MASTER

Inelastic Strong Interactions at High Energies

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N O T I C E

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Abstract

We have begun investigations in the area of Grand Unified Field Theories. We have studied various ways of breaking the $SU(5)$ symmetric theory of Georgi and Glashow. As usual, a 24 of Higgs breaks the symmetry from $SU(5)$ to $SU(3)_C \times SU(2) \times U(1)$. We have found that a 45 of Higgs is acceptable for breaking the symmetry from $SU(3)_C \times SU(2) \times U(1)$ to $SU(3)_C \times U(1)_{em}$. In addition we obtain phenomenologically correct quark-lepton mass ratios using renormalization-group techniques if there are 6 generations of particles in the theory.

Also our efforts directed at the development of approximate methods for extracting information from quantum field theories were continued.

The quantum mechanics of polynomial potentials, as a model for quantum field theories was investigated. A perturbation expansion for the energy levels and wave functions was constructed and has been proven to be convergent for arbitrary values of the coupling constants, in contrast to ordinary perturbation expansions that have a zero radius of convergence. The physical significance of the new perturbation expansions was explored both in the weak and strong coupling limits.

Grand Unified Field Theories

We have begun investigations in the area of Grand Unified Field Theories (GUTS). These theories attempt to unify the $SU(2) \times U(1)$ electroweak theory of Weinberg and Salam with the presumed $SU(3)$ color theory of the strong interactions, Quantum Chromodynamics. There are a number of possible unifying symmetry groups one could choose which contain $SU(3) \times SU(2) \times U(1)$ as a subgroup, the simplest of which is $SU(5)$ proposed by Georgi and Glashow.¹ Symmetry breaking in this $SU(5)$ theory has been intensively studied.^{2,3,4} At energies above the grand-unification mass, $E \sim 10^{16}$ GeV, the theory is symmetric, but as the energy is lowered the symmetry is broken, first to $SU(3) \times SU(2) \times U(1)$ near $E \sim 10^{16}$ GeV and then to $SU(3) \times U(1)_{em}$ near $E \sim 10^2$ GeV. The symmetry is broken by the Higgs mechanism, a $\underline{24}$ of Higgs, Φ , initiating the first breaking and then, conventionally, a $\underline{5}$, H , initiating the second breaking.

The masses of each generation of leptons and quarks are generated by the Yukawa couplings and certain mass ratios (m_e/m_d , m_u/m_s , m_τ/m_b) are easily calculated in the symmetry limit. Using renormalization-group techniques, these mass ratios can be calculated at low energies, $E \sim 10$ GeV, where reasonable phenomenological estimates of the ratios can be made, at least for the heavy particle ratio, m_τ/m_b . m_τ is the mass of the τ lepton and m_b is the mass of the bottom quark. The mass ratio at 10 GeV is a function of both the representation chosen for the H bosons and of the number of generations in the electroweak theory.

In Ref. (5) we have investigated this representation dependence in the $SU(5)$ theory of Georgi and Glashow. If the $\underline{5}$ representation is chosen for H , then $m_\tau/m_b = 1$ in the symmetry limit and one obtains

acceptable results for m_τ/m_b at $E \sim 10\text{GeV}$ if there are three generations.^{3,4} By examining the representation content of the Yukawa couplings of the fermions one can see that both a 5 or a 45 of Higgs will give mass to the fermions. If the theory is to be "natural" only one representation of Higgs is allowed⁶ and we chose to examine the consequences of a 45. The $SU(3) \times SU(2)$ content is acceptable. There is only one doublet of color-singlet states; the rest have color. The color-singlet doublet is to be identified with the Higgs doublet in the Weinberg-Salam model.

The complete Higgs potential with both a 24 and a 45 was studied in the tree approximation. It was determined that a range of parameters of the potential exists which allows for the correct symmetry breaking pattern; $SU(5) \rightarrow SU(3)_C \times SU(2) \times U(1) \rightarrow SU(3)_C \times U(1)_{em}$. The m_τ/m_b mass ratio in the symmetry limit was found to be 3 and was calculated at 10GeV using renormalization group techniques as a function of the number of generations. We obtained acceptable results for the mass ratio if 6 generations are included. For 5 generations the ratio is too small and for 7 generations the ratio is too large. There exists of course, the upper limit of 8 generations (number of flavors $< 33/2$) from the asymptotic freedom constraints of QCD.

The choice of a 45 for H has therefore been shown to be as acceptable as the conventional choice of a 5 within the domain of particle physics. A possible way of determining which representation of H is favored comes from cosmological arguments concerning the connection between the present relative helium abundance in the universe and the possible number of zero-mass neutrinos.⁷ The helium abundance limits the number of zero-mass neutrinos and therefore the number of generations to 3 or 4. If these

arguments are correct then the 5 of H would be favored.

The accompanying proposal describes the research we intend to carry out in the area of symmetry breaking in Grand Unified Field Theories.

New Perturbation Expansions for Polynomial Potentials

Ordinary perturbation expansions for theories with polynomial interaction terms always have a zero radius of convergence.^{8,9,10} Therefore, it has been impossible to extract meaningful information from quantum field theories (or quantum mechanics) in the medium or strong coupling regions by perturbation expansion alone.

We proved^{11,12} that in the quantum mechanics of polynomial potentials it is possible to split the Hamiltonian into free and interacting parts in such a way that a) the free part is exactly soluble, b) the interacting part is simple in terms of the eigenfunctions of the free part, i.e. in finite orders of the perturbation expansion only a finite number of easily calculable terms appear, c) the perturbation expansion is convergent for every physically admissible value of the coupling constants. The split is usually not unique and auxiliary parameters can be introduced. Naturally the sum of the perturbation series does not depend on these parameters but the rate of convergence of the series does and general considerations may allow one to pick optimum values of the parameters which would yield the fastest convergence.

A simple example is a perturbation series for the quartic oscillator

$$H = p^2 + x^4,$$

where

$$H_0 = (x^2 + p^2/\Omega^2)^2, \quad H' = p^2 - p^4/\Omega^4 - (x^2 p^2 + p^2 x^2)/\Omega^2,$$

and Ω is an auxiliary parameter.

In all the examples we investigated the convergence to the correct energy values (that had been numerically evaluated by the WKB method¹³)

was fast enough so that in third order of the perturbation expansion the error was less than one percent.¹¹

We investigated the convergence of our perturbation expansion by two different methods. We used recursion relations for the expansion coefficients to obtain an estimate of the Nth term for the Brillouin-Wigner series and the functional integral method of Lipatov^{9,10} to estimate the behavior of the Nth coefficient of the Rayleigh-Schroedinger series, E_N . The exact classical solution that dominates the functional integral (instanton) was found and the result was identical to that obtained for the Brillouin-Wigner series, namely we obtained the following asymptotic form of E_N for a quartic anharmonic oscillator of coupling strength g (choosing the "mass", the coefficient of the quadratic term, to be 1)¹²

$$E_N \sim \exp \left\{ - N^{3/7} g^{-4/7} (3/8)^{4/7} 7/3 \right\}. \quad (1)$$

Different choices of the free parameters of our perturbation expansion were used to investigate the weak and strong coupling limits. We could obtain a series that agreed asymptotically with the ordinary perturbation expansion up to order $M \sim 1/g$ (more exactly our expansion has the property that if its first L terms are expanded in a power series of g then the first L terms of the power series agree exactly with the first L terms of the ordinary perturbation expansion, i.e. expansion in powers of g), but while the divergence of the ordinary perturbation expansion becomes manifest above $N=M$, our expansion becomes strongly convergent for the same N values. One can see from Eq. (1) that the contribution of terms with $N>M$ is $O(\exp \{ -1/g \})$.

Another choice of the parameters allows one to investigate the strong coupling region. Our perturbation series provides a convergent expansion for the Taylor expansion coefficients of the Symanzik scaling function that controls the asymptotic behavior as follows

$$E = g^{1/3} f(\omega^2/g^{2/3}), \quad (2)$$

where f is the scaling function and ω^2 is the coefficient of the quadratic term of the Hamiltonian.

Further possibilities for the application of our method include extensions to coupled anharmonic oscillators, anharmonic lattices and possibly to field theories of higher than 1 dimension.

The work described in the present section was performed in collaboration with I.G. Halliday while the principal investigator was a Senior Visiting Fellow at the Imperial College of Science and Technology, London, England.

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