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**A Simple Model for an Upper Bound
on Density Fluctuations in a Turbulent Pinch**

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A SIMPLE MODEL FOR AN UPPER BOUND ON DENSITY FLUCTUATIONS IN A TURBULENT PINCH

by

Abram R. Jacobson

ABSTRACT

A rough scaling relationship for plasma density fluctuations driven by magnetic field disturbances is derived. The result is found to agree with an earlier theory based on acoustic waves "inflating" magnetic flux surfaces.

We will estimate the density excursions due to a component $\tilde{B}_{||}$ of the fluctuating magnetic field* parallel to the unperturbed field $\vec{B}_0$. To first order, the fluctuating magnetic pressure gradient is

$$\nabla_{||} p_{\text{mag}} \sim k_{||} \frac{2 \tilde{B}_{||} B_0}{8\pi} ,$$

where $k_{||}$ is the parallel wave number of the magnetohydrodynamic (MHD) disturbance. This is rewritten as

$$\nabla_{||} p_{\text{mag}} \sim k_{||} \frac{\tilde{B}_{||}}{B_0} v_A^2 \rho_0 ,$$

where v_A is the Alfvén speed and ρ_0 is the unperturbed mass density. Introducing the MHD fluid velocity parallel to the field, $\tilde{v}_{||}$, the force equation to first order is

* The tilde ($\sim$) above a quantity denotes the rms value of the fluctuating (i.e., mean = 0) component of that quantity.

$$\rho_0 \tilde{v}_{||} \sim |v_{||} \tilde{p}_{\text{mag}}|$$

or

$$\tilde{v}_{||} \sim \frac{k_{||}}{\omega} \frac{\tilde{B}_{||}}{B_0} v_A^2, \quad (1)$$

where ω is the MHD wave's angular frequency.

The linearized continuity equation gives

$$\frac{\partial \tilde{\rho}}{\partial t} \sim k_{||} \rho_0 \tilde{v}_{||}, \quad (2)$$

where $\tilde{\rho}$ is the fluctuating mass density. Combining Eqs. (1) and (2) yields a simple relationship between the driving magnetic perturbation and the resultant density excursion:

$$\frac{\tilde{\rho}}{\rho_0} \sim \left(\frac{k_{||} v_A}{\omega} \right)^2 \frac{\tilde{B}_{||}}{B_0}. \quad (3)$$

The quantity $\omega/k_{||}$ is just the MHD wave's phase velocity parallel to the field. In an infinite homogeneous cold plasma, $k_{||} v_A/\omega$ ordinarily would not exceed unity (considering the magnetosonic wave dispersion relation). However, in a bounded, inhomogeneous, and tightly sheared pinch, it is not ruled out that turbulent MHD waves can have parallel phase speeds slower than v_A . In that case Eq. (3) indicates that the "shallow" magnetic field fluctuations could drive "deeper" fluctuations in the density. Physically, the reason for this is that the magnetic piston travels more slowly and hence can collect more of the plasma.

We can now estimate an upper bound on the density fluctuations, using Eq. (3). The rapid electron thermal conduction along field lines ensures that the temperature remains effectively constant over a distance $1/k_{||}$ during the wave disturbance. Sound waves transmit pressure along the field direction at the sound speed, v_s . Since the disturbance is isothermal along the field direction, the density perturbation (due to the MHD wave) is eroded by sonic flow on

the time scale of an acoustic transit time. Therefore the increase of density fluctuations (as the phase speed $\omega/k_{||}$ is reduced) should saturate when

$$\left(\frac{\omega}{k_{||}}\right)_{\min} \sim v_s .$$

Substituting this into Eq. (3) gives an upper bound on the density fluctuations

$$\left(\frac{\tilde{\rho}}{\rho_0}\right)_{\max} \sim \frac{v_A^2}{v_s^2} \frac{\tilde{B}_{||}}{B_0} .$$

Using

$$\beta \equiv 2 v_s^2 / v_A^2 ,$$

this reduces to

$$\left(\frac{\tilde{\rho}}{\rho_0}\right)_{\max} \sim \frac{2}{\beta} \frac{\tilde{B}_{||}}{B_0} . \quad (4)$$

It is interesting to note that this result agrees with Robinson's acoustic turbulence model,¹ wherein density disturbances are pictured as inflating the magnetic field, just opposite to the approach taken here. Pressure balance requires

$$\tilde{p}_{\text{fluid}} = \tilde{p}_{\text{mag}}$$

which, with $\gamma = 1$ (isothermal), gives

$$\left(\frac{\tilde{\rho}}{\rho_0}\right)_{\text{pressure balance}} = \frac{2}{\beta} \frac{\tilde{B}_{||}}{B_0} .$$

REFERENCE

1. D. C. Robinson and M. G. Rusbridge, "Turbulent Density Fluctuations in Zeta," *Plasma Phy.* 11, 73-100 (1969).