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A RADIAL FOKKER-PLANCK MODEL FOR PLASMAS
CONFINED BY MAGNETIC MIRROR FIELDS

A. H. Futch

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PREFACE

The work described in the present paper was initiated almost ten years ago and was essentially completed in 1977 when quasilinear diffusion was added to the existing two-dimensional Fokker-Planck code for the distribution function, $f(v,r)$. An approximate treatment by T. B. Kaiser¹ had extended the one-dimensional quasilinear theory to include spatial diffusion in two dimensions perpendicular to the magnetic field. After adding quasilinear terms to the radial Fokker-Planck code (RFP), and setting the fluctuating potential in the code equal to probe measurements of this potential in the fan region of the 2XIIB experiment, calculated values of T_e decreased from classical Fokker-Planck values (1000 eV for 2XIIB) to the 50 eV to 200 eV range observed in 2XIIB and TMX experiments. Other plasma parameters such as the particle confinement times, radial density profiles, ion densities and energies were also in good agreement. As a result of the relatively large gyro-radius, radial transport in the Fokker-Planck model was observed to dominate the particle and power loss from the plasma core.

Preliminary results were discussed in an American Physical Society paper on October 30, 1978 at Colorado Springs, Colorado. The data from this poster session is presented without comment as Appendix A in this paper. In addition the author (memorandum A. H. Futch, June 25, 1979) showed that if the ion lifetime in the 2XIIB experiment was limited by electron drag and if T_e was constant or decreased with radius as measured experimentally, it followed from the drag formula that the ion lifetime decreases with radius. However, calculations of the ion lifetime from measurements of the radial density profile and assuming the only sources was beam ionization resulted in an ion lifetime that increased with radius. This suggested to the author that electron drag was not the only important loss process and that radially outward diffusion due to beam trapping and ion cyclotron fluctuations could play an important role. This unexpected n_i behavior vanished if an additional source term was introduced to account for radial transport. Appendix C is another memorandum showing a comparison between the calculated radial plasma-density profile and experimentally measured results for the plug in the TMX experiment. The data illustrates the effects of radial diffusion in creating a hollow density profile.

Since then the code has been used to calculate plasma parameters including radial transport in TMX, TMX-U, and in several other laboratories. This report is being issued in response to requests for description of the code and will serve to document the earlier work. At the present time the code is being used for modeling beam injection in the central cell of the thermal barrier tandem mirror TMX-U.

ABSTRACT

A time-dependent computer model has been developed for the spatially dependent distribution function $f(r,v,t)$. An orbit averaged Fokker-Planck equation treats Coulomb collisions and various atomic physics processes. The motivation for the present code is to provide an accurate treatment of gyro-radius effects and realistic beam geometries which have been neglected in previous Fokker-Planck codes. Finite width beams may be offset from the plasma center to determine beam injection for build-up studies. The inclusion of a finite gyro-radius is essential for studying radial transport due to both quasilinear fluctuations and charge exchange collisions. Charge exchange with energetic beam atoms and plasma erosion due to charge-exchange collisions with thermal gas are both included. These effects are major contributions to the loss of plasma in present experiments. Quasilinear diffusion terms include the effects of turbulent diffusion in the model.² The model has been used to study particle and energy confinement times in the 2XIIB experiment and in the mirror plug of the TMX experiment. Radial diffusion significantly changes the particle and energy confinement times in the plasma core. Calculated values of T_e decrease from classical Fokker-Planck values (1000 eV for 2XIIB) to the 50 eV to 200 eV range observed in 2XIIB and TMX experiments. Radial density profiles and n_r scaling with plasma radius and magnetic field agree with experimental data.

I. INTRODUCTION

The present code results from the merger of two separate lines of development: time-dependent Fokker-Planck codes describing Coulomb collision processes, and spatially dependent particle buildup codes. Fowler and Rankin (1962) and Futch and Killeen (1962) solved the Fokker-Planck equations for both ions and electrons with the assumption that the Fokker-Planck equation is separable. The evolution of numerical Fokker-Planck calculations was reviewed by Kuo-Petravic et al. in 1969. The time-dependent code developed at LLNL and described in detail by Killeen and Futch (1968) included the effects of charge exchange and time dependent buildup of plasmas formed by neutral-beam injection. The time-dependent LLNL code was extended to treat an arbitrary number of charged species (Futch, Holdren, Killeen, and Mirin (1972)). Parallel development at LLNL resulted in the development of two-dimensional, $f(v, \theta)$, multi-species Fokker-Planck codes (J. Killeen and A. A. Mirin, 1978 and J. Killeen et al., 1976).

Spatially dependent particle buildup codes for plasma production by neutral-atom injection evolved from an early code at LLNL (Futch, Heckrotte, Damm, Killeen, and Mish, 1962). Moir (unpublished) and Stallard (Stallard, UCRL-51784) studied the radial expansion (or erosion) caused by the competition between the trapping rate for ions at the plasma boundary and the loss rate due to charge exchange with thermal gas at the boundary. Their code included the radial dependence of the thermal gas and the trapping of secondary beams of neutral atoms resulting from charge exchange collisions, but failed to distinguish between particle density and guiding

center density in the trapping process. The present work generalizes the particle buildup code [Futch, Heckrotte, and Damm (1962)] to an equation for the distribution function $f(r,v,t)$, and includes to first order secondary neutral atom beams resulting from charge-exchange collisions of plasma ions with thermal gas and beam neutrals. The Coulomb processes are described by extending the multi-species Fokker-Planck code [Futch, Holdren, Killeen, and Mirin (1972)] to include the radial dimension.

Turbulent diffusion has been observed to play a key role in many mirror experiments (Bajborodov, et al., "Control Fusion and Plasma Physics," 2, 647 (1971); C. C. Damm, J. H. Foote, A. H. Futch, A. L. Gardner, and R. F. Post, Phys. Rev. Letters 13, 464 (1964); L. C. Kuo, E. G. Murphy, M. Petravic, and D. R. Sweetman, Phys. Fluids, 7, 988 (1964); F. H. Coensgen, W. F. Cummins, B. G. Logan, A. W. Molvik, W. E. Nexsen, T. C. Simonen, B. W. Stallard, and W. C. Turner, "Stabilization of a Neutral-Beam-Sustained, Mirror-Confined Plasma," Phys. Rev. Lett. 35, 1501 (1975). Calculations by Berk and Stewart² included quasilinear diffusion in velocity space to model DCLC turbulence in the 2XIIB experiment. Kaiser¹ suggested an approximate treatment for retaining the turbulence model of the 1D theory and extending the velocity space dimensionality to two directions perpendicular to the magnetic field. Baldwin, Berk, and Pearlstein developed a quasilinear model for the nonlinear saturation of the DCLC mode in mirror machines which was consistent with experimental observations in 2XIIB. [D. E. Baldwin, H. L. Berk, and L. D. Pearlstein, "Turbulent Lifetimes in Mirror Machines," Phys. Rev. Lett. 36, 1051 (1976).] We have included quasilinear diffusion in the present code using the formulation derived by Berk and Matsuda (p.103, UCRL-50002-77).

A principal contribution of the work is the inclusion of atomic physics trapping and loss processes including the finite orbit size of the ions in a two-dimensional orbit-averaged Fokker-Planck code, the code is capable of treating an arbitrary number of species and neutral beams of finite width. Diagnostics serve as a check on the conservation of particles and energy. They also display the details of particle and energy transfer within the plasma, and particle losses from the plasma. Finally, the good agreement with extensive experimental data, gives confidence in the use of the code for extrapolating to new regions.

The work is organized as follows; Section II describes the basic equations and assumptions. Section III discusses the computational procedure. A comparison with previous work is given in Section IV. The code results are compared with experimental data in Appendix A.

II. MATHEMATICAL MODEL

This section is organized as follows: the generalized Boltzmann equation is first expressed in terms of constants of the motion. Additional simplification is introduced by making the frequently used assumption of isotropy of the Rosenbluth potentials and by integrating the distribution over θ to obtain an equation independent of angle. The atomic physics and neutral gas equation are discussed in Sections 2 and 3. The electron equation is introduced in Section 4 and the quasilinear terms in Section 5. The complete guiding center equation is discussed in the final section.

1. Boltzmann's Equation

The Boltzmann equation with Fokker-Planck collision terms is the appropriate kinetic equation for describing the distribution function for charged particles in a magnetic field:

$$\frac{\partial f_a}{\partial t} + \underline{v} \cdot \frac{\partial f_a}{\partial \underline{r}} + \frac{\underline{F}}{m_a} \cdot \frac{\partial f_a}{\partial \underline{v}} = \left(\frac{\partial f_a}{\partial t} \right)_c + \left(\frac{\partial f_a}{\partial t} \right)_a + \left(\frac{\partial f_a}{\partial t} \right)_{q1} \quad (1)$$

where f_a is the distribution function for particles of specie "a", $\partial f_a / \partial t)_a$ represents the atomic physics terms, and $\partial f_a / \partial t)_c$ is the collision term. The Fokker-Planck collision term may be generalized to include turbulent diffusion by adding the appropriate quasilinear collision terms shown explicitly as the last term of Eq. (1). For solving Eq. (1) in mirror geometry, a spherical coordinate system is usually chosen for velocity space and a cylindrical coordinate system for physical space with z along the axis of symmetry and $\theta = 0$ corresponding to the magnetic field direction. For the present model, the following assumptions are made:

1. The usual square well model for the spatial variation of the ambipolar potential and magnetic field is adopted where the distribution functions are independent of the spatial coordinate along the magnetic field (z direction). For this model which represents the plasma in the uniform central region of a long mirror machine the magnetic mirrors and ambipolar potential effect only the boundary conditions in velocity space.

2. The only force acting on a particle other than collisions is the magnetic force resulting from a uniform magnetic field.
3. The distribution function is a function of radius, but independent of the azimuthal angle in physical space.

Under these conditions, Eq. (1) reduces to

$$\frac{\partial f_a}{\partial t} + v_r \frac{\partial f_a}{\partial r} = \left(\frac{\partial f_a}{\partial t} \right)_c + \left(\frac{\partial f_a}{\partial t} \right)_a + \left(\frac{\partial f_a}{\partial t} \right)_{q1} \quad (2)$$

Equation (2) may be simplified by observing that the second term on the left hand side is larger by the ratio Ω/v_c than the remaining terms where Ω is the ion cyclotron frequency and v_c is the coulomb collision frequency. This suggests an expansion of f_a in powers of v_c/Ω . Dropping the subscript "a", one sets

$$f = f_0 + f_1 + \dots \quad (3)$$

and equates like powers of v_c/Ω to obtain

$$v_r \frac{\partial f_0}{\partial r} = 0 \quad (4)$$

$$\frac{\partial f_0}{\partial t} + v_r \frac{\partial f_1}{\partial r} = \left(\frac{\partial f_0}{\partial t} \right)_c + \left(\frac{\partial f_0}{\partial t} \right)_a + \left(\frac{\partial f_0}{\partial t} \right)_{q1} \quad (5)$$

From Eq. (4), the zeroth order distribution function is independent of r .

In general, this means that

$$f_0 = f_0(E, \mu, P_\chi, t) \quad (6)$$

where E , the particle energy, μ , the magnetic moment, and P_χ , the canonical angular momentum, are all constants of the motion. For the square well model, E may be replaced by v , μ by θ , and P_χ by the guiding center radius, R . It follows from the formulae for μ , and P_χ that θ and R are constants of the motion if the magnetic field B is constant, i.e.,

$$\mu = \frac{E_\perp}{B} \quad (7)$$

and

$$P_\chi = \frac{eB}{2} (R^2 - \rho^2) \quad (8)$$

where ρ is the Larmor radius. The zero order distribution function becomes

$$f_0 = f_0(v, \theta, R, t) \quad (9)$$

Dividing Eq. (5) by v_r and integrating around a gyro-orbit gives

$$\frac{\partial f_0}{\partial t} \oint \frac{dr}{v_r} + \oint \frac{\partial f_1}{\partial r} dr = \oint \left[\left(\frac{\partial f_0}{\partial t} \right)_c + \left(\frac{\partial f_0}{\partial t} \right)_a + \left(\frac{\partial f_0}{\partial t} \right)_{ql} \right] \frac{dr}{v_r} \quad (10)$$

$$\frac{\partial f_0}{\partial t} = \frac{1}{\tau} \oint \left[\left(\frac{\partial f_0}{\partial t} \right)_c + \left(\frac{\partial f_0}{\partial t} \right)_a + \left(\frac{\partial f_0}{\partial t} \right)_{ql} \right] \frac{dr}{v_r} \quad (11)$$

where

$$\tau = \oint \frac{dr}{v_r} = \frac{2\pi}{\Omega} \quad (12)$$

The Fokker-Planck collision term (Rosenbluth et al., 1957) is

$$\left(\frac{\partial f_a}{\partial t} \right)_c = \frac{1}{2} \nabla_v \cdot [(\nabla_v f_a) \cdot D - f_a \nabla_v h_a] \quad (13)$$

$$\begin{aligned} &= \frac{1}{2v^2} \frac{\partial}{\partial v} \left\{ v^2 \left[D_{vv} \frac{\partial f_a}{\partial v} + \frac{D_{v\theta}}{v} \frac{\partial f_a}{\partial \theta} + \frac{D_{v\phi}}{v \sin \theta} \frac{\partial f_a}{\partial \phi} - f_a \frac{\partial h_a}{\partial v} \right] \right\} \\ &+ \frac{1}{2v \sin \theta} \frac{\partial}{\partial \theta} \left\{ \sin \theta \left[D_{\theta v} \frac{\partial f_a}{\partial v} + \frac{D_{\theta\theta}}{v} \frac{\partial f_a}{\partial \theta} + \frac{D_{\theta\phi}}{v \sin \theta} \frac{\partial f_a}{\partial \phi} - \frac{f_a}{v} \frac{\partial h_a}{\partial \theta} \right] \right\} \\ &+ \frac{1}{2v \sin \theta} \frac{\partial}{\partial \phi} \left\{ D_{\phi v} \frac{\partial f_a}{\partial v} + \frac{D_{\phi\theta}}{v} \frac{\partial f_a}{\partial \theta} + \frac{D_{\phi\phi}}{v \sin \theta} \frac{\partial f_a}{\partial \phi} - \frac{f_a}{v \sin \theta} \frac{\partial h_a}{\partial \phi} \right\} \end{aligned}$$

where

$$\underline{D} = \left[\begin{array}{l} \frac{\partial^2 g_a}{\partial v^2} , \frac{1}{v} \left(\frac{\partial^2 g_a}{\partial \theta \partial v} - \frac{1}{v} \frac{\partial g_a}{\partial \theta} \right) , \frac{1}{v \sin \theta} \left(\frac{\partial^2 g_a}{\partial \phi \partial v} - \frac{1}{v} \frac{\partial g_a}{\partial \phi} \right) \\ \frac{1}{v} \left(\frac{\partial^2 g_a}{\partial v \partial \theta} - \frac{1}{v} \frac{\partial g_a}{\partial \theta} \right) , \frac{1}{v} \left(\frac{\partial g_a}{\partial v} + \frac{1}{v} \frac{\partial^2 g_a}{\partial \theta^2} \right) , \frac{1}{v \sin \theta} \left(\frac{1}{v} \frac{\partial^2 g_a}{\partial \phi \partial \theta} - \frac{\cot \theta}{v} \frac{\partial g_a}{\partial \phi} \right) \\ \frac{1}{v \sin \theta} \left(\frac{\partial^2 g_a}{\partial v \partial \phi} - \frac{1}{v} \frac{\partial g_a}{\partial \phi} \right) , \frac{1}{v} \frac{\partial}{\partial \theta} \left(\frac{1}{v \sin \theta} \frac{\partial g_a}{\partial \phi} \right) , \frac{1}{v \sin \theta} \left(\sin \theta \frac{\partial g_a}{\partial v} + \frac{\cos \theta}{v} \frac{\partial g_a}{\partial \theta} \right. \\ \left. + \frac{1}{v \sin \theta} \frac{\partial^2 g_a}{\partial \phi^2} \right) \end{array} \right] \quad (14)$$

and the Rosenbluth potentials g_a and h_a are given by

$$g_a = \sum_b \frac{4\pi Z_a^2 Z_b^2 e^4 \ln \Lambda_{ab}}{m_a^2} \int f_b(\underline{v}') |\underline{v} - \underline{v}'| d\underline{v}' \quad (15)$$

$$h_a = \sum_b \frac{4\pi Z_a^2 Z_b^2 e^4 \ln \Lambda_{ab}}{m_a^2} \frac{(m_a + m_b)}{m_b} \int \frac{f_b(\underline{v}')}{|\underline{v} - \underline{v}'|} d\underline{v}' \quad (16)$$

The formulation for the Coulomb logarithm used here has been discussed previously (Futch, Holdren, etc. 1972). We make the frequently used assumption of isotropy for g_a and h_a (Ben Daniel and Allis (1962) Fowler and Rankin (1962), Futch and Killeen (1962), Kuo-Petravic et al. (1969)). With the isotropy assumption i.e.,

$$\frac{\partial g_a}{\partial \theta} = \frac{\partial h_a}{\partial \theta} = \frac{\partial g_a}{\partial \phi} = \frac{\partial h_a}{\partial \phi} = 0 \quad (17)$$

the tensor $\underline{D}$ greatly simplifies to

$$\underline{D} = \begin{pmatrix} \frac{\partial^2 g_a}{\partial v^2} & 0 & 0 \\ 0 & \frac{1}{v} \frac{\partial g_a}{\partial v} & 0 \\ 0 & 0 & \frac{1}{v} \frac{\partial g_a}{\partial v} \end{pmatrix} \quad (18)$$

and Eq. (14) becomes

$$\begin{aligned} \left. \frac{\partial f_a}{\partial t} \right|_c = & - \frac{1}{2v^2} \frac{\partial}{\partial v} \left[2v^2 \frac{\partial h_a}{\partial v} f_a - \frac{\partial}{\partial v} \left(v^2 f_a \frac{\partial^2 g_a}{\partial v^2} \right) + 2 \frac{\partial g_a}{\partial v} f_a \right] \\ & + \frac{1}{2 \sin \theta} \frac{\partial}{\partial \theta} \left[\frac{\sin \theta}{v^3} \frac{\partial g}{\partial v} \frac{\partial f_a}{\partial \theta} \right] \\ & + \frac{1}{2v \sin \theta} \frac{\partial}{\partial \phi} \left[\frac{1}{v^2 \sin \theta} \frac{\partial g_a}{\partial v} \frac{\partial f_a}{\partial \phi} \right] + \left. \frac{\partial f_a}{\partial t} \right|_{q1} . \end{aligned} \quad (19)$$

Changing the coordinate ϕ to the constant of motion coordinate R gives

$$\begin{aligned}
 \frac{\partial f_a}{\partial t} \Big|_c = & -\frac{1}{2} \frac{\partial}{\partial v} \left[\left(v^2 \frac{\partial h_a}{\partial v} + \frac{\partial g_a}{\partial v} \right) f_a - \frac{\partial}{\partial v} \left(\frac{v^2 f_a}{2} \frac{\partial^2 g_a}{\partial v^2} \right) \right] \\
 & + \frac{1}{2 \sin \theta} \frac{\partial}{\partial \theta} \left[\frac{\sin \theta}{v^3} \frac{\partial g_a}{\partial v} \frac{\partial f_a}{\partial \theta} \right] \\
 & + \frac{1}{2R} \frac{\partial}{\partial R} \left[\frac{1}{v^3 \sin^2 \theta} \frac{r^2 v^2}{\Omega^2 R} \frac{\partial g_a}{\partial v} \frac{\partial f_a}{\partial R} \right] + \frac{\partial f_a}{\partial t} \Big|_{q1} \quad (20)
 \end{aligned}$$

where we have used the relation

$$R^2 = r^2 + \rho^2 - 2r\rho \cos(\pi/2 + \phi) \quad (21)$$

with ϕ the angle between v_l and the r vector, $\hat{e}_r$, to obtain

$$\frac{\partial}{\partial \phi} = \frac{\partial R}{\partial \phi} \frac{\partial}{\partial R} = \frac{rv_r}{\Omega R} \frac{\partial}{\partial R} \quad (22)$$

Substituting the collision term into Eq. (11) and integrating around a gyro-orbit gives

$$\begin{aligned}
 \frac{\partial f_a}{\partial t} = & -\frac{1}{2} \frac{\partial}{\partial v} \left[f_a \left(v^2 \left\langle \frac{\partial h_a}{\partial v} \right\rangle + \left\langle \frac{\partial g_a}{\partial v} \right\rangle \right) - \frac{\partial}{\partial v} \left(\frac{v^2 f_a}{2} \left\langle \frac{\partial^2 g_a}{\partial v^2} \right\rangle \right) \right] \\
 & + \frac{1}{2 \sin \theta} \frac{\partial}{\partial \theta} \left[\frac{\sin \theta}{v^3} \left\langle \frac{\partial g_a}{\partial v} \right\rangle \frac{\partial f_a}{\partial \theta} \right] \\
 & + \frac{1}{2R} \frac{\partial}{\partial R} \left[\frac{1}{v^3 \Omega^2 R \sin^2 \theta} \left\langle r^2 v^2 \frac{\partial g_a}{\partial v} \right\rangle \frac{\partial f_a}{\partial R} \right] + \left\langle \frac{\partial f_a}{\partial t} \right\rangle_{q1} + \left\langle \frac{\partial f_a}{\partial t} \right\rangle_{ap} \quad (23)
 \end{aligned}$$

In Eq. (23), f_a is the zero order distribution function for species "a" and $\langle \rangle$ represents an average around the orbit, i.e.,

$$\langle \rangle = \frac{1}{\tau} \oint () \frac{dr}{v_r} \quad (24)$$

where τ is the orbit time. Multiplying Eq. (23) by $K_a \sin \theta d\theta$ and integrating over θ from θ_{LC} to $\pi - \theta_{LC}$, one obtains an equation independent of θ , i.e.,

$$\begin{aligned} \frac{\partial F_a}{\partial t} = & - \frac{1}{v^2} \frac{\partial}{\partial v} \left[\frac{1}{v^3 \Omega_R^2 \sin^2 \theta} \left\langle r^2 v^2 \frac{\partial g_a}{\partial v} \right\rangle \frac{\partial F_a}{\partial R} \right] \\ & + \left[\frac{K_a \sin \theta_{LC}}{v^3} \left\langle \frac{\partial g_a}{\partial v} \right\rangle \frac{\partial f_a}{\partial \theta} \right]_{\theta_{LC}} + \frac{1}{R} \frac{\partial}{\partial R} \left[\frac{1}{2v \Omega_R^2} \left\langle r^2 \cos^2 \phi \frac{\partial g_a}{\partial v} \right\rangle \frac{\partial F_a}{\partial R} \right] \\ & + \left(\frac{\partial F_a}{\partial t} \right)_{q1} + \left\langle \frac{\partial F_a}{\partial t} \right\rangle_{ap} \end{aligned} \quad (25)$$

where

$$F_a(v, R, t) = K_a \int_{\theta_{LC}}^{\pi - \theta_{LC}} f_a(v, \theta, R, t) \sin \theta d\theta \quad (26)$$

The normalization constant, K_a , is determined by the initial density and distribution function for each species, i.e.,

$$n_a(R, 0) = K_a \int_0^\infty F_a(v, R, 0) v^2 dv \quad (27)$$

In deriving Eq. (25) from Eq. (23) we have replaced the θ dependent gyroradius $\rho(\theta)$ by

$$\langle \sin \theta \rangle = \frac{\int \sin^2 \theta F(\theta) d\theta}{\int F(\theta) \sin \theta d\theta} \quad (28)$$

to obtain the orbit-averaged coefficients in Eq. (25).

To evaluate the second term on the right hand side of Eq. (25) requires a knowledge of $\partial f_a / \partial \theta \big|_{\theta_{LC}}$. In general, the θ dependence of f is an unknown quantity which may be represented by an infinite series of normal mode solutions which are functions of the mirror ratio R_m . In practice, the ambipolar potential ϕ has been taken into account by substituting for R_m an effective mirror ratio, R_{eff} , defined by

$$R_{eff} = R_m \left(1 \pm \frac{\phi}{E} \right)^{-1} \quad (29)$$

which enters as $\log_{10} R_{eff}$ in the expression for the plasma lifetime. The normal mode distribution may be varied from a broad distribution in θ to a "highly peaked" distribution by changing R_m . In the present model, the error resulting from the uncertainty in the angular dependence of the distribution function has been investigated by running the code for angular distributions varying from a broad normal mode distribution to a "highly peaked" distribution. The error (which will be discussed in a later section) resulting from the uncertainty in the θ dependence of the distribution function is small. This fortunate circumstance results from the fact that normal mode distributions for mirror ratios in the range from

1 to 5 are already well peaked and going to a more highly peaked or δ -function does not change the particle lifetime significantly. The procedure followed is to pick an angular distribution and perform the orbit average integrals in Eq. (25) using the chosen angular distribution and an average p calculated using Eq. (28). In practice we pick either the usual normal mode distribution or a highly peaked angular distribution if quasilinear processes are expected to dominate the diffusion processes.

Substituting for the Rosenbluth potentials and noting that the guiding center density, $D(R,v,t)$ is related to the lowest order distribution function $F(R,v,t)$ by $2\pi F_a(R,v,t) = D_a(R,v,t)$, Eq. (25) may be expressed in terms of the guiding center density as

$$\frac{\partial D_a}{\partial t}(R,v) = \frac{1}{v^2} \frac{\partial}{\partial v} \left[A_a D_a + B_a \frac{\partial D_a}{\partial v} \right] - \frac{C_a}{v^2} D_a + \frac{1}{R} \frac{\partial}{\partial R} \left[E_a \frac{\partial D_a}{\partial R} \right] + \frac{\partial D_a}{\partial t}_{ql} + \frac{\partial D_a}{\partial t}_{ap} \quad (30)$$

The coefficients are defined as follows:

$$A_a = \frac{\Omega \Gamma_a}{2\pi} \sum_b \oint \left[\left(\frac{Z_b}{Z_a} \right)^2 \left(\frac{M_a}{M_b} \right) K_b \ln \Lambda_{ab} N(F_b) \right] \frac{dr}{v_r}$$

$$B_a = \frac{\Omega \Gamma_a}{2\pi} \sum_b \oint \left(\frac{Z_b}{Z_a} \right)^2 K_b \ln \Lambda_{ab} \left[\frac{1}{3v} E(F_b) + \frac{v^2}{3} M(F_b) \right] \frac{dr}{v_r}$$

$$C_a = C_s = \frac{\Omega r_a}{2\pi} (\lambda_a/2v) \sum_b \oint \left(\frac{z_b}{z_a} \right)^2 K_b \ln \Lambda_{ab} \left[N(F_b) - \frac{1}{3v^2} E(F_b) + \frac{2v}{3} M(F_b) \right] \frac{dr}{v_r} \quad C_s \leq C_f$$

$$C_a = C_f = v^2 v_t / L_p R_m \quad C_s > C_f$$

$$E_a = \frac{r_a}{4\pi v \Omega R} \sum_b \oint \left(\frac{z_b}{z_a} \right)^2 K_b \ln \Lambda_{ab} \left[r^2 - \frac{(R^2 - r^2 - p^2)^2}{4p^2} \right] \times \left[N(F_b) - \frac{1}{3v^2} E(F_b) + \frac{2v}{3} M(F_b) \right] \frac{dr}{v_r}$$

where K_b is a normalization constant determined by the initial density and distribution for each species, i.e.,

$$n_b(0) = K_b \int_0^\infty F_b(v, 0) v^2 dv$$

and

L_p = plasma length

$$v_t = \sqrt{\frac{Z_a \phi}{m_a}}$$

$$M(F) = \int_v^\infty F(y, t) y dy$$

$$N(F) = \int_0^V F(y,t) y^2 dy$$

$$E(F) = \int_0^V F(y,t) y^4 dy$$

$$\Gamma_a = \frac{4\pi Z_a^4 e^4}{m_a^2}$$

$$\Omega = eB/m_a c = \frac{2\pi}{\oint \frac{dr}{v_r}}$$

and where we have used the empirical fit of Roberts and Carr (1960) for the coefficient of the scattering loss term in Eq. (25), i.e.,

$$\Lambda_a = \frac{2 \sin \phi_{LC}}{f_a} \frac{\partial f_a}{\partial \theta} = - (\log_{10} R_a)^{-1}$$

The "effective" mirror ratio, R_a , is defined by

$$R_a = \frac{R_m}{1 + Z_a e\phi/E_a(v)}$$

with R_m equal to the vacuum mirror ratio, and $E_a(v)$ the ion energy. The definition used for the Coulomb logarithm, $\ln \Lambda_{ab}$ is discussed in Ref. 6.

The usual assumption in calculating the collisional loss rate for mirror confinement is that an ion scattered into the loss cone leaves the plasma immediately. This assumption is not valid for low energy ions where

the transit time is longer than the collisional scattering time. Particle loss rates are expected to make a smooth transition from the usual mirror loss rate to transit time loss rates (Rognlien and Cutler, UCRL-84112). The coefficient, C_a , of Eq. (30) has been modified to provide a smooth transition from mirror confinement to transit time confinement by replacing C_a with the reciprocal of the transit time whenever $\tau_r > C_a^{-1}$.

2. Atomic Physics Source and Loss Terms

A major feature of the present work is the retention of finite Larmor radius terms in the description of atomic physics processes. If an energetic beam of neutral atoms is injected through a plasma, the neutral beam atoms will become trapped in the plasma by ionizing collisions with both plasma ions and electrons, and by charge exchange collisions with plasma ions. If the injection is perpendicular to the magnetic axis, the trapped ions will move in a plane perpendicular to the confining field; and the trapping processes may be represented by a two-dimensional model. Figure 1 shows the geometry of the problem. The trapped ions move in circular orbits of radius ρ . A neutral beam of width w and displaced a distance d above the magnetic axis passes through the confining field. The trapping and loss terms are described by

$$\frac{\partial D_a(R, v)}{\partial t} = K_2^a(R, v) - K_1^a(R, v) D_a(R, v) - \sum_k (\sigma_{cx}^k v) \langle n_0(r) \rangle D_a(R, v) \quad (31)$$

where

$$\langle n_0^k(r) \rangle = \frac{1}{\tau} \oint \frac{n_0^k(r) dr}{V_r} \quad (32)$$

and σ_{cx}^k is equal to either the molecular charge exchange cross section, σ_{cx}^m , or the atomic charge exchange cross section σ_{cx}^a depending on the gas species. The total trapping rate (ionization and charge-exchange) for species "a" is

$$K_2^a(R, v) = \sum_{\alpha} \frac{\delta(v - v_a^{\alpha})}{2\pi} \sum_b \frac{(\sigma v)_T}{v_a^{\alpha}} \left[\int_{\chi_2}^{\chi_3} J_a^{\alpha}(R, \chi) n_b(R, \chi) d\chi + \int_{\chi_4}^{\chi_1} J_a^{\alpha}(R, \chi) n_b(R, \chi) d\chi \right] \frac{1}{4\pi(v_a^{\alpha})^2} \quad (33)$$

where J_a^{α} is the current per cm^2 for beam component α of species a, and the factor $1/4\pi(v_a^{\alpha})^2$ occurs because the source is a delta function in velocity space. The integration over the angle χ in Eq. (33) accounts for the fractions of the azimuthal drift time within the beam, see Fig. 2. The summations indicated in Eq. (33) are over all beams (α) and over all plasma species (b). The α component of the beam velocity for specie "a" is v_a^{α} . The second term of Eq. (31) corresponds to the loss of plasma ions by charge-exchange collisions with neutral beam atoms. $K_1^a(R, v)$ is defined by

$$K_1^a(R, v) = \frac{1}{4\pi^2} \sum_b \sum_{\alpha} \frac{\sigma_{cx}^a v}{v_b^{\alpha}} \left[\int_{\chi_6}^{\chi_3} \int_{\beta_2}^{\beta_1} J_b^{\alpha}(R, \chi, \beta) d\beta d\chi + \int_{\chi_4}^{\chi_5} \int_{\beta_2}^{\beta_1} J_b^{\alpha}(R, \chi, \beta) d\beta d\chi \right] \quad (34)$$

The integration over the angle β accounts for the fraction of the gyro-motion within the beam and the integration over χ accounts for the fraction of azimuthal drift time within the beams. See. Fig. 4.

The last term of Eq. (31) is the loss of particles by charge-exchange collisions with thermal gas. The integration variables in Eqs. (33) and (34) are detailed in the following way.

$$h_1 = w/2 + d \quad u_1 = (h_1 - \rho)/R \quad u_3 = (h_1 + \rho)/R$$

$$h_2 = w/2 - d \quad u_2 = (h_2 + \rho)/R \quad u_5 = (h_2 - \rho)/R$$

then

$$\chi_1 = \sin^{-1} u_1 \quad -1 < u_1 < 1$$

$$= -\pi/2 \quad u_1 \leq -1$$

$$= \pi/2 \quad u_1 \geq 1$$

$$\chi_2 = \pi - \chi_1$$

$$\chi_3 = \pi + \sin^{-1} u_2 \quad u_2 < 1$$

$$\chi_3 = 3/2 \pi \quad u_2 > 1$$

$$\chi_4 = 3\pi - \chi_3 \quad u_3 > 1$$

$$x_5 = \sin^{-1} u_3 \quad u_3 < 1$$

$$x_5 = \pi/2 \quad u_3 > 1$$

$$x_6 = \pi - x_5$$

$$x_7 = \pi + \sin^{-1} u_3$$

Let

$$u_4 = \frac{|R \sin x| - h}{\rho} \quad \text{where } h = h_1 \quad 0 \leq x \leq \pi$$

$$= h_2 \quad \pi < x \leq 2\pi$$

$$\alpha = \cos^{-1} u_4$$

If $|u_4| < 1$, then

$$\beta_1 = \alpha + \pi/2 - x \quad 0 \leq x \leq \pi$$

$$\beta_1 = \alpha + 3/2\pi - x \quad \pi < x \leq 2\pi$$

$$\beta_2 = \beta_1 - 2\alpha$$

If $|u_4| > 1$

$$\beta_1 = 2\pi$$

$$\beta_2 = 0$$

$$\alpha = \pi$$

The attenuation of the primary neutral beams in passing through the plasma is given by

$$J_a^\alpha(r, x) = J_a^\alpha(x, y) \exp - \left(\sum_b \frac{(\sigma v)_T}{v_a^\alpha} \int_{x_1}^x n_b(x, y) dx \right) \quad (35)$$

with

$$x_1 = -\sqrt{R_p^2 - y^2}$$

and R_p the plasma radius.

In the foregoing equations, $(\sigma v)_T$ indicates the sum of the σv 's for ionization by ions and electrons and for resonant charge-exchange collisions. The relative velocity between the two colliding particles is v . The cross sections are represented by analytical fits to experimental data. Expressions given by Riviere (Nuclear Fusion 11, 363 (1971)) are used for the resonant charge exchange and proton ionization cross sections. The ionization cross sections have been generalized by appropriate mass and z dependence to model ionizing collisions by other ion species. The cross sections σ_{ex}^a and σ_i^a are the atomic charge-exchange and ionization cross section for hydrogen atoms, respectively. They are defined as follows:

$$\sigma_{cx}^a = \frac{0.6937 \times 10^{-14} \left[1 - 0.155 \log_{10} \left(\frac{E}{P} \right) \right]^2}{1 + 0.1112 \times 10^{-14} \left(\frac{E}{P} \right)^{3.3}} \text{ cm}^2 \quad (E \text{ in eV}) \quad (36)$$

$$\sigma_1^a = z^2 10^{-0.8712 [\log_{10} (E/P)]^2 - 8.156 \log_{10} (E/P) + 34.833} \quad (37a)$$

$$E \leq 150 \times 10^3 \text{ eV}$$

$$\sigma_1^a = \frac{3.6 \times 10^{-12} \log_{10} [0.1666 (E/P)] z^2}{E/P} \quad E > 150 \times 10^3 \text{ eV} \quad (37b)$$

where

P = the atomic mass

z = the atomic number.

The foregoing cross sections do not apply to electrons. For electron ionization, we assume that the electron velocity is much greater than the ion velocity and that the distribution function is Maxwellian. Under these conditions the electron ionization rate parameter may be represented by the following equations which are a function of the average electron energy only

$$\overline{(\sigma v)}_{ie} = A [1 - \exp (-0.00074 E^2)] + BE - CE^2 \quad E \leq 450 \text{ eV} \quad (38a)$$

where

$$A = 2.9297 \times 10^{-8}$$

$$B = 0.246862 \times 10^{-10}$$

$$C = 5 \times 10^{-14}$$

$$\overline{(\sigma v)}_{1e} = \frac{A}{E^{1/2}} (\ln E + B) \quad E > 450 \text{ eV} \quad (38b)$$

where

$$A = 1.62 \times 10^{-7}$$

$$B = -2.0618$$

3. The Electron Equations

Since the electrons differ only slightly from a Maxwellian distribution (Futch, Killeen, Holdren, Mirin, 1972) and have negligible Larmor radius, Pastukhov rate equations (Pastukhov, Nuclear Fusion 14, 3, 1974) with source terms are solved for the electron density, $n_e(R)$, and temperature, $T_e(R)$. The equation for $n_e(R)$ is

$$\begin{aligned} \frac{dn_e(R)}{dt} = & \sum_b \sum_a z_b J_b^a(R, \chi) \lambda_1(R) \sum_\beta \frac{(\sigma v)_i}{v_b} n_\beta(R) \\ & + \sum_\beta \sum_k n_\beta(R) n_0^k(R) (\sigma v)_i - \frac{n_e^2(R)}{n\tau} \end{aligned} \quad (39)$$

with

$$\lambda_1 = \frac{1}{\pi} [\sin^{-1} h_1/R + \sin^{-1} h_2/R] \quad (40)$$

$(\sigma v)_i$ = the appropriate σv for ionization by the plasma species

$$(n\tau)^{-1} = \frac{2}{\sqrt{\pi}} \frac{n_e^2(R)}{n_e(R)\tau_s} \frac{2R_m}{2R_m+1} \frac{\exp\left(-\frac{e\phi}{T_e}\right)}{\ln(4R_m+2)} \frac{T_e}{e\phi} \left[1 + \frac{T_e}{2e\phi} - \frac{1}{4} \frac{T_e}{e\phi} + \dots \right] \quad (41)$$

$$\tau_s = \frac{\sqrt{m} T^{3/2}}{\sqrt{2\pi n e^4} \lambda_c} \quad (42)$$

where λ_c is the Coulomb logarithm. The first term on the right hand side of Eq. (40) is the source of electrons due to neutral beam trapping and the second term is the electron source which results from ionization of thermal gas species $n_0^k(R)$.

The electron energy equation is

$$\frac{d}{dt} [n_e E_e] = 4\pi \int_0^\infty \sum_b v_{eb} F_e(v) v^2 dv + J_e E_{inj} - \frac{(\phi + T_e + E_I) n_e^2}{n\tau} \quad (43)$$

where E_{inj} is the energy at which electrons are injected into the plasma. The first term of Eq. (42) is the energy transferred to the electrons from the ion species. Here

$$v_{eb} = \frac{4\pi Z_b^2 e^4 \ln \Lambda_{eb}}{m_e} K_e K_b \left[\int_v^\infty F_b(y,t) y dy - \frac{m_e}{m_b} \int_0^v F_b(y,t) y^2 dy \right] \quad (44)$$

where K_e and K_b are the normalizing constants determined by the initial density and distribution for each species. The energy injected by the beam electrons is $J_e E_{inj}$. The two loss terms are the power lost by electrons scattered into the loss-cone and the power lost by ionization of cold gas. E_I is the energy required to ionize the neutral species and $(\phi + T_e)$ is the energy carried out by the hot electron which leaves when a cold electron is produced. The formula for $n\tau$ is given by Eq. (41).

4. Neutral Gas Equations

Molecular gas impinging on the plasma surface is converted into hydrogen ions and a flux of neutral atoms with energies of several eV (Franck-Condon atoms) within a thin boundary layer at the plasma surface.

Our model which neglects both the molecular gas and thermal ions resulting from the molecular dissociation assumes a radially directed flux of Franck-Condon atoms is created at the plasma surface. As these atoms penetrate the plasma, they undergo ionizing and charge-exchange collisions with plasma particles. The charge-exchange collision result in the formation of an additional flux of neutral atoms at the plasma ion energy which is in turn attenuated as it penetrates the plasma.

The radially dependent densities of Franck-Condon and charge exchange neutral atoms is given by

$$n_0^{FC}(R) = n_0^{FC}(R_p) \exp \left[-\sqrt{2} \int_R^{R_p} \sum_b n_b \frac{(\sigma v)_T}{v_{FC}} dR \right] \quad (45)$$

and

$$n_0^{cx}(R) = n_0^{FC}(R_p) (\text{RATIO}) \exp \left[-\sqrt{2} \int_R^{R_p} \sum_b n_b \frac{(\sigma v)_T}{v_{cx}} dR \left(1 - \exp - \sqrt{2} \int_R^{R_p} \sum_b n_b \frac{(\sigma v)_T}{v_{FC}} dR \right) \right] \quad (46)$$

respectively, where

$$\text{RATIO} = 0.5 \left[1 + (R_p - R)/R_p \right] (v_{FC}/v_{cx}) \frac{\int_R^{R_p} \sum_b n_b (\sigma v)_{cx} dR}{\int_R^{R_p} \sum_b n_b (\sigma v)_T^{FC} dR} \quad (47)$$

and,

v_{FC} = Franck-Condon velocity

v_{cx} = velocity of the charge exchange neutral atoms

R_p = plasma radius.

So far we have assumed that the neutral gas densities are independent of the axial coordinate z as would be the case for a plasma modeled by a uniform cylinder. We now generalize these results to model a non-uniform plasma column with a varying elliptical cross section in the axial direction. We model the z -dependent plasma by a gaussian profile, i.e.,

$$n^a(R, z) = n^a(R) e^{-(z/L_p)^2} \quad (48)$$

when L_p is the plasma e-folding length. We define $f(z)$ as the minor axis scale length to the midplane scale length. We can then replace dR in equations (45) through (47) with $f(z) dz$ to obtain the increased gas penetration along the thin side of the fan region. We then write the following expression for the radially dependent and z -averaged gas densities

$$\langle n_0^k(R) \rangle = \frac{\int_0^{z_m} e^{-(z/L_p)^2} n_0^k(R) \frac{dz}{B(z)}}{\int_0^{z_m} e^{-(z/L_p)^2} \frac{dz}{B(z)}} \quad (49)$$

For secondary beam components resulting from charge-exchange with plasma ions, the model described by Stallard (UCRL-51784) has been used.

The generalized form of his equations becomes

$$\begin{aligned}
 \frac{dJ_a(-z)}{dz} = & n_a \sum_b \frac{\sigma_a v_{cx}}{v_a} [F_{b,a} J_b(-z) + (1 - F_{b,a}) J_b(z)] \quad (a \geq 2) \\
 & + n_a \sum_b (\sigma_a v)_{cx} n_b(-z) \\
 & - \frac{J_a(-z)}{v_a} \left[n_a (\sigma v)_I + n_a (\sigma v)_{cx} (1 - F_{a,a}) + \sum_{b \neq a} n_b (\sigma v)_T \right] \\
 & + [1 - F_{a,a}] n_a (\sigma v)_{cx} \frac{J_a(z)}{v_a} \quad (50)
 \end{aligned}$$

The $F_{a,b}$'s are the forward scattering fraction described by Stallard.

Equation (50) is solved for the range $-R_w \leq z \leq R_w$. The secondary beams are included in the summations for the trapping and loss terms discussed earlier; see Eqs. (33) and (34).

5. Quasilinear Diffusion Terms

Berk (UCRL-80272) has derived the quasilinear diffusion equation of a plasma in a magnetic field. His theory takes into account spatial inhomogeneity of the magnetic field and includes the orbit and bounce motion of particles in a magnetic mirror field. A form of Berk's equation for the guiding center density in cylindrical coordinates is used here (Berk and

Matsuda - unpublished). The quasilinear diffusion terms were derived by using an expansion in the Larmor radius for the limit where the bounce frequency is less than the variation of the cyclotron frequency sampled by a particle. The quasilinear diffusion terms for the guiding center density become

$$\left. \frac{\partial D'_a(R, \theta, v)}{\partial t} \right|_{q1} = \left(2m\omega_{c1} \frac{\partial}{\partial v^2} + \frac{l}{\omega_{c1}} \frac{1}{R} \frac{\partial}{\partial R} \right) D_v(v^2, R) \left(2m\omega_{c1} \frac{\partial}{\partial v^2} + \frac{l}{\omega_{c1}} \frac{1}{R} \frac{\partial}{\partial R} \right) D'_a(R, \theta, v) \quad (51)$$

In Eq. (51), $D'_a(R, v, \theta)$ is the guiding center distribution function; l and m are azimuthal and harmonic mode numbers.

The quasilinear diffusion coefficient, D_v , is

$$D_v = \sum_m \left(\frac{e \tilde{\phi}_m(R)}{m_a} \right)^2 J_m^2(k_{\perp} v_{\perp} / \omega_{c1}) \Omega_m^{-1} \quad (52)$$

where $\tilde{\phi}_m(r)$ is the perturbed potential and Ω_m^{-1} is the correlation time given by

$$\Omega_m^{-1} = \frac{\Delta\omega}{(\omega - m\omega_{c1})^2 + \Delta\omega^2} \quad (53)$$

To obtain terms independent of θ , we multiply by $\sin \theta d\theta$ and integrate from θ_{LC} to $\pi - \theta_{LC}$ as before. The resulting equation for the quasilinear terms may be put in the form

$$\left(\frac{\partial D_a(R, v)}{\partial t} \right)_{q1} = \frac{1}{v^2} \frac{\partial}{\partial v} \left[A_{q1} D_a + B_{q1} \frac{\partial D_a}{\partial v} \right] - \frac{C_{q1}}{v^2} D_a + \frac{1}{R} \frac{\partial}{\partial R} \left[E_{q1} \frac{\partial D_a}{\partial R} + F_{q1} D_a + G_{q1} \right] \quad (54)$$

In Eq. (54),

$$D_a(R, v) = \int_{\theta_{LC}}^{\pi - \theta_{LC}} D'_a(R, \theta, v) \sin \theta \, d\theta \quad (55)$$

and the quasilinear coefficients are defined as follows:

$$A_{q1} = v \int D_v(v \sin \theta) \sin^2 \theta \cos \theta \frac{1}{D_a} \frac{\partial D'_a}{\partial \theta} \, d\theta + v \frac{m\ell}{R} \frac{\partial D_v}{\partial R} \sin \theta D'_a / D_a \, d\theta$$

$$B_{q1} = v^2 \int D_v(v \sin \theta) \sin^3 \theta D'_a / D_a \, d\theta$$

$$C_{q1} = 2 D_v(v \sin \theta_{LC}) \sin \theta_{LC} \cos^2 \theta_{LC} \frac{1}{D_a} \frac{\partial D'_a}{\partial \theta} \Big|_{\theta_{LC}} + \frac{m\ell}{R} (\alpha' - \beta' + v\gamma') \quad (56)$$

$$E_{q1} = \frac{k^2}{\omega_{ci}^2 R} \alpha$$

$$F_{q1} = \frac{m\ell}{v} \left[\frac{2\beta}{v} + \gamma + \frac{\delta}{v} \right]$$

$$G_{q1} = \frac{2m\ell}{v} \frac{\partial D_a}{\partial v} \alpha$$

with

$$\alpha = \int D_v \sin \theta \, D'_a/D_a \, d\theta$$

$$\beta = \int D_v \cos \theta \, \frac{\partial D_a}{\partial R}^{-1} \frac{\partial^2 D'_a}{\partial \theta \partial R} \, d\theta$$

$$\gamma = \int \frac{\partial D_v}{\partial v} \sin \theta \, D'_a/D_a \, d\theta$$

$$\delta = \int \frac{\partial D_v}{\partial \theta} \cos \theta \, D'_a/D_a \, d\theta \quad (57)$$

$$\alpha' = \int \frac{\partial D_v}{\partial v} \sin \theta \, D'_a/D_a \, d\theta$$

$$\beta' = \int \frac{\partial D_v}{\partial R} \cos \theta \, \frac{1}{D_a} \frac{\partial D'_a}{\partial \theta} \, d\theta$$

$$\gamma' = \int \frac{\partial^2 D_v}{\partial R \partial v} \sin \theta \, D'_a/D_a \, d\theta$$

If the real part of ω is near a harmonic of ω_{oi} , then $\Omega_m = \Delta\omega$. The Bessel functions of Eq. (52) are rapidly varying as the particle moves along its trajectory. To avoid the numerical difficulties associated with using Eq. (52) in a numerical code, an analytic expression representing the

envelope of the oscillations has been used by previous workers (Berk and Stewart, UCRL-78407) and is adopted here. The square of the Bessel function is represented by

$$J_m^2(z) = \frac{F_m(z) H(z)}{F_m(z) + H(z)} \quad (58)$$

with

$$F_m(z) = [(z/2)^m / m!]^2$$

and

$$H(z) = (\pi z)^{-1}$$

The radial dependence of $\phi_m^2(R)$ is modeled by the following function which vanishes at the origin and therefore has a zero particle flux at $R=0$:

$$\mathcal{F}(R) = \frac{(1 + \alpha) R^2}{R^2 + \alpha R_p^2} \quad (59)$$

with α an input number, one obtains

$$\frac{\partial D_v}{\partial R} = \frac{2 \alpha R_p^2}{(1 + \alpha) R^3} \mathcal{F}(R) D_v \quad (60)$$

Using Eq. (60), α' , β' , and γ' may be rewritten as

$$\begin{aligned}\alpha' &= \frac{2 \alpha R^2}{(1 + \alpha) R^3} \mathcal{F}(R) \alpha \\ \beta' &= \frac{2 \alpha R^2}{(1 + \alpha) R^3} \mathcal{F}(R) \beta \\ \gamma' &= \frac{2 \alpha R^2}{(1 + \alpha) R^3} \mathcal{F}(R) \gamma\end{aligned}\tag{61}$$

6. Complete Guiding Center Equation

Substituting in the quasilinear and atomic physics terms, Eq. (30) may be rewritten as

$$\begin{aligned}\frac{\partial D_a(R, v)}{\partial t} &= \frac{1}{v^2} \frac{\partial G_a}{\partial v} - \frac{C_a}{v^2} D_a + \frac{1}{R} \frac{\partial}{\partial R} [H_a + G_{q1}] + K_2^a(R, v) \\ &\quad - K_1^a(R, v) D_a - \sum_K (\sigma v)_{\text{cx}}^K \langle n_O^K(r) \rangle D_a\end{aligned}\tag{62}$$

where

$$G_a = A_a D_a + B_a \frac{\partial D_a}{\partial v}$$

$$H_a = F_{ql} D_a + E_a \frac{\partial D_a}{\partial R} \quad (63)$$

and the coefficients A_a , B_a , C_a , and E_a have been redefined to include the quasilinear coefficients, i.e.,

$$A_a = A_a + A_{ql}$$

$$B_a = B_a + B_{ql}$$

$$C_a = C_a + C_{ql}$$

and

$$E_a = E_a + E_{ql} \quad (64)$$

Equation (62) contains velocity diffusion terms and radial diffusion terms as well as space and velocity dependent source and loss terms. For a spatially uniform plasma, the guiding center equation reduces to the Fokker-Planck of Futch et al. (1972) with the quasilinear terms set equal to zero and when the transit time correction to the mirror loss rate is negligible, i.e., coefficient C of Eq. (62) becomes the mirror loss rate only.

III. NUMERICAL PROCEDURE

Equation (62) is solved by a two step process. The time-dependent change due to radial diffusion is solved first, i.e.,

$$\left. \frac{\partial D_a}{\partial t} (R, v) \right)_r = \frac{1}{R} \frac{\partial}{\partial R} [H_a + G_{q1}] \quad (65)$$

Integrating Eq. (65) over the volume gives

$$\int_{R=0}^{R=R_p} \left. \frac{\partial D_a}{\partial t} (R, v) \right)_r R dR = 0 \quad (66)$$

as long as $H_a = G_{q1} = 0$ for $R = 0$ and $R = R_p$. At the origin, this condition reduces to $\partial D_a / \partial R = 0$ since the form of ϕ_m^{-2} requires $F_{q1} = G_{q1} = 0$ for $R=0$ and $\partial D_a / \partial R = 0$ at the origin. For $R=R_p$ the appropriate boundary condition is $H_a + G_{q1} = 0$ for the radial flux to vanish. The boundary condition applied in the present model is $D_a(R, V) = 0$ for $R + \langle \sin \theta \rangle v / \Omega = 0$ where $\langle \sin \theta \rangle$ is averaged over the angular distribution. The radial grid is sufficiently large such that the wall loss is negligible. The radial flux to the wall is monitored by a term in the conservation equations discussed in the next section.

Equation (65) may be written as an implicit difference equation and then expressed as the linear algebraic system

$$-\alpha_L^{n+1} D_{L+1}^{n+1} + \beta_L^{n+1} D_L^{n+1} - \gamma_L^{n+1} D_{L-1}^{n+1} = \delta_L^n \quad (67)$$

as discussed in Ref. 6. The indicies n and L are the time and spatial indicies, respectively. Equation (67) is solved using the standard tridiagonal method of Richtmyer and Morton (1967), generalized to vector equations.

The solution of Eq. (65) is expressed in derivative form and added to Eq. (62) as a spatially dependent source term in place of the radial derivative terms. The resulting equation may be written in dimensionless form as

$$\frac{\partial D_a}{\partial t} = \frac{1}{x^2} \frac{\partial G_a}{\partial x} - \frac{C_a}{x^2} D_a + \left(\frac{\partial D_a}{\partial t} \right)_r + K_2^a - K_1^a D_a - \sum_k \sigma_{cx}^m v_0 x \langle n_0^k(r) \rangle D_a \quad (68)$$

with $x = v/v_0$. In the absence of source and loss terms the particle density, n_a , should remain constant in time. This condition gives the appropriate boundary condition for the solution of Eq. (68). That is

$$\frac{dn_a}{dt} = K_a \int_0^{x_{max}} \frac{1}{x^2} \frac{\partial}{\partial x} \left[A_a D_a + B_a \frac{\partial D_a}{\partial x} \right] x^2 dx = 0 \quad (69)$$

or

$$\left[A_a D_a + B_a \frac{\partial D_a}{\partial x} \right]_{x=0}^{x=x_{max}} = 0$$

for $t > 0$. For $X = 0$, the boundary condition reduces to $\partial D_a / \partial X = 0$ since A_a is zero at the origin.

Equation (68) is solved by the same procedure outlined above for Eq. (65) and discussed in Appendix B of Ref. 6.

At each time step, Eq. (67) is solved simultaneously with the electron equations. The particle distribution function, F_a , is obtained from the transformation

$$F_a(x, r, t) = \frac{2}{\pi} \int_{|R-\rho|}^{R+\rho} \frac{D_a(x, R, t) R dR}{\sqrt{4R^2 r^2 - (R^2 + r^2 - \rho^2)^2}} \quad (70)$$

The number density and average energy of each ion species are calculated from the equations

$$n_a(r, t) = K_a \int_0^\infty F_a(x, r, t) x^2 dx \quad (71)$$

$$E_a(r, t) = \frac{1}{2} m_a v_0^2 \int_0^\infty F_a(x, r, t) x^4 dx / \int_0^\infty f_a(x, r, t) x^2 dx \quad (72)$$

for each radial location. The plasma potential is computed for each time step of the calculation by requiring charge neutrality. A critical velocity $x_{cr}(t)$ is determined such that electrons with $x < x_{cr}(t)$ are not lost and those with $x > x_{cr}(t)$ can be lost by scattering into the loss cone. At each

time step and radial location the electron density is compared to the total ion charge density and the velocity x_{cr} modified accordingly.

We define Q^- as the electron charge density and Q^+ as the total ion charge density. The potential ϕ is determined as follows. For each time step and radial location $Q^-(t)$ and $Q^+(t)$ are computed, the old electron density $Q^-(\tau - \Delta\tau)$ is compared to the old ion density, $Q^+(\tau - \Delta\tau)$, and the change in the electron density is compared to that of the ions. Since charge neutrality is to be maintained, if $Q^-(\tau - \Delta\tau) \{ \gtrless \} Q^+(\tau - \Delta\tau)$ and $\Delta Q^- \{ \gtrless \} \Delta Q^+$, the ambipolar potential is {increased/decreased} by a small amount, and the time step is repeated. This cycling is continued until the condition $Q^- \{ \gtrless \} Q^+$ is obtained. The time dependent cross sections and the neutral gas equations are then solved before advancing to a new time step.

Conservation of Particles and Energy

A valuable feature of the present numerical model is the separate evaluation of particle and energy conservation equations. Conservation equations provide a check on the numerical accuracy of the code, and they are indispensable in providing a physical understanding of the complex interactions occurring in a spatially dependent Fokker-Planck code.

The particle equation for conservation of guiding center densities at R for species "a" is obtained by integrating Eq. (62) over $v^2 dv$ to obtain

$$K_a \int \frac{\partial D_a}{\partial t} (R, v) v^2 dv - \sum_a \int K_2^1(R, v) v^2 dv + K_a \int C_a D_a dv + K_a \int \frac{\partial D_a}{\partial t} r v^2 dv \quad (73)$$

$$+ K_a \int K_1^a(R, v) D_a v^2 dv - \sum_a \int K_2^{cx}(R, v) v^2 dv + \frac{2}{\pi} K_a \sum_l (av)_l^{1_{cx}} \langle n_l \rangle \int D_a v^2 dv$$

+ boundary loss = 0

where K_2^1 and K_2^{cx} are the ionization and charge exchange components of K_2^a , respectively. The velocity integrals are over the range from $v = 0$ to v_{max} . The neutral gas density, n_l , for species "l" averaged around the particle orbit is calculated by

$$\langle n_l \rangle = \frac{2}{\pi} \oint \frac{n_0^l(r) r dr}{[4R^2 r^2 - (R^2 + r^2 - \rho^2)^2]^{1/2}} \quad (74)$$

here integration limits are from $|R-\rho|$ to $R+\rho$. The last term of Eq. (73) represents particles that diffuse spatially beyond the plasma radius and are lost from the calculation. The number of particles lost in this manner are monitored by adding up the number of particles at each radial position after solving the guiding center equation but before the radius boundary conditions are applied and subtracting the particle sum after the radial boundary conditions are applied.

The energy conservation equation at radius R is obtained by multiplying Eq. (62) by $K_a m_a \frac{1}{2} v^2 dv$ and integrating to obtain

$$\begin{aligned}
 & \sum_a \left\{ \frac{m_a K_a}{2} \int \frac{\partial D_a(R, v)}{\partial t} v^4 dv - \sum_a \int K_2^a(R, v) E_a^\alpha v^2 dv + \frac{m_a K_a}{2} \int C_a D_a v^2 dv \right. \\
 & + \frac{m_a K_a}{2} \int \frac{\partial D_a}{\partial t} \Big|_r v^4 dv + \frac{m_a K_a}{2} K_1^a D_a v^4 dv - \frac{m_a}{2} \sum_a \int K_2^{cx}(R, v) v^4 dv \\
 & \left. + \frac{m_a K_a}{\pi} \sum_l (\sigma v)_l^{cx} \langle n_l \rangle \int D_a v^4 dv + \text{Energy lost to wall} \right\} + \frac{m_a K_a}{2} C_e F_e dv \\
 & + n_e \sum_l \sigma_{le}^l v n_l (E_l^i + T_e) = 0 \quad (75)
 \end{aligned}$$

where E_l^i is the energy lost in ionization of neutral species "l". The energy conservation equation has been summed over all ion species; the last two terms of Eq. (75) are the energy loss terms for electrons. Since Eq. (75) includes all species, energy transfer between species sum to zero.

Equations (73) and (75) are integrated over the plasma volume to give the global particle and energy equations.

IV. COMPARISON WITH PREVIOUS WORK

To provide additional checks on code validity, results were compared with previous calculations. The atomic physics source and loss terms were compared with Stallard's build-up code (B. W. Stallard, UCRL-51784 (1975)). For this comparison, the calculated values of n_i , ion and electron energies, etc. from the present code were used as inputs for the build-up code. The results of this comparison at the plasma center (i.e., $R=0$) are as follows:

	RFP <u>particles/cc-sec</u>	Build-up <u>particles/cc-sec</u>	Difference <u>%</u>
Total trapping (ionization plus charge exchange)	2.63×10^{14}	2.47×10^{14}	6.3
Net trapping (ionization)	1.13×10^{14}	1.14×10^{14}	0.9

In general, a comparison of RFP code results with previous Fokker-Planck calculations requires the "turning-off" of additional physics not included in previous results. Starting with a typical Fokker-Planck run for 2XIIB, quasilinear diffusion, thermal gas, half and third energy beam components were set equal to zero, and the gyro-radius was decreased by increasing the magnetic field. Figure 4 shows that the electron temperature on axis ($r=0$) approached the classical Fokker-Planck of approximately $0.09 E_i$ as additional physics terms were turned off and the gyro-radius, a_i , decreased. The primary reason T_e decreases is because the plasma confinement time τ_p decreases and since T_e is proportional to $\tau_p^{2/5}$, T_e decreases. Figure 5 compares the effective $n\tau$, $n\tau_{eff}$, as calculated from the RFP code with results from two previous Livermore Fokker-Planck codes, the Multi-Species Code⁶, and the Hybrid-1 Code⁹. $n\tau_{eff} = n^2/J$ where n is the plasma density and J is the trapping rate (particles/cm³-sec); both measured on axis ($r=0$). Because collisions are averaged around a particle orbit, a particle samples lower densities (for plasmas with a negative gradient) and $n\tau_{eff}$ is larger for lower magnetic fields. Therefore, the RFP results are plotted versus $1/B$ for comparison with the zero gyro-radius results of previous codes. For this comparison, the additional physics

terms not included in earlier Fokker-Planck codes are also set equal to zero as discussed earlier. The comparisons are within the 10 percent grid error expected from the various codes. The agreement improved when the number of velocity mesh points was doubled from 40 to 80.

Acknowledgments

The authors would like to acknowledge the early suggestion by Charles Damm that resonance charge exchange with injected beam atoms could lead to radial transport. We would like to thank Ira Bernstein for driving the Coulomb cross-field diffusion term and for many hours of helpful discussion. Art Mirin provided the basic Fokker-Planck package in a form suitable for the present code. The authors appreciate the many helpful discussions with the LLNL theory group including Don Pearlstein, Tom Rognlien, Herb Berk and others. Finally we would like to thank Bill Turner for his review of the manuscript.

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FIGURE CAPTIONS

- Figure 1. Geometry of neutral-atom injection experiment.
- Figure 2. Integration limits for source and loss terms.
- Figure 3. Integration limits corresponding to fraction of ion orbit within the beam.
- Figure 4. Comparison of the electron temperature on axis as additional physics terms are modified in the RFP code.
- Figure 5. Comparison of n_r with previous results.

A. H. Futch - Figure 1

A. H. Futch - Figure 2

4) INTEGRATION LIMITS FOR SOURCE TERM K_2

5) INTEGRATION LIMITS FOR SOURCE TERM K_3

A. H. Futch - Figure 3

A. H. Futch - Figure 4

A. H. Futch - Figure 5

APPENDIX A

ARCHER FUTCH

Colorado Springs

APS

1978

Quasilinear Diffusion and Electron Temperature Calculations in the 2XIIB Experiment--Archer H. Futch and Donald Freeman, Lawrence Livermore National Laboratory--Electron temperature measurements in the 2XIIB experiment are much lower than predictions based on spatially constant Fokker-Planck calculations unless additional heat losses are included for electrons (i.e., cold electron stream). The present paper describes a spatially dependent Fokker-Planck model which gives T_e close to experimental values without including streaming electrons. The calculated T_e drops from the spatially constant Fokker-Planck value (1 keV) to approximately 150 eV when quasilinear diffusion and finite radius effects are included. The calculated T_e is in good agreement with measurements when 2XIIB is operated under optimum conditions. The computer results are easily understood in terms of a simple model which gives $T_e \propto \tau_p^{2/5}$ where τ_p is the particle lifetime. Radial diffusion was also observed to stabilize and prevent the decrease in plasma radius which would otherwise occur as a result of surface charge exchange with thermal gas.

DERIVATION OF FOKKER-PLANCK EQUATION

Assumptions:

- 1) B_z is uniform in r and a square-well in z .
- 2) Collision frequency is much less than the gyrofrequency $\nu \ll \Omega$.
- 3) ν , R , are constants of the motion

$$\frac{\partial f}{\partial t} + v_r \frac{\partial f}{\partial r} = \left(\frac{\partial f}{\partial t} \right)_c, \quad \text{since } \dot{v} = \dot{R} = \dot{\theta} = 0.$$

Expanding f in terms of v/Ω : $f = f_0 + f_1 + \dots$

and equating like powers of v/Ω

$$\text{lowest order} \quad v_r \frac{\partial f_0}{\partial r} = 0$$

$$\text{first order} \quad \frac{\partial f_0}{\partial t} + v_r \frac{\partial f_1}{\partial r} = \left(\frac{\partial f_0}{\partial t} \right)_c$$

Integrating around a gyro-orbit:

$$\frac{\partial f_0}{\partial t} \oint \frac{dr}{v_r} + \oint \frac{\partial f_1}{\partial r} dr = \oint \left(\frac{\partial f_0}{\partial t} \right)_c \frac{dr}{v_r}$$

$$\frac{\partial f_0}{\partial t} = \frac{1}{\tau} \oint \left(\frac{\partial f_0}{\partial t} \right)_c \frac{dr}{v_r} = \tau = \oint \frac{dr}{v_r} = \frac{2\pi}{\Omega}$$

$$= \frac{1}{\pi} \int_{R-\rho}^{R+\rho} \frac{\left(\frac{\partial f_0}{\partial t} \right)_c r dr}{\sqrt{4R^2 \rho^2 - (r^2 - R^2 - \rho^2)^2}}$$

Where we have used the cos law, i.e., $r^2 = R^2 + \rho^2 - 2R\rho \cos \beta$.

By separating variables, the collision terms are modeled by an equation of the form

$$\begin{aligned} \frac{\partial D(R, v, t)}{\partial t} = & -\frac{1}{v^2} \frac{\partial}{\partial v} \left[v^2 \left\langle \frac{\partial h}{\partial v} \right\rangle D(R, v, t) - \frac{v^2}{2} \left\langle \frac{\partial^2 g}{\partial v^2} \right\rangle \frac{\partial D(R, v, t)}{\partial v} \right] - \frac{\lambda}{2v^3} \left\langle \frac{\partial g}{\partial v} \right\rangle D(R, v, t) \\ & + \left(A \frac{\partial}{\partial v} + \frac{B}{R} \frac{\partial}{\partial R} \right) D_v(R, v, t) \left(A \frac{\partial}{\partial v} + \frac{B}{R} \frac{\partial}{\partial R} \right) D(R, v, t) \\ & + K_2(R, v, t) \delta(v - v_D) - K_1(R, v, t) D(R, v, t) - D(R, v, t) \end{aligned}$$

$$\sigma_{cx} v \langle n_O(r) \rangle \quad (A-1)$$

where $h(r, v)$ and $g(r, v)$ are the isotropic form of the "Rosenbluth potentials" and $\langle () \rangle$ indicates the phase average around the particle orbit

$$\langle () \rangle = \frac{2}{\pi} \int_{R-\rho}^{R+\rho} \frac{() r dr}{\sqrt{4R^2 \rho^2 - (r^2 - R^2 - \rho^2)^2}}$$

and

$$D_v(R, v, t) = \sum_m \left(\frac{q\phi}{M_i} \right)^2 \frac{j_m^2 \left(\frac{k_i v}{\omega_{ci}} \right)}{\Delta \omega}$$

$K_2(R, v, t)$ are the collisional trapping terms and $K_1(R, v, t)$ corresponds to the loss of particles by resonance charge exchange with neutral beam atoms. The last term of Eq. (A-1) results from the loss of ions by charge exchange with thermal gas molecules or atoms.

After solving Eq. (A-1) for the guiding center density, $D(R, v, t)$, the real distribution function is obtained by the transformation

$$f(v, r, t) = \frac{2}{\pi} \int_{|r-\rho|}^{r+\rho} \frac{D(R, v, t) R dR}{\sqrt{4R^2 r^2 - (R^2 + r^2 - \rho^2)^2}}$$

the ion density is calculated by

$$n(r, t) = 4\pi \int f(v, r, t) v^2 dv$$

Pastukhov rate equations are solved for the electron density, $n_e(r)$, and temperature, $T_e(r)$

$$\frac{dn_e}{dt} = -\frac{2}{\sqrt{\pi}} \frac{n_e^2}{n_e} \frac{2R_m}{2R_m + 1} \frac{e^{-\frac{e\phi}{T_e}}}{\ln(4R_m + 2)} \frac{T_e}{e\phi} \left[1 + \frac{T_e}{2e\phi} - \frac{1}{4} \frac{T_e}{e\phi} + \dots \right] + J_e$$

$$\frac{d(n_e E_e)}{dt} = (\text{energy transfer from ions}) - \frac{(\phi + T_e)n^2}{\pi r} + J_e \cdot E_e^0$$

The plasma potential, $\phi(r,t)$ is computed at each time step by requiring charge neutrality.

ENERGY AND MAGNETIC FIELD SCALING OF QUASILINEAR
AND CHARGE-EXCHANGE DIFFUSION TERMS

$$\frac{\partial F}{\partial t} = \frac{A^2}{v^2} D_v \frac{\partial^2 F}{\partial v^2} + \frac{B^2}{r^2} D_v \frac{\partial^2 F}{\partial r^2} + D_{cx} \frac{\partial^2 F}{\partial r^2}$$

$$\propto \frac{\omega_{ci}}{v^2} D_v \frac{\partial^2 F}{\partial v^2} + \frac{k^2}{\omega_{ci}^2} D_v \frac{\partial^2 F}{\partial r^2} + \frac{\sigma_{cx}^A E_i}{B^2} \frac{\partial^2 F}{\partial r^2}$$

$$\underbrace{\propto \frac{B^2}{E_i} \frac{1}{B} \frac{\partial^2 F}{\partial v^2}}_{\text{End-loss}} + \underbrace{\frac{1}{v^2} \frac{1}{B} \frac{\partial^2 F}{\partial r^2}}_{\text{Q.L. Radial}} + \underbrace{\frac{\sigma_{cx}^A E_i}{B^2} \frac{\partial^2 F}{\partial r^2}}_{\text{cx Radial}}$$

End-loss Q.L. Radial cx Radial

$$\underbrace{\propto \frac{B}{E_i} \left(\frac{\partial^2 F}{\partial v^2} \right)}_{\frac{B}{E_i^2}} + \underbrace{\frac{1}{E_i B} \left(\frac{\partial^2 F}{\partial r^2} \right)}_{\frac{1}{B E_i r_0}} + \underbrace{\frac{\sigma_{cx}^A E_i}{B^2} \frac{\partial^2 F}{\partial r^2}}_{\frac{\sigma_{cx}^A E_i}{B^2 r_0^2}}$$

where $D_{cx} = J \sigma_{cx}^A \rho^2 / 4$

and $\rho^2 \propto E_i / B^2$

CALCULATED GLOBAL POWER BALANCE FOR HEAD-ON INJECTION WITH
450 AMPS IN THE 2XIIB EXPERIMENT

Total Beam = 450 amps
Net Beam Trapped = 68 amps

Losses

Ions

End-loss = 39 amps
cx and Radial Loss = 29 amps

Electrons

End-loss = 133 amps

Power Trapped

$$\text{Power} = \frac{68 \times 6.25 \times 10^{18} \times 15 \times 1.602 \times 10^{-9}}{10^7}$$

$$= 1 \text{ MW}$$

End-loss = 82 kilowatts
cx and Radial Loss = 918 kilowatts

The ion beam current and ion losses correspond to energetic ions only. Since the electron losses also include the ionization of thermal gas, the electron losses are greater than the ion losses.

RESULTS

Comparisons have been made between measured 2XIIB plasma parameters and calculated parameters for a large radius plasma (12 cm) with head-on injection. The addition of quasilinear (Q.L.) diffusion to the basic Fokker-Planck code causes major changes in plasma parameters since Q.L. loss rates are much greater than coulomb loss rates (~50 times greater) at equilibrium densities. A brief summary of some of the observations are the following:

1. Radial profiles were changed significantly when Q.L. diffusion was present. In previous runs without Q.L. diffusion the plasma decreased in radius as thermal gas eroded away the plasma surface. With Q.L. diffusion present the surface was stabilized by diffusion that moved the excess ions trapped in the plasma interior to the surface to compensate for the large charge exchange losses of the surface.
(See pages 4 and 5.)
2. Calculated values of plasma density and electron temperature are in agreement with measurements under optimum conditions. (See page 6.)
3. The effect on the electron temperature at the plasma center of removal of various physical effects from the code. (See page 7.)
4. The following very simple model for the electron temperature was developed to describe results from the code:

$$\frac{d(n_e E_e)}{dt} = n \left(\frac{w_i}{\tau_{ie}} \right) - (\phi + T_e) \frac{n^2}{n\tau}$$

At equilibrium

$$\frac{n\tau_e}{n} = \frac{(\phi + T_e)}{\left(\frac{w_i}{\tau_{ie}} \right)} = \frac{C\tau_e^{5/2}}{nW_i} \quad C = \text{constant}$$

$$T_e = C^{-2/5} (nW_i\tau_i)^{2/5}$$

In the plasma interior τ_e is equal to the hot ion lifetime, τ_i , and

$$T_e \propto (nW_i\tau_i)^{2/5}$$

The strong dependence of the ion lifetime with the magnetic field determines the broad features of 2XIIB plasma magnetic field scaling. Radial losses were found to dominate below 0.7 Tesla. The loss rates versus magnetic field, B, are shown on page 8.

5. Page 9 shows a comparison of the calculated T_e scaling, as compared with the experimental data for 10/25/77 and 10/20/77. Only shots which did not include guns #3 and #5 which pass through the plasma center are plotted.

6. Page 10 shows the line integral ion energy vs. B for both code results and measured data. The ion energy at the plasma center is also shown.
7. Since radial losses scale as either ρ (Q.L.) or ρ^2 (cx diffusion), the higher velocity particles have a shorter lifetime. This energy dependence explains the decreasing average ion energy as B decreases shown on page 10. Page 11 shows the effect of decreasing B on the velocity distribution function.

APPENDIX A FIGURE CAPTIONS

Figure A1. See Figure 1 of main text.

Figure A2. Calculated electron energy and density versus radius for head-on injection in the 2XIIB experiment and with quasilinear diffusion terms. Non-equilibrium results at 1.4 msec. Plasma radius stabilized by diffusion that moved the excess ions trapped in the plasma interior to the surface to compensate for large charge-exchange losses. Diffusion stabilizes the surface boundary.

Figure A3. Calculated ion energy and density versus radius for head-on injection in the 2XIIB experiment and with quasilinear diffusion terms. Non-equilibrium results at 1.4 m sec. Diffusion stabilizes the surface boundary.

Figure A4. Calculated electron energy and density versus radius for head-on injection in the 2XIIB experiment and without quasilinear diffusion terms. Non-equilibrium results at 1.3 msec. Decreasing plasma radius results from thermal gas erosion at the plasma surface.

Figure A5. Calculated ion energy and density versus radius for head-on injection in the 2XIIB experiment and without quasilinear diffusion terms. Non-equilibrium results at 1.3 msec. Decreasing plasma radius results from thermal gas erosion at the plasma surface.

Figure A6. Comparison of code results with experimental data for the 2XIIB experiment.

Figure A7. See Figure 4 of main text.

Figure A8. Calculated loss rates for the 2XIIB experiment at the plasma center versus magnetic field.

Figure A9. Electron temperature scaling versus magnetic field for the 2XIIB experiment.

Figure A10. Average ion energy versus magnetic field for the 2XIIB experiment.

Figure A11. Calculated distribution function versus magnetic field for the 2XIIB experiment.

Figure A12. Measured distribution function for 2XIIB versus magnetic field. Higher energy components decrease as the magnetic field decreases.

-A13-

A. H. Futch - Figure A1

See Figure 1 of main text.

A. H. Futch - Figure A2

A. H. Fitch Figure A3

A. H. Futch - Figure A4

A. 9 Fuleo - Figure A5

A. H. Futch - Figure A6

A.H. Futch - Figure A7

See Figure 4 of main text.

A. H. Futch - Figure A3

A. H. Futch - Figure A9

A. H. Futch - Figure A10

A. H. Futch - Figure A11

A. H. Futch - Figure A12

APPENDIX B

MEASURED PLASMA LIFETIME VERSUS

RADIUS FROM 2XIIB DATA

The radial dependence of the plasma lifetime is calculated from measurements of the radial and axial density profile, the plasma density, and the fraction of the neutral beam trapped. The plasma density is given by

$$n(r,z) = \hat{n} \exp [-(r/R_p)^2 - (z/L_p)^2] .$$

The plasma lifetime is calculated under equilibrium condition by

$$J(r) = \frac{n(r)}{\tau(r)} \quad \tau(r) = \frac{n(r)}{J(r)}$$

or

$$n\tau(r) = \frac{n(r)^2}{J(r)}$$

where $J(r)$ is the current trapped per cc by ionizing collisions. The geometry dependent factor of the trapped current is obtained from a computer

code using the measured density profile and measured fraction of the neutral beam trapped. Figure B1 shows the results of the calculation for head-on injection (Case 2 of "Energy Confinement in Mirror Plasmas Versus Radial Size"). The calculation assumes a rectangular shape for the beam instead of gaussian. If a gaussian shape were assumed, then the flat part of the curve near the center would tend to disappear.

If electron drag dominated hot ion losses, then n_i would be constant as a function of radius if T_e were constant. The increase of n_i with radius is inconsistent with the measured radial dependence of T_e . We therefore conclude that electron drag is not the dominate hot ion loss process (at least for this case).

At first glance, it seems surprising that the strong dependence shown in Fig. B1 should occur since a straightforward application of the electron drag formula gives $\tau(\text{drag}) = \tau(\text{measured})$; however, a straightforward application of the drag formula is probably not valid since a plasma ion makes many charge exchange collisions with the beam during a drag time - Nexsen and Rensink effect. (This is especially true near the plasma center.) Each charge exchange collision returns the particle to the injected beam energy and angle, thereby reducing the effect of electron drag. On the other hand a particle diffusing radially by quasilinear scattering moves further out by charge exchange with the beam.

Paul Drake has pointed out (MFE/CP/78-483) that the electron lifetime necessary to give typical electron temperature is $\approx 25 \mu\text{sec}$. Therefore, the contribution of radial losses to T_e scaling can be neglected for typical

2XII B plasmas. Optimum electron temperatures may be a factor of 2 higher than typical electron temperatures. The electron lifetime is then $\approx (2)^{5/2} \approx 6$ times longer. If the magnetic field is thus varied to obtain T_e versus R data (Clauser's data), then the ion lifetime appears necessary to explain the observed T_e dependence shown in Figure B2. The particle lifetime scaling should include radial loss terms.

APPENDIX B FIGURE CAPTIONS

Figure B1. $n\tau$ and τ versus radius for head-on injection--Case 2 of energy confinement paper.

Figure B2. Electron temperature scaling versus magnetic field.

A. H. Futch - Figure B1

A. H. Futch - Figure B2