\bar{V}$ in meters/second).

τ = time between successive velocity samples. This value is selected as desired. That is, if one wants one second spacing between observations of velocity, τ is set at one second (τ in seconds).

ρ = correlation coefficient between velocity samples spaced τ seconds apart. ρ may be computed from:

$$\rho = e^{-\frac{\bar{V}_T}{L}} \quad (4)$$

where $L = 25 Z_0^{-0.4} Z^C$ (L in meters) (5)

and $C = \text{Exp}(-0.025(\ln Z_0)^2 + 0.17 \ln Z_0 - 0.8)$ (6)

Z = height (agl) at which simulation is being performed
(Z in meters)

Z_0 = local surface roughness

$Z_0 = 0.005$ m smooth surface

$= 0.05$ m moderate terrain (7)

$= 0.5$ m rough terrain

The above representation for L is consistent with Reference 5.

A somewhat simpler but more conservative formula for L is used in this study's code development:⁽⁶⁾

$$L = 25 Z^{0.35} Z_0^{-0.063} \quad (L \text{ in meters}) \quad (8)$$

σ = the standard deviation of wind speed (rms) that may be computed from

$$\sigma \approx \frac{\bar{V}}{\ln(Z/Z_0)} \quad (\text{ref. 7}) \quad (9)$$

The next step in the simulation is to develop a method of selecting a random number from a normal frequency distribution that has zero mean and unit variance. This is accomplished by selecting two random numbers, U_i and U_j , from uniform distributions and performing the following operation that results in a random variable, N_i , whose frequency distribution is Gaussian.⁽⁸⁾

$$N_i = (-2 \ln U_i)^{1/2} \cos (2\pi U_j) \quad (10)$$

Each successive, N_i , is used in the following equation to compute a new longitudinal wind velocity, V_i .

$$V_i = \sigma \sqrt{1 - \rho^2} N_i + \bar{V}(1 - \rho) + \rho V_{i-1} \quad (11)$$

where V_{i-1} is the previous velocity obtained in the simulation.

To start a simulation the investigator may use any initial velocity desired, including the hourly mean value. Any standard random number generation scheme may be used to obtain the required U_i and U_j values.

A calculator program, based upon Equation 11, for simulating the longitudinal component is presented in the following section.

5.0 CALCULATOR PROGRAM FOR SIMULATING THE LONGITUDINAL COMPONENT OF ATMOSPHERIC TURBULENCE

This section contains a listing of a program to simulate the longitudinal component of turbulence. The required input information is listed in Table 5.1. The user enters these inputs by storing the values in the identified registers.

TABLE 5.1. Program Inputs

<u>Storage Register</u>	<u>Content</u>
1	τ = time desired between successive velocity samples
2	Z = the height above grade level at which the simulated wind history is desired
3	Z_0 = the local surface roughness Z_0 = 0.005 meters (smooth terrain) Z_0 = 0.05 meters (medium terrain) Z_0 = 0.5 meters (rough terrain)
4	$\bar{V}$ = the mean wind speed at the height at which the simulation is performed (the hourly average wind speed) in meters/second.

To run the program the operator presses GSB-1 for the first value of wind speed, then presses R/S for each successive value.

The program is listed in Table 5.2; the storage registers used by the program are listed in Table 5.3.

TABLE 5.2. Code for Simulation of Longitudinal Component of Turbulence for the HP-29C Hand Calculator

Step	Code		
1	15	13	01
2		24	2
3			73
4			03
5			05
6	14		64
7			02
8			05
9			61
10	24		03
11			73
12			00
13			06
14			03
15	14		64
16			71
17	23		08
18	15		74
19	24		01
20			61
21	24		04
22	23		05
23			61
24			32
25	15		42
26	23		.5
27	15		63
28			32
29			01
30			51
31	14		63

Computes L_u

Compute ρ

TABLE 5.2. (continued)

Step	Code		
32	24	04	
33		61	
34	24	02	
35	24	03	Computes
36		71	$\sigma\sqrt{1 - \rho^2}$
37	14	42	
38		71	
39	23	.4	
40		05	
41	14	63	Enters seed No.
42	15	62	
43	23	07	
44		09	
45		09	Stores 997 in
46		07	register 9
47	23	09	
48	15	13 02	
49		24 07	
50	24	09	Generates first
51		61	random No.
52	15	62	
53	23	06	
54	24	09	
55		61	Generates second
56	15	62	random No.
57	23	07	
58	15	73	
59		61	
60		02	
61		61	

TABLE 5.2. (continued)

Step	Code		
62	15	33	
63	14	53	
64	23	.0	
65	24	06	
66	14	42	Generates
67		02	Gaussian
68		61	Random No.
69		32	
70	14	63	
71	24	.0	
72		61	
73	23	.3	
74	24	05	
75	24	04	
76		41	
77	24	.5	
78		61	Calculates
79	24	04	new velocity
80		51	
81	24	.3	
82	24	.4	
83		61	
84		51	
85	23	05	
86		74	Displays new velocity
87	13	02	
88	15	12	

TABLE 5.3. Storage Register Contents^(a)

Storage Register	Contents
0	Not used
1	Time between samples (set prior to operation) = τ (second)
2	Height above ground for simulation = z (meters)
3	Surface roughness for local topography = z_0 (meters)
4	Mean wind speed at height of simulation (meters/second)
5	Used internally (becomes $V_{t+\tau}$)
6	Used internally to program (random no. 1, U_i)
7	Used internally to program (random no. 2, U_j)
8	Used internally to program = Lu
9	Used internally to program
.0	Used internally to program
.1	Not used
.2	Not used
.3	Used internally to program = N_i
.4	Used internally to program = $\overline{U} \sqrt{1 - \rho^2 / \ln(z/z_0)}$
.5	Used internally to program = ρ

(a) To run program press GSB-1 for first value; then press R/S for each successive value of wind speed.

6.0 FREQUENCY DISTRIBUTION AND CORRELATION OBTAINED FROM A SAMPLE SIMULATION

The simulation code presented in the text was used to generate a typical simulation and the statistical distributions acquired from the simulation.

The required inputs to the program were:

- $\tau = 1$ second between samples
- $Z = 25$ meters = height of simulated measurement
- $Z_0 = 0.05$ meters (medium terrain)
- $\bar{V} = 10$ meters/second = mean wind speed

Ten thousand sample points were generated from which the following statistics were computed:

- frequency distribution of longitudinal wind speed
- Autocorrelation function of longitudinal wind speed

Table 6.1 lists the frequency distribution of longitudinal wind speed obtained from the values generated by the simulation program along with the expected values calculated from a normal distribution. Figure 6.1 is a plot of the generated frequency distribution and the corresponding normal distribution. The two curves are extremely close in magnitude except that the simulation gave a few more large excursions than would be expected from the normal distribution. This is somewhat helpful, however, in that larger than normal excursions are expected in atmospheric turbulence.

TABLE 6.1. Frequency Distributions from Simulation and Theory

<u>V</u> <u>(m/s)</u>	<u>P(V)</u> _{sim}	<u>P(V)</u> _{normal}
3	0.000	0.000
4	0.000	0.000
5	0.000	0.002
6	0.008	0.011
7	0.048	0.044
8	0.121	0.115
9	0.169	0.204
10	0.242	0.248
11	0.210	0.204
12	0.139	0.115
13	0.038	0.044
14	0.018	0.011
15	0.004	0.002
16	0.001	0.000
17	0.002	0.000
18	0.000	0.000
19	0.000	0.000
20	0.000	0.000

Table 6.2 gives the tabular values of the autocorrelation function obtained from the simulated wind history and the expected negative exponential autocorrelation function.

Figure 6.2 provides a comparison of the two correlations. The two correlations are in excellent agreement, indicating that the simulation develops realistic correlation and spectral information.

FIGURE 6.1. Frequency Distributions from Example Simulation and Theory, Normal Distribution

TABLE 6.2. Autocorrelation Functions Derived from Simulated Wind History and Theory

Time Delay	Autocorrelation (Simulation)	Autocorrelation (Expected)
1	0.898	0.898
2	0.807	0.807
3	0.723	0.725
4	0.650	0.651
5	0.586	0.585
6	0.532	0.525
7	0.480	0.472
8	0.432	0.424
9	0.388	0.381
10	0.348	0.342
11	0.312	0.307
12	0.284	0.276
13	0.260	0.248
14	0.238	0.222
15	0.209	0.200
16	0.182	0.179
17	0.160	0.161
18	0.141	0.145
19	0.125	0.130
20	0.108	0.117

FIGURE 6.2. Autocorrelation Functions for Example Simulation and Theory (Negative Exponential)

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