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**RADIALLY RESOLVED SIMULATION OF A HIGH-GAIN
FREE ELECTRON LASER AMPLIFIER**

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Introduction

The tapered wiggler Free Electron Laser (FEL) amplifier has been proposed as an efficient device to convert the directed energy of a highly relativistic electron beam into coherent, short wavelength photons. To date, there have been many investigations,¹ both analytical and numerical, of the physics of FEL amplifiers, but such work in general has neglected or only roughly approximated two-dimensional effects (e.g. diffraction). In those few instances where the laser field has been radially resolved, the transverse motion and energy spread of the electron beam particles has been ignored. Here we present the results of a numerical model of an FEL amplifier in which both the laser beam and electron beam are fully resolved.

A number of two-dimensional effects must be considered in an FEL amplifier design. In many tapered wiggler designs, the wiggler length may be a half dozen or more Rayleigh lengths, Z_R , and thus diffractive losses will play an important role. As the laser beam propagates in the longitudinal direction, Z , the beam spreads out radially, thus lessening the depth of the ponderomotive well (or "buckets") in which trapped electrons execute their synchrotron motion. Furthermore, as the laser beam diffracts, its phase front will become curved. For some particles in highly elliptical betatron orbits, this curvature may lead to detrapping if the betatron frequency, k_β , is approximately half the synchrotron frequency, Ω_s .²

In an amplifier with strong gain over a Rayleigh length the above effects are reduced because core losses can be "replenished" and phase curvature minimized if the relative intensity gain decreases in the radial direction. In the limit of very strong gain on axis, these phase changes can be sufficient to lead to a self-focusing of the laser and possible detrapping.³

Radial resolution of the laser intensity and phase also allows us to consider different input profiles for the laser. For example, designs in which the laser beam comes to a focus one Rayleigh length or so in the wiggler may minimize deleterious curvature effects. Sharper than Gaussian or annular profiles may maximize conversion efficiencies in some situations. Although it may be possible to approximate such designs in a 1-D code, a 2-D code permits detailed study of the various tradeoffs involved.

In addition to the transverse gradients in the laser, there are important transverse variations in both the wiggler field and the electron beam. The increase of the magnetic field strength off-axis directly leads to a radial increase in the resonant particle energy, $\gamma_r mc^2$, where

$$\frac{\lambda_w}{2\gamma_r^2} (1 + a_w^2 + p_\perp^2) = \lambda_s \quad (1)$$

Here λ_s and λ_w are the laser and wiggler wavelengths, a_w is the wiggler vector potential in units mc/e , and $p_\perp$ is the transverse particle momentum in units mc . In a tapered wiggler design in which a_w is decreasing or γ_r is increasing, Eq. (1) can be satisfied at only one radius for a given Z , γ_r , and betatron orbit ellipticity.

The radial fall-off of the laser field intensity, when convolved with the electron beam profile and transverse energy spread, leads to a strong radial variation in the trapped current density. Since the source term for the laser field is directly proportional to the trapped current density, these variations will produce in general a non-gaussian profile beam and may also enhance self-focusing.

A mitigating factor to the above will be the betatron motion of the individual particles which, like diffraction, tends to "wash out" strong local radial gradients. Simultaneously, the rms beam radius will adiabatically

increase with γ in a tapered wiggler if the ratio a_w/γ decreases in the absence of external focusing. This increase may lead to detrapping and a lower conversion efficiency.

To study all of the above effects, we have developed a 2-D FEL amplifier computer code. In the remainder of this paper, we discuss the relevant physics and approximations employed in the code, and then report some numerical results concerning the sensitivity of moderate and high gain amplifiers to phase noise in the input laser field.

Simulation Code Description

At present our 2-D simulation code models helical tapered wigglers with constant λ_w . Our assumptions include (a) complete axisymmetry; (b) a monochromatic laser field; (c) all field and source quantities are time-independent (at a given Z); (d) all source terms change slowly in Z relative to a wiggler wavelength (i.e., $k_w \gg \Omega_s, k_B$); (e) all field quantities change slowly in Z relative to an optical wavelength (i.e., paraxial approximation or slowly varying envelope approximation); (f) the electron beam is highly relativistic, i.e. $\gamma^{-2}/2 \ll 1$; (g) the wiggler field contains only the fundamental mode at spatial frequency k_w and increases off-axis as the function $(1 + k_w^2 r^2/4)$; (h) Dirichlet boundary conditions are adopted for the laser field at $r = r_{\max}$ with $r_{\max}$ generally $\simeq (25 r_{\text{beam}})$.

As mentioned above, the laser field is propagated in Z under the slowly varying envelope approximation. The field is defined at discrete grid points and the parabolic equations for the real and imaginary components are numerically integrated by means of a Gear-type scheme.⁴ For error control set at 10^{-3}

or less, we have found that total (i.e., particle + field) energy conservation is significantly better than 1% over runs of many Rayleigh lengths. The code also has the capability to employ the moving finite element (MFE) method⁵ which was adapted to cylindrical symmetry by the last two authors. To date we have found that most FEL problems do not require the higher resolution obtainable by the MFE method and that only for extreme cases (e.g., Fresnel diffraction) in which strong transverse gradients appear does a moving grid give significantly greater accuracy. For moderate gain amplifiers, sharp gradients tend to diffuse radially and disappear before the electron beam particles can respond strongly.

In order to determine the source terms for the field equations, our simulation code tracks $O(5000)$ or more electron beam macroparticles in energy, γ , and relative phase, ψ . The equations of motion are those of [2] with the longitudinal electric field set to zero:

$$\frac{d\gamma}{dz} = - \frac{e_s a_w}{\gamma} \sin \psi \quad (2)$$

$$\frac{d\psi}{dz} = \frac{d\phi}{dz} + k_w - \frac{k_s}{2\gamma^2} (1 + a_w^2 + a_s^2 - 2a_s a_w \cos \psi) \quad (3)$$

Here e_s and a_s are the normalized laser field and vector potential while ϕ is the laser phase.

We advance these equations with an implicit ODE solver with variable step-size that is run in conjunction with the field solver in a predictor-corrector fashion. For each z -step, both solvers are called at least twice in order to ensure good convergence. For problems with little gain and with relative error controls of $\sim 3 \cdot 10^{-4}$, the code generally uses step-sizes $\geq 10^{-2} Z_R$. For higher gains (e.g., greater than one e-fold per Z_R), the

step-size is a few 10^{-3} ω_R and typical simulations require a couple minutes CPU time on a Cray-1.

For the runs discussed here, no particle betatron motion is explicitly included; hence each particle travels at constant radius. However, to model partially the expected effects of betatron motion, the particles are initialized at $z = 0$ with a finite longitudinal energy spread corresponding to the change in resonance energy, γ_r , from the axis to the beam radius:

$$\Delta\gamma \equiv \gamma_r(0) \cdot \frac{b_w^2 r_b^2}{4 (1 + a_w^2)} . \quad (4)$$

This initial energy spread is uniformly spread among the particles, independent of radius.

We have found that using ≥ 64 macroparticles per grid point gives a sufficiently low noise level for the field source terms which depend upon the averages of $\sin \psi/\gamma$ and $\cos \psi/\gamma$. For most runs, we actually use 256 macroparticles/grid point which gives good statistics even for very low trapping efficiencies.

As in most large simulation codes, diagnostics play a critical role in understanding the results. In addition to the normal plots of field and particle quantities versus r and Z , phase space (γ, ψ) plots of particle positions at a given r and Z give an indication of "bucket" size and where detrapping occurs. Histograms of the final γ show the overall trapping efficiency and can also be radially resolved to show where the trapping is most effective.

General Results

Over the last few months, we have used the simulation code to study various transverse effects in moderate and high gain amplifiers, concentrating mainly upon 10.6 μ and 0.5 μ laser wavelengths and multi- Z_R wiggler lengths. From these runs, we have come to the following general conclusions:

- (a) For both moderate and high gain parameters, the output laser radial profile is of extremely high quality, both in intensity and phase. This high quality occurs naturally due to diffraction which smooths over local discontinuities.
- (b) Once particles are trapped in the ponderomotive well, they generally remain so until either b_w approaches zero or diffractive losses strongly reduce e_s . The latter effect often lends to detrapping first occurring on-axis rather than at the e-beam edges.
- (c) We design our wiggler taper such that a "resonant" particle at $r = r_{\text{design}}$, $\gamma = \gamma_r$, and $\psi = \psi_r$ will decelerate at constant relative phase (i.e., $d\psi/dZ = 0$). We have found that our most efficient designs have $\psi_r = 0.3$ to 0.4 and $r_{\text{design}} = 0.6$ to $0.8 r_{\text{e-beam}}$. Choosing a larger ψ_r speeds up the deceleration but leads to a much reduced trapping efficiency, as does picking a smaller r_{design} . Although off-axis designs sometimes lead to a local minimum in trapping efficiency on-axis, diffraction fills in the laser beam and prevents any strongly annular profiles from developing.

- (c) Based upon a limited parameter survey, we achieve optimum gain with a laser beam waist radius $\sim 0.8 r_{e\text{-beam}}$ occurring $\sim 0.5 Z_R$ inside the wiggler. The delayed focus seems to allow the electrons time to bunch before beginning full deceleration and also lengthens the duration in Z of maximum laser intensity.
- (e) Much sharper than gaussian radial profiles (i.e., parabolic) for either the electron beam or laser beam improves gain. Final laser profiles in these cases are still fit well by gaussians.

Sensitivity To Noise

Strong amplification of input laser noise by high gain FEL amplifiers might severely limit their usefulness. We have begun a sensitivity study of this problem by initializing our input laser profile as follows

$$\phi(r, Z = 0) = \phi_0(r) + A_N \sin(2\pi r/\lambda_N), \quad (5)$$

where $\phi_0(r)$ is identically zero at the laser focus. If this phase modulation strongly grows, one would expect serious detrapping of particles executing full betatron motion in the curved ponderomotive wells. The lower the gain, the less danger the noise should present because of smoothing by diffraction.

We ran the simulation code with gaussian laser and parabolic e-beam profiles employing approximately 30 grid points within the laser's interior. The wiggler magnetic field strengths were taken from another, simpler 2-D design code. Both the high gain and moderate gain runs used

$r_{\text{laser}} = 1.8 \cdot 10^{-3} \text{ m}$	$\gamma_0 = 614.5$
$r_{\text{e-beam}} = 2.12 \cdot 10^{-3} \text{ m}$	$\Delta\gamma = 3.35$
$\lambda_w = 0.08 \text{ m}$	$Z_R = 20.36 \text{ m}$
$\lambda_s = 0.5 \mu$	

with the input laser intensity determined by the emittance of the e-beam. The current density in the high gain runs was approximately 20 times larger than in the moderate gain ones.

Table 1 lists the output characteristics of moderate and high gain runs. Despite quite large modulation of the input laser phase, the final output intensity only decreased by $\sim 30\%$ in the worst case. For the moderate gain runs, the final Strehl ratios⁶ are 0.98 or better, as compared with initial ratios which are lower than 0.6 (see Figs. 1-3). The noise-induced detrapping is strongest at the beam edges where the ponderomotive well is shallow and where a slight reduction in gain can have a large effect as shown by the differences between the 2-D histograms of Figs. 4 and 5.

For the high gain runs, the final Strehl ratios are lower but still indicate quite high quality. The output intensity is more sensitive to the noise wavelength λ_N , probably because the higher Ω_s in these runs allows the particles to couple more strongly to the noise before diffraction destroys the phase modulation. For the last high gain run in Table 1 in which $\lambda_N = r_{\text{e-beam}} = 2.12 \cdot 10^{-3} \text{ m}$, only the interior portion ($r < 0.5 \lambda_N$) of the e-beam became strongly trapped. Because this portion saw a smooth, half-sine wave variation in $\phi(0)$, the output Strehl ratio was quite high compared to the other high runs. In the limit of $\lambda_N \rightarrow \infty$, the e-beam will see a constant phase and the particles should be unaffected by the noise. In the limit $\lambda_N \rightarrow 0$, diffraction should quickly destroy the phase modulation and there should be little coupling except in cases of very high gain.

Table 1. Output characteristics of moderate and high gain runs.

A_N	λ_N	Gain	% Trapped	Final Strehl Ratio
<u>Moderate Gain ($4Z_R$ propagation)</u>				
0.0	-	245	81	0.9939
1.0	$0.7 \cdot 10^{-3}$ m	220	68	0.9937
1.0	$1.4 \cdot 10^{-3}$ m	192	63	0.9904
<u>High Gain ($0.5 Z_R$ propagation)</u>				
0.0	-	1780	75	0.914
1.0	$0.7 \cdot 10^{-3}$	1675	70	0.928
1.0	$1.4 \cdot 10^{-3}$	1710	74	0.874
1.0	$2.12 \cdot 10^{-3}$	1195	45	0.956

In summary, our preliminary numerical results indicate that FEL amplifiers are reasonably insensitive to input phase noise on the laser, and may actually "clean up" such noise under conditions of moderate and high gain.

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Figure 1. The final laser phase profile and the best-fit curve for spherical aberration for a moderate gain simulation run with no input phase noise. The final Strehl ratio of 0.994 indicates a nearly perfect quality beam. The ordinate is in radians.

Figure 2. The input laser phase profile for a moderate gain run with $\lambda_N = 1.4 \cdot 10^{-3}$ m and $A_N = 1$. The low Strehl ratio should be compared with the final Strehl ratio of Fig. 3.

Figure 3. The final phase profile of a moderate gain run with $\lambda_N = 1.4 \cdot 10^{-3}$ m. The large Strehl ratio indicates the phase noise has nearly completely disappeared.

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Figure 4. A (γ, r) contour plot of the final energies of the particles for the moderate gain run of Fig. 1 with $A_N = 0$. A contour level of 0.8 at a given γ and r signifies that 80% of the particles at this radius have final energies below γ .

Figure 5. Same as Fig. 4 but with $\lambda_N = 1.4 \cdot 10^{-3}$ m and $A_N = 1$. Note that the contour level of 0.6 has withdrawn to $r = 1.1 \cdot 10^{-3}$ m as compared with $r = 1.4 \cdot 10^{-3}$ of Fig. 4, indicating detrapping caused by the input laser phase noise.

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