DIRECT METHODS FOR SEISMIC PROFILING

by

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MASTER

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ABSTRACT

This group has been engaged in a coordinated research program in inverse problems over the period of this research contract. The program has evolved from formulation to analytical solution to implemented computer algorithms. There are two main lines of research in this program; a velocity inversion method, with application to seismic exploration, and a physical optics inverse scattering method for reflector mapping, with application to non-destructive evaluation. In each case, computer algorithms based on our theoretical results have been tested on real or testbed data from the area of the cited application. Research goals of both a theoretical and practical nature have been achieved with the aid of the support of this contract.

INTRODUCTION

Our research program under this contract has focused on two types of inverse problems, namely, the determination of velocity (or impedance) profiles in the earth and the imaging of reflectors. In both cases, our methods are soundly based on the governing equations of wave propagation. During this contract period, both techniques have evolved from theoretical formulation to practical computer implementation on real data sets. Progress on these two major programs will be described separately below. We remark, in passing, that during the past year, we have achieved the research goals stated in our proposal of a year ago.

We are pleased to report that our work has become widely recognized and is beginning to influence the research of other people. A list of the citations to our work on inverse problems is included in this report. In particular, we quote the following from the paper by Berryman and Greene:

"Clearly, it is desirable either to do migration without a predetermined velocity structure or to do inverse scattering while allowing the elastic parameters to vary horizontally. Much effort is currently being expended by many people to improve migration. However, relatively little effort is being directed towards practical applications of inverse scattering. *The work of Cohen and Bleistein (1979) is a notable exception.*" (Italics ours.)

We quote also from Weglein, Boyse and Anderson:

"A major achievement to the C & B [Cohen and Bleistein, 1979] approach is that their procedure has both clearly defined physical goals and clearly understood assumptions."

Finally, we remark that we have given a number of invited talks on our work at international meetings and University colloquia. These include colloquia in Departments of Electrical Engineering, Mechanical Engineering, Oceanography, Geophysics and Mathematics and also the following international meetings: The National Conference on Electromagnetic Scattering, the First International Symposium on Ultrasonic Materials Characterization, the International Meeting on Wave Phenomena in Oceanography, the International Meeting on Ocean Acoustics, and at the Society of Engineering Science Meeting. We have also spoken at two ARPA/AFML international meetings on non-destructive evaluation, the national meetings of IEEE, URSI, SIAM and SEG as well as the International Symposium on Ill-Posed Problems sponsored by the scientific offices of the Army and the Air Force.

Bleistein and Cohen have served on national panels, sponsored by both AFOSR and ONR. They have also been designated Class A consultants to the Office of Naval Research.

PROGRESS ON VELOCITY INVERSION

When a wave (acoustic, elastic, electromagnetic) propagates in a medium (such as the interior of the earth) it is distorted by the variations of the properties of that medium. The objective of this inverse problem is to determine these variations. The means of doing this is to observe the backscattered signals produced by known sources. We have derived an exact integral equation for the perturbation in velocity from some known reference level. This integral equation is non-linear because it contains a product of that perturbation with the wave field in the interior of the medium. That field is unknown, also, but satisfies a similar integral equation. In addition, these integral equations contain parameters which serve to characterize any desired source-receiver configuration.

For perturbations small enough to allow linearization of the system, we derive a linear integral equation for the perturbation in soundspeed. This equation is a Fredholm integral equation of the first kind. Unlike many first kind equations, the equation here is *not ill-conditioned* (the range space is non-compact). We have demonstrated that this theoretical continuous problem can be solved in a well conditioned way in the context of the real data restrictions: discretization, noise and limited bandwidth, spatial, and temporal range.

For the case in which the reference speed is taken as constant, we have an analytic solution of the linearized problem for back-scattered observations [1, 2]. This solution provides a fully three dimensional profile of the subsurface soundspeed variations when a two-dimensional array of observations on the upper surface is provided. Such grids of observations are rarely available at the present time. Thus, in practice, a line of observations is taken and it is implicitly assumed that the soundspeed varies only with depth and transverse direction of the array. Our analytic solutions still applies, with some simplification, under this assumption.

Computer implementations of our analytic solutions have been developed, taking into account the real world constraints mentioned above. In addition, it is typically the case that the low frequency data is absent. This restriction necessitates an additional *a priori* assumption about the medium. We have demonstrated that the particular assumption of a piecewise-constant, i.e., (non-horizontally) layered medium suffices to allow the determination of the profile in a well-conditioned manner. Our method is distinguished from other state-of-the-art techniques (such as migration) in that it provides estimates of the increments in velocity across the interfaces between layers, as well as a mapping of those interfaces.

Figures 1 - 18 demonstrate our computer implementation for the case of a linear array. Figure 1 shows an inclined plane for which synthetic data was generated for values of θ equal to 15° , 30° and 45° . Figures 2, 3 and 4 show the results of inversion. Figures 5 and 7 are synthetic reflection records for an anticline and a syn-

cline. Figures 6 and 8 are displays of the inversion output.

The Kirchhoff approximation was used to obtain modeling data for two cases in which diffraction effects are important, namely, a half plane and a buried-focus (i.e., a syncline whose center of curvature is below the observation surface.) The time records are shown in Figures 9 and 11, respectively, and the inversion mappings are shown in Figures 10 and 12, respectively.

A multi-layered synthetic was provided by the research group at Marathon Oil, to whom we are very grateful. They designed a subsurface configuration of velocity zones appropriate to a natural gas deposit. A two dimensional slice of that configuration is shown in Figure 13. Using their ray tracing program, the Marathon group generated backscattered data at eighty one stations spaced one hundred feet apart at the surface of the earth. They supplied us only this data, which is displayed in Figure 14; Figure 15 is our reconstruction of the subsurface medium. In performing this inversion we used a frequency range of 6 to 24 Hertz, which is typical for seismic profiling. In Figure 16, we show our estimates of various significant depths and velocities. The errors in the result vary between zero and four percent.

Recently, we have begun processing synthetics for the case of a surface array. Restrictions of cost for computer time and storage have forced us to confine our test to the two simple examples of a tilted plane and a spherical cap. The results of inversion are shown in Figures 17 and 18.

It should be noted here that in all cases except that discussed in detail above, the location and velocity increments produced by inversion were accurate to within 1%.

Over the past eighteen months, we have processed three sets of seismic data, two provided by Lamont Observatory and one by Marathon Oil. The results of processing that data are shown in Figures 19 - 23. Since this was not true amplitude data, true velocity increments cannot be directly measured. It is a virtue of our method that in such a case it still functions as a migration scheme, providing a subsurface map of the layers.

Figure 19 shows the results of the velocity inversion algorithm applied to the first data set provided by Lamont. The amplitudes of the traces are normalized to the peak amplitude over the entire data set. Since the ocean bottom reflection is at least five times as large as any other reflection, it dominates the output. The "ghost" of the ocean bottom arises from the signature of the source itself. The dimensions here are in hundreds of meters.

There is a hint of a reflector at the coordinates (-16, 31). In Figure 20, the data set from 3,000 meters to 4,500 meters is plotted, renormalized to its peak amplitude. The above mentioned reflector is clearly visible now, as well as its discontinuous extension down-dip to the right. With Figure 20 as a guide, the strongest part of this reflector may be seen in Figure 19.

The remaining two real data sets were received quite recently and analysis and processing of them has only begun. The preliminary output from the second tape provided by Lamont Observatory is shown in Figure 21. Finally, Figures 22 and 23 show the preliminary output from the data set provided by Marathon Oil Company.

For all three sets of data, the output is extremely smooth, demonstrating our claims above about the stability of our method.

Some comments about the computer processing are in order here. Our first program, designed to run on the Burroughs 6700 at the University of Denver, was admittedly crude and naive, with the run time proportional to the total number of input and output points. However, it did serve the purpose of confirming the validity of the method when applied to the earliest synthetics. In anticipation of receipt of the first tape from Lamont, we estimated the running time of our original algorithm. It was a totally unacceptable 4,000 hours!

We developed and debugged an entirely new and conceptually different computer algorithm to implement our theoretical result. The new running time estimate for the same data tape was 2.6 hours, 1/16 of the previous estimate!

At this time we moved our computing efforts outside of the University. We moved to a nearby CDC facility which, while not ideal, has provided us with vast improvements in turnaround time and reliable service when compared to the University computer. For the short term, this change set us back considerably, largely due to the necessity to change all our programs from Algol to Fortran. How-

ever, there is no doubt that our computer implementation has developed far more rapidly, now that the change has been accomplished.

On the CDC system, the time estimates for the above mentioned processing job was 10.7 minutes on the CDC 6600 and 4.3 minutes on the CYBER 76. Also, the cost per job has decreased dramatically, as well.

More recently, further improvements in the algorithm have been made, leading to even less processing time. The table below summarizes the processing time. As can be seen from the table, there is a pre-processing phase (Fast Fourier transform of the data set) followed by a processing step for which the time is proportional to the number of output data points. For the tapes Lamont 1 and 2, where more data points were processed in the same time, the improvement in processing time is readily apparent. The present rate of processing is approximately 2.3 msec/output data point.

For the first algorithm, processing time was recorded at a time per *total* number of data points (input plus output), at a rate of 5 msec/data point on the Burroughs 6700. While such a measure is not appropriate here because the algorithm is so different, we report it as a crude means of comparison. For the Marathon tape, which gives the largest time, the result is 4.3 nanosec/data point, approximately 1.1 million times faster.

Processing Time: CDC CYBER 76

Source tape	# of input data points	# of output data points	Preprocess time	Process time	Total time
Marathon	700,000	230,000	117	583	700
Lamont 2	600,000	68,000	78	154	232
Lamont 1	480,000	60,000	31	201	232

We have developed a much firmer (although not completely precise) grasp of the 'natural' parameters of the algorithm and the role they play in the accuracy and stability of our output.

Finally, as regards computation, we have made an important theoretical advance, which for real amplitude data, allows us to read velocity increments directly from the output. In contrast, the prior algorithm required a post-processing correction for the inclination of the reflecting surface. This, in turn was subject to the uncertainties of interpretation of the output in addition to the actual uncertainties of the output itself.

As satisfying as these results are, they are still limited to the case of constant reference speed. In many cases, this is not a sufficiently good approximation to support the subsequent linearization. With this as motivation research on inversion with non-constant reference speed has recently been undertaken. Here, the integral equation does not admit a closed-form analytic solution and numerical methods must be employed. The problem is sufficiently difficult that we have returned to the one-dimensional case. Maintaining the well-conditioned nature of the continuous problem after discretization is more difficult

than in the constant coefficient case.

An important start on this objective has been achieved in [3, 4], with application to a wide class of integral equations, well beyond the scope of the motivating problem. The essence of the method involves a careful analysis of the governing differential equation. This analysis has led to a method of constructing 'adapted' (unequally spaced) meshes so that the resulting system is extremely well-conditioned. In fact, the matrices turn out to be nearly orthogonal and the numerical results behave very much like those obtained in comparable constant reference speed problems.

The adaptation scheme was motivated by the observation that the greatest difficulty with inversion arises from the high frequency part of the kernel and the data. Thus, the part of the discrete matrix corresponding to high frequency needs to be most nearly orthogonal. This can be achieved by observing that in the high frequency limit, the WKB approximation provides an adequate characterization of the solutions of the governing equation of wave propagation. Thus, the spacing of the adapted mesh is dictated by uniform increments of phase of the WKB solution.

The essence of the results, here, may be illustrated with the aid of Figure 24. The solid line represents a sample reciprocal soundspeed. Results of one dimensional velocity inversion with (+) and without (-) adapting are indicated in the figure. The improvement in accuracy is quite apparent.

Another phase of our research was the study of the role of the Kirchhoff approximation in the inverse problem. Since real data is subjected to natural filtering processes such that only the 'high frequency' (in dimensionless variables) information reaches the receivers, the question posed was, "Can one deduce a high frequency inversion formula starting from a high frequency, i.e., Kirchhoff model, for direct scattering?" This turned out to be indeed possible, but the result was the same as would be obtained by filtering the data only while leaving the kernel of the wide-band integral equation unchanged. Because the kernel is not simplified, this Kirchhoff based derivation of the integral equation does not lead to a distinct inversion theory. However, it does have important implications for synthetic modeling. That is, in order to test inversion theories, it is necessary to generate scattering data which adequately models real world data, i.e., it should be high frequency band-pass data. The above cited result shows that when data of this type is derived, it suffices to use the Kirchhoff approximation in solving the direct scattering problem. We have taken advantage of this fact in producing the data for Figures 9 and 11. This work was described in [5].

Another frequently employed approximation, both in direct and inverse seismic problems, is the parabolic approximation. In recent years, the work on Jon Claerbout [6] has made this approximation so popular that its use is known as wave equation migration. Starting from the parabolic approximation and using constant reference speed, we produced an inversion formula and computer algorithm for which a few synthetics were done. Figures 25 and 26 repeat Figures 2 and 3

for this algorithm demonstrating the now discernable errors in geometry by including solid lines at the true positions of the layers.

The algorithm for this inversion is identical in structure to that based on the wave equation itself. Hence, the conclusion of our research is that the parabolic approximation affords no advantage in the seismic inverse problem. A paper based on this research appeared in Geophysics [7].

PROGRESS ON REFLECTOR MAPPING

When the objective is to locate and describe one (or a few) reflectors in an otherwise homogeneous medium, another type of inversion procedure is applicable. This is a high frequency method based on the Kirchhoff approximation in the farfield limit. The basis of the method is an identity relating the geometry of the reflector to the high frequency farfield scattering data. This identity is known as the Physical Optics Far Field Inverse Scattering Identity and is denoted by the acronym POFFIS.

In the earliest version [8 - 18], the geometry was identified by the characteristic function of the reflector (which, in that result, had to be assumed to be a closed domain). The characteristic function of the scattering domain is equal to unity in that domain and zero outside. The POFFIS identity related the Fourier transform of this characteristic function to the phase and range normalized far field scattering amplitude.

The frequency of the scattering data provides the magnitude of the spatial transform vector, while the angles of observation provide the directions of the spatial transform vector. In practice, the scattering data only provides information in a limited high frequency band and in some angularly restricted aperture. Furthermore, as a high frequency result, the POFFIS identity provides information of only questionable validity, at best, at low or moderate frequency.

In a series of papers [16 - 18], the theory of extracting partial information in a well-conditioned manner from this limited data set was developed. In essence, one can image that part of the reflector for which the specular returns are observed. Figures 27 - 33 are examples of reconstructions of all or part of the boundary of a region from Fourier data, frequency limited in all cases, angularly limited, as well, in Figures 28, 29, 30 and 32.

More recently, we have made a fundamental reassessment of this problem. Since it is the surface that is located, we developed an identity which directly relates the specular part of the surface to the scattering data [19]. This has the advantage that the theory is extended to open surfaces. In addition, it allows direct determination of the impedance of the reflector from inversion data. One can also observe in Figures 27 and 31 the phenomenon of 'image fading' which was a shortcoming of the earlier processing technique. This does not occur in the revised method.

This method is being used in an industrial project in non-destructive testing. In particular, backscattered data from an ellipsoidal void in a titanium sphere was provided by Rockwell International. An octant of data was provided and an octant of the surface of the void was produced - Figure 34.

For this first test on real data, the scatterer was, in fact a closed surface. However, in the near future, the method will be tested on scattering data from a crack, for which the new 'open surface' method should as well provide an image of the scattering

surface. It should be further noted here that this real data set severely tested the high frequency assumption of the basic identity. In the synthetics, the curvatures of the surfaces were all of order unity, so that the indicated values of magnitude of the transform vector (k) represent a measure of the dimensionless high frequency parameter. Typically, this was well above five. In preparation for the real data set, a computer algorithm was developed and tested in the range three to seven. In fact, for the real data set, the dimensionless 'high frequency' parameter ranged between 1.2 and 7!

The above cited paper on this new POFFIS identity for scattering surfaces also states a result for the case in which the scatter is not restricted to the far field. This result also has application to non-destructive testing in the case where flaws near the surface of a solid are to be detected.

It is gratifying that theoretical work supported by DOE has found practical application in this important field of nondestructive testing.

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FIGURE 1

FIGURE 2

FIGURE 3

DEPTH IN KILOFEET

0.0 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0

0.0 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0

DEPTH IN KILOFEET

FIGURE 4

FIGURE 5

FIGURE 6

FIGURE 7

FIGURE 8

FIGURE 9

FIGURE 10

FIGURE 11

FIGURE 12

ACTUAL MODEL

FIGURE 13

FIGURE 14

FIGURE 15

COMPUTED MODEL

FIGURE 16

FIGURE 17

FIGURE 18

EAST PACIFIC RISE-LINE 17

DEPTH SECTION 2100-4500 METERS

FIGURE 19

EAST PACIFIC RISE-LINE 17

DEPTH SECTION 3000-4500 METERS

FIGURE 20

SECOND LAMONT TAPE

DEPTH SECTION 4300-7000 METERS

CDP NUMBERS 201-301

FIGURE 21

SECOND MARATHON TAPE

LEFT HALF

DEPTH SECTION 100- 10000 FEET

FIGURE 22

SECOND MARATHON TAPE

RIGHT HALF

DEPTH SECTION 100-10000 FEET

FIGURE 23

FIGURE 24

FIGURE 25

FIGURE 26

$K_{\text{MIN}} = 10, K_{\text{MAX}} = 30, \text{RANGE} = 25$

FIGURE 27

$K_{\text{MIN}} = 10, K_{\text{MAX}} = 30, \text{ RANGE} = 25, \text{ ALPHA} = 120^\circ$

FIGURE 28

$K_{\text{MIN}} = 10, K_{\text{MAX}} = 30, \text{RANGE} = 28, \text{ALPHA} = 90^\circ$

FIGURE 29

$K_{\text{MIN}} = 10, K_{\text{MAX}} = 30, \text{RANGE} = 25, \text{ALPHA} = 60^\circ$

FIGURE 30

K Range 9-27

FIGURE 31

K Range 9 -27

Angular Range $0^{\circ} - 90^{\circ}$
 $180^{\circ} - 270^{\circ}$

FIGURE 32

K RANGE .5 - 3.5
FULL APERTURE IN θ

FIGURE 33

FIGURE 34

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FINANCIAL STATEMENT OF ESTIMATED COSTS

Special Research Support Agreement

University of Denver

EY-76-S-02-2482. *000

For the Period February 1, 1979 through January 31, 1980

1. Estimated Total Project Costs for the Current Period (Note A)	\$ 40,000
2. Total Amount Chargeable to DOE (Approved Support Percentage (100% of Item 1))	\$ 40,000
3. Cumulative Support Costs (Note C)	\$ 145,500
4. Estimated Total Cumulative Support Costs (Add lines 2 & 3)	\$ 185,500
5. Cumulative Support Ceiling (From Article III of the Contract)	\$ 185,500
6. Anticipated difference between the Estimated Total Cumulative Support Costs and the Cumulative Support Ceiling (lines 4 & 5) (Notes D & E) FREE SUPPORT	\$ <u>0</u>