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Yonathan Shapir
Physics Department
Brookhaven National Laboratory
Upton, New York 11973
U.S.A.

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Replica-Space Renormalization in Random-Field Systems

Yonathan Shapir
Physics Department
Brookhaven National Laboratory
Upton, New York 11973
U.S.A.

Abstract

The critical behavior of random-field systems is characterized by exponentially long relaxation times. They may differ by orders of magnitude and slow modes have to be considered quenched with respect to the faster ones. This requires a drastic modification of the renormalization process in which the degrees of freedom are integrated out. A new replica-space renormalization procedure, to carry out the coarse-graining of the time intervals, is presented. We relate by recursion-relations the effective reduced dimensions of consecutive time scales. Their stable fixed-points yield the apparent dimensionalities for the longest and shortest time scales, which are insensitive to the exact behavior on intermediate scales. For Ising systems we find that $d = 2$ is the lower critical dimension in both regimes. The thermal exponents for $d = 3$, in the short-time observable regime, are related to those of the pure system in $d = 2$. This is consistent with the observations in field-cooled random antiferromagnets of the correlation length exponent $\nu \approx 1$ (by neutron scattering) and of the symmetric logarithmic divergence in the specific-heat (by linear birefringence).

Many difficulties are encountered in the straightforward application of the equilibrium renormalization-group approach¹⁻⁵ to random-field (RF) systems (see ref. 6-9 for recent reviews). Its prediction of an effective reduction in dimension $d \rightarrow d-2$ for the critical properties is in disagreement with exact results on the lower critical dimension¹⁰ ($d_{lcd} = 2$ in Ising systems) and with critical exponents, measured over an intermediate range of temperatures, in three-dimensional systems (for recent reviews on the experimental side see ref. 11-13) realized by field-cooled random antiferromagnets¹⁴. Recently we have proposed an alternative approach of self-consistent dimensional reduction based on physical considerations (see Sec. II) of Aharony, Imry and Ma³. The dimensional-reduction which follows agrees with the experimental measurements (consistent with a reduced dimension $d = 2$ for the thermal exponents) and yields the correct critical dimensionalities^{8,15}. Elsewhere¹⁶, we have presented a derivation of these results from a self-similar replication procedure. In the present paper we elaborate further on this approach. It may be useful to other systems in which critical fluctuations are spread over many decades of time scales. In such systems the rescaling of the time intervals must account for the fact that slow fluctuations are practically quenched with respect to time scales of the faster fluctuations. We propose to perform time coarse-graining by a self-similar replication which amounts to a replica-space renormalization (RSR): matrices in this space will be iteratively decimated into scalars (similar to block-spins) or vice-versa depending on the regime.

In the usual RG picture⁵, depicted in Fig. 1a, the order-parameter is divided into two parts: $\phi(\vec{r}) = \phi_h(\vec{r}) + \psi(\vec{r})$. In a finite, but large, time t_0 we expect the decomposition to be different (Fig. 1b):

$$\psi_0(\vec{r}) \xrightarrow{\lambda^{(0)}} \phi_1(\vec{r}) + \psi_1(\vec{r}) . \quad (1)$$

$\phi_1(\vec{r})$ denotes many configurations (and we do not have to worry about the uniqueness of $\phi_h(\vec{r})$ which are absorbed into $\phi_1(\vec{r})$) the fluctuations among which will occur only on time scales $t > t_1$. $\psi_1(\vec{r})$, on the other hand, include all fluctuations with $t < t_1$. However, if we observe the behavior on a somewhat shorter (still large) time scale $t_2 < t_1$, each $\psi_1(\vec{r})$ will itself split into a frozen configuration $\phi_2(\vec{r})$ (with $t_2 < t < t_1$) and the faster ($t < t_2$) fluctuations and so on and so forth. The same picture is obtained for the spin-spin correlations. The RG result (Fig. 2a) is replaced by a more intuitive situation where this splitting between "frozen" and annealed components applies self-similarly on all time scales (Fig. 2b).

The structure in replica-space is naturally suggested from our discussion so far. The situation corresponding to the standard approach is shown in Fig. 3a where the scalar autocorrelations of the pure system are replaced by a matrix $n \times n$ ($n > 0$) which is a sum of a constant matrix of the frozen correlations and a scalar matrix of the connected thermal correlations:

$$\langle \phi^2 \rangle \xrightarrow{\lambda} q^{\alpha\beta} = Q \Pi_{\alpha\beta} + \tilde{q} \delta_{\alpha\beta} \quad (2)$$

where $\Pi_{\alpha\beta}$ is unity for each pair (α, β) .

The procedure of the self-similar replication will take each scalar diagonal element and will replace it again by an $n_k \times n_k$ ($n_k > 0$) matrix of the same structure as depicted in Fig. 3b.

$$\tilde{q}_{k-1} \delta_{\alpha_{k-1} \beta_{k-1}} \xrightarrow{\lambda(k-1)} [q_k \Pi_{\alpha_k \beta_k} + \tilde{q}_k \delta_{\alpha_k \beta_k}] \delta_{\alpha_{k-1} \beta_{k-1}} \quad (3)$$

One should note both the similarities and differences from Parisi's replica-symmetry breaking in long-range spin-glasses¹⁷.

Recursion-relations between critical exponents on different iterations were related¹⁶ through the scaling dimension of the internal RF, $\lambda(k)$. For the observable fast fluctuations they yield a dimensional-reduction by one both in $d = 3$ and $d = 2$ in agreement with experiments, simulations and exact results. Other glassy systems may be candidates to a similar approach and in particular short-range spin-glasses and random-anisotropic materials.

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Figure Captions

Fig. 1: The decomposition of the fluctuating order parameter to slow and fast components: (a) in the standard approach; (b) in the RSR scheme.

Fig. 2: The decomposition of the spin-spin correlations: (a) in the equilibrium RG; (b) in the multi-time scales approach.

Fig. 3: The corresponding structure of the matrix $q^{\alpha\beta}$ in replica-space: (a) in the standard RG; (b) in the RSR approach.

(a)

(b)

Fig. 1

$$\begin{array}{c} \phi \quad \phi \\ \hline \downarrow \lambda \\ \phi(1) \quad \phi(1) \quad + \quad \psi \quad \psi \\ \hline \times \end{array}$$

(a)

$$\begin{array}{c} \psi(0) \quad \psi(0) \\ \hline \downarrow \lambda^{(0)} \\ \phi(1) \quad \phi(1) \quad + \quad \psi(1) \quad \psi(1) \\ \hline \times \end{array} \quad \begin{array}{c} \downarrow \lambda^{(1)} \\ \phi(2) \quad \phi(2) \quad + \quad \psi(2) \quad \psi(2) \\ \hline \times \end{array} \quad \begin{array}{c} \downarrow \lambda^{(2)} \\ \vdots \\ \phi(k) \quad \phi(k) \quad + \quad \psi(k) \quad \psi(k) \\ \hline \times \end{array}$$

(b)

Fig. 2

$$\begin{array}{c} \overline{} \\ \downarrow \lambda \\ \left(\begin{array}{cccccc} - & + & * & & - & * & \dots & * \\ * & & - & + & * & & - & * & \dots & * \\ * & & * & & - & + & * & \dots & * \\ \vdots & & & & & & & & & \vdots \\ * & & & & & & - & + & * & \end{array} \right) \end{array}$$

(a)

$$\begin{array}{c} \overline{} \\ \downarrow \lambda^{(0)} \\ \left(\begin{array}{cccccc} - & + & * & & - & * & \dots & * \\ * & & - & + & * & & - & * & \dots & * \\ \vdots & & & & & & & & & \vdots \\ * & & & & - & * & & - & + & * \end{array} \right) \\ \downarrow \lambda^{(1)} \\ \left(\begin{array}{cccccc} - & + & * & - & * & \dots & * & & & \\ * & & - & + & * & \dots & * & + & \text{---} * & \dots & \circ \circ \circ \circ \\ \vdots & & & & \vdots & & & & & & \\ * & & * & & \dots & - & + & * & & & \\ & & \text{---} * & & & - & + & * & \dots & * & + \\ & & & & & \vdots & & & & & \\ & & & & & * & & - & + & * & \\ & & \circ & & & \circ & & & & & \\ & & \circ & & & \circ & & & & & \\ & & \circ & & & \circ & & & & & \end{array} \right) \end{array}$$

(b)

Fig. 3