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FUSION-PRODUCT TRANSPORT IN AXISYMMETRIC TOKAMAKS:
LOSSES AND THERMALIZATION

BY

LEE MIZENER HIVELY

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FUSION-PRODUCT TRANSPORT IN AXISYMMETRIC TOKAMAKS: LOSSES AND THERMALIZATION

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High-energy fusion-product losses from an axisymmetric tokamak plasma are studied. Prompt-escape loss fluxes (i.e. prior to slowing down) are calculated including the non-separable dependence of flux as a function of poloidal angle and local angle-of-incidence at the first wall. The flux is strongly peaked at near grazing incidence, suggesting that blistering-injected impurities can reduce reactor burn times by 50-70%. Non-prompt losses (during slowing down, without anomalous effects) are also calculated to be $\leq 0.1 \times (\text{prompt loss}) \times \sqrt{Z_{\text{eff}}}$ and consequently make a small contribution to impurity production. Sensitivity studies show the plasma-wall separation to be the strongest factor controlling prompt losses. Modest increases ($\leq 20\%$) in wall radius (plasma radius fixed) of a device, such as the ORNL-EPR, are found to reduce blistering-injected impurities to a level which should no longer limit the burn time.

Fusion-product (fp) thermalization and heating are calculated assuming classical slowing down. The present analytical model describes fast ion orbits and their distribution function in realistic, high- β , non-circular tokamak equilibria. First-orbit losses, trapping effects, and slowing-down drifts are also treated. By solving a 3-D (+ time) PDE, it is possible to obtain an invariant of

the slowing down process: $\mu/E = (\text{magnetic moment})/\text{energy} = \text{constant}$, and explicit expressions for the slowing-down drifts. Large banana-width effects give rise to a net co-going alpha particle current. The large banana-width orbits smear the energy deposition over large regions of the plasma. This causes the flux-surface-averaged ion heating rates to be 10-20% below in-situ rates on axis, but enhances the edge heating ≥ 10 -fold over in-situ deposition. While this result implies reduced alpha "ash" accumulation on axis, the reduced heating rate makes start-up and maintenance of ignition more difficult.

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I dedicate this thesis to the glory of God as revealed through Jesus Christ, with thanks to the many Christians who encouraged me and upheld me in prayer. My God-fearing parents, Otto and Mildred Hively, were unfailingly supportive; I thank them above all others for their steadfastness.

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1. INTRODUCTION

High-energy fusion-product (fp) transport (e.g., alpha particle transport in D-T plasmas) is a central issue in fusion reactor development. Important effects dependent on fp transport include: the fp heating profile, a crucial factor in ignited operation [1]; possible fp-induced instabilities [2]; the high-energy fp thermalization profile and subsequent "ash" build-up [3]; fp bombardment of the first wall and the resulting plasma contamination [4].

1.1 Fusion-Product Losses

Chapters 1-7 are concerned with fp wall bombardment and focus on two types of charged, high-energy fp losses from axisymmetric tokamak plasmas: prompt and non-prompt losses. The dominant means of fp escape is due to those particles whose birth orbits immediately intersect the wall, designated *prompt* losses. Some leakage also occurs as MeV fp's slow down in the plasma and pitch-angle-scatter or diffuse across the "unstable" stagnation orbit boundary [5]. This leakage is termed *non-prompt*. Such losses are important because high-energy bombardment causes wall erosion by blistering and sputtering. In addition to potentially limiting the wall lifetime, such radiation damage injects wall material into the fusion plasma and can cause excessive radiation emission.

In Chapter 2, the derivation of the guiding center motion is discussed and orbit characteristics studied. In Chapter 3, the generalized technique for finding the loss regions is presented; example calculations for loss regions in $(v_{\parallel}, v_{\perp})$ space are presented for TFTR.* In Chapter 4, the loss boundaries are used to calculate particle bombardment of the first wall as a function of poloidal angle; sample calculations are given for PLT,* TFTR, ORNL-EPR, and UMMAK I. To confirm the validity of the model, Appendix A compares results from this work [4] to recent calculations by Ohnishi et al. [6], Kolesnichenko et al. [7], Petrie and Miley [8], and Goldston [9].

The studies of Chapters 2-4 [4] show that typical peak-to-average ratios for alpha wall fluxes range from 1.4 to 2.3 with average fluxes of $\sim 10^{14}$ - 10^{15} alphas/m²-s. The resulting localized blistering [10-11] potentially represents a serious source of plasma impurities, as well as a significant wall erosion mechanism. This realization motivated further study of these effects. Since blistering and sputtering are sensitive to the incident velocity direction as well as the flux magnitude, the fp flux versus incident velocity direction at the wall is evaluated in Chapter 5. To better understand how to control fp bombardment, we consider the sensitivity of fp wall loading to various device parameters in Chapter 6. However, other losses, induced by toroidal field ripple, become dominant in EPR and power reactor-size plasmas and have been considered by others [8,12].

*See Table 2.1 for machine parameters.

Even so, axisymmetric losses are of interest for large reactors, because they form a lower bound, i.e. a theoretical limit for designers.

As well-confined fp's slow down in the Maxwellian background plasma, collisional pitch-angle-scattering into the velocity-space loss-region causes losses to the wall. Chapter 7 describes a crude model of this effect which consists of analytically solving the small banana-width, bounce-averaged, 2-D Fokker-Planck equation. Neglecting instability-induced losses, the relative contribution of these non-prompt losses is small. Still, the study of these losses is important as a "base case" comparison with possible instability-induced losses in future experiments, i.e. as a diagnostic for studying the slowing down mechanism(s). This formulation has been further extended in the following chapters, to model large banana-width orbits in non-circular tokamaks.

1.2 Fusion-Product Slowing-Down and Heating

Plasma heating by energetic, charged fusion products is a crucial issue for obtaining fusion reactor conditions. The evolution of alpha particle density, momentum, kinetic energy and heat flux is important in the study of start-up and ignition scenarios, as well as steady-state operation. Since fast ions have large banana-width guiding-center orbits in tokamaks, the usual locally-defined particle distribution function loses its usefulness. Accordingly, the fast ion collisional slowing-down problem has been formulated in a 3-dimensional space (two velocity-space coordinates and one spatial

coordinate) which describes these orbits in an axisymmetric, non-circular tokamak.

The present method consists of bounce-averaging the Fokker-Planck equation after the variables have been transformed to the three constants-of-motion (COM) which characterize the orbit of a collisionless particle [13]. The distribution function and its moments are found and since a time dependent 3-D equation is solved, it is also possible to derive scaling laws.

In Chapter 8, the theory of Rome and Peng [13] applicable to the present problem is summarized. Next, in Chapter 9, the formulation of the bounce-averaged Fokker-Planck equation is described and the slowing-down problem modeled. In Chapter 10 the source functions for 3.5-MeV alphas are calculated for sample cases, and the resulting implications discussed. Chapter 11 deals with the solution of the slowing-down problem, including both analytical and numerical results. Finally, Chapter 12 discusses the results of this work and presents the conclusions.

2. GUIDING-CENTER MOTION IN TOKAMAKS

Basic studies of neoclassical transport have been done by a number of workers [14-18]. These papers, as well as in Chapters 2-7 of this work, make seven basic assumptions: 1) the tokamak fields are axisymmetric, i.e. magnetic field ripple is neglected; 2) the particle gyroradius, r_g , is much smaller than the scale size of the magnetic field; 3) diamagnetic effects due to finite plasma pressure are negligible; 4) the plasma is collisionless, implying that no slowing down or pitch angle scattering occurs during one banana orbit; 5) the toroidal current density is a function of poloidal radius only; 6) the toroidal magnetic field is much larger than the poloidal field; 7) electric field effects on the orbits are negligible. Petrie [19-21] has analyzed the electric field effects due to fusion-product losses. His results show that the resulting $\vec{E} \times \vec{B}$ drift does not affect the fusion-product orbits, and that electrical potential (ϕ) retardation of fusion-product losses is negligible, i.e. $e\phi \leq kT \ll E$.

2.1 Previous Work

Prior work has concentrated on plasma transport [1] and gross features of high-energy ion losses [6,15-18]. The present work is the first to calculate wall loading fluxes [10]. In developing the techniques for this calculation, modifications and generalizations of the theory

were necessary. In particular, this work eliminates the small banana width approximation of Galeev and Sagdeev [14], and excludes the numerical procedures of Stringer [15], McAlees [16], Tenney [17] and Ohnishi et al. [6]. The latter methods require much computer time and would be difficult to extend to wall loading calculations. Rome et al. [18] calculate the velocity-space loss region for injected ions, but do not apply their results to compute wall loading. Specifically, they extend McAlees' work to analytically determine the pitch angle corresponding to the orbit tangent to a limiter at $r = a$ (the toroidal coordinates are defined in Fig. 2.1), and also derive an expression to numerically compute the pitch angle for the "fattest" banana orbit. Their results hold only for particles born in the equatorial plane with $R > R_0$. To extend the loss analysis over the whole plasma, Rome et al. note that the orbit equation can be used to map other points unto the corresponding points for $z = 0$, $R \geq R_0$; however, as they point out, this procedure is tedious in practice.

2.2 Large Banana-Width Orbits

The present study employs a more general analytic formulation to calculate orbits incident on a limiter at $r = a$ and/or a wall at $r > a$, from an arbitrary birth point. Our loss region algorithm extends that of Ref. 18 to include the effects of stagnation-orbits for $R > R_0$, orbits grazingly incident on the wall in the mid-plane for $R < R_0$ and also grazingly incident on the wall out of the mid-plane, $\chi \neq 0$ orbits for $R > R_0$, and elimination of disconnected orbits from the set of valid solutions (see Chapter 3). These effects are especially

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Figure 2.1 The toroidal coordinate geometry is shown here with $\hat{\phi}$ in the toroidal direction. The origin of the primed coordinates is located at the center of the wall cross section.

important for high-energy fusion products ($E > 1$ MeV) with birth points near the plasma edge (e.g. ORNL-EPR and UWMAK I) and can introduce errors $> 10\%$ in loss fraction (see Chapter 4).

The basic properties of banana orbits have been reviewed by Kadomtsev and Pogutse [22], and will not be repeated here. Under the assumptions stated at the beginning of this chapter, the Lagrangian of motion for a particle of mass, m , and charge, Ze , is:

$$L = \frac{m}{2} (\dot{r}^2 + r^2 \dot{\theta}^2 + R^2 \dot{\phi}^2) + Ze (\dot{r}A_r + r\dot{\theta}A_\theta + R\dot{\phi}A_\phi), \quad (2.1)$$

where electric fields are assumed to be negligible. From the axisymmetry condition, $\partial L / \partial \phi = 0$, together with $B_\theta \ll B_\phi$, the well-known canonical angular momentum is obtained: $P_\phi = mR_0 h v_\parallel + ZeR_0 h A_\phi$

where $h = R/R_0$, $v_\parallel = v \cos \chi$, and χ = the angle between $\vec{v}$ and $\vec{B}$. From the θ -component of $\vec{B} = \nabla \times \vec{A}$, one obtains $hA_\phi = - \int_0^r B_\theta(r) dr$, where the Knorr form [23] for the magnetic field is used: $\vec{B} = h^{-1} (\hat{\theta} B_\theta(r) + \hat{\phi} B_\phi)$, $B_\phi = \text{constant}$. The resulting expression for P_ϕ is:

$$P_\phi = mR_0 h v_\parallel - Ze\psi \quad (2.2)$$

where $\psi = R_0 \int_0^r B_\theta(r) dr$. Assuming a toroidal electric-current density of the form

$$J = J_0 \left[1 - \gamma \left(\frac{r}{a} \right)^N \right]^M, \quad (2.3)$$

the following expression for poloidal flux function, $\psi(r)$, is obtained with N real and M a positive integer:

$$\psi(r) = \begin{cases} \frac{\mu_0 I R_0}{2\pi} \sum_{j=0}^M \frac{(-\gamma)^j (r/a)^{jN+2}}{j!(M-j)!(jN+2)^2} / \sum_{j=0}^M \frac{(-\gamma)^j}{j!(M-j)!(jN+2)}, & r \leq a \quad (2.4) \\ \frac{\mu_0 I R_0}{2\pi} \left[\ln \frac{r}{a} + \sum_{j=0}^M \frac{(-\gamma)^j}{j!(M-j)!(jN+2)^2} / \sum_{j=0}^M \frac{(-\gamma)^j}{j!(M-j)!(jN+2)} \right], & r \geq a \quad (2.5) \end{cases}$$

where these equations are in rationalized mks units.

To obtain the equation of motion, the invariants $\mu = W_{\perp}/B$ and $E = W_{\perp} + \frac{1}{2} m v_{\parallel}^2$ are combined to obtain $v_{\parallel} = \pm \sqrt{2(E - \mu B_0)/h}/m$. As first done by Meka [24], this last expression is then substituted into (2.2) to obtain the orbital equation:

$$\left. \begin{aligned} x &= R_0 \left[\frac{1}{2} h_i \sin^2 \chi_i + \frac{1}{2} \sqrt{h_i^2 \sin^4 \chi_i + 4(P_{\phi} + Ze\psi(r))^2 / (mvR_0)^2 - 1} \right], \\ z &= \pm \sqrt{r^2 - x^2}, \end{aligned} \right\} \quad (2.6)$$

where the subscript "i" refers to the initial point of the orbit.

Equation (2.6) is the projection of the particle guiding center motion onto the x-z plane (see Fig. 2.1), and arises from toroidal axisymmetry (i.e. the ϕ -coordinate is ignorable.) It provides an especially convenient, analytical expression for generating and analyzing particle orbits.

Two useful properties of this orbital equation should be noted. First, express (2.6) as an equation for $x = x(r, \chi_i, B_{\phi}, B_{\theta})$. Then from (2.2) and (2.4)-(2.5), the symmetries with respect to the last three variables are obtained; namely:

$$x(r, \chi_1, \theta_\phi, \theta_\theta) = \begin{cases} x(r, \pi - \chi_1, -\theta_\phi, \theta_\theta) & (2.7a) \\ x(r, \pi - \chi_1, \theta_\phi, -\theta_\theta) & (2.7b) \\ x(r, \chi_1, -\theta_\phi, -\theta_\theta) & (2.7c) \end{cases}$$

Secondly, consider the ratio of ψ/mvR_0 for two different particles at a given poloidal radius and identical reactor parameters. From (2.4)-(2.5), we find:

$$\frac{\psi_1/m_1 v_1 R_0}{\psi_2/m_2 v_2 R_0} = \frac{Z_1}{Z_2} \sqrt{\frac{m_2 E_2}{m_1 E_1}} = \frac{Z_1}{Z_2} \frac{P_2}{P_1} \quad (2.8)$$

where P_i is the particle momentum. Combining (2.8) and (2.6) implies that identical orbits result when,

$$(Z_1/Z_2)(P_2/P_1) = 1. \quad (2.9)$$

Thus, identical fusion product orbits occur for two equally charged fusion products from the same reaction (e.g. for T and P from $D(d,p)T$), since then $P_1 = P_2$. Another consequence of this last condition is that protons and alphas of the same energy have essentially the same orbits. These properties will be used in Chapter 4, in discussing wall-loading profiles for various fusion products.

To calculate the loss region in velocity space, it is necessary to study the drift orbit properties. Figures 2.2 and 2.3 show orbits for $D\text{-He}^3$ fusion products (3.7-MeV α and 14.7-MeV p) in the TFTR weak compression (TFTR I) plasma (see Table 2.1 for machine parameters). $D\text{-He}^3$ is of interest because it avoids neutron- and tritium-radiation

Table 2.1: Tokamak Parameters

Machine	PLT[26]**	TFTR I[27] ⁺	TFTR II[27]	ORNL-EPR[28] [†]	UWMAK I[29] [®]
B_0 (T)	4.6	5.2	5.2	4.8	3.82
I (MA)	1	1	2.5	7.2	21
R_0 (m)	1.32	2.48	2.48	6.75	13.00
a (m)	0.45	0.54	0.85	2.25	5.00
r_w (m)	0.48	1.10	1.10	2.25	5.50
χ_0 (m)	0	0.17	0.17	0	0
T_{i0} (keV)	5	10	10	22.5*	11.1
T_i/T_{i0}	$1-.8(r/a)^2$	$1-.8(r/a)^2$	$1-.8(r/a)^2$	$1-(r/a)^2$	1
n_{i0} (m ⁻³)	1×10^{20}	1.6×10^{20}	1.6×10^{20}	8×10^{19}	1.2×10^{20}
n_i/n_{i0}	$1-(r/a)^2$	$1-(r/a)^2$	$1-(r/a)^2$	$[1-(r/a)^2]^{1/2}$	$[1-.99(r/a)^2]^{1/2}$
J/J_0	$1-(r/a)^2$	$1-(r/a)^2$	$1-(r/a)^2$	$1-(r/a)^2$	1
$q(a)$	3.53	3.06	3.03	2.50	1.75

* An ignited system is assumed, having a thermal power of 420 MW.

** Princeton Large Torus

⁺ Tokamak Fusion Test Reactor

[†] Oak Ridge National Laboratory-Engineering Power Reactor

[®] University of Wisconsin Tokamak-Design I

Figure 2.2 Neoclassical guiding-center orbits of a 3.7-MeV α , for various birth points on the x-axis (50, 30, -10 cm) in the TFTR weak compression plasma. The origin is positioned at the plasma column center ($R_0 = 248$ cm, $a \approx 54$ cm). The dashed semi-circle in each graph indicates the first wall position ($r_w = 110$ cm, $x_0 = 17$ cm). The number on each curve is the pitch angle at birth.

Figure 2.3 Neoclassical guiding-center orbits of a 14.7-MeV p, for various birth points on the x-axis (30,10,-30 cm) in the TFTR weak compression plasma. The origin is positioned at the plasma column center ($R_0 = 248$ cm, $a = 54$ cm). The number on each curve is the pitch angle at birth.

hazards in proposed fusion-product detection experiments [25]. Simultaneously, the alpha behavior is quite similar to the 3.5-MeV α from D-T. Note that only the $z \geq 0$ half of the orbits are plotted because (2.6) indicates symmetry in $\pm z$. The first wall position is indicated by the dashed line.

The details of the loss boundaries are considered next. For the 3.7-MeV α , the lower loss boundary in pitch-angle space, $\chi_{\min}$, intersects the wall at grazing incidence. The upper loss boundary, $\chi_{\max}$, corresponds to the "fattest" banana in Figs. 2.2b-2.2c, and to $\chi_{\max} = 180^\circ$ in Fig. 2.2a. The "fattest" banana is what Tenney [17] calls a stagnation orbit, and the point where this orbit touches the x-axis ($dz/dx = 0$), Tenney terms the stagnation point (Rome, et al. [18] refer to this point as the pinch point). At the stagnation point, x_s , the drift motion in $\hat{z}$ slows to zero, then reverses (stagnates) due to cancellation of the motion along the flux surface with the vertical drift (arising from ∇B and curvature effects). The point, x_s , is really an "unstable" stagnation point [30] since the particle takes an infinite time to drift there. This is to be distinguished from a "stable" stagnation point, which also has zero net vertical drift and which corresponds to the "point" orbit discussed in Chapter 3.

Additional features of these orbits are illustrated in Fig. 2.3. Particularly noteworthy are the stagnation orbits which occur for $R > R_0$ (see Figs. 2.3a-3b), caused by the same cancellation of vertical drift with motion along the flux surface. The existence of

unstable stagnation points for both $R < R_0$ and $R > R_0$ is interesting and has been discussed by Goldston [30], who elucidated the features of such orbits for fast, neutral-beam-injected ions in ATC. The $R > R_0$ unstable stagnation orbits for the 14.7-MeV p correspond to the lower loss boundary, $x_{\min}$. Also, in Fig. 2.3c is an instance in which $x_{\min} = 0$; and in Fig. 2.3a is a case for which $x_{\max} = 180^\circ$. Not shown in these figures are a small class of orbits which intersect the wall at grazing incidence for $R < R_0$, corresponding to $x_{\max}$.

Another feature of the $R < R_0$ stagnation orbits can be seen in the sequence of Figs. 2.2b-2.2c and 2.3b-2.3c: as the major radius of the initial point ($R_i = R_0 + x_i$) decreases, the major radius of the unstable stagnation point ($R_s = R_0 + x_s$) increases. Consequently, there is a critical position ($R_c^- = R_0 + x_c^-$) for which $x_i - x_s \rightarrow 0$. This situation corresponds to the orbit loop between the initial and stagnation points shrinking to a point as $x_i \rightarrow x_c^-$ for $x_i > x_c^-$. There is an analogous limit for $R > R_0$ stagnation orbits, since for these orbits R_s decreases as R_i increases; the corresponding critical position is $R_c^+ = R_0 + x_c^+$. For birth points on the x-axis at $(x_i, 0)$, outside the interval $x_c^- < x_i < x_c^+$, x_s coincides with x_i . Although the orbit loop, from birth to stagnation, shrinks to a point in this case, the "fattest banana" portion of the orbit can still carry the fast ion to the wall.

Finally there is a very specialized case of the stagnation orbit, illustrated in Fig. 2.4, for a 3.52-MeV α in TFTR I with $J/J_0 = 1 - (r/a)^{0.1}$.

Figure 2.4 Stagnation drift orbits of a 3.52-MeV α in the TFTR-I plasma with $J/J_0 = 1 - (r/a)^2$, for birth points on the x-axis at 8 cm (Fig. 2.4a) and 6.34 cm (Fig. 2.4b). A double stagnation orbit is shown in Fig. 2.4b.

In Fig. 2.4a, both the upper and lower loss boundaries are stagnation orbits for a birth point at $x_i = 8$ cm on the x-axis. As the x-value of the birth point decreases to $x_i = 6.34$ cm in Fig. 2.4b, the two orbits coalesce into a double stagnation orbit. For $x_i < 6.34$ cm, the $R < R_0$ stagnation orbit becomes the "fattest" banana, and the $R > R_0$ stagnation orbit becomes a counter-circulating orbit.

Fig. 2.4a suggests an interesting possibility for high-energy ion beam injection [31]. Neutral beam injection can be replaced by ion injection into the spatial and velocity regions bounded by a pair of stagnation orbits, like those of Fig. 2.4a. The ion injectors can be coupled to the tokamak via a bundle divertor. The ions can then enter the plasma along orbits corresponding to $x_2 < x < x_1$. It would be strongly advantageous to inject close to the outer stagnation orbit, $x = x_1$, for two reasons. First, any ions near the inner stagnation orbit, $x = x_2$, can easily charge exchange with neutrals near the plasma edge while moving slowly through the outboard stagnation point at $x = x_S^+$. Second, ions near the outer stagnation orbit will spend much time near the inboard stagnation point ($x = x_S^-$), and thus deposit more energy at the plasma core than at the plasma edge. The ions can be trapped in the plasma by pitch-angle scattering near the inboard stagnation point. Ions can also be pitch-angle scattered into loss orbits, but these orbits are well-defined, so lost particles might be recovered and re-injected. Although injection along stagnation orbits is a difficult problem, the advantage of ion injection offers a strong motivation for considering such a technique. While this is an intriguing problem for more study, it will not be discussed further.

3. LOSS-REGION BOUNDARIES

Having studied the drift orbit properties in Chapter 2, the determination of the loss region boundaries is next.

3.1 Grazing Orbit

First to be considered are orbits initiating at $(x_i, z_i) \leftrightarrow (r_i, \theta_i)$ and intersecting the wall at a position $(x_w, z_w) \leftrightarrow (r_w, \theta_w)$. Evaluating P_ϕ for this initial condition and substituting into (2.6) yields a quadratic in $\cos\chi$ which has the solution:

$$\cos\chi = \chi_{wi} \left[1 - \sqrt{(R_w/R_i)(1 - \chi_{wi}^2)} \right] \quad (3.1)$$

$$\text{where } \chi_{wi} = \frac{Ze}{mv} \left(\frac{\psi_w - \psi_i}{\chi_w - \chi_i} \right),$$

$$\text{and } \psi_w \equiv \psi(r_w), \psi_i \equiv \psi(r_i).$$

Real solutions require $|\chi_{wi}| \geq 1$ and $|\cos\chi| \leq 1$. Orbits grazingly incident on a circular wall, in the tokamak mid-plane are easily determined from (3.1). More generally, (3.1) can be used to obtain orbits incident on the wall out of the mid-plane by numerically finding a local minimum (maximum) in $\cos\chi$ versus poloidal angle, θ' , of the wall-orbit intersection for the upper (lower) loss boundary. For example, in TFTR, there is a class of guiding center orbits (0.15 - to 1-MeV α for $a = 85$ cm, $I = 2.5$ MA) which intersect the wall three times. The corresponding $\cos\chi$ vs θ' plot is shown in Fig. 3.1;

Figure 3.1 Plot of $\cos \chi$ versus poloidal angle, θ' , of wall-orbit intersections for a 1-MeV α in TFTR ($I = 2.5$ MA, $a = 85$ cm), having a birth position on the x-axis at $x_i = 72$ cm.

note that for $\cos\chi_1 < \cos\chi < \cos\chi_{\max}$, there are three θ' -values for one value of $\cos\chi$, corresponding to three wall-orbit intersections. However, the differential loss fraction, f_ℓ , for the interval Δ , is negligible ($f_\ell = \Delta/2 \leq .5\%$) so the loss boundaries are the maximum (minimum) in $\cos\chi$ as shown in Figure 3.1.

While the solution to (3.1) may satisfy the above mathematical conditions, it may also correspond to a disconnected branch of the orbit hitting the wall (see Fig. 3.2). It is necessary to test for this condition by moving along the orbit, from its birth point to the wall-orbit intersection position. For a valid intersection, the solution to (2.6) must be real for every r -value in the range, $r_i \leq r \leq r_w$. If a disconnected region does exist, i.e. no real solution to (2.6), then the grazing orbit solution must be rejected.

3.2 Stagnation Orbit

If the solution for a grazing orbit fails any of the above tests, the next loss boundary to consider is a stagnation orbit. At the unstable stagnation point $(x_s, 0)$, the orbit is characterized [18] by $dz/dx = 0$, or equivalently $dh/dr = \kappa/R_0$, where $\kappa = \text{sgn}(x_s)$. By evaluating P_ϕ at the unstable stagnation point (pitch angle = χ_s) and substituting into (2.6), a quadratic in $\cos\chi_s$ is obtained which has the solution:

$$\cos\chi_s = v_s \left(1 - \sqrt{1 - v_s^{-2}} \right), \quad v_s = \frac{Ze\kappa\psi'_s}{mv}, \quad (3.2)$$

where $\psi'_s \equiv d\psi/dr$ at $r = |x_s|$. A second equation for $\cos\chi_s$ is obtained

Figure 3.2 An example of a disconnected guiding-center orbit which grazingly intersects the TFTR wall.

from (3.1) for a particle initiating at $(x_s, 0) \leftrightarrow (r_s, \text{Arccos}(\text{sgn}(x_s)))$ with pitch angle, χ_s , and moving to $(x_i, z_i) \leftrightarrow (r_i, \theta_i)$; equating the result to (3.2) gives:

$$x_{si} \left[1 - \sqrt{(R_i/R_s)(1-x_{si}^{-2})} \right] - v_s \left(1 - \sqrt{1-v_s^{-2}} \right) \equiv G(x_s) = 0 \quad (3.3)$$

This last equation must be solved numerically for $r_s = |x_s|$, given the initial position. From symmetry, the pitch angle at initiation is:

$$\cos \chi = x_{si} \left[1 - \sqrt{(R_s/R_i)(1-x_{si}^{-2})} \right]. \quad (3.4)$$

Real solutions to (3.3) and (3.4) require $|x_{si}| \geq 1$, $|\cos \chi_s| \leq 1$ and $|v_s| \geq 1$.

A special case occurs when the stagnation orbit solutions are desired for an initial point $(x_i, 0)$ lying outside $x_c^- < x < x_c^+$. The corresponding pitch angle is obtained from (3.2) in the limit as

$x_s \rightarrow x_i$:

$$\cos \chi = v_i \left(1 - \sqrt{1-v_i^{-2}} \right), \quad v_i = \frac{Ze\kappa\psi_i}{mv} \quad (3.5)$$

As before $|v_i| \geq 1$ for real solutions to (3.5). More generally (3.5) determines the pitch angle for any orbit which is a point on the x-axis; for example, in Fig. 2.2b, the transition from $\chi = 70$ to $\chi = 90$ corresponds to a "point" orbit with $\chi = 80.51^\circ$. For birth points on the x-axis, with $x_i < x_c^-$ or $x_i > x_c^+$, the "fattest" banana (last trapped orbit) corresponds to a stagnation orbit, for which $x_s = x_i$, with a pitch angle determined by (3.5). A particle born at

$(x_C^\pm, 0)$ gyrates about that point until a perturbation (due to finite gyroradius effects or collisions) causes the particle to leave the x-axis, along a guiding-center path determined by (2.6). The values of $x_C^\pm$ are found by solving $dG/dx = 0$ at $x_s = x_i$, from (3.3); typical values of $x_C^\pm$ for TFTR are listed in Table 3.1. A useful property of these critical positions, $x_C^\pm$, is that the unstable stagnation point, x_s , occurs outside the interval $x_C^- \leq x \leq x_C^+$; this realization speeds the numerical search for x_s in (3.3). As with the grazing orbits, it is necessary to move along a stagnation orbit, beginning at the birth point, to assure that the path intersects the wall.

Table 3.1: Values of $x_C^\pm$ in TFTR I

Particle	$x_C^-(\text{cm})$	$x_C^+(\text{cm})$
3.52 MeV α	20.7	109.5
3.7 MeV α	-21.1	106.1
1.0 MeV α	-13.4	>127
14.7 MeV p	-33.8	51.2
3.0 MeV p	-19.6	121.0

3.3 Orbits with $\chi = 0$ (180°)

The final loss boundary to consider is $\cos\chi = \text{sgn}(x_i)$, i.e. $\chi = 0$ (180°), even if a valid stagnation orbit-wall intersection occurs. As before, the orbit must be traced from the birth point to the wall to verify intersection. If only a stagnation orbit-wall intersection exists, this is the appropriate loss boundary; likewise, if only a valid $\chi = 0$ (180°) intersection occurs. If both are valid intersections,

the latter is the correct loss boundary. Examples of these last two cases are shown in Fig. 3.3 for a 0.82-MeV He^3 in TFTR ($I=1\text{MA}$, $a=85\text{ cm}$).

3.4 Summary of Boundaries

Table 3.2 summarizes the various loss boundaries and the associated equations, used in calculating the loss region in velocity space. By solving an analytical form for the guiding-center drift-orbit equation, it is possible to find the pitch angle, χ , corresponding to an orbit (defined as type A) which has an arbitrary birth position and intersects the wall at any point. This includes a general form for χ , corresponding to x-type stagnation orbits (type B orbits), occurring for both $R < R_0$ and $R > R_0$. Study of the drift orbits [4,22] also shows that particles born near the inner (outer) plasma edge can have a minimum (maximum) loss orbit (type C orbit), for $\chi = 0$ (180°).

Table 3.2: Summary of Loss Boundaries

Type		Equation(s)	Figure(s) for Examples
A	Grazing	3.1	2.2
B	Stagnation	3.2-3.4	2.2-4
	Point	3.5	-
C	$\chi = 0, 180^\circ$	-	2.3a,3c

These orbit types are used to define the boundaries of the velocity-space loss-region for fp's escaping to the wall from any given birth point. If a type A orbit exists and is continuously connected between

Figure 3.3 Stagnation and $\chi = 180^\circ$ guiding center orbits for a 0.82-MeV He^3 in TFTR ($I = 1\text{ MA}$, $a = 85\text{ cm}$). Fig. 3.3a shows an example of only the $\chi = 180^\circ$ orbit intersecting the wall for a birth point at (57 cm, 63.17 cm). Fig. 3.3b shows an example of both the stagnation and $\chi = 180^\circ$ orbits hitting the wall from an initial point of (-42.5 cm, 73.61 cm).

the birth point and wall, it is the appropriate loss boundary to the outboard (inboard) edge of the wall. If not, B, and then C, are successively examined. If both B and C satisfy these criteria, then orbit C is the appropriate loss boundary. When these conditions for particle loss are combined, the loss region in velocity space is obtained. An example for alpha losses from the TFTR I plasma is shown in Fig. 3.4 for a parabolic current density profile. It is noteworthy that losses occur across the entire plasma but only for energies above 350 keV. Fig. 3.4 differs from the plots of Rome et al. [18], because in the present calculations the wall is removed from the plasma edge. The intersection of the loss region with the $v_{||}$ axis occurs for $\cos\chi = \pm 1$, which implies $|x_{w1}| = 1$, from (3.1). The resulting velocity is:

$$v_{\max} = \frac{Ze}{mR_0} \left| \frac{\psi_w - \psi_i}{h_w - h_i} \right| \quad (3.6)$$

where $h_w = (R_0 + x_w)/R_0$ and x_w corresponds to the inner and outer edges of the first wall; similarly, h_i corresponds to the initial point.

Using these results in the next chapter, the wall loading from each birth point is calculated for all wall segments between the wall-orbit intersections, corresponding to the loss boundaries, $x_{\min} \leq x \leq x_{\max}$. If all these boundaries fail to exist, there is no loss region for the given birth point, i.e., the fp is totally contained.

For later reference it is convenient to define the zero loss region (ZLR); all orbits originating inside the ZLR are completely confined and cannot reach the wall. The ZLR can occur in two ways

Figure 3.4 Loss region in $v_{\parallel}$ - $v_{\perp}$ space for an α incident on the first wall from birth points on the x-axis in the TFTR-I plasma. The assumed form for current density is $J = J_0[1-(r/a)^2]$. Numbers of each curve in the upper graph indicate birth points (for $R < R_0$) as the ratio $(-x_i/a)$; on the lower figure the numbers indicate birth points (for $R > R_0$) as $(+x_i/a)$.

(see Fig. 3.5): either being bounded by the inner loop of an unstable stagnation orbit which barely intersects the outboard edge of the wall, or being bounded by the second loop of a double stagnation orbit [4]. In the extreme case, the loss fraction, F_g , becomes zero as the ZLR covers the whole plasma for a sufficiently large plasma-wall separation.

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Figure 3.5 In the top figure the zero loss region is bounded by the inner loop of the stagnation orbit which grazingly intersects the outboard edge of the first wall; orbits originating inside the ZLR cannot reach the wall. In the bottom figure, the zero loss region is bounded by the second loop of a double stagnation orbit.

4. FUSION-PRODUCT LOSSES TO FIRST WALL

The fraction of particles lost to the wall from a given birth point is:

$$f_L = \frac{\int_{\cos\chi_{\max}}^{\cos\chi_{\min}} 2\pi d(\cos\chi)}{\int_{-1}^1 2\pi d(\cos\chi)} \frac{1}{2}(\cos\chi_{\min} + \cos\chi_{\max}). \quad (4.1)$$

The trapping fraction can be calculated as an immediate consequence of (4.1). For a birth point having zero loss region, the first trapped orbit occurs for $\chi = 90^\circ$ and the last for the fattest banana. From (4.1), these conditions determine the trapping fraction:

$$f_T = -\frac{1}{2} \cos\chi_S, \quad (4.2)$$

where $\cos\chi_S$ corresponds to the stagnation orbit, defined by (3.3)-(3.5). When a birth point has a non-zero loss region, the overlap between $\chi_{\min} < \chi < \chi_{\max}$ and $90^\circ < \chi < \chi_S$ must be excluded from f_T . The more general form for f_T is then:

$$f_T = \begin{cases} 0, & \text{for } \chi_{\min} \leq 90^\circ < \chi_S \leq \chi_{\max} & (4.3a) \\ -\frac{1}{2} \cos\chi_{\min}, & \text{for } 90^\circ < \chi_{\min} \leq \chi_S \leq \chi_{\max} & (4.3b) \\ -\frac{1}{2} \cos\chi_S, & \text{for } f_L = 0, \text{ or } 90^\circ < \chi_S \leq \chi_{\min} < \chi_{\max} & (4.3c) \end{cases}$$

Figs. 2.3a-c illustrate (4.3a) and Figs. 2.2b-2.2c illustrate (4.3b).

The net loss fraction is:

$$F_L = \frac{\text{total fusion products lost from plasma}}{\text{total fusion products produced}}$$

$$= \int_V f_L n_1 n_2 \langle \sigma v \rangle_{12} R r dr d\theta d\phi \bigg/ \int_V n_1 n_2 \langle \sigma v \rangle_{12} R r dr d\theta d\phi. \quad (4.4)$$

This work assumes only Maxwellian fusions [32] with particle density profiles of the form $n = n_0 [1 - \gamma_N (r/a)^L]^K$ and background temperature profiles like $T = T_0 [1 - \gamma_T (r/a)^Q]^P$. Loss fraction results discussed below assume no fusion product scrape-off by a limiter. When losses to the limiter dominate, the limiter position can be used to define the wall; this has been done for example, in our calculations for ORNL-EPR (see Table 2.1).

4.1 Wall Loading Profile

Another important result of this analysis is the wall loading profile i.e. the incident fusion product flux, $d\dot{n}/dA$, as a function of poloidal angle (calculated in units of $\#/\text{m}^2\text{-s}$):

$$P \equiv \frac{d\dot{n}}{dA} = \frac{\text{total fusion products lost to the wall between } \theta' \text{ and } \theta'+d\theta'}{\text{toroidal wall area between } \theta' \text{ and } \theta'+d\theta'}$$

$$= \int_V n_1 n_2 \langle \sigma v \rangle_{12} f_L(r, \theta, \theta') R r dr d\theta d\phi \bigg/ \int_S R r d\theta d\phi, \quad (4.5)$$

where $f_L(r, \theta, \theta')$ is the loss fraction from a birth point at (r, θ) to a wall position between θ' and $\theta'+d\theta'$. Both (4.4) and (4.5) have been

evaluated numerically, and in particular (4.5) has been computed for equal intervals around the wall, i.e. between θ'_k to θ'_{k+1} , as indicated in Fig. 2.1. Finite gyroradius motion causes the fusion product to be deposited at a minimum distance r_g from the guiding center-wall intersection, with a random phase. The resulting wall loading is strongly smoothed. It is therefore appropriate to choose $\Delta\theta' = \theta'_{k+1} - \theta'_k$ so that $r_w \Delta\theta' = 2 r_g$. A second effect to be considered here is that all the losses occur in the direction of the vertical drift due to ∇B and curvature. Consequently, the losses occur only to the upper- or only to the lower-half plane of the tokamak. The denominator of (4.5) can be evaluated analytically for a strip of area around the torus:

$$\Delta S(\theta'_k, \theta'_{k+1}) = 2\pi[r_w^2(\sin\theta'_{k+1} - \sin\theta'_k) + R_o r_w(\theta'_{k+1} - \theta'_k)] \quad (4.6)$$

The loss fraction in (4.5) is computed as $f_\ell(r, \theta, \theta'_k \rightarrow \theta'_{k+1})$ using (4.1) where $\cos x_{\min}$ and $\cos x_{\max}$ correspond to wall-orbit intersections at θ'_k and θ'_{k+1} as calculated from (3.1). Only first bounce (prompt) losses are considered here; wall bombardment due to fusion products that pitch-angle scatter into the loss region during slowing down, not included here, must ultimately be considered. Rough estimates are that pitch-angle losses are as much as an order of magnitude lower than prompt losses. On the other hand, anomalous effects such as shear-Alfven-wave-induced fast ion losses, analyzed by Chan and Sigmar [33], could cause significant localized effects. For this

case, particles pitch-angle scatter by increasing the maximum poloidal radius of the drift orbit, $r_{\max}$, and thus tend to preferentially bombard the wall in the mid-plane.

4.2 Results

Wall loading profiles for 3.52-MeV alpha losses from PLT, TFTR, ORNL-EPR and UWMAK I (see Table 2.1 for machine parameters) are shown in Fig. 4.1. See Table 4.1 for a summary of the results for each curve (the peak-to-average flux ratio is designated as P/A and the average flux is given as $\bar{F}$). In PLT and TFTR I, loss contributions occur over the entire plasma. For the larger plasmas, there are successively larger zero-loss, core regions. In TFTR II, the outer 55% of the plasma contributes to wall loading, dropping to 30%

Table 4.1: Summary of 3.52-MeV Alpha Losses for Figure 4.1

Machine	Curve 1 TFTR I	Curve 2 ORNL-EPR	Curve 3 TFTR II	Curve 4 PLT	Curve 5 UWMAK I
F_{α}	0.13	3.1×10^{-3}	0.014	0.48	1.4×10^{-4}
P/A	1.9	1.8	1.4	1.4	2.3
$\bar{F}$	4.5×10^{15}	1.5×10^{15}	1.1×10^{15}	1.1×10^{15}	1.6×10^{14}

for ORNL-EPR, and to 4% for UWMAK I. This is as expected, since the poloidal gyroradius is comparable to r_w in PLT and TFTR I, less than r_w in TFTR II, and much less than r_w in ORNL-EPR and UWMAK I. Also, note that PLT can be used to simulate TFTR II losses for poloidal wall angles below 135° .

Figure 4.1 Flux of 3.52 MeV alphas on the first wall versus poloidal angle for several representative tokamaks. The loss fraction, F_L , peak-to-average flux ratio, P/A , and average flux, $\bar{P}$ (#/m²-sec), are summarized in Table 4.1.

Losses for various fusion products in PLT are shown in Fig. 4.2. Here, the normalized flux is obtained by dividing the particle flux by the total-fusions-per-second, S , for the given reaction, i.e., the denominator of (4.4). See Table 4.2 for the Figure key. The curves for 3.7-MeV and 3.52-MeV alphas, 3.03-MeV p, and 1.01-MeV T are essentially identical because (2.9) is exact for the last two cases and approximately true for all four. The 14.7-MeV p is poorly confined because the poloidal gyroradius is larger than r_w , while the 0.82-MeV He^3 is well confined because its poloidal gyroradius is much smaller than r_w . Similar results are obtained for various fusion products in the other tokamak machines.

Table 4.2: Summary of Fusion-Product Losses for Figure 4.2

Curve	Fusion-Product	F_{ℓ}	P/A	$\bar{F}$	S
1	$\text{D}(\text{He}^3, \text{p})\text{He}^4$ 14.70 MeV p	0.99	1.6	9.1×10^{11}	1.1×10^{13}
2	$\text{D}(\text{t}, \text{n})\text{He}^4$ 3.52 MeV He^4	0.48	1.4	1.1×10^{15}	2.9×10^{16}
3	$\text{D}(\text{He}^3, \text{p})\text{He}^4$ 3.70 MeV He^4	0.46	1.4	4.3×10^{11}	1.2×10^{13}
4	$\text{D}(\text{d}, \text{p})\text{T}$ { 3.03 MeV p 1.01 MeV T	0.44	1.5	1.5×10^{13}	4.2×10^{14}
5	$\text{D}(\text{d}, \text{n})\text{He}^3$ 0.82 MeV He^3	0.07	1.8	2.2×10^{12}	4.2×10^{14}

The wall loading profiles are also strongly influenced by profiles of n , T and J . This profile dependence is strongest in a plasma that has no zero-loss core, i.e., when the fusion product losses are a function of the complete spatial profile. Plasmas having the largest

Figure 4.2 Normalized fusion product wall flux for DT, DD and D-He³ reactions in PLT versus poloidal angle. The loss-fraction, F_L , peak-to-average flux ratio, P/A , average flux, $\bar{F}$ (#/m²-sec), and total fusions, S (#/sec), are summarized for each curve in Table 4.2.

zero loss core show the least variation in wall loading as the spatial profiles are changed. Examples of the former case are TFTR I and PLT; UWMAK I is an example of the latter.

4.2.1 Variation with T_i -Profile

Figure 4.3 illustrates the effect of varying the T-profile in TFTR I, from $T/T_0 = 1$, to $T/T_0 = 1 - .8(r/a)$. The peak alpha flux values vary by a factor of 3.7 over this range of T-profiles. The constant-T curve produces the highest overall wall loading because the fusion reactivity is constant over the whole plasma, causing the highest fusion rate. The linear-T profile concentrates fusions at the plasma core, producing the lowest net fusion rate, and consequently the lowest wall loading values. The position of the flux peaks varies only slightly, from $\theta' = 58^\circ$ for $T/T_0 = 1$, to $\theta' = 52^\circ$ for the linear-T plot. The flux peak is shifted to a lower θ' , for a more centrally peaked profile, because losses from the plasma core are concentrated at the outboard portion ($R > R_0$) of the wall. Fusion products born in the plasma fringe move along orbits which hit the wall over a broad range of poloidal angles (compare Fig. 2.2a and 2.2c). Flatter T-profiles enhance these fringe losses, thus broadening the wall loading curve and shifting the peak flux to higher θ' .

4.2.2 Variation with n_i -Profile

The wall bombardment in TFTR I has been calculated for various n-profiles, ranging from $n/n_0 = 1$ to $n/n_0 = 1 - (r/a)$. The resulting alpha flux curves for $n/n_0 = 1 - (r/a)^L$ are essentially identical to the

Figure 4.3 Wall loading flux (3.52-MeV α) in TFTR-I versus poloidal wall angle, for various T-profiles. The n- and J-profiles are as given in Table 2.1.

corresponding flux curves (within 10%) for $T/T_0 = 1 - .8(r/a)^Q$. The peak fluxes (F_{pk}) vary by a factor of 3.4 in this case, with the position of these peaks (in θ') identical to the corresponding T-profile flux peaks. This near superposition of wall loading curves occurs because the fusion source rate changes in the same way as the T- and n- profiles are varied.

4.2.3 Variation with Current Profile

Figure 4.4 shows alpha wall loading dependence on J-profile in TFTR I. The extremes in profile are $J/J_0 = 1$, and $J/J_0 = 1 - (r/a)^{.01}$ which is very peaked about $r = 0$; between these two limits, peak fluxes vary by a factor of 2.7. The peaked-J profile produces the largest B_θ at a given poloidal radius (c.f. eqns. 2.4-2.5), yielding the best fusion product confinement, i.e. lowest wall loading. The maximum flux position is strongly dependent on J-profile, decreasing by 40° as $J(r)$ progresses from a constant to very localized at $r = 0$. The constant-J enhances losses from the plasma edges so that the alpha flux curve is flatter and has a maximum for larger θ' . The peaked-J de-emphasizes the fringe losses, shifting the maximum flux position to smaller θ' .

4.2.4 Variations with Geometric Parameters

Geometric factors also are a strong factor in fusion-product loss curves. The variables to be considered are R_0 , a , r_w and $q(a)$. Since wall loading sensitivity to these parameters is stronger in larger machines the example used here is ORNL-EPR.

Figure 4.4 Wall loading flux (3.52-MeV α) in TFTR-I versus poloidal wall angle, for various J-profiles. The T- and n-profiles are as given in Table 2.1.

Figure 4.5 is a plot of alpha wall loading in ORNL-EPR for various q -values, ranging from 1.5 to 3.5, resulting in a maximum flux variation by a factor of 32. This very strong flux dependence on q arises because the banana width is proportional to q . The maximum flux position increases slightly with increasing q ($\theta' = 42^\circ$ for $q = 1.5$, $\theta' = 50^\circ$ for $q = 3.5$) because the poorer confinement increases losses for $r \sim a$. This same effect also flattens the wall loading curve somewhat for higher q -values.

The effect of varying r_w is shown in Fig. 4.6 for ORNL-EPR. As the wall radius is increased from nominal ($r_w = a$) to $r_w = 1.3a$, the maximum flux decreases by a factor of 1/4300. This huge variation arises because fewer of the alphas can reach the wall as r_w increases, i.e. the zero-loss core grows from 55% to 80% of the plasma volume. As the zero-loss core increases, the lossy region becomes concentrated in a new-moon shaped sliver at the outboard plasma edge. Consequently, losses are increasingly peaked at lower values of θ' .

Figure 4.7 illustrates the influence of minor radius variations on wall loading in ORNL-EPR. As the minor radius increases from 100 cm to 350 cm, the peak flux decreases by 2 orders of magnitude. This strong dependence results from the zero-loss region increasing from 0 to 70% of the plasma volume as the minor radius increases by a factor of 3.5. Large fusion rates in the plasma core easily outweigh the smaller plasma volume for small minor radii, yielding large alpha wall fluxes. For larger a -values, the combination of a large zero-loss core and low fusion rates in the lossy fringe produce the low wall

Figure 4.5 Wall loading flux (3.52-MeV α) in ORNL-EPR versus poloidal wall angle, for various q -values. Other parameters are as given in Table 2.1, while I varies consistently with changing q .

Figure 4.6 Wall loading flux (3.52-MeV α) in ORNL-EPR versus poloidal wall angle, for various values of r_w . All other parameters are as given in Table 2.1.

Figure 4.7 Wall loading flux (3.52-MeV α) in ORNL-EPR versus poloidal wall angle, for various values of minor radius. Other parameters are as shown in Table 2.1 while I varies consistently with $q=\text{constant}$.

loadings. The position of maximum flux, θ' , decreases with increasing minor radius because the new moon-shaped, lossy plasma fringe concentrates losses toward the outboard edge.

Figure 4.8 demonstrates the change in normalized wall flux (particle flux divided by total-fusions-per-second, as in Fig. 4.2) as the major radius of ORNL-EPR is varied. This form of the plot was chosen because the curves coalesce. That is, peak flux is roughly proportional to the plasma volume ($4\pi^2 R_0 a^2$); so, increasing R_0 by a factor of 3.1 causes F_{pk} to increase by a factor of 3.6. The additional increase in maximum flux is caused by a decrease in the size of the zero-loss core region as R_0 increases. The zero-loss core size decreases as R_0 is increased because total current is inversely proportional to R_0 for q fixed. The peak flux position is essentially constant as R_0 changes. Coalescence of normalized flux curves does not occur in Fig. 4.7 because the peak flux rapidly decreases for increasing minor radius (see also Table 4.3).

Table 4.3: Summary of 3.52-MeV Alpha Losses for Figure 4.8

R_0	$F_L \times 10^3$	$F_{pk} \times 10^{-15}$	$\bar{F} \times 10^{-15}$	P/A	$S \times 10^{-20}$
350	1.6	1.3	0.80	1.6	0.77
1100	5.3	4.7	2.6	1.8	2.4

Figure 4.8 Normalized wall loading flux (3.52-MeV α) in ORNL-EPR versus poloidal wall angle, for various values of major radius. Other parameters are as given in Table 2.1, while I varies consistently with q =constant. The loss fraction, $F_{L\alpha}$, peak-to-average flux ratio, P/A , average flux, $\bar{F}$ (#/m²-sec) and total fusions (#/sec) are summarized for each curve in Table 4.2.

5. FLUX VERSUS ANGLE-OF-INCIDENCE

Blistering-induced plasma impurity generation by high-energy fp bombardment is sensitive to the angle of incidence, as well as the particle flux. This effect has been modeled for prompt losses, including all the relevant particle drifts (neo-classical), motion along the magnetic field, and random gyro-phase.

5.1 Theory

The flux-per-steradian, $dF/d\Omega$, at the wall is:

$$\frac{dF}{d\Omega} = \frac{d\dot{n}}{dA d\Omega} = \int_V \frac{n_1 n_2 \langle \sigma v \rangle}{2\pi (\cos \alpha_k - \cos \alpha_{k+1}) \Delta A} f_{\ell}(\theta'_k \rightarrow \theta'_{k+1}, \alpha_k \rightarrow \alpha_{k+1}) R dr d\theta d\phi \quad (5.1)$$

Here $f_{\ell}(\theta'_k \rightarrow \theta'_{k+1}, \alpha_k \rightarrow \alpha_{k+1})$ is the differential loss fraction reaching the wall for $\theta'_k \leq \theta' \leq \theta'_{k+1}$, for angles of incidence $\alpha_k < \alpha < \alpha_{k+1}$, where

$$f_{\ell}(\theta'_k \rightarrow \theta'_{k+1}, \alpha_k \rightarrow \alpha_{k+1}) = f_{\ell}(\theta'_k \rightarrow \theta'_{k+1}) \times \rho(\Omega; \vec{r}_i) \Delta\Omega. \quad (5.2)$$

Details of the derivation are described in Appendix B. The function $\rho(\Omega; \vec{r}_i)$ is defined as the probability per unit angle that an alpha born at $\vec{r}_i$ will strike the wall with gyrophase angle Ω . It is found to be

$$\rho(\Omega; \vec{r}_i) = |1 - v_G \sin \Omega / v_D| / 2\pi, \quad (5.3)$$

where $\Omega = 0$ corresponds to grazing incidence when only gyro-motion is considered. The velocities v_D and v_G are the radial components (in the wall frame) of the streaming-plus-drift motion and gyro-velocity, respectively. These velocities have been evaluated using expressions derived by Galeev and Sagdeev [14]. While the *integrand* of (5.1) is separable into a form like $f_g(\vec{r}; \theta') \rho(\alpha, \theta'; \vec{r}) \langle \sigma v(\vec{r}) \rangle$, the *integral* averages over all possible birth positions. Thus, the flux distribution is not separable into independent functions of α and θ' , as done in Refs. 9 and 36. In particular, the spatial dependence of v_D/v_G must be included in this averaging process; this is discussed further below.

Recent analyses by Bauer et al. [9] and Belikov et al. [36] also consider this problem but use an average case probability distribution function (i.e. no spatial variation in v_D/v_G to calculate flux-versus-incidence-direction). For $v_D/v_G \ll 1$, $\rho(\Omega; \vec{r}_i)$ in Eq. (5.3) reduces to the linear probability function used in Ref. 9. While this assumption is good for reactor-sized machines, it breaks down for near-term devices that are of interest here. The present analysis includes spatial variations in v_D/v_G and yields fluxes as a superposition of sinusoidal profiles with significant fluxes at near-grazing incidence. This technique also allows for an offset, X_0 , between the centers of

the plasma and wall cross sections (due to compression) and permits the poloidal wall radius to be arbitrarily larger than the plasma radius.

For particles to impact on the wall, the net radial velocity in the wall frame must be outward, yielding

$$\sin \Omega < v_D/v_G \equiv \sin \Omega_0. \quad (5.4)$$

The allowed range of gyrophase angles is then

$$0 \leq \Omega^* \leq \Omega < 2\pi + \Omega_0, \quad (5.5)$$

where Ω^* is defined by

$$(\Omega^* - \Omega_0)v_D/v_G = \cos \Omega_0 - \cos \Omega^*. \quad (5.6)$$

When $v_G < v_D$, (5.3) remains valid, with $\Omega^* \leq \Omega \leq 2\pi$ and (5.6) being replaced by

$$\Omega^*v_D/v_G = 1 - \cos \Omega^*. \quad (5.7)$$

However, this case applies to a very small class of particles ($< 10^{-6}$ of total). Equations (5.6) ~ (5.7) are solved numerically using a Newton-Raphson search, with a starting guess for Ω^* of $(3\pi/2) - \Omega_0$.

Erosion due to sputtering by leaking background D-T ions can remove the surface so rapidly that build-up of a critical helium concentration and hence blistering, is prevented. (Sputtering by 3.5-MeV alphas is negligible [5] based on extrapolations of data by

Vernickel [37].) Bauer, et al. [9] first noted this problem, while Fenske et al. [38] have considered the θ' -dependence of alpha flux, found here for an average plasma edge temperature ~ 60 eV.

Reference 38 finds that blistering is possible for many materials for $20^\circ \leq \theta' \leq 100^\circ$, if the ratio of average alpha flux to background D-T flux is $\leq 0.1\%$. At other poloidal angles, D-T sputtering dominates (this is discussed further at the end of this chapter). In the present study, we have only considered blistering effects because in principle, improved plasma confinement and/or divertors can reduce D-T fluxes (and the consequent sputtering) to arbitrarily low levels. In this view, high-energy alpha losses represent a lower bound on wall-generated plasma impurities.

5.2 Results

Figures 5.1-5.4 show the results of our calculations for PLT, TFTR, ORNL-EPR and UWMAK-I. (See Table 5.1 for parameters). The resulting value of $dF/d\Omega$ rises from near zero at $\alpha = 90^\circ$ (grazing), peaks at $\alpha = \alpha_c$ (some critical angle), then falls abruptly to zero for $\alpha < \alpha_c$. The rise in $dF/d\Omega$, as α decreases from 90° to α_c , is due to the sinusoidal probability distribution (c.f. eqn. (5.3)), since α varies sinusoidally in Ω , for $v_D/v_G \ll 1$. The abrupt drop in $dF/d\Omega$ occurs because $\alpha_c = (\Omega^*)_{\max}$ is determined by the maximum value of v_D/v_G from (5.6). In all cases, the peak value of $dF/d\Omega$ occurs at or near the poloidal wall position corresponding to the peak flux, F_{pk} .

Figure 5.1 Flux of 3.5-MeV alphas versus incident velocity direction at various poloidal wall angles in PLT.

Figure 5.2 Flux of 3.5-MeV alphas versus incident velocity direction at various poloidal wall angles in TFTR-I.

Figure 5.3 Flux of 3.5-MeV alphas versus incident velocity at various poloidal wall angles in ORNL-EPR.

Figure 5.4 Flux of 3.5-MeV alphas versus incident velocity direction at various poloidal wall angles in UWMAX-I.

Table 5.1: Tokamak Parameters and Wall Loading Summary

Machine	PLT[26]	TFTR-I[27] ⁺⁺	ORNL-EPR[28]	UWMK-1[29]	JEPR[6]	T-20[7]
B_0 (T)	4.6	5.2	4.8	3.82	4.0	3.5
I (MA)	1	1	7.2	21	3.67	6.09
R_0 (m)	1.32	2.48	6.75	13.00	4.90	5.00
a (m)	0.45	0.54	2.25	5.00	1.50	2.00
r_w (m)	0.48	1.10**	2.25	5.50	1.50	2.00
T_{i0} (keV)	5	10	22.5*	11.1	8	15
T_i/T_{i0}	$1-.8(r/a)^2$	$1-.8(r/a)^2$	$1-(r/a)^2$	1	1	$1-(r/a)^2$
n_{i0} (m ⁻³)	1×10^{20}	1.6×10^{20}	8×10^{19}	1.2×10^{20}	1.1×10^{20}	5×10^{19}
n_i/n_{i0}	$1-(r/a)^2$	$1-(r/a)^2$	$[1-(r/a)^2]^{1/2}$	$[1-.99(r/a)^2]^{1/2}$	$[1-(r/a)^3]^{1/2}$	$1-(r/a)^2$
J/J_0	$1-(r/a)^2$	$1-(r/a)^2$	$1-(r/a)^2$	1	1	$1-(r/a)^2$
F_α	0.48	0.13	3.1×10^{-3}	1.4×10^{-4}	0.19	3.3×10^{-3}
F (#/m ² -s)	1.1×10^{15}	4.5×10^{15}	1.5×10^{15}	1.6×10^{14}	3.1×10^{16}	2.1×10^{14}
P/A [†]	1.4	1.9	1.7	2.3	1.6	1.8

*An ignited system is assumed here, having a thermal power of 420 MW.

**There is also in offset of $X_0 = 0.17$ m for this case.

†P/A is the ratio of peak-to-average alpha flux.

++This is the high compression scenario in TFTR; the high current case is discussed in Ref. 4.

5.3 Application

To illustrate the significance of these results, blistering-induced plasma impurity generation has been estimated via a model developed by Fenske et al. [38]. A Thomas-Fermi form of nuclear-stopping, along with Brice's [39] electronic-stopping formula, are used to calculate the average projected range and straggling of 3.5-MeV alphas. The rate of helium concentration increase in near-surface regions of a steel first wall is obtained as a function of θ' and depth into the wall assuming a Gaussian projected range profile.

Under the assumption that prompt alpha losses dominate blistering formation (i.e., neglecting intermediate energy alphas, synergistic effects, etc.), the onset of rupture and flaking is taken to occur at a critical helium concentration value, taken here as 30 atomic-percent. The corresponding onset time is t_0 , with a blister thickness equalling the depth of peak concentration, x_{pk} . The values of t_0 and x_{pk} vary with poloidal wall angle; for example in ORNL-EPR, the maximum x_{pk} is 1 μm at $\theta' = 86^\circ$ (see Fig. 5.5). The smallest onset time is $t_0 \sim 4000$ hours at $\theta' = 56^\circ$. Thus exfoliation begins at $\theta' = 56^\circ$ in ORNL-EPR and gradually spreads to other θ' -values.

The average rate of impurity generation, $\dot{n}_I$, after many burn cycles (i.e., after sufficient operation so that blisters cover the whole surface area) is

Figure 5.5 Depth, x_{pk} , of peak helium concentration and onset time, t_0 , for blistering as a function of poloidal wall angle, θ' , in ORNL-EPR (taken from Ref. 38).

$$\dot{n}_I = \frac{A_v f_s}{V f_t} \int \frac{x_{pk} dA}{t_0} \quad (5.8)$$

Here, $\int dA$ is taken over the upper (or lower) half of the wall area, and V is the plasma volume. Also, A_v is the atomic volume of iron; f_s is the fraction of the blistered surface which reaches the plasma as impurities (here taken as unity); f_t is the fraction of the onset time, during which flaking occurs (taken as 0.1, based on Fig. 11 of Ref. 40).

The effect of blistering-induced impurities must be evaluated by transport modeling. However, the most sophisticated models are inadequate to describe experimental results, even when neoclassical as well as anomalous terms are included [41]. Here, a global model is used:

$$\frac{\partial n_I}{\partial t} = \dot{n}_I + n_I/\tau_I, \quad (5.9)$$

where τ_I is the particle transport time. Depending on the details of the model (including boundary conditions), τ_I can be either positive or negative [41]. Consequently, the extreme case is assumed: all the impurities remain in the plasma ($\tau_I = \infty$), trivially yielding $n_I = \dot{n}_I t$. The accumulating impurities can then quench the burn by replacing D-T ions, assuming fixed β and fully stripped low-Z impurities. The characteristic time, τ_L , for

quenching the burn corresponds to the fusion rate decreasing to $1/e$ of its nominal value. The resulting time is:

$$\tau_L = 0.393 \bar{n}_e / (Z_I + 1) \dot{n}_I, \quad (5.10)$$

where Z_I is the impurity charge, and $\bar{n}_e$ is the average electron density. Typical results of this calculation, shown in Table 5.2 for ORNL-EPR and UWMAK-1, indicate that $\tau_L < \tau_{\text{burn}}/2$ for a steel wall.

Table 5.2: The Effect of Blister-Injected Impurities

Parameter	ORNL-EPR	UWMAK-1
$\bar{n}_e (\text{m}^{-3})$	4×10^{19}	8×10^{19}
$\tau_{\text{burn}} (\text{s})$	100	5400
$\tau_L (\text{s})$	47	2310
$\Delta_w (\tau_L = 10\tau_b)$	0.17	0.028
$\Delta_w (\tau_L = 20\tau_b)$	0.20	0.031
$\Delta_I (\tau_L = 10\tau_b)$	0.92	0.26
$\Delta_I (\tau_L = 20\tau_b)$	1.23	0.29

Fenske et al [38] have examined the combined effect of MeV α -blistering (using the spatial and angle-of-incidence profiles obtained in the present work) and thermal D-T sputtering on impurity generation. Figure 5.6 shows the resulting impurity level in ORNL-EPR [normalized to the impurity level due to a D-T flux of 5×10^{17} ions/($\text{m}^2 \cdot \text{s}$)] versus accumulated reactor operating time.

Figure 5.6 Impurity levels in ORNL-EPR are plotted versus accumulated operating time (taken from Ref. 38).

The dotted curve in Fig. 5.6 assumes only blistering-injected impurities, while the dashed line indicates the sputtering-induced impurity level. The solid line shows the impurity level due to blistering and simultaneous D-T sputtering, accounting for increased blister formation time, as well as the decreased blister-skin thickness. Increasing the D-T flux reduces the relative effect of blistering until the surface is eroded by sputtering at a rate too fast to allow blister formation. This trend is shown by the chain-dotted curve in Fig. 5.6. These results show that the combined effect of blistering and sputtering typically produces $\leq 50\%$ increase in the impurity generation rate over blistering alone, potentially limiting the burn time to $\tau_L < \tau_{\text{burn}}/3$. Thus, it is concluded that blistering by MeV alphas, with or without DT sputtering, might reduce the burn time by 50-70%.

The surface heating effect of MeV alphas and keV DT ions is not included in this analysis. A comprehensive review of plasma-wall interactions, including such effects, has been done recently by McCracken and Stott [122].

6. WALL LOADING SENSITIVITY

Since blistering-injected plasma impurities may limit the reactor burn time, it is important to consider ways to reduce alpha bombardment of the first wall. Previous workers [4,6] have examined the sensitivity of alpha losses, but only over narrow parameter ranges for a few devices. Our present work studies a broad range of devices and a wide spectrum of parameters which are important for designing practical reactors.

6.1 Approach

Our approach is to consider variations in the key parameters: major and minor plasma radii, wall radius, and q-value. For this purpose, a detailed study of wall loading profiles for various tokamaks is not feasible due to the computation time required. Instead, the parameters required for first wall design are considered: peak and average fluxes. The peak flux, F_{pk} , is used as a key indicator because this determines the worst region for wall effects. Also, the ratio of peak-to-average fluxes, P/A , is insensitive to the parameter variations considered here, as demonstrated through the relation to the ratio, F_{pk}/F_{ℓ} , namely:

$$0.4 \leq (0.3-0.4) \times P/A = (F_{pk}/F_{\ell})(2\pi r_w R_0/S) \leq 1.2 \quad (6.1)$$

See Appendix C for details of this scaling argument. These limits

follow because F_ℓ (c.f. Eq. 4.4) and F_{pk} (c.f. Eq. 4.5) have the same functional form. (The loss fraction is related to the average flux, $\bar{F}$, because $\bar{F}$ is equal to $F_\ell \times S/\Delta A$, where S is the total fusion source rate, c.f. the denominator of Eq. 4.4.) Since F_{pk} is closely related to both F_ℓ and $\bar{F}$ via Eq. (6.1), we concentrate on changes in F_{pk} . For reference, typical loss fractions are included in Table 5.1.

6.2 Results

Figure 6.1 shows F_{pk} for 3.5-MeV alphas versus the wall radius, r_w , for a sequence of typical tokamaks ranging from near-term experiment to large reactors (other parameters fixed, c.f. Table 5.1). Since the wall loadings are quite sensitive to the current density profile [4] (difficult to measure and not controllable at this stage of tokamak development), results are plotted for several different profiles, namely: $J/J_0 = 1, 1 - (r/a)^2$, and $1 - (r/a)^{0.1}$. A uniform current density yields the poorest fp confinement, while the centrally peaked current density produces the best (c.f. Sec. 4.2.3). Increasing r_w (with constant minor plasma radius) allows more space between the plasma and wall for alphas to orbit back into the plasma, reducing the wall flux. Increasing r_w in small devices ($r_w, a \leq \rho_\theta$, where ρ_θ = poloidal gyroradius) causes F_{pk} to decrease slowly. However, for larger devices ($r_w, a > 2\rho_\theta$) like ORNL-EPR and UWMAK-I, there is up to a 10^4 -fold decrease in F_{pk} for a 50% increase in r_w .

Figure 6.1 Variation in peak 3.5-MeV α -flux, F_{pk} , versus wall radius, r_w , for various tokamaks and current density profiles. Dots on the various curves indicate design points as given in Table 5.1.

Changes in F_{pk} versus q -values are shown in Fig. 6.2, where q is the "safety factor" $aB_0/R_0B_\theta(a)$. Strong changes ($\sim 10^3$ -fold increase) are obtained by increasing q from 1 to 5. This can be simply understood [4] by noting that $\cos\chi$ in Eqn. (3.1) is proportional to $q r_g/a \sim 1/I$, where r_g is the gyroradius. Thus the loss region will decrease as the plasma current increases. That is, by increasing q , the current decreases, which increases losses due to the larger banana width.

Figure 6.3 is a plot of F_{pk} as the minor radius, a , varies such that $(r_w - a)$ is constant with q (a) also held fixed. Up to a 100-fold decrease occurs as the minor radius is increased by a factor of 4. This is a result of a growing zero-loss region in the plasma as the minor radius increases. Since $\cos\chi$ is proportional to $q r_g/a$, increasing the minor radius with q constant, decreases the losses.

Variations in peak flux with major radius, R_0 , for q fixed, are shown in Fig. 6.4. Weak changes (≤ 3 -fold increase) are obtained for two-fold increases in R_0 . This increase occurs because the zero-loss region decreases (i.e., more fp's can escape to the wall) as R_0 increases due to the total current being proportional to R_0^{-1} for fixed q .

6.3 Application

These results have important implications for cost-effective first wall design. A modest increase in r_w produces drastic

Figure 6.2 Variation in peak 3.5-MeV α -flux, F_{pk} , versus $q(a)$ for various tokamaks and current density profiles. Dots on the various curves indicate design points as given in Table 5.1.

Figure 6.3 Variation in peak 3.5-MeV α -flux, F_{pk} , versus minor plasma radius, a , for $q(a)$ and $(r_w - a)$ held constant for various tokamak and current density profiles. Dots on the various curves indicate design points as given in Table 5.1.

Figure 6.4 Variation of peak 3.5-MeV α -flux, F_{pk} , versus major plasma radius for $q(a)$ fixed, for various tokamak and J-profiles. Dots on the various curves indicate design points as given in Table 5.1.

reductions in the first-bounce fp losses to the wall and consequent plasma contamination. However, increasing r_w must be balanced against the increased cost of the larger blanket and magnet size required. Increasing the plasma current is also helpful, but less effective. Changes in the major and minor radii have the least effect.

To demonstrate an application of these studies, the effect of varying the plasma-wall separation and the plasma current without impurity removal is studied. The fractional increase in the design value of r_w , required to obtain τ_L equal to a multiple of the burn time (e.g., $\tau_L = 10\tau_b$), is:

$$\Delta_w(\tau_L) = \Delta r_w / r_w. \quad (6.2)$$

Similarly, the fractional increase in the plasma current, required to achieve this goal is designated as:

$$\Delta_I(\tau_L) = \Delta I / I. \quad (6.3)$$

Corresponding values of Δ_w and Δ_I are shown in Table 5.2. For the ORNL-EPR design, a 20% increase in r_w yields a 20-fold decrease in the α flux with a corresponding reduction in blistering. Increasing the plasma current is less effective.

Increasing the plasma-wall separation appears "acceptable" from a cost point of view. For example in a recent TNS reactor design ($a = 1.2\text{m}$, $R_0 = 5\text{m}$, $B_T = 5.3\text{T}$, $P_{\text{fus}} = 1140 \text{ MW}_T$), a Δ_w - value of 20% raises the capital cost by 9% [42]. On the other hand, to accept a

shorter burn time for this sample case requires increasing the plasma-wall separation by 40% to maintain fixed values of thermal wall loading and shut-down time. Consequently, the added capital cost to control α -generated impurities appears to be a good investment compared to the increased expense of a shorter burn time. A more sophisticated means of control is to take advantage of the α -flux peak at the outboard edge of the wall. This is done by leaving the plasma-wall separation unchanged at the inboard edge (i.e., $r_w - \chi_0 - a = \text{constant}$), while increasing it at the outboard edge (i.e., $r_w + \chi_0 - a$ increasing). This avoids conflicts with space constraints at the tokamak centerline (e.g., the larger poloidal radius of magnet coils, shielding and blanket) that would be encountered in the first method. However, present economic models cannot assess this approach [42].

7. ESTIMATE OF NON-PROMPT LOSSES

As well-confined fp's slow down in the Maxwellian background plasma, they can be lost to the wall by collisional pitch-angle scattering (PAS) into the loss region. For wall loading effects, it is important to estimate the size of the non-prompt losses relative to prompt bombardment. Assuming classical PAS (i.e., neglecting instability-induced losses during slowing down), it is found that the relative contribution of non-prompt losses is small. Still, the study of these losses is important as a "base case" comparison with instability-induced losses in future experiments, i.e., as a diagnostic [5,43,44] for studying the slowing down mechanism(s). Also, synergistic effects due to energy dependence and the angle-of-incidence of fp's reaching the wall may modify impurity evolution significantly.

7.1 Theory

Since fp energies considered here are much larger than the background plasma temperature ($E > 50$ keV), up-scattering due to velocity-space diffusion and losses by charge-exchange are neglected. In accord with previous work [18], small banana-width orbits and very untrapped particles are assumed. Under the above assumptions and in steady state, the resulting two-dimensional, bounce-averaged Fokker-Planck equation (derived by Rome et al. [18]) simplifies to:

$$\frac{1}{v^2} \frac{\partial}{\partial v} \left[(v^3 + v_c^3) g \right] + \frac{m_i v_c^3 z_{\text{eff}}}{2m v^3} \frac{\partial}{\partial \zeta} \left[(1 - \zeta^2) \frac{\partial g}{\partial \zeta} \right] + s \tau_s = 0 \quad (7.1)$$

where $g = g(v, \zeta)$ = fp velocity-space distribution function,

m = fp mass,

Ze = fp charge,

$v_c^3 = \frac{3\sqrt{\pi}}{4} \frac{m_e}{m_i} v_e^3$ = critical velocity corresponding to equal slowing contributions by background ions and electrons.

m_e, m_i = electron and background ion masses, respectively.

$v_e = \sqrt{2kT_e/m_e}$, for electron temperature, T_e .

$z_{\text{eff}} = \sum_j n_j Z_j^2 / n_e$ = effective charge of the background plasma,

$\zeta = \cos \chi$, evaluated when the drift orbit crosses the mid-plane ($\theta=0$)

$\tau_s = \frac{3\pi^{3/2} \epsilon_0^2 m_e v_e^3}{n_e Z^2 e^4 \ln \Lambda}$ = the Spitzer slowing down time for electronic charge q

s = fp source rate.

These approximations are reasonable for counter-circulating fp's, because the trapping boundary usually coincides with the edge of the loss region; thus, the spatial deviation from a given flux surface doubles upon crossing this boundary [18]. Further, counter-circulating fp's, which scatter toward the loss region, have a slowing-down drift that is outward in r ; co-circulating fp's tend to drift inward [18,45]. This inward, slowing-down drift of co-circulating ions, together with the orbit topology [13] causes the

PAS across the $x_{\min}$ loss-region boundary to be zero. Thus, non-prompt fp losses occur predominantly across the $x_{\max}$ boundary, which is usually the x-type stagnation orbit. The small banana-width assumption (equivalent to neglecting variations in P_{ϕ}) tends to underestimate PAS for these counter-going particles. (Inclusion of the effect of large-banana widths requires a 3-D Fokker-Planck formulation: 2-D in velocity space (v, ζ) and 1-D in coordinate space. This is discussed in Chapters 8-11.)

The non-prompt loss fraction is estimated by integrating the second term in (7.1) over an element of velocity space. This is true, since the second term describes pitch-angle scattering which transports particles across the loss boundaries. The first term in Eqn. (7.1) describes slowing down in velocity magnitude only, i.e., motion parallel to the loss boundaries shown in Fig. 7.1. The result is

$$\frac{\delta F_{np}}{\delta E} = \frac{1}{S \delta E} \int_V 2\pi R r dr d\theta \int_{v+\Delta v}^v \pi dv \frac{m_i Z_{eff}^2 v^3}{m v \tau_s} (1-\zeta^2) \left. \frac{\partial g}{\partial \zeta} \right|_{\zeta_1}, \quad (7.2)$$

where $\delta E = \frac{m}{2} [(v+\Delta v)^2 - v^2]$. The outer integral is evaluated over the plasma volume, V , and S is the total fusion source rate. The derivative of the distribution function in Eqn. 7.2, $\partial g / \partial \zeta$, is obtained by solving Eqn. 7.1 analytically with $g = 0$ at the boundary of the loss region. For this purpose, the loss region is approximated as shown in Fig. 7.1. As discussed above, only the $x_{\max}$

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Figure 7.1 Velocity-space boundary conditions used in non-prompt loss calculation.

boundary is important, so the required constraint is $g(\zeta_1) = 0$,

where $\zeta_1 = \cos \chi_{\max}$. The result is:

$$\left. \frac{\partial g}{\partial \zeta} \right|_{\zeta_1} = \sum_v \frac{n_1 n_2 \langle \sigma v \rangle_{12} f_c \tau_s}{4\pi (v_0^3 + v_c^3)} \left(\frac{2v+1}{v(v+1)} \right) \left(\frac{\partial P_v}{\partial \zeta} / \frac{\partial P_v}{\partial v} \right)_{\zeta_1} \quad (7.3)$$

$$\times \left(\frac{v_0^3 + v_c^3}{v_0^3 + v_c^3} \right) \left(\frac{v_0^3}{v_c^3} \times \frac{v_0^3 + v_c^3}{v_0^3 + v_c^3} \right)^{v(v+1)m_i Z_{\text{eff}}/6m},$$

where v_0 = initial fp velocity magnitude (corresponding to energy, E_0),

$P_v(\zeta)$ = Legendre function,

v = spectrum of eigenvalues, obtained from $P_v(\zeta_1) = 0$,

$f_c = \frac{1}{2}(1 + \cos \chi_{\max})$ = differential fraction of fp's confined, having been born at (r, θ) with maximum loss angle, $\chi_{\max}$.

Equation (7.2) can also be obtained by viewing PAS losses as non-re-entrant losses in velocity space. Using $g = 0$ at the loss-region boundary, the fp velocity-space current density, J_i , is [46-47]:

$$J_i = A_i g - D_{ij} \partial g / \partial v_j = -D_{ij} \partial g / \partial v_j, \quad (7.4)$$

where A_i is the kinematic coefficient. The net loss is then obtained by integrating J_i over an interval of velocity-space loss-region surface using the well-known form for the velocity-space diffusion tensor, D_{ij} , for an isotropic, Maxwellian plasma [46-47].

The method of calculation is as follows. First, the velocity-space loss region, $\chi_{\min} \leq \chi \leq \chi_{\max}$, is determined for a given fp

birth point in the plasma, (r_i, θ_i) . Second, for the $x_{\max}$ orbit, $\zeta = \cos \chi|_{\theta=0}$, at $(r_{\min}, \theta=0) \leftrightarrow (x_{\min}, z=0)$, is obtained via a numerical, step-wise search. If the drift orbit intersects the mid-plane for $x_{\min} \leq 0$, the loss contributions due to this orbit are neglected for two reasons: 1) such orbits are a small class of the total (less than a few percent), and 2) orbits that form the $x_{\max}$ boundary and have $x_{\min} < 0$ will have slowing-down drifts that are *inward* in r . The latter effect causes PAS across the $x_{\max}$ loss boundary to be negligible, as in the case of co-circulating particles. Third, the derivative $\partial g / \partial \zeta$, is evaluated at $\zeta = \zeta_1$, using Eqn. (7.3). The Legendre functions of non-interger order, $P_\nu(\zeta)$, are determined using routines from the CERN Program Library [48]. The derivative, $\partial P_\nu / \partial \zeta$, is evaluated using [49]:

$$\frac{\partial P_\nu}{\partial \zeta} = \frac{\nu(P_{\nu-1} - \zeta P_\nu)}{1-\zeta^2} = \frac{\nu P_{\nu-1}(\zeta)}{1-\zeta^2},$$

since $P_\nu(\zeta) = 0$. While $\partial P / \partial \nu$ can be evaluated analytically [50], central differences are computationally faster:

$$\frac{\partial P_\nu}{\partial \nu} = \frac{P_{\nu+\Delta\nu} - P_{\nu-\Delta\nu}}{2\Delta\nu},$$

for $\Delta\nu = 5 \times 10^{-6}$. Finally, the integral in eqn. (7.3) is evaluated over the plasma volume.

7.2 Results

Typical results for the non-prompt alpha loss-fraction-per-keV in PLT and TFTR-1, are shown in Figs. 7.2-7.3. Since the drift orbit physics is the same here as for the prompt flux (depending on the direction of the toroidal B-field), the non-prompt losses occur either to the upper-half or to the lower-half surface of the device. The corresponding flux-per-keV, averaged over the upper-(lower-) half surface, is shown, as a companion ordinate, in these Figures.

Also shown in Figs. 7.2-7.3 are the cumulative particle loss fraction, v_p , defined as:

$$v_p = \int_{E_0}^E \frac{\delta F_{np}}{\delta E} dE, \quad (7.5)$$

and the cumulative energy loss fraction, v_E , given as:

$$v_E = \int_{E_0}^E \left(\frac{E}{E_0} \right) \frac{\delta F_{np}}{\delta E} dE. \quad (7.6)$$

As expected, both the v_p and v_E rise monotonically with decreasing alpha energy. The fastest change occurs close to the birth energy, indicating that MeV losses are dominant. This is discussed further below.

In PLT (Fig. 7.2), the loss-fraction-per-keV falls abruptly as the fp energy decreases below the birth energy, rises slowly to a

Figure 7.2 The lower figure shows the non-prompt alpha loss fraction per keV and average flux per keV versus particle energy in PLT. The upper figure shows v_p and v_E (cumulative particle- and energy-loss fractions, respectively) versus alpha energy in PLT.

Figure 7.3 The lower figure shows the non-prompt alpha loss fraction per keV and average flux per keV versus particle energy in TFTR-I. The upper figure shows v_p and v_E (cumulative particle- and energy-loss fractions, respectively) versus alpha energy in TFTR-I.

maximum at $v \sim 2.7 v_c$, and then decreases monotonically. The abrupt drop in $\delta F_{np}/\delta E$, for $E < E_0$ is expected, due to particles in orbits very near the prompt-loss boundaries entering the loss region. The maximum at $v \sim 2.7 v_c$ occurs because PAS increases more rapidly than the loss region is decreasing. Below 700 keV, the decreasing loss region dominates PAS, yielding a monotonic drop in the losses. In larger machines like TFTR (Fig. 7.3), the loss region is completely dominant over PAS, giving a monotonic decrease in $\delta F_{np}/\delta E$ with decreasing fp energy. The results for ORNL-EPR and UWMAC-I are very similar to Fig. 7.3, and therefore are not shown.

The dependence of non-prompt losses on Z_{eff} can be gauged as follows. The asymptotic values of the v_p and v_E (i.e., as the fp speed approaching the minimum value at which escape can occur), are defined to be v_p^* and v_E^* respectively. The average energy, $\bar{E}$, of the non-prompt losses is obtained by combining (7.5) and (7.6).

$$\bar{E} = E_0 (v_E^*/v_p^*) . \quad (7.7)$$

Both v_p^* and v_E^* rise monotonically as Z_{eff} increases (see Fig. 7.4) with $v_p^* \leq 0.1 \times F_k \times \sqrt{Z_{eff}}$. The value of $\bar{E}$ is constant with respect to Z_{eff} , indicating that increasing Z_{eff} yields a uniform rise in PAS for all velocities. In all cases, $\bar{E}$ is greater than 1.4 MeV; for TFTR, ORNL-EPR, and UWMAC-I, $\bar{E}$ is above 2.6 MeV. These large average energies indicate that most non-prompt losses occur near the birth energy (i.e. the loss region shrinks rapidly with decreasing speed).

Figure 7.4 Asymptotic values of the cumulative particle- and energy-loss fractions (ν_p^* and ν_E^* , respectively) versus Z_{eff} for various tokamaks.

7.3 Comparison to Ohnishi's Work

Recently, Ohnishi et al., have also modeled alpha losses during slowing-down and pitch-angle-scattering [35] using a Monte Carlo method. They determine losses in JEPR (see Table 5.1) for a constant current density and temperature, with

$$n_i = 10^{20} [1 - 0.8(r/a)^2] (m^{-3}).$$

A comparison of the present results with their work [c.f. Fig. 9 of Ref. 35] is given in Table 7.1. The accuracy of our model was estimated by calculating non-prompt losses for extreme cases of

Table 7.1: Non-Prompt Alpha Loss Fraction

<Z>	Loss Fraction, %		Ratio (Ref. 35/Present)
	Ref. 35	Present Work	
1	1.41	0.56	2.5
2	2.06	0.81	2.5
3	2.67	1.00	2.7
4	3.43	1.16	3.0
5	4.14	1.31	3.2

slowing down on constant r -values at $r_{\max}$ and r_i [51]; the variation among these cases is less than a factor of 2 (see Appendix D).

Ref. 35 includes the effects of slowing-down drifts and large banana-width orbits, while our estimate excludes these effects. Consequently,

the factor of 2 difference between our results and Ref. 35 is not surprising. However, slowing-down drifts and large banana-width effects are independent of Z_{eff} [116]. Thus, it is puzzling that both the present work and Ref. 19 find non-prompt losses proportional to $\sqrt{Z_{\text{eff}}}$, while Ohnishi et al. find that these losses increase linearly in Z_{eff} . This difference is unresolved to date [115].

As discussed above, non-prompt losses are one-tenth of the prompt losses, and thus the former are unimportant consideration in the evaluation of alpha heating, even for large Z_{eff} . In contrast, Ohnishi et al. [35] conclude that non-prompt losses significantly reduce alpha heating. This is because they assumed constant current-density and particle-density. When more realistic parabolic J- and n-profiles are used, the losses from the EPR-size device considered by Ref. 35 are comparable to losses in ORNL-EPR.

8. SLOWING-DOWN IN NON-CIRCULAR TOKAMAKS

To fully describe the collisional slowing-down of fast ions in non-circular tokamaks, the formulation of Chapter 7 has been extended to include large banana-width effects.

8.1 Previous Work

Fast ion dynamics have been studied extensively in the small banana-width limit. In the so-called "retardation theory" [1,47, 52-56], energetic ions are viewed as test particles undergoing collisional relaxation in a uniform background plasma. Ion and electron heating rates are obtained from the "instantaneous heating" or "in-situ" deposition model (for example, see Refs. 57-58) by assuming that the energy relaxation time is nearly zero, and thus all the energy is deposited on the flux surface where the particle is born. Numerous workers [18,45,59-76] have calculated fast ion distributions using the small banana-width, bounce-averaged Fokker-Planck equation, including the effects of speed-diffusion, pitch-angle scattering, the velocity-space loss-region, toroidal electric field, impurities, charge exchange, and time delay in heating due to finite slowing down time. Trapped particle contributions have been examined analytically by Cordey [45] and via Monte Carlo calculations by Killeen et al [77] and Goldston [99], yielding inward (outward) slowing-down drifts (relative to the plasma center) for co-(counter) going ions. (The present work finds that these

results need clarification; see Chapter 11.) The additional effect of large energy transfer (LET) interactions has been modeled by the multi-group method [78-80] and by quantum mechanics, including nuclear and coulomb scattering plus spin [81]. However, for alpha particles, LET effects are small for $T_i \sim T_e \leq 20$ KeV, as inferred from Figs. 4-5 of Ref. 82. Still, the above models are inadequate for describing alpha particle effects because the small banana-width approximation is used.

More recently, the detailed properties of large banana-width orbits have been studied [4,13,16-17,35,54,117], and corresponding neoclassical transport models developed. The early attempts to model α heating used only the initial orbits ("single orbit" approximation), described as circles [15] and as finite width bananas [83]. Later, Patrie and Miley [8] calculated alpha power deposition using a spatial- and phase-group method to follow the evolving guiding-center orbits during the slowing-down process. In the light of the present work, the drifts obtained by Ref. 43 are consistent with Refs. 29-30 (see Ch. 11). Ohnishi et al. [35] examined alpha losses to the first wall during slowing down and pitch-angle scattering using a Monte Carlo Technique. Recently, Mikkelsen and Post [84] have developed a Monte Carlo alpha heating code, including first-orbit losses, finite slowing down time, and orbit drifts. Using these techniques, it is relatively easy to obtain moments of the distribution function because integration

smooths out discrete particle effects, but large amounts of computer time (because of slow numerical convergence) are required to find the distribution function itself (for comparison with charge-exchange data). In all the studies cited here, anomalous slowing effects (e.g. Refs. 2 and 33) are ignored.

The present work is based on classical slowing down, which has been well tested by neutral injection experiments at energies, $E \leq 100$ keV. More recently, fusion-produced tritium ($E \leq 1$ MeV) experiments in PLT [100] have demonstrated thermalization consistent with the classical model. Since fast ion transport scales [4] like the (charge)/(momentum), the 1-MeV T results apply to 3.5-MeV alphas as well (c.f. eqn. 2.9).

The present method consists of bounce-averaging the Fokker-Planck equation after the variables have been transformed to the three constants-of-motion (COM) which characterize the orbit of a collisionless particle [13]. This is the only treatment, known to us, that accurately describes fast ion orbits and their distribution function in realistic, high- β , non-circular equilibria like those of Ref. 85. However, Mikkelsen and Post can do such calculations in principle. Recently, Kolesnichenko and Yavorskii [113] have modeled 3.5-MeV alpha losses from non-circular tokamaks, but they did not study slowing down effects. In addition, our model treats first-orbit losses, trapping effects and slowing-down drifts. The COM approach does find the distribution function, and since a 3-D (+ time) equation is solved, it is possible to derive scaling laws.

8.2 Guiding-Center Equation

Three constants-of-motion (COM) are required to fully describe the guiding-center motion of a collisionless particle in an axisymmetric tokamak. The usual COM are E (the energy), μ (the magnetic moment), and P_ϕ (the toroidal canonical angular momentum), but this set is inconvenient for our problem. Following Rowe and Peng [13], we pick v (the ion speed), ψ_x (the maximum value of the poloidal flux function along the guiding-center orbit, where ψ is increasing from the magnetic axis), and ζ (the cosine of the angle between the parallel component of plasma current and the ion velocity, i.e. $\zeta = \vec{J}_\parallel \cdot \vec{v} / (|\vec{J}_\parallel| v)$ at ψ_x). The remainder of this chapter summarizes the pertinent results of Ref. 13, which will then be applied to the present problem.

First to be considered are the COM in terms of E , μ , and P_ϕ , from which the guiding center motion can be derived. The toroidal canonical angular momentum is conserved through first order in gyroradius [86] when written in terms of guiding-center coordinates. Assuming ideal MHD equilibrium [85], P_ϕ can be written in mks units as:

$$P_\phi = m v \eta F / B - Ze \psi = m v \gamma \zeta F_x / B_x - Ze \psi_x, \quad (8.1)$$

where m and Ze are the ion mass and charge, $1/\gamma = \sqrt{1 - (v/c)^2}$,

c = speed of light, $F = F(\psi) = R B_\phi$. Also, η is the cosine of the local pitch angle (i.e. $\eta = \vec{J}_\parallel \cdot \vec{v} / (|\vec{J}_\parallel| v)$ evaluated at ψ), and the subscript "x" in (8.1) denotes evaluation at ψ_x . A cylindrical coordinate system (R, ϕ, z) is used. Quantities obtained from

the MHD equilibrium [85] include: the poloidal flux function, ψ , the magnitude of the magnetic field, B , the function $F(\psi)$, and their derivatives. The magnetic moment can be rewritten in terms of these same variables, yielding:

$$(1-\zeta^2)/B_x = (1-\eta^2)/B. \quad (8.2)$$

Rewriting (8.2), the parallel velocity at any point on the orbit is $v = v_\eta = \pm v \sqrt{1-(1-\zeta^2)B/B_x}$, which can be substituted into (8.1) to obtain the orbit equation, $B = B_0(\psi)$:

$$B = B_x / \left\{ \frac{1}{2} (1-\zeta^2) + \left[\frac{1}{4} (1-\zeta^2)^2 + (F_x/F)^2 [\zeta - (G_x c/v_T) \times (1-\psi/\psi_x)]^2 \right] \right\} \quad (8.3)$$

where the dimensionless constant, $G_x = ZeB_x\psi_x/mcF_x$. Picking the COM coordinates (v, ζ, ψ_x) uniquely defines an orbit, which is parameterized by ψ over the interval $\psi_n \leq \psi \leq \psi_x$. Here, ψ_n is the minimum value of ψ on the orbit.

There is a one-to-one correspondence between the spatial coordinates (R, z) and the (B, ψ) coordinates of the guiding center. Here, the $B = \text{constant}$ and $\psi = \text{constant}$ surfaces constitute a non-orthogonal system which can be inverted to find (R, z) . This is particularly difficult in the equatorial plane ($z=0$) where the B - and ψ - surfaces are parallel. To establish the allowed domain in B - ψ space for all possible orbits, we first plot (see Fig. 8.1) B

Figure 8.1 The correspondence between orbits in (a) (B, ψ) space and (b) configuration (R, z) space (taken from Ref. 13), with the resulting orbit loci in phase space for $v = \text{constant}$. Up-down symmetry permits only $z > 0$ part of orbits.

along the equatorial plane versus ψ (normalized to the ψ -value at the plasma edge, ψ_{edge}). In Fig. 8.1a, the curve segment between points 1 and 2, labelled $(B-\psi)_{\text{IN}}$, is obtained from Fig. 8.1b by moving (on the equatorial plane) from point 1 (at the inboard edge of the equilibrium) to point 2 (at the magnetic axis). Similarly, in Fig. 8.1a, the curve segment between points 2 and 3, labelled $(B-\psi)_{\text{OUT}}$, is obtained from Fig. 8.1b by moving from point 2 to 3 (at the outboard edge of the plasma). Particle orbits move monotonically in ψ , intersecting the $(B-\psi)_{\text{EQ}}$ curve (composed of $(B-\psi)_{\text{IN}}$ and $(B-\psi)_{\text{OUT}}$) usually only twice: at (B_x, ψ_x) and at (B_n, ψ_n) . Several examples of orbit trajectories in $B-\psi$ space are shown in Fig. 8.1a with the corresponding configuration space orbits in Fig. 8.1b[13]. The sequence of co-going orbits (evolving from co-circulating to pinch, trapped, and o-type stagnation) is obtained by decreasing ζ from +1, for a fixed value of ψ_x on the outboard side of the magnetic axis. The corresponding points in $\psi_x-\zeta$ phase space ($v = \text{constant}$) are shown in Fig. 8.1c. Similarly, the sequence of counter-going orbits (evolving from counter-circulating to counter x-type stagnation) is obtained by increasing ζ from -1, for a fixed value of ψ_x on the inboard side of the magnetic axis.

The COM-space topology in the $v = \text{constant}$ plane, is shown in Fig. 8.1c. The boundaries (loss, stagnation orbit) and regions (forbidden, trapped particle) determine the transport mechanisms for

fast ions. Consequently, the detailed features of the (v, ζ, ψ_x) space are examined next.

8.3 Stagnation Orbit Boundary

The stagnation orbits (the counter x-type and o-type stagnation) occur when the $(B-\psi)$ orbit trajectory is tangent to $(B-\psi)_{EQ}$ at ψ_x :

$$B'_{ex} = B'_{ox} = \frac{2B_x \zeta}{1+\zeta^2} \left(\frac{\zeta F'_x}{F_x} - \frac{G_x \zeta}{vY\psi_x} \right) \quad (8.4)$$

Here, B'_{ex} indicates the ψ -derivative (at ψ_x) of the *equilibrium* field in the equatorial plane; B'_{ox} is the ψ -derivative of the *orbit* equation (8.3) at ψ_x . The o-type stagnation orbit (a 'point' orbit in both $B-\psi$ and $R-z$ space) occurs when the curvature and $\vec{B} \times \nabla B$ drifts exactly cancel the poloidal motion along a field line. In Fig. 8.1c, the boundaries ABCD and FG are given by (8.4). The region between these two boundaries is forbidden; corresponding orbits in $(B-\psi)$ space lie below $(B-\psi)_{OUT}$ or above $(B-\psi)_{IN}$ and thus do not constitute physically realizable trajectories in $(R-z)$ space. Since orbits can also be labeled by v, ψ_n , and $\lambda = (\vec{J}_n \cdot \vec{v}) / (|\vec{J}_n| v)$ at ψ_n , the forbidden region also serves to make the labeling unique by grouping all the ambiguous points associated with (v, λ, ψ_n) into the forbidden region.

The second important internal boundary is the locus of pinch orbits (line HI in Fig. 8.1c), occurring when the $(B-\psi)_{\text{orbit}}$ trajectory is tangent to $(B-\psi)_{\text{EQ}}$ at the pinch point, ψ_p and is given by:

$$\begin{aligned} & \eta_p^4 \{-4B_x B_p^3 F_x^2 + B_x^2 [(1-\psi_p/\psi_x)(2B_p F_p' - F_p B_p') + 2B_p F_p]^2\} \\ & + \eta_p^2 \{-4(B_p - B_x) B_p^3 F_x^2 - 2B_x^2 B_p' F_p (\psi_x - \psi_p) [(1-\psi_p/\psi_x)(2B_p F_p' - F_p B_p') + 2B_p F_p]\} \\ & + (B_x B_p' F_p (1-\psi_p/\psi_x))^2 = 0 \end{aligned} \quad (8.5)$$

and

$$\zeta = \sqrt{1 - (1 - \eta_p^2) B_x / B_p} \quad (8.6)$$

The subscript 'p' indicates evaluation at the pinch point (see Figs. 8.1a-b). As before, primed variables are derivatives with respect to ψ . Equation (8.5) is solved by choosing a point (B_x, ψ_x) in the equatorial plane (outboard side), then searching (numerically) through values of (B_p, ψ_p) lying in the equatorial plane (inboard side). The corresponding value of η_p^2 is given by solving the bi-quadratic equation (8.5), $c_1 \eta_p^4 + c_2 \eta_p^2 + c_3 = 0$:

$$\eta_p = -\frac{c_2}{2c_1} \left[1 - \operatorname{sgn} \left(\frac{c_2}{2c_1} \right) \sqrt{1 - 4c_1 c_3 / c_2^2} \right] \quad (8.7)$$

from which ζ is found using (8.6). Note that the minus sign is used

in (8.7) so that $|\eta_p| \leq 1$. The search is successful when the desired value of v/c is obtained from

$$u = \frac{YV}{c} = G_x \eta_p / \left[\frac{\eta_p^2 B_x}{F_x} \left(\frac{F'_p}{B_p} - \frac{F_p B'_p}{B_p^2} \right) - \frac{1}{2} (1 - \zeta^2) \frac{F_p B'_p}{F_x B_p} \right], \quad (8.8)$$

$$\text{with } v/c = u / \sqrt{1+u^2}. \quad (8.9)$$

The end of the pinch boundary (point H in Fig. 8.1c) is found when the inner (counter-going) loop of the x-type stagnation orbit shrinks to a point ($\psi_n = \psi_p$ at point B in Fig. 8.1c) and occurs when $B''_{ep} = B''_{op}$ in the equatorial plane (inboard side). The resulting condition is a biquadratic in η_p (the cosine of the pitch angle at ψ_p), $c_1^4 \eta_{pe}^4 + c_2^2 \eta_{pe}^2 + c_3 = 0$:

$$\begin{aligned} & \eta_{pe}^4 [4F''_p/F_p - 4B'_p F'_p/B_p F_p - 2B''_p/B_p + 3(B'_p/B_p)^2] \\ & + \eta_{pe}^2 [6(B'_p/B_p)^2 - 2B''_p/B_p - 4B'_p F'_p/B_p F_p] - (B'_p/B_p)^2 = 0 \end{aligned} \quad (8.10)$$

which has the solution:

$$\eta_{pe} = - \sqrt{-\frac{c_2^2}{2c_1^4} \left[1 - \operatorname{sgn}\left(\frac{c_2^2}{2c_1^4}\right) \sqrt{1 - 4c_1^4 c_3^2 / (c_2^4)^2} \right]}. \quad (8.11)$$

As before, the minus sign in (8.11) is required for $|\eta_{pe}| \leq 1$.

It is necessary to (numerically) search for the value of ψ_p which produces the desired value of v/c from (8.9) and:

$$u = \frac{YV}{c} = G_p n_{pe} / [n_{pe}^2 F'_p / F_p - \frac{1}{2}(1+n_{pe}^2) B'_p / B_p]. \quad (8.12)$$

The corresponding values of ψ_x and ξ are found by searching for the ψ_x crossing via the orbit equation:

$$B = B_p / \left\{ \frac{1-n_p^2}{2} + \left\{ \left(\frac{1-n_p^2}{2} \right)^2 + \left(\frac{F_p}{F} \right)^2 \left[n_p - \frac{G_p c}{v_Y} \left(1 - \frac{\psi}{\psi_p} \right) \right]^2 \right\}^{\frac{1}{2}} \right\}, \quad (8.13)$$

where $G_p = ZeB_p \psi_p / mcF_p$.

8.4 Connectivity of Orbit Topology

The next important feature of the orbit topology is the connectivity of the two halves of phase space via scattering across the pinch orbit. Figure 8.2 illustrates the ways that scattering occurs among orbits adjacent to the pinch orbit. The arrows, labelled 1 in Figs. 8.2a-b, correspond to scattering from a counter x-type stagnation orbit (...), across the pinch orbit (—), to a fattest-banana (---). The arrows, labelled 2, denote scattering from a fattest banana (---), across the pinch orbit to either a counter x-type stagnation (...) or to an outermost kidney (—) orbit. Finally, the arrows, labelled 3, indicate scattering from an outermost kidney orbit (—), across the pinch orbit (—), to a fattest banana (---). It will be necessary to account for all these processes, in the solution for alpha slowing down, by conserving the resultant velocity-space particle fluxes.

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Figure 8.2 The pinch orbit in (R, z) space, together with neighboring orbits is shown in (a). The corresponding directions of the particle fluxes in (ψ_x, ζ) space is depicted in (b) for $v=\text{constant}$.

8.5 Loss Boundaries

The last important feature is the loss boundary (lines CDE and FIJ in Fig. 8.1c). Losses occur when an orbit intersects the wall at ψ_w , yielding $\psi_x \geq \psi_w$ as the condition for the boundaries DE and FIJ in Fig. 8.1c. Particles which drift or scatter across the boundary CD, in Fig. 8.1c, are also lost to the wall. This corresponds to an orbit evolving from counter-circulating, to counter x-type stagnation (the inner loop of a pinch), to banana (arrow 1 in Fig. 8.2). It is the trapped orbit (fattest banana), which has $\psi_x > \psi_w$ and causes the particle to be lost to the wall.

9. FOKKER-PLANCK EQUATION OF SLOWING DOWN

Here, we find the distribution function in non-local coordinates, (v, ζ, ψ_x) , meaning that particles are assigned to an orbit, rather than the distribution function being given at a local point in configuration- and velocity-space. It is then possible to describe the evolution of the orbits (e.g. slowing down), assuming the slowing-down time, τ_s , is long compared to a bounce time, τ_b . Beginning with the drift-kinetic equation, and expanding in multiple time scales as done in Ref. 18, we obtain a bounce-averaged, drift-kinetic equation on the τ_s time-scale:

$$\frac{\partial f(v, \zeta, \psi_x)}{\partial t} = \frac{1}{\tau_b} \oint d\tau \{ C[\psi; f(v, \zeta, \psi_x)] - \frac{e}{m} \vec{E} \cdot \frac{\partial f}{\partial \vec{v}} + s \} \quad (9.1)$$

The bounce average on the RHS of (9.1) includes all the large banana-width effects. (The details of evaluating these bounce integrals are described in Appendix E.) Here, C is the Fokker-Planck operator, which is time-averaged around an entire bounce orbit (i.e. bounce averaged), and s is the fast ion source. The particle flows in (9.1) arise from collisions and from the electric field, $\vec{E}$, associated with the ohmic heating current. The collision operator, $C(f)$, is a function of the local variables, $(R, z, v, \eta) \leftrightarrow (B, \psi, v, \eta)$, while f is labelled non-locally by the COM variables (v, ζ, ψ_x) . To use (9.1), it must be transformed into the COM space.

9.1 Transformation of Collision Operator

The Fokker-Planck operator used here describes fast ion binary encounters with the background plasma. Fast ion interactions with themselves are neglected since the fp density is much less than the background density. Further, assuming azimuthal velocity-space symmetry for a $1/r^2$ force-law allows the Rosenbluth-MacDonald-Judd form [87] of the collision operator to be written as:

$$C(f) = \frac{-1}{v^2} \frac{\partial}{\partial v} \left[f \left(v^2 \frac{\partial H}{\partial v} + \frac{\partial G}{\partial v} \right) \right] + \frac{1}{2v^2} \frac{\partial}{\partial v^2} \left(v^2 f \frac{\partial G}{\partial v^2} \right) + \frac{1}{2v^3} \frac{\partial G}{\partial v} \frac{\partial}{\partial \eta} \left[(1-\eta^2) \frac{\partial f}{\partial \eta} \right], \quad (9.2)$$

where G and H are the Rosenbluth potentials:

$$G = \sum_{\alpha} \Gamma_{\alpha} \int d^3 v' |\vec{v} - \vec{v}'| f_{\alpha}(v'), \quad (9.3)$$

$$H = \sum_{\alpha} \Gamma_{\alpha} \frac{m_{\alpha} + m}{m_{\alpha}} \int d^3 v' f_{\alpha}(v') / |\vec{v} - \vec{v}'|. \quad (9.4)$$

The index, α , indicates a sum over all background species, with

$$\Gamma_{\alpha} = \frac{Z_{\alpha}^2 Z^2 e^4 \ln \Lambda}{4\pi \epsilon_0^2 m^2}. \quad (9.5)$$

The local derivatives in (9.2) must be transformed to COM coordinates using the chain rule:

$$\left. \frac{\partial}{\partial \eta} \right|_{Rzv} = \left. \frac{\partial}{\partial \eta} \right|_{B\psi v} = \left. \frac{\partial \zeta}{\partial \eta} \right|_{B\psi v} \left. \frac{\partial}{\partial \zeta} \right|_{\psi \psi_x v} + \left. \frac{\partial \psi_x}{\partial \eta} \right|_{B\psi v} \left. \frac{\partial}{\partial \psi_x} \right|_{\psi v \zeta} \quad (9.6)$$

$$\left. \frac{\partial}{\partial v} \right|_{Rz\eta} = \left. \frac{\partial}{\partial v} \right|_{B\psi \eta} = \left. \frac{\partial}{\partial v} \right|_{\psi \psi_x \zeta} + \left. \frac{\partial \zeta}{\partial v} \right|_{B\psi \eta} \left. \frac{\partial}{\partial \zeta} \right|_{\psi \psi_x v} + \left. \frac{\partial \psi_x}{\partial v} \right|_{B\psi \eta} \left. \frac{\partial}{\partial \psi_x} \right|_{\psi v \zeta}, \quad (9.7)$$

where the subscripts indicate the variables held constant during differentiation.

Substituting (9.6) - (9.7) and (8.2) into (9.2) gives:

$$\begin{aligned} C(f) = & \frac{1}{v^2} \frac{\partial}{\partial v} \left[f \left(v^2 \frac{\partial H}{\partial v} + \frac{\partial G}{\partial v} \right) \right] + \frac{1}{2v^2} \frac{\partial^2}{\partial v^2} \left(v^2 f \frac{\partial^2 G}{\partial v^2} \right) \\ & + \frac{1}{2v^3} \frac{\partial G}{\partial v} \left(\frac{\partial \zeta}{\partial \eta} \right)^2 \frac{B}{B_x} \frac{\partial}{\partial \zeta} \left[(1 - \zeta^2) \frac{\partial f}{\partial \zeta} \right] \\ & + A_1 \frac{\partial f}{\partial \zeta} + A_2 \frac{\partial f}{\partial \psi_x} \quad (9.8) \\ & + A_3 \frac{\partial^2 f}{\partial \zeta \partial \psi_x} + A_4 \frac{\partial^2 f}{\partial \psi_x^2} + A_5 \frac{\partial^2 f}{\partial \zeta^2} \\ & + \frac{1}{v^2} \left[\frac{\partial \zeta}{\partial v} \frac{\partial}{\partial v} \left(v^2 \frac{\partial^2 G}{\partial v^2} \frac{\partial f}{\partial \zeta} \right) + \frac{\partial \psi_x}{\partial v} \frac{\partial}{\partial v} \left(v^2 \frac{\partial^2 G}{\partial v^2} \frac{\partial f}{\partial \psi_x} \right) \right], \end{aligned}$$

where the coefficients A_i are:

$$\begin{aligned}
 A_1 = & -\frac{1}{v^2} \frac{\partial \zeta}{\partial v} \left(v^2 \frac{\partial H}{\partial v} + \frac{\partial G}{\partial v} \right) + \frac{1}{2} \frac{\partial^2 G}{\partial v^2} \left(\frac{\partial \zeta}{\partial v} \frac{\partial}{\partial \zeta} \frac{\partial \zeta}{\partial v} + \frac{\partial \psi_x}{\partial v} \frac{\partial}{\partial \psi_x} \frac{\partial \zeta}{\partial v} + \frac{\partial}{\partial v} \frac{\partial \zeta}{\partial v} \right) \\
 & + \frac{1}{2v^3} \frac{\partial G}{\partial v} (1-\zeta^2) \left(\frac{\partial \zeta}{\partial \eta} \frac{\partial}{\partial \zeta} + \frac{\partial \psi_x}{\partial \eta} \frac{\partial}{\partial \psi_x} \right) \left(\frac{B}{B_x} \frac{\partial \zeta}{\partial \eta} \right)
 \end{aligned} \quad (9.9)$$

$$\begin{aligned}
 A_2 = & -\frac{1}{v^2} \frac{\partial \psi_x}{\partial v} \left(v^2 \frac{\partial \psi}{\partial v} + \frac{\partial G}{\partial v} \right) + \frac{1}{2} \frac{\partial^2 G}{\partial v^2} \left(\frac{\partial \zeta}{\partial v} \frac{\partial}{\partial \zeta} \frac{\partial \psi_x}{\partial v} + \frac{\partial \psi_x}{\partial v} \frac{\partial}{\partial \psi_x} \frac{\partial \psi_x}{\partial v} + \frac{\partial}{\partial v} \frac{\partial \psi_x}{\partial v} \right) \\
 & + \frac{1}{2v^3} \frac{\partial G}{\partial v} \left[(1-\zeta^2) \left(\frac{\partial \zeta}{\partial \eta} \frac{\partial}{\partial \zeta} + \frac{\partial \psi_x}{\partial \eta} \frac{\partial}{\partial \psi_x} \right) \left(\frac{B}{B_x} \frac{\partial \psi_x}{\partial \eta} \right) \right. \\
 & \left. - 2 \left(\frac{\partial \zeta}{\partial \eta} \frac{\partial \psi_x}{\partial \eta} \frac{B}{B_x} \right) \zeta \right],
 \end{aligned} \quad (9.10)$$

$$A_3 = \frac{\partial \zeta}{\partial v} \frac{\partial \psi_x}{\partial v} \frac{\partial^2 G}{\partial v^2} + \frac{1}{v^3} \frac{\partial \zeta}{\partial \eta} \frac{\partial \psi_x}{\partial \eta} \frac{B}{B_x} (1-\zeta^2) \frac{\partial G}{\partial v}, \quad (9.11)$$

$$A_4 = \frac{1}{2} \left(\frac{\partial \psi_x}{\partial v} \right)^2 \frac{\partial^2 G}{\partial v^2} + \frac{1}{2v^3} \left(\frac{\partial \psi_x}{\partial \eta} \right)^2 \left(\frac{B}{B_x} \right) (1-\zeta^2) \frac{\partial G}{\partial v}, \quad (9.12)$$

$$A_5 = \frac{1}{2} \left(\frac{\partial \zeta}{\partial v} \right)^2 \frac{\partial^2 G}{\partial v^2}. \quad (9.13)$$

The derivatives in the chain rule are found by taking $\frac{\partial}{\partial \eta} \Big|_{B\psi v}$ and

$\frac{\partial}{\partial v} \Big|_{B\psi \eta}$ of (8.1) and (8.2), then solving the resulting pairs of 2

equations in 2 unknowns for $\partial \zeta / \partial \eta$, $\partial \psi_x / \partial \eta$, $\partial \zeta / \partial v$ and $\partial \psi_x / \partial v$:

$$\frac{\partial \zeta}{\partial \eta} = \frac{\left(\frac{B_x^2}{F_x} \right) \left\{ \frac{B'_{ex} F}{B_x B} (1 - \zeta^2) + \frac{2F_x}{F} \left[\frac{e}{mv_Y} - \zeta \left(\frac{F_x}{B_{ex}} \right) \right] \left[\zeta - \frac{G_x c}{v_Y} \left(1 - \frac{\psi}{\psi_x} \right) \right] \right\}}{(1 + \zeta^2)(B'_{ex} - B'_{ox})}, \quad (9.14)$$

$$\frac{\partial \psi_x}{\partial \eta} = \frac{2B_x^2}{BF_x} \left\{ \frac{F_x^2 B}{FB_x} \left[\zeta - \frac{G_x c}{v_Y} \left(1 - \frac{\psi}{\psi_x} \right) \right] - F\zeta \right\} / (1 + \zeta^2)(B'_{ex} - B'_{ox}), \quad (9.15)$$

$$\frac{\partial \zeta}{\partial v} = \frac{-G_x c \gamma B'_{ex}}{v^2 (B'_{ex} - B'_{ox})} \left(\frac{1 - \zeta^2}{1 + \zeta^2} \right) \left(1 - \frac{\psi}{\psi_x} \right), \quad (9.16)$$

$$\frac{\partial \psi_x}{\partial v} = \frac{2G_x c \gamma B_x}{v^2 (B'_{ex} - B'_{ox})} \left(\frac{\zeta}{1 + \zeta^2} \right) \left(1 - \frac{\psi}{\psi_x} \right). \quad (9.17)$$

The result of this transformation into the COM space is a 3-dimensional, second-order, partial differential equation (PDE) with only the coefficients being bounce-averaged. In contrast, without bounce-averaging, the result is a 4-dimensional, second-order, PDE.

9.2 Simplifying Assumptions

To examine 3.5-Mev alpha particle slowing-down, several additional approximations are possible. Speed-diffusion is important for energies above the initial alpha energy and when:

$$E_\alpha/E_i \leq (m/m_e)^{1/3} (m/m_i)^{2/3} Z_{\text{eff}}^{2/3} = 26.7 Z_{\text{eff}}^{2/3} \quad (9.18)$$

Both limits are unimportant for plasma heating by alphas so speed-diffusion is neglected. In fact, Cordey [65] finds the correction due to speed-diffusion to be $\sim T_i/E_0$, which is a negligible correction ($< 10^{-3}$) in this problem. Charge-exchange losses due to impurities could be significant [88], however the reactor-grade plasmas of interest here require that Z_{eff} (and thus such impurities) be minimized: $Z_{\text{eff}} \sim 1-2$. Consequently, charge exchange losses are neglected because the cross sections are very low for MeV alphas in hydrogenic plasmas. Also, the pitch angle scattering term is small for low Z_{eff} and $v > v_c$, where v_c is the speed at which the slowing drag due to ions equals that due to electrons, with $E_c = \frac{1}{2} m v_c^2 \sim 32.1 T_e$ for α 's in a 50-50 D-T plasma (c.f. eqn. 4.13 of Ref. 18). For these purposes, by the time alphas have slowed to $E \leq E_c$, 90% of their energy has been deposited in the plasma, so the scattering contribution is also dropped. Petrie and Miley [8] have explicitly evaluated the contribution due to pitch-angle scattering and found it to be small. Electric field effects are important only for circulating particles after a large number of bounce periods: $N_b \sim \tau_s/\tau_b \sim 10^5$. This corresponds to energies, $N_b e\phi \leq 10^5$ eV, where ϕ is the toroidal loop potential driving the ohmic current, which is also insignificant for heating calculations. The simplified form describing the alpha slowing-down becomes:

$$\frac{\partial f}{\partial t} = \frac{1}{v^2} \left(\frac{\partial}{\partial v} \tilde{U} f + \widetilde{U \frac{\partial \zeta}{\partial v}} \frac{\partial f}{\partial \zeta} + \widetilde{U \frac{\partial \psi_x}{\partial v}} \frac{\partial f}{\partial \psi_x} \right) + \tilde{s} \quad , \quad (9.19)$$

where $U = - \left(v^2 \frac{\partial H}{\partial v} + \frac{\partial G}{\partial v} \right)$, and the bounce-average, $\tilde{D}$, of some quantity, D , is $\tilde{D} = \frac{1}{\tau_b} \int_0^{\tau_b} d\tau D$. While low Z_{eff} was assumed in deriving this last equation, all the terms in (3.19) are rigorously independent of Z_{eff} . The resulting 3-dimensional first-order partial differential equation, together with the boundary conditions, in (v, ζ, ψ_x) space is straight-forward to solve.

10. ALPHA PARTICLE SOURCE FUNCTION

The first step in solving (9.19) is determining the initial conditions, i.e. the alpha source rate, at $v = v_0$, where v_0 is the birth speed corresponding to 3.5 MeV. This is done by integrating (9.19) over an element of 6-space, d^3vd^3x , from $v_0 - \Delta v \leq v \leq v_0 + \Delta v$, then taking the limit $\Delta v \rightarrow 0$. Integrating by parts, and noting that there cannot be an accumulation of particles in any volume element, the result is:

$$f(v_0, \zeta_0, \psi_{x0}) = \tilde{a}(v_0, \zeta_0, \psi_{x0}) / 4\pi \tilde{U}(v_0, \zeta_0, \psi_{x0}). \quad (10.1)$$

The 6-space volume element used above is most easily found through the pair of transformations $(R, z, v, \eta) \rightarrow (B, \psi, v, \eta) \rightarrow (\psi, \psi_x, v, \zeta)$, yielding

$$d^3vd^3x = (2\pi v^2 dv d\zeta)(2\pi R d\psi_x d\psi) / J_1 \left(\frac{B, \psi}{R, z} \right) J_2 \left(\frac{\psi_x, \zeta}{B, \eta} \right), \quad (10.2)$$

where the Jacobians are

$$J_1 \left(\frac{B, \psi}{R, z} \right) = R |\vec{B} \cdot \nabla B|, \text{ and} \quad (10.3)$$

$$J_2 \left(\frac{\psi_x, \zeta}{B, \eta} \right) = 2B_x^3 F[1 - \frac{1}{2}(1 - \zeta^2)B/B_x] / F_x B^3 (1 + \zeta^2) |B'_{ex} - B'_{ox}|. \quad (10.4)$$

Combining (10.2) - (10.4), d^3vd^3x can be written as:

$$d^3vd^3x = 4\pi v^2 J dv d\psi_x d\zeta \left(\frac{d\tau'}{d\psi} \right) d\psi, \quad (10.5)$$

where J is the 'net' Jacobian of the transformation

$$J = \pi F_x (1 + \zeta^2) |B'_{ex} - B'_{ox}| / B_x^3. \quad (10.6)$$

The term $(d\tau'/d\psi)$ is obtained from the bounce-time differential,

$$d\tau' = d\tau / \left(\frac{2e}{\gamma m v^2} \right), \text{ where}$$

$$\tau_b = \oint d\tau = 2 \int_{\psi_n}^{\psi_x} \frac{d\psi}{|\vec{v}_d \cdot \nabla \psi|} = \frac{2e}{\gamma m v^2} \int_{\psi_n}^{\psi_x} \frac{B^3 d\psi}{F |\vec{B} \cdot \nabla B| [1 - B(1 - \zeta^2)/2B_x]}, \quad (10.7)$$

and $\vec{v}_d$ is the particle guiding-center drift velocity. The extra factor of two in (10.5) and in (10.7) is due to up-down symmetry in $\pm z$. The remaining problem is to evaluate (10.1).

10.1 Calculation of Integrals

Our calculations are performed as follows. First, a point in (v, ζ, ψ_x) space, corresponding to a particular orbit, is chosen, so as to lie outside the forbidden region defined by (8.4). Next, the sequence of points, $(\psi_0, B_0(\psi_0; v, \zeta, \psi_x))$, are calculated along this orbit between $\psi_n \leq \psi_0 \leq \psi_x$. The value of ψ_n is found numerically as the point at which the orbit in B - ψ space crosses the $(B-\psi)_{EQ}$ curve (c.f. Fig. 8.1a). Third, for each point, (B_0, ψ_0) , the corresponding spatial coordinate, (R_0, z_0) , is found numerically by solving:

$$\left. \begin{aligned} B_e(R_0, z_0) &= B_0 \\ \psi_e(R_0, z_0) &= \psi_0 \end{aligned} \right\}, \quad (10.8)$$

using the non-linear equation solver HYBRD1[89]. Here, the poloidal flux function for the equilibrium, ψ_e , is found by Lagrangian interpolation of ψ -values given on an (R_i, z_j) mesh. The equilibrium $\vec{B}$ -field, $B_e = |\vec{B}_e|$ is similarly found from [85]:

$$\vec{B}_e = \left[-\hat{R} \frac{\partial \psi}{\partial z} + \hat{z} \frac{\partial \psi}{\partial R} + \hat{\phi} F(\psi) \right] / R, \text{ so that} \quad (10.9)$$

$$B_e = \sqrt{|\nabla \psi|^2 + F^2} / R. \quad (10.10)$$

Also, at each spatial point on the orbit, $\vec{B} \cdot \nabla B$ is obtained:

$$\begin{aligned} \vec{B} \cdot \nabla B = & \frac{1}{BR^4} \frac{\partial \psi}{\partial z} \left[F^2 + \left(\frac{\partial \psi}{\partial R} \right)^2 + \left(\frac{\partial \psi}{\partial z} \right)^2 \right] + \frac{1}{BR^3} \left\{ \frac{\partial \psi}{\partial R} \frac{\partial \psi}{\partial z} \left(\frac{\partial^2 \psi}{\partial z^2} - \frac{\partial^2 \psi}{\partial R^2} \right) \right. \\ & \left. + \frac{\partial^2 \psi}{\partial R \partial z} \left[\left(\frac{\partial \psi}{\partial R} \right)^2 - \left(\frac{\partial \psi}{\partial z} \right)^2 \right] \right\}, \end{aligned} \quad (10.11)$$

and used to evaluate the integrand of (10.7). Finally, the bounce-averaged quantities $\tilde{s}$ and $\tilde{U}$ in (10.1) are determined numerically (see Appendix E).

10.2 Results

Figure 10.1 shows contours of constant τ_b in (ψ_x, z) space for 3.5-MeV alphas in a D-shaped tokamak reactor having $\bar{B} = 0.8\%$. In comparing this to Fig. 8.1c, all the features of the phase space topology are present. The prominent aspects include the absence of orbits in the forbidden region (blank portion about $z = 0$ and near

Figure 10.1 Contours of constant bounce time, $\tau_b(\mu s)$, in (ψ_x, ζ) space for 3.5-MeV alphas in a D-shaped tokamak equilibrium ($\beta = 0.8\%$).

$\psi_x/\psi_e = 0$), $\tau_b \rightarrow \infty$ along the left-hand side of the forbidden region (for counter-going, x-type stagnation orbits, curve BCD in Fig. 8.1c), the transition to finite τ_b at the end of the pinch locus (point H in Fig. 8.1c), and the transition to finite τ_b at the x-type to o-type stagnation-orbit transition (point B in Fig. 8.1c). Hinton et. al. [98] have examined the transport for particles on or near x-type stagnation orbits and found that a banana-plateau regime model is required, since $\tau_b \rightarrow \infty$. Here, we assume that slowing down drifts carry fast ions across such regions rapidly enough that the contribution from plateau-like regimes is small. As expected, circulating ions ($\zeta \rightarrow \pm 1$) have the shortest τ_b , since they circulate along an average flux surface. The τ_b -value for circulating orbits ($\zeta \sim \pm 1$) increases with increasing ψ_x because the distance that the ion travels around its average flux surface increases. Also, as expected, $\tau_b \sim 1/v$, even for 3.5-Mev alphas; this is because the distance along the orbit, i.e. the connection length, is essentially independent of v . These features of τ_b in (ψ_x, ζ) space thus reinforce the orbit topology concepts discussed in Chapter 8.

Another set of important results, which are readily calculated at this stage, are the flux-surface-averaged source rates for alpha density and current. At the birth energy, (9.1) becomes:

$$\left. \frac{\partial f}{\partial t} \right|_{v=v_0} = \tilde{s} \quad (10.12)$$

from which the flux-surface-averaged alpha source rate, $\langle \dot{n}_\alpha \rangle$ is:

$$\langle \dot{n}_\alpha \rangle = \int \left. \frac{\partial f}{\partial t} \right|_{v_0} d^3 v d^3 x \bigg/ \int d^3 x = \int \tilde{s} d^3 v d^3 x \bigg/ \int_{\psi_1}^{\psi_2} d^3 x, \quad (10.13)$$

and the parallel component of flux-surface-averaged alpha current rate, $\langle \dot{J}_\parallel \rangle$, is:

$$\langle \dot{J}_\parallel \rangle = \int \left. \frac{\partial f}{\partial t} \right|_{v_0} e v_\parallel d^3 v d^2 x \bigg/ \int d^2 x = \int \tilde{s} e v_\parallel \frac{d^3 v d^3 x}{2\pi R} \bigg/ \int_{\psi_1}^{\psi_2} d^2 x. \quad (10.14)$$

The integrals $\int_{\psi_1}^{\psi_2} d^3 x$ and $\int_{\psi_1}^{\psi_2} d^2 x$ are the volume and area, respectively,

between the flux surfaces labelled by ψ_1 and ψ_2 , where $d^2 x = dR dz = d^3 x / 2\pi R$. The corresponding integral over $d^3 v d^3 x$ also requires that the $d\psi$ piece have the limits $\int_{\psi_1}^{\psi_2} \left(\frac{d\tau}{d\psi} \right) d\psi$, with a point-wise source rate, s :

$$s = n_1(\psi) n_2(\psi) \langle \sigma v(T(\psi)) \rangle_{12} \delta(v - v_0) / 4\pi v^2. \quad (10.15)$$

Making these substitutions into (10.13)-(10.14) yields:

$$\langle \dot{n}_\alpha \rangle = \int_0^{\psi_e} d\psi_x \int_{-1}^1 d\zeta \widetilde{n_1 n_2 \langle \sigma v \rangle} J(\psi_x, \zeta) \int_{\psi_1}^{\psi_2} \left(\frac{d\tau}{d\psi} \right) d\psi \bigg/ \int_{\psi_1}^{\psi_2} d^3 x, \quad (10.16)$$

$$\langle \dot{J}_\parallel \rangle = \frac{e v_0}{2\pi} \int_0^{\psi_e} d\psi_x \int_{-1}^1 d\zeta \widetilde{n_1 n_2 \langle \sigma v \rangle} J(\psi_x, \zeta) \int_{\psi_1}^{\psi_2} \left(\frac{d\tau}{d\psi} \right) \frac{n d\psi}{R} \bigg/ \int_{\psi_1}^{\psi_2} d^2 x. \quad (10.17)$$

These integrals have been evaluated numerically with an error of $\leq 1\%$ using 128×128 equal intervals in (ψ, ζ) space. We assume a 50-50 D-T plasma with parabolic density profiles:

$$n_D = n_T = n_i(\psi) = n_0[1 - .99(\psi/\psi_e)^2] \quad (10.18)$$

The temperature is self-consistently determined from the pressure profile, $p(\psi)$, used to determine the equilibrium:

$$T_i(\psi) = p(\psi)/4kn_i(\psi), \quad (10.19)$$

where k is the Boltzmann constant. The electron density, n_e , is taken as $n_e = 2n_D$; the electron temperature is assumed equal to the ion temperature, T_i . The central temperature was taken at 15 keV, and the corresponding n_0 determined from (10.19), with $\langle \sigma v \rangle$ taken from Ref. 32.

Figure 10.2 shows $\langle \dot{n}_\alpha \rangle$ versus ψ/ψ_{edge} for two D-shaped equilibria (See Table 10.1). For comparison, the point-wise source rate, $\dot{n}_\alpha = n_1 n_2 \langle \sigma v \rangle_{12}$, is also plotted. The particle source rate difference, ϵ , is given in Table 10.1 for each case, where

$$\epsilon = (\dot{n}_\alpha - \langle \dot{n}_\alpha \rangle) / \langle \dot{n}_\alpha \rangle. \quad (10.20)$$

Since $\epsilon \leq 5\%$ for these cases for $\psi/\psi_{\text{edge}} \leq 0.4$, it is concluded that the "in-situ" deposition model is adequate for determinations of the total (ion+electron) interior heating. This is because the heating rate goes like $\dot{E} \sim E_0 \langle \dot{n}_\alpha \rangle$, where $E_0 = \frac{1}{2} m v_0^2$.

Figure 10.2 Flux-surface-averaged particle source rate, $\langle \dot{n}_\alpha \rangle$, and current source rate, $\langle \dot{J}_\parallel \rangle$, for 3.5-MeV α -phas in D-shaped ETF equilibria (see Table 10.1).

Table 10.1. Non-Circular Tokamak Parameters

	Low β	Medium β	
$\bar{\beta}$	0.008	0.075	
$B_p(a)$	0.389	2.688	} given from equilibrium data
$q(a)$	3.207	3.353	
$J_{p1}(MA)$	5.78	5.95	
$B_0(T)$	5.3	4.3	
$R_0(m)$	5.035	5.356	
$\psi_{edge}(Wb)$	3.675	2.985	
$\epsilon(\%)$	4.7	5.4	} calculated in present work
$\langle \dot{I}_u \rangle (MA/s)$	0.021	0.81	
$\tau_s(s)$	0.9	0.1	
$\tau_s \langle \dot{I}_u \rangle (MA)$	0.02	0.08	
$\langle I_u \rangle (kA)^*$	6.3	26.	

* For a discussion of convergence of $\langle I_u \rangle$, see Appendix G.

Figure 10.2 also shows $\langle \dot{J}_\parallel \rangle$ versus ψ/ψ_{edge} , with the values of $\langle \dot{I}_\parallel \rangle$ given in Table 10.1. Small banana-width models are inadequate to describe this current, which is a large banana-width effect. McNally [90] has previously estimated the alpha-driven current using scaling arguments. However, for the purposes of this section, the current is estimated as the current source rate times the characteristic slowing time:

$$I_\parallel \sim \langle \dot{I}_\parallel \rangle \tau_s \quad (10.21)$$

where τ_s is the momentum slowing down time (cf. eqn. 11.11). The corresponding estimates for sample cases are given in Table 10.1. In Chapter 11, we improve on this estimate by calculating $\langle I_\parallel \rangle$ as the first moment of the slowing down distribution.

11. SLOWING DOWN SOLUTION

11.1 Method of Solution

Application of the method of characteristics to (9.19) yields the slowing-down distribution. For $v < v_0$, (9.19) can be rewritten as:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \dot{v} \frac{\partial f}{\partial v} + \dot{\zeta} \frac{\partial f}{\partial \zeta} + \dot{\psi}_x \frac{\partial f}{\partial \psi_x} = \frac{f}{v^2} \frac{\partial \tilde{U}}{\partial v}, \quad (11.1)$$

where the slowing-down drift rates are found using (9.16)-(9.17):

$$\dot{v} = \frac{-\tilde{U}}{v^2} \quad (11.2)$$

$$\dot{\zeta} = \frac{G_x c \gamma B'_{ex}}{v^4 \psi_x (B'_{ex} - B'_{ox})} \left(\frac{1 - \zeta^2}{1 + \zeta^2} \right) \widetilde{U(\psi_x - \psi)}, \quad (11.3)$$

$$\dot{\psi}_x = \frac{-2G_x c \gamma B_x}{v^4 \psi_x (B'_{ex} - B'_{ox})} \left(\frac{\zeta}{1 + \zeta^2} \right) \widetilde{U(\psi_x - \psi)}. \quad (11.4)$$

Integration of (11.1), using (11.2) to write, $dt = -v^2 dv / \tilde{U}$, gives:

$$f = f(v_0, \zeta_0, \psi_{x0}) \tilde{U}(v_0, \zeta_0, \psi_{x0}) / \tilde{U}(v, \zeta, \psi_x), \quad (11.5)$$

which can be further simplified by substitution from (10.1):

$$f = s(v_0, \zeta_0, \psi_{x0}) / 4\pi \tilde{U}(v, \zeta, \psi_x). \quad (11.6)$$

Note that (11.6) makes sense only when the coordinates (v, ζ, ψ_x) lie along the characteristic obtained by solving the system of ordinary differential equations (11.2)-(11.4) with the initial condition $(v_0, \zeta_0, \psi_{x0})$.

11.1.1 The Invariant, μ/E

An important property of the slowing down distribution can be derived from this solution. By taking the ratio of (11.3) and (11.4) and integrating the resulting exact differential, the result is:

$$\mu/E = [1-\zeta^2(t)]/B_x(t) = \text{constant.} \quad (11.7)$$

Physically, this invariant arises because there is a negligible amount of pitch angle scattering during the slowing down. Alternately, the multiple time-scale discussion by Trubnikov [46] can be used to infer (11.7). This result can also be found by following Hazeltine's derivation [96] of the gyro-averaged derivative of μ , beginning with the Lorentz equations and including slowing-down losses. Meka [97, 121] previously obtained (11.7) for α 's slowing in low- B tokamaks assuming the Knorr-form [23] for the magnetic field.

11.1.2 Slowing Down Drifts

The direction of the drifts in (11.3)-(11.4) is also important:

$$\text{sgn}(\dot{\zeta}) = \text{sgn}\left(\frac{B'_{ex}}{B'_{ex} - B'_{ox}}\right) = \begin{cases} +, R_x > R_0 \text{ and } B'_{ex} > 0 \\ -, \text{ otherwise} \end{cases} \quad (11.8a)$$

$$\text{sgn}(\dot{\psi}_x) = \text{sgn}\left(\frac{-\zeta}{B'_{ex} - B'_{ox}}\right) = \begin{cases} +, R_x > R_0 \text{ and } B'_{ex} > 0 \text{ and } \zeta < 0 \\ -, \text{ otherwise} \end{cases} \quad (11.8b)$$

As illustrated in Fig. 11.1, fast ions approach some "nominal" flux surface as they slow down to form small banana-width (thermalized) orbits. Since co-circulating and trapped orbits have ψ_x shifted outward in R , the corresponding $\dot{\psi}_x$ -drift is inward ($\dot{\psi}_x < 0$). Likewise, since counter-circulating orbits have ψ_x shifted inward in R , the $\dot{\psi}_x$ -drift is toward the magnetic axis ($\dot{\psi}_x < 0$). The positive $\dot{\psi}_x$ drift is a new (though small) finite- β effect occurring only for counter-going ions, where $R > R_0$ in the portion of the minimum- B well for which $B'_{ex} > 0$. The corresponding region of COM space is small, lying between $\zeta = 0$ and the right-hand o-type stagnation orbit boundary. The $\dot{\zeta}$ -drift occurs to conserve μ/E in the presence of the $\dot{\psi}_x$ -drift. These results are consistent with the inward drifts for co-going particles ($\dot{\psi}_x < 0$ for $\zeta > 0$) found by Cordey [45] and Killeen et al [77], but are in apparent contradiction with the outward drifts for counter-going ions found by Refs. 45 and 77. However, this discrepancy is due to an ambiguity in definitions for $\zeta = v_{||}(\psi=\psi_x)/v$, used here, and $\xi = v_{||}(\theta=0)/v$, used in Refs. 18, 45 and 77. The definition for $\xi > 0$ corresponds to $\dot{\psi}_x$, which is consistent with the usage here, but $\xi < 0$ corresponds to evaluation at ψ_n for which the drift is outward (c.f. Fig. 11.1) and is consistent with Refs. 45 and 77. The results of Petrie and Miley [8] also show a net inward, slowing-down drift by using a model which apportions all the slowing contributions at the two points $\psi = \psi_x$ and $\psi = \psi_n$.

The magnitude of the drifts is an important result. In the limit $v_{ion} \ll v \ll v_e$, the parameter, U , in (9.19) becomes:

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--- NOMINAL FLUX SURFACE
 -.- FAST ION ORBIT
 — THERMALIZED ORBIT

CO-GOING

TRAPPED

COUNTER-GOING

Figure 11.1 Representation of fast ion approaching some nominal flux surface during thermalization for (a) co-going, (b) trapped, and (c) counter-going ions.

$$U = (v_c^3 + v^3)/\tau_s, \quad (11.9)$$

$$v_c^3 = \frac{3\sqrt{\pi}}{4} \frac{m_e}{m_i} v_e^3, \quad (11.10)$$

$$\tau_s = \frac{3\pi^{3/2} \epsilon_0^2 m_e m v_e^3}{Z^2 e^4 n_e \ln \Lambda}. \quad (11.11)$$

As before, m_e and m_i are the electron and background ion masses; n_e is the electron particle density, and $v_e = \sqrt{2kT_e/m_e}$. Combining (11.9)-(11.11) with (11.2-11.4), and applying simple scaling arguments, one finds for typical ETF equilibria:

$$\frac{\dot{\psi}_x}{\psi_x - \psi_n} \sim \alpha_\psi \frac{\dot{v}}{v}, \text{ with } 0.25 < \alpha_\psi < 0.7, \quad (11.12)$$

$$\dot{\zeta} \sim \alpha_\zeta \frac{\dot{v}}{v_0}, \text{ with } 0 < |\alpha_\zeta| < 0.05. \quad (11.13)$$

Since $(\psi_x - \psi_n)$ is a measure of the poloidal gyroradius, (11.12) implies that the net slowing-down drift in ψ_x is $\sim 1/2$ of a banana width in slowing from velocity, v , to thermal. This is consistent with the arguments of the previous paragraph. The ζ -drift, while small, is sufficient to eliminate any net thermal α -current. Numerical calculations show that (11.12)-(11.13) are valid from 3.5 MeV to thermal energies. Since the poloidal gyroradius is proportional to velocity, we find

$$\psi_{x0} - \psi_x(t) \sim \alpha_\psi (\psi_x - \psi_n)_0 (1 - v/v_0), \quad (11.14)$$

where $(\psi_x - \psi_n)_0$ indicates $(\psi_x - \psi_n)$ at $v = v_0$, and similarly that:

$$z(t) - z_0 \sim \alpha_z (1 - v/v_0). \quad (11.15)$$

Thus, the coefficients α_ψ and α_z characterize not only the drift rates but the total drifts in slowing to thermal speeds. These results apply to both of the equilibria listed in Table 10.1.

11.1.3 Method of Characteristics Solution

With the above properties in hand, the system (11.2)-(11.4) can be reduced to the solution of a single differential equation. Since the flux-surface-averaged quantities (c.f. eqns. 11.21-11.25) require a knowledge of the distribution function over (v, z, ψ_x) space, it is unnecessary to know the explicit time dependence. (Even for non-steady-state problems, not of interest here, the explicit time dependence of $f(v, z, \psi_x, t)$ must be found only if the time-scale for evolution of the equilibrium is $\leq \tau_s$.) We therefore choose the speed, v , as the independent variable; this choice permits the differential equations to be rewritten as:

$$\frac{dz}{dv} = \frac{\dot{z}}{\dot{v}} = \frac{-G_x c \gamma B'_{ex}}{v^2 \psi_x (B'_{ex} - B'_{ox})} \left(\frac{1 - z^2}{1 + z^2} \right) \frac{\widetilde{U(\psi_x - \psi)}}{\widetilde{U}}, \quad (11.16)$$

$$\frac{d\psi_x}{dv} = \frac{\dot{\psi}_x}{\dot{v}} = \frac{2G_x c \gamma B_x}{v^2 \psi_x (B'_{ex} - B'_{ox})} \left(\frac{z}{1 + z^2} \right) \frac{\widetilde{U(\psi_x - \psi)}}{\widetilde{U}} \quad (11.17)$$

In addition, (11.7) can be used to eliminate (11.16):

$$\zeta = \pm \sqrt{1 - \frac{B_x}{B_{x0}} (1 - \zeta_0^2)} \quad , \quad (11.18)$$

in which the sign ambiguity is resolved from:

$$\text{sgn}(\zeta^{i+1}) = \begin{cases} +, R_x > R_0 \text{ and } \zeta^{(i)} > 0, \\ +, R_x > R_0, \zeta^{(i)} < 0, \text{ and } |\zeta^{(i+1)}| > |\zeta^{(i)}|, \\ -, R_x < R_0, \end{cases}$$

where $\zeta^{(i)}$ corresponds to $v^{(i)}$ and $\zeta^{(i+1)}$ to $v^{(i)} + \Delta v$ in the numerical integration. When $R_x > R_0$, $\zeta^{(i)} < 0$, and $|\zeta^{(i+1)}| \leq |\zeta^{(i)}|$, the sign ambiguity is resolved by numerically integrating (11.16); this is a very small class of cases, corresponding to orbits having $\tilde{\psi}_x > 0$ (discussed above). The solution to (11.17)-(11.18) is of the form:

$$\psi_x = \psi_x(v_i; v_0, \zeta_0, \psi_{x0}), \text{ and} \quad (11.19)$$

$$\zeta = \zeta(v_i; v_0, \zeta_0, \psi_{x0}). \quad (11.20)$$

Solving (11.16)-(11.17) completely describes the slowing-down problem, including the large banana width effects and slowing-down drifts in realistic non-circular, high- β equilibria.

11.1.4 Calculation of Integrals

To numerically evaluate the flux-surface averages, it is easiest to find $f(v_i, \zeta_j, \psi_{xk})$ on a specified rectilinear mesh, where $v_i = i\Delta v$, $\zeta_j = j\Delta\zeta$, $\psi_{xk} = k\Delta\psi_x$ for integral values of (i, j, k) . This is done by taking $(v_i, \zeta_j, \psi_{xk})$ as the starting point and integrating (11.17) backward in time to $v = v_0$, with corresponding values $\zeta(v_0) = \zeta_0$ and $\psi_x(v_0) = \psi_{x0}$. Using the source rate, $\tilde{s}(v_0, \zeta_0, \psi_{x0})$, found in Chapter 10, the distribution-function is obtained from (11.6). At each step in the v -integration, it is necessary to check that $\psi_x \leq \psi_{\text{edge}}$; if $\psi_x > \psi_{\text{edge}}$, then the starting point lies in a region of zero distribution function. That is, the inward slowing-down drifts carry the fast ions about half of a banana width inside the plasma edge, leaving no fast ions at the edge. The important flux-surface-averaged quantities are as follows: the alpha particle density profile,

$$\langle n_\alpha \rangle = \int f_\alpha d^3x d^3v \bigg/ \int_{\psi_1}^{\psi_2} d^2x, \quad (11.21)$$

the alpha-particle current density profile,

$$\langle J_\alpha \rangle = \int ev_\parallel f_\alpha d^2x d^3v \bigg/ \int_{\psi_1}^{\psi_2} d^2x, \quad (11.22)$$

the profile of alpha heating power per ion,

$$\langle P_e \rangle = \int f_\alpha m v \dot{v}_e d^3x d^3v \bigg/ \int_{\psi_1}^{\psi_2} n_e d^2x, \quad (11.23)$$

$$\langle P_i \rangle = \int f_{\alpha} m v \dot{v}_i d^3 x d^3 v / \int_{\psi_1}^{\psi_2} n_i d^3 x, \quad (11.24)$$

and the net alpha particle current,

$$\langle I_{\alpha} \rangle = \int e v n f_{\alpha} d^2 x d^3 v, \quad (11.25)$$

where $\dot{v}_i = \widetilde{v^3/\tau_s}$, and $\dot{v}_e = \widetilde{v_c^3/\tau_s}$, are the deceleration rates due to ions and electrons, respectively. These integrals are evaluated in the same way as the source rate integrals in Chapter 10, c.f. (10.16)-(10.17).

11.1.5 Boundary Conditions

The boundary conditions, at the o-type and x-type stagnation boundaries, are the only remaining aspect of this problem. Considering first the o-type stagnation boundary (lines A-B and F-G in Fig. 8.1c), the corresponding surface in (v, ζ, ψ_x) space can be written using (8.4):

$$W = B'_{ex} - B'_{ox}(v, \zeta, \psi_x) = 0 \quad (11.26)$$

where B'_{ox} is given by the RHS of (8.4). The normal to this surface is ∇W , while the tangent to a slowing-down characteristic is obtained from (11.2)-(11.4):

$$\frac{d\vec{v}}{dt} = \dot{v}\hat{v} + \dot{\zeta}\hat{\zeta} + \dot{\psi}_x\hat{\psi}_x. \quad (11.27)$$

Evaluating the dot product, $\nabla W \cdot d\vec{v}/dt$, using (11.26), and the condition that $B_{ox} = B_{on}$ for an o-type stagnation orbit, gives

$$\nabla W \cdot d\vec{v}/dt = \frac{\partial W}{\partial v} \dot{v} + \frac{\partial W}{\partial \zeta} \dot{\zeta} + \frac{\partial W}{\partial \psi_x} \dot{\psi}_x = 0. \quad (11.28)$$

This condition implies that characteristics in the vicinity of the o-type stagnation boundary (surface) move parallel to the boundary, i.e. have no component perpendicular to it. Consequently, no conservation condition is needed at this boundary. Looking next at the counter-going x-type boundary (line BCD in Fig. 8.1c), one sees from (11.8) that slowing drifts take particles away from this boundary because ζ decreases monotonically. Thus, conservation conditions are not needed at counter-going x-type stagnation boundary. Finally, at the (x-type) pinch boundary (line HI in Fig. 8.1c), the ζ -drifts (c.f. eqn. 11.8) carry the characteristics in the direction of decreasing ζ (arrows labelled 3 in Fig. 8.2b). Since adjacent segments across the pinch boundary are contiguous, velocity-space fluxes are trivially conserved. Here, as is typically the case in method-of characteristics solutions, the boundary conditions are implicitly included in the solution.

11.2 Results

The alpha particle profiles (c.f. eqns. 11.21-11.25) have been evaluated for three models. First, for comparative purposes, in-situ deposition is used to determine the electron and ion heating rate profiles:

$$(P_e)_{IS} = E_0 \int_{\psi_1}^{\psi_2} G_e n_i n_2 \langle \sigma v \rangle_{12} d^3 x / \int_{\psi_1}^{\psi_2} n_e d^3 x, \quad (11.29)$$

$$(P_i)_{IS} = E_0 \int G_i n_i n_2 \langle \sigma v \rangle_{12} d^3 x / \int_{\psi_1}^{\psi_2} n_i d^3 x. \quad (11.30)$$

Here, $G_e = 1 - G_i$, and G_i is the fraction of power into ions found by Stix [54]:

$$G_i = \left(\frac{v_c}{v_0} \right)^2 \int_0^{(v_0/v_c)^2} \frac{dx}{1+x^{3/2}}. \quad (11.31)$$

For $v_0/v_c > 1$, (11.31) can be conveniently expanded in a Taylor series and integrated to yield:

$$G_i = \left(\frac{v_c}{v_0} \right) \left(1 - \frac{1}{4y} + \frac{1}{7y^2} - \frac{1}{10y^3} + \frac{1}{13y^4} - \dots \right), \quad (11.32)$$

where $y = (v_0/v_c)^3$. For $v_0/v_c > 2$, (11.32) converges well; for $v_0/v_c < 2$, (11.31) is evaluated numerically. In the second model, the ζ - and ψ_x -drifts are set to zero, to demonstrate the size of their effect on the profiles. The distribution function in this approximation is:

$$f(v, \zeta, \psi_x) = s(v_0, \zeta_0, \psi_{x0}) / 4\pi \tilde{U}(v, \zeta_0, \psi_{x0}). \quad (11.33)$$

Finally, the large-banana-width slowing-down distribution, including ξ - and ψ_x -drifts is obtained by integrating (11.17) via a second-order Runge-Kutta method.

The results of these three models for various equilibria (c.f. Table 10.1) are shown in Figs. 11.2-11.3. The prominent peakedness of the in-situ heating curves is strongly weighted by the alpha source function, $\dot{n}_\alpha = n_1 n_2 \langle \sigma v \rangle_{12}$, as the alphas slow down at their birth positions. Large banana-width orbits smear the heating deposition over large regions of the plasma (c.f. Fig. 10.2). This finite-banana spreading causes the flux-surface-averaged profiles without drifts ($\dot{\psi}_x=0$, $\dot{\xi}=0$) to be less centrally peaked, but substantially above the in-situ curve at the edge ($\psi/\psi > 0.6$). The flux-surface-averaged profiles including drifts ($\dot{\psi}_x \neq 0$, $\dot{\xi} \neq 0$) lie below the second case since the inward ψ_x -drift causes slightly more core heating while depleting the edge.

The net positive alpha current (in the direction of the plasma current) is due to finite banana-width effects (see Figs. 11.2-11.3). From the values of $\langle I_\alpha \rangle$ shown in Table 10.1, we find an improved estimate for the alpha current is:

$$I_\alpha \sim .3 \langle I_\alpha \rangle \tau_S. \quad (11.34)$$

The current contributions due to co- and counter-going orbits tend to cancel, leaving mainly the trapped contribution (consistent with Cordey's result [45]). For large banana-width trapped orbits, the path length along the co-going part is longer than the counter-going

Figure 11.2. Alpha particle density, current and heating profiles in a D-shaped ETF ($\bar{\beta} = 0.008$) reactor. In-situ deposition is indicated by P_e ; flux-surface-averaged profiles, without the ψ_x and ζ -drifts, are labelled with the subscript "ND," with the ψ_x and ζ -drifts, the subscript "D" is used.

Figure 11.3. Alpha particle density, current and heating profiles in a D-shaped ETF ($\bar{\beta} = 0.075$) reactor. In-situ deposition is indicated by $P_{e,i}$; flux-surface-averaged profiles, without the ψ_x and ζ -drifts, are labelled with the subscript "ND," with the ψ_x and ζ -drifts, the subscript "0" is used.

part. This results in a net co-current, since crudely speaking, the travel time goes like (connection length)/v. More realistically, most trapped particles have a net co-current (as described above), while a few trapped ions have a net counter-current. The x-stagnation orbits are an example of the latter because the bounce time contribution becomes infinite at the pinch point, where the motion is counter-going. From eqn. E.13 (see Appendix E), the resulting current is then $I_s \propto N_s Z e v \eta_p < 0$, where N_s is the number of particles on the x-stagnation orbit and η_p is the cosine of the pitch angle at the pinch point. However, the bounce time is very long compared to the coulomb scattering time, implying that most ions are scattered off x-stagnation orbits. Summing these co- and counter-current contributions over all of phase space (c.f. eqn. 11.25) yields a net co-going current. As expected, the additional effects of the negative ζ -drift decreases the alpha current, in comparison to the no-drift case. The inward ψ_x -drift depletes the current density at the edge, while making the inner $\langle j_{||} \rangle$ profile more peaked.

The background plasma response is not included in the calculation of the alpha particle current. Ohkawa [119] first suggested that fast ion injection in the co-going direction could form a toroidal plasma current. More recently, Hirshman [91] used a variational technique to find the Green's function for the plasma current response to fast ions, while Cordey et al. [92] obtained the induced plasma current via a perturbation method. The additional effect of

trapped electrons in the banana regime has been evaluated by Start et al. [93]. The net fast-ion-driven current in a circular cross-section tokamak is [114]:

$$I_{\text{driven}} = I_{\text{fast}} \left[1 - \frac{Z}{Z_{\text{eff}}} + \frac{1.46 Z}{Z_{\text{eff}}} \left(\frac{r}{R} \right)^{3/2} A(Z_{\text{eff}}) - \frac{1.2}{Z_{\text{eff}}} \left(\frac{v}{v_e} \right)^2 \right], \quad (11.35)$$

where $A(Z_{\text{eff}}) \approx 1 + 0.7/Z_{\text{eff}}$ [60], Z is the fast ion charge, and v_e is the electron thermal speed. The first term on the RHS of (11.35) is the α -current, $\langle I_\alpha \rangle$, calculated above. The second term is the "shielding current" due to streaming of electrons, caused by collisional momentum transfer from the fast ions. The third term describes the effect of banana-trapped electrons. The last term accounts for electron-electron collisions, for the fact that the electron distribution cannot be represented by a displaced-Maxwellian, and for the velocity dependence of the friction force between the electrons and fast ions. The α -driven current, in turn, can serve as a seed current for the bootstrap effect [94]. Considering practical heating rates and seed current values, the total current, I_{tot} , might become approximately 10 times the seed current, I_{driven} [95]. This is significant since the α -current might then drive part of the steady-state plasma current, reducing the volt-sec requirements for a reactor [90], though not affecting the volt-sec requirements for start-up.

Stacey and Sigmar [117] have determined the full response of a non-circular axisymmetric tokamak plasma to particle and momentum sources. They find that a net toroidal momentum input produces an

additional contribution to the toroidal current density, beyond those discussed above. One such source would be the α -driven, bootstrap-enhanced current, I_{tot} . Alpha particle losses to the tokamak first wall are another current input [16] (due to a net loss of momentum). However, such losses are unimportant ($< 1\%$ contribution) in the large plasmas of interest here. A proper evaluation of these effects including the non-circular cross section is beyond the scope of this work.

12. SUMMARY AND CONCLUSIONS

12.1 Importance of Alpha Particle Losses

An analytical model has been used to calculate the fusion product loss profile on the first wall of an axisymmetric tokamak. This model eliminates the cumbersome and time-consuming numerical procedures and uses a more general loss region than previous workers [14-18]. The key assumptions used were toroidal axisymmetry and the Knorr form for the poloidal and toroidal $\vec{B}$ -components. This analytical formulation also yields some useful symmetry, (2.7), and scaling laws, (2.8-2.9). In addition, knowledge of loss orbits may have other important consequences. For example, as suggested in Chapter 2, a potentially attractive fueling concept might employ ion injection into the spatial and velocity regions bounded by a pair of stagnation orbits.

The erosion mechanisms, namely blistering (primary) and sputtering, are strongly dependent on the energy- and angular-distribution of the incident fp's. In particular, we find the fp flux-versus-angle-of-incidence rises sinusoidally from near zero at grazing incidence ($\alpha=90^\circ$), peaks at some critical angle α_c , then falls rapidly to zero for $\alpha < \alpha_c$. Blistering occurs when the helium concentration at any depth into the wall materials exceeds a critical value. The resulting blistering-induced impurities can potentially limit burn times to $< 50\%$ of their design value. If successive generations of blisters occur, this effect will continue

throughout the life of the wall; if not, it is still necessary to cope with the first generation of impurities. The corresponding structural damage to the first wall is negligible: $\sim 30\mu\text{m/yr}$ in TFTR I and $\sim 1\mu\text{m/yr}$ in UNMAK I. A more complete evaluation of this problem including synergistic effects of MeV- α blistering and sputtering, the keV-DT sputtering must await extended experimental results into these regimes.

The non-prompt loss is $\leq 0.1 \times (\text{prompt loss}) \times \sqrt{Z_{\text{eff}}}$, with MeV-alphas being predominant. Since the banana-width of high-energy fp drift-orbits decreases during slowing down, the lower energy component of these losses is peaked more toward the outboard edge of the wall. Consequently, the effect of non-prompt MeV-alpha losses relative to prompt losses is negligible. KeV-to-100-KeV α losses are also negligible because the unconfined D-T flux to the wall is large in comparison. However, the significant non-prompt flux reaching the wall can be used for measuring Z_{eff} [44] as well as diagnosing [43-44] the slowing-down mechanism(s), i.e., classical versus anomalous slowing-down. Appendix F discusses a proposed experiment to measure such fusion product losses.

12.2 Control of Alpha Loss Effects

Having studied the physics of fp wall bombardment, it is possible to suggest ways to decrease its effects, which may be particularly serious in light of potential limitations on reactor burn time due to blistering-injected impurities. First, the effective

wall loading flux can be halved by reversing the toroidal field and plasma current after a convenient number of containment cycles.

This is because fp losses occur only in the direction of the vertical drift due to ∇B and curvature. Second, the regions of peak flux are a strong function of poloidal wall angle, suggesting special wall materials and/or design for rapid wall replacement to reduce blistering. Third, sensitivity studies show that a modest increase in wall-plasma separation dramatically reduces the fp wall loading. However, the increase in r_w must be balanced against the larger size of the blanket and magnets, i.e., cost. Increasing the plasma current is also useful, but less effective.*

12.3 Large Banana-Width Effects During Thermalization

Using the formulation developed by Rome and Peng [13], large banana-width slowing down (without anomalous effects) has been modeled for realistic, high-B, non-circular tokamak equilibria. First orbit losses, trapping effects, and slowing-down drifts are treated in the present model by analytically solving a 3-D (+ time) PDE. While not done here, our formulation is sufficiently general to include charge-exchange, pitch-angle scattering, speed-diffusion, losses during thermalization, electric field effects, impurities,

*The above techniques are to control fp's reaching the wall. An alternate approach, not analyzed here, is the control of impurities after leaving the wall. Examples include: a shielding divertor to extract-wall-generated impurities before they enter the plasma, and a gas blanket, between the plasma and wall to shield the plasma from entering impurities.

time-dependence, and large energy transfer collisions. Scaling laws governing the collisional slowing process are obtained, including the explicit signs of the slowing drifts, consistent with previous results [8,45,77], but requiring clarification of the pitch-angle conventions used by Cordey [45] and Killeen et al. [77]. An invariant, μ/E , governs the collisional slowing when pitch-angle scattering is negligible. By using this invariant in steady state, the fp distribution function can be found analytically by the method of characteristics, leaving a 1-D, first-order ODE to be solved numerically.

Zeroth-, first-, and second-order velocity moments of this distribution function yield the fp density, current, and heating profiles respectively. As compared to the commonly used in-situ deposition model, the flux-surface-averaged profiles are less centrally peaked, but are substantially above the in-situ values at the edge ($\psi/\psi_{\text{edge}} > 0.6$), due to smearing by the large banana-width orbits. The decreased α -heating, in comparison to in-situ deposition, implies reduced ion heating rates (by 15-20%), making start-up and maintenance of ignition more difficult. However, the lower central alpha density (in comparison to in-situ) indicates a reduced alpha ash accumulation on axis, somewhat relaxing the impurity removal problem.

There is a net co-going alpha particle current, due to finite banana width effects. When the background plasma response

is included in the calculation, bootstrap effects might give up to a 10-fold enhancement. The resulting α -driven current might serve to replace part of the steady-state plasma current, thus reducing the volt-sec requirements for a reactor.

12.4 Recommendations for Future Work

While this work has answered several important questions about alpha particle losses and slowing, additional questions have been raised as a result. The below suggestions are therefore included to guide later work.

- a) The Rome-Peng formulation [13] for alpha motion should be applied to calculate prompt alpha losses in non-circular tokamaks. This is significant because the plasma-wall separation varies in a complex way, e.g. a rectangular wall cross-section with an elliptical plasma. It is particularly important to obtain the alpha bombardment versus poloidal wall angle, since onset time and position for blistering-injected impurities are a sensitive function of the flux variation. Loss orbits can be parameterized by $(v, \psi_{wall}, n_{wall})$, where n_{wall} is the cosine of the pitch angle at the wall. The alpha flux is then found by integrating over all possible pitch angles at the given wall position, weighted by the alpha source rate. Sensitivity studies, similar to those done in this work, should be done to provide design information for reactor applications.
- b) While this work has demonstrated a net co-going alpha current due to large banana-width effects, no attempt was made to optimize this current. The α -driven, bootstrap-enhanced current, I_{tot} , should be maximized in order to relax the volt-second restriction on tokamak reactor burn time. The decreased α -heating, in comparison to in-situ deposition, should also be further studied. For plasma temperatures above 15 keV, the reduced ion heating is particularly important because the long ion-electron equilibration time,

$\tau_{eq} > \tau_s$, implies that the electron and ion heating become decoupled. This effect should be examined (i.e. alpha heating as one component of the time-dependent electron/ion transport) and will probably represent a trade-off against the largest possible value of I_{tot} .

- c) As noted in Chapter 8, the guiding-center expansion of P_ϕ is accurate through first order in gyroradius [86]. This expansion should be extended for $\beta > 0.1$, and the corresponding corrections added to the Rome-Peng formulation to examine high-energy ion effects. This is particularly important in spheromak configurations (for example, see Ref. 120), typically having $\beta > 50\%$.
- d) Since fusion product transport is such a crucial factor in reactor design, forthcoming charged fusion-product detection experiments on tokamaks need to be accurately modeled. The Suvikhin slowing mode [47] should be used. Pitch angle-scattering, charge-exchange, $\vec{E}$ -field effects, speed-diffusion, and time-evolution are important for obtaining the energy dependence of the detector response. This same formulation also applies to neutral beam injection in non-circular tokamaks. The resulting Fokker-Planck equation is a 3-D, second-order, PDE (see Chapter 9) and poses a major computational problem.
- e) Nuclear elastic scattering needs to be included in tokamak studies, in addition to large banana-width effects. The resulting large energy transfer (LET) events are a negligible modification to heating in DT devices (scattering probability $< 8\%$). However, the resultant large angle scattering in velocity direction might produce significantly enhanced non-prompt losses, thus modifying fusion-product detection results. LET heating effects in a deuterium or catalyzed-deuterium plasma are important. For example, the LET scattering probability is 37% for a 3-MeV proton slowing to 250 keV in a 20 keV deuterium plasma. In a device like TFTR, LET non-prompt losses can be modeled using a Monte Carlo method superimposed on the Petrie-Miley code, SYMALF, for continuous slowing down [8]. In non-circular tokamaks, the method developed here can be modified to include nuclear elastic scattering by the same sort of Monte Carlo technique.

These recommendations are certainly not exhaustive, but depend in large part on accurate, fast computing. This last aspect represents the largest challenge and the greatest opportunity for creativity in such work.

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APPENDIX A: COMPARISON WITH OTHERS' CALCULATIONS

Recent calculations of 3.5-MeV alpha transport in axisymmetric tokamaks [4,6-9] have modeled losses to the first wall. In this appendix, the results from these studies are compared [118]. Prompt loss fractions are reported in all cases. Three of the references [4,7,9] calculate wall loading profiles, but only two [4,6] consider the sensitivity to various machine parameters.

A.1 Comparison of Models

A summary of the various calculational models is given below.

Ohnishi et al. [6] numerically integrate the differential equations describing the guiding-center motion of 24,000 groups of alphas which intersect the wall (at the plasma edge). Kolesnichenko et al. [7] expand an analytical form for the drift orbit equation to obtain analytical expressions for loss fraction, F_L , and flux, P , as a function of poloidal wall angle, assuming a homogeneous plasma. They also calculate losses numerically for an inhomogeneous plasma, via the analytical forms for the drift motion. Petrie and Miley [8] use a spatial-and phase-group method similar to Ref. 6, but follow particles using an analytical form for the guiding-center motion. Hively and Miley [4] find the velocity-space loss-region from an analytical form for the alpha drift motion, then numerically integrate the losses over the plasma. Finally, Bauer et al. [9]

calculate losses via a Monte Carlo method, by randomly selecting initial birth points and pitch angles, weighting with the local fusion rate, and integrating around the orbits.

A.2 Comparison of Results

The results of Ohnishi et al. [6] for an EPR-size fusion reactor ($R_0 = 4.90\text{m}$, $a = 1.90\text{m}$, $I = 3.67\text{MA}$, $B_0 = 40\text{T}$, $T_{i0} = 8\text{keV}$) have been compared to those of Ref. 4. For F_L versus q , we obtain $F_L \sim q^k$, for $1.3 < k < 1.8$, with k decreasing with increasing q . This is consistent with Ref. 6 for $q > 2$; as pointed out by Ohnishi [34], the discrepancies noted for $q < 2$ apparently arise from techniques he employed in the numerical integration of the drift equations, and the assumption that $B_\phi \gg B_0$, which is not strictly true for $q \sim 1$. Our results [4] also compare closely to Ref. 6, for F_L versus a ($F_L \sim a^{-2}$) and F_L versus R_0 ($F_L \sim R_0^{1.5-2}$) when plasma current, I , is held constant. As a point of clarification, the results of Ref. 6 are for constant current ($I = 3.67\text{MA}$), rather than for $q(a)$ constant as one might infer from Table I of Ref. 6 [34].

However, there are discrepancies in a "base case" comparison among loss fractions read from Figs. 3-5 of Ref. 6 (See Table A.1). The differences within the third and fourth columns of Table A.1 are too large to be caused by statistical fluctuations in the model or by reading errors. Further, the results of Ref. 6 do not appear to be consistent with other studies. A comparison among results due

to Petrie and Miley [8], results of our work [4], Monte Carlo calculations by Goldston using the method of Ref. 9, and those of Ref. 6, are shown in Table 5.2. The correspondence among Refs. 4, 8 and 9 is good while the values of Ohnishi et al. [6] are quite different. These discrepancies are unresolved to date [34]. Consequently, we find that while the underlying scalings agree, the absolute numbers disagree.

The results of Ref. 4 and calculations by Goldston using the method of Ref. 9 have been compared to those of Kolesnichenko, et al. [7]. Figure A.1 shows the alpha flux versus poloidal wall angle for a homogeneous plasma in the T-20 tokamak (i.e., n -, J -, and T - profiles all being constant); the correspondence of the three methods is excellent. In Fig. A.2, we compare results for all parabolic profiles in T-20. Significant discrepancies between the values of Ref. 7 and the other two are seen, for poloidal wall angles larger than 40° ; the "knee" in results of Ref. 7 is particularly difficult to reconcile with the results of the other two methods. We hypothesize that this "knee" might be due to improper inclusion of contributions from disconnected orbits [4].

A.3 Comparison Summary

Calculations [4,6-9] of 3.5-MeV alpha flux to the first wall of an axisymmetric tokamak have been compared. Excellent agreement is found among Refs. 4, 8 and 9. Inconsistencies are found between Ref. 6 and Refs. 4, 8, 9, and between Ref. 7 and Refs. 4, 8, 9.

Table A.1: Net Loss Fractions (%) Read from Ref. 6

Figure	Assumed Profiles for T and n			
	$T/T_0=1$	$T/T_0=1$	$T/T_0=1-.8\left(\frac{r}{a}\right)^4$	$T/T_0=1-.8\left(\frac{r}{a}\right)^4$
	$n/n_0=\sqrt{1-\left(\frac{r}{a}\right)^3}$	$n/n_0=1-.8\left(\frac{r}{a}\right)^2$	$n/n_0=1$	$n/n_0=1-.8\left(\frac{r}{a}\right)^2$
3	8.2	6.3	3.4	1.3
4	8.3	6.4	2.3	1.7
5	8.3	6.4	1.4	1.3

Table A.2: Comparison of Net Loss Fractions(%) Obtained by Various Methods, for the EPR-size Tokamak Studies by Ref. 6

Profile	Ref. 6	Ref. 8	Ref. 4	Ref. 9
$T/T_0=1$				
$n/n_0=\sqrt{1-(r/a)^3}$	8.2% [†]	18.6%	19.3%	19.6%
$T/T_0=1-.8(r/a)^4$				
$n/n_0=1-.8(r/a)^2$	1.3-1.7%*	8.2%	8.0%	not available

*Range indicates values for different graphs of Ref. 6.

[†]We note that in a more recent study by Ohnishi, et al. [35], this value is quoted as 18%.

Figure A.1 Comparison of 3.5-MeV α -flux versus poloidal wall angle, θ , for a homogeneous plasma in T-20, as calculated by Kolesnichenko, et al. [7], our previous work [4], and Bauer et al. [9].

Figure A.2 Comparison of 3.5-MeV α -flux versus poloidal wall angle for parabolic n-, T- and J-profiles in T-20, as calculated by Ref. 4, Ref. 7, and Ref. 9.

APPENDIX B: INCIDENT VELOCITY MODEL

For fp's born at a given position in the plasma, $(r_i, \theta_i) \leftrightarrow (x_i, z_i)$, the velocity-space loss region, $x_{\min} \leq x \leq x_{\max}$, is found using the method of Chapter 3. The extremes of the wall-orbit intersection points, $\theta'_{\min} \leq \theta' \leq \theta'_{\max}$ (corresponding to $x_{\min}$ and $x_{\max}$) are found using the orbit-following method of Chapter 3. At the midpoint of each wall interval, $\theta'_{\min} \leq \theta'_k < \theta'_{k+1} \leq \theta'_{\max}$, the pitch-angle of the particle hitting the wall, χ_k , is found using (by symmetry from (3.1)):

$$\cos \chi_k = x_{iw} \left[1 - \sqrt{(R_i/R_w)(1-x_{iw}^{-2})} \right],$$

where $x_{iw} = x_{wi}$ and (r_k, θ_k) is found from simple geometric considerations with $\bar{\theta}'_k = \frac{1}{2}(\theta'_k + \theta'_{k+1})$:

$$r_k = \sqrt{x_o^2 + r_w^2 + 2r_w x_o \cos \bar{\theta}'_k},$$

$$\theta_k = \text{Arccos}[(x_o + r_w \cos \bar{\theta}'_k) / r_k].$$

The differential loss fraction corresponding to this wall interval, $\theta'_k < \theta' < \theta'_{k+1}$ is the same as that in the integrand of (4.4), and will be denoted by $f_{\ell}(\theta'_k + \theta'_{k+1})$. From the pitch angle, χ_k , the incident velocity is found as the sum of velocities due to ∇B , curvature, gyromotion, and streaming along $\vec{B}$:

$$\begin{aligned}\vec{v} = & \hat{r}(-v_d \sin \theta_k) + \hat{\theta}[-v_d \cos \theta_k + (B_\theta/B)v \cos \chi_k] \\ & + \hat{\phi}[(B_\phi/B)v \cos \chi_k] + \vec{v}_g, \end{aligned} \quad (\text{B.1})$$

where v_d is the velocity due to ∇B and curvature [14]:

$$v_d = (\mu B/m + v_\perp^2)/R_0 \omega_{c0} = v^2(1 + \cos^2 \chi_k)/2R_0 \omega_{c0}.$$

The variable ω_{c0} is the average gyrofrequency, eB_0/m . The gyro-velocity, $\vec{v}_g$, has a random phase, and a magnitude,

$$|\vec{v}_g| = v_\perp = v \sin \chi_k.$$

Inclusion of this random phase is a key aspect of this model.

The total velocity $\vec{v}$ is transformed into the wall frame (see Fig. 8.1) by:

$$\vec{v} = \hat{r}'(v_r \cos \xi + v_\theta \sin \xi) + \hat{\theta}'(-v_r \sin \xi + v_\theta \cos \xi) + \hat{\phi}'v_\phi, \quad (\text{B.2})$$

where (v_r, v_θ, v_ϕ) are the components of (B.1) and $\xi = \theta'_k - \theta_k$. For each interval of gyrophase, the angle, α , between $\vec{v}$ and the wall surface normal is found. The portion of the incident flux associated with α is accumulated for each angular interval,

$$\alpha_{\min} < \alpha_{\min} + \Delta\alpha < \dots < \alpha_\ell < \alpha_{\ell+1} < \dots < \pi/2,$$

for $\Delta\alpha = (\pi/2 - \alpha_{\min})/30$, where $\alpha_{\min}$ is some minimum angle. Finally, the flux-per-steradian for each wall segment, $\theta'_k \leq \theta' \leq \theta'_{k+1}$, and

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Figure B.1 The lower figure shows the coordinate system used to study the flux-versus-incident-velocity at the wall. The upper figure shows the details of this coordinate system.

angle-of-incidence interval, $\alpha_k \leq \alpha \leq \alpha_{k+1}$, is obtained by integrating the contributions over the whole plasma. This can be written as:

$$\frac{dF}{d\Omega} = \frac{dn}{dAd\Omega} = \int \frac{n_1 n_2 \langle \sigma v \rangle_{12} f_k(\theta'_k + \theta'_{k+1}, \alpha_k \rightarrow \alpha_{k+1}) R r dr d\theta d\phi}{2\pi(\cos \alpha_k - \cos \alpha_{k+1}) \Delta A} \quad (B.3)$$

The result of this calculation is that the flux-per-steradian can be calculated as a function of the angle-of-incidence interval,

$\alpha_k \leq \alpha \leq \alpha_{k+1}$, for a fixed wall position $\theta'_k \leq \theta' \leq \theta'_{k+1}$

As stated above, a crucial part of this analysis is inclusion of the random gyrophase. First, note that the gyrovelocity, $\vec{v}_g$, is perpendicular to drift velocity along a field line, i.e., $\vec{v}_g \perp \vec{v}_\parallel$, where

$$\vec{v}_\parallel = (\hat{\theta} B_\theta + \hat{\phi} B_\phi) v \cos \chi_k / B. \quad (B.4)$$

These velocities can be written in spherical coordinates (see Fig. B1) as

$$\vec{v}_\parallel = v_\parallel (\hat{\theta}' \sin \alpha' \cos \nu' + \hat{\phi}' \sin \alpha' \sin \nu' + \hat{r}' \cos \alpha'), \quad (B.5)$$

$$\vec{v}_g = v_g (\hat{\theta}'' \sin \alpha'' \cos \nu'' + \hat{\phi}'' \sin \alpha'' \sin \nu'' + \hat{r}'' \cos \alpha''), \quad (B.6)$$

where α'' and ν'' are spherical angles in the (r'', ϕ'', θ'') frame, and where (B.4) has been transformed according to (B.2). Since $\hat{\theta}' = \hat{\theta}''$, etc., $\vec{v}_g \cdot \vec{v}_\parallel = 0$ allows ν'' to be found from ν', α' and α'' by:

$$\nu'' = \nu' \pm \text{Arccos} (-\cot \alpha' \cot \alpha'') . \quad (\text{B.7})$$

As shown in Fig. 81, $\Omega = 0$ is taken as lying in the $\theta'' - \phi''$ plane, and thus $\nu'' = \nu' - \pi/2$ has been chosen as defining the $\Omega = 0$ direction, $\hat{\Omega}$, by substituting for ν'' and $\alpha'' = \pi/2$ in (B.6) :

$$\hat{\Omega} = (\hat{\theta}'' \cos \nu'' + \hat{\phi}'' \sin \nu'') = (\hat{\theta}' \sin \nu' - \hat{\phi}' \cos \nu') .$$

The gyrophase angle is found from $\cos \Omega = \hat{\Omega} \cdot \vec{v}_g / v_g$. When this result is combined with (B.7), α'' is found in terms of α' and Ω :

$$\alpha'' = \text{Arccos} (-\sin \Omega \sin \alpha') . \quad (\text{B.8})$$

Substituting (B.8) into (B.7), ν'' can be found:

$$\nu'' = \nu' - \text{sgn}(\cos \Omega) \text{Arccos} \left[\frac{\cos \alpha' \sin \Omega}{\sqrt{1 - \sin^2 \alpha' \sin^2 \Omega}} \right] . \quad (\text{B.9})$$

The $\hat{r}'$ component of incident particle velocity is then:

$$v_{r'} = v_{\parallel} \cos \alpha - v_d \sin \bar{\theta}'_k + v_g \cos \alpha'' , \quad (\text{B.10})$$

and thus the angle of incidence (azimuthal angle measured from $\hat{r}'$) is obtained from:

$$\cos \alpha = v_{r'} / |\vec{v}| . \quad (\text{B.11})$$

For particles to impact on the wall, we require $v_{r'} > 0$. From (B.10) and (B.8) the resulting constraint is:

$$\sin \Omega < v_D / v_G \equiv u = \sin \Omega_0 , \quad (\text{B.12})$$

where $v_D = v_n \cos \alpha' - v_d \sin \bar{\theta}_k'$, and $v_G = v_g \sin \alpha'$. The numerator of the RHS of (B.12) is the net radial drift in the primed frame, which must be in the $+\hat{r}'$ direction to reach the wall, implying $u \geq 0$.

To obtain the probability, $p(\Omega)$ associated with the angle of incidence, (B.10) is rewritten as

$$v_{r*} = v_D - v_G \sin \Omega, \quad (\text{B.13})$$

where $\Omega = \Omega_0 + \omega_c t$. The values of v_D and v_G are constant over one gyroperiod, so (B.13) can be integrated to give the radial position of the particle. The distance from the particle to the wall is then:

$$\Delta r = \omega_c^{-1} [v_D(2\pi\delta - \Omega + \Omega_0) - v_G(\cos \Omega - \cos \Omega_0)], \quad (\text{B.14})$$

where $v_{r*}(t=0) = 0$, $\Delta r(t=0) = 2\pi v_D \delta / \omega_c$, for $0 \leq \delta \leq 1$. Since the probability associated with Δr is uniformly distributed ($p(\delta)=1$), the impact condition, $\Delta r = 0$, implies that

$$p(\Omega) = p(\delta) \times |d\delta/d\Omega| = \left| 1 - \frac{\sin \Omega}{u} \right| / 2\pi, \quad (\text{B.15})$$

for a range of gyrophase angles, $0 \leq \Omega^* \leq \Omega \leq 2\pi + \Omega_0$, where Ω^* is defined by

$$u(\Omega^* - \Omega_0) = \cos \Omega_0 - \cos \Omega^*. \quad (\text{B.16})$$

We solve (B.16) numerically, using a Newton-Raphson search, with a starting guess for Ω^* of $3\pi/2 - \Omega_0$. (While the derivation detailed here applies for $u < 1$, these same results apply for $u > 1$.)

Thus, given Ω , the portion of differential loss fraction associated with α , found from (B.11), is:

$$f_g(\theta'_k \rightarrow \theta'_{k+1}, \alpha_l \rightarrow \alpha_{l+1}) = f_g(\theta'_k \rightarrow \theta'_{k+1}) \times p(\Omega)\Delta\Omega, \quad (\text{B.17})$$

where $\Delta\Omega = (2\pi + \Omega_0 - \Omega^*)/60$. To summarize this development, for each value of gyrophase, Ω , the angle-of-incidence α , is found from (B.11); the associated differential loss fraction is found from (B.17) and used in eqn. (B.3).

APPENDIX C: SCALING OF F_{pk}/F_ℓ

The crucial factor in both F_ℓ and F_{pk} is the differential loss fraction, f_ℓ . In general, both the $\cos\chi_{\min}$ and $\cos\chi_{\max}$ terms in f_ℓ can be written in the form of (3.1). Then, a scaling law can be obtained by rewriting the χ_{wi} term of (3.1) as a dimensionless form, via substitution from (2.4)-(2.5):

$$\cos\chi = \left(\frac{R_0}{\rho_\theta}\right) \hat{\chi}_{wi} \left\{ 1 - \sqrt{\frac{h_w}{h_i} \left[1 - \left(\frac{\rho_\theta}{R_0}\right)^2 \hat{\chi}_{wi}^2 \right]} \right\} \equiv \left(\frac{R_0}{\rho_\theta}\right) \gamma_{wi}, \quad (C.1)$$

$$\begin{aligned} \rho_\theta &= mv/eB_\theta = \text{poloidal gyroradius}, \\ \hat{\chi}_{wi} &= (\hat{\psi}_i - \hat{\psi}_w) / (\hat{r}_i \cos\theta_i - \hat{r}_w \cos\theta_w), \\ \hat{\psi} &= \psi(r) / (\mu_0 I R_0 / 2\pi). \end{aligned}$$

As before the subscripts "i" and "w" indicate the initial and wall intersection points of the drift orbit, respectively; $\hat{r}$ is r/a .

The net loss fraction can now be rewritten* as:

* Note two key dimensionless parameters appearing in this analysis, namely (a/R_0) and (ρ_θ/R_0) . The latter parameter is identical to the parameter, P , used by Rome et al. [18]. As expected, these same parameters arise from Newton's equations of motion for this system. One additional parameter, (a/r_g) also arises (r_g is the particle gyroradius in the toroidal field, B_0), but can be eliminated by noting that (2.6) is independent of B_0 .

$$F_L = \left(\frac{R_0}{\rho_\theta \hat{S}} \right) \int_0^1 \hat{n}_1 \hat{n}_2 \langle \sigma v \rangle_{12} \hat{r} d\hat{r} \int_0^\pi d\theta (Y_{\min} - Y_{\max}) \left[1 + \left(\frac{a}{R} \right) \hat{r} \cos \theta \right], \quad (C.2a)$$

$$\hat{S} = 2\pi \int_0^1 \hat{n}_1 \hat{n}_2 \langle \sigma v \rangle_{12} \hat{r} d\hat{r}. \quad (C.2b)$$

The terms $Y_{\min}$ and $Y_{\max}$ in (C.2) refer to the functional part of $\cos \chi_{\min}$ and $\cos \chi_{\max}$, respectively, and $\hat{n} = n/n_0$.

The peak flux F_{pk} can be scaled similarly. From (4.6), the denominator of (4.5) can be approximated as:

$$\Delta A = 2\pi r_w R_0 \Delta \theta' (1 + r_w \cos \theta'_{pk} / R_0) = 2\pi r_w R_0 h_{pk} \Delta \theta', \quad (C.3)$$

where θ'_{pk} is the poloidal wall angle corresponding to the peak flux position. Substituting (C.1) and (C.3) into (4.5), yields

$$F_{pk} = \left(\frac{R_0 a^2 n_{10} n_{20}}{\rho_\theta r_w h_{pk} \Delta \theta'} \right) \int_0^1 \hat{n}_1 \hat{n}_2 \langle \sigma v \rangle_{12} \hat{r} d\hat{r} \int_0^\pi d\theta \left[Y^{(k)} - Y^{(k+1)} \right] \times \left[1 + \frac{a}{R_0} \hat{r} \cos \theta \right]. \quad (C.4)$$

The terms $Y^{(k)}$ and $Y^{(k+1)}$ refer to the functional portion of $\hat{\chi}_{wi}$, corresponding to $\cos \chi_k$ and $\cos \chi_{k+1}$, (c.f. Eqn. C.1) where θ'_k and θ'_{k+1} are the end points of the interval that includes the peak flux.

Since (C.2) and (C.4) have identical functional forms, the scaled ratio is on the order of unity. For the cases considered here, this implies that

$$0.4 \leq (F_{pk}/F_g)(2\pi r_w R_o/S) \approx (0.3-0.4) \times P/A \leq 1.2, \quad (C.5)$$

where S is the total fusion rate, c.f. the denominator of (4.4), and P/A the peak-to-average fp flux ratio.

APPENDIX D: DETAILS OF NON-PROMPT CALCULATION

A major part of the non-prompt loss calculation involves evaluation of the appropriate special functions and the associated accuracy of the final result. The Legendre functions, $P_\nu(\zeta)$, are determined using routines from the CERN Program Library [48]. The derivative, $\partial P_\nu / \partial \zeta$, is evaluated using the well-known equation [49]:

$$\frac{\partial P_\nu}{\partial \zeta} = \frac{\nu(P_{\nu-1} - \zeta P_\nu)}{1 - \zeta^2} = \nu P_{\nu-1}(\zeta) / (1 - \zeta^2),$$

since $P_\nu(\zeta_1) = 0$. The derivative, $\partial P_\nu / \partial \nu$, is evaluated using central differences:

$$\frac{\partial P_\nu}{\partial \nu} = \frac{P_{\nu+\Delta\nu} - P_{\nu-\Delta\nu}}{2\Delta\nu}.$$

While $\partial P_\nu / \partial \nu$ can be evaluated analytically [50], the form given above is much faster computationally and is accurate to $< 10^{-4}\%$ for $\Delta\nu = 5 \times 10^{-6}$. The eigenvalues of $P_\nu(\zeta_1) = 0$ are found numerically, by first detecting a zero crossing via a step-wise search, then by successive linear interpolations until $|P_\nu(\zeta_1)| < 10^{-9}$. To speed the calculations, these eigenvalues are found for $-1 < \zeta \leq 0$ in steps of 0.01; the results are stored for each value of ζ . A four-point Lagrangian interpolation is used to obtain eigenvalues for

intermediate values of ζ_1 . The resulting set of $\{v_n\}$ has an average error $\leq 10^{-4}\%$. It is necessary to use one hundred terms in (5.5) to obtain a convergence within 0.1%.

To compute the $\int dv$ at a given (r_i, θ_i) in (7.2), the value of $v_{\min}$ (c.f. Fig. 7.1) is found, being the largest v for which the loss region is zero. Non-zero contributions to (7.2) occur only for $v > v_{\min}$. The smallest such $v_{\min}$ occurs for $(r_i, \theta_i) = (a, 0)$, and is designated $v_{\min}$. Each velocity step is then decremented by the ratio:

$$v/(v-\Delta v) = (v_0/v_{\min})^{1/50},$$

yielding a convergence, with respect to Δv , of $\leq 0.1\%$. The spatial grid in (7.2) is 54×45 for $0 \leq r \leq a$ and $0 \leq \theta \leq \pi$ respectively, giving a net computational accuracy in $\delta F_{np}/\delta E$ of better than 1%.

The model discussed in Chapter 7 corresponds to each fast ion slowing down on the circular flux surface at its birth radius. Since the small banana-width approximation was used as a simplifying assumption, it is useful to also consider limiting cases of the model to establish bounds on the "nominal" (slowing down at $r = r_i$) result [51]. The first limit is to allow the particle to slow down on the flux surface corresponding to the minimum poloidal radius, $r = r_{\min}$. This is the radius, $(r_{\min}, \theta = 0) \leftrightarrow (x_{\min}, z = 0)$, at which ζ is evaluated. As described above, if $x_{\min}$ is non-positive, the contributions due to this orbit are neglected. The corresponding loss region is found via the algorithm of Chapter 3, assuming a

"birth" point at $(r_{\min}, \theta = 0)$. The confined fraction, f_c , and eigenvalues, v_n , are found as before using

$$\zeta = \cos \chi_{\max}(r_{\min}, \theta = 0) \quad .$$

For the second limiting case, we assume that the particle slows down at $r = r_{\max}$ of the drift orbit. Since the $r_{\max}$ value of grazing and $\chi = 0(\pi)$ orbits have maximum poloidal radii $r_{\max} \geq r_w$, their contributions are neglected. Thus, only orbits circulating inside the inner loop of an "unstable" stagnation orbit will contribute in this limit. We further assume that if $r_{\max} > a$, the particle is rapidly lost by ripple trapping or charge exchange. The loss region is evaluated for $(r_{\max}, \theta = 0)$ and the corresponding ζ -value used in determining f_c and v_n is:

$$\zeta = \cos \chi_{\max}(r_{\max}, \theta = 0).$$

The remainder of the model for these limiting cases is identical to that previously discussed.

Figure D1 shows a comparison of non-prompt losses in PLT for slowing down at $r = r_i$ and at $r = r_{\min}$, with $\langle Z \rangle = 1$; no $r_{\max}$ curve is shown because $r_{\max} > a$ for all the fp loss-orbits in PLT. For $v > v_c$, the $r_{\min}$ result lies slightly above the r_i curve, since $T(r_{\min}) > T_i(r_i)$ implies that $v_c(r_{\min}) > v_c(r_i)$. The larger v_c -value, in turn, indicates more PAS losses since v_c^3 appears in the numerator of (7.2). For $v < v_c$, the $r_{\min}$ curve lies below the r_i

Figure D.1 Non-prompt alpha loss fraction per keV and average flux per keV versus particle energy in PLT, for two cases: the ion slows down on its minimum poloidal drift radius, $r_{\min}$, and on its birth radius, r_i . In PLT, all 3.5-MeV alpha orbits have $r_{\max} > a$, and thus this case is omitted (see text).

result because the smaller loss-region, evaluated at $(r_i, \theta = 0)$, dominates over the PAS losses.

A similar comparison for TFTR with $\langle Z \rangle = 1$ is shown in Fig. D2. The $r_{\min}$ curve lies above the r_i curve for the same reason as given above. For $v \sim v_{\min}$, the $r_{\min}$ result does fall slightly below the r_i curve, because the smaller loss region finally dominates. The $r_{\max}$ losses are much above the r_i curve because the large loss-region, evaluated at $(r_{\max}, \theta = 0)$ is dominant. This yields a second maximum, similar to Fig. 7.2, because the large loss-region decreases slowly in comparison to the rising PAS contribution. For PLT, TFTR, ORNL-EPR, and UWMAK-I, we obtain

$$0.9 \leq v_p^*(r_{\min})/v_p^*(r_i) \leq 1.2,$$

$$0.9 \leq v_E^*(r_{\min})/v_E^*(r_i) \leq 1.2,$$

$$0.7 \leq v_p^*(r_{\max})/v_p^*(r_i) \leq 1.2,$$

$$0.5 \leq v_E^*(r_{\max})/v_E^*(r_i) \leq 0.8,$$

$$1.0 \leq \bar{E}(r_{\min})/\bar{E}(r_i) \leq 1.1,$$

$$0.6 \leq \bar{E}(r_{\max})/\bar{E}(r_i) \leq 0.8.$$

As functions of $\langle Z \rangle$, the above ratios are constants within 5%.

Figure D.2 Non-prompt alpha loss fraction per keV and average flux per keV versus particle energy in TFTR-I, for 3 cases: 1) the ion slows down on its maximum poloidal drift radius, $r_{\max}$, 2) on its minimum radius, $r_{\min}$, and 3) on its birth radius, r_i .

APPENDIX E: EVALUATION OF BOUNCE INTEGRALS

In evaluating dt and the resultant bounce-averaged quantities, it is necessary to contend with the fact that $\vec{B} \cdot \nabla B$ is zero in the equatorial plane. This is because surfaces of constant- ψ and constant- B are both vertical at $z = 0$, implying that $\vec{B}$ is perpendicular to ∇B there. In particular, a Taylor series expansion of ψ about $z = 0$ gives:

$$\partial\psi/\partial z = z \left(\frac{\partial^2\psi}{\partial z^2} \right)_{z=0}, \quad (\text{E.1})$$

$$\partial^2\psi/\partial z\partial R = z \left(\frac{\partial^3\psi}{\partial z^2\partial R} \right)_{z=0} \quad (\text{E.2})$$

For use in (10.7), (10.11) must be evaluated using (E.1)-(E.2), but these quantities must be determined along the orbital trajectory. This is done by Taylor-series expanding ψ and B along the orbit, about $(B_x, \psi_x) \leftrightarrow (R_x, z=0)$ and $(B_n, \psi_n) \leftrightarrow (R_n, z=0)$, then solving the resulting pair of equations for z (about $z=0$):

$$z = \left\{ \frac{\sqrt{\psi_x - \psi}}{\sqrt{\psi - \psi_n}} \right\} \times \sqrt{2 \left| \frac{(B'_e - B'_0)}{B'_e} \right| / \left| \left(B'_e \frac{\partial^2\psi}{\partial z^2} - \frac{\partial^2 B}{\partial z^2} \right) \right|}} \quad (\text{E.3})$$

where $\sqrt{\psi_x - \psi}$ or $\sqrt{\psi - \psi_n}$ is used when near ψ_x or ψ_n respectively, and the derivatives are evaluated at $(R_x, z=0)$ or $(R_n, z=0)$ respectively. The

term $(B-B_x)$ arises in the expansion at $\psi = \psi_x$ and is rewritten as $B'_{ox}(\psi-\psi_x)$; similarly, $(B-B_n)$ becomes $B'_{on}(\psi-\psi_n)$, where B'_{ox} is given by the RHS of (8.4) and

$$B'_{on} = \frac{B_x n (n F'_n / F_n - e B_n / m v_Y F_n)}{\sqrt{\left(\frac{1-\zeta^2}{2}\right)^2 + \eta^2 (B_x/B_n)^2}} \quad (\text{E.4})$$

As before B'_{on} designates $\partial B_o / \partial \psi$ at ψ_n and likewise for F'_n . The cosine of the local pitch angle, η , is found from (8.1):

$$\eta = \frac{F_x}{F_n} \frac{B_n}{B_x} \left[\zeta - \frac{G_x c}{v_Y} \left(1 - \frac{\psi_n}{\psi_x} \right) \right]. \quad (\text{E.5})$$

Combining (10.11) and (E.1)-(E.3) yields

$$\begin{aligned} \vec{B} \cdot \nabla B \Big|_{z=0} &= \frac{\sqrt{\psi_x - \psi}}{\sqrt{\psi - \psi_n}} \times \sqrt{2 \left| (B'_e - B'_o) / \left(B'_e \frac{\partial^2 \psi}{\partial z^2} - \frac{\partial^2 B}{\partial z^2} \right) \right|} \\ &\times \frac{1}{8R^4} \left\{ \frac{\partial^2 \psi}{\partial z^2} \left[F^2 + \left(\frac{\partial \psi}{\partial R} \right)^2 + R \frac{\partial \psi}{\partial R} \left(\frac{\partial^2 \psi}{\partial z^2} - \frac{\partial^2 \psi}{\partial R^2} \right) \right] \right. \\ &\left. + R \frac{\partial^3 \psi}{\partial z^2 \partial R} \left(\frac{\partial \psi}{\partial R} \right)^2 \right\}. \quad (\text{E.6}) \end{aligned}$$

Thus, both (10.7) and bounce-averaged quantities, $\tilde{D} = \frac{1}{\tau_b} \oint d\tau D$, reduce to integrals of the form $\int I d\psi / |\vec{B} \cdot \nabla B|$, with $\vec{B} \cdot \nabla B$ having square-root singularities at ψ_n and ψ_x . Such integrals can be

evaluated by subtracting the singularities from I , and doing the singular pieces analytically:

$$\int_{\psi_n}^{\psi_x} \frac{I d\psi}{|\vec{B} \cdot \nabla B|} = \int_{\psi_n}^{\frac{1}{2}(\psi_n + \psi_x)} d\psi \left(\frac{I}{|\vec{B} \cdot \nabla B|} - \frac{I_n}{c_n \sqrt{\psi - \psi_n}} \right) + \int_{\frac{1}{2}(\psi_n + \psi_x)}^{\psi_x} d\psi \left(\frac{I}{|\vec{B} \cdot \nabla B|} - \frac{I_x}{c_x \sqrt{\psi_x - \psi}} \right) \quad (E.7)$$

$$+ 2 \left(\frac{I_n}{c_n} + \frac{I_x}{c_x} \right) \sqrt{\frac{\psi_x - \psi_n}{2}},$$

where I_n and I_x are the values of I , evaluated at ψ_n and ψ_x , respectively. Similarly, c_n and c_x are the coefficient portions of (E.6) determined at ψ_n and ψ_x , respectively. McAlees [16] found this same square-root singularity for concentric, circular tokamak equilibria, while the present form applies without restriction.

An important special case is the evaluation of τ_b and bounce-averages for stagnation orbits. Near o-type stagnation orbits, the orbit equation (8.3) can be expanded in a Taylor series about ψ_n and about ψ_x . The product $(B - B_n) \times (B - B_x)$ can then be expressed in terms of $(\psi - \psi_n)$, $(\psi - \psi_x)$ and the derivatives B'_{ox} and B''_{ox} , yielding to first-order

$$B - B_x = B'_{ox} \times \begin{cases} \psi_n - \psi, & \text{for } \psi_n \leq \psi \leq \frac{1}{2}(\psi_n + \psi_x), \\ \psi - \psi_x, & \text{for } \frac{1}{2}(\psi_n + \psi_x) \leq \psi \leq \psi_x \end{cases} \quad (E.8)$$

Combining (10.11), (E.8) and (E.1)-(E.3), we find the first two terms on the RHS of (E.7) become zero in the limit $\psi_n \rightarrow \psi_x$. Further, by examining the orbit equation expanded to second order, the difference $(B'_{ex} - B'_{ox})$ is:

$$B'_{ex} - B'_{ox} = \left(\frac{\psi_x - \psi_n}{2} \right) (B''_{ex} - B''_{ox}), \text{ as } \psi_n \rightarrow \psi_x. \quad (E.9)$$

Substituting (E.6) and (E.9) into (E.7) yields

$$\tau_b(\psi_n \rightarrow \psi_x) = \frac{16eB_x^4 R_x^4}{\gamma m v^2 F_x (1+\zeta^2)} \times \sqrt{2 \left| \left(B'_{ex} \frac{\partial^2 \psi}{\partial z^2} - \frac{\partial^2 B}{\partial z^2} \right) / (B''_{ex} - B''_{ox}) \right|} \\ \left/ \left\{ \frac{\partial^2 \psi}{\partial z^2} \left[F_x^2 + \left(\frac{\partial \psi}{\partial R} \right)^2 + R \frac{\partial \psi}{\partial R} \left(\frac{\partial^2 \psi}{\partial z^2} - \frac{\partial^2 \psi}{\partial R^2} \right) \right] \right. \right. \\ \left. \left. + R_x \frac{\partial^3 \psi}{\partial z^2 \partial R} \left(\frac{\partial \psi}{\partial R} \right)^2 \right\} \right. \quad (E.10)$$

where B''_{ox} is $\partial^2/\partial \psi^2$ of (8.3) at ψ_x :

$$B''_{ox} = \frac{2B_x}{1+\zeta^2} \left[\left(\frac{B'_{ex}}{B_x} \right)^2 (2+\zeta^2) + \left(\frac{F''_x}{F_x} - 3 \left(\frac{F'_x}{F_x} \right)^2 \right) \zeta^2 + \frac{4F'_x}{F_x} \frac{G_x \zeta}{\psi_x} \right. \\ \left. - \left(\frac{G_x}{\psi_x} \right)^2 \right], \quad (E.11)$$

and the derivatives in (E.10)-(E.11) are determined at $(R_x, z=0)$.

Bounce averages are easily found as

$$\tilde{D}(\psi_n \rightarrow \psi_x) = \frac{2}{\tau_b} \int_{\psi_n}^{\psi_x} d\tau D = D_x . \quad (\text{E.12})$$

As the counter-going x-type stagnation boundary is approached,

$B'_{ex} - B'_{ox} \rightarrow 0$, while $(\psi_x - \psi_n)$ remains finite, implying that

$\tau_b(x\text{-stagn.}) \rightarrow \infty$. This applies to pinch (x-type stagnation)

orbits as well. Bounce averages are also easily found in this limit,

in effect becoming the averaged quantity weighted by a Dirac-delta

function at the x-type stagnation point:

$$\tilde{D}(x\text{-stagn}) = \begin{cases} D_x, & \text{for counter x-type stagnation orbit,} \\ D_p, & \text{for pinch orbit.} \end{cases} \quad (\text{E.13})$$

For cases other than these special ones, τ_b and $\tilde{D}$ must be evaluated numerically, as described above.

APPENDIX F: PROPOSED FUSION-PRODUCT DETECTION EXPERIMENT

The measurement of fusion product (fp) leakage from PLT and TFTR plasmas has been proposed [25] as a critical experiment for future reactor development. Fusion rates are sufficient to carry out such experiments with either deuterium or D-³He plasmas in these devices. This would provide important data about fp transport prior to contamination of TFTR with tritium for D-T experiments (making experimental access much more difficult). Such an experiment would provide the first actual data about the accuracy of present fusion product transport calculations and the possible occurrence of instabilities driven by the high-energy alphas. This is particularly true in light of the prediction by Chan and Sigmar [33] of large non-prompt losses due to α -driven (low frequency) Alfvén waves. More recently, Molvig and Sigmar [102] have predicted that α -driven (high frequency) electrostatic instabilities will cause anomalous slowing down at a rate much faster than collisional. Enhanced α losses, due to trapping in B-field ripple [8,103] have also been predicted. The detection of anomalously small or large losses can be used to identify the slowing down mechanism(s). By understanding this physics, engineering designs can be modified to control excess losses or enhance anomalously fast slowing, in order to achieve reactor conditions.

F.1 Introduction

There is much previous experience in charged fp detection. This includes fast ions from laser driven plasmas [104-107], from z-pinches [108], and from a plasma-focus device [109]. To date, however, no such experiment has been performed on a tokamak. Therefore, we have proposed [25] that a charged fp measurement be done on near term devices like PLT and TFTR. In particular, measurement of prompt losses from the plasma will constitute a direct verification of the drift-orbit theory used in fp transport models. Non-prompt losses are excluded in the present work because that analysis would require an evaluation of the fp distribution function as the plasma discharge progresses in time. Measurement of fp's from D-D plasmas in PLT will minimize the radiation complications due to tritium and neutrons. Also, for D-D plasmas the fp flux is low, minimizing fp-driven instabilities. Other secondary effects include $\vec{B}$ -field ripple losses ($< 8\%$ for 0.82 MeV He^3 [8] as the worst case) and uncertainties in the profiles plasma current density and background ion temperature and density. The latter effects are negligible for the purposes of this study.

Hively and Miley [44,110] have analyzed an experiment involving two detectors (one at the top and one at the outboard side of the torus; see Fig. F.1), shielded, and facing away from the plasma (to minimize background radiation), so that fusion products can enter along gyro-helices [111]. The analysis is for prompt

losses and while similar to the previous analysis for wall bombardment [4] is complicated by the need to evaluate two factors: (a) the differential fraction, f_g , of fusion products reaching the vicinity of the detector from each interval of plasma volume, and (b) the fraction of (a) that can gyro-orbit (with proper gyrophase and pitch angle) through the collimator defined by the shielding. With a 5-keV D-D plasma in PLT, for example, the total source is 4.2×10^{14} $^3\text{He}/\text{sec}$, producing an average wall loading flux of 3×10^8 to 3×10^9 $^3\text{He}/\text{cm}^2/\text{sec}$ [4], assuming a net loss fraction of 0.1-1.0. A shielding assembly like the one analyzed here (10 mm^2 detector area) accepts 4-40% of the incident ^3He particles, thus detecting 10^6 - 10^9 $^3\text{He}/\text{sec}$ (10^3 - 10^6 $^3\text{He}/\text{sec}$ for 1-keV DD plasma). Hence, the measurement is not a question of having a sufficiently large fp source, but one of detector signal-to-noise ratio (an experimental problem).

F.2 Detection Physics

The detector geometry used here, as first suggested by Hendel [111], is a variation on a Katsumata probe [112]). In Katsumata's case, the ion gyro-orbits were small in comparison to the detector size, permitting a simple analysis. However, fp gyro-orbits are large compared to the detector size, due to radiation shielding requirements, complicating the analysis considerably. Energy discrimination is also possible by a sequence of foils placed over the detector to eliminate particles according to energy. Consequently,

the proposed measurement could measure the time history of (incoming particles/sec) versus energy.

We begin by noting that the ∇B and curvature drifts must be upward for fp's to reach to top detector. This requires that the toroidal field go into the page in Fig. F.1; for this purpose, the $+\hat{\phi}$ direction is taken into the page. The corresponding gyromotion, looking in the direction of $+\hat{\phi}$ is counterclockwise. Consequently, the side detector must "look" downward (See Fig. F.1) for the gyromotion to carry a particle around the radiation shield and into the detector. By the same reasoning, the top detector must "look" away from the torus centerline.

F.3 Detector Response

The number of particles-detected-per-second, $\dot{N}$, is a summation of the contributions over the plasma volume:

$$\dot{N} = \int_V d^3r \, n_1 n_2 \langle \sigma v \rangle_{12} \int_D dA \, P_d(df_\ell/dA); \quad (F.1)$$

where p_d is the detection probability associated with (b), above; the integral $\int_D dA(\dots)$ is taken over the area in the R-z plane through which the drift-orbit guiding centers must pass so that fp's can gyro-orbit around the radiation shield and be detected; and df_ℓ/dA is the prompt-loss-fraction-per-unit-area in the vicinity of the detector.

Detectable particle trajectories are determined by integrating the equations of motion backward in time. That is, under time reversal symmetry, fp's reaching the detector are equivalent to those starting at the detector and moving freely out past the radiation shield. For these purposes, the time-dependent fp guiding-center motion is described integrating the Galeev-Sagdeev equations [14] for dr/dt and $r d\theta/dt$. For detection, the particle path computed by this method cannot intersect the inner surface of the conical radiation shield.

The detection probability arises as a natural consequence of which particle orbits are detectable. Firstly, since the area interval is a short segment rather than a strip around the entire torus, axisymmetry implies a proportionately smaller number of particles reach δA , from the plasma, namely:

$$P_1 = \delta A / 2\pi R \Delta_r, \quad (F.2)$$

where Δ_r is the grid size over which (F.1) is numerically evaluated. Secondly, since the particles are born with random gyro-phase, the particles reach the detector over a proportionate section of the gyrohelix pitch length, $2\pi r_g \cos \chi$:

$$P_2 = \delta A / 2\pi \Delta_r r_g \cos \chi. \quad (F.3)$$

A third factor enters because some drift orbits intersect the wall (or limiter) before reaching the detector ($\leq 20\%$ correction

in some cases). This factor is unity if the pitch angle at birth lies outside the loss region (see Chapter 3). If however, the pitch angle lies in the interval $x_{\min} \leq x \leq x_{\max}$ (c.f. eqn. 4.1), P_3 must be reduced appropriately. Combining these conditional probabilities yields the net detection probability, used in (F.1):

$$P_d = \frac{P_3(\delta A/\Delta_r)^2}{4\pi^2 r_g R \cos \chi} \quad (\text{F.4})$$

Finally, the loss-fraction-per-unit-area is calculated by finite differences:

$$\frac{df_g}{dA} = \frac{\Delta f_g}{\Delta A}, \quad (\text{F.5})$$

where ΔA is an interval in area in the r - θ plane and f_g is the fraction of particles passing through that area.

F.4 Results and Discussion

Our calculations have examined the sample case of 0.82-MeV He^3 from Maxwellian D-D reactions in PLT. The PLT parameters used are:

$$R_0 = 132 \text{ cm}, a = 45 \text{ cm}, r_w = 48 \text{ cm}, x_0 = 0, J/J_0 = 1 - (r/a)^2,$$

$$T(\text{keV}) = 5[1 - (r/a)^2],$$

$$n(\text{cm}^{-3}) = 10^{14}[1 - .8(r/a)^2].$$

For a centerline temperature of 1 keV, and 3 keV, the values for $\dot{N}$ are down by factors of $\sim 10^3$ and ~ 10 respectively. The plane of the detector is taken as the R - ϕ and z - ϕ planes for the side and top detectors, respectively. In both cases the detector center lies 48 cm from the plasma center. The examination of this case was chosen because He^3 confinement has the widest variation for PLT parameters; the net loss fraction, F_ℓ varies from $F_\ell \sim 1$ for $I = 0.2$ MA to $F_\ell \sim .3$ for $I = 0.6$ MA [4]. The results are shown in Fig. F.2 for $0.2 \text{ MA} \leq I \leq .6 \text{ MA}$ and for $B_0 = 20, 30$ and 40 kG at the top and side detectors. To explain the shape of $\dot{N}$ versus plasma current for the side detector, it is necessary to consider two effects. First, since $B_\theta/B_0 \ll 1$, the radiation shield permits detection of particles having a range of pitch angles (typically $0.6 \leq \sin \chi \leq .99$), independent of I (detection probabilities are a function of both I and B_0). The crucial drift orbits within this range of pitch angles are those reaching farthest into the plasma. Examples of such orbits (projected on the R - z plane) in PLT for 0.82-MeV He^3 are shown in Fig. F.3, for various plasma currents. As I increases, the banana width of the neoclassical orbit decreases, holding the motion closer to the (circular) flux surface. Consequently, more particles from the high temperature plasma core become detectable as the current increases. Thus, $\dot{N}$ rises for I increasing. The second effect is that the loss fraction decreases with increasing current ($F_\ell \sim I^{-1}$), due to the decreasing banana

Figure F.2 Number of 0.82-MeV ^3He seen (relative units) is plotted versus plasma current for various center-line toroidal field values (kG) in PLT (deuterium plasma).

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Figure F.3 The drift orbits shown here are for the pitch angle, $\chi = \arcsin(.65) \sim 40^\circ$, as well as intermediate values; the number on each curve is the pitch angle at the detector. Orbits having $\chi > 90^\circ$ are not shown because these trapped orbits lie mostly outside the plasma and intersect the wall.

width, $\rho_\theta \sim B_\theta^{-1} \sim I^{-1}$. The net effect is that as I increases, $\dot{N}$ initially rises rapidly, peaks, then falls monotonically. A similar argument can explain the $\dot{N}$ vs I curves for the top detector. The variation of $\dot{N}$ with B_0 is easily explained by (F.4). Here the detection probability varies as $p_d \sim r_g^{-1} \sim B_0$, implying that $\dot{N}$ increases as B_0 . As expected, it is also found that $\dot{N} \sim \langle \alpha v \rangle$, corresponding to the average ion temperature of the plasma. Thus $\dot{N}$ is a measure of the α source function, as expected.

Having modeled the detector physics and calculated a sample case, it is finally necessary to ask what the detector actually measures. Referring to Fig. F.3, at low plasma currents the detector samples the fusion-product source function at the outboard plasma edge. Increasing I permits examination of successively more of the plasma interior. However, the sampling is integrated over all the intermediate drift orbits as well. This is crudely analogous to the integrated emission measurement of microwave, uv, X-rays, etc. along chords through the plasma. Abel inversion of the $\dot{N}$ versus I measurement, while in principle possible, is severely complicated by the drift orbit geometry. An alternate refinement of the detection scheme would be an added slit in the detector to admit only a well-defined range of pitch angles. The resulting measurement would then sample the thermonuclear source function along the corresponding band of drift orbits. Abel inversion of the latter measurement, while still quite complicated, seems more

promising. Further refinements and theoretical analysis of this type cannot be justified without experimental verification of this work and further improvement of the detection technique.

APPENDIX G: SLWDN9 USER'S GUIDE

G.1 Code Description

The code, SLWDN9, calculates flux-surfaced-averaged values of alpha density, current, and electron/ion heating profiles in realistic, non-circular tokamak plasmas. See Figure G.1 for program flow diagram. The code is written in FORTRAN and executes on the CRAY-1 machine at the Magnetic Fusion Energy Computer Center (MFECC) sponsored by the U. S. Department of Energy, in Livermore, CA. The typical run time is 1-1.2 cpu hours. This guide will assume familiarity with the MFECC machines, with the system architecture, and with the utility routines. Online documentation can answer system-related questions and is available on the MFECC CDC-7600 by using the DOCUMENT utility.

There are two files required to execute the code. The FORTRAN source is obtained by using FILEM:

FILEM RDS 1352 SLWDN9

The input data file describes the tokamak equilibrium; two examples are available by using FILEM:

FILEM RDS 1352 TNS008 TNS075

The first data file, TNS008, is for a D-shaped, ETF equilibrium with $\bar{\beta} = 0.008$, a major radius of 5.04 m, and a elongation of 1.6. The second data file, TNS075, is for a D-shaped, ETF equilibrium having $\bar{\beta} = 0.075$, a major radius of 5.36 m, and an elongation of 1.6.

Section G.4 describes the formats and data for the input file. The default input/output filenames are TNS008 and L032 respectively.

Figure G.1 SLW0N9 Flow Chart.

To change these filenames, the "CALL LINK" statement on line 62 of SLWDN9 must be appropriately changed. To compile and execute SLWDN9, use

```
RCFT I = SLWDN9, L = 0, GO, LIB = (F')/T V
```

where the allotted time, T, should be ≥ 90 min. The time value, V, should be chosen keeping in mind that this code requires $\sim 1200K$ octal words of memory. Section G.5 discusses the output, including an example which corresponds to the input file, TNS008.

G.2 Inherent Limitations of the Code

There are several restrictions on the use of this code. First, the model which describes fast alpha guiding-center motion is accurate to first-order in gyroradius, requiring $\bar{\beta} < 0.1$. Also, the restrictions on code runtime and memory require that the various alpha profiles (density, current, etc.) be accumulated in no more than 16 intervals in ψ . For example, the flux-surface-averaged alpha density is computed for $0 \leq \psi \leq \psi_1$, $\psi_1 \leq \psi < \psi_2$, etc., and similarly for the other profiles. The approximations used in this allotment among ψ -bins place inherent limitations on the numerical accuracy of the parallel component of alpha current, $\langle I_{\parallel} \rangle$. The value of $\langle I_{\parallel} \rangle$ computed using 16 ψ -bins (i.e., $0 \leq \psi/\psi_{\text{edge}} \leq \frac{1}{16}$, $\frac{1}{16} \leq \psi/\psi_{\text{edge}} \leq \frac{1}{8}$, etc.) is typically 8% larger than when the whole plasma is spanned by a single ψ -bin. The latter value is more accurate because the approximations used to apportion the contributions among each ψ -interval create $\sim 0.5\%$ error per ψ -bin. Only the convergence of the total alpha current is affected by these

limitations. If the total current is important, rather than profile information, line 170 should be changed to

$$NPSI = 1$$

The whole plasma is then spanned by the single ψ -bin described above.

The final limitation on the use of SLWDN9 is related to convergence of $\langle I_\alpha \rangle$ as the number of phase space intervals changes. The phase space labels used to parameterize each orbit are v (ion speed), ψ_x (the maximum value of the poloidal flux function along the guiding-center orbit, where ψ is increasing from the magnetic axis), and ζ (the cosine of the angle between the parallel component of plasma current $\vec{J}_\parallel$, and the ion velocity, $\vec{v}$). The various alpha particle profiles are obtained by integrating moments of the α distribution function over this phase space. In particular, convergence of $\langle I_\alpha \rangle$ is demonstrated in Table G.1 as the number of phase space intervals (N_v , N_ζ , N_{ψ_x}), corresponding to (v , ζ , ψ_x), is successively doubled.

Table G.1: Convergence of $\langle J_\parallel \rangle$ (kA)

	N_v	N_ζ	N_{ψ_x}	$\bar{\beta} = 0.008$	$\bar{\beta} = 0.075$
I_1	8	16	8	10.01	67.85
I_2	16	32	16	8.399	52.36
I_3	32	64	32	7.480	42.63
I_∞	(limit $\rightarrow \infty$)			6.3	26.2

For $(N_V, N_\zeta, N_{\psi_x}) = (4, 8, 4)$ the numerical methods are too crude to give meaningful results. For phase space intervals greater than $(32, 64, 32)$, the runtime of the code is prohibitively long.

For successive doubling of the number of phase space intervals, there is a geometric convergence (refer to Table G.1):

$$I_2 = I_1 - \Delta I, \quad (G.1)$$

$$I_3 = I_1 - \Delta I - \xi \Delta I, \quad (G.2)$$

$$\vdots$$

$$I_\infty = I_1 - \Delta I - \xi \Delta I - \xi^2 \Delta I \dots = I_1 - \Delta I / (1 - \xi). \quad (G.3)$$

Solving (G.1) and (G.2) for ΔI and ξ yields:

$$\xi = \frac{I_2 - I_3}{I_1 - I_2}, \quad \Delta I = I_1 - I_2. \quad (G.4)$$

The value of I_∞ is obtained by applying (G.3) and (G.4) to the data in Table G.1; this is the "converged" value for $\langle I_\infty \rangle$. The default values for $(N_V, N_\zeta, N_{\psi_x})$ are $(32, 64, 32)$ yielding convergence to better than 1% for the alpha particle density, and heating profiles. To use a different number of intervals, lines 166-168 should be changed. For example, to determine I_1 in Table G.1, these lines become:

NV = 8

NZ = 16

NPM = 8

The program can then be compiled and run as described above.

G.3 Additional Options

There are other options available for user control of SLWDN9. Different fuel cycles and fusion products can be investigated by making

the appropriate alterations. For example, to calculate 3-MeV proton slowing down effects in a deuterium plasma, change lines 145-147. The new fusion-product is described by its mass (MASS) in amu, charge (Z) in multiples of the protonic charge, and birth energy (EN) in MeV:

MASS = 2.

Z = 1.

EN = 3.03

To select the deuterium plasma and corresponding fusion rates, change line 158 to

ISV = 3

The settings of ISV are as follows: 1 for D- ^3He , 2 for D-T, 3 for $\text{DD} \rightarrow \text{p} + \text{T}$, and 4 for $\text{DD} \rightarrow \text{n} + ^3\text{He}$. The background ion density profile is parameterized by $n_i/n_{i0} = (1. - \text{GN} * (\psi/\psi_{\text{edge}}) ** \text{NN}) ** \text{SN}$. Here GN, NN, and SN are all real, so to model a profile like $n_i/n_{i0} = [1 - .9(\psi/\psi_{\text{edge}})^2]^{1/2}$, lines 155-157 become:

GN = 0.9

NN = 2.0

SN = 0.5

Finally, the default value of central ion temperature (15 keV) can be modified at line 159; for example, to use 20 keV:

TMAX = 20.

The corresponding T - profile is determined from the pressure profile (used to generate the equilibrium) and the chosen n-profile, using the ideal gas law with $n_i = n_e$.

G.4 Description of Input

The input file characterizes the $1\frac{1}{2}$ -D, non-circular tokamak equilibrium, determined on an R-z mesh. The first two lines of input summarize the equilibrium set and are displayed as a part of the output. The remainder of the file describes the profiles of plasma pressure, $p(\psi)$ and F-value, $F(\psi) = RB_\phi$, plus the poloidal flux function $\psi(R,z)$. A detailed deliniation of the input data is shown in Table G.2 with a corresponding example shown in Table G.3. Sets of equilibrium data, other than the examples given here, can be obtained from Martin Peng or Dennis Strickler (ORNL).

G.5 Description of Output

The output file consists of two sections: a summary of the runtime and input information, and then the code results. Table G.4 discusses the details on a line-by-line basis and Table G.5 displays an example.

Table G.2: Input Variables for SLWDN9

Line(s)	Variable	Description
1	BETAT	Average toroidal beta
1	QPSI	q-value at the plasma edge
1	TIP	Total plasma current (amps/10.)
1	BT	Magnitude of the toroidal $\vec{B}$ -field (gauss) at R_0
2	RAXIS	Approximate value of major radius, R_0 (meters)
2	PAXIS	Value of poloidal flux function (webers) at $(R_0, 0)$
2	PWALL	Value of poloidal flux function (webers) at wall
2	BETAP	Average poloidal beta
3	NOCONS	Number of ψ -values used to tabulate $F(\psi)$, $dF/d\psi$, and $p(\psi)$; see below.
3	NCFT	Number of spline coefficients for $F(\psi)$
3	NCFT1	Number of spline coefficients for $p(\psi)$
4-44	PSITAB	Sequence of NOCONS ψ -values, ψ_i (in webers), used to tabulate $F(\psi)$, $dF/d\psi$, and $p(\psi)$; see above.
4-44	FTAB	Sequence of NOCONS F-values at ψ_i , i.e., $F(\psi_i)$ in webers/m
4-44	FPTAB	Sequence of NOCONS values of $dF/d\psi$ at ψ_i in m^{-1}
4-44	PTAB	Sequence of NOCONS $p(\psi)$ -values at ψ_i , i.e., $p(\psi_i)$ in newtons
45-54	XFSQ	Sequence of NCFT normalized ψ -values (ψ/ψ_{wall}) used to spline fit $F(\psi)$
45-54	CF	Sequence of NCFT spline coefficients for $F^2(\psi)$ in $(\text{webers})^2/m^2$
55-64	XPRS	Sequence of NCFT1 normalized ψ -values (ψ/ψ_{wall}) used to spline fit $p(\psi)$ in newtons
55-64	CPMKS	Sequence of NCFT1 spline coefficients for $p(\psi)$
65	JMAX	Number of equally spaced R-values, over which $\psi(R, z)$ is tabulated

Table G.2: (continued)

Line(s)	Variable	Description
65	KMAX	Number of equally spaced z -values, over which $\psi(R,z)$ is tabulated
66-69	RJ	Array of R_j -values (in meters) in ascending order over which $\psi(R,z)$ is tabulated
97-122	ZK	Array of z_k -values (in meters) over which $\psi(R,z)$ is tabulated: $-\Delta z/2, \Delta z/2, 3\Delta z/2, \dots$
123-4003	PSI	Array of ψ -values (in webers), $\psi(R_j, z_k)$, ordered like $\psi(R_{\min}, -\Delta z/2), \dots, \psi(R_k^{\max}, -\Delta z/2), \dots, \psi(R_{\min}, z_{\max}), \dots, \psi(R_{\max}, z_{\max})$

Table G.3: Example of an Input Data File

Line					
1	7.53569315E-03	3.20692909E+00	5.78062641E+05	5.29872354E+04	
2	5.00000000E+00	-3.95362033E+00	-1.09311402E-01	3.88501212E-01	
3	41 10 10				
4	-3.95362033E+00	2.70123748E+01	-2.42677006E-01	2.99784001E+05	
...	...				
44	-1.09311402E-01	2.64936177E+01	-1.44062964E-03	0.	
45	0.	7.29992330E+02			
46	0.	7.24470036E+02			
47	0.	7.14880269E+02			
48	0.	7.04843659E+02			
49	3.3333333E-01	7.01904300E+02			
50	6.66666667E-01	7.01913795E+02			
51	1.00000000E+00	0.			
52	1.00000000E+00	0.			
53	1.00000000E+00	0.			
54	1.00000000E+00	0.			
55	0.	2.99796655E+05			
56	0.	2.41823371E+05			
57	0.	1.40451232E+05			
58	0.	3.27832142E+04			
59	3.3333333E-01	1.38877745E+02			
60	6.66666667E-01	-1.48443630E+01			
61	1.00000000E+00	0.			
62	1.00000000E+00	0.			
63	1.00000000E+00	0.			
64	1.00000000E+00	0.			
65	154 126				
66	3.79210526E+00	3.80789474E+00	3.82368421E+00	3.83947368E+00	3.85526316E+00
...	...				
96	6.16052632E+00	6.17631579E+00	6.19210526E+00	6.20789474E+00	
97	-7.98387097E-03	7.98387097E-03	2.39516129E-02	3.99193548E-02	5.58770468E-02
...	...				
121	1.90814516E+00	1.92411290E+00	1.94008065E+00	1.95604839E+00	1.97201613E+00
122	1.98798387E+00				
123	3.64325860E-02	-3.65162777E-02	-1.09855019E-01	-1.83597116E-01	-2.57687493E-01
124	-3.32071842E-01	-4.06695466E-01	-4.81505106E-01	-5.56448342E-01	-6.31473465E-01
...	...				

Table G.4: Output from SLWDN9

Line(s)	Variable	Description
1	SLWDN9	Version of code, as distinguished from earlier versions (e.g., SLWDN8)
1	02/02/80C	Run 02 Feb 1980 on C-machine (CRAY-1)
1	07:37:46	Time of run (PST)
2	BETAT	Average toroidal beta of plasma
2	BETAP	Average poloidal beta
2	Q(A)	Q-value at plasma edge
2	IO	Total plasma current (amps)
2	BT	Magnitude of toroidal field at $(R_0, 0)$
3	RO	Major radius (meters) at magnetic axis
3	RIN	Major radius (meters) at inboard edge
3	ROUT	Major radius (meters) at outboard edge
3	PEDGE	ψ -value at plasma edge (webers)
4	Z	Fusion-product charge (multiples of proton)
4	MASS	Fusion-product mass (amu)
4	ENERGY	Fusion-product birth energy (MeV)
4	VEL	Fusion-product birth speed (m/s)
5	NV	Number of phase space intervals in speed, v
5	NPM	Number of phase space intervals in ψ_x
5	NZ	Number of phase space intervals in ζ
5	NINT	Number of ψ -intervals to span plasma
5	NPSI	Number of ψ -bins
5	NVOL	Number of ψ -intervals to obtain volume and area integrals
5	STOL	Search tolerance termination criterion

Table G.4: (continued)

Line(s)	Variable	Description
6-23	P/PWL	Normalized ψ -value (ψ/ψ_{wall}) at which subsequent variables are evaluated
6-23	DV	Amount of plasma volume (m^3) contained in this ψ -bin
6-23	T	Ion temperature (keV)
6-23	NI	Ion density ($\#/\text{m}^3$)
6-23	NP	Number of fusion products born ($\#/\text{m}^3\text{-s}$)
6-23	NA	Flux-surface-averaged number of alphas ($\#/\text{m}^3$)
6-23	JA	Flux-surface-averaged current density (amps/m^2)
6-23	GE	Flux-surface-averaged heating rate (keV/s-electron) of electrons, accounting for large banana-width orbits (LRG BANAN) and IN-SITU deposition
6-23	GI	Same as GE, but for ions
24	DV	Total plasma volume (m^3)
24	NP	Total fusion-product rate ($\#/\text{s}$)
24	NA	Total number of fusion products in plasma
24	JA	Total alpha particle current (amps)
24	GE	Total electron and ion heating rates for both the large banana-width (LRG BANAN) and IN-SITU deposition models
25	CPU	Central processing time (sec)
25	ID	Input/output time (sec)
25	SYS	System overhead time (sec)
25	TOT	Total execution time for this run (sec)

Table G.5: Example of an Output Data File

SLWDN9 ON: 02/02/80C AT 07:37:46 PACIFIC TIME
 PLASMA--BETAT=0.008, BETAP=0.389, Q(A)=3.207, IO(AMPS)=5780626., BT(G)=52987.
 SIZE: RO(M)=5.035, RIN(M)=3.860, ROUT(M)=6.145 PEDGE(W)=3.675
 FUSION PRODUCT: Z=2., MASS(AMU)=4.00 ENERGY(MEV)=3.52, VEL(M/S)=1.303E+07
 NV= 32, NPM= 32, NZ= 64, NINT=1024, NPSI= 16, NVOL=128, STOL= 1.0E-05

#	P/PWL	DV:M3	T:KEV	NI:#/M3	NP:#/M3/S	NA:#/M3	JA:A/M2	GE:KEV/S/ELECTRON		GI:KEV/S/ION	
	0.0		15.00	6.24E+19	2.578E+17			LRG BANAN	IN-SITJ	LRG BANAN	IN-SITU
1	0.030	8.95	14.24	6.23E+19	2.327E+17	2.216E+17	1.402E+03	9.462E+00	1.023E+01	2.346E+00	2.916E+00
2	0.090	9.19	12.84	6.19E+19	1.853E+17	1.735E+17	2.934E+03	8.329E+00	8.399E+00	1.875E+00	2.153E+00
3	0.149	9.45	11.58	6.10E+19	1.429E+17	1.262E+17	3.257E+03	6.898E+00	6.712E+00	1.392E+00	1.549E+00
4	0.209	9.72	10.42	5.97E+19	1.065E+17	9.013E+16	3.696E+03	5.581E+00	5.213E+00	1.015E+00	1.083E+00
5	0.269	10.02	9.37	5.79E+19	7.631E+16	5.836E+16	3.537E+03	4.364E+00	3.921E+00	6.600E-01	7.316E-01
6	0.329	10.34	8.39	5.57E+19	5.230E+16	3.681E+16	2.770E+03	3.100E+00	2.843E+00	4.302E-01	4.753E-01
7	0.388	10.68	7.47	5.31E+19	3.399E+16	2.283E+16	2.089E+03	2.285E+00	1.973E+00	2.661E-01	2.942E-01
8	0.448	11.07	6.61	5.00E+19	2.072E+16	1.308E+16	1.462E+03	1.533E+00	1.299E+00	1.569E-01	1.718E-01
9	0.508	11.50	5.80	4.64E+19	1.168E+16	7.343E+15	8.658E+02	1.046E+00	8.014E-01	8.807E-02	9.319E-02
10	0.568	11.99	5.02	4.25E+19	5.951E+15	3.876E+15	5.825E+02	6.431E-01	4.548E-01	4.890E-02	4.601E-02
11	0.627	12.56	4.28	3.81E+19	2.651E+15	1.788E+15	3.301E+02	3.995E-01	2.309E-01	2.197E-02	2.002E-02
12	0.687	13.23	3.56	3.32E+19	9.758E+14	8.268E+14	1.921E+02	2.313E-01	1.001E-01	1.057E-02	7.282E-03
13	0.747	14.05	2.85	2.79E+19	2.692E+14	2.895E+14	1.021E+02	1.168E-01	3.425E-02	3.681E-03	2.027E-03
14	0.807	15.07	2.16	2.22E+19	4.656E+13	8.113E+14	4.352E+01	5.932E-02	8.072E-03	9.300E-04	3.714E-04
15	0.866	16.43	1.47	1.60E+19	3.364E+12	1.458E+13	9.312E+00	2.339E-02	9.949E-04	1.418E-04	3.317E-05
16	0.926	18.40	0.80	9.41E+18	2.807E+10	1.003E+12	8.613E-01	5.662E-03	3.294E-05	7.140E-06	6.998E-07
	192.65				8.343E+18	7.172E+18	7.507E+03	4.751E+06	4.705E+06		
EXECUTION TIMES(CPU, IO,SYS,TOT):					3.668E+03	3.083E+00	4.053E+00	3.675E+03			

VITA AND PUBLICATIONS

Lee M. Hively was born (December 30, 1947) and raised in Williamsport, PA. His undergraduate studies at the Pennsylvania State University were completed in 1970 with a B.S. in Engineering Science, a B.S. in Mathematics and a B.A. in General Arts and Sciences plus a minor in physics. In 1971, he finished an M.S. in Physics at the University of Illinois. He then began full-time work with Western Electric Company as a Member of Research Staff in millimeter waveguide processing, while continuing part-time graduate studies at Princeton University and Rutgers University. He later obtained a U.S. patent (#3,944,963) on an ultrasonic measuring device for waveguides. He returned to the University of Illinois in 1974 to complete his doctoral studies in High-energy fusion-product transport in tokamaks with a Ph.D. in Nuclear Engineering (1980).

Mr. Hively has made substantial contributions to numerous conference papers and has published two articles in Nuclear Fusion, with a third paper submitted (see attached list of publications). He has also participated in preparation and teaching of the 1976 and 1977 ANL Faculty Conferences on Fusion. In April 1978, he was elected a member of Sigma Xi, the Scientific Research Society of North America. He received the first ANS Student Award for Fusion Science and Engineering in June 1978.

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