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**Comparison of Zero-Dimensional and One-Dimensional
Thermonuclear Burn Computations
for the Reversed-Field Pinch Reactor (RFPR)**

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COMPARISON OF ZERO-DIMENSIONAL AND ONE-DIMENSIONAL THERMONUCLEAR BURN COMPUTATIONS FOR THE REVERSED-FIELD PINCH REACTOR (RFPR)

by

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ABSTRACT

Conceptual fusion reactor designs of the Reversed-Field Pinch Reactor (RFPR) have been based on profile-averaged zero-dimensional (point) plasma models. The plasma response/performance that has been predicted by the point plasma model is re-examined by a comprehensive one-dimensional (radial) burn code that has been developed and parametrically evaluated for the RFPR. Agreement is good between the zero-dimensional and one-dimensional models, giving more confidence in the RFPR design point reported previously from the zero-dimensional analysis.

I. INTRODUCTION

A comprehensive conceptual design of a fusion power reactor based on the Reversed-Field Pinch (RFP) has recently been published.¹ The plasma physics model used to estimate the thermonuclear burn dynamics and fusion yield was based on a zero-dimensional or "point-properties" formulation. This "point" model effectively averaged all crucial plasma properties (i.e., field, density, and temperature profiles) in accordance with the Bessel-function model^{2,3} for magnetic field profiles.

As part of a continuing parametric analysis and refinement of the RFPR concept, a more realistic one-dimensional (radial) plasma model has been developed⁴ and applied to the point design reported in Ref. 1. The results of this comparison and possible alterations in future RFPR design points are reported herein. Before reporting the results of these parametric analyses, however, background items that are pertinent to the RFP are reviewed.

II. BACKGROUND

A. Physics of the RFP

Like the tokamak, the RFP is a toroidal, axisymmetric confinement device. Both systems use a combination of poloidal, B_θ , and toroidal, B_ϕ , magnetic fields to confine a plasma in a minimum energy state. For both systems the poloidal field is created by inducing through transformer action a large toroidal current, I_ϕ , within the plasma column; the B_ϕ field results from current flowing in external coils. Figure 1 schematically depicts the field and pressure profiles across the plasma minor radius for both tokamak and RFP systems. Toroidal equilibrium in both systems can be achieved by either using a conducting shell near the plasma, an external vertical field, or a combination of both schemes. The RFP requires a conducting shell for plasma stabilization against unstable MHD modes with wavelengths in excess of the shell radius, r_w , whereas the tokamak is not necessarily subjected to this requirement. Localized MHD modes in the RFP are suppressed by the strongly sheared magnetic fields caused by a slight reversal of the B_ϕ field at the plasma edge (Fig. 1). Although the tokamak does not require a conducting shell near the plasma column, stabilization of the kink instability establishes specific requirements on the relative magnitude of B_θ , B_ϕ , the plasma radius, r_p , and the major radius of the torus, R . Specifically, for the tokamak the safety factor, $q = (r_p/R)(B_\phi/B_\theta)$, must be greater than unity, which assures that MHD kink modes with wavelengths in excess of the major circumference will be stable. Experimental values of $q \sim 2-3$ are required for stable plasma operation. The RFP, on the other hand, operates with q less than unity; the q -value actually falls through zero and becomes negative outside the plasma region, $r > r_p$. The presence of a passive conducting shell in the RFP replaces the $q > 1$ stability criterion with one that requires $(dq/dr) \neq 0$; the variation of the plasma/field shear should not exhibit a minimum in the region enclosed by the conducting shell. The reactor

Fig. 1.

Comparison of magnetic and pressure profiles for a $dq/dr \neq 0$ stabilized RFP and a $q > 1$ stabilized tokamak.

disadvantages associated with (passive) wall stabilization or (active) coil stabilization are countered by the advantages the RFP approach exhibits when not constrained by the $q > 1$ criterion. Imposition of the $q > 1$ constraint implies small values of R/r_p and B_θ/B_ϕ , which in turn create the following problems for tokamaks:

- Since the plasma pressure is essentially held by the B_θ field, the ratio of plasma pressure to total field pressure is small, implying a poor utilization of magnetic field energy per unit of fusion yield.
- Since B_θ/B_ϕ is limited and since practical coil design establishes physical limits on the toroidal field, B_ϕ , the plasma current, I_ϕ , is limited. This limit imposed on I_ϕ generally precludes significant ohmic heating, thereby necessitating more complex and less efficient plasma heating schemes (i.e., neutral atom beams, radio-frequency heating).

- The $q > 1$ constraint also enforces limits on the plasma aspect ratio, R/r_p . In addition to obvious engineering and system problems that accompany the design of low-aspect ratio reactor tori, relatively large inhomogeneities result in the toroidal field, B_ϕ , that in turn lead to numerous trapped-particle instabilities and enhanced transport of particles and/or energy from the plasma. Low aspect ratios also lead to a less efficient coupling of the poloidal field energy to the plasma.
- Generally, the $q > 1$ constraint forces the utilization of the highest possible toroidal fields, B_ϕ , and, therefore, causes a difficult magnet design task and the storage of considerable quantities of magnetic energy per unit of contained plasma energy.

In a sense, therefore, the RFP approach "differentiates away" the $q > 1$ constraint imposed on tokamaks and in its place requires $(dq/dr) \neq 0$. The positive implications of this alternative stability criterion are:

- The aspect ratio, R/r_p , can be chosen solely on the basis of engineering considerations and convenience:
- The β limits predicted for the RFP are at least 10 to 50 times greater for $q > 1$ systems if ideal MHD theories are used. The use of resistive theories reduce this factor from 10-50 to 3-10, which still represents a significant improvement.
- The plasma may be brought to ignition with ohmic heating alone with the poloidal field system, which is also in place to confine the plasma pressure.
- The confinement of high-to-moderate β plasma is achieved primarily by poloidal fields, B_θ , which characteristically decrease with increased distance from the plasma, thereby considerably reducing fields and stresses at the magnets.
- The use of field reversal near the plasma edge for the $dq/dr < 0$ RFP configuration makes possible a "vacuum" (low current) region to be established between the plasma and first wall.

Although implications of these improvements are significant from the reactor viewpoint, they are accompanied by the perceived need for a passively conducting first wall. Additionally, the energy that must be expended in establishing and maintaining the near minimum-energy RFP configuration is not known, but if this setup/sustenance energy is significant, operation as an ignition device is made more difficult. Lastly, little or no consideration has been given to the physics implications of fueling and ash-removal systems required for a steady-state operation; the RFP design presented in Ref. 1 is based on a long-pulsed (25-30 s) batch-burn operation. The favorable energy

balance (recirculating power fraction is 0.17) computed for the batch-burn mode of operation reflects the efficient use of magnetic field energy by the RFP, although technological issues associated with pulsed superconducting magnets and energy transfer/storage systems will require further development and study.

The plasma issues of stability and field reversal, as they impact on the RFPR model, have been reviewed in Ref. 1. Stable field profiles within the plasma, an example of which is illustrated in Fig. 2, are modeled by Bessel functions. These profiles are integrated over the plasma radius to give the time-dependent, point model. Generally, ideal MHD stable profiles have been found using numerical methods if the following three constraints are imposed:

- net positive toroidal flux: $\int_0^{r_w} B_\phi 2\pi r dr > 0$ (1)
- poloidal β limit: $\beta_\theta < 0.5 + \beta_\theta(B_\phi = 0)$ (2)
- Suydam criterion: $r[(d \ln v / dr)^2 + [dp/dr] \beta_\theta / B_\phi^2] > 0$, (3)

where r_w is the radius of the conducting shell, $\beta_\theta(B_\phi = 0)$ is the local β at the zero point of the toroidal field, p is the plasma pressure, and $d \ln v / dr$ is

Fig. 2.

Comparison of stable RFP profiles computed numerically with the Bessel function profiles used to generate the point RFPR plasma model.

the magnetic field shear, with $v = B_\theta / r B_\phi = 1/qR$. The first two conditions have been imposed on all RFPR burn simulations, whereas the third condition cannot be imposed directly on the point model used; the Bessel function profiles (Fig. 2), however, are generally a very good approximation to profiles that satisfy the Suydam criterion. A major objective of the one-dimensional calculation presented herein is to examine this latter issue.

The question of toroidal field reversal has also been addressed in Ref. 1 and probably represents the single greatest physics uncertainty for the RFPR. The reactor computations assume that field reversal occurs spontaneously and is maintained automatically throughout the burn period by an unspecified relaxation process or instability. Self-reversal of the toroidal field is an experimental fact, but the associated energy loss is not known. The energy loss associated with this sustained self-reversal, therefore, was assumed equal to loss rates measured from large tokamak experiments, which indicate that the energy confinement times equal approximately 200 Bohm diffusion times. That self-reversal occurs is not in question; the self-reversed pinch state has been observed in many RFP experiments over the last 25 years. Recent theories^{2,3} of relaxed states have led to a substantial increase in fundamental understanding of this minimum-energy, field-reversed state. Given any arbitrary dissipation mechanism, these theories predict that a plasma surrounded by a flux conserving shell will relax to a minimum-energy, force-free, zero- β state. This minimum-energy state is described by the Bessel function model given in Fig. 2. Numerical methods have confirmed this behavior for high- β , reversed-field plasmas. The key descriptive parameters in the Taylor theory² are the pinch parameter, Θ , and the reversal parameter, F , where

$$\Theta = B_\theta(r_w) / \langle B_\phi \rangle \quad (4)$$

$$F = B_\phi(r_w) / \langle B_\phi \rangle \quad (5)$$

$$\langle B_\phi \rangle = (2/r_w^2) \int_0^{r_w} B_\phi r dr \quad (6)$$

Figure 3 shows the locus of minimum energy states as described on the $F - \Theta$ diagram; both the analytic ($\beta = 0$) Taylor state² and the numerical high- β states are shown. The desired field-reversed state corresponds to $F < 0$ and $1.2 < \Theta < 1.6$. It is noted that high- β RFP states have been observed both experimentally and numerically for higher Θ values, but the Taylor theory predicts an ultimate relaxation to the minimum energy states given on Fig. 3; both relaxation mechanism and associated time constants for this sustained, self-reversal of toroidal field are not well understood at this time. It is noted that the minimum-energy tokamak state is described by the Taylor theory as the point where $F = 1$, for which $\Theta \approx r_p/R$. The penalties incurred for operation near this state, however, have been previously discussed.

Fig. 3.
Region of stable, minimum-energy states displayed on $F - \Theta$ diagram for both low- β (Taylor) and high- β (numerical) RFP plasmas.

B. Description of RFPR Based on Point Plasma Model

To the casual observer a magnetically confined fusion reactor appears as an intertwined array of coolant ducts penetrating an almost inaccessible toroidal assembly of superconducting magnets. If, however, the toroidal aspect ratio can be made sufficiently large to approximate a linear geometry and to allow the use of nearly cylindrical blanket modules, this problem of accessibility can be considerably reduced. Furthermore, if the primary plasma confinement system can be combined with the major plasma heating scheme, large and complex appendages can be eliminated from the torus, and the reactor system becomes even less complicated. Finally, if the appreciable plasma pressures (30-60 MPa) can be supported primarily by poloidal magnetic fields, which characteristically decrease in strength as the minor radius increases, the low-field superconducting coils can be removed from the vicinity of the blanket/shield without a serious increase in stored magnetic energy; the accessibility problem is thereby further reduced. The RFP combines all three of these requirements; arbitrary aspect ratio, combined heating and confinement system, and low-field magnets. These features, additionally, exist for a scheme whose physics admits the potential for a regenerative, near minimum-energy, high- β plasma state. These physics characteristics directly and favorably impact the reactor design that has been presented in Ref. 1. Potential problem areas for the RFPR include the presently perceived need for a ~ 20 -mm-thick copper first wall for short-time (~ 0.1 s) plasma stabilization and the need for pulsed (~ 0.1 -s risetime, 25-30 s dwell time) magnetic energy transfer and storage.

The time-dependent burn model, plasma and engineering energy balances, and the stability/equilibrium criteria upon which this RFPR design is based are described in Ref. 1 and summarized in Table I. Table II gives a summary description of key reactor parameters.

Upon induction of the 20-MA (5.4 MA/m^2) toroidal current in ~ 0.1 s, the plasma ohmically heats to ignition in ~ 3 s, as shown by the results of the RFPR point burn model in Fig. 4. The plasma subsequently burns for 15 s at 20 keV to yield a fuel burnup fraction, f_B , of 0.5. The end of the burn is determined by the ion temperature dropping below 8 keV, as the plasma losses begin to exceed the alpha-particle heating. At this point the plasma is expanded to the wall by opening switches in both poloidal and toroidal coil systems. Current in the poloidal field coil is again reversed by using a

TABLE I
RFPR REACTOR (POINT PLASMA) PHYSICS MODEL

- Pressure-balance model using average properties
 - $B_{\theta} = A_{\theta} J_1(\alpha r)$
 - $B_{\phi} = A_{\phi} J_0(\alpha r)$
- Three species
 - Maxwellian ions and electrons
 - Fokker-Planck alpha particles
- Ohmic heating using modified classical resistivity
- Bremsstrahlung, cyclotron, line radiations
- Tokamak start-up code (separate)
- Anomalous (Bohm) radial transport
 - Typically, $\tau_E = 200 \tau_{BOHM}$
- Time-dependent properties (temperatures, densities, radius)
- Circuit model option
- First-wall heat transport and stress response
- Detailed reactor energy balance
- Direct interface with comprehensive costing model
- Stability criteria: $\Theta = B_{\theta}(r_w) / \langle B_{\phi} \rangle \cong 2$, $F = B_{\phi}(r_w) / \langle B_{\phi} \rangle \cong 1$, $\beta_{\theta} < 0.4$

homopolar motor/generator set as a transfer element, this action resulting in a current flowing in the coil between burn pulses. The toroidal field energy left untrapped within the plasma is extracted from the reactor and stored in the homopolar between burn pulses. Magnetic field trapped in the plasma at the termination of the burn is assumed to be thermally dissipated and delivered as heat to the blanket through the first wall. Neutral DT gas is added at this point to promote a controlled plasma quench and dilution of the alpha-particle ash. Continuous pumping by the vacuum system readies the plasma chamber for the next burn pulse during the 5-s dwell period. Table III gives a summary energy balance for the burn cycle depicted in Fig. 4.

Throughout the burn cycle, including the startup and approach to ignition, the energy confinement time was fixed at 200 times the instantaneous Bohm diffusion time, this scaling being deduced^{1,5} from existing tokamak experimental data. Both in magnitude and in terms of functional scaling, this loss rate is sufficient to control the plasma β and leads to a stable burn

TABLE II
SUMMARY DESCRIPTION OF RFPR DESIGN PARAMETERS

PARAMETER	VALUE
First-wall radius, r_w (m)	1.5
Major radius, R (m)	12.7
Toroidal plasma current, I_ϕ (MA)	20.0
Toroidal field at the coil, $B_{\phi c}$ (T)	2.0
Poloidal field at the coil, $B_{\theta c}$ (T)	2.0
Toroidal coil energy, $W_{B\phi}$ (GJ)	3.7
Poloidal coil energy, $W_{B\theta}$ (GJ)	11.0
Field rise time, τ_R (s)	0.1
Burn time, τ_B (s)	21.6
Cycle time, τ_C (s)	26.6
Average fuel burnup, f_B	0.5
Average plasma density, $n(10^{20}/m^3)$	2.0
Average plasma temperature, $T_{i,e}$ (keV)	15
14.1-MeV neutron current at first wall, I_W (MW/m ²)	2.7
Engineering Q-value, Q_E	5.8
Recirculating power fraction, $\epsilon = 1/Q_E$	0.17
Average blanket power density, P_{BLK} (MW/m ³)	4.7
Average system power density, P_{SYS} (MW/m ³)	0.9
Total thermal power, P_{TH} (MWt)	3000
Net electrical power, P_E (MWe)	750
Net plant efficiency, $\eta_p = \eta_{TH}(1-\epsilon)$	0.25

trajectory. Simultaneously, the burn trajectory in $F - \Theta$ space, as seen from Fig. 4, follows closely the predictions of the Taylor minimum-energy model. The physics operating point summarized in Fig. 4 and Tables II and III represents the culmination of an extensive parameter search which used as an object function the plant capital and power costs.¹ This point-plasma design point is re-examined here in terms of a one-dimensional burn code.

Fig. 4.

RFPR (superconducting coils, air-core system) burn parameters using an energy confinement time $\tau_E = 200 \tau_{BOHM}$. The burn trajectory is in good agreement with that required by the high- β model for a minimum-energy configuration (Fig. 3).

III. ONE-DIMENSIONAL RFPR THERMONUCLEAR BURN

Like the point model previously described, the one-dimensional model treats a three-particle quasi-neutral gas of electrons, DT-ions and alpha particles. The embedded magnetic field has a toroidal and a poloidal component, B_ϕ and B_θ , respectively. Radial field diffusion and plasma convection in this azimuthally symmetric system are modeled under a magnetohydrostatic approximation, and thermal conduction for ions and electrons are computed separately. The plasma is heated ohmically, and radiation losses include bremsstrahlung and electron cyclotron channels. A Fokker-Planck model is used to partition the alpha-particle energy appropriately between DT ions and electrons. A number of approximations were used to describe the spatial distribution of both energy deposition and mass accumulation for the alpha particles.

TABLE III
SUMMARY ENERGY BALANCE FOR A 21.6-s BURN AND A 26.6-s CYCLE TIME

PARAMETER	VALUE (MJ/m)
Initial plasma energy	0.05
Final plasma energy	2.5
Radiation energy ^a	28.1
Ohmic heating energy	7.1
Plasma energy loss (conduction) ^b	147.5
Plasma expansion energy	0.7
Eddy current losses in the blanket/shield	1.5
Magnetic-field energy lost at end of burn cycle ^c	21.5
Magnetic-field energy transfer losses ^d	8.1
Fusion neutron energy	792.
Auxiliary energy requirements ^e	14.3

^abremsstrahlung and line radiation

^bbased on an energy confinement time equal to 200 Bohm diffusion times

^cassumed to be thermally dissipated

^dbased on a 95% efficient inductive/capacitive transfer from the homopolar motor/generator (capacitive) to and from the magnets

^ethe cryogenic system required for the superconducting magnets consumes 21% of the auxiliary power

A. Radial Transport

The radial burn code uses an implicit Lagrangian meshing technique that has been developed explicitly for modelling plasma processes on a time scale that is long in comparison to the sound transit time; inertial terms in the MHD equations are, therefore, ignored, and plasma motion is thereby determined by a quasi-static pressure balance, $\vec{J} \times \vec{B} = \vec{\nabla} p$. This approximation infers that the quasi-static equilibrium profiles are stable, and this assumption is examined periodically by a separate algorithm. The pressure balance constraint is expressed by the following integral equation for systems that exhibit both axial and azimuthal invariance.

$$p + B^2/2\mu_0 = (2/r^2) \int_0^r (p + B^2/2\mu_0) r' dr' \quad , \quad (7)$$

where p is the total plasma pressure, $B^2 = B_\phi^2 + B_\theta^2$, and mks units are consistently used except for the temperature, $T(\text{keV})$.

Plasma parameters are computed as functions of time by a two-step method. First, the Lagrangian mesh is fixed in space for the computation of all diffusion, radiation, and heating processes. Secondly, the Lagrangian mesh is adjusted in space to satisfy the pressure balance constraint given by Eq. (7).

The first phase of this two-step process, wherein all transport is computed, is described by the following expressions for ion density n_i , alpha-particle density n_α , energy of the j th species T_j ($j=e, i, \alpha$), and magnetic field.

$$\frac{\partial n_i}{\partial t} + \vec{\nabla} \cdot \vec{\Gamma}_i = S_i - 2f(1-f) \langle \sigma v \rangle n_i^2 \quad (8A)$$

$$\frac{\partial n_\alpha}{\partial t} + \vec{\nabla} \cdot \vec{\Gamma}_\alpha = f(1-f) \langle \sigma v \rangle n_i^2 f_\alpha \quad (8B)$$

$$\frac{3}{2} n_j (10)^3 e \frac{\partial T_j}{\partial t} + p_j \vec{\nabla} \cdot \vec{\Gamma}_j = -\vec{\nabla} \cdot \vec{q}_j + Q_j \quad (8C)$$

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \left[-\vec{v} \times \vec{B} + \frac{1}{en_e} \vec{R}_{TH} + \eta_{||} J_{||} \vec{e}_{||} + \eta_{\perp} J_{\perp} \vec{e}_{\perp} \right] \quad (8D)$$

The fluid velocity with respect to the Eulerian coordinates is $\vec{v}$, $\vec{\Gamma}_j = n_j \vec{v}_j$ is the particle flux, S_j is an ion source term, f is the tritium fraction, f_α is an alpha-particle confinement efficiency, $\vec{q}_j$ is a thermal heat flux, Q_j is a heat source, and $\vec{R}_{TH}$ represents a thermal force. The parallel and perpendicular resistivities, $\eta_{||}$ and $\eta_{\perp}$, are treated either classically or with anomalous effects, and $e(1.602(10)^{-19} \text{ J/eV})$ is the electronic charge.

B. Pressure Balance

Upon evaluating the transport (Eqs. (8)) for a given time step at each radial position, the new values of temperature, density and fields will not satisfy pressure balance (Eq. (7)). The Lagrangian mesh is adjusted adiabatically as a $\gamma = 5/3$ ideal gas with the new radial positions being selected to satisfy the Lagrangian constraint. This procedure is executed in conjunction with an appropriate boundary condition, such as maintaining a given plasma radius or current. An appropriate energy balance and variable time step is used to assure a rapidly-executing and accurate model.

C. Stability Model

Two methods are used to monitor the RFP stability in the context of the one-dimensional model. Local modes are assumed to saturate and are monitored using a Suydam criterion.⁶ A Rayleigh-Ritz type energy principle is used to monitor global modes. Both methods for monitoring stability are derived from the ideal MHD equations using a cylindrical approximation.

If the plasma is assumed to extend radially, the one-dimensional energy principle assumes the following form⁷

$$\delta W = (\pi/2) \int_0^{r_p} [f(d\xi/dr)^2 + g\xi^2] dr \quad , \quad (9)$$

where,

$$f = \frac{r(krB_\phi + mB_\theta)^2}{\mu_0 [(kr)^2 + m^2]} \quad (10A)$$

$$g = \frac{2(kr)^2}{(kr)^2 + m^2} \left(\frac{dp}{dr} \right) + \frac{[krB_\phi + mB_\theta]^2 [(kr)^2 + m^2 - 1]}{\mu_0 r [(kr)^2 + m^2]} \\ + \frac{2(kr)^2 [(kr)^2 B_\phi^2 - m^2 B_\theta^2]}{\mu_0 r [(kr)^2 + m^2]^2} \quad . \quad (10B)$$

The axial wave number is k , the mode number is m and ξ is the radial displacement. In order to avoid the singularity in ξ when f approaches zero the transform $\psi = \xi\sqrt{f}$ proves useful.⁸ Equation (9) transforms to

$$\delta W = (\pi/2) \int_0^R \left\{ (d\psi/dr)^2 - (\psi/f)(df/dr)(d\psi/dr) + \left[(df/dr)^2/4f^2 + g/f \right] \psi^2 \right\} dr \quad (11)$$

The parameter ψ is then expanded into a Fourier Series according to

$$\psi = \sum_{i=0}^{n_1} C_i \sin(\alpha_i r) \quad , \quad (12)$$

where $\alpha_i = (i+1)\pi/R$. The constant C_0 is normalized to unity, and the remaining constants, $C_i (i=1, \dots, n)$, are computed to minimize the energy δW by requiring that $\partial \delta W / \partial C_i = 0$ for $i=1, \dots, n$. The resulting set of n inhomogeneous equations and unknowns is inverted and solved for C_i . If the resulting value of δW is negative, the associated plasma configuration is considered unstable. This δW stability check is applied to the one-dimensional burn computations for $m = 0$, $k \neq 0$ and $m = 1$, $-30/R < k < 30/R$.

The Suydam criterion, as embodied in Eq. (3), is included in the more general energy principle stated by Eq. (9). These Suydam modes, however, have eigenfunctions that are asymptotically singular at rational surfaces. Since the mesh size used by the one-dimensional computation is finite and determined by the previously described transport computation, eigenfunctions that are more localized than the mesh size will not be detected by the global energy-principle analysis. Consequently, the global stability analysis is able to find gross modes that may also occur simultaneously with Suydam modes, which are assumed to saturate while possibly leading to enhanced transport. The global energy-principle analysis also quantifies the validity of the assumption that Suydam modes are localized in that the modes will be detected if the associated eigenfunctions extend over a "macroscopic" (i.e., mesh size) volume of the plasma.

D. Anomalous Transport

As will be shown, a large region of the plasma is found to be unstable to local Suydam modes for the plasma conditions selected for the point plasma operating parameters. The resulting instabilities are assumed to remain localized and to be manifested as enhanced transport. Following a similar study,⁹ Bohm transport is assumed to describe these regions of Suydam instability. Specifically, the perpendicular resistivity, $\eta_{\perp}$, was modified relative to the classical value, $\eta_{\perp c}$, according to

$$\eta_{\perp} = \eta_{\perp c} + f_T f_s C B^2 D_B / n_e \quad , \quad (13)$$

where f_T is a triangular function that spreads the region of enhanced transport over five adjacent mesh points (~ 10 Larmor radii) around the location of Suydam instability, the constant C is taken as 10^{-4} , and f_s is given by

$$f_s = \begin{cases} 0 & \text{if } \epsilon < 1 \\ (\epsilon-1)/\epsilon & \text{if } \epsilon > 1 \end{cases}$$

$$\epsilon = - \frac{dp/dr}{(B_{\theta}^2 / 8r\mu_0)(dp/dr)^2} \quad (14)$$

$$P = rB_{\phi}/B_{\theta} = 1/\nu \quad .$$

By means of this algorithm, the increased perpendicular resistivity forces the current to flow more parallel to magnetic field lines, thereby reducing the pressure gradients (i.e., $\vec{J} \times \vec{B} = \nabla p$) that initiated the Suydam instability and increased radial transport of particles and energy.

IV. RESULTS

In examining dimensional effects on the thermonuclear yield and overall plasma performance, three computational cases were considered:

- Case A: The one-dimensional model was forced to follow exactly the radius trajectory predicted by the zero-dimensional model.
- Case B: The more physically correct assumption of constant radius and moderately programmed toroidal current was examined.
- Case C: With the plasma radius fixed, regions of Suydam instability were subjected to significantly enhanced transport in order to model a flattening of the temperature profile (e.g., Eq. (14)).

Similar to the point model, the radial energy transport for Cases A and B is set equal to 200 Bohm times. Cases A-C assume that both the alpha-particle energy and mass are deposited locally. A fourth case, Case D, is identical to Case A, except the alpha particle mass and energy is deposited across the plasma radius according to the ion density profile. Two additional cases have also been considered: Case E fixes the plasma radius and deposits both alpha-particle mass and energy according to the ion density profile, whereas Case F fixes the plasma radius, distributes the alpha-particle energy according to the fusion reaction rate, and distributes the thermalized alpha-particles according to the ion density profile. Table IV summarizes the essential elements of the six simulation cases described above.

A. Exact Comparison Between Zero- and One-Dimensional Models: forced Plasma Radius Trajectory.

In order to match the point model upon which the RFPR design given in Tables II and III is based, the plasma was confined to a trajectory $r_p(t)$ given by

$$r_p(t) = r_p(t=0)/\sqrt{1-\beta_\theta} \quad , \quad (15)$$

where the plasma radius, r_p , is defined by the toroidal field reversal point, and β_θ is the maximum poloidal beta. Initial attempts to duplicate the point model by holding the toroidal current, I_ϕ , constant were not as successful as computations that used the forcing function given by Eq. (15), although the time variation in I_ϕ when r_p is specified amounts to only a few percent. The initial temperature distribution was assumed uniform, and the initial density

TABLE IV
SUMMARY OF CONDITIONS ASSUMED FOR ONE-DIMENSIONAL SIMULATIONS

<u>Case</u>	<u>Radius Trajectory</u>	<u>Alpha-Particle Energy Distribution</u>	<u>Alpha-Particle Mass Distribution</u>	<u>Energy Transport</u>	<u>Burn time^(b) τ_p (s)</u>
A ^(a)	moving	n^2	n^2	$200 \tau_{BOHM}$	21.6
B	fixed	n^2	n^2	$200 \tau_{BOHM}$	25
C	fixed	n^2	n^2	$200 \tau_{BOHM}$ plus Suydam turbulence	25
D	moving/fixed (optimized)	n	n	$200 \tau_{BOHM}$	21.6
E	fixed	n	n	$200 \tau_{BOHM}$	25
F	fixed	n^2	n	$200 \tau_{BOHM}$	25

(a) closest simulation of zero-dimensional model

(b) quench time taken as 5 s

and toroidal field profiles within the plasma region were modeled by appropriate Bessel functions

$$n(r,0) = n(0,0) J_0^2(0.94\alpha r) \quad (16)$$

$$B_\phi(r,0) = B_\phi(0,0) J_0(\alpha r) \quad , \quad (17)$$

where $\alpha = 2.405/r_p(0)$. Using the above initial conditions, the initial poloidal field profile is computed to satisfy pressure balance.

Figure 5 compares the averaged ion and electron temperatures obtained with the one-dimensional model for Case A with the predictions of the zero-dimensional model. Shown also is the time dependence of the peak poloidal beta and the fractional burnup of DT fuel. The radial dependence of temperatures and fields 12-s into the ignited burn is depicted in Fig. 6. In addition to illustrating the radial density profile at 12 s, Fig. 7 gives the radial dependence of the shear parameter, $P = 1/\dot{v}$, and the Suydam parameter,

Fig. 5.

Comparison of averaged parameters versus time as predicted by the Case A one-dimensional simulation and the zero-dimensional model.

f_g (Eq. (14)). It is noted that for the parameters shown, $f_g > 0$, and the plasma is unstable to local Suydam modes over its entire volume. Lastly, Fig. 8 compares the neutron, transport, and radiation energy flux at the first wall for the Case A simulation.

The differences between the point and one-dimensional models illustrated in Figs. 5-8 can be attributed to differences in assumptions. First, the point model assumes a uniform plasma temperature, where the radially varying temperatures and associated enhanced fusion reactivity (and alpha-particle heating) in the central plasma region results in a higher average temperature

Fig. 6.

Temperature and field profiles for the Case A simulation at 12.0 s into the burn phase.

shortly after ignition (Fig. 5). Consequently, the one-dimensional model burns out more rapidly, resulting in a lower value for f_B (Fig. 5). This lower fuel burnup (batch or unrefueled burn is assumed for both models) can be attributed to differences in alpha-particle heating and fuel density profiles as the burn evolves.

It is noted that the point model assumes that the alpha-particle energy deposition occurs uniformly across the plasma radius and that the fuel density maintains its initially specified radial profile (Eq. (16)). The one-dimensional model assumes in situ alpha-particle heating and fuel burnup,

Fig. 7.

Profiles of density, pitch parameter, P , and Suydam parameter, f_s , for the Case A simulation at 12.0 s into the burn phase.

resulting in a comparatively higher alpha-particle buildup and subsequently lower fuel fraction in the central region of the plasma. Since a major portion of the fusion reactions occur in this hotter, more dense central region, the one-dimensional simulation burns out more rapidly with a resulting total fuel burnup fraction that is lower.

The point burn model must specify a magnetic field profile to assure pressure balance and cannot account for resistive decay of the toroidal field B_ϕ . In forcing a radius trajectory, $r_p(t)$, the one-dimensional model must force the toroidal current, I_ϕ , to decay, as resistive losses consume B_ϕ .

Fig. 8.

Comparison of first-wall neutron, thermal conduction, and radiation fluxes for the Case A one-dimensional simulation and the zero-dimensional model.

field. This artifact results in somewhat less ohmic heating, although this effect is small.

The radial profiles shown in Figs. 6 and 7 remain relatively unchanged throughout the burn period. Unlike earlier simulations,⁴ both the temperature and pressure show a maximum on the plasma centerline because of the assumed Bohm-like thermal conduction. The fact that the bulk plasma is unstable to Suydam modes, however, must be reflected in an enhanced transport, the implications of which is examined by Case C. Table V summarizes crucial performance parameters for both the zero-dimensional and Case A one-dimensional simulations. Of particular interest is the plasma Q-value, Q_p , defined in Table V as the ratio of total fusion yield, $1.25W_N$, to the sum of the total ohmic dissipation, W_{OHM} , and the field energy stored in the plasma, W_{BF} , at the end of the burn phase but before plasma current rundown and quench. It is seen that for the conditions under which Case A was

TABLE V

SUMMARY COMPARISON OF ENERGY BALANCE FOR ZERO-AND ONE-DIMENSIONAL BURN MODELS

PARAMETER (MJ/m)	ZERO-D	ONE-D ^f CASE A	ONE-D ^g CASE B	ONE-D ^h CASE C	ONE-D ⁱ CASE D (OPTIMIZED)
Burn time τ_B (s)	21.6	21.6	24.6	25.0	22.
Initial plasma energy, W_{pi}	0.05	0.7	0.8	0.8	.99
Final plasma energy, W_{pf}	2.5	1.9	3.0	3.1	3.68
Radiation energy ^a , W_{RAD}	28.1	14.3	14.8	13.9	19.3
Ohmic heating energy, W_{OHM}	7.1	8.9	10.4	12.4	8.93
Plasma energy loss (conduction), W_{COND}	147.5 ^b	93.0	115.2	114.5	146.7
Plasma expansion energy, W_{pv}	0.7	1.1	0.0	0.0	0.0
Eddy current losses in the blanket/shield, W_{EDDY}	1.5	---	---	---	---
Magnetic-field energy inside first wall at end of burn cycle, W_{Bf}	51.6 ^c	45.0	45.1	45.5	30.6
Magnetic-field energy transfer losses, W_{ETS}	8.1 ^d	---	---	---	---
Fusion Neutron Energy, W_N (14.1 MeV)	676.5	410.	493.5	469.7	646.2
Auxiliary energy re- quirements, W_{AUX}	14.3 ^e	---	---	---	---
$Q_p = (\text{fusion energy}) /$ (ohmic + field energy)	14.4	9.5	11.1	10.1	20.4

^aBremsstrahlung and line radiation^bbased on an energy confinement time equal to 200 Bohm diffusion times^ca significant fraction of this energy would be recovered during plasma current rundown and quench (~ 50% recovered in RFPR reactor design)^dbased on a 95% efficient inductive/capacitive transfer from the homopolar motor/generator (capacitive) to and from the magnets^ethe cryogenic system required for the superconducting magnets consumes 21% of the auxiliary power^fradius forced to follow zero-dimensional case^gfixed plasma radius, Bohm conduction^hfixed plasma radius, Suydam turbulenceⁱfixed plasma radius, Bohm conduction

evaluated, a factor of ~ 1.5 decrease in Q_p relative to the point model is predicted. This significant difference in Q_p illustrates the net influence of profile effects, with the reduction in Q_p a direct result of the lower fusion yield because of differences in the evolution of the fuel density profile during the burn, as previously discussed. As for any model, these differences in predicted Q_p values are sensitive to modelistic assumptions. Some of these assumptions are examined further.

B. Comparison of Constant Radius Cases With and Without Enhanced Transport, Cases B and C.

The one-dimensional model was evaluated for a programmed rather than a constant toroidal current, using the formalism for Suydam turbulent transport represented by Eqs. (13) and (14). A comparison for the constant radius case with and without Suydam turbulence is presented in Fig. 9, where the time dependence of average temperature, β_0 , and f_B is given. The radial profiles of temperatures and fields with and without Suydam turbulence are depicted in Fig. 10, whereas Fig. 11 gives the radial dependence of densities, pitch parameter, P , and Suydam parameter, f_S . Key plasma parameters for Cases B and C are also summarized on Table V.

Even without Suydam turbulence enforced a greater fraction of the plasma volume becomes Suydam stable (Fig. 11) for the constant-radius Case B. Simulation of Suydam turbulence by causing anomalous Bohm-like transport to occur in Suydam unstable regions (Eqs. (13) and (14)) flattens the density profiles in the center of the plasma, causing a similar leveling of the pressure profile and pushing the plasma towards the marginal Suydam stability region.

Comparison of the Suydam stability parameter, f_S , for Cases B and C shows that enhanced anomalous transport transforms the plasma from one of uniform Suydam instability to a pointwise, stochastic violation of the Suydam criterion. As an unstable plasma region is stabilized by anomalous transport, pressure gradients in adjacent regions are increased, causing these latter regions to become unstable. The net result is a randomized or statistical violation of the Suydam criterion in regions of weak shear and an enhanced transport. These computational results indicate that, like the tokamak, a major contribution to the plasma confinement is provided by the plasma edge regions.

Fig. 9.

Comparison of Case B and Case C one-dimensional simulations showing the influence of Suydam turbulence on the average temperatures; poloidal beta, and burnup fraction. For f_B and β_θ , no difference was observed.

The magnetic fields maintain a continuous topological structure despite the enhanced transport associated with the Case C conditions. Perturbations in both field and pitch parameter profiles are small compared to density perturbations (Fig. 11). This result is caused by scaling the anomalous transport through the perpendicular resistivity, $\eta_\perp$ (Eqs. (13) and (14)), rather than through both perpendicular and parallel components; this method implicitly assumes that turbulence results in enhanced transport directly, rather than by only increasing collisionality.

Despite the significant differences in transport models used in Cases B and C, the plasma Q-value is increased only $\sim 20\%$ from the Case A comparison case; a factor of 1.3 then separates the predictions of the one-dimensional

Fig. 10.
Comparison of Case B (no turbulence) and Case C (with turbulence) one-dimensional simulations in terms of temperature and field profiles that have evolved at ~ 10 s into the burn.

and zero-dimensional models. If Case B is re-evaluated with alpha-particle deposition proportional to ion density, Q_p increases slightly from 11.1 to 11.6. The reasons for this depressed Q -value are associated with the premature burnout of the plasma central core, the lower fuel burnup and a diminished thermonuclear yield. Increasing the burn time results in marginally improved Q -values. If Case B is simulated with a 40% increase in filling pressure, Q_p increases from 11.1 to 17.4, however. Both thermal and neutron first-wall flux limits will eventually intervene as the Q -value is

Fig. 11.

Comparison of Case B (no turbulence) and Case C (with turbulence) one-dimensional simulations in terms of density, P , and f_s profiles that have evolved at ~ 10 s into the burn phase.

increased by raising the plasma density and correspondingly the plasma beta. This issue is addressed quantitatively in the following section.

C. Parametric Studies Leading to Potentially New RFPR Operating Points.

In order to ameliorate the adverse effects on the plasma Q -value associated with central core burnout the effects of both a number of alternative alpha-particle thermalization models and variations in plasma density and magnetic field levels were examined. The resulting sensitivity of Q_p to these assumptions is summarized. For all cases, particle transport per se was treated only within the classical Lagrangian framework, as described in Sec. III. A. Anomalous particle transport could possibly permit a larger burnup fraction and Q_p for a batch-burn operating mode; this question is also

addressed briefly. Some form of refueling may be required in order to retrieve the favorable f_B and Q_p values predicted by the zero-dimensional model and used as a basis to estimate the RFPR engineering parameters,¹ although modest change in the toroidal magnetic fields are shown to also lead to increased Q values.

1. Dependence of Q_p on Plasma Density and Alpha-Particle Models

As indicated in Sec. IV. B and Table V, the mechanism by which alpha-particle energy and mass is distributed over the plasma radius has a minor influence on the plasma Q -value. Figure 12 quantifies this effect, wherein the dependence of Q_p on line density is shown for a range of alpha-particle thermalization models (Table IV). Figure 12 also illustrates the dependence of the 14.1-MeV first-wall loading on line density. Setting $\sim 4\text{--}5 \text{ MW/m}^2$ as an upper limit on the neutron wall loading, values of Q_p in the range 15-20 are expected, which is significantly greater than the value (14.4) used for the point-plasma RFPR design.¹ General trends and observations not reflected by Fig. 12 are:

- The higher density cases generally tend to burn out faster than the lower density cases. This behavior would result in a higher instantaneous neutron and thermal loading at the first wall for a fixed quench period of $\tau_Q \approx 5 \text{ s}$.
- The models that allow the plasma radius (i.e., the reversal point) to move in some cases predicted the plasma radius contacts the fixed wall radius ($r_W = 1.5 \text{ m}$).
- The higher density cases generally achieved higher plasma temperatures.
- Cases A and D at a line density above $4.5(10)^{21} \text{ m}^{-1}$ did not ignite.

2. Optimization of RFPR Design Point by the One-Dimensional Model

The re-optimization of the RFPR design point by the one-dimensional model is the primary motivation for this study. Overall reactor system considerations are not within the scope of this work, and a comparison with the point model is used to gauge whether or not the predictions of the one-dimensional model are favorable or not. Establishing a 14.1-MeV neutron wall loading near the reference design¹ (2.7 MW/m^2), while maintaining a

Fig. 12.

Dependence of plasma Q-value, Q_p , and 14.1-MeV neutron wall loading on line density and alpha-particle models for the five cases summarized in Table IV.

similar power output, fixes the plasma line density at $1.5(10)^{21} m^{-1}$. Using Case D (with fixed radius) listed in Table IV, the magnetic field levels were adjusted until the Q-value reached a maximum. A plot of Q_p versus $B_{\phi 0}$, the initial bias magnetic field on axis, is shown in Fig. 13 and illustrates that the Q-values increase as $B_{\phi 0}$ decrease because of the reduced quantity of

Fig. 13.

Dependence of plasma Q-value and associated maximum poloidal beta on the initial toroidal bias field.

magnetic field remaining in the plasma at the end of the burn. If $B_{\phi 0}$ is reduced below ~ 5.8 T, plasma ignition does not occur. A tabular comparison between the optimized point-model calculation and the one-dimensional optimized calculation is shown in Table V as Case D. As noted in Fig. 13, the poloidal beta increases from 0.39 to 0.51 as the magnetic fields are reduced; although high, this beta still remaining within stability limits as predicted by ideal MHD theory. If lower beta limits are enforced, the overall RFPR efficiency decreases somewhat, although detailed parametric studies¹ show that the attendant decrease in system performance (and cost) does not become serious until the maximum allowable β_θ falls below ~ 0.15 .

The time dependent electron and ion temperatures for the $Q_p = 20.4$ case shown in Fig. 13 are plotted in Fig. 14, along with the fractional burnup, f_B , and poloidal beta, β_θ , which peaks at 0.5. The plasma power response is also given in Fig. 14 with the total integrated output being within 5% of that used

Fig. 14.

Time dependence of average plasma temperatures, beta, and DT fuel burnup fraction for the optimum design point. Also shown are the neutron, radiation and conduction powers expressed per unit first-wall area.

in the RFPR reactor design.¹ The radial dependence of the magnetic fields is shown in Fig. 15 for $t = 0, 10, 20$ s. At $t = 0$ the plasma pressure is

Fig. 15.

Radial profiles of the magnetic fields, particle densities, ion and electron temperatures, magnetic field pitch and the Suydam stability parameter (f_s) for the optimal design point at various times.

essentially zero, and the magnetic field profiles describe a pressureless plasma state. Plasma expansion occurs as the plasma heats, and the magnetic fields near the center necessarily drop in order to maintain pressure balance. This behavior is illustrated in Fig. 15 by the curves for $t = 10$ -s. Near the end of the burn ($t \approx 20$ s) the plasma is again cool, and the fields again relax toward the center, although resistive dissipation tends to flatten the profiles. The effect of plasma expansion is also exhibited by the electron density profiles depicted in Fig. 15. Once again, a general flattening of the profiles (in this case the electron density) results during the 20-s burn

cycle. The effects of fuel burnup are illustrated in Fig. 15 by the decreasing ion density and corresponding increase in alpha-particle density as the burn proceeds. Finally, the pitch of the magnetic fields versus radius is shown in Fig. 15 along with a graphic illustration of the regions of Suydam stability violation. Only a small region of the pinch is initially unstable to this pressure dependent instability, which tends to grow as the plasma temperature is increased. At $t = 10$ s the plasma temperature is nearly maximum, and the largest area of Suydam violation occurs. This area of Suydam instability is reduced as the plasma temperature decreases near the end of the burn, although relaxation of the profiles also has contributed to the degradation of the stability. The existence of a turbulent Suydam-unstable central region is common to axisymmetric configurations of this type (both the tokamaks and the RFP) and is not expected to limit severely the confinement properties because of the stable buffer region that exists between the turbulent plasma core and first wall. This behavior was in fact shown numerically by this study (Sec. III.D) in which the inducement of turbulence and enhanced transport in regions of Suydam violation did not produce significantly different plasma performance. The transport loss corresponding to $200 \tau_{\text{BOHM}}$ dominates any other induced losses required to force the profiles into a nearly Suydam-stable configuration.

V. CONCLUSIONS

Agreement is good between the zero-dimensional and one-dimensional RFPR burn models. This agreement gives more confidence in the RFPR design point reported earlier¹ on the basis of a zero-dimensional analysis. The minor differences that have emerged between the two models can be attributed primarily to profile effects. In a sense, this agreement between the one-dimensional and zero-dimensional models is fortuitous in that both models are based on similar physics assumptions; a number of these assumptions remain to be validated or disproved by experiment. The one-dimensional model, however, represents an important tool by which the sensitivity of RFPR performance to important physics assumptions can be parametrically evaluated. Given below is a synopsis of key conclusions that have been drawn from this comparison study.

- The one-dimensional model indicates the importance of local ignition and fuel burn-out. Although the observed center-line burnout causes some modification of the burn trajectory, as estimated from the zero-dimensional model, the overall plasma performance, remains essentially unaltered.
- The plasma performance appears to be relatively insensitive to the alpha-particle thermalization and transport model, provided a major fraction of the total alpha-particle energy is delivered to the plasma.
- The one-dimensional model shows that a substantial portion of the plasma is unstable to local Suydam modes. Attempts to induce an enhanced or turbulent transport associated with this local instability had little effect on the overall plasma performance. The Suydam-induced transport causes a stochastic variation in the local violations of the Suydam criterion, and this transport was induced through the perpendicular electrical resistivity. The resistivity related transport was small compared to the consistently assumed $\sim 200 \tau_{\text{BOHM}}$ enhanced thermal conductivity.
- The global stability tests indicates good stability against gross $m = 0,1$ modes for all cases examined.
- The zero-dimensional model tends to over estimate the plasma current required to ignite the plasma. The one-dimensional model, on the other hand, allows a local ignition to occur at the plasma center line. This local ignition rapidly propagates radially outward. Consequently, the one-dimensional ignition can occur at lower fields, and improved values of Q_p would be predicted by the one-dimensional simulations.
- Differences in the plasma Q-values, Q_p , arise between the fixed versus the variable radius cases. An increasing plasma radius results in an increased beta (B^2 decreases faster than the plasma pressure as the radius increases). For a fixed (given) beta limit, therefore, constant radius operation yields a higher Q-value. This mode of operation, however, may complicate the reactor operation somewhat because the plasma currents must be more carefully controlled to achieve a constant radius operation.
- The one-dimensional burn model has led to an improved reactor design point that is characterized by greater latitude and flexibility in achieving a given performance level.
- The fundamental conclusion that the RFPR can operate efficiently and economically in the considerably simpler batch-burn (unrefueled) mode remains unchanged if not firmer.
- The largest uncertainty associated with the RFPR prognosis rests with the assumed physics of particle/energy transport; this uncertainty can not be resolved by code development at any level-of-effort, although careful parametric studies can at least establish limits for this uncertainty.
- The one-dimensional model has been developed and tested against the global, zero-dimensional model, rendering more confidence in each, within the uncertainties of the physics assumptions. The more elaborate and flexible one-dimensional model can now be used to examine effects and parameter

variations heretofore inaccessible by the zero-dimensional model. For example:

- alpha-particle thermalization, transport, orbit effects, losses, etc.
- impurity transport
- better evaluation of models that couple particle/energy/field transport and the local state of MHD stability
- model RFPR startup (low-beta, field reversal) and shutdown (field energy recovery, plasma cooling) processes
- model refueling and ash-removal schemes that would be required for sustained thermonuclear burns

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