Calculation of Pressure Distribution in Vacuum Systems*
Using a Commercial Finite Element Program

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Abstract

The finite element method has proven to be a very useful tool for calculating pressure distributions in complex vacuum systems. A number of finite element programs have been developed for this specific task. For those who do not have access to one of these specialized programs and do not wish to develop their own program, another option is available. Any commercial finite element program with heat transfer analysis capabilities can be used to calculate pressure distributions.

The approach uses an analogy between thermal conduction and gas conduction with the quantity temperature substituted for pressure. The thermal analogies for pumps, gas loads and tube conductances are described in detail. The method is illustrated for an example vacuum system. A listing of the ANSYS data input file for this example is included.

I. INTRODUCTION

A good understanding of the pressure distribution within a vacuum system is essential to the economical selection and placement of pumps and chambers. As the complexity of the vacuum system increases, so does the complexity of the calculations required to characterize the pressure distribution.

For this reason a number of finite element programs have been developed to analyze the pressure in vacuum systems. In many cases existing commercial finite element programs can be used to analyze pressure distributions. When they are applicable, commercial finite element programs have the advantages of wide distribution and well developed user interfaces.

A commercial finite element program may be used for vacuum analysis in the high vacuum region. In the free-molecular-flow region an analogy can be made between gas flow and linear thermal conduction. One technique for implementing the analogy is described in this paper. It will be illustrated with an example for a simple vacuum system.

II. DEVELOPMENT

A. Basic Vacuum Relationships

In general the pressure distribution in a vacuum system is determined by the release, flow, and pumping of gas. For most systems, the pressure can be analyzed with the pumping equation:

$$Q = Sp + V\frac{dp}{dt} \qquad (1)$$

$$\text{where} \quad Q = gas\ sources\ (Torr\text{-}\frac{l}{s})$$

$$S = effective\ pumping\ speed\ (\frac{l}{s})$$

$$V = system\ volume\ (l)$$

$$p = pressure\ (Torr)$$

Using the pumping equation the pressure can be expressed as:

$$p = \frac{Q}{S} + p_0 e^{-(\frac{S}{V})t} \qquad (2)$$

The exponential decay term in (2) represents the pumping of "volume" gas within the system. After a sufficient period of time, this term can be neglected and the pumping equation reduces to:

$$p = \frac{Q}{S} \qquad (3)$$

The gas flow rate, or throughput, q , between two points can be expressed as:

$$q = C(p_2 - p_1) \qquad (4)$$
$$\text{where}\ C = gas\ conductance\ (\frac{l}{s})$$

In the free-molecular-flow region, C is independent of pressure.

DISCLAIMER

B. Thermal Analogy

For each quantity and relationship expressed in equations (3) and (4), a thermal conduction analogy exists. These vacuum/thermal analogies are summarized in Table 1.

Table 1 Vacuum - Thermal Relationships

p	Pressure	Torr	T	Temperature	°C
Q	Gas Sources	$\dfrac{Torr \times l}{s \times cm^2}$	$\ddot{q}$	Heat Sources	W/cm^3
C	Gas Conductance	l/s	kA/l	Thermal Conductance	W/°C

The analogy for a pump with a pumping speed S is a conduction element with a thermal conductance equal to S and with a temperature boundary condition, T = 0, at the free end.

III. IMPLEMENTATION

A. Basic Element

The two node conduction link is the most common element to use. The element has one degree of freedom, temperature, at each node. It can represent a linear temperature distribution. The input variables for the element are cross-sectional area, material thermal conductivity and volumetric heat generation rate. The element length is determined by the location of the two nodes. These quantities can be manipulated in a variety of ways to achieve the proper gas conductance and outgassing rate for the vacuum component it is representing.

B. Property Selection

The following is one typical sequence of steps for choosing the element thermal properties:

- Choose nodal locations corresponding to the physical length of the components being modeled. Thus, a plot of the model serves as a visual verification of the geometry.

- Set the volumetric heat generation rate equal to the outgassing rate of the vacuum component.

- Set the cross sectional area of the element equal to the surface area per unit length of the vacuum component.

- Calculate the material thermal conductivity so that the thermal conductance per unit length will be equal to the gas conductance per unit length.

The technique is quite straightforward for components such as long tubes where the aperture conductance is not significant. However for short tubes and closely spaced apertures, gas transmission probabilities should be considered when specifying the element properties.

C. Pumps

Any lumped pumps in the system can be modeled by a conduction link with a thermal conductance equal to the pumping speed. Temperatures at the free ends of the pump elements are specified as T = 0. The heat generation rate for pump elements should be zero.

D. Other Options

A variety of other thermal elements can be used to model features such as distributed pumps and concentrated gas loads.

E. Model Solution

The output from the model is temperatures at the element nodes. These temperatures are equal to the corresponding pressure at that point of the vacuum system.

IV. EXAMPLE

A. Description

The thermal analogy technique is used to evaluate the pressure distribution of the vacuum system shown in Fig. 1. The system consists of a 500 cm long circular tube with one change in cross section. Two pumps are attached to the tube. One pump is attached directly to the tube and the other is attached to a 100 cm long branch. The gas conductances are determined for air at room temperature. A constant thermal desorption rate of 3×10^{-12} Torr-l/cm² is used.
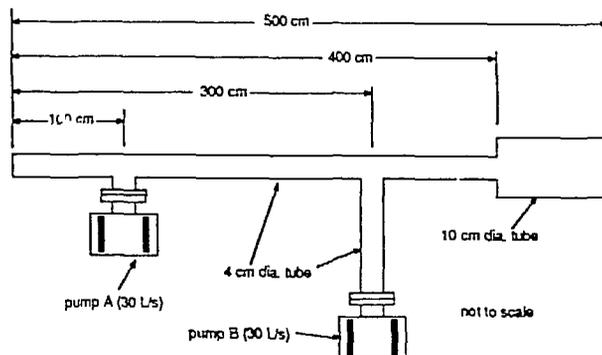

Fig. 1 Example Vacuum System

B. Vacuum Model

The element mesh for the model is shown in Fig. 2. A listing of the ANSYS [2] data input file appears in Fig. 4.
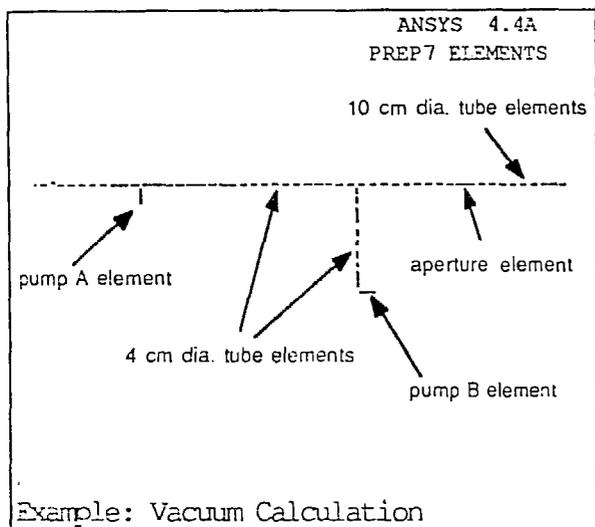

Fig. 2 Vacuum Model

C. Model Results

The pressure distribution in the 500 cm long tube is shown in Fig. 3. These results could be used to identify higher pressure regions and to indicate design changes to improve the overall system vacuum.

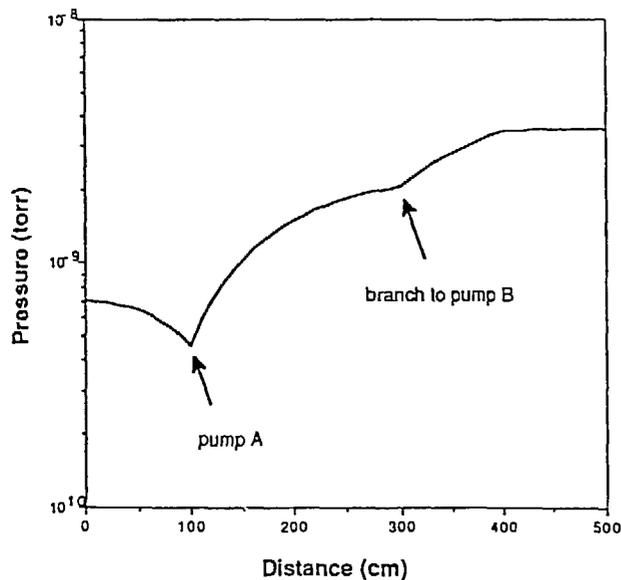
Fig. 3 Pressure in 500 cm Long Tube

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L,2,3,1 $L,5,6,1                                  *pump sections
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Fig. 4 Listing of Data Input File

V. CONCLUSION

The finite element method can be a valuable aid in the design of complex vacuum systems. In the high vacuum region an analogy exists between gas conduction and linear thermal conduction. In many situations a commercial finite element package with thermal analysis capabilities can be used to calculate the pressure distribution in a vacuum system.

VI. REFERENCES

[1] G. L. Weister and R. W. Carlson, *Vacuum Physics and Technology*, Orlando: Academic Press, 1979, pp. 6-23, 506-508

[2] ANSYS finite element package from Swanson Analysis Systems Inc., Johnson Road, P. O. Box 65, Houston, PA 15342, USA.