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Attenuation of Shock Waves in Copper and Stainless Steel

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ABSTRACT

By using shock pins, data were gathered on the trajectories of shock waves in stainless steel (SS-304L) and oxygen-free-high-conductivity copper (OFHC-Cu). Shock pressures were generated in these materials by impacting the appropriate target with thin (~ 1.5 mm) flying plates. The flying plates in these experiments were accelerated to high velocities (~ 4 km/s) by high explosives. Six experiments were conducted, three using SS-304L as the target material and three experiments using OFHC-Cu as the target material. Peak shock pressures generated in the steel experiments were approximately 109, 130, and 147 GPa and in the copper experiments, the peak shock pressures were approximately 111, 132, and 143 GPa. In each experiment, an attenuation of the shock wave by a following release wave was clearly observed.

An extensive effort using two characteristic codes (described in this work) to theoretically calculate the attenuation of the shock waves was made. The efficacy

of several different constitutive equations to successfully model the experiments was studied by comparing the calculated shock trajectories to the experimental data. Based on such comparisons, the conclusion can be drawn that OFHC-Cu enters a melt phase at about 130 GPa on the principal Hugoniot. There was no sign of phase changes in the stainless-steel experiments. In order to match the observed attenuation of the shock waves in the SS-304L experiments, it was necessary to include strength effects in the calculations. It was found that the values for the parameters in the strength equations were dependent on the equation of state used in the modeling of the experiments.

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INTRODUCTION

The physics of matter subjected to high pressure is an area of research that is of interest to many different branches of engineering, chemistry, and physics. High pressure experimental and theoretical research cover a very wide range of pressures from just a few tenths of a gigapascal (GPa)[†] in the study of pressure vessels, to pressures of nearly a hundred gigapascals in experimental work using high pressure anvils, to the extremely high pressures in the theoretical study of the death of a star.^{1,2} The present work is both an experimental and calculational investigation of the behavior of materials subjected to moderately high pressures (on the order of 100 GPa).

In the experimental work, these pressures were generated by strong shock waves and data were gathered on peak shock strengths and attenuation of these shock waves. Specifically, these experiments were designed to gather data relevant to the equations of state for oxygen-free-high-conductivity (OFHC) copper and a commercial grade of stainless steel. The stainless steel used in these experiments was SS-304L. The calculational effort centered on modeling the experiments and comparing the calculational results with the data to draw conclusions about the constitutive equations used in the modeling of the experiments.

[†] The unit of pressure in this work is the gigapascal (GPa). One pascal is equal to one newton per square meter.

Experiments involving pressures above ambient conditions have a long history. A brief but interesting historical account of high pressure experiments is given by Bridgman.³ The experiments referenced by Bridgman were all static experiments with maximum pressures ranging from .2 to .6 GPa. In this century Bridgman did some of the most significant work in high pressure experiments studying a wide range of physical phenomena. He routinely used pressures in the range of 1-2 GPa and achieved pressures as high as 10 GPa in the study of the compressibilities of solids. These experiments laid an important foundation for later work.

Experimental methods using shock waves to produce high pressures began to receive much attention with the advent of World War II. In this country much of this work was started at Los Alamos by Goranson and others.^{4,5} The mathematical framework needed for the understanding of shock waves was laid by Rankine, Hugoniot, Hadamard, and others.⁶ Following World War II, high pressure physics research continued with much emphasis placed on dynamic experiments using shock waves. In the 1950's the upper limit on the pressures that could be achieved in static experiments was around 10 GPa. The attractiveness of experiments using high explosives to generate shock waves was that a pressure regime nearly an order of magnitude greater than that in static experiments was open to researchers. An excellent review of the experimental shock wave work through 1979 is given by Davison and Graham.⁷ Before 1960, the majority of the experimental shock-wave

work was concerned with measuring quantities associated with the shock wave such as the speed of the shock wave, and the velocity imparted to the material by the shock wave. The need to better understand the behavior of material shocked to high pressures led investigators to measure other quantities beside the shock speed and particle velocity, such as sound speeds in the high pressure state and the decay of the shock wave. Among the earliest work along this line was that of Al'tshuler and others.⁸ Many others have performed experiments measuring similar quantities but using different techniques.

It was work along such lines that provided the impetus and inspiration for the present experimental work. By measuring the attenuation of a shock wave in a material, data are gathered on the sound speed and hence the compressibility of the material in a high pressure state. When such data are used in conjunction with calculational modeling of the experiments, inferences may be drawn about the equation of state and other aspects of material response. The present experimental data and calculational modeling indicate that material rigidity or strength is an important ingredient in describing the material response of stainless-steel to shock pressures of at least 150 GPa and probably higher. For copper, however, the present experiments suggest a purely bulk equation of state is adequate for modeling the material response of copper at shock pressures of 110 GPa. There is some indication that the copper may well be undergoing a phase transformation

to a liquid state in the experiments performed at still higher pressures.

Shock waves and fluid dynamics are intimately connected and chapter 2 lays much of the theoretical foundation needed for the understanding of shock waves and the interaction of shock waves with the flow equations of fluid dynamics. In chapter 3 the experiments are described in some detail. The manner in which the raw data are analyzed is also described in chapter 3. The calculational modeling of the shock-attenuation experiments is discussed at length in chapter 4. The implications provided by the comparisons of the calculational results with the experimental data for the physical models is also covered in chapter 4. Additional details about the codes used for the calculations and other aspects of the physics involved in the modeling of these experiments are given in the appendices.

THEORY

2A. Basic Assumptions

In working with materials that are dynamically shocked, four assumptions are made concerning the nature and structure of the shock wave and the state of the material behind the shock front. Before proceeding with any analysis, an elaboration of these assumptions is necessary.

The foremost assumption in the present investigation is that the shock front is infinitesimally thin, which implies that the shock process is discontinuous. In reality, the shock front has a finite thickness, but experimental estimates of the thickness of shock fronts have very small values. Experiments using laser-induced shocks in aluminum⁹ have shown the thickness of the shock front to be less than $0.7\text{ }\mu\text{m}$ at a shock pressure of 200 GPa. The very thin nature of the shock front supports the treatment of the shock wave as a discontinuity.

Another important assumption concerns the state of the material once it has been shocked. It is assumed that the material is in local thermodynamic equilibrium and may be described by a suitable equilibrium constitutive equation or equation of state. The basis for this assumption is partially drawn from the fact that pressure-volume data taken by shock-wave measurements extrapolates fairly well to lower pressure data taken statically.¹⁰

The last two assumptions are concerned with the nature of the flow behind the shock front. The first assumption is that the flow is adiabatic and this implies that heat conduction is being ignored. The whole process of shocking a material and then releasing down to ambient pressures occurs in less than $10 \mu\text{s}$ in these experiments. This is just too short a time scale for heat conduction to have any significant effect. Using even very high estimates of the thermal conductivity, such as $2 \times 10^{-9} \text{ ergs}/(\text{cm-s-K})$, the energy flow per unit area due to heat conduction is still at least three orders of magnitude lower than that associated with the flow equations given in section 2C. The second assumption pertaining to the flow is that viscosity effects may be safely ignored. The extremely small thickness of a shock front implies that the coefficient of viscosity is on the same order of magnitude as that for a fluid under normal conditions.¹¹ This in turn suggests that viscous effects may be neglected. The neglect of these effects will insure that the system of partial differential equations governing the flow will be a first-order system.

2B. The Rankine-Hugoniot Equations

The Rankine-Hugoniot equations^{10,12,13} relate kinematic and thermodynamic variables between an initial state and a final state of a material that has been shocked. Consider a planar shock traversing a material that is initially at rest. The shock wave will travel at a wave velocity, U_s , with respect to the unshocked

material. In the shocked state, the material acquires a particle velocity, u_p . Across the shock front conservation of mass, momentum, and energy must hold. These conservation laws lead to the Rankine-Hugoniot equations. The Rankine-Hugoniot equations are

$$\rho_o U_s = \rho(U_s - u_p), \quad (2.1)$$

$$P - P_o = \rho_o U_s u_p, \quad (2.2)$$

and

$$E - E_o = \frac{1}{2}(P + P_o)\left(\frac{1}{\rho_o} - \frac{1}{\rho}\right). \quad (2.3)$$

In these equations, ρ is the density, P is the pressure, and E is the specific internal energy. The zero subscripted variables refer to the material in the initial or unshocked state, while the unscripted variables refer to the material in the shocked state. If strength effects are important such that the material can support a deviatoric stress, then the component of the total stress that results in a force normal to the shock front replaces the pressure in equations (2.2) and (2.3).

Experimentally it is found that many materials obey the following empirical relation between shock velocity and particle velocity^{10,14,15}:

$$U_s = c_o + s u_p. \quad (2.4)$$

The parameters c_o and s are constants characteristic of the material being shocked. These parameters also depend on the initial conditions, but in general they are

insensitive to slight changes of the initial conditions. When equation (2.4) is used in conjunction with the Rankine-Hugoniot equations, a relatively simple expression can be written for the principal Hugoniot. The principal Hugoniot is the locus of all points in the P - ρ plane that can be reached by a single shock from the initial state P_o, ρ_o . With the use of equation (2.4), the expression for the pressure on the principal Hugoniot is

$$P = P_o + \rho_o c_o^2 \eta (1 - s\eta)^{-2}, \quad (2.5)$$

where

$$\eta = 1 - \frac{\rho_o}{\rho}. \quad (2.6)$$

Similarly the particle velocity and specific internal energy can also be expressed in terms of the density:

$$u_p = c_o \eta (1 - s\eta)^{-1}, \quad (2.7)$$

and

$$E = E_o + \frac{\eta P_o}{\rho_o} + \frac{1}{2} c_o^2 \eta^2 (1 - s\eta)^{-2}. \quad (2.8)$$

Frequently, the reference energy E_o is chosen to be zero, and the initial pressure term P_o is often ignored because it is anywhere from three to six orders of magnitude less than the pressures of interest in shock experiments.

2C. Flow Behind the Shock Front

After the shock front has passed a region of the material, this portion of the material is in a compressed state at high pressure or stress. Further flow occurs as the shocked region releases down to ambient pressure. This flow is always continuous provided the material is normal in the sense of the Bethe-Weyl conditions.¹⁶ There is the possibility of supporting a rarefaction shock if the material undergoes a marked phase change such that at a lower pressure the material can support a faster longitudinal or bulk sound speed than at a higher pressure; however, in this case the material is not normal in the sense of the Bethe-Weyl conditions. If the material does not exhibit this behavior, the whole release process will be continuous. Discontinuities may exist in the spatial gradients of the flow variables, but the flow variables such as pressure, density, energy and particle velocity will be spatially continuous behind the shock front.

The derivation of the flow equations proceeds from the consideration of a control volume and the conservation of mass, momentum, and energy. In the absence of body forces and dissipative mechanisms, the flow equations are

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} = 0, \quad (2.9)$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} + \frac{1}{\rho} \frac{\partial P}{\partial x_i} = 0, \quad (2.10)$$

and

$$\frac{\partial E}{\partial t} + u_i \frac{\partial E}{\partial x_i} + \frac{P}{\rho} \frac{\partial u_i}{\partial x_i} = 0. \quad (2.11)$$

The indices run from 1 to 3, and the summation convention is used for repeated indices. The term u_i is the material velocity in the i^{th} direction. For a material that can support a deviatoric stress, equation (2.9) remains the same, but equations (2.10) and (2.11) become

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} + \frac{1}{\rho} \frac{\partial \sigma_{ji}}{\partial x_j} = 0, \quad (2.12)$$

and

$$\frac{\partial E}{\partial t} + u_i \frac{\partial E}{\partial x_i} + \frac{\sigma_{ij}}{\rho} \frac{\partial u_j}{\partial x_i} = 0. \quad (2.13)$$

The term σ_{ij} is a component of the stress tensor. The sign convention has been chosen such that the sum of the principal stresses is positive in compression.

In the present investigation, the flow is essentially one-dimensional with no flow in the lateral directions. The stress tensor, when reduced to its principal values, will have one component that can be considered as a force per unit area normal to the shock front and two components that are tangential to the shock. From symmetry arguments, the tangential components must be equal. This greatly simplifies the flow equations. For plane flow in one dimension, the governing system of partial differential equations is

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0, \quad (2.14)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{\partial P}{\partial x} = 0, \quad (2.15)$$

and

$$\frac{\partial E}{\partial t} + u \frac{\partial E}{\partial x} + \frac{P}{\rho} \frac{\partial u}{\partial x} = 0. \quad (2.16)$$

If the material can support a stress, then P must be replaced in equations (2.15) and (2.16) by σ_n , the normal component of the stress tensor.

Equations (2.14)-(2.16) form a system of partial differential equations that are functions of two independent variables, x and t . Once an equation of state has been specified then finite differencing techniques can be applied to this system of equations to numerically solve for configurations later in time. A slightly different approach is to apply the method of characteristics^{17,18} to equations (2.14)-(2.16). The method of characteristics essentially transforms the system from three partial differential equations to a system of six first-order, ordinary differential equations.

In the method of characteristics, solutions to the flow are evolved along certain characteristic directions in the x - t plane. For equations (2.14)-(2.16), three characteristic directions are defined and they are

$$\frac{dx}{dt} = u, \quad (2.17)$$

$$\frac{dx}{dt} = u + c, \quad (2.18)$$

and

$$\frac{dx}{dt} = u - c. \quad (2.19)$$

Along each characteristic direction, a compatibility equation holds that specifies the manner in which the dependent variables must change. For the three characteristic directions that have been defined, the compatibility equations are, respectively,

$$dE = \frac{P}{\rho^2} d\rho, \quad (2.20)$$

$$dP = -\rho c du, \quad (2.21)$$

and

$$dP = \rho c du. \quad (2.22)$$

The quantity c in these equations can be identified as a bulk sound speed, and is defined by the following equation:

$$c^2 = \left(\frac{\partial P}{\partial \rho} \right)_E + \frac{P}{\rho^2} \left(\frac{\partial P}{\partial E} \right)_\rho. \quad (2.23)$$

Initial conditions or boundary conditions must be specified along a line in the x - t plane which is nowhere tangent to a characteristic direction in the domain of the problem. Numerical differencing techniques can then be applied to the system of equations (2.17)–(2.22).

If strength effects are to be modeled, the method of characteristics can still be used. The characteristic directions remain the same, but the compatibility equations are changed. Modeling of strength effects are explained in more detail

in section 2F. In appendix A, two computer programs using the method of characteristics to solve for the flow behind the shock front, as well as the trajectory of the shock front, are described.

There are many existing hydrodynamic codes that have been written for the problem of compressible fluid flow in one dimension. By and large the majority of such hydrodynamic codes use finite differencing techniques applied to the Lagrangian form¹⁹ of equations (2.14)-(2.16). Generally speaking, these codes use artificial viscosity techniques to follow a shock front. Artificial viscosity techniques do not solve the Rankine-Hugoniot equations in order to follow a shock. By using the method of characteristics, the Rankine-Hugoniot equations are solved exactly to relate the conditions at the shock front to the continuous flow immediately behind the shock front. It was for this reason that the computer programs described in appendix A were developed.

2D. Equations of State

In any quantitative modeling of shock wave attenuation, it will be necessary to have an equation of state giving the pressure as a function of the specific internal energy and density. The equation of state is important because wave speeds in the rarefaction region are determined by it. The basis on which an equation of state

may be constructed, as well as some specific equations of state, will be discussed in this section.

Knowledge of the qualitative nature of the structure of metals forms a basis from which an equation of state may be constructed. A simple approach is to consider the metal as a crystal lattice with the valence electrons forming an electron gas. In this approach, the interaction between lattice points and the free electrons is not explicitly taken into account. The total specific internal energy can then be written as the sum of three terms²⁰:

$$E = E_c + E_t + E_e . \quad (2.24)$$

The first term, E_c , arises from the potential of the crystal and is assumed to be a function of specific volume or density only. The zero-point vibrational energy of the lattice may be included in this first term, as well as any other contribution to the internal energy which can be assumed to be a function of volume only. The second term, E_t , is the contribution to the specific internal energy from thermal vibrations of the lattice. The last term, E_e , is the thermal contribution of the electron gas to the specific internal energy.

The assumption that the total internal energy can be written as the sum of three terms leads to a similar equation for the pressure. From thermodynamic theory, the contribution to the total pressure from the first term in equation (2.24)

is

$$P_c = - \frac{dE_c}{dv} . \quad (2.25)$$

In this equation v is the specific volume, and the total derivative of E_c is taken since E_c is assumed to be a function of specific volume only. Equation (2.25) is frequently called the cold compression curve since this is the pressure the material would exhibit along a zero-degree isotherm. If the temperature is not too great, then the electronic term in equation (2.24) may be neglected; but even if it is not neglected, the total pressure may still be written as

$$P = P_c(v) + P_t(v, T) . \quad (2.26)$$

In equation (2.26) T is the absolute temperature, and P_t represents all the thermal contributions to the pressure.

An interesting form for the equation of state can be derived by taking the partial derivative of equation (2.26) with respect to the temperature holding the specific volume constant:

$$\left(\frac{\partial P}{\partial T} \right)_v = \left(\frac{\partial P_t}{\partial T} \right)_v . \quad (2.27)$$

The left-hand side of equation (2.27) can be rewritten using the thermodynamic definition of the Gruneisen parameter and the definition of the specific heat at constant volume. The thermodynamic definition of the Gruneisen parameter²¹ is

$$\gamma = \frac{1}{\rho} \left(\frac{\partial P}{\partial E} \right)_v . \quad (2.28)$$

The defining equation for the specific heat at constant volume is

$$c_v = T \left(\frac{\partial S}{\partial T} \right)_v = \left(\frac{\partial E}{\partial T} \right)_v, \quad (2.29)$$

where S is the specific entropy. Use of these equations suggests that the equation for the total pressure may be written in the following manner:

$$P = P_c + \int \gamma \rho c_v dT. \quad (2.30)$$

If the Gruneisen parameter is assumed to be independent of temperature, then $\gamma \rho$ can be taken outside the integral in equation (2.30), and the equation becomes

$$P = P_c + \gamma \rho \int c_v dT. \quad (2.31)$$

Now the integral in equation (2.31) is nothing more than the difference between the total specific internal energy and the energy on the cold compression curve. Replacing the integral by this difference, the equation of state becomes

$$P = P_c + \gamma \rho (E - E_c). \quad (2.32)$$

Equation (2.32) is the form for the Mie-Gruneisen equation of state.^{10,13,21} To arrive at equation (2.32) it was necessary to assume that γ was independent of temperature. For a metal this implies that electronic effects are neglected, and that the metal remains in a single phase.

If E is known for a locus of states in the P - ρ or P - v plane, then equation (2.32) leads to a first-order differential equation for E_c . One such locus of states

that is known is the principal Hugoniot. If the material obeys a linear U_s-u_p relation, then equation (2.5) gives the pressure on the principal Hugoniot and equation (2.8) gives the specific internal energy. Assuming that E_o and P_o can be neglected, the differential equation for E_c is

$$\frac{\gamma}{v} E_c + \frac{dE_c}{dv} = \gamma \frac{c_o^2 \eta^2}{2(1-s\eta)^2} - \frac{c_o^2 \eta}{v_o(1-s\eta)^2}, \quad (2.33)$$

where η is given by equation (2.6). The functional dependence of γ on the specific volume must still be known before equation (2.33) can be integrated to find E_c ; however, it is not necessary to find E_c to arrive at an equation of state. Using equation (2.33), equation (2.32) can be written as

$$P = \frac{\gamma}{v} E_c + \frac{c_o^2 \eta}{v_o(1-s\eta)^2} - \frac{\gamma}{2v} \frac{c_o^2 \eta^2}{(1-s\eta)^2}. \quad (2.34)$$

The functional dependence of γ on v or ρ is still not specified in equation (2.34). The derivation of equation (2.34) is based on the following assumptions: the material obeys a linear U_s-u_p relation given by equation (2.4), electronic effects may be neglected, and the Gruneisen parameter depends only upon density or specific volume and is thus independent of temperature or specific internal energy.

A slightly different way of deriving equation (2.32) uses a more explicit treatment of a crystal as an assembly of harmonic oscillators. The Helmholtz free energy for an assembly of harmonic oscillators^{22,23} is given by

$$A = A_o + \frac{kT}{m} \sum_i \ln(1 - \exp(-\frac{h\nu_i}{kT})). \quad (2.35)$$

In this equation, k is Boltzmann's constant, h is Planck's constant, m is the mass, and ν_i is the frequency for the i^{th} normal mode of vibration. The term A_o is dependent on the specific volume only and is identical to the term E_c in equation (2.24). The sum is over all normal modes. The pressure is given by

$$P = -\left(\frac{\partial A}{\partial v}\right)_T = -\frac{dA_o}{dv} - \frac{1}{mv} \sum_i \frac{h\nu_i}{\exp\left(\frac{h\nu_i}{kT}\right) - 1} \frac{v}{\nu_i} \left(\frac{\partial \nu_i}{\partial v}\right)_T. \quad (2.36)$$

When the frequencies are independent of temperature, the specific internal energy is

$$E = A_o + \frac{1}{m} \sum_i \frac{h\nu_i}{\exp\left(\frac{h\nu_i}{kT}\right) - 1}. \quad (2.37)$$

Now make the assumption that all frequency changes vary in the same manner, and define the Gruneisen parameter by

$$\gamma = -\frac{v}{\nu_i} \frac{d\nu_i}{dv}. \quad (2.38)$$

Using equations (2.37) and (2.38), equation (2.36) may be written as

$$P = -\frac{dA_o}{dv} + \frac{\gamma}{v}(E - A_o). \quad (2.39)$$

This equation is in the same form as equation (2.32).

In deriving equation (2.32), one assumed that the Gruneisen parameter was independent of temperature. In deriving equation (2.39), one assumed that the normal frequencies were independent of temperature. A second assumption, that

the normal frequencies all varied in exactly the same manner with respect to changes in density or specific volume, was also needed. The thermodynamic definition of the Gruneisen parameter, equation (2.28), and the definition given by equation (2.38) are identical under these assumptions.

An interesting point to consider is the definition of the Gruneisen parameter given by equation (2.38) and the maximum frequency postulated in Debye theory. There are many different expressions relating the maximum or cutoff frequency in Debye theory to the Gruneisen parameter and to bulk properties.²⁴ Some of these theories are discussed in greater detail in appendix D. A relatively simple formula for the Gruneisen parameter can be found when it is assumed that the normal frequencies of a material are only linearly dependent on the density. To arrive at this formula replace the specific volume by the density in equation (2.38). Then take the derivatives of both sides with respect to the density. Assuming that the second derivative of the frequencies with respect to the density is zero, leads to the following differential equation for the Gruneisen parameter:

$$\frac{d(\gamma\rho)}{d\rho} + \left(\frac{\gamma\rho}{\rho}\right)^2 = 0. \quad (2.40)$$

The solution to equation (2.40) is

$$\gamma\rho = \frac{\gamma_o\rho_o}{(1 + \gamma_o\eta)}. \quad (2.41)$$

This specific form for the Gruneisen parameter may now be substituted into equation (2.34) to give the following for the equation of state:

$$P = \frac{\gamma_o \rho_o}{1 + \gamma_o \eta} \left[E - \frac{(c_o \eta)^2}{2(1 - s\eta)^2} \right] + \frac{\rho_o c_o^2 \eta}{(1 - s\eta)^2}. \quad (2.42)$$

When equation (2.42) is used for the equation of state, the sound speed is independent of the specific internal energy. Equation (2.42) was developed and used by Rice²⁵ in his analysis of some Hugoniot data for NaCl that he had taken. Equation (2.42) will be referred to as the Rice equation of state.

When the compression of the material is not too great, the functional dependence of the Gruneisen parameter on the density as given by equation (2.40) may be approximated by the equation

$$\gamma = \frac{\gamma_o \rho_o}{\rho}. \quad (2.43)$$

It turns out that equation (2.43) is a reasonable representation of the functional dependence of the Gruneisen parameter on the density as determined from experiments on porous aluminum.²⁶ Equation (2.43) is frequently used in high-pressure research. Another aspect of equation (2.43) is that it simplifies the equation of state. The term that is linear in the internal energy no longer involves the density. With equation (2.43) inserted into equation (2.34), the equation of state becomes

$$P = \gamma_o \rho_o E + \rho_o c_o^2 \eta \frac{1 - \frac{1}{2} \gamma_o \eta}{(1 - s\eta)^2}. \quad (2.44)$$

Equations (2.42) and (2.44) are two equations of state that are used in examining the present experiments. Both these equations of state are based on a linear U_s-u_p relation and reference the principal Hugoniot. The analytical simplicity of these equations makes them particularly useful in hydrodynamic computations. The deficiencies in these equations of state are fairly obvious. They do not model phase changes, and the Gruneisen parameter as defined in equation (2.28) is assumed to be a function of density or specific volume only. More complicated equations of state can be used. In addition to equations (2.42) and (2.44) two other equations of state are used to model the experiments. They are the Tillotson equation of state and the Gray equation of state.

The Tillotson equation of state²⁷ was constructed to be a general analytical equation of state valid for a wide pressure range from 0 to a maximum pressure of approximately 10^5 GPa. It is thought that approximately 10^3 GPa is the minimum pressure where the Thomas-Fermi or Thomas-Fermi-Dirac equation of state^{28,29} is applicable; thus the pressure range of 10^3 to 10^5 GPa for the Tillotson equation of state overlaps a region where the Thomas-Fermi equation of state is believed to be valid. Tillotson constructed his equation of state to extrapolate smoothly from the pressure region where Hugoniot data has been taken to the Thomas-Fermi region.

The form for the Tillotson equation of state is very similar to the Mie-

Gruneisen form; specifically, the form is

$$P = \gamma \rho E + f(\rho). \quad (2.45)$$

The major difference between the Tillotson and the Mie-Gruneisen equation of state is that the parameter γ in equation (2.45) is now a function of both density and the specific internal energy. In the Tillotson equation of state, the parameter γ is given by the equation

$$\gamma = \alpha + \frac{(\gamma_o - \alpha)B}{E(1 - \eta)^2 + B}. \quad (2.46)$$

In this equation B is an adjustable parameter, E is the specific internal energy, and γ_o is the Gruneisen parameter in the ambient state. In the limit of high internal energies, equation (2.46) gives a limiting value of α for γ . For the Tillotson equation of state, the value of α for most materials is one-half.

The last equation of state used in the modeling of these experiments is the the Gray eos^{30,31} (equation of state). The Gray eos is a multiphase equation of state that takes into account solid, liquid, and gas phases of a material. The contribution of an electron gas in a metal to the specific internal energy is also taken into account. The Gray eos proceeds from the assumption that a suitable equation of state can always be written in the form

$$P = P_c + \frac{\Gamma}{v}(E - E_c), \quad (2.47)$$

where the terms P_c and E_c refer to quantities on the zero-degree isotherm. At this point the Gray equation of state is quite similar to the Mie-Gruneisen form (equation (2.32)). The difference between the two equations is that Γ in equation (2.47) is dependent on the internal energy or temperature as well as the density, while the parameter γ in equation (2.32) is assumed to be a function of density only.

In order to determine the behavior of the cold curve for the Gray equation of state, equation (2.47) is abandoned and equation (2.32) is used to relate the principal Hugoniot to the cold curve; that is P_H , the pressure on the principal Hugoniot, and E_H , the specific internal energy on the principal Hugoniot, replace P and E in equation (2.32). For a material that follows a linear U_s - u_p relation, equation (2.33) is the differential equation for E_c . To solve for E_c it is assumed in the Gray eos that γ has a linear dependence on η :

$$\gamma = \gamma_o - a\eta. \quad (2.48)$$

The parameters γ_o and a are material constants. Instead of using an exact solution to equation (2.33), the Gray eos uses an approximate solution for E_c whose principal term is

$$E_c = \frac{c_o^2 \eta^2}{2(1 - s\eta)} \left[1 + \frac{s\eta}{3} + \frac{s^2 \eta^2}{6} \left(1 - \frac{\gamma_o}{s} \right) \right]. \quad (2.49)$$

It is apparent there is a slight inconsistency in the derivation of the Gray equation

of state. Temperature effects are included in writing equation (2.47) but then are ignored in deriving equation (2.49).

Referencing the principal Hugoniot to the cold curve by use of equations (2.32) and (2.48) enables one to rewrite equation (2.47) in the following manner:

$$P = P_H \left(1 + \frac{1}{2} (\gamma_0 + a\eta) \frac{\eta}{1 - \eta} \right) + \rho E (\gamma_0 + a\eta) + \rho (\Gamma + \gamma_0 + a\eta) (E - E_c). \quad (2.50)$$

When the equation of state is written in this form, it is clear that the first two terms of equation (2.50) represent a Mie-Gruneisen form for the equation of state in which the Gruneisen parameter is given by equation (2.48). The nonlinear effects of the specific internal energy or temperature are included in the third term and this term may be viewed as a correction term to a Mie-Gruneisen equation of state.

In the Gray equation of state it is necessary to calculate a temperature in order to calculate the correction term in equation (2.50). The temperature is calculated from the specific internal energy. For a material in the solid phase in this eos, the relation of the temperature to the specific internal energy is given by

$$E = E_c + c_v T + \frac{1}{2} g T^2. \quad (2.51)$$

The second term, $c_v T$, in equation (2.51) corresponds to E_t in equation (2.24) and the third term of equation (2.51) to E_e in equation (2.24). The specific heat

at constant volume, c_v , refers to the lattice only and is assumed to be constant at the classical Dulong and Petit value as long as the material remains in the solid phase. The term, $\frac{1}{2}gT$, is the electronic specific heat. Equations similar to equation (2.51) which take into account the latent heat of phase transformations are used when the material is in a liquid or dense gas phase to calculate the temperature. The correction term of the Gray eos for a material in the solid phase can be found from the equation

$$(\Gamma - \gamma_o + a\eta)(E - E_c) = \frac{1}{2}gT^2(\gamma_e - \gamma_o + a\eta). \quad (2.52)$$

The parameter γ_e is an electronic Gruneisen parameter and is usually assumed to be equal to two-thirds for most metals in the Gray eos. Equation (2.52) completes the specification of the Gray eos for a material in the solid phase.

2E. The Release Wave and the Gruneisen Parameter

For one-dimensional planar motion, an interesting relation can be derived between the slope of the principal Hugoniot, the bulk or longitudinal wave speed of the head of the rarefaction fan, and the Gruneisen parameter. This relation is based on the assumption that the initial part of the release wave is an isentropic process. If strength effects are present, this assumption implies that the initial release is an elastic process. For simplification, strength effects will be ignored.

From thermodynamics, one has that changes in the specific internal energy are governed by the equation

$$dE = TdS + \frac{P}{\rho^2}d\rho, \quad (2.53)$$

where S is the specific entropy. Similarly, changes in internal energy along the principal Hugoniot are given by

$$dE = \frac{\eta}{2\rho_o}dP + \frac{P}{2\rho^2}d\rho. \quad (2.54)$$

Substituting the right-hand side of equation (2.53) for dE in equation (2.54) leads to the equation

$$TdS + \frac{P}{\rho^2}d\rho = \frac{\eta}{2\rho_o}dP + \frac{P}{2\rho^2}d\rho. \quad (2.55)$$

If the specific entropy is expressed in terms of P and ρ , then

$$TdS = \frac{1}{\gamma\rho}dP + T\left(\frac{\partial S}{\partial\rho}\right)_P d\rho, \quad (2.56)$$

but

$$T\left(\frac{\partial S}{\partial\rho}\right)_P = -\frac{1}{\gamma\rho}\left(\frac{\partial P}{\partial\rho}\right)_S = -\frac{c^2}{\gamma\rho}. \quad (2.57)$$

The thermodynamic definition of the Gruneisen parameter (equation (2.28)) was used in deriving equations (2.56) and (2.57). With the use of equations (2.56) and (2.57), equation (2.55) can be written as

$$\frac{1}{\gamma\rho}dP - \frac{c^2}{\gamma\rho}d\rho = \frac{\eta}{2\rho_o}dP + \frac{P}{2\rho^2}d\rho. \quad (2.58)$$

Equation (2.58) holds along the principal Hugoniot in the P - ρ plane. Dividing by $d\rho$ and rearranging terms, equation (2.58) becomes³²

$$c^2 = \left(1 - \frac{\eta\gamma\rho}{2\rho_0}\right) \left(\frac{\partial P}{\partial \rho}\right)_H + \frac{P_H}{2\rho^2} \gamma\rho. \quad (2.59)$$

In equation (2.59), the subscript H refers to quantities on the principal Hugoniot or to derivatives taken along the Hugoniot. Equation (2.59) gives the square of the bulk sound speed along the principal Hugoniot; or, phrased in another manner, the slope of any isentrope intersecting the principal Hugoniot is given by equation (2.59).

A more illustrative representation of equation (2.59) is

$$\left(\frac{\partial P}{\partial \rho}\right)_S = \left(\frac{\partial P}{\partial \rho}\right)_H - \frac{\gamma\rho}{2} \left[\left(\frac{\partial P}{\partial \rho}\right)_H \frac{\eta}{\rho_0} - \frac{P_H}{\rho^2} \right]. \quad (2.60)$$

Since the Gruneisen parameter is a positive quantity, the term inside the brackets in equation (2.60) determines whether the slope of the Hugoniot is greater or less than the slope of the isentrope in the P - ρ plane. To determine the sign of the quantity in the brackets, consider the intersection point between the principal Hugoniot and an isentrope. For any increase in density, the change in specific internal energy along the principal Hugoniot must be given by equation (2.53). Along the isentrope the change in specific energy is also given by equation (2.53) but the specific entropy term is identically zero. The change in specific internal

energy along the principal Hugoniot must also be given by equation (2.54). This implies the following must hold:

$$\frac{\eta}{2\rho_o} dP + \frac{P}{2\rho^2} d\rho \geq \frac{P}{\rho^2} d\rho, \quad (2.61)$$

or

$$\frac{\eta}{\rho_o} \left(\frac{\partial P}{\partial \rho} \right)_H \geq \frac{P_H}{\rho^2}. \quad (2.62)$$

From this it can be seen that the slope of the principal Hugoniot is greater than the slope of the intersecting isentrope. The equality sign in equation (2.61) holds at the foot of the Hugoniot, and thus the isentrope that goes through the point ρ_o will have the same slope as the principal Hugoniot at this point. The influence of the Gruneisen parameter on the speed of the head of any release wave is readily apparent in equation (2.60). For a given shock state on the principal Hugoniot the larger the Gruneisen parameter is, the smaller the sound speed becomes.

It is interesting to apply equation (2.59) to a material which has a linear U_s - u_p relation. For such a material the pressure on the principal Hugoniot is given by equation (2.5). The derivative of the pressure with respect to the density along the Hugoniot is

$$\left(\frac{\partial P}{\partial \rho} \right)_H = \left(\frac{\rho_o c_o}{\rho} \right)^2 \frac{1 + s\eta}{(1 - s\eta)^3}. \quad (2.63)$$

This leads to the following equation for the sound speed along the Hugoniot:

$$c^2 = \left(\frac{\partial P}{\partial \rho} \right)_s = \left(\frac{\rho_o c_o}{\rho} \right)^2 \frac{(1 + s\eta - s\eta^2 \gamma \rho / \rho_o)}{(1 - s\eta)^3}. \quad (2.64)$$

No special assumptions had to be made concerning the Gruneisen parameter in deriving equation (2.64). It is interesting to compare this equation with the bulk sound speed predicted by a Mie-Gruneisen equation of state. For this equation of state the Gruneisen parameter is assumed to be independent of temperature or specific internal energy. Using equation (2.23) to calculate the sound speed from equation (2.34), one has that the slope of the isentrope is given by

$$c^2 = \left(\frac{\partial P}{\partial \rho} \right)_S = \left(\frac{\rho_o c_o}{\rho} \right)^2 \frac{1 + s\eta - s\eta^2 \gamma \rho / \rho_o}{(1 - s\eta)^3} + \left[E - \frac{c_o^2 \eta^2}{2(1 - s\eta)^2} \right] \left[\frac{d(\gamma \rho)}{d\rho} + \left(\frac{\gamma \rho}{\rho} \right)^2 \right]. \quad (2.65)$$

The first term on the right-hand side of equation (2.65) is identical to equation (2.63). As long as

$$\frac{d(\gamma \rho)}{d\rho} + \left(\frac{\gamma \rho}{\rho} \right)^2 > 0, \quad (2.66)$$

then the effect of shock heating is to increase the sound speed on the release isentrope for a given density. If γ is given by equation (2.41), then the slope on a release isentrope is independent of shock heating.

2F. Strength Effects

In the previous sections the problem of shock compression and subsequent release to ambient pressure has been approached with the explicit assumption

that a hydrodynamic description is adequate. This can only be justified a posteriori. If a hydrodynamic description is not appropriate, then the effects of material rigidity or strength must be taken into account. This opens a virtual Pandora's box of possible constitutive equations and material response functions.† These difficulties are further complicated by material symmetry. To eliminate this last complication, the discussion will be restricted to the idealized case of isotropic materials. A natural division of material response functions are those materials which are viscid (stresses are rate-dependent) and those that are inviscid (stresses are rate-independent). When dealing with a viscid material, the set of equations governing any flow process is a system of second-order partial differential equations. Flow for an inviscid material is described by a system of first-order partial differential equations. The assumptions that a material is isotropic and inviscid will limit the scope of possible constitutive equations but a great variety of material models is still possible. As in the hydrodynamic case, the assumption that a material is isotropic and inviscid can only be justified a posteriori. The first order of business, in view of the complexities involved, is to develop a clear picture of the simpler isotropic and rate independent materials.

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† The nomenclature (stress, strain, plasticity, viscid materials, etc.) that is used in this section as well as the scope of possible constitutive relations and material response functions can be found in books or articles on continuum mechanics. Some good sources are listed in references 33-36.

In dealing with strength, the obvious first step is to write the flow equations using the stress tensor instead of the pressure as was done in section 2C. Material isotropy was assumed in the argument in that section concerning the normal and tangential components of the stress tensor for one-dimensional flow. It must be emphasized that the primary difference between the hydrodynamic system and the system in which strength is important is the introduction of non-zero deviatoric stresses. Although a stress tensor must now be used, the concept of pressure can still be retained by defining a pressure in terms of the first invariant of the stress tensor as

$$P = \frac{1}{3} I_{\sigma} = \frac{1}{3} (\sigma_n + 2\sigma_t), \quad (2.67)$$

where I_{σ} is the first invariant of the stress tensor. For the present case of one-dimensional flow, σ_n is the component parallel to the flow and σ_t is the component perpendicular to the flow. The sign convention for the stress tensor is the same as that in section 2C. With the use of equation (2.67), the deviatoric stresses can be defined as

$$\tau_n = \sigma_n - P \quad (2.68)$$

and

$$\tau_t = \sigma_t - P. \quad (2.69)$$

From these definitions it is seen that the following equation holds for the deviatoric

stresses:

$$\tau_n + 2\tau_t = 0. \quad (2.70)$$

With the deviatoric stresses given by equations (2.68) and (2.69), the set of partial differential equations governing the flow behind the shock front can be written as

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0, \quad (2.71)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{\partial(P + \tau_n)}{\partial x} = 0, \quad (2.72)$$

and

$$\frac{\partial E}{\partial t} + u \frac{\partial E}{\partial x} + \frac{P + \tau_n}{\rho} \frac{\partial u}{\partial x} = 0. \quad (2.73)$$

It is now assumed that the pressure defined in equation (2.67) and used in equations (2.72) and (2.73) is given by an equation of state similar to one of those in section 2D. Once a constitutive equation is specified for the deviatoric stress τ_n , the system of partial differential equations governing the flow can be solved. The remainder of this section will be devoted to the description of a suitable model for the deviatoric stress.

The mechanical behavior of solids and metals in particular can be qualitatively classified in one of two broad categories, elastic behavior and plastic behavior.³⁷ The simplest descriptions of each phenomenon are fundamentally different, but each can be described by rather simple examples. A perfectly elastic material is deformed according to Hooke's law when a force or stress is applied

to the material. When the force or stress is relieved, the material returns to its original shape. For a material that is perfectly plastic there is no deformation until the stress field reaches a certain magnitude. The material then flows undergoing a deformation. When the stress is relieved from a plastic material, the flow ceases and the material retains a permanent deformation. All metals exhibit to some extent the features of both elasticity and plasticity. The constitutive equation for τ_n should reflect some type of elastic-plastic behavior.

Elasticity and plasticity have been studied from the point of view of solid-state physics and from a phenomenological viewpoint. Historically, the phenomenological approach is the oldest of the two approaches and the more pervasive in the formulation of elastic-plastic models. An important factor in the phenomenological description of an elastic-plastic material is the concept of a yield strength or yield surface. Examples of a yield surface or yield criteria are the Tresca and the von Mises yield conditions.³⁸ A yield condition is the mathematical expression, written in terms of the stress tensor, of the experimental observation that a material can only support a certain maximum stress field before it flows plastically or fails catastrophically. The Tresca yield condition is written in terms of the principal stress components of the stress tensor as

$$\sigma_1 - \sigma_3 = Y, \quad (2.74)$$

where σ_1 is the maximum value of the principal stress components and σ_3 is the

minimum value. The von Mises yield condition is written in terms of the stress deviators as

$$\frac{3}{2}\tau_{ij}\tau_{ij} = Y^2. \quad (2.75)$$

In equations (2.74) and (2.75) the parameter, Y , is the yield strength in simple tension. For a problem in which deformation only occurs in one dimension, these yield conditions both give

$$|\tau_n| = \frac{2}{3}Y. \quad (2.76)$$

Equations (2.74) and (2.75) represent the conditions that the stress tensor must meet when the material fails. These are limiting values of the stress tensor or the deviatoric stress tensor; thus, the deviatoric stress of the material will always be limited in absolute magnitude and equation (2.76) can be written as

$$|\tau_n| \leq \frac{2}{3}Y. \quad (2.77)$$

In general, the yield conditions expressed in equations (2.74) and (2.75) resulted from experimental studies performed at much lower static pressures than the peak pressures in these experiments. Even though this is true, it will be assumed that equation (2.77) can be extrapolated to high pressure regions as long as the material remains in the solid phase. The yield strength may be considered to be a function of thermodynamic variables such as density or strain, internal energy or temperature, or it may depend on the stress-strain history such as in

work hardening. It may also depend on a dynamic property such as strain rate, but it is simplest to treat Y in equation (2.77) as a constant.

The yield condition of equation (2.77) can be considered as the criterion for separating the material response into two regions. The first region occurs when the inequality sign holds in equation (2.77). For this region the material has some sort of elastic response. The problem is to incorporate some elastic model in with the bulk properties that are assumed to dominate. Depending on the sign of the velocity gradient and whether τ_n is at an upper or lower yield limit, the material may also respond elastically when the equality sign in equation (2.77) holds. When the material is in the plastic state, the equality sign in equation (2.77) holds.

A physical quantity that is used in the modeling of solids is the quantity called strain. Depending upon the circumstance and what one wants to model, there are many different definitions of strain. Fortunately, the case of deformation in one dimension makes the definition of strain relatively easy. The definition that is appropriate for this type of deformation is the natural or logarithmic strain. As in the case of the stress tensor, the strain tensor will be resolved into one normal component and two tangential components. The tangential components will be equal from symmetry arguments. The normal component, ε_n , is given by the

equation

$$\varepsilon_n = \ln\left(\frac{\rho_o}{\rho}\right), \quad (2.78)$$

and the tangential components by the equation

$$\varepsilon_t = 0. \quad (2.79)$$

One approach to the description of an elastic-plastic material is to assume that each of the total strain components may be considered to be the sum of an elastic term and a plastic term. This implies the following equations:

$$\varepsilon_n = e_n + \lambda_n = \ln\left(\frac{\rho_o}{\rho}\right), \quad (2.80)$$

and

$$\varepsilon_t = e_t + \lambda_t = 0. \quad (2.81)$$

The quantity e denotes the elastic strains and λ denotes the plastic strains. It is also assumed that the plastic strains do not produce a volume change and this assumption results in the equations

$$\lambda_n + 2\lambda_t = 0 \quad (2.82)$$

and

$$e_n + 2e_t = \ln\left(\frac{\rho_o}{\rho}\right). \quad (2.83)$$

From these equations it is possible to define strain deviators. It is clear from equation (2.82) that the plastic strain deviator is identical to the plastic strain.

This is not true for the elastic strain deviator, ξ , and the components of the elastic strain deviator are given by the equations

$$\xi_n = e_n - \frac{1}{3} \ln\left(\frac{\rho_o}{\rho}\right) \quad (2.84)$$

and

$$\xi_t = e_t - \frac{1}{3} \ln\left(\frac{\rho_o}{\rho}\right). \quad (2.85)$$

The deviators are needed since the constitutive equations for τ_n in simple elasticity theory are given in terms of the normal component of the elastic strain deviator.

The strength model may be summarized up to this point as dividing the response of a material to a deformation into two broad categories, an elastic response and a plastic one. These two types of material response are separated by a yield condition. For one-dimensional deformation, the constitutive equation for τ_n in the plastic region must satisfy the equality constraint of equation (2.77). When τ_n satisfies the inequality in equation (2.76), the material is responding elastically but equation (2.77) no longer provides a constitutive equation for τ_n . Appealing to simple elasticity theory,³⁹ a constitutive equation for τ_n in terms of the elastic strain deviator is

$$\tau_n = -2\mu\xi_n. \quad (2.86)$$

Another constitutive equation that has been given for τ_n in terms of ξ_n is,⁴⁰

$$\tau_n = -2\mu \frac{\rho}{\rho_o} \xi_n. \quad (2.87)$$

The minus sign has been added to these equations to agree with the sign convention of section 2C that stress is positive in compression. The term μ is the shear or rigidity modulus and may be assumed to be a constant.

Equations (2.86) and (2.87) are constitutive equations from elasticity theory that have been applied to one-dimensional deformations. In particular they have been applied to small deformations and quasi-static problems. For the present problem, equations (2.86) and (2.87) will not be used in the forms given but will be generalized to give the deviatoric stress rates. This is similar to hypoelasticity. The generalizations for the deviatoric stress rate in the elastic region are

$$\frac{d\tau_n}{dt} = -2\mu \frac{d\xi_n}{dt} \quad (2.88)$$

and

$$\frac{d\tau_n}{dt} = -2\mu \frac{\rho}{\rho_o} \frac{d\xi_n}{dt} . \quad (2.89)$$

Now equation (2.88) can be derived from equation (2.86) when the rigidity modulus is constant but equation (2.89) certainly cannot be directly derived from equation (2.87). When the material is flowing plastically, the deviatoric stress rate is

$$\frac{d\tau_n}{dt} = 0 , \quad (2.90)$$

assuming that the yield strength is constant. Equations (2.88) and (2.89) may be written in terms of the density. Recognizing that the plastic strain rates are iden-

tically zero in the elastic regime, equation (2.84) may be expressed as a function of the total strain and the plastic strain and then differentiated to arrive at

$$\frac{d\xi_n}{dt} = -\frac{2}{3\rho} \frac{d\rho}{dt} . \quad (2.91)$$

Equation (2.88) then becomes

$$\frac{d\tau_n}{dt} = \frac{4}{3} \frac{\mu}{\rho} \frac{d\rho}{dt} \quad (2.92)$$

and equation (2.89) may be rewritten as

$$\frac{d\tau_n}{dt} = \frac{4}{3} \frac{\mu}{\rho_o} \frac{d\rho}{dt} . \quad (2.93)$$

Equations (2.77), (2.90), and (2.92) or (2.93), allow a complete description of τ_n in a region where the flow is continuous.

An interesting sidelight to these strength models is the prediction of a Hugoniot elastic limit.⁴¹ At the Hugoniot elastic limit the normal component of the deviatoric stress will satisfy equation (2.76) but the jump conditions across the shock can be found from the equations governing the flow of an elastic material. Considering the yield strength and the rigidity modulus to be constant and the initial state to be at normal density with zero stress, the equations across the shock front can be approximated as

$$u_p = U_s \frac{w}{1 + w} , \quad (2.94)$$

$$\rho = \rho_0(1 + w), \quad (2.95)$$

$$E = \frac{1}{2} U_s^2 \left(\frac{w}{1 + w} \right)^2, \quad (2.96)$$

and

$$P = \rho_0 U_s^2 \left(\frac{w}{1 + w} \right) - \frac{2}{3} Y. \quad (2.97)$$

The parameter w in equations (2.94) through (2.97) is given by,

$$w = \frac{Y}{2\mu}. \quad (2.98)$$

A quick perusal of these equations reveals that the density at the Hugoniot elastic limit is not fixed by the shock speed but rather by the strength parameters which result from the constitutive equation (2.92) or (2.93). The normal stress at the Hugoniot elastic limit is given by

$$\sigma_n = P + \frac{2}{3} Y. \quad (2.99)$$

To determine the stress at the Hugoniot elastic limit, the pressure in equation (2.97) must be known. Because equations (2.94)–(2.97) are not sufficient to solve for all the unknowns, the specification of the equation of state is the necessary fifth equation that allows the determination of all relevant quantities at the Hugoniot elastic limit.

The phenomenological strength models related in this section are mainly introduced to describe the effects that non-zero deviatoric stresses may have on

wave propagation in a very high pressure regime. They are not intended to be detailed descriptions of either plasticity or elasticity. The strength models are being added in a rather ad hoc fashion to the equations of state of section 2D in an attempt to furnish a more complete description of material behavior than is possible in a purely hydrodynamic model. The fact that the equations of state reference the principal Hugoniot and the principal Hugoniot is really a measurement of a normal stress component casts some doubt on the accuracy of the equations of state. However, it should be noted that most principal Hugoniot data have been taken at states well beyond the Hugoniot elastic limit and in these pressure states it is expected that the stress deviator should be just a small percentage of the total stress. This last statement is only true if it is assumed that the yield strength does not increase by orders of magnitude with increasing pressure. In any case, one would expect that bulk properties must dominate in determining the principal Hugoniot and the inclusion of strength models should not greatly alter the pressure calculation along the principal Hugoniot at these very high pressures.

The situation is much different when one considers the possible effects that a strength model has on the speed of a release wave. If the initial release from a state on the principal Hugoniot is elastic then the longitudinal sound speed, c_ℓ , is given by the equation

$$c_\ell^2 = c_\beta^2 + \frac{4}{3} \frac{\mu}{\rho} + \frac{\tau_n}{\rho^2} \left(\frac{\partial P}{\partial E} \right)_\rho. \quad (2.100)$$

The term c_β in equation (2.100) is the bulk sound speed and is given by the right-hand side of equation (2.23). Equation (2.100) is valid when the stress rate is given by equation (2.92). For the stress rate given by equation (2.93), c_ℓ is given by

$$c_\ell^2 = c_\beta^2 + \frac{4}{3} \frac{\mu}{\rho_0} + \frac{\tau_n}{\rho^2} \left(\frac{\partial P}{\partial E} \right)_\rho. \quad (2.101)$$

The bulk sound speed increases as the density increases; thus, the difference between the longitudinal and bulk sound speed will decrease when the rigidity modulus is treated as a constant. At zero or ambient pressure the bulk sound speed may be twenty to thirty percent smaller than the longitudinal sound speed, and although this difference decreases in these particular strength models with increasing pressure, the difference in the sound speeds is still appreciable.

A last remark must be made on the propagation of waves in the plastic region. In this region the stress rate is given by equation (2.90). This leads to the following equation for the sound speed in the plastic state:

$$c_\ell^2 = c_\beta^2 + \frac{\tau_n}{\rho^2} \left(\frac{\partial P}{\partial E} \right)_\rho. \quad (2.102)$$

Since the normal component of the deviatoric stress is limited in magnitude by equation (2.77) and the isentropic bulk modulus is usually very much greater than the yield strength, it is seen that wave speeds in the plastic region are nearly the same as wave speeds calculated in models that do not include strength effects.

THE EXPERIMENTS

3A. Experimental Method and Design

There is more than one method by which one may measure the speed of a release wave in a material, but the method chosen for these experiments was the "overtaking-relaxation" method.⁸ This method involves producing a strong shock wave of constant velocity in a target material and then introducing a rarefaction or release wave into the target that overtakes the shock and attenuates or "relaxes" it. By observing the point at which the attenuation first occurs, one can determine the speed of the wave if the spatial and temporal origins of the wave are known.

The actual experimental technique entails accelerating a thin flyer plate to some velocity and letting it collide with a thicker target plate. If the velocity of the flyer plate is sufficiently high, a strong shock wave is produced that travels forward into the target plate and another strong shock wave is also produced that travels backward into the flyer plate. When the shock wave in the flyer plate reaches the back surface of the flyer, a release wave is created that travels forward through the flyer and into the target plate, where it eventually overtakes and then attenuates the shock wave in the target. A schematic of this process in a t (time) versus x (position) diagram is shown in figure 1 (appendix I). A flyer plate of thickness, h , traveling at velocity, u_o , impacts a target plate at $t = 0$, $x = 0$. This creates the forward-traveling shock wave with velocity U_s shown in the target plate. When

the backward-traveling shock wave in the flyer plate reaches the back surface of the flyer plate, a release wave is created at $t = t_1$, $x = x_1$. The head or lead characteristic of the release wave is shown overtaking the shock wave in the target plate at $t = t_3$, $x = x_3$. Experimentally, the quantities that were measured were the trajectory of the flyer plate just before it collided with the target plate and the trajectory of the shock wave in the target plate. It must be mentioned that figure 1 is for the idealized case in which there are no strength effects in the high pressure state. Strength effects are often assumed to be negligible even when the material remains in the solid phase upon shock loading.

The trajectory of the flyer plate and the trajectory of the shock wave in the target plate were measured by means of electrical contactor pins.⁴² These pins are coaxial in nature with an inner conductor and an outer jacket or sleeve that serves as a ground. When the sleeve is shorted to the inner conductor, an electrical signal is produced. In the present experiments the inner conductor was set back approximately 0.025 mm with respect to the sleeve. To measure the trajectory of the flyer, holes were drilled through the target plate and the ends of the contactor pins protruded through these holes. A thin nickel cap covered the end of each pin to prevent pre-shortening from ionized gases or ions ejected from the surface of the flyer plate prior to mechanical contact of the flyer plate with the pins. To measure the shock trajectory in the target plate, flat-bottomed holes were drilled in the

target and an uncapped pin was butted against the bottom of each hole.

Figure 2 is a layout drawing of the experiments. The explosive driver system in each experiment was comprised of a detonator, a P/120 HE (High Explosive) lens, and a cylindrical slab of HE. The slab of HE had a nominal diameter of 304 mm and a nominal thickness of 50.8 mm. The purpose of the P/120 HE lens was to initiate a planar detonation in the cylindrical slab of HE. The detonation products from the slab of HE then drove the flyer plate across a 25.4-mm gap where the flyer impacted the target plate. A machined ring of mild steel was used to hold the HE slab. This steel ring had an inside lip on which the HE slab rested. The lip was 6.35-mm thick and provided a standoff between the slab and the flyer plate. The flyer plate also had a thin sheet (~ 0.38 mm) of polyethylene attached to the back surface of the flyer plate. The purpose of the polyethylene sheet and the 6.35-mm standoff was to reduce the peak shock pressure in the flyer plate and to provide a smoother acceleration of the flyer plate. The mild steel ring was bolted to the flyer plate and to an aluminum standoff ring. The thickness of the aluminum standoff ring in all experiments was 25.4 ± 0.014 mm. This whole arrangement then bolted to an end plate in which the target plate fitted. By using these machined pieces, careful alignment of the HE, flyer plate, and target plate was achieved. The experiments were made such that the explosive driving system, flyer plate, and the impact surface of the target plate were all in an evacuated

region. The evacuation greatly reduced the amount of mass from gases or air between the flyer plate and target plate. Typical pressures in the cannisters at time of firing were 500 millitorr.

To control the peak shock pressure in the target plate, the velocity of the flyer plate had to be controlled. The velocity of the flyer could be controlled by changing the amount of HE, the type of HE, the thickness of the flyer plate, or the length of run between the flyer and target. In the present experiments, it was decided that the velocity of the flyer plate could be most easily controlled by changing the type of high explosive used for the cylindrical HE slab. Three different types of HE were used in these experiments, and are listed in table 1 (appendix H). The explosives used were a TNT explosive, a RDX-based explosive, and a HMX-based explosive.⁴³ The Comp B explosive listed in table 1 is the RDX-based explosive and PBX-9404 is the HMX-based explosive. Properties that can be used to characterize these explosives are the initial density and the detonation or Chapman-Jouget (CJ) pressure.⁴⁴ The PBX-9404 has the greatest initial density and CJ pressure while the TNT has the lowest initial density and lowest CJ pressure of the three explosives used in these experiments. Before any experiment was conducted involving the measurement of shock attenuation in a target plate, a preliminary experiment was conducted to characterize the kinematics of the flyer plate for a given explosive driving system.

3B. The Flying Plate Experiments

The purpose of the flyer experiments was threefold. First, the timing and velocity of the flyer plate for a particular explosive driving system had to be determined. Second, the planarity of the flyer plate and the shock wave produced by the collision of the flyer with the target plate had to be examined. Third, if the system were sufficiently planar, then a region free from edge perturbations had to be found.

A basic experimental problem using explosively driven flyer plates for one-dimensional shock experiments is the lack of planarity of the system. Two perturbations are prevalent, tilt of the flying plate and bowing of the flying plate. The amount of tilt can be minimized by careful alignment of the system. The present experiments were designed to be axially symmetric about a line drawn from the center of the detonator through the center of the flyer plate and target plate. The bowing of the flyer plate appears to be radially dependent with respect to this axis of symmetry. To check on these perturbations the data from each flying plate experiment were fit to the following equations:

$$t = a_o + a_x x + a_y y + a_z z + a_r r, \quad (3.1)$$

$$l = a_o + a_x x + a_y y + a_z z + a_r r^2. \quad (3.2)$$

If the radial dependence of the fit was found to be negligible, the data were then

fit to the equation

$$t = a_o + a_x x + a_y y + a_z z . \quad (3.3)$$

The coordinate system was chosen such that the x -axis coincided with the axis of symmetry; further, the impact surface of the flyer plate and target was chosen as the plane $x = 0$. For this coordinate system, r is defined by

$$r^2 = y^2 + z^2 . \quad (3.4)$$

In each flying plate experiment, the target or witness plate was made of 6061 aluminum. Although no attempt was made to measure shock wave attenuation in the aluminum witness plates, pins were placed in the witness plates to check on the shape of the shock wave produced by the collision of the flyer plate with the witness plate. At least three planes or levels of pins were used in each flying plate experiment. One group of pins was placed to monitor the shape of the flyer plate just before collision with the target plate. A second group was set flush with the impact surface (the plane $x = 0$) and a third group was placed in the witness plate to monitor the shock wave. The level of pins in the impact surface served a dual role in each flyer experiment; first, this level was used in determining the shape and velocity of the flyer plate and then was used along with the last level of pins in determining the shape and velocity of the shock wave. When using the pins in the $x = 0$ plane in this manner, the assumption is implicitly made that the

shock wave is created the instant the flyer plate makes contact with the witness or target plate.

By their very nature pin data are Eulerian measurements and not Lagrangian measurements, in that they provide a time record for a particular point in space and do not follow a material point in the flyer plate or witness plate. When pin data are fit to equations (3.1), (3.2), or (3.3), the pin measurements are being used in the most propitious manner. Specifically, the pin measurements are being used for interpolation to determine isochrones for the shock front or the front surface of the flyer plate. Although the material velocity of any particular point in the surface of the flyer plate is not determined, a velocity for the surface can be found from the fitted data. The magnitude of a velocity in a direction normal to the surface for equation (3.1) is given by the equation

$$u = [a_x^2 + (a_y + a_r y r^{-\frac{1}{2}})^2 + (a_z + a_r z r^{-\frac{1}{2}})^2]^{-\frac{1}{2}}. \quad (3.5)$$

For equations (3.2) and (3.3) the equations for velocity are, respectively,

$$u = [a_x^2 + (a_y + 2a_r y)^2 + (a_z + 2a_r z)^2]^{-\frac{1}{2}}, \quad (3.6)$$

and

$$u = [a_x^2 + a_y^2 + a_z^2]^{-\frac{1}{2}}. \quad (3.7)$$

It was found that in all the present flying plate experiments that the parameter a_x completely dominates all other parameters in equations (3.5)–(3.7) in determining the speed of the flyer plate.

An interesting point to consider is the comparison of the shape of the shock front to the shape of the flyer plate. This comparison can be made by fitting the pin data from the $x = 0$ plane independently of the pin data from the witness plate. The fitting equations should reflect the fact that within a given level of pins, the x -values are nominally constant. A suitable set of such equations are

$$t = a_o + a_y y + a_z z + a_r r, \quad (3.8)$$

$$t = a_o + a_y y + a_z z + a_r r^2, \quad (3.9)$$

and

$$t = a_o + a_y y + a_z z. \quad (3.10)$$

If the only component of material velocity of the flyer plate is in the x direction and is independent of material position in the flyer, then the shock front should show the same tilt and bowing as the flyer plate. The shapes of the fronts are compared in tables 2b, 3b, and 4b.

The first flying plate experiment was RS-2301. The high explosive used to drive the flyer was TNT. A total of one hundred and fifty-six contactor pins were used in this experiment. Fifty-four capped pins were placed flush with the front surface of the target or witness plate. Another fifty-four uncapped pins were butted against the free surface of the witness plate. The aluminum witness plate had an average thickness of 15.22 mm. Of the remaining forty-eight pins, twenty-four capped pins were placed approximately 1-mm away from the impact surface

and the other twenty-four pins were placed approximately 2-mm away from the same surface. The pins were thus grouped into four planes, nominally at $x = -2$, $x = -1$, $x = 0$, and $x = 15.22$ mm. Within each plane the pins were further placed in circular arrays. These arrays were concentric about the x -axis. For the $x = 0$ plane the innermost array had a radius of 30 mm and the array with the greatest radius had a radius of 95 mm. For the plane at $x = -2$ mm and the plane at $x = -1$ mm the arrays extended from a radius of 50 mm to a radius of 95 mm. The pins in the 15.22-mm plane covered an area with an inner radius of 2 mm and an outer radius of 110 mm. Three pins were equally spaced (angular separation of 120°) in the 110-mm radius array. The next circular array was at 105 mm with three pins placed in this array. Unfortunately the signals from these pins failed to be recorded and no time data for pins with a radius greater than 100 mm were recorded.

For the second experiment in the series of flying plate experiments only three levels or planes of pins were used. For experiment RS-2307, the planes were -1.5 , 0 , and 5 mm. A total of one hundred and fourteen pins were used in this experiment. Thirty-eight pins were placed in each level. Within each plane the pins were positioned in concentric arrays about the x -axis as was done in experiment RS-2301. In each plane the area covered by the pins was an annulus with an inner radius of 20 mm and an outer radius of 110 mm. The pin array for

the last flying plate experiment, RS-2310, in the series was the same as experiment RS-2307 except that the first level or plane of pins was nominally set at $x = -1.2$ mm.

In fitting the data to any of the equations such as (3.1), (3.2), or (3.3), it was assumed that positioning errors were negligible. The method of least squares was then used to determine the coefficients of the equations. This entailed minimizing R with respect to the parameters a_o , a_x , etc. R is defined by the equation

$$R = \sum_{i=1}^N [t_i - f(x_i, y_i, z_i)]^2. \quad (3.11)$$

In equation (3.11), N is the number of data points and f can be any of the functions represented in equations (3.1)–(3.3) or (3.8)–(3.10). The standard deviation, σ , of the fit was then taken to be

$$\sigma = \left(\frac{R}{N - N_a} \right)^{\frac{1}{2}}, \quad (3.12)$$

where N_a is the number of coefficients or parameters in the equation used for the fit.

The results from the flying plate experiments are presented in tables 2, 3, and 4. Each table lists the values of the coefficients for that equation that best fit the data for that particular experiment. The error bars for each parameter were calculated from the standard error matrix using a 90% confidence level.

Experiment RS-2301 was fitted best by equation (3.2). Experiment RS-2307 was best fit by equation (3.3). For experiment RS-2310 it was found that the fitting equation needed both the linear and quadratic term in the radius to give the best representation of the data. Figure 3 plots average times for each circular array in the $x = 0$ plane against the radius for each of the three flying plate experiments.

3C. Shock Trajectory Data Analysis

As stated in section 3A, the principal measurement in each shock-wave attenuation experiment was the trajectory of the shock wave in the target plate. The trajectory data consisted of a set of discrete points. The analyses of these data proceeded from certain assumptions concerning the state of the flyer plate and the nature of the shock wave introduced into the target plate. For the data reduction discussed in this section it was assumed that the flyer plate had uniform properties; that is, all material points in the flyer plate had the same particle velocity, density, and specific internal energy. This assumption then implied that a shock wave of constant velocity would be introduced into the target plate and would remain at a constant velocity until it was overtaken by the release wave. These assumptions form the premise on which the data analysis of each shock-wave experiment was based. Further discussion and quantitative refinement of these idealized assumptions are given in chapter 4.

The placement of the shock pins in the target plate was made on the basis of the flyer plate experiment for that particular explosive driving system. In general each shock-wave attenuation experiment had from fifteen to twenty levels of shock pins. These levels were spaced approximately 1-mm apart in the x direction. Every experiment had from four to six shock pins in each level. Further details about the positions of the shock pins for each experiment are given in appendix G.

The main difficulty in analyzing these shock wave experiments is determining the point at which the attenuation of the shock wave first begins. An example of shock trajectory data is plotted in figure 4. The measured arrival time, t , of the shock wave at a depth, x , into the target plate is plotted against x in this figure. The data are from experiment RS-2302. As can be seen in this plot, the amount of attenuation does not appear to be great. To better show that attenuation is indeed present, a plot of " δt " versus x is given in figure 5. The curvature of the data that is attributable to a decrease in the velocity of the shock wave is readily apparent in this plot.

The term " δt " is a calculated quantity which is based on a fit to the experimental data. The first step in calculating δt is to fit the first few levels of pin data to the equation

$$t = a_o + a_x x. \quad (3.13)$$

On the basis of this fit a calculated time, tc_i , is found for every shock pin. Once

this has been done, an initial value of δt_i for each shock pin is then found from the equation

$$\delta t_i = t_i - tc_i, \quad (3.14)$$

where t_i is the observed or measured time for the i^{th} shock pin.

The next step in arriving at a final answer for δt is the calculation of a shock velocity at the position of each shock pin. For the first few levels of pin data that are fit to equation (3.13), the shock velocity is simply

$$U_s = \frac{1}{a_x}. \quad (3.15)$$

For the remainder of the pin data the calculation of a shock velocity is a little more involved.

All shock pin data not explicitly fit to equation (3.13) can be considered to be in the attenuation region where the shock wave is decaying. The δt_i data in this attenuation region may be fit to a polynomial or a spline function of the independent variable x . It was decided that a suitable function for fitting the data would be a spline function with two break points. The choice of the position of the first of these break points is discussed in section 3D and in chapter 4. From strictly a statistical point of view the position of the break points is somewhat arbitrary but is clear that the first break point, x_1 , should always be assumed to lie between the last level of pin data fit to equation (3.13) and the next level of pins.

This first break point, x_1 corresponds to x_3 in figure 1. The only requirement for the second break point, x_2 , is that it should be greater than or equal to the first break point. A combination of both cubic and quadratic polynomials were used in fitting the δt data. A physical basis for the choice of cubic or quadratic polynomials and the position of x_2 is also discussed in section 3D.

When the data are analyzed in this manner the data are divided into three regions or zones. In the first zone ($x_i \leq x_1$), the appropriate data are fitted to equation (3.13). On the basis of this fit a tc_i is calculated for each datum. Then a δt_i is found from equation (3.14). In the second zone ($x_1 < x_i \leq x_2$), the δt_i data are fit to the equation

$$\begin{aligned} \delta t = & b_0 + b_1(x - x_1) + b_2(x - x_1)^2 \\ & + b_3(x - x_1)^3. \end{aligned} \quad (3.16)$$

If a quadratic polynomial is used in this zone then b_3 is identically set to zero.

For the third zone ($x_2 < x_i$), the δt_i data are fit to the equation

$$\begin{aligned} \delta t = & c_0 + c_1(x - x_2) + c_2(x - x_2)^2 \\ & + c_3(x - x_2)^3. \end{aligned} \quad (3.17)$$

As in the second zone, if a quadratic fit is used then the coefficient for the cubic term is set to zero.

The coefficients in equations (3.16) and (3.17) are not independent and are subject to constraints. The first constraint is that δt should be continuous in x . Since the first zone ($x \leq x_1$) is the region where δt is nominally zero, the continuity of δt implies that b_0 must be zero in equation (3.16). The second and last constraint is that once the shock wave is formed the velocity of the shock wave is continuous in x . This second constraint is equivalent to insisting that the first derivative of δt with respect to x is continuous. This implies that b_1 in equation (3.16) must also be zero. These continuity conditions on δt and its first derivative imply that at the point $x = x_2$ the following relations must hold:

$$c_0 = b_2(x_2 - x_1)^2 + b_3(x_2 - x_1)^3, \quad (3.18)$$

and

$$c_1 = 2b_2(x_2 - x_1) + 3b_3(x_2 - x_1)^2. \quad (3.19)$$

It is possible to obtain a velocity for the shock wave from these fits to the data in a straightforward manner. In the first region the shock velocity is simply given by (3.15). If in the second and third zones the representation for δt is

$$\delta t = f(x), \quad (3.20)$$

then the shock wave velocity at any point in the attenuation region is given by

$$U_s = \left(\frac{df(x)}{dx} + a_x \right)^{-1}. \quad (3.21)$$

In equation (3.21) the parameter a_x is the same parameter that is in equation (3.15).

After a shock velocity has been found for each pin, an average value for x is calculated for each group of pins that are in the same level. The measured time for each pin is then adjusted to the average x value for that particular pin level. Letting x_i be the average value of x for a group of pins in a given level, the new adjusted time, t'_i , is given by

$$t'_i = t_i + \frac{x_i}{U_i} - \frac{x_1}{U_i} . \quad (3.22)$$

The shock velocity, U_i , is calculated from either equation (3.15) or equation (3.21) depending on whether the datum is in the shock-attenuation region or not.

The t'_i are not the final adjustments that are made to the measured times. Equation (3.20) shows that within each pin level all measured times in that pin level have been adjusted to an average value of x for that group of pins. Within each group or level of pins there is still much scatter in the data. Much of this scatter can be attributed to tilt or bowing or both. To account for these perturbations each level of data is fit to either equations (3.8) or (3.10). If a given level of data does not have enough pins to fit to these equations, then that level of pin data is simply ignored. Based on these fits, average values for these perturbation parameters are found. The final adjusted times, $\tilde{t}_i$, are then calculated from

$$\tilde{t}_i = t'_i - a_y y_i - a_z z_i - a_r r_i . \quad (3.23)$$

These new adjusted times are an attempt to take into account the non-planarity of the shock. The $\tilde{t}_i$ are then fit to equation (3.13) and the δt_i data are recalculated from equation (3.14) using $\tilde{t}_i$ instead of t_i . The plot of δt in figure 5 was made using this analysis technique. Figure 6 is a δt plot using the raw times t_i instead of the adjusted times, $\tilde{t}_i$. It is very apparent there is much more scatter in the data when no attempt is made to take into account the perturbations to the planarity of the shock wave.

3D. The Shock-Attenuation Experiments

The analysis procedure described in section 3C does not give any quantitative means for the determination of the initial overtaking point. Section 3C merely describes the handling of the data when the initial overtaking point, x_1 , and the second break point, x_2 , are given. It is possible to include x_1 and x_2 in the equations of section 3C as unknowns that are to be determined by a minimization of some objective function such as the sum of the squares of the residuals. The problem with such a technique is that it leads to a highly nonlinear fitting routine. Another problem is a program such as this is dependent to some extent on the continuity of the objective function with respect to the unknown parameters. In the present case the objective function is not continuous with respect to x_1 and x_2 . Besides these problems, it is not clear whether the scatter in the data would allow

an accurate determination of these break points. This question will be examined in some detail in the present section.

It is desirable to have some quantitative guide for judging or choosing where the point x_1 might lie. The starting point for determining where this point might be is the consideration of the assumptions stated in section 3C for the analysis of the data. Specifically, the assumption that the flyer plate has uniform properties is needed here. When the additional assumption is made that the flyer plate is at normal density and zero pressure just before it collides with the target plate, one can derive a rather simple formula for the point x_1 . For a flyer and target plate made of the same material, the equation for x_1 is

$$x_1 = h \frac{c + U_s - u_p}{c - U_s + u_p} . \quad (3.24)$$

In this equation, h is the thickness of the flyer plate and c is the sound speed of the lead characteristic in the release wave. The shock speed in equation (3.24) does not have to be measured from the shock-wave trajectory data. In each experiment the trajectory of the flyer plate is also measured and the flyer plate velocity may be calculated from this data. The shock speed in the target may then be calculated on the basis of the flyer plate velocity. By making reasonable estimates about the maximum and minimum values for c in equation (3.24), quantitative estimates for the minimum and maximum values of the overtaking point may be made.

A maximum value for c in equation (3.24) can be found by considering the nature of the shock process. The shock represents an irreversible process, while the release wave is assumed to be isentropic. This implies that in the P - ρ plane any isentrope that intersects the principal Hugoniot must have a slope less than the slope of the Hugoniot at the point of intersection. For a material whose principal Hugoniot is adequately described by a linear U_s - u_p relation, equation (2.59) or equation (2.60) can be used to find a maximum value for c .

A minimum value for c can also be found from equation (2.59) or (2.60). In appendix D it is shown that in the absence of any phase changes, γ will decrease as the density increases along the principal Hugoniot. A reasonable estimate for the minimum value of c can thus be found by replacing γ by γ_o in equation (2.60).

In each shock-wave attenuation experiment the pin data may be divided into two separate sets. One set of pin measurements pertains to the flyer plate while the second set is the t - x data for the shock wave in the target plate. The analysis of this first set of pin data is the same as the analysis of the data of the flying plate experiments in section 3B. The pin data for the flyer plate in each experiment are fit to equation (3.1). From this fit the velocity of the flyer plate can be calculated. Based solely on the calculated flyer plate velocity, the initial shock speed and the minimum and maximum values of x_1 in the target plate can be predicted. Table 5 gives the measured flyer plate velocity for each shock-attenuation experiment, the

initial shock speed as predicted from the flyer plate velocity, and the minimum and maximum values of x_1 using equations (2.59) and (2.60) to calculate maximum and minimum sound speeds. From this table it is very noticeable that the values for x_1 are smaller in the copper experiments than in the stainless-steel experiments. This occurs because of the higher shock speed in stainless steel as opposed to copper for comparable pressures.

The δt plots do result from a fitting procedure as explained in section 3C. The division of the shock-wave data into three zones ($x_i \leq x_1, x_1 < x_i \leq x_2, x_2 < x_i$) permits a sharp demarcation of the data into two different regions. The first zone is the region where the shock is assumed to be moving at a constant velocity. The second region is composed of the second and third zones where it is assumed that the shock wave is being attenuated by the release wave. The separation of this second region into two zones by the break point, x_2 , is partly a convenience for fitting the adjusted δt data by equations (3.16) and (3.17). The fitting or smoothing functions are of course quadratic or cubic polynomials.

Obviously, the choice of where x_1 lies has an effect on the calculation of δt . Consider for the moment when the true position of x_1 is x^* . If x_1 is now chosen such that x_1 is very much greater than x^* , then one would expect the calculation of the initial shock velocity would be too low and the values of δt for the zones where $x_i > x_1$ would also be too low. If the assumption that an initially flat-

topped shock is introduced into the target is correct, it would be expected that the shock velocity calculated when x_1 is much smaller than x^* would be very close to the value of the shock velocity when all the pins with $x_i < x^*$ are used to calculate it. The values for δt in the attenuation region would also be expected to be very similar in these two cases.

The position of the second break point, x_2 , and the option of cubic or quadratic polynomials for the fitting or smoothing functions in the second and third zones have a negligible effect on the calculation of δt ; however, they do have a pronounced effect on the calculation of shock velocities in these zones. The choices of the position of x_2 and the option of the fitting functions are thus not strictly a matter of convenience, but can be made from an examination of the shock velocity plots.

For experiment RS-2302, table 5 suggests that the earliest observation of the overtaking point should occur between the 7 and 8-mm pin levels. Figure 7 is a δt - x plot for this experiment in which the first break point is at 7.8 mm and the second break point is at 8.5 mm. The dashed line in the figure is the result of fitting equations (3.16) and (3.17) to the δt data in the attenuation region. In this particular plot the quadratic forms for equations (3.16) and (3.17) were used. The shock velocity as calculated from these fits is shown in figure 8. Referring to figure 7, it is fairly safe to say that those pins at the 11-mm level and beyond are

definitely in the region where the shock wave is decaying. Following the general shape of the data in the attenuation region, it is probably safe to conclude that the pins at the 10-mm level are also in the attenuation region. What is difficult to conclude is whether the pins in the 8- and 9-mm levels should be in the attenuation region.

As previously stated, the general effect of changing the degree of the fitting functions and the position of the point x_2 can dramatically change the shape of the shock velocity curve while having a negligible effect on the δt - x plot. Figure 9 plots δt in which the first and second break points are the same as in figure 7 but cubic polynomials have been used in zones two and three. The δt data have changed imperceptibly, but the velocity graph (figure 10) based on these new fits has changed considerably. The shock velocity in the first zone is the same as that calculated for the first zone in figure 8. The difference is the shock velocity in the attenuation region. Once the attenuation of the shock wave has begun no positive acceleration of the shock wave is expected. Thus the large increase in the shock velocity seen in the second zone is physically unreasonable. If a simple substitution is made in zone two of a quadratic polynomial instead of a cubic polynomial, one then gets figure 11 for the shock velocity. This simple change of the fitting function for zone two has eliminated the acceleration of the shock wave and produced a smooth velocity curve as can be seen in this plot.

The main problem with using a cubic polynomial in either zone two or three is the possibility of the δt data having too much scatter and the cubic polynomial following the scatter in the data. This is what happened in figure 10. In the same figure, the large amount of data in zone three has produced a reasonably smooth portion of the velocity curve even though a cubic polynomial has been used for this zone. The suitability of using a quadratic or cubic polynomial is thus dependent not only on the scatter in the data but on the amount of data in the zone as well. An example of this is seen in figures 12 and 13. In these figures, x_1 is the same as in figure 9 and cubic polynomials were used in the attenuation region but the second break point has been moved from 8.5 to 11.5 mm. A comparison of the velocity plots in figures 10 and 13 shows that by simply changing the position of the second break point a much smoother velocity curve has been achieved.

When x_1 is varied in the interval between two levels of pin data, there is essentially no change in the δt data and no change in the calculated shock velocity of zone one. The variation of x_1 only produces a change in the δt data and the initial shock velocity when the number of pin levels in the first zone changes. This effect can be seen when figure 7 is compared with figure 14. The δt data for figure 14 is the same as in figure 7 even though x_1 has been moved from 7.8 to 7.1 mm. When the first break point is moved to 4.5 mm, the δt points do indeed change as a comparison of figure 14 with figure 15 shows. The changes in the data occur

because the calculated value for the shock velocity in the first zone has changed. When only the first four pin levels as opposed to the first seven pin levels are used in zone one, a lower initial shock velocity is calculated for experiment RS-2302 and this in turn produces the noticeable differences in the δt plots of figures 14 and 15. The shock velocity plot for x_1 at 4.5 mm and x_2 at 8.5 mm using quadratic polynomials in the attenuation region is shown in figure 16. Just as in figure 10, zone two in this plot shows a positive acceleration of the shock wave.

The maximum value of x_1 given in table 5 for experiment RS-2302 implies that the pins in the 10-mm level may belong to the unperturbed region instead of the region where the shock wave is decaying. A qualitative examination of figure 7 suggested this was not the case. When x_1 is taken to be at 10.2 mm, x_2 at 10.6 mm, and quadratic polynomials are used in zones two and three, one arrives at figure 17. The shock velocity plot for this fit is shown in figure 18. A comparison of figure 17 with figure 7 reveals that the δt data have not changed very much, but the shock velocity in the attenuation region has changed dramatically. There is a large and extremely rapid drop in the shock velocity in zone two and the discontinuity in the slope of the shock velocity at the second break point has become more pronounced. A shift of the first break point from 10.2 to 9.2 leads to similar results shown in figures 19 and 20. Attempts at smoothing the velocity plots can be made by moving x_2 and going to a cubic fitting function for zone

three. With x_1 at 10.2 mm, x_2 at 12.5, and a change from a quadratic to a cubic polynomial in zone three one gets the shock velocity curve shown in figure 21. The large and rapid decrease in shock velocity in zone two is still present and a positive acceleration of the shock wave has now appeared in zone three. A similar result for x_1 at 9.2 mm is seen in figure 22. When x_2 is left at 12.5 mm and the fitting functions are left as a quadratic polynomial in the second zone and a cubic polynomial in the third zone but x_1 is moved to 8.2 mm, one obtains figures 23 and 24. The δt plot for x_1 at 8.2 mm is not dissimilar from the δt plots with x_1 at 10.2 and 9.2 mm, but the behavior of the shock velocity shows a much smoother curve. The question is whether one expects a smooth decay or attenuation of the shock. Just on the basis of the δt plots alone one might not be able to conclude much about the position of x_1 , but when the velocity plots are also considered the situation is considerably altered. Assuming that the sharp decreases in the shock velocity seen in figures 18 and 20 are not the expected behavior, one may conclude the 9-and 10-mm pin levels are in the attenuation region in experiment RS-2302.

The initial shock velocities calculated for zone one and the standard deviations for some of the fits to the data for experiment RS-2302 are listed in table 6. The column headed by the term "spline" refers to the polynomials that were used in the attenuation region. An entry in this column such as "2-3" means that a

quadratic polynomial was used in zone two and a cubic polynomial in zone three. The term σ_1 is the standard deviation of the fit in the first zone or unperturbed region and σ_2 is the standard deviation in the attenuation region. The term σ is the standard deviation of the overall fit.

Although the table does not contain as much information as the δt and shock velocity plots, it does show some quantitative information that is difficult to discern from the plots. Even though the fits with the first break point less than 7 mm in experiment RS-2302 may be rejected on the arguments of the maximum speed of the release wave, they are included in the table as illustrating how the shock speed calculation in the unperturbed region is changing as more pin levels are added to the first zone. The statistics of the fits with x_1 between 7.0 and 9.0 mm show there is essentially no statistical difference among these fits and one cannot distinguish the position of x_1 on the basis of these fits alone. The additional tools for the acceptance or rejection of the first break point are physical arguments on the region where x_1 might lie and the expected manner in which the shock wave will be attenuated. It should be noted that the smallest standard deviation for the data of experiment RS-2302 is around 7.1 ns and this must be considered as the inherent scatter of the data for this experiment.

The fact that the variation of x_1 in the interval between successive pin levels does not change the shock velocity in zone one or the δt plots indicates that an

adequate choice for the initial analysis of the pin data may be made by choosing x_1 to lie halfway between two successive pin levels. To arrive at velocity plots, one can make the rather arbitrary choice that x_2 will be 1 mm away from x_1 . Since this choice will mean that the second zone will have a paucity of data, one can insist on a quadratic polynomial in zone two to prevent the scatter in the data from having an undue influence on the shock velocity plot. The choice of a quadratic or cubic polynomial for zone three may not have the same influence as in zone two but again a quadratic polynomial may be chosen as being less susceptible to strange accelerations of the shock wave in zone three if there is a good deal of scatter in the data for this zone. An illustration of using such a procedure for experiment RS-2302 is shown in figures 25 through 30. The δt plots and their respective velocity plots show essentially the same behavior as that determined previously. The remainder of the shock-wave experiments will be analyzed in this simplified manner.

The next experiment after RS-2302 using stainless steel as the target plate was experiment RS-2309. This shot produced the greatest shock speeds measured in any of the shock-attenuation experiments. Table 7 lists data for some of the fits to the data of experiment RS-2309. The first of these fits had x_1 at 5.5 mm and in successive fits the first break point was stepped in 1-mm increments out to a maximum value of 9.5 mm. Only quadratic polynomials were used in these

fits. Just a cursory examination of this table reveals an increase in the overall standard deviation for the fit with x_1 at 8.5 mm. Figure 31 shows the reason for this increase: the 8-mm pin level should not be included in zone one, but rather it should be included with the pin data in the attenuation region. Figure 32 shows the $\delta t-x$ plot for the fit with x_1 at 7.5 mm and figure 33 is the shock velocity plot for this fit. The deceleration of the shock wave in zone two is large and certainly suggests that the pins at the 7-mm level might also lie in the attenuation region. Figures 34 and 35 are the plots for the fit in table 7 with x_1 at 6.5 mm. The velocity plot in figure 35 shows a more gradual decrease of the shock velocity in zone two than does the fit with x_1 at 7.5 mm but an even smoother velocity plot is produced when x_1 is moved to 5.5 mm. This last plot shown in figure 36 is somewhat disturbing since it implies that the 6-mm pins should be in the attenuation region, which contradicts the assumption made about the maximum sound speed in deriving the values for the minimum position of x_1 in table 5. A discussion of this point will be reserved for chapter 4.

Table 8 lists data for fits of experiment RS-2315, which was the last of the stainless-steel experiments conducted in this research. The situation for this experiment is somewhat similar to experiment RS-2309. The plots associated with the fit that has x_1 at 8.5 mm are shown in figures 37 and 38. These plots indicate that both the 8- and 9-mm pin levels are in the attenuation region and not zone

one. The δt data and shock velocity graphs for the fit with x_1 at 7.5 mm are displayed in figures 39 and 40. Even these plots look a little suspicious as though the 7-mm pin level should be in the attenuation region. The plots for x_1 at 6.5 mm are in figures 41 and 42. This fit gives a smoother shock velocity plot than the fit with x_1 at 7.5 mm. As was the case in experiment RS-2309, these plots suggest that the overtaking point may occur before the minimum value predicted in table 5.

The first experiment using OFHC copper for the target plate was experiment RS-2306. The initial shock velocity and other quantities for some fits to the experimental data of RS-2306 are listed in table 9. The respective δt - x plot and the associated shock velocity plot for each of these fits are shown in figures 43 through 50. The minimum value for x_1 listed in table 5 is 6.4 mm. The fit with x_1 at 6.5 mm gives as small a standard deviation as seen in table 9 and the shock velocity curve in figure 46 is the smoothest of the velocity curves derived from these fits. The fit with x_1 at 5.5 mm shows an acceleration of the shock wave in zone two, which implies that not enough pin data have been included in the first zone. The fit with x_1 at 8.5 mm shows the large deceleration of the shock wave in zone two that seems to be typical of these plots when too many pin levels are included in the unperturbed region. The fit with x_1 at 7.5 mm is somewhat similar to the fit with x_1 at 8.5 mm but the shock velocity in zone two does not

show as great or rapid a drop in the velocity, as is seen in figure 50, which makes it difficult to judge whether the 7-mm pin level should be in zone one or not.

Due to a slight error in machining the copper target plate for experiment RS-2308, the first pin level was nominally set at 1.3 mm with subsequent levels machined at 1-mm increments; thus the midway point between successive pin levels occurs at .8 instead of at .5 as in the previous experiments. The first fit listed in table 10 for RS-2308 starts with a value for x_1 that is less than the minimum value of x_1 listed in table 5 for RS-2308. The δt and shock velocity plots in figures 51 and 52 for this fit certainly seem to agree with table 5 that a 4.8 mm position for x_1 is too small. When the first break point is moved to 5.8 mm, the calculated value for the initial shock velocity increases from 7.19 km/s to 7.24 km/s. The δt plot for x_1 at 5.8 mm is shown figure 53. The interesting thing about the shock velocity plot in figure 54 is that zone two shows only a very gradual decrease in the shock velocity. This is certainly suggestive that a 5.8-mm position for x_1 may not be great enough. Figures 55 and 56 are for the fit with x_1 at 6.8 mm. Figure 56 for this fit shows an extremely smooth shock velocity curve. The plots for the 8.8-mm fit are shown in figures 57 and 58. These plots indicate that the 8.3-mm pin level should be in zone two or three. The next to the last fit (x_1 at 7.8 mm) is plotted in figures 59 and 60. This fit does not give as smooth a shock velocity curve as the 6.8-mm fit but there is enough ambiguity

in the plots that one cannot say in a definitive manner whether the 7.3-mm pin level should be in the attenuation region or not.

Figure 61 through figure 68 are the plots associated with fits given in table 11 for experiment RS-2314. This experiment is quite similar to the other two copper experiments. The fit using x_1 at 8.5 mm indicates that the 8-mm pin level is in the attenuation region and not zone one. The plots with the first break point at 5.5 mm seem to indicate that indeed this is too small a value for the overtaking position. The better overall fits are achieved with x_1 at 6.5 and 7.5 mm.

The analyses of the shock wave experiments have been intended to accurately find the position at which the shock wave is overtaken by the release wave in each experiment. Unfortunately, the scatter in the data has prevented an accurate determination of the point x_1 in any of the experiments. There is, however, sufficient resolution of the data that some conclusions can be drawn about a region or interval in which the overtaking of the shock wave by the release wave probably occurred. This region or interval for each experiment can be determined using only the analysis techniques in this chapter. A brief summary of the results extracted from these experiments is given in table 12. This table lists the interval or region for each experiment in which the release wave probably overtook the shock wave as well as the speeds of the flyer plate and the shock wave as calculated from the pin measurements. The values for the shock speeds are actually

weighted values associated with the unperturbed region of each experiment. For the copper experiments the point x_1 could only be determined within a 2-mm interval with the region being essentially the same for each experiment. In the stainless-steel experiments there is a little more variation of the overtaking region and in experiment RS-2315 the overtaking interval appears to be determined to within 1 mm as opposed to a 2-mm interval for the other experiments. The experiments seem to suggest that the speed of the lead characteristic in stainless steel may be greater than the maximum sound speed given by the slope of the principal Hugoniot while in copper this is probably not the case.

There are some anomalies in the data that need more investigation. The shock speeds measured in these experiments are generally one to two per cent above the values predicted by the impedance-matching calculations using the measured velocities of the flyer plates. The one exception to this is experiment RS-2308, which has a measured shock velocity about one per cent lower than the predicted value in table 5. This is not all that suprising when one considers the uncertainty in the flyer plate velocities but this could be intimately connected to one of the basic assumptions being made in the analyses of these experiments. An examination of the δt plots for the data in zone one shows that in most of these plots there appears to be some curvature of the data in this region. This could be caused by a small pressure gradient and hence a small density gradient existing in the

flyer plate at the time it collides with the target plate. This would mean that the assumption that the flyer plate has uniform properties is erroneous. These questions concerning the state of the flyer and the curvature of the data will be addressed in more detail in the next chapter.

COMPUTER MODELING OF THE EXPERIMENTS

4A. Preliminaries

In the analyses of the shock trajectory data in section 3D some questions arose that could not be answered based on that simplified approach to the data. A more thorough examination of the experimental data can be accomplished by resorting to computer modeling of the experiments in which the trajectory of the shock wave was measured. Problems concerning the curvature of the data or the shape of the velocity curve in the attenuation region can all be addressed in a calculational investigation.

Of the questions raised in section 3D, one of the most fundamental concerns the state of the flyer at the time it collides with the target plate. The basic assumptions that the flyer plate has uniform properties and is at normal density at the time it collides with the target plate must be questioned in light of the apparent curvature exhibited in many of the δt plots for the data in the supposedly undisturbed region. The importance of the initial state of the flyer plate is difficult to emphasize too much since the modeling of the experiments is dependent upon the initial conditions.

When the flyer plate is assumed to have uniform properties, it explicitly follows that the shocks introduced into the flyer plate and target plate initially move at constant velocity. It also follows that the rarefaction wave created when

the shock wave in the flyer plate reaches the free surface of the flyer will be a simple-centered wave. If the target and flyer are made of the same material and have the same initial states, then the rarefaction fan will remain a simple-centered wave until it overtakes the shock wave in the target plate. The modeling of shock attenuation is considerably simplified when one can consider the problem as a simple-centered wave overtaking a shock wave. Even when the target and flyer are dissimilar materials, the assumption that the flyer plate initially has no gradients in it means that analytic methods may still be used to calculate the overtaking point as was done in section 3D. Numerical methods must still be used to calculate the decay of the shock wave, but the fact that an analytic expression exists for the overtaking position is a great benefit.

For the case when gradients do exist in the flyer plate at the time it collides with the target plate, shock waves cannot propagate in either the flyer or the target plate at constant velocities but must show accelerations as a result of the gradients in the flyer. Once the shock wave traveling in the flyer plate has reached the back surface of the flyer, the release wave no longer travels into a constant state region and thus the rarefaction fan cannot be a simple wave. There is also no simple analytic expression for the overtaking point and the whole problem from the propagation of the shock waves to the subsequent decay of the shock wave in the target plate must be solved numerically. The modeling of these experiments

will use both assumptions concerning the initial state of the flyer. When the assumption is made that the initial state in the flyer is not uniform, the initial condition of the flyer plate will be calculated in the manner shown in appendix E.

Another question of fundamental importance is one concerning the maximum speed of the release wave. In section 3D a simple argument was made that the slope of the principal Hugoniot in the P - ρ plane is the limiting factor in determining the maximum speed in the rarefaction fan. The stainless-steel experiments seemed to contradict this hypothesis. The argument of section 3D concerning the maximum speed of the release wave is valid for a material behaving as an inviscid fluid, but it is not valid for a material described by the constitutive models of section 2F. These constitutive models have non-zero deviatoric stresses and a discontinuity in the wave speeds at an elastic-plastic boundary. This results in a release process that is fundamentally different from that of a material that cannot support a deviatoric stress. Making the assumption that on the principal Hugoniot the normal component of the deviatoric stress is at an upper yield limit implies that in the initial stages of the unloading process the material will be in an elastic state. In this elastic state the slope of the release path in the P - ρ or σ - ρ plane may be greater than the slope of the principal Hugoniot. Eventually the material goes into a plastic state and the sound speeds are then essentially determined by bulk properties alone. For such material models, a compressive wave for a state already

on the principal Hugoniot would not take the material into an elastic regime but into a plastic state and the wave speeds would once again be determined by bulk properties only.

There is some difficulty in sorting out the influence of the strength models and the influence of only the equations of state on the attenuation of a shock wave. Even the consideration of the simpler case in which strength effects are not modeled presents a rather complicated flow in the attenuation of a shock wave. An heuristic approach to the problem may be taken by considering the situation of a simple-centered wave overtaking a shock wave. A diagram for such a flow is shown in figure 69. This figure is similar to figure 1 but certain regions have been labeled and some additional characteristic lines have been drawn. Region I is the region which has not yet been hit by the shock wave. The line labeled " U_s " is the path or trajectory of the shock wave. Region II represents a material region sitting at peak shock pressure, particle velocity, etc. The state of the material is uniform throughout region II. The simple-centered release wave occupies region III. Region IV is a final state which is characterized by zero pressure in this region. The line labeled " $u-c$ " is a left-going characteristic given by equation (2.19) which originates from the overtaking point x_1, t_1 . This line is also the left boundary for region V. The right boundary for region V is the line labeled " u ", which is the material or particle path for the point at which the shock wave

was intercepted by the lead characteristic of the release wave. In region III the right-going characteristics have a constant slope; but, once the release wave enters region V, the wave no longer remains a simple wave. The specific entropy can be considered to be constant in regions II, III, IV, and V. In region VI the entropy is different along each particle path since each particle has been shocked to a different state on the principal Hugoniot. The right-going characteristics are not shown in regions V and VI except for the solid line, which is the hypothetical continuation of the dashed line of region III.

In region III each right-going characteristic has a constant value of u and c . The particle velocity on each right-going characteristic can be found from the equation

$$u = u_2 + \int_{P_2}^P \frac{1}{\rho c} dP. \quad (4.1)$$

In this equation u_2 and P_2 are the particle velocity and pressure in region II. Equation (4.1) is the compatibility equation for the left-going characteristics of region III. Every left-going characteristic in region III originated in region II. Because all the material in region II and region III has seen the same peak shock conditions, equation (4.1) may be written as

$$u = u_2 + \int_{\rho_2}^{\rho} \frac{c}{\rho} d\rho. \quad (4.2)$$

By the same token the specific internal energy on each right-going characteristic

may be found from the equation

$$E = E_2 + \int_{\rho_2}^{\rho} \frac{P}{\rho^2} d\rho. \quad (4.3)$$

For a general equation of state in which the pressure is given as a function of specific internal energy and density, the sound speed will in general be a function of these same quantities as well. Equations (4.2) and (4.3) may be integrated numerically to find the conditions along each right-going characteristic of region III.

In region V, the simple-wave solution is no longer appropriate because the left-going characteristics no longer originate in region II but in region VI. Any point in region V may be considered as being intersected by three different characteristics, a right-going characteristic, a left-going characteristic, and a particle path. The right-going characteristic may be considered as originating from the left boundary of region V and thus the compatibility equation for the right-going characteristic may be written as

$$u = u_3 + \int_{P_3}^P \frac{1}{\rho c} dP, \quad (4.4)$$

where u_3 and P_3 are the particle velocity and pressure for the appropriate point on the left boundary. Along the left-going characteristic the particle velocity is given by

$$u = u_s + \int_{P_s}^P \frac{1}{\rho c} dP. \quad (4.5)$$

In this equation u_3 and P_3 represent the particle velocity and pressure for the point of origin of the left-going characteristic on the decaying portion of the shock wave. The last characteristic direction is the particle path and the compatibility equation along this path in region V is again given by equation (4.3). The fact that the material in region V has all been shocked to the same peak pressure implies that the right-going characteristic can be considered as being isentropic and equation (4.4) can be written as

$$u = u_3 + \int_{\rho_3}^{\rho} \frac{c}{\rho} d\rho. \quad (4.6)$$

The replacement of dP by $c^2 d\rho$ cannot be made in equation (4.5) because part of the integration occurs in region VI. The left-going characteristics in region VI traverse different particle paths which are also different isentropes. The integral in equation (4.5) can be split into two integrals, one for region V and one for region VI. For that integral in region V the replacement of dP by $c^2 d\rho$ may be made.

The integrals presented thus far are only the compatibility equations and they hold only along the characteristic directions or paths. In order to integrate them numerically it is necessary to know the specific paths of the characteristics. The differential forms for these paths are given by equations (2.17)–(2.19). The complexity of the problem is apparent and this is why it is necessary to resort to numerical techniques to solve the problem of shock attenuation. In some special

cases the problem is amenable to analytic methods but for a general equation of state the problem is only tractable using numerical techniques.

A further understanding of the problem may be accomplished by resorting to a model in which the equation of state is a function of density only. When the equation of state is a function of density alone, the energy equation is no longer necessary and a particle path is not a characteristic direction. There are just two characteristic directions and they are given by equations (2.18) and (2.19). For such an equation of state, regions V and VI of figure 69 can be considered as one region since the specific internal energy is not needed in determining the pressure.

Some interesting questions concerning the behavior of the characteristics in the wave interaction region can be addressed with this model. One question is whether the right-going characteristics of the simple wave can remain lines of constant slope in region V with constant values of u , c , and ρ ? The first step in answering this question is to assume that the characteristics are straight lines everywhere and find what requirements are placed on the equation of state by this assumption. The changing conditions at the shock front must be related to the changing conditions in the wave interaction region. For initial conditions of zero pressure and zero particle velocity ahead of the shock, equations (2.1) and (2.2) can be solved to give

$$u^2 = P \left(\frac{1}{\rho_o} - \frac{1}{\rho} \right). \quad (4.7)$$

Equation (4.7) holds at the shock front. Along a left-going characteristic behind the shock, the change of the particle velocity with respect to the density is given by

$$\frac{du}{d\rho} = \frac{c}{\rho}. \quad (4.8)$$

Equation (4.8) is valid for any left-going characteristic in any region because it is explicitly assumed that the pressure is a function of density only. If the right-going characteristics are to remain straight lines in region V with constant values of u , c , and ρ , then equations (4.7) and (4.8) may be used to arrive at a differential equation for P :

$$\frac{1}{P} \frac{dP}{d\rho} = \frac{\rho_o}{\rho(\rho - \rho_o)}. \quad (4.9)$$

The solution of equation (4.9) results in an equation of state which does not satisfy the Bethe-Weyl conditions in that the second derivative of pressure with respect to density is less than zero. A material that has such an equation of state cannot support a compressive shock wave. The conclusion is that for an equation of state with pressure as a function of density only and satisfying the Bethe-Weyl conditions, the $u + c$ characteristics cannot maintain constant values of u , c , and ρ in region V.

Such conclusions cannot be so easily reached when the equation of state exhibits an energy dependence. When the equation of state exhibits an energy dependence, then equation (2.11) must be included in the system of partial dif-

ferential equations that govern the flow. The additional complications that arise from the inclusion of the energy equation make a simple analysis impossible. Another analysis of the attenuation problem using a highly specialized equation of state that includes an energy dependence is given in appendix B. The equation of state discussed in appendix B results in an analytical solution for the trajectory of the shock wave.

4B. Hydrodynamic and Simple Strength Modeling

The experiments using SS-304L for the target plate will be modeled first because the modeling of the copper experiments is dependent on knowing the behavior of the stainless-steel flyer plate. The first step in modeling these experiments is to start with a simple hydrodynamic model and assume that the flyer plate is at normal density and zero pressure at the time it collides with the target plate. This implies that figure 69 gives the correct qualitative description of the shock attenuation process. The program SHKSIM described in appendix A can then be used to calculate the decay of the shock wave. The primary goal of this first step is to test the hypothesis of whether or not the attenuation of the shock wave can be suitably modeled as a simple-centered release wave overtaking a shock wave.

Important considerations in setting up calculations on the code SHKSIM are

the initial conditions. One input parameter is the thickness of the flyer plate. Another input parameter is the velocity of the flyer plate. This input parameter can be taken as the measured value of the velocity of the flyer but it is not. In general the pin data for the flying plate are not as good as the pin data for the trajectory of the shock wave in the target plate. For this reason the velocity of the flyer plate taken from the pin data monitoring the flyer is not used for the calculations done with the code SHKSIM. Rather, the flyer plate velocity is adjusted as an input parameter so that the shock velocity given by the code in the region before the release wave catches the shock, agrees with a shock velocity determined from the experimental data for the target plate.

The results of a calculation by the code SHKSIM for experiment RS-2302 are shown in figure 70. For the comparison of the calculational results to the experimental data some choices have to be made as to the location of the first and second break points for the analysis of the pin data. Whenever possible the first break point for the analysis of the pin data was chosen to coincide with the initial overtaking point given by the code calculation. In some instances such a choice for the first break point was not warranted. For each δt - x plot that shows a comparison between a calculation and experimental data there is a table listing information about the analysis of the data and the calculation shown in the plot. Information pertaining to the code calculation and the data analysis for figure 70

are listed in table 13.

In the last section some questions concerning the behavior of right-going characteristics in the wave interaction region were raised. For the particular calculation shown in figure 70 these questions can now be answered. This calculation used equation (2.44) for the equation of state. For the $u + c$ characteristics that were specifically followed in the calculation there was very little change in the slope of any characteristic through the wave interaction region. Referring to figure 69, the hypothetical continuation of a right-going characteristic from region III through regions V and VI is shown with a decreasing value of $u + c$. The behavior of a right-going characteristic is just the opposite in the present calculation. The value of $u + c$ on a right-going characteristic increases as the characteristic traverses the wave interaction region. A specific example of this is given in table 14. The starting values are the values of variables on the characteristic when it is at the left boundary of region V in figure 69. The ending values for the characteristic occur when it intersects the shock front. The characteristic listed in table 14 intersected the shock front at 19.27 mm. The calculation ended when the shock front reached 20.0 mm. The increase in $u + c$ from table 14 is well under one percent. As might be expected from the work in appendix F, both the pressure and specific internal energy decrease along the characteristic while density increases. These results should not be generalized to other calculations using different equations of

state since the details of how much the variables change is highly dependent on the equation of state.

Figure 70 shows a large discrepancy between the experimental data and the calculational results. The calculation does not show as much attenuation of the shock wave as is seen in the experiment. This discrepancy could have its origin in the equation of state used in the calculation or could be due to the basic assumptions of the modeling. To address the question concerning the equation of state, comparisons between the data of RS-2302 and calculations using the other equations of state are shown in figures 71–73. Each of these calculations also shows a noticeable discrepancy in comparison with the experimental data. None of the equations of state predict as much attenuation as is seen experimentally. The best agreement of a calculation with experiment RS-2302 is shown in figure 71. This calculation used the Rice equation of state in which the sound speed is independent of the specific internal energy. The examination of the respective tables for these plots reveals that this equation of state is the only one that predicts the overtaking of the shock wave by the release wave before the 9-mm pin level in this experiment.

The discrepancy between the calculations and the experimental data of RS-2302 is indicative of a fundamental inadequacy of the modeling; however, it is possible that something could be wrong with the data of RS-2302. To answer this

question a set of purely hydrodynamic calculations using the code SHKSIM was performed for experiments RS-2315 and RS-2309. The comparisons between these calculations and the experiments are shown in figures 74-81. Information for the data analysis in each figure and other information pertaining to the calculation are given in the appropriate table (tables 18-25). The results from these calculations are qualitatively the same as the calculational results shown in figures 70-73. None of these calculations show as much attenuation of the shock wave as is seen in either experiment. Again the best agreement between a calculation and an experiment is achieved when the Rice equation of state is used in the modeling of the experiment.

The fact that none of these preliminary calculations accurately match any of the experiments does suggest that something is indeed wrong with the modeling. There are other possible explanations for the lack of agreement between the calculations and the experiments. Again one such hypothesis for the source of the discrepancy is that the data are in error. If this is true, then it must be highly systematic since all the stainless-steel experiments show the same qualitative behavior in comparisons with code calculations. Another possible source of errors is numerical problems in the codes, but the work of appendix B contradicts this hypothesis. Assuming that these are not the sources of the discrepancies, then one is left with the original questions concerning the basic assumptions of the modeling.

Either the equations of state are not giving the correct bulk compressibilities at these high pressures or the assumptions of modeling the attenuation of the shock wave as a strictly hydrodynamic process is incorrect. Now the equations of state do a good job reproducing the principal Hugoniot data and there is no a priori reason to expect that the equations of state would be grossly inadequate for computing bulk sound speeds. This suggests that the basic assumption of modeling the attenuation process as a simple-centered release wave in which strength is not important must be questioned.

Further illumination of this problem can be gained by an examination of the shock velocities from the code calculations and comparing them against the shock velocities derived from the pin data. A comparison of the experimental and calculational shock velocities of figure 70 is shown in figure 82. Similar comparisons of the shock velocities for figure 74 and figure 78 are shown in figures 83 and 84. The solid line in each figure is the shock velocity from the fit to the data and the dashed line is from the code calculation. For experiment RS-2302 shown in figure 82, the velocity curve from the code calculation seems to parallel the experimentally determined shock curve but is just translated. For the other two experiments the calculated shock velocity curves appear to be converging toward the experimental curves. Anything in the modeling that would cause the release wave to come in earlier or faster would thus enhance the agreement between the

calculations and the experiments.

The next step in modeling these experiments is to include strength effects. A suitable starting point is the strength modeling in section 2F. The constitutive equations for strength given in that section are examples of simple elastic-plastic models. As stated previously an elastic-plastic model implies there is a region defined by a yield surface within which the material behaves in an elastic manner. Outside this region the material behaves plastically. The material in this region behaves more like a fluid but one in which deviatoric stresses still exist and satisfy the yield condition. The effects of such a simple strength model in the code can be anticipated. The most noticeable effect of an elastic-plastic model is that the speed of the lead characteristic in the release wave should increase as discussed in section IIF. This is easily seen in equations (2.100) and (2.101), which give the longitudinal sound speeds for a material obeying the constitutive equations of section 2F. Another noticeable effect of the strength models will occur in a simple-centered release wave. The rarefaction or release wave will separate into two distinct parts. This happens because of the discontinuity in wave speeds when the material goes from an elastic state into a plastic state; thus, the elastic part of the rarefaction fan will outrun the plastic part.

Of the equations of state used in the purely hydrodynamic calculations for shock attenuation in stainless steel, the Rice equation of state gave the best agree-

ment with the experiments. Using this equation of state and equation (2.93) for the stress rate, the effects of the constitutive equations for strength in a code calculation of the attenuation of a shock wave are shown in figures 85 and 86. The calculation and data shown in these figures are for experiment RS-2302. Table 26 lists parameters for the calculation and the analysis of the pin data for the plots displayed in these figures. The code employed for the calculation was the code SHKSIB since the code SHKSIM cannot handle a strength model. The immediate consequences of using a simple strength model in a calculation for experiment RS-2302 are easily seen in these figures. The overtake position occurs much earlier and the plateau or shoulder caused by the discontinuity in the wave speeds between the elastic and plastic regions is clearly defined in the shock velocity plot of figure 86. Figure 85 can be directly compared with figure 71, which shows a purely hydrodynamic calculation for experiment RS-2302 that uses the same equation of state. Although the overtake position does occur much earlier in figure 85 than in figure 71, the overall agreement between the data and the calculational results is not significantly improved. Part of the explanation for this can be found from an examination of the shock velocity plots for the two calculations. Figure 87 shows the shock velocities associated with figure 71. In comparing figures 86 and 87, the calculation with strength rapidly approaches the solution which does not include strength.

In making comparisons between the data and calculations and displaying them in $\delta t-x$ plots, some mention must be made of the sensitivity of the comparisons to the choice of the first break point for the analysis of the data. For figure 85 the first break point was chosen to be at 7.1 mm. This meant that the first seven levels of pin data were used in determining the initial shock velocity in the target plate. This was done in spite of the fact that the calculation suggested a better choice for the first break point was at 6.9 mm. If only the first six pin levels are used for the first zone of the data analysis, there is a significant reduction in the value for the initial shock velocity of experiment RS-2302. A calculation performed to match this value for the initial shock velocity does show better agreement with the data. An example of this is shown in figure 88. Undoubtedly the better agreement results from the lower value for the initial shock velocity of the data. An examination of the shock velocity plot in figure 89 still shows divergence of the calculated shock velocity curve from the experimental curve. For this reason the value of the shock velocity calculated with the first seven pin levels is probably the better value. Another clue for this assumption is that the pins at the 1-mm pin level do appear to be early. If these pins are deleted from the first zone of the data for the determination of the shock velocity, then the 2-6-mm pin levels give an initial shock velocity of 7.36 km/s. This agrees with the shock velocity calculated from the data for figure 85.

Another example of a calculation using a strength model is shown in figure 90. The calculation shown in this figure for experiment RS-2315 uses the same equation for the stress rate and the same equation of state as the calculation shown in figure 85. Figure 90 can be compared with figure 75, which shows a calculation for the same experiment but one which does not have strength. Still another calculation using the same equation of state and strength model as the calculations plotted in figures 85 and 90 is given in figure 91 for RS-2309. Figure 79 shows a comparable calculation for RS-2309 in which strength was not used. Again the most noticeable effect of the strength model is the movement of overtaking point. The calculations with strength show a little better agreement with the data for the intermediate pin levels than the calculations without strength but at the higher pin levels there is little difference between the two types of calculations. This is also confirmed by an examination of the shock velocities for figures 90 and 91. The shock velocities for figure 90 are shown in figure 92 and those for figure 91 are plotted in figure 93.

Some interesting trends can be seen in the strength calculations that have been done thus far for the three stainless-steel experiments. These trends are most apparent in the shock velocity plots. In experiments RS-2315 and RS-2309, the initial attenuation of the shock wave shows a definite divergence of the experimental and computational shock velocity curves. Just the opposite behavior

is seen in the velocity curves in figure 86 for RS-2302. Some reservation must be taken before making too much of this trend because the experimental shock velocity curves are highly dependent on the scatter of the data as well as the data analysis.

To make sure there was nothing peculiar to these strength calculations, another set of strength calculations was done using a different equation of state. In figures 94-99 are shown experimental and calculational results for each of the stainless-steel experiments. The code calculations for these figures used the Tillotson equation of state but the exact same strength model as that used previously. The same qualitative effects are seen in going from calculations without strength to calculations with strength for the Tillotson equation of state as were seen for the calculations using the Mie-Gruneisen equation of state.

Yet another question that must be addressed is the one concerning the condition of the flyer plate at the time it impacts the target plate. So far all the calculations that have been shown have been done using the initial conditions that the flyer plate is at normal density, zero pressure, zero internal energy, and a uniform particle velocity. The work in appendix E shows how a density and pressure gradient may be estimated for the flyer plate. The effect of such gradients in the flyer plate will now be examined by repeating a strength calculation with gradients in the flyer plate for the initial conditions.

The initial conditions for the flyer plate are calculated in accordance with the procedure described in appendix E. For experiment RS-2302 it is assumed that the back boundary condition of the flyer plate is a pressure of 1.3 GPa. The front boundary condition is zero pressure. A linear density gradient in the flyer plate is assumed and each material point is assumed to be on a release isentrope from a shock pressure of 7.5 GPa. This assumption plus equation (e9) fixes, ρ , E , and P at each point in the flyer. To fix the material or particle velocity in the flyer, some value that lies within the error bars of the measured velocity of the flyer plate is used as the velocity at the front boundary of the flyer. The particle velocity at each succeeding point in the plate is then determined by finding the difference in the Riemann integrals for each point and the front surface and subtracting this difference from the chosen velocity for the front surface.

An example of a calculation with these initial conditions in the flyer plate is shown in figure 100. A shock velocity plot from this calculation is shown in figure 101. These nonuniform initial conditions in the flyer plate do not have a large effect on the calculation of shock attenuation but there are definite effects. One effect is that the overtake position changes by about 0.1 mm. Another effect is there is slightly more dispersion in the initial part of the release wave. Also the strength of the shock wave grows slightly before it is overtaken by the release wave. This last effect is readily seen as an increase in the shock velocity in figure

101. This qualitatively explains why in many of the experiments an increase in the shock velocity of the first zone of the data is seen as more pin levels are added to this zone. In estimating the initial conditions for the flyer plate, the greatest uncertainty is probably the choice of the particle velocity gradient across the flyer plate at the moment of impact. There is some ambiguity in calculating δt for code calculations in which the flyer plate is given nonuniform conditions. This occurs because the shock velocity is not constant in the region before the release wave overtakes the shock. In general, δt for the code calculations is found from the equation

$$\delta t = t - \frac{x}{U_s} \quad (4.10)$$

The choice of U_s in equation (4.10) when the initial conditions of the flyer plate are uniform and the same as the target plate, except for particle velocity, is unambiguous. In this case, it is just the initial shock velocity calculated from an impedance match solution for the impact of the flyer plate into the target plate. For a calculation with nonuniform initial conditions in the flyer, a nominal value of the shock velocity in the region free from attenuation is used in equation (4.10).

The modeling that has been done up to this point for the stainless-steel experiments shows that strength is important in the code calculations. The simple strength model does produce better agreement between the code calculations and the data. The inclusion of nonuniform initial conditions in the flyer plate does seem

to qualitatively explain the behavior of the pin data in the first zone. Before going to more complicated or elaborate strength models for use in the computations for the stainless-steel experiments, it is worth while to do some calculations for the copper experiments to see if they follow the same trend as the calculations for the stainless-steel experiments. This will be done even though none of the stainless-steel experiments has been closely matched by calculations. In the calculations for the copper experiments, the Rice equation of state and equation (2.93) will be used as the constitutive equations for the flyer plate because these equations produced the best match with the stainless-steel data.

The logical calculation for the first of the copper experiments is to use the same type of equation of state and strength model that were the most successful in the modeling of the stainless-steel experiments. For copper the rigidity modulus and yield strength are much smaller than for stainless steel so it is expected that the simple strength model used thus far will have less of an effect in calculations for the attenuation of a shock wave in copper than in the modeling of shock-wave attenuation in stainless steel. A $\delta t-x$ plot for the pin data of experiment RS-2306 as well as a calculation from SHKSJB that models this experiment are shown in figure 102. The code calculation does a fair job in matching the data. A plot of the shock velocities from figure 102 is shown in figure 103. It is very apparent from this figure that the strength model in the calculation for RS-2306

is responsible for only a very small amount of the attenuation of the shock wave. When the calculation is repeated without a strength model for the copper target plate, the calculation matches the data nearly as well as with a strength model. The repeated calculation is shown in figure 104.

The calculations for experiment RS-2306 are much like the calculations for the stainless-steel experiments in that the calculations that come the closest in matching the data use the Rice equation of state. Calculations for RS-2306 using the other equations of state are shown in figures 105–107. These calculations also use a strength model for the copper target. The fact that the yield strength for copper is only 0.12 GPa and is treated as a constant in these calculations means that the strength model has an almost negligible effect on the attenuation of a shock wave. The only appreciable effect of the strength model is to move the location of the overtaking point. Another calculation for RS-2306 using the Tillotson equation of state but no strength in the copper is shown in figure 108. Comparison of figures 108 and 106 again demonstrates the very small effect of the strength model in the copper calculations.

The next set of copper calculations were for experiment RS-2314. Since the strength model has been shown to have such a small effect in the calculations for the attenuation of a shock wave in the copper target, no strength model was used in the modeling of the target plate in these calculations. The surprising thing

about this set of calculations for experiment RS-2314 is that the equation of state that worked best in matching the data of RS-2306 did not give the best agreement for RS-2314. The results of the calculations for experiment RS-2314 are shown in figures 109-112. Additional information about the calculations and the analysis of the data for the figures can be found in the appropriate tables. The striking feature of the calculation shown in figure 109 is that it shows greater attenuation of the shock wave than is seen in the experiment. This is quite a reversal of the calculational results for experiment RS-2306. Although the equation of state used for the calculation in figure 109 gave the best agreement with the data of any of the calculations for experiment RS-2306, it definitely did not predict too much attenuation for that experiment. In experiment RS-2314 this equation of state definitely gives too much attenuation. The better agreement between the calculational results and the data of RS-2314 is seen in figures 110 and 112. The calculations shown in these plots use the Mie-Gruneisen equation of state with $\gamma\rho$ a constant and the Gray equation of state, respectively. These equations of state give very similar results. Lastly, the calculation using the Tillotson equation of state shown in figure 111 still produces too little attenuation of the shock.

Further insight into the behavior of the code calculations in comparison to the experimental data can be gained by examining the shock velocity curves. The shock velocity curves for the calculation and the data of figure 109 are shown

in figure 113. The calculation shows a more rapid and precipitous drop in the shock velocity from the initial overtake point than the data. This is in contrast to figure 114, which shows good agreement between the shock velocity curves of the calculation and the data for the initial part of the attenuation. The δt - x plot for figure 114 is shown in figure 110.

Plots of δt versus x and shock velocities versus x for experiment RS-2308 are shown in figures 115-122. Figure 115 shows that the calculation using the Rice equation of state again predicts too much attenuation of the shock wave. The calculation using the Mie-Gruneisen equation of state in which $\gamma\rho$ is constant is shown in figure 117 and does a better job of matching the data. This calculation is very similar to the calculation using the Gray equation of state shown in figure 121, but these calculations now seem to be showing a little too much attenuation. The best job of matching the data of RS-2308 with a calculation is seen in figure 119. The calculation in this figure uses the Tillotson equation of state.

4C. Additional Modeling

All the calculations shown thus far have been done with the parameters of the strength models and the equations of state fixed by zero pressure data and principal Hugoniot data. No attempt has been made to adjust any of the parameters to make the calculations fit the shock-attenuation data. The calculations in

section 4B were done to show the general behavior of the material models in the codes and to give some insight in determining what may need improvement.

It must be remembered that these experiments follow the attenuation of a shock wave over a limited pressure range. Maximum pressure in any of the experiments is not above 150 GPa and the minimum pressure at a shock front is around 50 GPa. In this pressure regime the comparisons between the calculations and the experimental data showed some interesting trends. Within the limited modeling done in section 4B, quite different behavior is seen in the stainless-steel experiments as compared to the copper experiments. Qualitatively, the results of the last section indicate that the combination of a strength model with the equation of state that gives the greatest isentropic bulk modulus along the principal Hugoniot does the best job in matching the data of the stainless-steel experiments. In the copper experiments this is not the case. The general trend seen in the copper experiments is to go to material models that give lower wave velocities as the peak pressures in the experiments increase. This response suggests the copper may be in a melt phase in experiments RS-2314 and RS-2308. If the copper is not in a mixed phase region on the principal Hugoniot, then it may well be melting on the release isentrope. This would agree with work reported on the phase line of copper.⁴⁵

The possibility of a material melting on the principal Hugoniot or on a release

isentropes point out a rather glaring deficiency in three of the four equations of state discussed in chapter 2. Of the equations of state discussed in that chapter only the Gray equation of state takes into account the phase transformation of melting. Also the Gray equation of state is the only one that explicitly includes electronic effects, although in a relatively simple fashion. None of the calculations using the Gray equation of state showed the copper entering a melt region. This could be because the Gray equation of state uses an approximation of the Lindemann law for melting and for some materials this melting law has been found to give rather high values for the pressure at which melting begins on the principal Hugoniot.^{4C}

The other aspect of the modeling that is lacking is the treatment of the rigidity modulus and the yield strength. In the code calculations performed in the last section, the strength parameters were simply treated as constants whenever a strength model was employed. On the basis of elasticity theory, it is probably a better assumption to consider the rigidity modulus as being proportional to the bulk modulus. In many high pressure experiments, increases in the rigidity modulus as well as the yield strength have been reported.⁴⁷⁻⁴⁹ Also some account of the reduction of the yield strength along an isobar as the specific internal energy or temperature increases should be considered. A good example of a more comprehensive treatment of strength parameters is given in reference 49.

Before actually invoking any new material models it is interesting to explore in greater detail the interplay of the simple strength models and the equation of state. While many experiments have been done that report values of the yield strength and the rigidity or shear modulus at high pressure, it is important to remember that in most cases the values that are reported are dependent on the equation of state used to interpret the data or upon other assumptions made about the bulk response of the material. In analyzing such experiments, the difficulty lies in trying to sort the bulk properties from those that give rise to deviatoric terms. The present experiments suffer from the same problem but the main problem with the present experiments is the ambiguity in the determination of the initial overtaking point. There are times when it is clear that strength models are not needed. This is the situation for experiment RS-2308 in which it is fairly obvious that no strength model is needed to explain the attenuation of the shock wave in the copper target. In making this statement, the implicit assumption is made that one of the equations of state is correctly modeling the bulk response of the material.

In the modeling performed in section 4B in which the rigidity modulus and the yield strength were treated as constants with values appropriate for ambient conditions, it was seen that only one of the four equations of state given in chapter II gave what can be considered as adequate agreement with the experimental data

for stainless steel. If the rigidity modulus and the yield strength are now treated as adjustable parameters then better agreement between the data and calculations using the other equations of state can be obtained. This then is a curve-fitting exercise that is intended to give insight into the interplay of the constitutive equations for strength and the equations of state.

An example of changing the values for the strength parameters to achieve better agreement between the calculations and the data of experiment RS-2302 is shown in figures 123–125. For the three calculations in these figures, the strength model is the same as that used previously. The Tillotson equation of state was used to model bulk response in each calculation. In each succeeding figure the yield strength in the calculation has been increased until a reasonably good fit to the data has been achieved. Shown in figures 126–128 are comparisons of the shock velocity plots for these calculations and the experiment. The interesting item is the behavior of the shock velocity in these new calculations as revealed in figures 126–128. As the yield strength increases, the elastic-plastic plateau or shoulder occurs at a lower value of the shock velocity. There is also the appearance of additional structure in the shock velocity curve beyond the plateau region that becomes more pronounced as the yield strength increases.

The influence of the increased yield strength can be split into two principal effects. The first effect is that in the simple-centered release wave, the elastic

part of the release wave encompasses a larger drop or change in the normal stress and the density. For the strength model used in these calculations, the change in density across the elastic part of the release wave is given by

$$\Delta \rho = \frac{Y_0}{\mu} \rho_0. \quad (4.11)$$

The second effect is related to the first but now brings into play the plastic part of the release wave. Because of the increased yield strength and the increased attenuation of the shock wave by the elastic part of the release wave, there is a greater portion of the wave interaction region that will support elastic wave speeds. For a release wave overtaking a shock wave moving to the right, this will mean that a $u + c$ characteristic of the rarefaction wave that has a plastic wave speed will encounter earlier that portion of the wave interaction region that supports elastic wave speeds. This in turn attenuates the shock at a faster rate.

In the work of section 4B, the calculations using the Tillotson equation of state produced the poorest agreement of any of the equations of state with the data of the stainless-steel experiments. The plots in figures 123–125 show that by adjusting the values of the strength parameters, the calculational modeling using the Tillotson equation of state can be made to fit the data of RS-2302 reasonably well. Although the comparison of the data and the calculation in figure 125 is pretty good, one might be hesitant to say that the comparison of

the shock velocities in figure 128 is as good. When the same type of adjustment of the strength parameters is done for the calculations for experiments RS-2315 and RS-2309 that use the Tillotson equation of state, there is better agreement shown between the calculations and the data in the shock velocity plots than in figure 128 for experiment RS-2302. Figure 129 shows δt - x plots of the calculational results and the experimental data for RS-2315. The shock velocity from the calculation and the shock velocity from the pin data are shown in figure 130. Both the δt data and the shock velocity from the pin data have been matched fairly well by the calculation. Another example of forcing a better match to the data by adjusting the strength parameters is shown in figures 131 and 132 for RS-2309. The calculation shown in these figures was also done using the Tillotson equation of state.

In order to achieve a closer match between the data of the stainless-steel experiments and the calculations using the Tillotson equation of state some rather large increases in the yield strength have been needed. The rigidity modulus has not been increased by more than a factor of two but the yield strength has been increased by more than an order of magnitude. The same type of curve fitting that has been done using the Tillotson equation of state can be done with any of the other equations of state. When this is done it is found that the adjustment of the yield strength to force a fit to the data is dependent on the equation of state. A

specific example of this is shown in figures 133–138. All the calculations shown in these figures use the equation of state in which the Gruneisen parameter is given by equation (2.41). It is significant that in order to get the close match to the data in these new calculations the yield strength has been reduced by a factor of two as compared to the calculations using the Tillotson equation of state. The fact that the rigidity or shear modulus has essentially the same values as those used in the calculations with the Tillotson equation of state just reflects the uncertainty in the determination of the overtaking point in these experiments.

These examples of adjusting the values of the parameters of the strength models to gain better agreement between the calculations and the experimental data, while to some extent are exercises in curve fitting, do show how the choice of an equation of state can influence the values of the parameters of the strength model. While the values of the rigidity modulus and the yield strength have been changed, at no time were the values of the parameters in the equations of state changed. In the calculations of section 4B where the deviatoric strength was always less than one percent of the total stress, there was little influence of the strength model on the calculation of the principal Hugoniot when it is based on the equation of state and the strength model. When the yield strength is allowed to go as high as 4.76 GPa, the deviatoric stress can be nearly four or five percent of the total stress at the shock front in the pressure regime of interest in these

experiments. This large yield strength can cause a significant change or shift in the calculation of the Hugoniot. An example of this is given by considering a 100 GPa shock in stainless steel using the Tillotson equation of state. For the purely hydrodynamic case, the values for the density, specific internal energy, shock velocity, and particle velocity on the Hugoniot are, respectively, 10.46 g/cc, 1.546 GPa-cc/g, 7.196 km/s, and 1.243 km/s. If the yield strength is now increased to 4.5 GPa, then the values for the same quantities are respectively, 10.39 g/cc, 1.515 GPa-cc/g, 7.268 km/s, and 1.231 km/s. Although the deviatoric stress is 3% of the total stress when the yield strength is 4.5 GPa, none of the other quantities change by 3%. The density and particle velocity each decrease by about 1%. The internal energy decreases by about 2% and the shock velocity increases by about 1%. In general, principal Hugoniot data is thought to be good to about 1%. Thus the influence of such a high yield strength is just great enough that one should consider making changes to the equations of state to allow for the increased yield strength; however, no changes will be made to the equations of state at this time.

In yet other experiments⁵⁰ where a material has been both released and reshocked from the same high pressure state, there is some hope in trying to sort the bulk properties from the deviatoric properties; but even these experiments have their problems. Most experiments at pressures greater than 10 GPa that investigate strengths of materials have been done using wave propagation tech-

niques. Whenever these techniques are employed, the interpretation of the data is again dependent on the equation of state and other aspects of the constitutive modeling. For the present experiments not all constitutive models pertinent to the examination or modeling of the experiments will be investigated. Such possible material response as viscoelasticity and viscoplasticity will be ignored.

It would be desirable if a single equation of state and strength model could be found that fits all the stainless-steel and copper data. Up to now the constitutive equations for strength have been added to the hydrodynamic model in a rather ad hoc fashion. There has been no attempt to relate the strength parameters to the equation of state in any manner except to treat the strength parameters as constants. The first problem that arises is to find a model that relates the strength parameters to the equation of state in a consistent manner. One such model that does this is given by Steinberg, Cochran, and Guinan.⁴⁹ An alternative to that model is to generalize linear elasticity theory. For an isotropic material, linear elasticity theory gives a relation between the rigidity modulus and the isothermal bulk modulus:

$$\mu = \frac{3}{2} \left(\frac{1 - 2\nu}{1 + \nu} \right) K. \quad (4.12)$$

In this equation ν is Poisson's ratio and K is the isothermal bulk modulus. Equation (4.12) is appropriate for small isothermal deformations. The analogue of equation (4.12) for the propagation of waves in a linear elastic medium occurs

when the isentropic bulk modulus is substituted for the isothermal bulk modulus in equation (4.12). Because the isentropic bulk modulus can be directly calculated from the equation of state, equation (4.12) becomes

$$\mu = \frac{3}{2} \left(\frac{1 - 2\nu}{1 + \nu} \right) \rho c^2, \quad (4.13)$$

where c^2 is given by equation (2.23). In doing this, ν in equation (4.13) can now be identified as the isentropic value of Poisson's ratio. To continue with this model it is necessary to relate the yield strength to other parameters. The yield strength can be related to the rigidity modulus by assuming that the ratio of the yield strength to the rigidity modulus is constant:

$$Y = Y_o \frac{\mu}{\mu_o}. \quad (4.14)$$

This equation is taken from reference 49 when work hardening and strain rate effects are ignored. If the yield strength increases at a faster rate than the rigidity modulus, then equation (4.14) can be changed to

$$Y = Y_o \left(\frac{\mu}{\mu_o} \right)^b. \quad (4.15)$$

In equation (4.15), the parameter b may be assumed to be a constant. With these equations for μ and Y , the strength parameters are intimately connected to the equation of state.

The problem of treating thermal effects and how they should be incorporated in the constitutive equations for strength still remains. Two different approaches

may be taken with respect to the calculation of temperature. Temperature may be directly incorporated into the equation of state as is done in the Gray equation of state or a temperature may be calculated from the equation of state solely for use in calculating the parameters μ and Y in the strength model. Obviously in the latter case, the phenomenon of melting cannot be properly treated with an equation of state based only on a single phase of the material. In this latter situation, the temperature is being used as a means to turn off strength effects as the temperature approaches the melting temperature. When the temperature is not directly calculated in the equation of state, then a separate reference curve such as a zero-degree isotherm must be found to allow the calculation of temperature at higher densities and pressures.

One manner of incorporating temperature effects in the strength model is to assume that ν in equation (4.13) depends on the temperature. One requirement of the strength model is that μ and Y should be zero when the material is at the melt temperature. This implies that ν should be one-half when the temperature T is equal to T_m , the temperature at which melting begins. This in turn suggests that ν has a functional dependence on the ratio of T to T_m :

$$\nu = \nu\left(\frac{T}{T_m}\right). \quad (4.16)$$

The melting temperature, T_m , can be calculated from one of the melting formulas discussed in appendix C. Equation (4.16) may be expanded in a power series but

a particularly simple form for ν may be written as

$$\nu = \nu_1 + \nu_2 \left(\frac{T}{T_m} \right)^x. \quad (4.17)$$

Equation (4.17) is intended to approximate the general behavior of Poisson's ratio. If there is sufficient isobaric data, one can assume a truncated power series for equation (4.16) and calculate the coefficients for the power series. If there is not sufficient data, then equation (4.17) should qualitatively describe the behavior of ν . While a material might show a highly skewed dependence of ν on the temperature at zero pressure there is the possibility that ν might exhibit a more linear dependence at higher pressures. Thus one could make the exponent in equation (4.17) dependent on the pressure.

In order to evaluate the parameters ν_1 and ν_2 in equation (4.17), one needs to know the value of the exponent, x , and an initial value of ν . If ν has an initial value of ν_0 at the temperature T_0 , then ν_1 and ν_2 must satisfy the following:

$$\nu_1 + \nu_2 = \frac{1}{2}, \quad (4.18)$$

and

$$\nu_1 + \nu_2 \left(\frac{T_0}{T_m} \right)^x = \nu_0. \quad (4.19)$$

The difficult problem lies in trying to determine the value of the exponent, x , in equation (4.17). A metal such as a steel might show less than a 1% change in ν for

a temperature rise of 100 K. This might represent a 20 to 30% change in the ratio of the temperature to the melting temperature. Thus for steel, a minimum value for x is probably in the range of 3 to 4. Longitudinal and transverse sound speed data for copper⁵¹ indicate a minimum value for x that is in the same range. These values for x are taken from data measured under ambient pressure conditions.

Examples of calculations using a strength model given by equations (4.13), (4.15), and (4.17), are plotted against the appropriate stainless-steel experiment in figures 139–144. The equation of state used for this set of calculations was the one by Rice given by equation (2.42). This strength model has a significant dependence on the calculation of Poisson's ratio, which in turn depends on the calculation of the temperature and the melting temperature. The melting temperature was calculated using the Kraut-Kennedy melting formula discussed in appendix C. Instead of using a value of 2.9 for A in equation (c11) as suggested in the appendix, a higher value of 4.9 was used. The reason for doing this was that work on iron gives the intersection of the phase boundary of the solid with the principal Hugoniot at around 240 GPa.⁵² If the stainless steel hits the melting region in the same pressure regime, then a value of 2.9 for A is too low. For this equation of state and a value of A in the Kraut-Kennedy formula of 4.9, melting begins on the principal Hugoniot at about 220 GPa. Equally important in this strength model is the calculation of the temperature. The temperature for equation (4.17) was

calculated from equation (2.51). The cold curve or zero-degree isotherm needed for equation (2.51) and the room-temperature isotherm needed for the Kraut-Kennedy melting formula were both directly calculated from the Rice equation of state. The value for c_v in equation (2.51) was the Dulong and Petit value and the value of g for stainless steel was taken to be 1.74×10^{-8} GPa-cc/(K²-g). After calculating both T and T_m one still needs the exponents for equation (4.17) and (4.15). Instead of using an exponent of 3 or greater, a value of 2 was used for x in equation (4.17). A value of 1.5 was used for b in equation (4.15).

The plots in figures 139–144 show that this model for the strength parameters does a fairly good job of matching the data for the stainless-steel experiments. In the previous calculations using constant values for μ and Y it was found that these parameters had to be adjusted for each experiment in order to obtain a reasonably good fit between a calculation and the data. In this new model for the strength parameters, some fine-tuning of the model can be achieved by adjusting the exponents in equations (4.15) and (4.17). When x is changed in equation (4.17), ν_1 and ν_2 must also be adjusted according to equations (4.18) and (4.19). In this model for the strength parameters the magnitude of the yield strength can be controlled to some extent by the value of b in equation (4.15). Thus if the equation of state is changed and it is found that it is necessary to increase the value of the yield strength, this can be accomplished by increasing the value of b .

When the Kraut-Kennedy formula was used for the calculation of the melting temperature in the code calculations shown in figures 139-144, it was not possible to match both the zero pressure data for the phase line and still have the phase line intersect the principal Hugoniot of stainless-steel at 220 GPa. The melting formula given by equation (c16) overcomes this problem. It is clear from the manner in which this strength model is formulated that the yield strength and the rigidity modulus are sensitive to the calculation of Poisson's ratio. When the melting formula is changed, this of course has an influence on the calculation Poisson's ratio and hence the rigidity modulus and the yield strength. The effects of changing from the Kraut-Kennedy melting formula to equation (c16) are shown in figures 145- 150. In general the two melting laws produce about the same results but when equation (c16) is used for the melting law, the value of Poisson's ratio is slightly higher. The examination of the appropriate tables shows that the use of equation(c16) moves the overtake position about 0.1 mm.

Although the melting formula does have an effect on the calculations, a relatively larger effect is seen when the exponent in equation (4.17) is changed. When this exponent for the calculations shown in figures 145-150 is changed from 2 to 1, there is a significant increase in the value of Poisson's ratio along the principal Hugoniot. This produces lower values of the rigidity modulus and the yield strength. Examples of using a value of 1 instead of 2 for the exponent in equation

(4.17) are shown in figures 151-156. The comparisons show that the match to the data is actually a little better for the intermediate pin levels of experiment RS-2302 when the exponent is 1 instead of 2; however, for experiments RS-2315 and RS-2309 the new calculations shown in figures 153 and 155 do not agree with the data quite as well as the previous calculations.

There is one aspect of wave propagation at high pressure that has not yet been discussed. In the many experiments performed at high pressures that examine release waves, the elastic-plastic plateau or shoulder that is so clearly defined in the calculations using strength, has not been observed experimentally.⁴⁹ The lack of a clear separation between the elastic and plastic regions of the release wave has been attributed to the Bauschinger effect. The Bauschinger effect⁵³ is a phenomenon noted in stress-strain tests where there is reverse loading or cyclic loading of the test specimen. The modeling of this phenomenon in a hydrodynamics code can be quite complicated.⁵⁴ Suffice it to say that there are two effects in the modeling of this phenomenon that have an influence on the present problem. The first effect is that the discontinuity in wave speeds in going from the elastic regime to the plastic regime is effectively eliminated. This wipes out the plateau region seen in the shock velocity plots. The second effect is there is more dispersion to the elastic regime of the release wave. This in turn obviates the need for the yield strengths to be as large as they are in the calculations that use the adjustment

of the yield strength to match experimental data. For the calculations shown in figures 139–144, the modeling of the Bauschinger effect would decrease the value of b from 1.5 for these calculations, but just how much this decrease would be is hard to say.

With this new strength model for the stainless steel, attention can now be turned to the modeling of the copper experiments. Changing to the strength model of stainless steel used for the calculations in figures 145–150 does not change any of the conclusions reached previously about the copper experiments. The modeling that was done indicated that each of the separate equations of state did a fair job of matching one of the copper experiments without the need of a strength model. The Rice equation of state matched the data of experiment RS-2306 while the Gray and the Mie-Gruneisen equation of state matched the data of experiment RS-2314. For the copper experiment with the highest pressure, RS-2308, the Tillotson equation of state did the best job of matching the experimental data. The fact that a different equation of state is used in matching the data of each succeeding copper experiment and each successive equation of state gives a higher compressibility would seem to indicate that the copper is indeed undergoing a phase change.

The most difficult aspect of the modeling situation for copper is trying to include a liquid phase of the material in the equation of state. In lieu of a sat-

isfactory procedure for doing this, parameters in the equation of state can be adjusted to achieve a match between the calculations and the experimental data. Up to this point no parameters in the equation of state have been adjusted to force an agreement between calculations and experiments. The strength parameters, μ and Y , have been used in this manner, but no parameters in the equations of state have been used to do this. If either the Rice or the Mie-Gruneisen equation of state is used to do this, there are only four parameters in the equation of state to work with. If the equation of state is to reproduce the principal Hugoniot, then all parameters in the Rice or the Mie-Gruneisen equation of state except one are fixed by the principal Hugoniot. The one exception is the Gruneisen parameter, γ_0 . By adjusting this parameter for the calculations of each of the copper experiments, then the general trend for the sound speeds of the release waves in the copper experiments can be ascertained.

A particularly interesting calculational exercise can be carried out using the Mie-Gruneisen equation of state to match the data of the copper experiments. A Mie-Gruneisen form based on a linear U_s-u_p relation is quite useful for doing this. What makes it so useful is that the Gruneisen parameter can be adjusted in such an equation of state without having any influence on the principal Hugoniot; that is, the principal Hugoniot derived from the equation of state is independent of the Gruneisen parameter. Using equation (2.44) for the equation of state of copper it

is found that a value of 1.3 for γ_0 in a code calculation matches the data for experiment RS-2306. The experiment next higher in pressure, RS-2314, is matched quite well with a value of 2.0 for the Gruneisen parameter. For experiment RS-2308 it is found that a value of 2.5 is needed for the Gruneisen parameter in order to match the data. The comparisons between these calculations and the experimental data are shown in figures 157-162. The material model for the stainless steel in these calculations is the same as that used in the calculations for figures 145-150.

4D. Further Discussion

The original goal of the experiments has been stated as being an accurate measurement of the sound speed in copper and stainless steel at high pressures. Although this primary goal was not obtained because of the scatter in the data, the experimental data proved to have sufficient resolution to be very useful for comparisons with the calculational modeling. The experiments have enough resolution and are simple enough conceptually that any model of material behavior can be checked against the present data.

Two rather general statements can be made about the behavior of copper and stainless steel at the pressures in these experiments. For the stainless steel, it appears that the modeling of strength parameters is important to shock pressures

of at least 150 GPa and probably higher. For the OFHC copper, strength modeling is not as important in the same pressure range; but it does appear that the copper is undergoing a phase transition to a liquid state.

The basis for making these statements about strength for stainless steel and the melting of copper is founded on comparisons of the calculational results with the experimental data. It was clear from the calculations of the last chapter that a match to the data of the stainless-steel experiments could not be done without invoking a strength model. This must be qualified in that a match could not be obtained with a purely hydrodynamic calculation unless one were willing to alter the parameters in the equation of state. An example of this is the calculation for experiment RS-2302 shown in figure 70. This calculation used a Mie-Gruneisen equation of state and a value of 2.07 for γ_o . There was no strength model in the calculation and the calculation did not match the experiment. If this calculation is now repeated but with a value of 0.8 for the Gruneisen parameter, a very good match to the data is achieved. The calculation for this new value of γ_o is shown in figures 163 and 164. The point that must be made is that calculations for RS-2315 and RS-2309 using the same equation of state and the same value of 0.8 for γ_o do not match the experimental data. Thus a simple change of γ_o from its nominal value at zero pressure in purely hydrodynamic calculations cannot account for the variability seen in the speeds of the release waves in the three experiments. By

switching to a strength model in which the strength parameters are related to the equation of state and by using the Rice equation of state, a moderately good fit to all three stainless-steel experiments was obtained (see figures 145-150).

It is of course possible to make the argument that strength effects are not important for the stainless steel and that the lack of agreement between the experiments and the hydrodynamic calculations in which there is no strength model merely points out the deficiencies in the equations of state. There are reasons for not accepting this argument. One reason is that strength effects are seen unequivocally in static pressure experiments and low pressure dynamic experiments. Another reason for rejecting the argument that the whole problem lies with the equation of state is that the present data can be explained by using a quite reasonable strength model. As long as the material remains in the solid phase, there is no reason not to expect possible strength effects. These arguments must not be interpreted as saying there are no deficiencies in the equations of state. There are still questions concerning the bulk response of material at high pressure that must be addressed.

A principal question raised by this work is, "Which equation of state is the most appropriate for the description of a metallic solid under high pressures?". To some extent the answer to this question is subjective and much depends upon the assumptions made about such diverse quantities as the strength parameters, the

zero-degree isotherm, phase lines, and temperatures along the principal Hugoniot.

The calculations in this chapter demonstrate the interplay of the constitutive equations for strength and the equations of state. When Y_o was used as an adjustable parameter for the calculations, it was found that the different equations of state required different values of the yield strength in order to match the same data. If one believed that the equation of state that needed the lowest value of the yield strength was the correct or most appropriate equation of state, then one would use the Rice equation of state.

Another consideration in choosing an equation of state is the behavior of the lattice Gruneisen parameter. In both the Mie-Gruneisen and the Rice equations of state the lattice definition of the Gruneisen parameter and the thermodynamic definition are the same. In the Gray and the Tillotson equations of state the thermodynamic definition is not assumed to be a function of density only. Of the three forms (equations (2.41), (2.43), and (2.48)) for the lattice definition of the Gruneisen parameter, the form (equation (2.48)) used for the Gray equation of state agrees the best with the work done in appendix D. On this basis alone one might choose the Gray equation of state over the others but there are other aspects of the Gruneisen parameter that are also important. Both the form given by equation (2.41) and the form in which $\gamma\rho$ is a constant give lower values of γ at densities greater than normal density than equation (2.48). As shown in appendix

D, the form for the Rice equation of state gives much lower values of the Gruneisen parameter than the other forms and as a consequence the pressure and specific internal energy along the zero-degree isotherm are greater for compressed states than along the zero-degree isotherms given by the other equations of state. As a result of the higher energies for the zero-degree isotherm, when equation (2.51) is used to calculate temperatures along the principal Hugoniot, the Rice equation of state gives appreciably lower values of the temperature than the other three equations of state discussed in chapter 2.

Each of the equations of state has its own idiosyncracies and arguments can be made for or against using a particular equation of state. The Rice equation of state gives lower temperatures and compressibilities along the principal Hugoniot than the other equations of state. In matching code calculations to the stainless-steel data, the Rice equation of state needs lower values of the yield strength and the rigidity modulus than the other equations of state. Although these facts suggest that the Rice equation of state may be better for the description of a metal in a highly compressed solid state than the other equations of state discussed in section 2D, there are other pressure regimes and thermodynamic states where a different equation of state is clearly preferable.

One area in which this is true is the region where Thomas-Fermi theory is believed to hold. A deficiency that the Mie-Gruneisen, Rice, and Gray equations

of state all have is the density singularity in the zero-degree isotherm. This singularity is a direct result of a linear U_s-u_p relation. The limit of the maximum density for these three equations of state is given by

$$\rho = \rho_0 \left(\frac{s}{s-1} \right)^s. \quad (4.20)$$

Because the value of s for most materials is around 1.5, equation (4.20) limits the maximum density to about three times the normal density. This limit is well below the densities allowed in the Tillotson equation of state and so the Tillotson equation of state is expected to be more appropriate for pressure regimes in the Thomas-Fermi region.

The example just presented for preferring one equation of state over another is not that pertinent to the present study but is shown for illustrative purposes. The reason that it is not relevant is that pressures in the Thomas-Fermi region are at least one or two orders of magnitude greater than the pressures generated in these experiments. It does show, however, that at times a theoretical basis for a choice about the equation of state exists. It cannot be emphasized too greatly that of the equations of state used in these calculations, there are no clearcut theoretical justifications for saying one equation of state is preferable over another in the pressure regimes of these experiments. At best, one can place an emphasis on a certain aspect of the equation of state such as the behavior of the lattice Gruneisen

parameter and then use this as a basis for making a choice about the equation of state.

Many of the shortcomings of the equations of state have been presented in chapter 2. A farther elaboration of some of the problems and uncertainties of the equations of state can only be discussed in the context of the manner in which the equations of state are being used. The equations of state in the computer calculations are used to give two quantities of critical importance, the pressure as a function of the density and the specific internal energy, and the bulk sound speed or isentropic compressibility as a function of the same quantities. The uncertainties in the equations of state do not lie so much in the pressure as in the bulk sound speed.

The measurements in the experiments are the trajectories of shock fronts and the shock fronts are followed from maximum pressures of less than 150 GPa to minimum shock pressures of around 50 GPa. Because of the range of pressures followed in the experiments, the states of interest in the P - ρ plane are never very far from the principal Hugoniot. The farthest point of interest off the Hugoniot may be taken as a pressure of approximately 50 GPa on a release isentrope that intersects the principal Hugoniot at 150 GPa. This holds for both the copper experiments and the stainless-steel experiments. All the equations of state faithfully reproduce the principal Hugoniot data for both copper and stainless steel

and they all give release isentropes from 150 GPa that are quite similar. There are some noticeable differences on the release isentropes in the P - ρ plane but they occur as the isentropes approach zero pressure. It is for these reason that one is led to the conclusion that the calculation of the pressure from the equations of state is not a problem.

Even though all four equations of state give release isentropes in the P - ρ plane that agree quite well in the pressure range of these experiments, the situation for the bulk sound speed is not as good. Restricting the loci of states to be considered to the principal Hugoniot, the agreement of the bulk sound speed among the different equations of state is pretty good for pressures below 50 GPa. The largest difference is seen between the Rice and the Tillotson equations of state. At 50 GPa the difference is 1.5% for stainless steel and for copper the difference is 1.8%. As the shock pressure increases, the difference in sound speeds between the Rice equation of state and the Tillotson equation of state increases. When the shock pressure reaches 130 GPa, the difference in the sound speed has grown to 5.3% for stainless steel and 6.5% for copper. It is the differences in the bulk sound speeds when integrated over time that accounts for the very appreciable differences seen in the calculations of the attenuation of a shock wave when different equations of state are used.

All the equations of state used in this work can be considered as being semi-

empirical equations of state since all are based on matching the principal Hugoniot of a material. This means all the equations of state will give reliable answers for shock states. The differences among the equations of state arise in the calculation of the compressibility or the bulk sound speed. This uncertainty or variation in the sound speeds actually exists as well in the calculation of the pressure from the equations of state but is masked to a certain extent because of the semi-empirical nature of the equations of state. Consider for the moment equation (2.26). If the pressure on the principal Hugoniot is substituted for the left-hand side of equation (2.26), the known quantities are the total pressure, density, and specific internal energy along the principal Hugoniot. What is not known is the correct partitioning of the right-hand side of equation (2.26) between the cold pressure, P_c , and the thermal pressure, P_t . Each equation of state has a different partition between the cold pressure and the thermal pressure and it is this aspect of the equations of state that leads to the variations seen in the sound speeds.

The general ideas of modeling strength for the calculations pertaining to these high pressure experiments are extrapolations and generalizations of the phenomenologic descriptions of elasticity and plasticity. In this respect the correct modeling of strength parameters at high pressures is much less certain than the equations of state.

Different models or treatments of the strength parameters have been used

in the computer calculations. One treatment has been simply to treat the yield strength, Y , and the rigidity modulus, μ , as constants. The comparison of the calculational results with the experimental data has shown this to be unsatisfactory and some type of model that allows Y and μ to vary as the pressure changes is needed. One method or treatment for the strength parameters that allows this is the model given in reference 49. Another model for the strength parameters is the generalization of the Steinberg, Cochran, and Guinan model that is given in section 4C as embodied in equation (4.13) through equation (4.17). In the Steinberg, Cochran, and Guinan model the rigidity modulus has a linear dependence on the pressure. In the model for the strength parameters given in section 4C, the rigidity modulus is related to the isentropic compressibility as is done in linear elasticity. One might assume that the model in which the pressure is used instead of the compressibility in calculating the rigidity modulus would be less sensitive to the vagaries of the equation of state but this is not the case. The rigidity modulus in both models is used in calculating the longitudinal sound speed and in both models the bulk sound speed is needed for the calculation of the longitudinal sound speed. This shows that in order to match experimental data on longitudinal sound speeds, both models for the rigidity modulus are still dependent on the isentropic compressibility.

The combination of the strength model given in section 4C and the Rice

equation of state has proven to give a satisfactory fit to all three stainless-steel experiments. A more stringent test of this material model is to compare it with actual sound speed data taken at high pressures. In making such a comparison no inferences can be drawn about the yield strength but by making a comparison with such data, a good quantitative check on the sound speed given by the material model can be made.

A good set of sound speed data for tantalum has been reported by Brown and Shaner.⁵⁵ In order to see if the model embodied in equations (4.13)–(4.17), which has been successful in modeling the behavior of stainless steel, will be equally successful for tantalum, it is necessary to first determine the many parameters needed in the Rice equation of state and in the strength model. The parameters applicable to tantalum in the Rice equation of state will be taken as follows; $\rho_o = 16.68$ g/cc, $c_o = 3.293$ km/s, $s = 1.307$, and $\gamma_o = 1.65$. The values for c_o and s have been reported by Nellis and Mitchell.⁵⁶ The value for the Gruneisen parameter is a nominal value taken from the literature.

The next set of parameters for which values are needed are for equation (c16), which gives the melting temperature as a function of the density. The density of tantalum in the solid phase is 15.22 g/cc at a melting temperature of 3270 K. A value of -0.05 for δ_o gives excellent agreement with the experimental value of 58 K/GPa for the slope of the phase line in the T - P plane.⁵⁷ The parameter, δ_1 , in

equation (c16) determines where the phase line of the solid intersects the principal Hugoniot. The value for this parameter cannot be determined from experiments done under normal pressure conditions. If δ_1 is assumed to be zero, then the model gives the intersection of the Hugoniot and the phase line at 219 GPa. Based on their sound speed measurements, Brown and Shaner determined the intersection of the phase line and the Hugoniot to occur at around 295 GPa. A value of 1.66 for δ_1 give an intersection at 294 GPa.

The last piece of information that is needed has to do with the determination of the parameters in equation (4.17). It may be assumed that Poisson's ratio is linear in the ratio of the temperature to the melting temperature. This fixes the exponent in equation (4.17) and leaves only ν_1 and ν_2 to be determined. Any one of three pieces of data may be used to determine these parameters. The three pieces of data are the longitudinal sound speed, Poisson's ratio, and the rigidity modulus. There is a discrepancy between the bulk sound speed given by the Rice equation of state at zero pressure and the bulk sound speed of tantalum based on longitudinal and transverse sound speed measurements. The bulk sound speed derived from the longitudinal and transverse sound speed measurements is about 3.40 km/s which is somewhat higher than the value of 3.293 km/s for c_o in the Rice equation of state. This implies that all three pieces of data (longitudinal sound speed, rigidity modulus, and Poisson's ratio) cannot be matched simultaneously

at zero pressure in this model. If the measured value of the rigidity modulus is used to determine the values of ν_1 and ν_2 , then one cannot match either the longitudinal sound speed or Poisson's ratio. If one uses a value of 69.5 GPa for the rigidity modulus in this model, then the calculated value for Poisson's ratio is 0.329 and the calculated value for the longitudinal sound speed is 4.04 km/s. This compares with reported values of 0.342 for Poisson's ratio and 4.14 km/s for the longitudinal sound speed.⁵⁸

The calculated sound speeds from this model are compared with the experimental data of Brown and Shaner in table 68. Also shown in this table are sound speeds using an exponent of $\frac{3}{2}$ in equation (4.17) instead of 1. This produces somewhat higher sound speeds but both sets of calculated sound speeds agree fairly well with the measured values. No attempt has been made to compare calculated sound speeds with the measured values in the melt region. This was not done because the model given in section 4C does not address any questions concerning the equation of state in the liquid region.

The greatest difficulty in treating the liquid region is knowing what the compressibility of the material in the liquid state should be. Few definitive statements about the equation of state in the liquid region can be made and the statements that can be made are more qualitative in nature than quantitative. In many cases the assumption is made that the same equation of state for the solid phase may

be used in the liquid phase as well at high pressures. By making this assumption, there is no change in the isentropic compressibility of the material in going from the solid phase into the mixed phase region. Some questions about the mixed phase region can be answered based on knowing the phase line of the solid. Using equation (c13) for the change in the specific entropy along the phase line of the solid, it can be determined if there should be a change in the isentropic compressibility or equivalently the sound speed of the material on entering the coexistence or mixed phase region. If the change in the specific entropy along the boundary of the solid and the coexistence region is zero, then there is no change in the sound speed on going from the solid to the mixed phase region. If the specific entropy increases on the phase line of the solid with increasing pressure and

$$\gamma T_m < \rho c^2 \frac{dT_m}{dP_m}, \quad (4.21)$$

then the sound speed will show a discontinuity on entering the mixed phase region and will be lower than in the solid phase. If the specific entropy change is negative and the inequality given by equation (4.21) is true, then the sound speed in the mixed phase region will be greater than the sound speed in the solid phase.

It is interesting to apply these arguments to the material model that has been used for calculating the sound speeds of tantalum given in table 68. At 297 GPa on the principal Hugoniot the reported sound speed of tantalum is 6.37 km/s and at 293 GPa the reported sound speed is 6.65 km/s. If it is assumed that the state

at 293 GPa is a pure solid phase with almost no contribution from the rigidity modulus to the sound speed, then the model given for the solid phase of tantalum has the correct qualitative features. The tantalum data supports the idea that the specific entropy along the phase of the solid is increasing with increasing pressure. One consequence of this interpretation is that the Gruneisen parameter as defined by equation (2.28) must undergo a discontinuous change from the solid to the mixed phase region since the sound speed is undergoing a discontinuous change. Whether the Gruneisen parameter increases or decreases under this phase change can be determined from the work in section 2E. When the sound speed decreases from the solid to the mixed phase region, the Gruneisen parameter must increase.

The general discussion that has been given for tantalum pertains equally well to copper. The unfortunate aspect of the present copper experiments is that the sound speeds at the peak shock pressures are not well determined and all the data for the shock wave in the target plate must be examined. In reviewing the set of calculations that are shown in figures 157-162, the fact that γ_o was increased to 2.5 in order to match the data of experiment RS-2308 certainly implies that the copper has at least partially melted in this experiment. It is intriguing that the value of 2.5 for γ_o is close to the value of the Gruneisen parameter used in the equation of state for copper in the liquid state in reference 45. The value of the Gruneisen parameter was also increased in going from the calculation for

experiment RS-2306 shown in figures 157 and 158 to the calculation shown in figures 159 and 160 for experiment RS-2314. However, the low value for γ_0 in the calculation for RS-2306 can be explained as compensating for the lack of a strength model in the calculation.

In modeling the copper experiments different conclusions may be reached depending on the form of the equation of state used to model the bulk response of copper. Now according to the Mie-Gruneisen equation of state in which $\gamma\rho$ is a constant, the modeling of the present copper experiments indicates the phase line of the solid for copper intersects the principal Hugoniot between 131 and 144 GPa. If the Rice equation of state is used to model the copper experiments, the modeling indicates that incipient melting occurs slightly before 131 GPa on the principal Hugoniot. The conclusions that result from the modeling performed with the Mie-Gruneisen equation of state agree somewhat better with the reported value of 133 GPa for the intersection of the phase line and the principal Hugoniot of copper than when the Rice equation of state is used; however, the modeling performed with the Rice equation of state is not inconsistent with a two-phase region beginning at 130 or 131 GPa on the principal Hugoniot. The magnitude of the differences between the results of these two equations of state is not great but the impact on the interpretation of experiment RS-2314 is quite dramatic. For the Mie-Gruneisen equation of state, the conclusion is that the copper has not yet

entered a mixed phase region where the liquid phase is present but is still in a pure solid phase. For the Rice equation of state the conclusion is that the peak shock pressure has taken the copper into the melt region in this experiment.

When using the Rice equation of state a very pronounced difference is seen in the phase line of copper as opposed to the phase line of tantalum. In appendix C, a value of 0.56 for δ_1 in equation (c16) gives the intersection of the phase line with the principal Hugoniot at approximately 140 GPa for copper. If this intersection point is now lowered to 131 GPa, then δ_1 must be lowered to -0.11 . In the modeling of tantalum the specific entropy along the phase line is increasing with increasing pressure near the intersection point of the phase line with the Hugoniot. In copper with the value of -0.11 for δ_1 , the specific entropy along the phase line of the solid is always decreasing with increasing pressure. This would seem to imply that the sound speed might actually increase in going to the melt region but whether this happens or not depends on the inequality given in equation (4.21). For tantalum, equation (4.21) holds true, but for copper this inequality at 131 GPa on the Hugoniot does not hold true and thus the sound speed is still expected to decrease on going from the pure solid phase to the mixed phase region.

One last sidelight to modeling with the Rice equation of state and the Mie-Gruneisen equation of state for the copper experiments is that the expected change

in the sound speed in going from the pure solid phase to the mixed phase region may be estimated. For the Rice equation of state the sound speed would be expected to change from about 7.5 km/s in the solid phase to about 7.2 km/s. With the Mie-Gruneisen equation of state the change would be from about 7.2 km/s in the solid phase to about 7.0 km/s in the mixed phase region. When more accurate data become available, it may well be possible to distinguish which equation of state is more appropriate for copper in this pressure regime.

SUMMARY

When the shock attenuation data are examined closely and compared with the results from the calculational modeling, different trends are seen in the decay of shock waves in stainless steel as compared against the attenuation of shock waves in copper. It is found that the experimental data for the stainless steel cannot be explained without invoking strength effects and that strength effects are important to shock pressures of at least 150 GPa. Although there is no evidence of a melt phase in the experiments for the stainless steel, it can be inferred from the modeling done in section 4C, that the intersection of the phase line of the solid with the principal Hugoniot in the neighborhood of 220 GPa is probably a good estimate for SS-304L. In copper it is found that strength effects are not needed in order to explain the experimental results. The data for copper is consistent with copper entering a melt state on the principal Hugoniot slightly above 130 GPa. This is in substantial agreement with the value of 133 GPa reported in reference 45. The intersection point given in reference 45 was determined from Hugoniot data taken on porous copper samples.

The experimental data has proved valuable when used in conjunction with the characteristic codes to analyze material behavior. The experimental data can be used to test any proposed material descriptions. Material models have been discussed in some detail in chapters 2 and 4. One of the most intriguing aspects

of this present work is the differences seen in modeling with the Rice equation of state and the modeling with the standard Mie-Gruneisen equation of state. Significant differences are implied in the strength modeling depending on which equation of state is used to model bulk response. Further work still needs to be done on the equation of state of copper in the liquid state.

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APPENDICES

A. Characteristic Codes for One-Dimensional Flow

Two computer codes were written to specifically help in analyzing and modeling the present experiments. The codes are based on applying differencing techniques to the equations for the characteristic directions and their associated compatibility equations. The main difficulty in using computer codes based on the method of characteristics is that they often require much specialized logic. The advantage of characteristic codes is that they can treat discontinuities, such as shocks, exactly. The codes were written to solve the problem of shock attenuation in a semi-infinite target plate impacted by a thin flyer plate. The target is assumed to occupy the space $x \geq 0$. The codes use a constant time step technique.†

In this technique, initial conditions are specified at different spatial positions or grid points but at the same initial time. A time step is then calculated which is used in updating the flow variables and the state variables at each grid point along the characteristic direction given by equation (2.17). A new time step is then calculated to update the variables at each grid point, and thus the problem is advanced in time.

The same basic ideas and differencing techniques appertain to both the code

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† The basic idea of using a constant time step technique in a characteristic code was taken from reference 17.

SHKSIM and the code SHKSJB but there are some fundamental differences. The code SHKSIM is more specialized than the code SHKSJB and cannot handle anything but the hydrodynamic problem of a simple-centered release wave overtaking a shock wave. This means the code SHKSIM cannot handle the problem when the flyer plate is a different material from the target plate or when the flyer plate is the same material but has a different initial thermodynamic state than the target. The main aspect of the code SHKSIM, that the code SHKSJB does not have, is that the code SHKSIM can follow as many of the right-going characteristics given by equation (2.18) through the wave interaction region as needed. The code SHKSJB cannot follow more than two of the $u + c$ characteristics in the release wave.

In solving the problem of a simple-centered release wave overtaking a shock wave, the code SHKSIM makes extensive use of Riemann integrals. The right-going characteristics and all values for quantities at grid points throughout the release wave can simply be calculated from equations (4.1)–(4.3) and the characteristic equation (2.18). Thus the assumption that the problem can be treated as a simple-centered release wave overtaking a shock means that the initial configuration (that is, the configuration at the moment the lead characteristic of the release wave just catches the shock front) can be calculated without resorting to finite differencing techniques applied to the characteristic equations and the

compatibility equations. To farther calculate the time evolution of the problem, the code SHKSIM resorts to the same type of differencing techniques as the code SHKSJB.

The code SHKSJB is more versatile than the code SHKSIM in that it can do the problem of a dissimilar material impacting the target and it can also handle problems with material strength. The main disadvantage of the code SHKSJB is that finite differencing techniques are applied to the problem the moment the flyer plate makes contact with the target plate. In the code SHKSIM such techniques are not employed until the release wave actually overtakes the shock wave. Because of this difference the code SHKSJB takes more computer time to run a problem than the code SHKSIM.

The structure of the coding in which strength effects are modeled is slightly different and more complicated than the purely hydrodynamic case. Since the coding for the hydrodynamic problem is so similar in both SHKSIM and SHKSJB, it will be discussed first. After the shock wave has been overtaken by the release wave, there are just two principal algorithms that need to be considered. One of these algorithms updates the variables at each material or grid point while the other algorithm updates the trajectory of the shock front in the target.

For updating variables at material grid points, the characteristic direction given by equation (2.17) is followed. Consider at time t^0 a grid point at the

spatial position x_j^0 . The superscript will refer to a time index and the subscript to a spatial index. The material or particle velocity at the grid point is u_j^0 . The density and specific internal energy are ρ_j^0 and E_j^0 . The pressure and sound speed can be calculated from the equation of state. For a time step given by Δt , the new time is

$$t^1 = t^0 + \Delta t. \quad (a1)$$

A simple forward differencing scheme is used to update the position of the grid point:

$$x_j^1 = x_j^0 + u_j^0 \Delta t. \quad (a2)$$

This is the difference equation associated with equation (2.17). To find the new material velocity, use is made of equations (2.18) and (2.19) and their associated compatibility equations (2.21) and (2.22). For a point to the left of x_j^0 , a right-going characteristic can be found that emanates from the point x_l and ends at the point x_j^1 . The difference equation for the right-going characteristic is

$$x_l + (u_l + c_l)\Delta t = x_j^1. \quad (a3)$$

If the nearest grid point to the left of x_j^0 is x_i^0 then a simple linear interpolation formula can be used to calculate a particle velocity and sound speed at the point x_l . The interpolation formula for u_l is given by

$$u_l = \frac{(u_j^0 - u_i^0)(x_l - x_i^0)}{(x_j^0 - x_i^0)} + u_i^0. \quad (a4)$$

A similar equation is used to calculate c_l . Using these interpolation formulas, equation (a3) can be solved for x_l . For a point to the right of x_j^0 , a left-going characteristic can be found that emanates from the point x_r and ends at the point x_j^1 . The difference equation that corresponds to equation (2.19) is

$$x_r + (u_r - c_r)\Delta t = x_j^1. \quad (a5)$$

Linear interpolation can be used to find u_r and c_r in terms of x_r and then equation (a5) can be used to solve for x_r .

To find the new particle velocity at the grid point x_j^1 , it is necessary to invoke the compatibility equations. Along the right-going characteristic, a forward differencing equation that corresponds to equation (2.21) can be written:

$$P_j^1 - P_l = -\rho_l c_l (u_j^1 - u_l). \quad (a6)$$

Since x_l is known, linear interpolation can be used to solve for the pressure, density, sound speed, and particle velocity at x_l . Along the left-going, the difference equation analogous to equation (2.22) is

$$P_j^1 - P_r = \rho_r c_r (u_j^1 - u_r). \quad (a7)$$

Subtracting equation (a7) from equation (a6) one arrives at the following equation for the new particle velocity:

$$u_j^1 = \frac{\rho_l c_l u_l + \rho_r c_r u_r + P_l + P_r}{\rho_r c_r + \rho_l c_l}. \quad (a8)$$

Once the new particle velocity is found, it is an easy matter to use either equation (a6) or (a7) to solve for the pressure at the new grid position x_j^1 .

It only remains to calculate the new values of density and specific internal energy. The fact that the new pressure at x_j^1 is known, can be used to find the new density and specific internal energy. From the equation of state, any infinitesimal change in pressure can be written as

$$dP = \left(\frac{\partial P}{\partial \rho} \right)_E d\rho + \left(\frac{\partial P}{\partial E} \right)_\rho dE. \quad (a9)$$

When the characteristic equation (2.17) is followed, the compatibility equation given by equation (2.20) must hold. Since the beginning and ending pressure for the material point x_j are known, then equations (a9) and (2.20) can be considered as a system of first-order partial differential equations which may be solved for $d\rho$ and dE . An algorithm using a fourth-order Runge-Kutta method is employed to solve for dE and $d\rho$.

In tracking shock fronts, the code SHKSJB must follow two shock fronts while the code SHKSIM must track only one shock. Both codes use an impedance-match solution for finding the shock conditions in the target and the flyer but because of the initial conditions that are assumed in the code SHKSIM, the solution is considerably more simple. The code SHKSIM only follows the shock front in the target plate and the shock velocity is constant until the shock is overtaken by the

release wave. In the code SHKSJB, the left-going shock in the flyer plate must be followed as well as the right-going shock wave in the target plate. Because SHKSJB allows for nonuniform initial conditions, numerical techniques for advancing the shocks are employed from the time the flyer impacts the target. The following description of the algorithm for advancing the right-going shock in the target is essentially the same for both codes. The one difference is that the state of the material ahead of the shock is uniform in the code SHKSIM while the code SHKSJB allows for a nonuniform state.

The algorithm for the advancement of the shock keeps track of the shock velocity and, of course, the position of the shock front. The algorithm also tracks the particle velocity, density, and specific internal energy immediately ahead of the shock and immediately behind it. The advancement of the shock front employs a forward differencing scheme similar to the main algorithm for the advancement of the position of a grid point. At time t^0 , the position of the shock front is x_s^0 . At time t^1 the new position of the shock front is given by

$$x_s^1 = x_s^0 + U_s^0 \Delta t, \quad (a10)$$

where U_s^0 is the shock velocity at time t^0 .

The new position of the shock front is the starting point for updating all other quantities at the shock front. The next step is to update the particle velocity,

density, and specific internal energy immediately ahead of the shock front to the new position of the shock. The algorithm for doing this follows the algorithm for updating a material grid point and does not involve the shock-jump or Rankine-Hugoniot equations.

After the quantities immediately ahead of the shock front have been updated, it is necessary to find the new values of the same quantities immediately behind the shock front. For a right-going shock wave, only $u + c$ characteristics from points to the left of the shock wave can overtake the shock. Neither u characteristics nor $u - c$ characteristics from any point to the left of the shock front can intercept the shock front. A right-going characteristic can be found that emanates from a position, x_l , that is to the left of the old position of the shock front and ends at the new position of the shock front. This is given by the equation

$$x_s^1 = x_l + (u_l + c_l)\Delta t. \quad (a11)$$

When linear interpolation is used to write u_l and c_l in terms of x_l , equation (a11) can be used to solve for x_l . The compatibility equation that holds along this right-going characteristic can be written as

$$P_s^1 - P_l = -\rho_l c_l (u_s^1 - u_l), \quad (a12)$$

where P_s^1 and u_s^1 are, respectively, the pressure and particle velocity at the new position of the shock front. The quantities P_l , ρ_l , u_l , and c_l are all known from

linear interpolation. There are now five unknowns at the shock front, the shock velocity, the particle velocity, the pressure, the density, and the specific internal energy. Equation (a12) and the Rankine-Hugoniot equations form a system of four equations. To find the shock and particle velocities at the new shock position, a fifth equation must be used. This fifth equation is the equation of state.

To solve for the new values at the shock front let χ be the new shock velocity at x_j^1 . The quantities subscripted with a will refer to the known quantities immediately ahead of the shock front. The quantities subscripted with b will refer to the new unknown variables immediately behind the shock front. The quantities with the subscript l are the same ones as those defined in equations (a11) and (a12). On the basis of the shock-jump conditions, four relations can be written:

$$v_b = u_a + \frac{P_l - p_a + \rho_l c_l (u_l - u_a)}{\rho_a (\chi - u_a) + \rho_l c_l}, \quad (a13)$$

$$\rho_b = \rho_a \frac{\chi - u_a}{\chi - u_b}, \quad (a14)$$

$$E_b = E_a + \frac{P_a}{\rho_a} \frac{u_b - u_a}{\chi - u_a} + \frac{1}{2} (u_b - u_a)^2, \quad (a15)$$

and

$$P_b = P_a + \rho_a (\chi - u_a) (u_b - u_a). \quad (a16)$$

As stated previously, the necessary fifth equation to solve for the five unknowns is the equation of state in which the pressure, P_b , is written as a function of the density, ρ_b , and the specific internal energy, E_b .

The solution for the new value of the shock velocity, χ , and hence the other quantities, proceeds by defining a new function $g(\chi)$ which is the pressure given by the equation of state subtracted from equation (a16):

$$g(\chi) = P_b - P(\rho_b(\chi), E_b(\chi)). \quad (a17)$$

Starting with the old value of the shock velocity, a Newton-Rhapson search method can be applied to equation (a17) to find the new value of the shock velocity. Then the new values for other quantities can be found from the shock-jump conditions.

The method of characteristics can still be used when strength is modeled but the differencing equations become a little more involved. When strength is being modeled there is a fourth equation governing the flow. The difficulty lies in the fact that this fourth equation has abrupt changes in it depending on whether the flow is in an elastic regime or a plastic one. In an elastic regime the fourth equation is given by an equation such as equation (2.92) or (2.93) and in a plastic regime the fourth equation is one such as equation (2.76). With the modeling of strength effects, the characteristic directions are given by the equations

$$\frac{dx}{dt} = u, \quad (a18)$$

$$\frac{dx}{dt} = u + c_\ell, \quad (a19)$$

and

$$\frac{dx}{dt} = u - c_\ell. \quad (a20)$$

When there are four equations governing the flow, there should be four characteristic directions. When strength is modeled, equation (a18) is actually a double root of the solution for the characteristic directions. The term c_ℓ in these equations is not the bulk sound speed but rather the longitudinal sound speed. Again the term c_ℓ depends on whether the flow is elastic or plastic. For flow in an elastic regime, c_ℓ is given by equation (2.100) or equation (2.101) while for flow in a plastic regime, equation (2.102) is appropriate for c_ℓ .

When strength is modeled there are still only three distinct characteristic directions to deal with which are the same as in the hydrodynamic problem, but the compatibility equations are increased to four in number. These equations are altered by the addition of the deviatoric stress term. Along the characteristic direction given by equation (a18) there are now two compatibility equations. One equation is for the specific internal energy and is given by

$$dE = \frac{(P + \tau_n)}{\rho^2} d\rho. \quad (a21)$$

The other equation is for the change in the deviatoric stress and is given by an equation such as equation (2.92) when the flow is in an elastic region. If allowance is made for a variable yield strength, then the compatibility equation for equation (a18) in a plastic flow region is

$$d\tau_n = \pm \frac{2}{3} dY. \quad (a22)$$

The plus sign is appropriate when the deviatoric stress is at an upper yield limit and the minus sign applies when the deviatoric stress is at a lower yield limit.

The compatibility equation for equation (a19) is

$$dP + d\tau_n + \rho c_\ell du = 0, \quad (a23)$$

and for the last characteristic equation, the compatibility equation is

$$dP + d\tau_n - \rho c_\ell du = 0. \quad (a24)$$

The obvious complication in the compatibility equations is the addition of the stress deviator and the question of when the flow is in a plastic regime or in an elastic one. One interesting sidelight about writing the compatibility equations in this manner is that there is no additional conceptual difficulty in assuming that μ and Y may be functions of ρ and E as opposed to assuming they are constants.

With the inclusion of strength, the code also keeps track of the stress deviator at each material point or grid point as well as the other hydrodynamic variables such as density, specific internal energy, and pressure. The procedure for updating the variables at a grid point begins in the same manner as in the hydrodynamic case. The new position of the grid point is found by using equation (a2). The code then uses forward differencing equations (a19) and (a20) and linear interpolation to find the interpolation points x_l and x_r . Difference equations appropriate for equations (a23) and (a24) are then used to find the new particle velocity. Next

the new total stress is found from these equations. It must be emphasized that the new stress is now known, neither the new pressure nor the new value of the stress deviator is known. At this point the code checks to see if the flow is such that the change in the total stress for this material point will be an elastic flow or not. If the flow is not elastic, then equation (a22) is used to calculate the change in the stress deviator. If the yield strength is constant, then there is no change in the stress deviator and knowing the new total stress is equivalent to knowing the new value of the pressure. In this situation, the same type of Runge-Kutta algorithm as for the hydrodynamic case can then be used to solve for $d\rho$ and dE . If the yield strength is not constant or the flow is elastic, a Runge-Kutta method can still be used. The reason such a method can still be employed is that in either of these two cases, $d\tau_n$ may be written as

$$d\tau_n = g(\rho, E)d\rho. \quad (a25)$$

Of course in writing $d\tau_n$ in this manner, the assumption is being made that if Y and μ are not constants they may be assumed to be functions of ρ and E .

In advancing the shock front in the strength case, the assumption is made that the shock process always takes a material to a stress state such that the stress deviator is at the upper yield limit. With this assumption, the stress deviator at the shock front is always given by

$$\tau_n = \frac{2}{3}Y. \quad (a26)$$

The attenuation process for shocks occurs in the same manner as in the hydrodynamic case except it is now complicated by the effects of material strength. The new position of the shock front is given by equation (a10). The compatibility equation for the right going characteristic that overtakes the shock is

$$P_s^1 + \frac{2}{3}Y - P_l - \tau_l + \rho_l c_l(u_s^1 - u_l) = 0. \quad (a27)$$

The interpolation point x_l is found from equation (a11) where c_l is calculated as an elastic sound speed. Equation (a27), the equation of state, and the Rankine-Hugoniot equations in which the pressure has been replaced by the normal component of the stress tensor form a system of equations which is used to solve for the new shock velocity at the new position of the shock front. The system of equations is formulated in the exact same manner as in the hydrodynamic case so that a Newton-Raphson search may be used to find the new value of the shock velocity.

The codes SHKSIM and SHKSJB are written in Fortran. Listings of the codes can be obtained by writing to

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B. An Analytical Solution for Shock Attenuation

To check on both the logic and accuracy of the computer codes SHKSIM and SHKSJB, results from the codes can be compared to an analytical solution⁵⁹ of the problem of a shock being attenuated by a release wave. To arrive at this exact solution, the problem must be carefully formulated. Consider a homogeneous material initially at rest that occupies the half-space $x \geq 0$. At time $t = t_1$, a pressure is instantaneously applied to the material boundary at $x = 0$. This sudden application of pressure produces a shock wave of constant velocity in the body. Assuming the material obeys equation (2.4) relating shock velocity to particle velocity, then equations (2.1), (2.2), and (2.3) can be used to solve for the other thermodynamic and kinematical variables if any one of the five variables, ρ , E , P , U_s , or u_p , is given. These steady-state conditions are maintained until $t = 0$, when the pressure at the material boundary is reduced to zero. This introduces a rarefaction that eventually overtakes and decays the strength of the shock wave. The position of the shock wave and the pressure and particle velocity of any point in the body can be found analytically at any time if the flow equations are written in terms of Lagrangian coordinates, and the proper equation of state is chosen. This problem is directly analogous to the problem of a thin flyer plate striking a target plate, provided the flyer and the target plate have the same equations of state and the same initial densities and specific internal energies.

The equation of state that must be used for this analytical solution is the Walsh equation of state.⁵⁹ It is interesting to note that although Walsh found an analytical solution for the problem of shock attenuation, he never invoked the specific internal energy and in fact did not need an equation of state giving P as a function of ρ and E . Rather, he postulated that the isentropic compressibility is given by

$$\rho^2 c^2 = \rho_o^2 c_o^2 + 4s\rho_o P \quad . \quad (b1)$$

To obtain results from the computer codes, P must be given as a function of ρ and E . From equation (b1), an equation of state can be formulated. Equation (b1) can be integrated along an isentrope to give the pressure as a function of the density:

$$P = \frac{\rho_o c_o}{4s} \left[A \exp\left(-\left(\frac{4s\rho_o}{\rho}\right)\right) - 1 \right] . \quad (b2)$$

In equation (b2), A is a constant of integration that varies from isentrope to isentrope. The constant A can be evaluated from the fact that equation (b2) must agree with equation (2.5) when the isentrope intersects the principal Hugoniot; thus,

$$A = \left(\frac{1 + s\eta_h}{1 - s\eta_h} \right) \exp\left(\frac{4s\rho_o}{\rho_h}\right), \quad (b3)$$

where η_h is given by

$$\eta_h = 1 - \frac{\rho_o}{\rho_h}, \quad (b4)$$

and ρ_h is the density at which the isentrope intersects the Hugoniot. To find the intersection density ρ_h , the specific internal energy can be integrated along the isentrope from the point ρ, E to the point ρ_h, E_h where E_h is the specific internal energy at which the isentrope intersects the principal Hugoniot. This leads to the following equation for η_h :

$$\begin{aligned} 2s\eta_h + \left(\frac{2s\eta_h}{1 - s\eta_h} \right)^2 - \left(\frac{1 + s\eta_h}{1 - s\eta_h} \right)^2 \left(1 - \exp(4s(\eta - \eta_h)) \right) \\ = 2s\eta + 8E \left(\frac{s}{c_0} \right)^2. \end{aligned} \quad (b5)$$

Once η_h is found from equation (b5), then A and hence I' are easily calculated. The equation of state algorithm used in the codes uses an iterative scheme to calculate η_h from equation (b5).

The test problem chosen for checking the code was a flyer plate 1.5-mm thick impacting a 20-mm thick target plate of the same material. Physical parameters appropriate for SS-304 were used in the equation of state. Problems were run using two different impact velocities for the flying plate and these velocities approximately bracketed the actual velocities of the flying plates seen in the experiments. One set of calculations was run using an impact velocity of 3.7 km/s and another set of calculations was run using a velocity of 4.65 km/s. An examination of the effects of zoning or distance between material points was also made.

A comparison between a numerical solution from the code SHKSIM and the analytical solution for shock attenuation is shown in figure 163. The particular

problem shown is for an impact velocity of 3.7 km/s for the flyer plate. The initial zoning used for the code calculation was 0.05 mm between points. The plot shows that the numerical results from SHKSIM are in excellent agreement with the analytical solution. For this problem, the overtaking point or the point at which the lead characteristic of the release wave intersects the shock wave occurs at 9.486 mm. There is of course exact agreement between the code calculation and the analytical solution on the value of the overtaking point. For the intersection of other characteristics with the shock wave the agreement between the numerical solution and the analytical solution is no longer exact but it is still very good. For a shock position of 20 mm, the δt value from the numerical solution is approximately 0.3 ns less than the δt value from the analytical solution.

Although the plot is not shown, the agreement between the numerical solution from SHKSIM and the analytical solution is equally as good when the impact velocity of the flying plate is raised to 4.65 km/s. There is a slight increase in the difference between the δt values at 20 mm but only by about 0.1 ns. All the calculations performed in chapter 4 using the code SHKSIM had finer zoning than the calculation shown in figure 163. The initial zoning for the calculations in chapter 4 using the code SHKSIM was 0.025 mm. From the test problems it was found that when the distance between the points was cut in half, the difference or the discrepancy between the δt values of the numerical solution and the analytical

solution was roughly cut in half. Because of the fine zoning used for the SHKSIM calculations in chapter 4 and the good agreement between code results and the analytical solutions shown here, the numerical errors for the calculations in figures 71–87 are probably negligible.

The test calculations using the code SHKSJB do not show as good agreement with the analytical solutions as the calculations using SHKSIM. Undoubtably, this occurs because of the difference in the initial or starting conditions for the two codes. In the code SHKSIM, the code starts from the point at which the lead characteristic of the release wave has just intersected the shock. For the code SHKSJB, the entire evolution of the release wave must be numerically calculated by applying a finite differencing scheme to the system of characteristic equations. An example comparing the numerical results from SHKSJB to an analytical solution is plotted in figure 164. The test problem for this figure is the same as that in figure 163. The initial zoning for the code calculation is 0.05 mm between material points. The difference in the δt values at a shock position of 20 mm is approximately 6 ns, which is a considerably larger discrepancy than the result from SHKSIM. In going to a greater velocity of 4.65 km/s for the flyer plate the difference between the code result and the analytical solution at 20 mm grows to approximately 8 ns. When the zoning is made finer for the code SHKSJB, there is better agreement between the code results and the analytical solution but the

agreement is still not as good as that seen by using the code SHKSIM. Figure 165 shows the results for the test problem with an impact velocity of 4.65 km/s but one in which the initial zoning for the code calculation has been decreased from 0.05 mm to 0.025 mm. The difference in the δt values at a shock position of 20 mm is now about 4 ns. The difference between the numerically calculated shock velocity and the shock velocity from the analytical solution as a function of shock position can be seen in figure 166. The numerical results shown in this figure are from the same code calculation shown in figure 165.

In the calculations performed in chapter 4 using the code SHKSJB, the width between point for the initial zoning was never greater than 0.025 mm. Whenever a calculation involved a strength model, the initial width of the zoning was decreased to 0.02 mm. From these results, the numerical effects of the zoning on the calculations can be estimated. A conservative estimate is the effects of the zoning are never greater than 5 ns in any of the δt plots based on the computer calculations of chapter 4. Effects this great would only occur at or around the 20-mm level. For calculations ending at a 16-mm shock position, the effect of the zoning would be less than 3 ns. The initial overtaking point in the SHKSJB calculations shows almost no dependence on the initial zoning as long as the flyer plate has uniform initial conditions.

C. Melting on the Principal Hugoniot

In section 2F the effects of strength were described in terms of an elastic-plastic model of a solid. Central to this model was the concept of a yield strength. Once a material has melted it will permanently deform under any shear stress no matter how small. This implies the material has zero yield strength. If the peak shock pressure is above the pressure on the principal Hugoniot at which melting occurs, then one would imagine that the hydrodynamic description of the shock attenuation process would be valid.

Shock processes are not isentropic and irreversible work is associated with the shocking of any material. After a material has released from a shocked state to zero or ambient pressure, this irreversible work is manifested as a heating of the material. The shock heating leaves the material with a residual temperature T_r , which is greater than the initial temperature T_o . The rise in temperature also leaves the material with a density ρ_r , which for normal materials is less than the initial density ρ_o .

Two different thermodynamic paths can be considered in reaching the final temperature T_r . If the initial pressure can be taken as zero, then a zero-pressure isobar can be considered as the first path. Along this path the change in the specific internal energy in going from the temperature T_o to the temperature T_r

is given by

$$\Delta E = \int_{T_0}^{T_r} c_p dT, \quad (c1)$$

where c_p is the specific heat at constant pressure. The second path can be broken into two steps. The first step is the shock itself and the second step is the subsequent release to zero pressure. The energy change associated with the shock can be found from equation (2.3). Since the release process is isentropic, the change in the specific internal energy in going from a density ρ on the principal Hugoniot to the final density ρ_r is given by

$$\Delta E_2 = \int_{\rho}^{\rho_r} \frac{P}{\rho^2} d\rho. \quad (c2)$$

If ΔE_1 is the the change in the specific internal energy due to the shock, then the following equation must hold:

$$\Delta E = \Delta E_1 + \Delta E_2. \quad (c3)$$

By means of equation (c3), any final state on the zero-pressure isobar can be related to a state on the principal Hugoniot.

For most solid materials, the melting point increases with increasing pressure. For such materials, equation (c3) and a simple equation of state such as equation (2.44) can be used to calculate a minimum pressure on the principal Hugoniot below which one would expect the substance to always remain in the solid phase.

Let T_{m0} be the melting temperature for a material at zero pressure. The maximum change in the specific internal energy that can be tolerated by the material without entering into a mixed phase region can be determined from equation (c1) with T_r replaced by T_{m0} . By knowing ΔE , ρ_r , the principal Hugoniot, and the equation of state, it is possible to calculate a pressure on the principal Hugoniot that results in the final or residual temperature T_{m0} . This calculation is sensitive to the equation of state and the parameters used in the equation of state. In table 69 are listed some results of calculating pressures on the principal Hugoniot that produce the residual temperature T_{m0} . The pressures that are calculated for the copper used in these experiments are in the same range as the experimental pressures. For the stainless steel, the calculated pressures are higher than the pressures generated in these experiments.

The pressures calculated in table 69 are only an estimate of where melting might be important in the release stage. To see if melting might occur in the high pressure state, it is necessary to extend the solid-liquid phase lines and determine where they intersect the principal Hugoniot. A simple first step is to use the Clapeyron-Clausius formula. Along the solid-liquid phase line in the P - T plane one has that

$$\frac{dP}{dT} = \frac{\Delta S}{\Delta v}, \quad (c4)$$

where ΔS and Δv are the change in the specific entropy and specific volume on

melting at constant pressure. The easiest assumption to make is that the slope of the phase line is constant. This implies that

$$dP = B dT, \quad (c5)$$

where B is now a constant. Equation (c5) can be integrated to yield the melting temperature as function of pressure:

$$T_m = T_{m0} + \frac{P}{B}. \quad (c6)$$

It is now necessary to know the temperature along the principal Hugoniot in order to determine the intersection point between the phase line (equation(c6)) and the principal Hugoniot.

To calculate temperatures along the principal Hugoniot consider the specific entropy expressed in terms of the specific volume and temperature. The thermodynamic equation is

$$T dS = c_v dT + T \left(\frac{\partial S}{\partial v} \right)_T dv. \quad (c7)$$

Using equation (2.28) for the definition of the Gruneisen parameter, equation (c7) may be written as

$$T dS = c_v dT + c_v T \frac{\gamma}{v} dv. \quad (c8)$$

Equation (c8) is to be integrated along the principal Hugoniot. The left-hand side of equation (c8) can be found from equation (2.55) and for a material that

obeys a linear U_s-u_p relation, dP along the principal Hugoniot can be found from equation (2.5). Thus, along the principal Hugoniot equation (c8) becomes

$$\frac{\rho_o c_o^2}{\rho^2} \frac{s\eta^2}{(1-s\eta)^3} = c_v dT - c_v T \frac{\gamma \rho}{\rho^2} d\rho. \quad (c9)$$

Equation (c9) can be integrated numerically; but, before it can be integrated, the functional form for the Gruneisen parameter and the specific heat must be specified. Using equation (2.43) for the Gruneisen parameter and the classical Dulong and Petit value for the specific heat at constant volume, the phase line for copper intersects the principal Hugoniot above 400 GPa. When the assumption is made that the percentage volume change and the entropy change are constant on melting, the phase line for copper intersects the principal Hugoniot at a considerably lower pressure of approximately 230 GPa. Effects of the electron gas can be included by writing the specific heat in equation (c9) as in equation (2.51). When this is done the calculated temperatures along the principal Hugoniot are lowered and the intersection of the phase line and the principal Hugoniot occurs at a higher pressure.

Yet another way of extending the phase line in the P - T plane is to use the Lindemann equation for melting.^{60,61} The Lindemann formula can be expressed

as

$$T_m = A\rho^{2/3} \Theta^2, \quad (c10)$$

where A is a material constant and Θ is the Debye temperature. If Θ and ρ are known, then the melting temperature can be determined from equation (c10). The results given in appendix D can be used to find the Debye temperature as a function of η . Because the internal energy along the principal Hugoniot is known as a function of η , it is possible to calculate the temperature on the principal Hugoniot. Figures 167 and 168 are temperature calculations for copper and stainless steel based on the results given in appendix D. In general the temperature calculations based on the results of appendix D give a higher temperature for a given Hugoniot pressure than does a temperature calculation based on equation (c8). The melting curve given by equation (c10) and temperature calculations based on appendix D predict that melting begins on the principal Hugoniot at about 180 GPa for copper and at about 250 GPa for stainless steel.

Another melting formula that has had some success describing melting is the Kraut-Kennedy law.^{62,63} The formulation of this melting law relates the melting temperature to the compression on the room-temperature isotherm. It is given by the equation

$$T_m = T_{m0}(1 + A\eta). \quad (c11)$$

T_{m0} is the melting temperature at zero pressure. The parameter A is a constant characteristic of the material in question. The term η in equation (c11) is given by equation (2.6) but ρ is now the density on the isotherm at room temperature

for a given pressure and thus η in equation (c11) is actually a function of the pressure.

Since η is to be taken along the isotherm at room temperature, knowledge of ΔS and Δv at melting allows the determination of the constant A in equation (c11). From these considerations the value of A can be approximated as

$$A \approx \frac{\rho_o c_o^2}{T_{mo}} \frac{\Delta v}{\Delta S}. \quad (c12)$$

If values of 33 K/GPa and 31 K/GPa are used for $\Delta v/\Delta S$ for copper and stainless steel, respectively, then A for copper is 3.4 and A for stainless steel is 2.9. The value of T_{mo} for copper is 1356 K and for SS-304L a value of 1700 K was used. When electronic effects are included, the copper phase line intersects the principal Hugoniot at about 140 GPa. For the stainless steel the intersection occurs at about 175 GPa.

Yet another melting formula can be derived by using equation (c8). For the solid along the melting boundary, equation (c8) can be written as

$$T_m \frac{dS}{d\eta} = c_v \frac{dT_m}{d\eta} - c_v T_m \frac{\gamma \rho}{\rho_o}. \quad (c13)$$

In equation (c13) all derivatives are taken along the phase line of the solid. Both c_v and γ are known for the solid. It is informative to rewrite equation (c13) as

$$\frac{1}{T_m} \frac{dT_m}{d\eta} = \frac{\gamma \rho}{\rho_o} + \frac{1}{c_v} \frac{dS}{d\eta}. \quad (c14)$$

Now make the assumption that the following expansion may be made for the entropy of the solid along the phase line:

$$\frac{1}{c_v} \frac{dS}{d\eta} = \delta_0 + 2\delta_1(\eta - \eta_o). \quad (c15)$$

The expansion has been taken around η_o , where η_o is the value for the solid at melting under normal conditions. With γ given by equation (2.41), equation (c14) may now be integrated to yield

$$T_m = T_{mo} \frac{1 + \gamma_o \eta}{1 + \gamma_o \eta_o} \exp(\delta_0(\eta - \eta_o) + \delta_1(\eta - \eta_o)^2). \quad (c16)$$

The inclusion of two constants in equation (c15) allows for a good deal of flexibility in fitting the data. The constant δ_0 can be chosen from knowing the values of ΔS and Δv at the normal melting temperature and then δ_1 can be chosen on the basis of the intersection of the phase line with the principal Hugoniot. For copper with a value of -0.07 for δ_0 and a value of 0.56 for δ_1 , equation (c16) predicts an intersection of the copper phase line with the principal Hugoniot at 140 GPa. For the stainless steel with values of -1.09 for δ_0 and 3.31 for δ_1 , equation (c16) predicts an intersection of the phase line and the principal Hugoniot at 200 GPa. If the phase line for SS-304L intersects the principal Hugoniot at 220 GPa, then δ_1 must be changed from a value of 3.31 to 3.89 . The temperature on the Hugoniot was calculated using the cold curves from the Rice equation of state and equation (2.51) to calculate the temperature.

Perhaps the most satisfying way of calculating the pressure on the principal Hugoniot at which melting begins would be to know the equation of state for the liquid phase as well as the solid phase. By knowing the equation of state for the liquid state, one could formulate the Gibbs free energy for each phase and thus calculate the phase lines. McQueen and others used experimental data on porous copper to empirically formulate an equation of state for the liquid phase of copper.⁴⁵ Based on this equation of state, they estimated that melting would begin on the principal Hugoniot for OFHC copper at 133 GPa. This pressure is within the range of the present experiments. Both the Kraut-Kennedy and the Rice melting formula (equation (c16)) agree fairly well with this value.

D. The Gruneisen Parameter and the Cold Curve

When a solid is modeled as a collection of harmonic oscillators, then the Gruneisen parameter can be defined by equation (2.38). In Debye theory all frequencies are assumed to be independent of temperature or specific internal energy. This means that the maximum or cutoff frequency postulated in Debye theory may be substituted for ν_i in equation (2.38). Next the question arises about the relation between the maximum frequency and bulk properties. Three prevalent relations that express the Debye cutoff frequency, ν_x , in terms of quantities on the cold compression curve can all be expressed by the relation²⁴

$$\nu_x \propto \rho^{(q-2)/3} \left(\rho^2 \frac{d}{d\rho} (P_c \rho^{-2q/3}) \right)^{\frac{1}{2}}. \quad (d1)$$

In equation (d1) P_c is the pressure on the cold compression curve or zero-degree isotherm.

With ν_x replacing ν_i in equation (2.38), the equation for the Gruneisen parameter can be written as

$$\gamma = \frac{\eta}{2} \frac{(P_c(1-\eta)^{2q/3})''}{(P_c(1-\eta)^{2q/3})'}. \quad (d2)$$

In this equation the single prime represents the first derivative of the quantity within the brackets with respect to η and the double prime is the second derivative.

When $q = 0$ in equation (d2), the equation gives the Slater formula⁶⁴ for the

Gruneisen parameter. When $q = 1$ and $q = 2$, the equation gives the Dugdale-McDonald formula⁶⁵ and the “free-volume” formula respectively for the Gruneisen parameter.

Equations (2.33), (2.64), and (d2) can be used to form a system of equations that can be numerically integrated to determine the cold curve and the Gruneisen parameter at high pressure. The system of equations is

$$E'_c = \frac{P_H}{\rho_o} - B(E_H - E_c), \quad (d3)$$

$$G' = 2B - \frac{2}{3} \frac{q-2}{1-\eta}, \quad (d4)$$

and

$$B' = \left(\frac{c_H^2}{(1-\eta)^2} - E''_c \right) (E_H - E_c)^{-1} - B^2. \quad (d5)$$

In equation (d3)–(d5) the prime again denotes differentiation with respect to η . All variables subscripted by H refer to quantities on the principal Hugoniot. The quantity B is given by the equation

$$B = \frac{\gamma}{1-\eta}, \quad (d6)$$

and the variable G is given by

$$G = \ln(A E'_c (1-\eta)^{2q/3})', \quad (d7)$$

where the parameter A is a constant.

Before this system of equations can be integrated, it is necessary to specify initial values for the quantities appearing in these equations. The initial value for c_H is c_0 as given in equation (2.4). The thermodynamic definition of the Gruneisen parameter can be used to find the initial value for it and hence the initial value for B . The initial value for E_c is a little more difficult to calculate. The reference energy can be chosen such that E_H has an initial value of zero. This choice for E_H implies that the initial value for E_c will be given by the equation

$$E_c = -\frac{3Nk\Theta}{a_w} D_E(\chi_D). \quad (d8)$$

In equation (d8) D_E is the Debye energy function, a_w is the atomic weight, N is Avogadro's number, and Θ is the Debye temperature.

The variable for which it is difficult to calculate an initial value is E_c'' . Instead of trying to calculate E_c'' on the basis of zero-pressure isobar data, E_c'' can be calculated from equation (d5) if the initial value of B' is known. The initial value B' can be estimated from zero-pressure data or values of B' can be calculated based on assumptions about the initial behavior of the Gruneisen parameter. One assumption that can be made is that the derivative of γ with respect to ρ is zero at $\rho = \rho_0$. In this case one would have that B' has an initial value equal to B . A second assumption that can be made is that the derivative of $\gamma\rho$ with respect to ρ or η is zero at $\rho = \rho_0$ in which case the initial value for B' would be zero.

The last quantity that must still be specified is q . This quantity is treated as a constant and if one of the three models is invoked in equation (d2) then it is treated as an integer constant; however, the value for q must not be chosen arbitrarily. The reason that q must not be arbitrarily chosen is that there is a second-order contact between the Hugoniot and the isentrope that goes through the foot of the Hugoniot. This continuity between the isentrope and the Hugoniot forces constraints on many of the initial values of the variables in equations (d3)–(d5) and the value of q . The second-order contact between the Hugoniot and the isentrope can be expressed by the equation

$$\left(\frac{\partial^2 P}{\partial \rho^2}\right)_S = \left(\frac{\partial^2 P}{\partial \rho^2}\right)_H, \quad (d9)$$

where equation (d9) holds for $\rho = \rho_o$. For a material that has a linear U_s - u_p relation, the initial value for the right-hand side of equation (d9) is given by

$$\left(\frac{\partial^2 P}{\partial \rho^2}\right)_H = \frac{(4s - 2)c_o^2}{\rho_o}. \quad (d10)$$

By making use of these equations and the additional assumption that E_c may be ignored in comparison to c_o^2 , an equation for q can be derived⁶⁶:

$$q = \frac{3}{2}\left(4s - 2\gamma_o - \frac{4}{3}\right). \quad (d11)$$

There are many different thermodynamic quantities that can be examined by the integration of equations (d3)–(d5). Examples of calculating the temperature

along the principal Hugoniot for copper and stainless steel are shown in figures 167 and 168. The initial values necessary for the integration of equations (d3)–(d5) are given in table 70. In figure 169 the Gruneisen parameter for copper is plotted as a function of η . Also shown in this figure are curves calculated from equations (2.41), (2.43) and (2.48). As can be seen in these plots equation (2.41) gives much lower values for the Gruneisen parameter at high densities than does equation (2.43). The numerical integration of equations (d3)–(d5) predicts yet higher values for the Gruneisen parameter than does equation (2.43). The analytic form for γ given by equation (2.48) agrees somewhat better with the integration of equations (d3)–(d5) than does equation (2.43). Lastly the pressure on the zero-degree isotherm is plotted against η in figure 170. Three different zero-degree isotherms are drawn in this figure. One curve is derived from the integration of equations (d3)–(d5), another from the Rice equation of state, and the last from the Mie-Gruneisen equation of state. For reference purposes the principal Hugoniot is also shown. Similar plots for stainless-steel SS-304L are shown in figures 171 and 172.

E. Initial Conditions in the Flyer Plate

The initial conditions in the flyer plate can be estimated by taking advantage of what is measured in each experiment. Specifically, these measurements are the thickness of the flyer plate, the speed of the plate at the time it impacts the target, and the distance the flyer plate has traveled. The distance the flyer plate has traveled is nominally the same in each experiment at 25.4 mm. From these measurements an average driving pressure for the flyer plate can be calculated. This average pressure is given by the equation

$$\tilde{P} = \rho_o h \frac{u_f^2}{2d}. \quad (e1)$$

In this equation h is the initial thickness of the flyer plate, d is the distance the flyer plate has traveled, ρ_o is initial density of the flyer, and u_f is velocity of the flyer just before it impacts the target.

The derivation of equation (e1) assumes a constant acceleration of the flyer plate. It would then be expected that equation (e1) would overestimate the pressure on the back side of the flyer plate at the time it collides with the target plate. To refine the estimate, the detonation products of the HE may be treated as an ideal gas. The pressure in the gas may be related to the average driving pressure by the equation

$$\frac{1}{d} \int_l^{l+d} P dx = \tilde{P}. \quad (e2)$$

The term d in equation (e2) is the same as that in equation (e1). Assuming that the expansion of the gas is adiabatic, equation (e2) may be integrated to yield the equation

$$\frac{P_i}{1-\gamma} \left((y+1) \left(\frac{y}{y+1} \right)^\gamma - y \right) = \tilde{P}, \quad (e3)$$

where

$$y = \frac{l}{d}. \quad (e4)$$

The term P_i represents an initial pressure and the term l represents some thickness of the HE gas that actually takes part in driving the flyer. Throughout this appendix, the parameter γ in equation (e3) is the ratio of specific heats for an ideal gas and is not the Grüneisen parameter as it is elsewhere.

In order to calculate the final pressure at the interface of the flyer plate and the detonation products of the explosive, it is necessary to know either P_i or l . The term l can be estimated as

$$l = \tilde{c} t. \quad (e5)$$

The time t in this equation can be taken as the time it takes the flyer plate to travel the distance d . The term $\tilde{c}$ is an average sound speed in the HE gas and may be found from the average pressure. All this leads to the equation

$$\left(\frac{l}{d} \right)^2 = \frac{2\gamma h \rho_o}{d \rho_g}, \quad (e6)$$

where ρ_g is a density of the HE detonation products. To find a minimum value for l and hence y , a maximum value for ρ_g can be found from the equation

$$\rho_g = \rho_i \frac{\gamma + 1}{\gamma} . \quad (e7)$$

The quantity ρ_i in equation (e7) is the density of the HE before it is detonated. After l is found from equation (e6) then P_i can be calculated from equation (e3). The final pressure is then given by the equation

$$P_f = P_i \left(\frac{y}{y + 1} \right)^\gamma . \quad (e8)$$

With the calculation of P_f , the boundary conditions for the pressure on the flyer plate are known. The initial pressure P_i may be assumed to be a shock pressure from which the flyer plate has released. Instead of assuming or choosing a specific type of pressure gradient in the flyer plate, a linear density gradient in the flyer plate is assumed. The density gradient in the flyer plate is chosen such that

$$\int \rho dx = \rho_o h . \quad (e9)$$

The density and specific internal energy at the front surface of the flyer plate are such that the calculation of the pressure from the equation of state for the flyer will give zero pressure. The density and specific internal energy at the back surface will give the pressure P_f for the same calculation.

To evaluate P_i and P_f it is necessary to know the value of γ for the HE detonation products. The value of γ does depend on the type of high explosive but a typical value for a generic HE is 2.7. Table 71 lists values for P_i and P_f for each of the different type explosives used in these experiments. A nominal thickness of 1.5 mm was used for the thickness of the flyer plate in calculating the pressures P_i and P_f .

F. Equation of State Parameters and Sound Speeds

As stated previously, the parameters for the equations of state are mainly determined from principal Hugoniot data and zero pressure data. Principal Hugoniot data for both SS-304L stainless steel and OFHC copper are described by equation (2.4) very well. In the Mie-Gruneisen, the Rice, and the Gray equations of state, a linear U_s - u_p relation is assumed in deriving the equation of state. The parameters c_o and s are explicitly used in these equations of state. Only in the Tillotson equation of state is the parameter s not explicitly used. The parameter c_o is used in the Tillotson equation of state in determining the adiabatic bulk modulus at normal density and zero pressure. Reported values,³² of 3.940 km/s and 1.489 were used for c_o and s , respectively, for copper. For the stainless steel c_o and s were derived from a fit of U_s to u_p data.⁶⁶ The values used for c_o and s for stainless steel were 4.566 km/s and 1.485, respectively.

Other parameters needed for the equations of state are the normal density of the materials and the value of the Gruneisen γ at zero pressure. For the stainless steel ρ_o was taken to be 7.903 gm/cc and ρ_o for copper was taken to be 8.930 gm/cc. The initial values of the Gruneisen γ or γ_o were determined from the thermodynamic definition which leads to the equation

$$\gamma_o = \frac{c_o^2 \beta}{c_p}. \quad (f1)$$

In equation (f1) β is the coefficient of thermal expansion and c_p is the specific heat capacity at constant pressure. Using values of $49.5 \times 10^{-6}/\text{K}$ for β and 3.839×10^6 ergs/gm for c_p , equation (f1) gives a value of 2.002 for the Gruneisen parameter for copper. Using values of 44.1×10^{-6} for β and 4.442×10^6 ergs/gm for c_p , γ_o for SS-304L is 2.070. With the specification of γ_o , all the equation of state parameters are determined for the Rice and the Mie-Gruneisen equations of state. Additional information about the other parameters needed for the Gray and the Tillotson equations of state can be found in the appropriate reference.

There is one aspect of the formulation of the Gray equation of state in the present work that slightly deviates from that given by Royce.³⁰ Instead of assuming that α in equation (2.48) is determined from the equation

$$\gamma_o - \alpha = \frac{1}{2}, \quad (f2)$$

it is assumed that α is determined from the singularity produced by the linear U_s-u_p relation. More particularly, this singularity leads to a limiting value for γ in equations (d3)–(d5). When equation (2.48) is forced to agree with this limiting value, α can be found from the equation

$$\gamma_o - \frac{\alpha}{s} = 2(s-1). \quad (f3)$$

The principal Hugoniot calculated from these equations of state agree very well with experimental data. Comparisons of the principal Hugoniot calculated

from each equation of state and shock data for stainless steel⁶⁶ and copper³² are shown in figures 173–180. These comparisons are plotted in the P - ρ plane and each plot is limited in magnitude to 200 GPa. This more than covers the pressure range of interest in the present experiments. Also shown in each plot are release isentropes from 150 GPa and 80 GPa. In figures 181–188 the specific internal energy is plotted as a function of the pressure for the principal Hugoniot and for the release isentropes from 150 GPa and from 80 GPa. Each of the equations of state does a good job of matching the experimental Hugoniot data, but this is to be expected. What is more interesting than the calculations of the Hugoniots is the behavior of the isentropes. There are slight but definite differences in the release isentropes. These differences are the most noticeable in the E - P plots.

In calculating the decay of a shock wave, two quantities of principal importance are the bulk sound speed and the particle velocity. As figures 173–188 show, all the equations of state give nearly identical shock loci and release isentropes that are all very similar in the P - ρ and the E - P planes. The one quantity in which significant differences arise occurs in the calculation of c , the bulk sound speed. Although $\left(\frac{\partial P}{\partial \rho}\right)_H$ along the principal Hugoniot is essentially the same for each equation of state, this is not true for $\left(\frac{\partial P}{\partial \rho}\right)_S$ along the Hugoniot. This term is, of course, identical to c^2 . Calculations of the bulk sound speed as a function of the shock pressure from each equation of state are shown in figure 189 for stain-

less steel. Figure 190 shows the corresponding plots for copper. At low shock pressures, there is little difference among the values of c calculated from the different equations of state. As the pressure increases on the principal Hugoniot, the differences in the calculated sound speeds grow appreciably.

It is clear from figures 189 and 190 that above zero pressure on the Hugoniot, the Rice equation of state gives the largest sound speeds on the Hugoniot for a given pressure. A markedly different behavior is seen in the release isentropes. It must be remembered that the Rice equation of state is the only one of the equations of state in which the sound speed has no dependence on the specific internal energy. As the pressure decreases on the release isentropes, the effects of shock heating on the calculation of the bulk sound speed in the other equations of state become very apparent. Figure 191 shows the calculated sound speeds for SS-304L on the release isentropes from 150 GPa for each of the equations of state. In the pressure range of 60 to 80 GPa the Rice isentrope is intersected by all the other isentropes so that below about 60 GPa the Rice equation of state gives the lowest sound speed of any of the equations of state for a given pressure. Figure 192 reveals that the calculated sound speeds for copper exhibit the same behavior. In figures 193 and 194 the sound speeds along the release isentropes from 80 GPa are shown.

G. Pin Data

The pin data for the experiments in which the attenuation of a shock was measured are given in tables 72–77. For every pin there is a pin identification number, a measured time, and a measured position given in terms of x , y , and z . Those pin numbers appended by a small 'c' identifies the pin as a capped pin. The capped pins were used to measure the trajectory of the flyer plate just before impact with the target plate. In some instances a pin failed to give a signal. In such cases the measured time is replaced by 'np' (no pulse). In still other cases pin times were recorded but were so far out line with other times that they were dropped from the data analysis. Such times are marked by an asterisk. In experiment RS-2302 some pins were used solely to check on the planarity of the shock and were not used in the actual analysis of the decay of the shock wave. These pins are marked by a dagger. The reference system for the pin positions is given in chapter 3.

H. Tables

Table 1.

Brief Description of the Experiments

Experiment	Driving	Shot
Number	Explosive	Description
RS-2301	TNT	Flying Plate
RS-2302	TNT	STM* in SS-304L
RS-2306	TNT	STM in OFHC-Cu
RS-2307	PBX-9404	Flying Plate
RS-2308	PBX-9404	STM in OFHC-Cu
RS-2309	PBX-9404	STM in SS-304L
RS-2310	Comp. B	Flying Plate
RS-2314	Comp. B	STM in OFHC-Cu
RS-2315	Comp. B	STM in SS-304L

* Shock Trajectory Measurement

Table 2a.

RS-2301: Parameters for the Fit

$$t = a_o + a_x x + a_y y + a_z z + a_r r^2$$

Nominal Planes	-2, -1, 0	0, 15.22
No. of Data Points	94	96
a_o (μs)	$1.643 \pm .004$	$1.643 \pm .005$
a_x ($10^{-1} \mu s/mm$)	$2.765 \pm .022$	$1.155 \pm .003$
a_y ($10^{-3} \mu s/mm$)	$-0.213 \pm .036$	$-0.237 \pm .045$
a_z ($10^{-3} \mu s/mm$)	$-0.125 \pm .035$	$-0.112 \pm .043$
a_r ($10^{-5} \mu s/mm$)	$-0.694 \pm .072$	$-0.695 \pm .072$
σ (μs)	0.011	0.013
u (km/s)	$3.617 \pm .029$	$8.656 \pm .021$

Table 2b.

RS-2301: Parameters for the Fit

$$t = a_o + a_y y + a_z z + a_r r^2$$

Nominal Plane	0	15.22
No. of Data Points	49	47
a_o (μs)	$1.645 \pm .005$	$3.400 \pm .006$
a_y ($10^{-3} \mu s/mm$)	$-0.264 \pm .036$	$-0.237 \pm .045$
a_z ($10^{-3} \mu s/mm$)	$-0.130 \pm .052$	$-0.093 \pm .073$
a_r ($10^{-5} \mu s/mm$)	$-0.734 \pm .096$	$-0.666 \pm .109$
σ (μs)	0.011	0.014

Table 3a.

RS-2307: Parameters for the Fit

$$t = a_o + a_x x + a_y y + a_z z$$

Nominal Planes	-1.5, 0	0, 5
No. of Data Points	75	74
a_o (μs)	$1.937 \pm .004$	$1.937 \pm .004$
a_x ($10^{-1} \mu s/mm$)	$2.404 \pm .037$	$1.081 \pm .011$
a_y ($10^{-3} \mu s/mm$)	$0.106 \pm .054$	$-0.156 \pm .052$
a_z ($10^{-3} \mu s/mm$)	$-0.141 \pm .055$	$-0.138 \pm .051$
σ (μs)	0.015	0.014
u (km/s)	$4.159 \pm .065$	$9.248 \pm .091$

Table 3b.

RS-2307: Parameters for the Fit

$$t = a_o + a_y y + a_z z$$

Nominal Plane	0	5
No. of Data Points	37	37
a_o (μs)	$1.937 \pm .004$	$2.477 \pm .004$
a_y ($10^{-3} \mu s/mm$)	$0.161 \pm .077$	$0.156 \pm .078$
a_z ($10^{-3} \mu s/mm$)	$-0.136 \pm .077$	$-0.156 \pm .076$
σ (μs)	0.014	0.014

Table 4a.

RS-2310: Parameters for the Fit

$$t = a_0 + a_x x + a_y y + a_z z + a_1 r + a_2 r^2$$

Nominal Planes	-2, -1, 0	0, 15.22
No. of Data Points	94	96
a_0 (μ s)	$3.312 \pm .013$	$3.305 \pm .015$
a_x ($10^{-1} \mu$ s/mm)	$2.179 \pm .034$	$1.050 \pm .010$
a_y ($10^{-3} \mu$ s/mm)	$0.164 \pm .041$	$0.114 \pm .047$
a_z ($10^{-4} \mu$ s/mm)	$0.166 \pm .409$	$0.298 \pm .462$
a_1 ($10^{-3} \mu$ s/mm)	$1.537 \pm .439$	$1.719 \pm .474$
a_2 ($10^{-5} \mu$ s/mm)	$-1.132 \pm .319$	$-1.235 \pm .350$
σ (μ s)	0.011	0.012
u (km/s)	$4.590 \pm .072$	$9.520 \pm .087$

Table 4b.

RS-2310: Parameters for the Fit

$$t = a_0 + a_y y + a_z z + a_1 r + a_2 r^2$$

Nominal Plane	0	5
No. of Data Points	37	34
a_0 (μ s)	$3.296 \pm .018$	$3.831 \pm .024$
a_y ($10^{-3} \mu$ s/mm)	$0.107 \pm .060$	$0.119 \pm .076$
a_z ($10^{-4} \mu$ s/mm)	$0.074 \pm .589$	$0.482 \pm .757$
a_1 ($10^{-3} \mu$ s/mm)	$1.997 \pm .596$	$1.590 \pm .786$
a_2 ($10^{-5} \mu$ s/mm)	$-1.414 \pm .439$	$-1.136 \pm .582$
σ (μ s)	0.011	0.013

Table 5.
Measured Flyer Plate Velocities and
Predicted Shock Speeds

Experiment	Flyer Velocity (km/s)	Shock Speed (km/s)	x_1 minimum (mm)	x_1 maximum (mm)
RS-2302	3.65	7.28	7.8	10.6
RS-2309	4.53	7.93	6.6	9.5
RS-2315	4.15	7.65	7.0	9.8
RS-2306	3.69	6.65	6.4	8.8
RS-2308	4.63	7.34	5.6	8.3
RS-2314	4.10	6.95	7.0	9.8

Table 6.
 δt Fits for RS-2302

U_s (km/s)	x_1 (mm)	x_2 (mm)	Spline Option	σ_1 (ns)	σ_2 (ns)	σ (ns)
7.25 ± 0.06	4.5	8.5	2-2	4.9	8.6	8.2
7.24 ± 0.05	5.5	8.5	2-2	5.9	9.1	8.5
7.31 ± 0.05	6.5	8.5	2-2	7.9	7.6	7.4
7.35 ± 0.04	7.1	8.5	2-2	7.8	6.7	7.1
7.35 ± 0.04	7.8	8.5	3-3	7.8	6.7	7.1
7.35 ± 0.04	7.8	11.5	3-3	7.8	6.8	7.1
7.34 ± 0.03	8.2	12.5	2-3	7.8	6.6	7.1
7.36 ± 0.03	9.2	10.6	2-2	7.9	6.5	7.2
7.34 ± 0.03	10.2	10.6	2-2	8.0	6.5	7.3

Table 7.

 δt Fits for RS-2309

U_a (km/s)	x_1 (mm)	x_2 (mm)	Spline Option	σ_1 (ns)	σ_2 (ns)	σ (μ s)
8.03 ± 0.06	5.5	6.5	2-2	3.8	4.3	4.1
8.01 ± 0.05	6.5	7.5	2-2	3.8	4.2	4.1
8.02 ± 0.05	7.5	8.5	2-2	4.2	4.7	4.4
8.00 ± 0.04	8.5	9.5	2-2	4.2	6.3	5.2
7.95 ± 0.04	9.5	10.5	2-2	5.2	6.5	5.7

Table 8.
 δt Fits for RS-2315

U_s	x_1	x_2	Spline	σ_1	σ_2	σ
(km/s)	(mm)	(mm)	Option	(μ s)	(ns)	(ns)
7.68 ± 0.06	5.5	6.5	2-2	5.4	7.1	6.6
7.72 ± 0.05	6.5	7.5	2-2	5.3	7.1	6.5
7.73 ± 0.05	7.5	8.5	2-2	5.2	7.4	6.4
7.72 ± 0.04	8.5	9.5	2-2	5.3	8.1	6.7
7.67 ± 0.04	9.5	10.5	2-2	5.9	7.8	6.7

Table 9.

 δt Fits for RS-2306

U_s (km/s)	x_1 (mm)	x_2 (mm)	Spline Option	σ_1 (ns)	σ_2 (ns)	σ (ns)
6.67 ± 0.06	5.5	6.5	2-2	7.5	8.2	8.0
6.71 ± 0.05	6.5	7.5	2-2	7.4	8.3	8.0
6.70 ± 0.05	7.5	8.5	2-2	7.0	8.8	8.1
6.68 ± 0.04	8.5	9.5	2-2	7.3	9.2	8.5

Table 10.
 δt Fits for RS-2308

U_s (km/s)	x_1 (mm)	x_2 (mm)	Spline Option	σ_1 (ns)	σ_2 (ns)	σ (ns)
7.19 ± 0.06	4.8	5.8	2-2	3.9	6.0	5.3
7.24 ± 0.05	5.8	6.8	2-2	4.0	6.1	5.4
7.24 ± 0.05	6.8	7.8	2-2	3.9	6.4	5.4
7.22 ± 0.04	7.8	8.8	2-2	4.2	6.5	5.4
7.19 ± 0.04	8.8	9.8	2-2	4.7	6.2	5.4

Table 11.
 δt Fits for RS-2314

U_s (km/s)	x_1 (mm)	x_2 (mm)	Spline Option	σ_1 (ns)	σ_2 (ns)	σ (ns)
7.05 ± 0.06	5.5	6.5	2-2	6.4	7.6	7.2
7.04 ± 0.05	6.5	7.5	2-2	5.9	8.2	7.3
7.07 ± 0.05	7.5	8.5	2-2	6.0	8.1	7.1
7.06 ± 0.04	8.5	9.5	2-2	5.8	8.7	7.2

Table 12.

Probable Overtake Region

Experiment	Flyer Velocity (km/s)	Shock Speed (km/s)	Shock Pressure (GPa)	Overtake Region (mm)
RS-2302	3.65 ± 0.09	7.35 ± 0.07	~ 109	7 – 9
RS-2309	4.54 ± 0.15	8.02 ± 0.05	~ 147	5 – 7
RS-2315	4.15 ± 0.10	7.72 ± 0.05	~ 130	6 – 7
RS-2306	3.69 ± 0.09	6.71 ± 0.06	~ 111	6 – 8
RS-2308	4.63 ± 0.14	7.23 ± 0.04	~ 143	6.3 – 8.3
RS-2314	4.10 ± 0.10	7.06 ± 0.04	~ 132	6 – 8

Table 13.

RS-2302: Data Analysis and Code Parameters for Figure 70

Data Analysis Parameters

First Break Point	8.5 mm*
Second Break Point	9.5 mm
U_s for Zone 1	7.34 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Cubic

Code Parameters—SHKSIM

Flyer Plate Velocity	3.73 km/s
Mie-Gruneisen EOS for Flyer	
No Strength Model for Flyer	
Shock Velocity in Target	7.34 km/s
Overtake Position	9 $\pm$ 5 mm
Mie-Gruneisen EOS for Target	
No Strength Model for Target	

- * The first break point for the data analysis does not coincide with the overtake position from the code calculation.

Table 14.
Variables on a Right-Going Characteristic

	Starting Values	Ending Values
u (km/s)	1.287	1.289
c (km/s)	6.656	6.700
ρ (g/cc)	9.763	9.866
E (GPa-cc/g)	1.058	0.831
P (GPa)	66.33	66.02

Table 15.

RS-2302: Data Analysis and Code Parameters for Figure 71

Data Analysis Parameters

First Break Point	8.8 mm
Second Break Point	9.5 mm
U_s for Zone 1	7.34 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Cubic

Code Parameters—SHKSIM

Flyer Plate Velocity	3.73 km/s
Rice EOS for Flyer	
No Strength Model for Flyer	
Shock Velocity in Target	7.34 km/s
Overtake Position	8.78 mm
Rice EOS for Target	
No Strength Model for Target	

Table 16.

RS-2302: Data Analysis and Code Parameters for Figure 72

Data Analysis Parameters

First Break Point	8.5 mm*
Second Break Point	9.9 mm
U_0 for Zone 1	7.34 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Cubic

Code Parameters—SHKSIM

Flyer Plate Velocity	3.71 km/s
Tillotson EOS for Flyer	
No Strength Model for Flyer	
Shock Velocity in Target	7.34 km/s
Overtake Position	10.01 mm
Tillotson EOS for Target	
No Strength Model for Target	

* The first break point for the data analysis does not coincide with the overtake position from the code calculation.

Table 17.

RS-2302: Data Analysis and Code Parameters for Figure 73

Data Analysis Parameters

First Break Point	8.5 mm*
Second Break Point	9.8 mm
U_s for Zone 1	7.34 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Cubic

Code Parameters—SHKSIM

Flyer Plate Velocity	3.74 km/s
Gray FOS for Flyer	
No Strength Model for Flyer	
Shock Velocity in Target	7.34 km/s
Overtake Position	9.80 mm
Gray EOS for Target	
No Strength Model for Target	

* The first break point for the data analysis does not coincide with the overtake position from the code calculation.

Table 18.

RS-2315: Data Analysis and Code Parameters for Figure 74

Data Analysis Parameters

First Break Point	7.2 mm*
Second Break Point	8.6 mm
U_s for Zone 1	7.73 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSIM

Flyer Plate Velocity	4.26 km/s
Mie-Gruneisen EOS for Flyer	
No Strength Model for Flyer	
Shock Velocity in Target	7.73 km/s
Overtake Position	8.64 mm
Mie-Gruneisen EOS for Target	
No Strength Model for Target	

- * The first break point for the data analysis does not coincide with the overtake position from the code calculation.

Table 19.

RS-2315: Data Analysis and Code Parameters for Figure 75

Data Analysis Parameters

First Break Point	7.1 mm*
Second Break Point	7.9 mm
U_s for Zone 1	7.73 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSIM

Flyer Plate Velocity	4.26 km/s
Rice EOS for Flyer	
No Strength Model for Flyer	
Shock Velocity in Target	7.73 km/s
Overtake Position	7.86 mm
Rice EOS for Target	
No Strength Model for Target	

* The first break point for the data analysis does not coincide with the overtake position from the code calculation.

Table 20.

RS-2315: Data Analysis and Code Parameters for Figure 76

Data Analysis Parameters

First Break Point	7.1 mm*
Second Break Point	9.1 mm
U_s for Zone 1	7.73 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSIM

Flyer Plate Velocity	4.25 km/s
Tillotson EOS for Flyer	
No Strength Model for Flyer	
Shock Velocity in Target	7.73 km/s
Overtake Position	9.13 mm
Tillotson EOS for Target	
No Strength Model for Target	

- * The first break point for the data analysis does not coincide with the overtake position from the code calculation.

Table 21.

RS-2315: Data Analysis and Code Parameters for Figure 77

Data Analysis Parameters

First Break Point	7.2 mm*
Second Break Point	8.9 mm
U_s for Zone 1	7.73 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSIM

Flyer Plate Velocity	4.28 km/s
Gray EOS for Flyer	
No Strength Model for Flyer	
Shock Velocity in Target	7.73 km/s
Overtake Position	8.91 mm
Gray EOS for Target	
No Strength Model for Target	

- * The first break point for the data analysis does not coincide with the overtake position from the code calculation.

Table 22.

RS-2309: Data Analysis and Code Parameters for Figure 78

Data Analysis Parameters

First Break Point	6.9 mm*
Second Break Point	8.3 mm
U_s for Zone 1	8.02 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSIM

Flyer Plate Velocity	4.65 km/s
Mie-Gruneisen EOS for Flyer	
No Strength Model for Flyer	
Shock Velocity in Target	8.02 km/s
Overtake Position	8.26 mm
Mie-Gruneisen EOS for Target	
No Strength Model for Target	

- * The first break point for the data analysis does not coincide with the overtake position from the code calculation.

Table 23.

RS-2309: Data Analysis and Code Parameters for Figure 79

Data Analysis Parameters

First Break Point	6.9 mm*
Second Break Point	7.5 mm
U_s for Zone 1	8.02 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSIM

Flyer Plate Velocity	4.65 km/s
Rice EOS for Flyer	
No Strength Model for Flyer	
Shock Velocity in Target	8.02 km/s
Overtake Position	7.46 mm
Rice EOS for Target	
No Strength Model for Target	

* The first break point for the data analysis does not coincide with the overtake position from the code calculation.

Table 24.

RS-2309: Data Analysis and Code Parameters for Figure 80

Data Analysis Parameters

First Break Point	6.9 mm*
Second Break Point	8.8 mm
U_s for Zone 1	8.02 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSIM

Flyer Plate Velocity	4.65 km/s
Tillotson EOS for Flyer	
No Strength Model for Flyer	
Shock Velocity in Target	8.02 km/s
Overtake Position	8.77 mm
Tillotson EOS for Target	
No Strength Model for Target	

* The first break point for the data analysis does not coincide with the overtake position from the code calculation.

Table 25.

RS-2309: Data Analysis and Code Parameters for Figure 81

Data Analysis Parameters

First Break Point	6.9 mm*
Second Break Point	8.5 mm
U_s for Zone 1	8.02 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSIM

Flyer Plate Velocity	4.67 km/s
Gray EOS for Flyer	
No Strength Model for Flyer	
Shock Velocity in Target	8.02 km/s
Overtake Position	8.55 mm
Gray EOS for Target	
No Strength Model for Target	

* The first break point for the data analysis does not coincide with the overtake position from the code calculation.

Table 26.

RS-2302: Data Analysis and Code Parameters for Figure 85

Data Analysis Parameters

First Break Point	7.1* mm
Second Break Point	8.5 mm
U_s for Zone 1	7.35 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	3.74 km/s
Rice EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa
Shock Velocity in Target	7.35 km/s
Overtake Position	6.94 mm
Rice EOS for Target	
Strength Equation (2.93) for Target	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa

* The first break point for the data analysis does not coincide with the overtake position from the code calculation.

Table 27.

RS-2302: Data Analysis and Code Parameters for Figure 88

Data Analysis Parameters

First Break Point	6.9 mm
Second Break Point	8.5 mm
U_s for Zone 1	7.31 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	3.69 km/s
Rice EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa
Shock Velocity in Target	7.31 km/s
Overtake Position	6.97 mm
Rice EOS for Target	
Strength Equation (2.93) for Target	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa

Table 28.

RS-2315: Data Analysis and Code Parameters for Figure 90

Data Analysis Parameters

First Break Point	6.5 mm
Second Break Point	7.5 mm
U_s for Zone 1	7.72 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	4.23 km/s
Rice EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa
Shock Velocity in Target	7.71 km/s
Overtake Position	6.47 mm
Rice EOS for Target	
Strength Equation (2.93) for Target	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa

Table 29.

RS-2309: Data Analysis and Code Parameters for Figure 91

Data Analysis Parameters

First Break Point	6.3 mm*
Second Break Point	7.5 mm
U_s for Zone 1	8.01 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	4.64 km/s
Rice EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa
Shock Velocity in Target	8.02 km/s
Overtake Position	6.26 mm
Rice EOS for Target	
Strength Equation (2.93) for Target	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa

Table 30.

RS-2302: Data Analysis and Code Parameters for Figure 94

Data Analysis Parameters

First Break Point	7.5 mm
Second Break Point	8.5 mm
U_s for Zone 1	7.35 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Cubic

Code Parameters—SHKSJB

Flyer Plate Velocity	3.71 km/s
Rice EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa
Shock Velocity in Target	7.34 km/s
Overtake Position	7.54 mm
Rice EOS for Target	
Strength Equation (2.93) for Target	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa

Table 31.

RS-2315: Data Analysis and Code Parameters for Figure 96

Data Analysis Parameters

First Break Point	7.1 mm
Second Break Point	8.5 mm
U_s for Zone 1	7.73 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	4.25 km/s
Tillotson EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa
Shock Velocity in Target	7.74 km/s
Overtake Position	7.12 mm
Tillotson EOS for Target	
Strength Equation (2.93) for Target	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa

Table 32.

RS-2309: Data Analysis and Code Parameters for Figure 98

Data Analysis Parameters

First Break Point	6.9 mm
Second Break Point	7.5 mm
U_s for Zone 1	8.01 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	4.63 km/s
Tillotson EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa
Shock Velocity in Target	8.01 km/s
Overtake Position	6.99 mm
Tillotson EOS for Target	
Strength Equation (2.93) for Target	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa

Table 33.

RS-2302: Data Analysis and Code Parameters for Figure 91

Data Analysis Parameters

First Break Point	6.8 mm
Second Break Point	7.5 mm
U_s for Zone 1	7.36 km/s*
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	variable
Rice EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa
Shock Velocity in Target	variable
Overtake Position	6.84 mm
Rice EOS for Target	
Strength Equation (2.93) for Target	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa

* The 1-mm pin level has been deleted for the calculation of U_s .

Table 34.

RS-2306: Data Analysis and Code Parameters for Figure 102

Data Analysis Parameters

First Break Point	6.1 mm
Second Break Point	7.5 mm
U_s for Zone 1	6.71 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Cubic

Code Parameters—SHKSJB

Flyer Plate Velocity	3.77 km/s
Rice EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa
Shock Velocity in Target	6.71 km/s
Overtake Position	6.07 mm
Rice EOS for Target	
Strength Equation (2.93) for Target	
Constant μ	47.7 GPa
Constant Y_0	0.12 GPa

Table 35.

RS-2306: Data Analysis and Code Parameters for Figure 104

Data Analysis Parameters

First Break Point	6.9 mm
Second Break Point	7.5 mm
U_s for Zone 1	6.71 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	3.76 km/s
Rice EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa
Shock Velocity in Target	6.71 km/s
Overtake Position	6.90 mm
Rice EOS for Target	
No Strength Model for Target	

Table 36.

RS-2306: Data Analysis and Code Parameters for Figure 105

Data Analysis Parameters

First Break Point	6.4 mm
Second Break Point	7.5 mm
U_s for Zone 1	6.71 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	3.77 km/s
Rice EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa
Shock Velocity in Target	6.71 km/s
Overtake Position	6.41 mm
Mie-Gruneisen EOS for Target	
Strength Equation (2.93) for Target	
Constant μ	47.7 GPa
Constant Y_0	0.12 GPa

Table 37.

RS-2306: Data Analysis and Code Parameters for Figure 106

Data Analysis Parameters

First Break Point	6.6 mm
Second Break Point	7.5 mm
U_s for Zone 1	6.71 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	3.76 km/s
Rice EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa
Shock Velocity in Target	6.72 km/s
Overtake Position	6.65 mm
Tillotson EOS for Target	
Strength Equation (2.93) for Target	
Constant μ	47.7 GPa
Constant Y_0	0.12 GPa

Table 38.

RS-2306: Data Analysis and Code Parameters for Figure 107

Data Analysis Parameters

First Break Point	6.5 mm
Second Break Point	7.5 mm
U_o for Zone 1	6.71 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	3.77 km/s
Rice EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa
Shock Velocity in Target	6.71 km/s
Overtake Position	6.49 mm
Gray EOS for Target	
Strength Equation (2.93) for Target	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa

Table 39.

RS-2306: Data Analysis and Code Parameters for Figure 108

Data Analysis Parameters

First Break Point	6.8 mm
Second Break Point	7.5 mm
U_s for Zone 1	6.70 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	3.75 km/s
Rice EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa
Shock Velocity in Target	6.71 km/s
Overtake Position	7.84 mm
Tilltson EOS for Target	
No Strength Model for Target	

Table 40.

RS-2314: Data Analysis and Code Parameters for Figure 109

Data Analysis Parameters

First Break Point	6.4 mm
Second Break Point	7.5 mm
U_s for Zone 1	7.04 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	4.23 km/s
Rice EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa
Shock Velocity in Target	7.05 km/s
Overtake Position	6.42 mm
Rice EOS for Target	
No Strength Model for Target	

Table 41.

RS-2314: Data Analysis and Code Parameters for Figure 110

Data Analysis Parameters

First Break Point	6.9 mm
Second Break Point	7.5 mm
U_s for Zone 1	7.04 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	4.23 km/s
Rice EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa
Shock Velocity in Target	7.05 km/s
Overtake Position	6.97 mm
Mie-Gruneisen EOS for Target	
No Strength Model for Target	

Table 42.

RS-2314: Data Analysis and Code Parameters for Figure 111

Data Analysis Parameters

First Break Point	7.4 mm
Second Break Point	8.5 mm
U_o for Zone 1	7.07 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	4.25 km/s
Rice EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa
Shock Velocity in Target	7.06 km/s
Overtake Position	7.37 mm
Tillotson EOS for Target	
No Strength Model for Target	

Table 43.

RS-2314: Data Analysis and Code Parameters for Figure 112

Data Analysis Parameters

First Break Point	7.1 mm
Second Break Point	8.5 mm
U_s for Zone 1	7.07 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	4.25 km/s
Rice EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa
Shock Velocity in Target	7.06 km/s
Overtake Position	7.10 mm
Gray EOS for Target	
No Strength Model for Target	

Table 44.

RS-2308: Data Analysis and Code Parameters for Figure 115

Data Analysis Parameters

First Break Point	6.4 mm
Second Break Point	7.8 mm
U_s for Zone 1	7.24 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	4.49 km/s
Rice EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa
Shock Velocity in Target	7.24 km/s
Overtake Position	6.26 mm
Rice EOS for Target	
No Strength Model for Target	

Table 45.

RS-2308: Data Analysis and Code Parameters for Figure 117

Data Analysis Parameters

First Break Point	6.8 mm
Second Break Point	7.8 mm
U_s for Zone 1	7.24 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	4.49 km/s
Rice EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa
Shock Velocity in Target	7.24 km/s
Overtake Position	6.82 mm
Mie-Gruneisen EOS for Target	
No Strength Model for Target	

Table 46.

RS-2308: Data Analysis and Code Parameters for Figure 119

Data Analysis Parameters

First Break Point	7.2 mm
Second Break Point	7.8 mm
U_s for Zone 1	7.24 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	4.50 km/s
Rice EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa
Shock Velocity in Target	7.24 km/s
Overtake Position	7.25 mm
Tillotson EOS for Target	
No Strength Model for Target	

Table 47.

RS-2308: Data Analysis and Code Parameters for Figure 121

Data Analysis Parameters

First Break Point	6.9 mm
Second Break Point	7.8 mm
U_s for Zone 1	7.24 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	4.50 km/s
Rice EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	75.13 GPa
Constant Y_0	0.34 GPa
Shock Velocity in Target	7.24 km/s
Overtake Position	6.97 mm
Gray EOS for Target	
No Strength Model for Target	

Table 48.

RS-2302: Data Analysis and Code Parameters for Figure 123

Data Analysis Parameters

First Break Point	7.1 mm
Second Break Point	8.5 mm
U_s for Zone 1	7.35 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Cubic

Code Parameters—SHKSJB

Flyer Plate Velocity	3.71 km/s
Tillotson EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	100.17 GPa
Constant Y_0	0.85 GPa
Shock Velocity in Target	7.36 km/s
Overtake Position	7.06 mm
Tillotson EOS for Target	
Strength Equation (2.93) for Target	
Constant μ	100.17 GPa
Constant Y_0	0.85 GPa

Table 49.

RS-2302: Data Analysis and Code Parameters for Figure 124

Data Analysis Parameters

First Break Point	7.1 mm
Second Break Point	8.5 mm
U_s for Zone 1	7.35 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Cubic

Code Parameters—SHKSJB

Flyer Plate Velocity	3.68 km/s
Tillotson EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	100.17 GPa
Constant Y_0	1.70 GPa
Shock Velocity in Target	7.35 km/s
Overtake Position	7.12 mm
Tillotson EOS for Target	
Strength Equation (2.93) for Target	
Constant μ	100.17 GPa
Constant Y_0	1.70 GPa

Table 50.

RS-2302: Data Analysis and Code Parameters for Figure 125

Data Analysis Parameters

First Break Point	7.2 mm
Second Break Point	8.5 mm
U_s for Zone 1	7.35 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Cubic

Code Parameters—SHKSJB

Flyer Plate Velocity	3.64 km/s
Tillotson EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	100.17 GPa
Constant Y_0	3.40 GPa
Shock Velocity in Target	7.36 km/s
Overtake Position	7.24 mm
Tillotson EOS for Target	
Strength Equation (2.93) for Target	
Constant μ	100.17 GPa
Constant Y_0	3.40 GPa

Table 51.

RS-2315: Data Analysis and Code Parameters for Figure 129

Data Analysis Parameters

First Break Point	6.5 mm
Second Break Point	7.5 mm
U_s for Zone 1	7.72 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	4.13 km/s
Tillotson EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	125.22 GPa
Constant Y_0	4.08 GPa
Shock Velocity in Target	7.71 km/s
Overtake Position	6.51 mm
Tillotson EOS for Target	
Strength Equation (2.93) for Target	
Constant μ	125.22 GPa
Constant Y_0	4.08 GPa

Table 52.

RS-2309: Data Analysis and Code Parameters for Figure 131

Data Analysis Parameters

First Break Point	6.1 mm
Second Break Point	7.5 mm
U_s for Zone 1	8.01 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	4.53 km/s
Tillotson EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	150.26 GPa
Constant Y_0	4.76 GPa
Shock Velocity in Target	8.01 km/s
Overtake Position	6.12 mm
Tillotson EOS for Target	
Strength Equation (2.93) for Target	
Constant μ	150.26 GPa
Constant Y_0	4.76 GPa

Table 53.

RS-2302: Data Analysis and Code Parameters for Figure 133

Data Analysis Parameters

First Break Point	6.6 mm
Second Break Point	7.5 mm
U_s for Zone 1	7.37 km/s*
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Cubic

Code Parameters—SHKSJB

Flyer Plate Velocity	3.74 km/s
Rice EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	100.17 GPa
Constant Y_0	1.36 GPa
Shock Velocity in Target	7.37 km/s
Overtake Position	6.59 mm
Rice EOS for Target	
Strength Equation (2.93) for Target	
Constant μ	100.17 GPa
Constant Y_0	1.36 GPa

* The 1-mm pin level has been deleted for the calculation of U_s .

Table 54.

RS-2315: Data Analysis and Code Parameters for Figure 135

Data Analysis Parameters

First Break Point	6.2 mm
Second Break Point	7.5 mm
U_s for Zone 1	7.71 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	4.21 km/s
Rice EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	100.17 GPa
Constant Y_0	1.70 GPa
Shock Velocity in Target	7.72 km/s
Overtake Position	6.20 mm
Rice EOS for Target	
Strength Equation (2.93) for Target	
Constant μ	100.17 GPa
Constant Y_0	1.70 GPa

Table 55.

RS-2309: Data Analysis and Code Parameters for Figure 137

Data Analysis Parameters

First Break Point	5.5 mm
Second Break Point	6.5 mm
U_s for Zone 1	8.03 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	4.63 km/s
Rice EOS for Flyer	
Strength Equation (2.93) for Flyer	
Constant μ	150.26 GPa
Constant Y_0	2.04 GPa
Shock Velocity in Target	8.03 km/s
Overtake Position	5.55 mm
Rice EOS for Target	
Strength Equation (2.93) for Target	
Constant μ	150.26 GPa
Constant Y_0	2.04 GPa

Table 56.

RS-2302: Data Analysis and Code Parameters for Figure 139

Data Analysis Parameters

First Break Point	5.7 mm
Second Break Point	8.5 mm
U_s for Zone 1	7.37 km/s*
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Cubic

Code Parameters—SHKSJB

Flyer Plate Velocity	3.73 km/s
Rice EOS for Flyer	
Strength Equation (2.92) for Flyer	
Equations (4.13), (4.15), and (4.17) for μ and Y	
μ_0 , Y_0	0.34, and 75.13 GPa
Melting Formula (c11) for T_m	
Shock Velocity in Target	7.37 km/s
Overtake Position	5.68 mm
Rice EOS for Target	
Strength Equation (2.92) for Target	
Equations (4.13), (4.15), and (4.17) for μ and Y	
μ_0 , Y_0	0.34, and 75.13 GPa
Melting Formula (c11) for T_m	

* U_s is calculated from the 2-mm through 6-mm pin levels.

Table 57.

RS-2315: Data Analysis and Code Parameters for Figure 141

Data Analysis Parameters

First Break Point	5.4 mm
Second Break Point	7.5 mm
U_s for Zone 1	7.72 km/s*
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	4.23 km/s
Rice EOS for Flyer	
Strength Equation (2.92) for Flyer	
Equations (4.13), (4.15), and (4.17) for μ and Y	
μ_0, Y_0	0.34, and 75.13 GPa
Melting Formula (c11) for T_m	
Shock Velocity in Target	7.72 km/s
Overtake Position	5.45 mm
Rice EOS for Target	
Strength Equation (2.92) for Target	
Equations (4.13), (4.15), and (4.17) for μ and Y	
μ_0, Y_0	0.34, and 75.13 GPa
Melting Formula (c11) for T_m	

* U_s is calculated from the 1-mm through 6-mm pin levels.

Table 58.

RS-2309: Data Analysis and Code Parameters for Figure 143

Data Analysis Parameters

First Break Point	5.4 mm
Second Break Point	6.5 mm
U_{∞} for Zone 1	8.03 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	4.63 km/s
Rice EOS for Flyer	
Strength Equation (2.92) for Flyer	
Equations (4.13), (4.15), and (4.17) for μ and Y	
μ_0, Y_0	0.34, and 75.13 GPa
Melting Formula (c11) for T_m	
Shock Velocity in Target	8.03 km/s
Overtake Position	5.43 mm
Rice EOS for Target	
Strength Equation (2.92) for Target	
Equations (4.13), (4.15), and (4.17) for μ and Y	
μ_0, Y_0	0.34, and 75.13 GPa
Melting Formula (c11) for T_m	

Table 59.

RS-2302: Data Analysis and Code Parameters for Figure 145

Data Analysis Parameters

First Break Point	5.8 mm
Second Break Point	8.5 mm
U_s for Zone 1	7.37 km/s*
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Cubic

Code Parameters—SHKSJB

Flyer Plate Velocity	3.73 km/s
Rice EOS for Flyer	
Strength Equation (2.92) for Flyer	
Equations (4.13), (4.15), and (4.17) for μ and Y	
μ_0, Y_0	0.34, and 75.13 GPa
Melting Formula (c16) for T_m	
Shock Velocity in Target	7.37 km/s
Overtake Position	5.79 mm
Rice EOS for Target	
Strength Equation (2.92) for Target	
Equations (4.13), (4.15), and (4.17) for μ and Y	
μ_0, Y_0	0.34, and 75.13 GPa
Melting Formula (c16) for T_m	

* U_s is calculated from the 2-mm through 6-mm pin levels.

Table 60.

RS-2315: Data Analysis and Code Parameters for Figure 147

Data Analysis Parameters

First Break Point	5.6 mm
Second Break Point	7.5 mm
U_s for Zone 1	7.72 km/s*
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	3.73 km/s
Rice EOS for Flyer	
Strength Equation (2.92) for Flyer	
Equations (4.13), (4.15), and (4.17) for μ and Y	
μ_0, Y_0	0.34, and 75.13 GPa
Melting Formula (c16) for T_m	
Shock Velocity in Target	7.72 km/s
Overtake Position	5.56 mm
Rice EOS for Target	
Strength Equation (2.92) for Target	
Equations (4.13), (4.15), and (4.17) for μ and Y	
μ_0, Y_0	0.34, and 75.13 GPa
Melting Formula (c16) for T_m	

* U_s is calculated from the 1-mm through 6-mm pin levels.

Table 61.

RS-2309: Data Analysis and Code Parameters for Figure 149

Data Analysis Parameters

First Break Point	5.6 mm
Second Break Point	7.5 mm
U_s for Zone 1	8.03 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	4.64 km/s
Rice EOS for Flyer	
Strength Equation (2.92) for Flyer	
Equations (4.13), (4.15), and (4.17) for μ and Y	
μ_0 , Y_0	0.34, and 75.13 GPa
Melting Formula (c16) for T_m	
Shock Velocity in Target	8.03 km/s
Overtake Position	5.57 mm
Rice EOS for Target	
Strength Equation (2.92) for Target	
Equations (4.13), (4.15), and (4.17) for μ and Y	
μ_0 , Y_0	0.34, and 75.13 GPa
Melting Formula (c16) for T_m	

Table 62.

RS-2302: Data Analysis and Code Parameters for Figure 151

Data Analysis Parameters

First Break Point	6.2 mm
Second Break Point	8.5 mm
U_s for Zone 1	7.37 km/s*
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Cubic

Code Parameters—SHKSJB

Flyer Plate Velocity	3.74 km/s
Rice EOS for Flyer	
Strength Equation (2.92) for Flyer	
Equations (4.13), (4.15), and (4.17) for μ and Y	
μ_0, Y_0	0.34, and 75.13 GPa
Melting Formula (c16) for T_m	
Shock Velocity in Target	7.36 km/s
Overtake Position	6.15 mm
Rice EOS for Target	
Strength Equation (2.92) for Target	
Equations (4.13), (4.15), and (4.17) for μ and Y	
μ_0, Y_0	0.34, and 75.13 GPa
Melting Formula (c16) for T_m	

*The 1-mm pin level has been deleted in the calculation of U_s .

Table 63.

RS-2315: Data Analysis and Code Parameters for Figure 153

Data Analysis Parameters

First Break Point	5.9 mm
Second Break Point	7.5 mm
U_s for Zone 1	7.72 km/s*
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	3.74 km/s
Rice EOS for Flyer	
Strength Equation (2.92) for Flyer	
Equations (4.13), (4.15), and (4.17) for μ and Y	
μ_0, Y_0	0.34, and 75.13 GPa
Melting Formula (c16) for T_m	
Shock Velocity in Target	7.72 km/s
Overtake Position	5.95 mm
Rice EOS for Target	
Strength Equation (2.92) for Target	
Equations (4.13), (4.15), and (4.17) for μ and Y	
μ_0, Y_0	0.34, and 75.13 GPa
Melting Formula (c16) for T_m	

* U_s is calculated from the 1-mm through 6-mm pin levels.

Table 64.

RS-2309: Data Analysis and Code Parameters for Figure 155

Data Analysis Parameters

First Break Point	6.0 mm
Second Break Point	7.5 mm
U_s for Zone 1	8.03 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	4.65 km/s
Rice EOS for Flyer	
Strength Equation (2.92) for Flyer	
Equations (4.13), (4.15), and (4.17) for μ and Y	
μ_0, Y_0	0.34, and 75.13 GPa
Melting Formula (c16) for T_m	
Shock Velocity in Target	8.03 km/s
Overtake Position	5.97 mm
Rice EOS for Target	
Strength Equation (2.92) for Target	
Equations (4.13), (4.15), and (4.17) for μ and Y	
μ_0, Y_0	0.34, and 75.13 GPa
Melting Formula (c16) for T_m	

Table 65.

RS-2306: Data Analysis and Code Parameters for Figure 157

Data Analysis Parameters

First Break Point	6.7 mm
Second Break Point	8.5 mm
U_s for Zone 1	6.71 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	3.76 km/s
Rice EOS for Flyer	
Strength Equation (2.92) for Flyer	
Equations (4.13), (4.15), and (4.17) for μ and Y	
μ_0 , Y_0	0.34, and $\gamma \leq 13$ GPa
Melting Formula (c16) for T_m	
Shock Velocity in Target	6.71 km/s
Overtake Position	6.65 mm
Mie-Gruneisen EOS for Target	
γ_0 changed to	1.3
No Strength Model for Target	

Table 66.

RS-2314: Data Analysis and Code Parameters for Figure 159

Data Analysis Parameters

First Break Point	6.7 mm
Second Break Point	8.5 mm
U_s for Zone 1	7.04 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	4.22 km/s
Rice EOS for Flyer	
Strength Equation (2.92) for Flyer	
Equations (4.13), (4.15), and (4.17) for μ and Y	
μ_0 , Y_0	0.34, and 75.13 GPa
Melting Formula (c16) for T_m	
Shock Velocity in Target	7.04 km/s
Overtake Position	6.76 mm
Mie-Gruneisen EOS for Target	
γ_0 changed to	2.0
No Strength Model for Target	

Table 67.

RS-2308: Data Analysis and Code Parameters for Figure 161

Data Analysis Parameters

First Break Point	7.1 mm
Second Break Point	8.8 mm
U_0 for Zone 1	7.24 km/s
Polynomial Fit for Zone 2	Quadratic
Polynomial Fit for Zone 3	Quadratic

Code Parameters—SHKSJB

Flyer Plate Velocity	4.50 km/s
Rice EOS for Flyer	
Strength Equation (2.92) for Flyer	
Equations (4.13), (4.15), and (4.17) for μ and Y	
μ_0 , Y_0	0.34, and 75.13 GPa
Melting Formula (c16) for T_m	
Shock Velocity in Target	7.25 km/s
Overtake Position	7.11 mm
Mie-Gruneisen EOS for Target	
γ_0 changed to	2.5
No Strength Model for Target	

Table 68.

Experimental and Calculated Sound Speeds for Tantalum

Pressure	Experimental c_l	Calculated c_l	Calculated c_l
		$x = 1$	$x = 1.5$
GPa	km/s	km/s	km/s
0	4.14	4.04	4.05
152	6.23	6.23	6.38
211	6.55	6.50	6.64
229	6.61	6.56	6.68
255	6.75	6.63	6.71
293	6.65	6.70	6.70

Table 69.

Hugoniot Pressure and Residual Temperature

Material	EOS	Hugoniot Pressure	Residual Temperature
OFHC-Cu	Rice	~130 GPa	~1356 K
OFHC-Cu	Mie-Gruneisen	~140 GPa	~1356 K
SS-304L	Rice	~160 GPa	~1700 K
SS-304L	Mie-Gruneisen	~185 GPa	~1700 K

Table 70.

Initial Values for Equations (d3)–(d5)

Parameter	OFHC-Cu	SS-304L
E_H (GPa-cc/gm)	0.0	0.0
E_c (GPa-cc/gm)	-0.0763	-0.0923
R	2.002	2.070
B'	0.0	0.0
c_H (km/s)	3.940	4.566

Table 71.

Pressures at the HE-Flyer Interface

HE	ρ_o	u_f	$\tilde{P}$	$\frac{l}{d}$	P_i	P_f
	gm/cc	km/s	GPa		GPa	GPa
TNT	1.6	3.7	3.2	1.1	7.5	1.3
Comp B	1.7	4.1	3.9	1.0	9.4	1.5
PBX-9404	1.8	4.6	4.9	1.0	12.0	1.9

Table 72.
Pin Data for RS-2302

Pin No.	<i>t</i> (μ s)	<i>x</i> (mm)	<i>y</i> (mm)	<i>z</i> (mm)	Pin No.	<i>t</i> (μ s)	<i>x</i> (mm)	<i>y</i> (mm)	<i>z</i> (mm)
1c	1.111	-1.498	62.5	0.0	38	3.315	12.97	48.2	57.5
2c	1.318	-0.746	62.2	6.5	39	3.469	13.98	44.1	60.7
3c	1.538	0.0	61.1	13.0	40	3.620	14.95	39.7	63.6
4 [†]	2.064	3.98	57.9	23.4	41	3.774	15.99	35.2	66.2
5 [†]	2.212	4.97	54.1	31.2	42	3.919	16.96	30.5	68.5
6 [†]	3.624	14.96	40.2	47.9	43	4.127*	17.96	25.7	70.5
7 [†]	3.789	15.96	29.3	55.2	44	4.247	18.96	20.7	72.1
8c	1.129	-1.502	-17.2	60.1	45	4.426	19.99	15.6	73.4
9c	1.535	0.0	-23.4	57.9	46	1.648	0.96	-15.6	73.4
10 [†]	2.071	3.97	-33.1	53.0	47	1.795	1.96	-20.7	72.1
11 [†]	2.213	5.02	-40.2	47.9	48	1.926	2.98	-25.7	70.5
12 [†]	3.631	14.95	-54.1	31.3	49	2.073	3.97	-30.5	68.5
13 [†]	3.788	15.95	-57.9	23.4	50	2.219	5.00	-35.2	66.2
14 [†]	3.783	15.96	-57.9	-23.4	51	2.350	5.99	-39.7	63.6
15 [†]	3.616	14.96	-54.1	-31.3	52	2.465	6.96	-44.1	60.7
16 [†]	2.202	5.00	-40.2	-47.9	53	2.598	7.95	-48.2	57.5
17 [†]	2.062	3.96	-33.1	-53.0	54	2.732	8.99	-52.1	54.0
18c	1.530	0.0	-23.4	-57.9	55	2.877	9.97	-55.7	50.2
19c	1.041*	-1.509	-17.2	-60.1	56	3.024	10.96	-59.1	46.2
20 [†]	3.777	15.97	33.1	-53.0	57	3.171	11.97	-62.2	41.9
21 [†]	3.621	14.95	40.2	-47.9	58	3.320	12.96	-65.0	37.5
22 [†]	2.205	5.00	54.1	-31.3	59	3.464	13.96	-67.4	32.9
23 [†]	2.065	3.96	57.9	-23.4	60	3.616	14.94	-69.5	28.1
24c	1.519	0.0	61.1	-13.0	61	3.776	15.98	-71.3	23.2
25c	1.321	-0.748	62.2	-6.5	62	3.930	16.96	-72.8	18.1
26	np	0.96	75.0	2.6	63	4.081	17.96	-73.9	13.0
27	1.782	1.97	74.6	7.8	64	4.248	18.96	-74.6	7.8
28	1.923	2.96	73.9	13.0	65	4.404	19.99	-75.0	2.6
29	2.052	3.99	72.8	18.1	66	4.398	19.99	-75.0	-2.6
30	2.192	4.96	71.3	23.2	67	4.233	18.96	-74.6	7.8
31	2.322	5.96	69.5	28.1	68	4.088	17.95	-73.9	-13.0
32	2.466	6.96	67.4	32.9	69	3.918	16.95	-72.8	18.1
33	2.615	7.97	65.0	37.5	70	3.771	15.98	-71.3	-23.2
34	2.745	8.99	62.2	41.9	71	3.603	14.95	-69.5	-28.1
35	2.892	9.98	59.1	46.2	72	3.458	13.99	-67.4	-32.9
36	3.033	10.99	55.7	50.2	73	3.304	12.96	-65.0	-37.5
37	3.173	11.97	52.1	54.0	74	3.161	11.97	-62.2	-41.9

Table 72 continued

Pin No.	t (μ s)	x (mm)	y (mm)	z (mm)	Pin No.	t (μ s)	x (mm)	y (mm)	z (mm)
75	3.005	10.97	-59.1	-46.2	106c	1.095	-1.516	87.5	0.0
76	2.870	9.97	-55.7	-50.2	107c	1.318	-0.751	87.0	9.1
77	2.722	8.97	-52.1	-54.0	108c	1.492	0.0	85.6	18.2
78	2.587	7.96	-48.2	-57.5	109 [†]	2.042	3.98	81.2	32.8
79	2.435	6.97	-44.1	-60.7	110 [†]	2.185	4.98	75.8	43.8
80	2.308	5.97	-39.7	-63.6	111 [†]	3.613	14.96	56.2	67.0
81	2.179	5.00	-35.2	-66.2	112 [†]	3.764	15.96	46.4	74.2
82	2.095	3.96	-30.5	-68.5	113c	1.098	-1.492	-24.1	84.1
83	1.909	2.98	-25.7	-70.5	114c	1.707*	0.0	-32.8	81.1
84	1.775	1.97	-20.7	-72.1	115 [†]	2.060	3.96	-46.4	74.2
85	1.624	0.95	-15.6	-73.4	116 [†]	2.175	4.97	-56.2	67.0
86	4.380	19.99	15.6	-73.4	117 [†]	3.619	14.95	-75.8	43.8
87	4.218	18.96	20.7	-72.1	118 [†]	3.783	14.95	-81.1	32.8
88	4.075	17.96	25.7	-70.5	119 [†]	3.745	15.95	-81.1	-32.8
89	3.902	16.96	30.5	-68.5	120 [†]	3.595	14.94	-75.8	-43.8
90	3.755	15.97	35.2	-66.2	121 [†]	2.313	4.97	-56.2	-67.0
91	3.598	14.96	39.7	-63.6	122 [†]	2.032	3.96	-46.4	-74.2
92	3.449	13.98	44.1	-60.7	123c	1.508	0.0	-32.8	-81.1
93	3.306	12.96	48.2	-57.5	124c	1.089	-1.511	-24.1	-84.1
94	3.151	11.98	52.1	-54.0	125 [†]	3.739	15.95	46.4	-74.2
95	3.019	10.97	55.7	-50.2	126 [†]	np	14.97	56.2	-67.0
96	2.895	9.96	59.1	-46.2	127 [†]	2.173	4.97	75.8	-43.8
97	2.754	8.96	62.2	-41.9	128 [†]	2.039	3.99	81.1	-32.8
98	2.598	7.97	65.0	-37.5	129c	1.508	0.0	85.6	-18.2
99	2.463	6.97	67.4	-32.9	130c	1.300	-0.735	87.0	-9.1
100	2.321	5.97	69.5	-28.1					
101	2.201	5.00	71.3	-23.2					
102	2.049	3.96	72.8	-18.2					
103	1.921	2.98	73.9	-13.0					
104	1.778	1.97	74.6	-7.8					
105	1.642	0.96	75.0	-2.6					

Table 73.
Pin Data for RS-2309

Pin No.	t (μ s)	x (mm)	y (mm)	z (mm)	Pin No.	t (μ s)	x (mm)	y (mm)	z (mm)
1c	1.647	-1.007	40.0	0.0	38	3.812	15.030	-43.3	-25.0
2c	np	0.0	39.1	8.3	39c	1.851	0.0	-25.0	-43.3
3c	1.643	-1.011	-20.0	34.6	40	1.977	1.016	-20.3	-45.7
4c	1.857	0.0	-26.8	29.7	41	2.103	2.013	-15.5	-47.6
5c	1.619	-0.991	-20.0	-34.6	42	2.222	3.010	-10.4	-48.9
6c	1.844	0.0	-12.4	-38.0	43	2.347	4.010	-5.2	-49.7
7c	np	0.0	50.0	0.0	44	2.478	5.028	0.0	-50.0
8	1.994	0.997	49.7	5.2	45	2.606	6.024	5.2	-49.7
9	2.119	2.012	48.9	10.4	46	2.727	7.041	10.4	-48.9
10	2.242	3.000	47.6	15.5	47	2.858	8.030	15.5	-47.6
11	2.365	3.988	45.7	20.3	48	2.992	9.017	20.3	-45.7
12	2.494	5.027	43.3	25.0	49	3.128	10.032	25.0	-43.3
13	2.620	6.009	40.5	29.4	50	3.258	11.029	29.4	-40.5
14	2.752	7.018	37.2	33.5	51	3.395	12.009	33.5	-37.2
15	2.871	8.022	33.5	37.2	52	3.540	13.023	37.2	-33.5
16	3.003	9.014	29.4	40.59	53	3.681	14.010	40.5	-29.4
17	3.135	10.043	25.0	43.3	54	3.828	15.029	43.3	-25.0
18	3.270	11.028	20.3	45.7	55c	1.643	-0.992	60.0	0.0
19	3.406	12.005	15.5	47.6	56c	1.873	0.0	58.7	12.5
20	3.539	13.012	10.4	48.9	57c	1.643	-1.001	-30.0	52.0
21	3.680	14.031	5.2	49.7	58c	1.858	0.0	-40.1	44.6
22	3.826	15.048	0.0	50.0	59c	1.640	-0.994	-30.0	-52.0
23c	1.860	0.0	-25.0	43.3	60c	1.854	0.0	-18.5	-57.1
24	1.983	1.023	-29.4	40.5	61c	1.863	0.0	70.0	0.0
25	2.106	2.037	-33.5	37.2	62	1.983	0.999	69.6	7.3
26	2.187	2.712	-37.2	33.5	63	2.107	2.019	68.5	14.6
27	2.358	4.009	-40.5	29.4	64	2.237	3.012	66.6	21.6
28	2.482	5.029	-43.3	25.0	65	2.360	4.016	63.9	28.5
29	2.600	6.015	-45.7	20.3	66	2.489	5.021	60.6	35.0
30	2.725	7.018	-47.6	15.5	67	2.608	6.015	56.6	41.1
31	2.859	8.020	-48.9	10.4	68	2.731	7.036	52.0	46.8
32	2.989	9.026	-49.7	5.2	69	2.866	8.015	46.8	52.0
33	3.116	10.032	-50.0	0.0	70	3.000	8.987	41.1	56.6
34	3.246	11.036	-49.7	-5.2	71	3.132	10.030	35.0	60.6
35	np	12.018	-48.9	-10.4	72	3.260	11.015	28.5	63.9
36	3.525	13.038	-47.6	-15.5	73	3.389	11.982	21.6	66.6
37	3.665	14.030	-45.7	-20.3	74	3.500	12.752	14.6	68.5

Table 79 continued

Pin No.	t (μ s)	x (mm)	y (mm)	z (mm)
75	3.671	14.006	7.3	69.6
76	3.816	15.022	0.0	70.0
77c	1.845	0.0	-35.0	60.6
78	1.974	1.013	-41.4	56.6
79	2.105	2.005	-46.8	52.0
80	2.221	2.998	-52.0	46.8
81	2.339	4.006	-56.6	41.1
82	2.473	5.037	-60.6	35.0
83	2.590	6.016	-63.9	28.5
84	2.715	7.038	-66.6	21.6
85	2.842	8.035	-68.5	14.6
86	2.980	9.024	-69.6	7.3
87	3.116	10.049	-70.0	0.0
88	3.250	11.047	-69.6	-7.3
89	3.380	12.011	-68.4	-14.6
90	3.518	13.027	-66.6	-21.6
91	3.666	14.029	-63.9	-28.5
92	3.800	15.042	-60.6	-35.0
93c	1.829	0.0	-35.0	-60.6
94	1.960	1.023	-28.5	-63.9
95	2.090	2.044	-21.6	-66.6
96	2.220	3.019	-14.6	-68.5
97	2.339	4.014	-7.3	-69.6
98	2.457	5.012	0.0	-70.0
99	2.599	6.022	7.3	-69.6

Pin No.	t (μ s)	x (mm)	y (mm)	z (mm)
100	2.716	7.027	14.6	-68.4
101	2.850	8.027	21.6	-66.6
102	2.981	9.022	28.5	-63.9
103	3.120	10.041	35.0	-60.6
104	3.256	11.045	41.1	-56.6
105	3.390	12.014	46.8	-52.0
106	3.525	13.034	52.0	-46.8
107	3.672	14.029	56.6	-41.1
108	np	15.042	60.6	-35.0
109c	1.632	-1.015	80.0	0.0
110c	np	0.0	78.3	16.6
111c	1.620	-0.990	-40.0	69.3
112c	1.836	0.0	-53.5	59.5
113c	1.608	-0.995	-40.0	-69.3
114c	1.836	0.0	-24.7	-76.1

Table 74.
Pin Data for RS-2315

Pin No.	t (μs)	x (mm)	y (mm)	z (mm)	Pin No.	t (μs)	x (mm)	y (mm)	z (mm)
1c	1.394	-1.012	40.0	0.0	38	3.680	15.000	-43.3	-25.0
2c	1.637	0.0	39.1	8.3	39c	1.672	0.0	-25.0	-43.3
3c	1.415	-1.009	-20.0	34.6	40	1.805	0.991	-20.3	-45.7
4c	1.656	0.0	-26.8	29.7	41	1.928	1.989	-15.5	-47.6
5c	1.424	-1.002	-20.0	-34.6	42	2.056	2.982	-10.4	-48.9
6c	1.670	0.0	-12.4	-38.0	43	2.183	3.972	-5.2	-49.7
7c	1.641	0.0	50.0	0.0	44	2.317	4.985	0.0	-50.0
8	1.772	0.993	49.7	5.2	45	2.447	5.992	5.2	-49.7
9	1.903	2.012	48.9	10.4	46	2.571	7.015	10.4	-48.9
10	2.031	2.975	47.6	15.5	47	2.704	7.983	15.5	-47.6
11	2.148	3.936	45.7	20.3	48	2.841	8.979	20.3	-45.7
12	2.284	4.987	43.3	25.0	49	2.977	9.981	25.0	-43.3
13	2.411	5.984	40.5	29.4	50	3.111	10.979	29.4	-40.5
14	2.547	7.002	37.2	33.5	51	3.238	11.966	33.5	-37.2
15	2.672	7.996	33.5	37.2	52	3.391	12.994	37.2	-33.5
16	2.818	8.987	29.4	40.5	53	3.519	13.989	40.5	-29.4
17	2.940	9.993	25.0	43.3	54	3.666	14.998	43.3	-25.0
18	3.088	10.989	20.3	45.7	55c	1.396	-1.008	60.0	0.0
19	3.223	11.987	15.5	47.6	56c	1.629	0.0	58.7	12.5
20	3.381	13.003	10.4	48.9	57c	1.399	-1.005	-30.0	52.0
21	3.516	14.000	5.2	49.7	58c	1.634	0.0	-40.1	44.6
22	3.663	15.009	0.0	50.0	59c	1.421	-0.998	-30.0	-52.0
23c	1.655	0.0	-25.0	43.3	60c	1.661	0.0	-18.5	-57.1
24	1.785	0.996	-29.4	40.5	61c	1.638	0.0	70.0	0.0
25	np	2.002	-33.5	37.2	62	1.754	0.978	69.6	7.3
26	2.038	3.008	-37.2	33.5	63	1.881	2.004	68.5	14.6
27	2.172	3.972	-40.5	29.4	64	2.012	2.986	66.6	21.6
28	2.308	4.984	-43.3	25.0	65	2.143	3.990	63.9	28.5
29	2.427	5.976	-45.7	20.3	66	2.264	4.993	60.6	35.0
30	2.553	6.997	-47.6	15.5	67	2.390	5.972	56.6	41.1
31	2.697	7.997	-48.9	10.4	68	2.521	6.983	52.0	46.8
32	2.836	9.010	-49.7	5.2	69	2.656	7.973	46.8	52.0
33	2.965	9.99	-50.0	0.0	70	2.791	8.991	41.1	56.6
34	3.109	10.998	-49.7	-5.2	71	2.916	10.011	35.0	60.6
35	3.258	11.913	-48.9	-10.4	72	3.060	10.972	28.5	63.9
36	3.402	13.009	-47.6	-15.5	73	3.210	11.982	21.6	66.6
37	3.531	14.000	-45.7	-20.3	74	3.352	12.991	14.6	68.5

Table 74 continued

Pin No.	t (μ s)	x (mm)	y (mm)	z (mm)	Pin No.	t (μ s)	x (mm)	y (mm)	z (mm)
75	3.499	13.984	7.3	69.6	100	2.549	6.982	14.6	-68.4
76	3.656	15.006	0.0	70.0	101	2.686	7.982	21.6	-66.6
77c	1.623	0.0	-35.0	60.6	102	2.823	8.991	28.5	-63.9
78	1.752	0.994	-41.4	56.6	103	2.956	9.995	35.0	-60.6
79	1.888	2.003	-46.8	52.0	104	3.093	10.957	41.1	-56.6
80	2.022	2.992	-52.0	46.8	105	3.227	11.962	46.8	-52.0
81	2.141	3.969	-56.6	41.1	106	3.374	12.986	52.0	-46.8
82	2.288	5.009	-60.6	35.0	107	3.507	13.974	56.6	-41.1
83	2.409	5.993	-63.9	28.5	108	3.648	14.974	60.6	-35.0
84	2.549	7.001	-66.6	21.6	109c	1.376	-1.006	80.0	0.0
85	2.668	7.995	-68.5	14.6	110c	1.615	0.0	78.3	16.6
86	2.816	8.992	-69.6	7.3	111c	1.396	-1.007	-40.0	69.3
87	2.941	9.997	-70.0	0.0	112c	1.624	0.0	-53.5	59.5
88	3.084	10.972	-69.6	-7.3	113c	1.404	-0.996	-40.0	-69.3
89	3.233	11.981	-68.4	-14.6	114c	1.648	0.0	-24.7	-76.1
90	3.386	12.993	-66.6	-21.6					
91	3.530	13.996	-63.9	-28.5					
92	3.684	15.005	-60.6	-35.0					
93c	1.653	0.0	-35.0	-60.6					
94	1.774	0.976	-28.5	-63.9					
95	1.902	1.996	-21.6	-66.6					
96	2.030	2.996	-14.6	-68.5					
97	2.168	3.977	-7.3	-69.6					
98	2.306	5.006	0.0	-70.0					
99	2.426	5.992	7.3	-69.6					

Table 75.

Pin Data for RS-2306

Pin No.	t (μ s)	x (mm)	y (mm)	z (mm)	Pin No.	t (μ s)	x (mm)	y (mm)	z (mm)
1c	0.994	-1.174	63.5	2.2	38	1.757	2.965	-30.5	68.5
2c	1.310	0.0	62.9	8.8	39	1.615	1.989	-35.2	66.2
3c	1.323	0.0	-27.8	57.1	40	1.450	0.955	-39.7	63.6
4c	1.009	-1.207	-33.6	53.9	41	1.316	1.990	-44.1	60.7
5c	1.333	0.0	-39.1	50.0	42	1.771	2.969	-48.2	57.5
6c	1.322	0.0	-35.5	-52.6	43	1.909	3.958	-52.1	54.0
7c	0.990	-1.203	-29.8	-56.1	44	2.060	4.958	-55.7	50.2
8c	1.313	0.0	-23.8	-58.9	45	2.203	5.936	-59.1	46.2
9c	1.308	0.0	63.3	-4.4	46	2.349	6.927	-62.2	41.9
10	1.437	0.967	75.0	2.6	47	2.512	7.954	-65.0	37.5
11	1.599	1.973	74.6	7.8	48	2.666	8.952	-67.4	32.9
12	1.745	2.965	73.9	13.0	49	2.820	9.969	-69.5	28.1
13	1.903	3.964	72.8	18.1	50	2.982	10.950	-71.3	23.2
14	2.043	4.967	71.3	23.2	51	3.140	11.925	-72.3	18.1
15	2.189	5.939	69.5	28.1	52	3.307	12.960	-73.9	13.0
16	2.349	6.934	67.4	32.9	53	3.474	13.912	-74.6	7.8
17	2.508	7.959	65.0	37.5	54	3.645	14.967	-75.0	2.6
18	2.661	8.935	62.2	41.9	55	3.825	15.951	-75.0	-2.6
19	2.817	9.954	59.1	46.2	56	3.041	14.923	-74.6	-7.8
20	2.973	10.958	55.7	50.2	57	3.466	13.905	-73.9	-12.0
21	3.143	11.928	52.1	54.0	58	3.303	12.965	-72.8	-18.1
22	3.325	12.970	48.2	57.5	59	3.124	11.899	-71.3	-23.2
23	3.494	13.927	44.1	60.7	60	2.973	10.937	-69.5	-28.1
24	3.660	14.985	39.7	63.6	61	2.823	9.949	-67.4	-32.9
25	3.821	15.946	35.2	66.2	62	2.657	8.924	-65.0	37.5
26	3.652	14.968	30.5	68.5	63	2.499	7.951	-62.2	-41.9
27	3.469	13.925	25.7	70.5	64	2.349	6.924	-59.1	-46.2
28	3.295	12.979	20.7	72.1	65	2.197	5.921	-55.7	-50.2
29	3.123	11.945	15.6	73.4	66	2.045	4.958	-52.1	-54.0
30	2.972	10.971	10.4	74.3	67	np	3.940	-48.2	-57.5
31	2.808	9.979	5.2	74.8	68	1.741	2.942	-44.1	-60.7
32	2.657	8.962	0.0	75.0	69	1.606	1.958	-39.7	-63.6
33	2.492	7.965	-5.2	74.8	70	1.452	0.941	-35.2	-66.2
34	2.346	6.959	-10.4	74.3	71	1.615	1.981	-30.5	-68.5
35	2.194	5.951	-15.6	73.1	72	1.734	2.960	-25.7	-70.5
36	np	4.984	-20.7	72.1	73	1.904	3.943	-20.7	-72.1
37	1.912	3.965	-25.7	70.5	74	2.046	4.965	-15.6	-73.4

Table 75 continued

Pin No.	t (μ s)	x (mm)	y (mm)	z (mm)
75	2.192	5.933	-10.4	-74.3
76	2.345	6.947	-5.2	-74.8
77	2.497	7.957	0.0	-75.0
78	2.642	8.934	5.2	-74.8
79	2.803	9.958	10.4	-74.3
80	2.965	10.948	15.6	-73.4
81	3.125	11.926	20.7	-72.1
82	3.299	12.962	25.7	-70.5
83	3.460	19.930	30.5	-68.5
84	3.643	14.961	35.2	-66.2
85	3.818	15.956	39.7	-63.6
86	3.827*	14.953	44.1	-60.7
87	3.486	13.925	48.2	-57.5
88	3.309	12.982	52.1	-54.0
89	3.128	11.939	55.7	-50.2
90	2.973	10.967	59.1	-46.2
91	2.817	9.979	62.2	-41.9
92	2.658	8.968	65.0	-37.5
93	2.508	7.979	67.4	-32.9
94	2.338	6.957	69.5	-28.1
95	2.181	5.950	71.3	-23.2
96	2.060	4.977	72.8	-18.1
97	1.894	3.957	73.9	-13.0
98	1.743	2.971	74.6	-7.8
99	1.596	2.003	75.0	-2.6

Pin No.	t (μ s)	x (mm)	y (mm)	z (mm)
100c	0.977	-1.185	87.4	3.1
101c	1.302	0.0	87.0	9.1
102c	1.316	0.0	-41.1	77.3
103c	0.984	-1.171	-46.4	74.2
104c	1.294	0.0	-51.4	70.8
105c	1.291	0.0	-46.4	-74.2
106c	0.972	-1.168	-41.1	-77.3
107c	1.287	0.0	-35.6	-79.9
108c	1.310	0.0	87.4	-3.1

Table 76.
Pin Data for RS-2308

Pin No.	t (μ s)	x (mm)	y (mm)	z (mm)	Pin No.	t (μ s)	x (mm)	y (mm)	z (mm)
1c	1.450	-1.002	40.0	0.0	38	3.838	15.340	-43.3	-25.0
2c	1.667	0.0	39.1	8.3	39c	1.671	0.0	-25.0	-43.3
3c	1.463	-0.995	-20.0	34.6	40	1.802	1.320	-20.3	-45.7
4c	1.689	0.0	-26.8	29.7	41	1.937	2.342	-15.5	-47.6
5c	1.468	-0.996	-20.0	-34.6	42	2.076	3.342	-10.4	-48.9
6c	1.680	0.0	-12.4	-38.0	43	2.210	4.326	-5.2	-49.7
7c	np	0.0	50.0	0.0	44	2.348	5.323	0.0	-50.0
8	1.768	1.309	49.7	5.2	45	2.486	6.327	5.2	-49.7
9	1.920	2.331	48.9	10.4	46	2.631	7.359	10.4	-48.9
10	2.046	3.321	47.6	15.5	47	2.771	8.324	15.5	-47.6
11	2.185	4.309	45.7	20.3	48	2.909	9.332	20.3	-45.7
12	2.314	5.302	43.3	25.0	49	3.057	10.361	25.0	-43.3
13	2.459	6.319	40.5	29.4	50	3.203	11.354	29.4	-40.5
14	2.589*	7.346	37.2	33.5	51	3.355	12.348	33.5	-37.2
15	2.724*	8.334	33.5	37.2	52	3.511	13.344	37.2	-33.5
16	2.858*	9.326	29.4	40.59	53	3.661	14.336	40.5	-29.4
17	np	10.319	25.0	43.3	54	3.829	15.350	43.3	-25.0
18	3.177	11.358	20.3	45.7	55c	1.411	-0.995	60.0	0.0
19	3.332	12.340	15.5	47.6	56c	1.635	0.0	58.7	12.5
20	3.493	13.351	10.4	48.9	57c	1.429	-0.992	-30.0	52.0
21	3.651	14.336	5.2	49.7	58c	1.661	0.0	-40.1	44.6
22	3.820	15.371	0.0	50.0	59c	1.450	-0.988	-30.0	-52.0
23c	1.664	0.0	-25.0	43.3	60c	1.659	0.0	-18.5	-57.1
24	1.784	1.300	-29.4	40.5	61c	1.614	0.0	70.0	0.0
25	1.934	2.346	-33.5	37.2	62	1.748	1.314	69.6	7.3
26	2.077	3.329	-37.2	33.5	63	1.883	2.344	68.5	14.6
27	2.207	4.315	-40.5	29.4	64	2.024	3.331	66.6	21.6
28	2.348	5.339	-43.3	25.0	65	2.164	4.325	63.9	28.5
29	2.490	6.327	-45.7	20.3	66	2.300	5.320	60.6	35.0
30	2.637	7.339	-47.6	15.5	67	2.435	6.314	56.6	41.1
31	2.773	8.364	-48.9	10.4	68	2.590	7.360	52.0	46.8
32	2.906	9.332	-49.7	5.2	69	2.732	8.348	46.8	52.0
33	3.061	10.337	-50.0	0.0	70	2.871	9.350	41.1	56.6
34	3.214	11.346	-49.7	-5.2	71	3.015	10.354	35.0	60.6
35	3.363	12.322	-48.9	-10.4	72	3.169	11.325	28.5	63.9
36	3.520	13.347	-47.6	-15.5	73	np	12.331	21.6	66.6
37	3.685	14.322	-45.7	-20.3	74	3.479	13.358	14.6	68.5

Table 76 continued

Pin No.	<i>t</i> (μ s)	<i>x</i> (mm)	<i>y</i> (mm)	<i>z</i> (mm)	Pin No.	<i>t</i> (μ s)	<i>x</i> (mm)	<i>y</i> (mm)	<i>z</i> (mm)
75	3.632	14.365	7.3	69.6	100	2.601	7.343	14.6	-68.4
76	3.798	15.370	0.0	70.0	101	2.743	8.343	21.6	-66.6
77c	1.643	0.0	-35.0	60.6	102	2.880	9.338	28.5	-63.9
78	1.763	1.331	-41.4	56.6	103	3.029	10.337	35.0	-60.6
79	1.923	2.367	-46.8	52.0	104	3.168	11.346	41.1	-56.6
80	2.052	3.343	-52.0	46.8	105	3.321	12.355	46.8	-52.0
81	2.201	4.348	-56.6	41.1	106	3.489	13.359	52.0	-46.8
82	2.332	5.348	-60.6	35.0	107	3.634	14.330	56.6	-41.1
83	2.478	6.342	-63.9	28.5	108	3.792	15.370	60.6	-35.0
84	2.613	7.352	-66.6	21.6	109c	1.393	-0.987	80.0	0.0
85	2.763	8.340	-68.5	14.6	110c	1.602	0.0	78.3	16.6
86	2.894	9.334	-69.6	7.3	111c	1.426	-0.988	-40.0	69.3
87	3.053	10.355	-70.0	0.0	112c	1.643	0.0	-53.5	59.5
88	3.202	11.331	-69.6	-7.3	113c	np	-0.998	-40.0	-69.3
89	3.360	12.347	-68.4	-14.6	114c	1.660	0.0	-24.7	-76.1
90	3.517	13.360	-66.6	-21.6					
91	3.665	14.343	-63.9	-28.5					
92	np	15.355	-60.6	-35.0					
93c	1.646	0.0	-35.0	-60.6					
94	1.778	1.307	-28.5	-63.9					
95	1.920	2.336	-21.6	-66.6					
96	2.058	3.330	-14.6	-68.5					
97	2.199	4.303	-7.3	-69.6					
98	2.328	5.331	0.0	-70.0					
99	2.464	6.333	7.3	-69.6					

Table 77.

Pin Data for RS-2314

Pin No.	t (μ s)	x (mm)	y (mm)	z (mm)	Pin No.	t (μ s)	x (mm)	y (mm)	z (mm)
1c	1.422	-0.999	40.0	0.0	38	3.857	15.000	-43.3	-25.0
2c	1.666	0.0	39.1	8.3	39c	1.627	0.0	-25.0	-43.3
3c	1.419	-1.001	-20.0	34.6	40	1.773	1.013	-20.3	-45.7
4c	1.656	0.0	-26.8	29.7	41	1.920	2.029	-15.5	-47.6
5c	1.385	-1.002	-20.0	-34.6	42	2.060	3.003	-10.4	-48.9
6c	1.633	0.0	-12.4	-38.0	43	2.202	4.002	-5.2	-49.7
7c	1.675	0.0	50.0	0.0	44	2.347	5.003	0.0	-50.0
8	1.808	0.947	49.7	5.2	45	2.487	6.017	5.2	-49.7
9	1.959	1.996	48.9	10.4	46	2.621	7.016	10.4	-48.9
10	2.097	2.974	47.6	15.5	47	2.779	8.015	15.5	-47.6
11	2.236	3.967	45.7	20.3	48	2.925	8.984	20.3	-45.7
12	2.386	4.984	43.3	25.0	49	3.069	10.003	25.0	-43.3
13	2.533	5.979	40.5	29.4	50	3.234	10.989	29.4	-40.5
14	2.672	6.988	37.2	33.5	51	3.397	11.995	33.5	-37.2
15	2.816	7.953	33.5	37.2	52	3.567	13.006	37.2	-33.5
16	2.961	8.960	29.4	40.5	53	3.744	14.004	40.5	-29.4
17	3.118	9.977	25.0	43.3	54	3.913	15.007	43.3	-25.0
18	3.257	11.001	20.3	45.7	55c	np	-0.977	60.0	0.0
19	3.430	11.990	15.5	47.6	56c	1.693	0.0	58.7	12.5
20	3.600	13.000	10.4	48.9	57c	1.418	-0.988	-30.0	52.0
21	3.756	14.003	5.2	49.7	58c	1.658	0.0	-40.1	44.6
22	3.893	15.016	0.0	50.0	59c	1.381	-1.009	-30.0	-52.0
23c	1.662	0.0	-25.0	43.3	60c	1.628	0.0	-18.5	-57.1
24	1.807	1.005	-29.4	40.5	61c	1.658	0.0	70.0	0.0
25	1.937	2.005	-33.5	37.2	62	1.802	1.018	69.6	7.3
26	2.072	2.994	-37.2	33.5	63	1.954	2.011	68.5	14.6
27	2.209	3.971	-40.5	29.4	64	2.098	2.999	66.6	21.6
28	2.361	5.014	-43.3	25.0	65	2.232	4.016	63.9	28.5
29	2.491	5.998	-45.7	20.3	66	2.382	5.023	60.6	35.0
30	2.626	7.008	-47.6	15.5	67	2.528	6.008	56.6	41.1
31	np	8.002	-48.9	10.4	68	2.671	7.038	52.0	46.8
32	2.925	9.000	-49.7	5.2	69	2.810	8.020	46.8	52.0
33	3.070	10.015	-50.0	0.0	70	2.947	9.002	41.1	56.6
34	3.223	11.008	-49.7	-5.2	71	3.114	10.020	35.0	60.6
35	3.370	11.992	-48.9	-10.4	72	3.273	11.025	28.5	63.9
36	3.539	13.008	-47.6	-15.5	73	3.423	12.043	21.6	66.6
37	3.694	13.992	-45.7	-20.3	74	3.581	13.039	14.6	68.5

Table 77 continued

Pin No.	<i>t</i> (μ s)	<i>x</i> (mm)	<i>y</i> (mm)	<i>z</i> (mm)	Pin No.	<i>t</i> (μ s)	<i>x</i> (mm)	<i>y</i> (mm)	<i>z</i> (mm)
75	3.754	14.036	7.3	69.6	100	2.615	7.023	14.6	-68.4
76	np	15.083	0.0	70.0	101	2.748	8.001	21.6	-66.6
77c	1.658	0.0	-35.0	60.6	102	np	9.009	28.5	-63.9
78	1.782	1.022	-41.4	56.6	103	3.057	10.021	35.0	-60.6
79	1.920	2.021	-46.8	52.0	104	3.215	11.011	41.1	-56.6
80	2.045	3.007	-52.0	46.8	105	3.376	11.980	46.8	-52.0
81	2.192	4.035	-56.6	41.1	106	3.532	13.012	52.0	-46.8
82	2.334	5.031	-60.6	35.0	107	np	14.020	56.6	-41.1
83	2.476	6.032	-63.9	28.5	108	np	15.037	60.6	-35.0
84	2.609	7.040	-66.6	21.6	109c	1.426	-0.994	80.0	0.0
85	2.747	8.011	-68.5	14.6	110c	1.660	0.0	78.3	16.6
86	2.887	9.002	-69.6	7.3	111c	1.418	-0.986	-40.0	69.3
87	2.914	10.031	-70.0	0.0	112c	1.652	0.0	-53.5	59.5
88	3.192	11.042	-69.6	-7.3	113c	1.376	-1.001	-40.0	-69.3
89	3.348	11.995	-68.4	-14.6	114c	1.626	0.0	-24.7	-76.1
90	np	13.026	-66.6	-21.6					
91	3.513	13.012	-63.9	-28.5					
92	3.684	14.057	-60.6	-35.0					
93c	1.609	0.0	-35.0	-60.6					
94	1.737	0.998	-28.5	-63.9					
95	1.902	2.027	-21.6	-66.6					
96	2.041	2.994	-14.6	-68.5					
97	2.181	3.978	-7.3	-69.6					
98	2.322	5.009	0.0	-70.0					
99	2.467	5.998	7.3	-69.6					

I. Figures

Figure 1: A diagram showing the wave interaction for a flyer plate hitting a semi-infinite target. The figure is for the idealized case of a simple-centered release wave overtaking and attenuating a sharp shock.

Figure 2: Schematic diagram for a typical experiment.

Figure 3: Average times for a given radius in the plane $x = 0$ for the three flying plate experiments.

Figure 4: Observed times, t_i , plotted against pin position, x_i , in the target plate for experiment RS-2302.

Figure 5: An example of the adjusted times, δt_i , plotted against pin position, x_i , for experiment RS-2302.

Figure 6: A plot of δt versus x in which no attempt has been made to account for the non-planarity of the shock wave for experiment RS-2302.

Figure 7: A $\delta t - x$ plot for experiment RS-2302. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 7.8$ mm and the second break point is at 8.5 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 8: A plot of shock velocity, U_s , versus position, x , in the target plate taken from the fit to the data shown in figure 7 for experiment RS-2302.

Figure 9: A $\delta t - x$ plot for experiment RS-2302. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 7.8$ mm and the second break point is at 8.5 mm. Cubic polynomials have been used for the fits in the second and third zones.

Figure 10: A U_s versus x plot for RS-2302 taken from the fit to the data shown in figure 9.

Figure 11: A U_s versus x plot for RS-2302. The shock velocity curve is taken from a fit to the data shown in figure 9 that has the same break points but with a quadratic polynomial in the second zone instead of a cubic polynomial.

Figure 12: A $\delta t - x$ plot for experiment RS-2302. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 7.8$ mm and the second break point is at 11.5 mm. Cubic polynomials have been used for the fits in the second and third zones.

Figure 13: A U_s versus x plot for RS-2302 taken from the fit to the data shown in figure 12.

Figure 14: A $\delta t - x$ plot for experiment RS-2302. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 7.1$ mm and the second break point is at 8.5 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 15: A $\delta t - x$ plot for experiment RS-2302. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 4.5$ mm and the second break point is at 8.5 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 16: A U_s versus x plot for RS-2302 taken from the fit to the data shown in figure 15.

Figure 17: A $\delta t - z$ plot for experiment RS-2302. The dashed line is a fit to the adjusted δt data. The first break point is at $z = 10.2$ mm and the second break point is at 10.6 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 18: A U_s versus x plot for RS-2302 taken from the fit to the data shown in figure 17.

Figure 19: A $\delta t - x$ plot for experiment RS-2302. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 9.2$ mm and the second break point is at 10.6 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 20: A U_s versus x plot for RS-2302 taken from the fit to the data shown in figure 19.

Figure 21: A U_s versus x plot taken from a fit to the data of RS-2302. The first break point is at 10.2 mm and the second at 12.5 mm. A quadratic polynomial has been used for zone two and a cubic polynomial in zone three.

Figure 22: A U_s versus x plot taken from a fit to the data of RS-2302 using the same functional fitting forms as those used for figure 21. The second break point is still at 12.5 mm but the first break point has been moved to 9.2 mm.

Figure 23: A $\delta t - x$ plot for experiment RS-2302. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 8.2$ mm and the second break point is at 12.5 mm. A quadratic polynomial have been used for the fit in the second zone and a cubic polynomial in the third zone.

Figure 24: A U_s versus x plot for RS-2302 taken from the fit to the data shown in figure 23.

Figure 25: A $\delta t - x$ plot for experiment RS-2302. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 7.5$ mm and the second break point is at 8.5 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 26: A U_s versus x plot for RS-2302 taken from the fit to the data shown in figure 25.

Figure 27: A $\delta t - x$ plot for experiment RS-2302. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 8.5$ mm and the second break point is at 9.5 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 28: A U_s versus x plot for RS-2302 taken from the fit to the data shown in figure 27.

Figure 29: A $\delta t - x$ plot for experiment RS-2302. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 9.5$ mm and the second break point is at 10.5 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 30: A U_s versus x plot for RS-2302 taken from the fit to the data shown in figure 29.

Figure 31: A $\delta t - x$ plot for experiment RS-2309. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 8.5$ mm and the second break point is at 9.5 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 32: A $\delta t - x$ plot for experiment RS-2309. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 7.5$ mm and the second break point is at 8.5 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 33: A U_s versus x plot for RS-2309 taken from the fit to the data shown in figure 32.

Figure 34: A $\delta t - x$ plot for experiment RS-2309. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 6.5$ mm and the second break point is at 7.5 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 35: A U_s versus x plot for RS-2309 taken from the fit to the data shown in figure 34.

Figure 36: A U_s versus x plot taken from a fit to the data of RS-2309 using the same functional fitting forms as those used for figure 35. The second break point is now at 6.5 mm but the first break point has been moved to 5.5 mm.

Figure 37: A $\delta t - x$ plot for experiment RS-2315. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 8.5$ mm and the second break point is at 9.5 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 38: A U_s versus x plot for RS-2315 taken from the fit to the data shown in figure 37.

Figure 39: A $\delta t - x$ plot for experiment RS-2315. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 7.5$ mm and the second break point is at 8.5 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 40: A U_s versus x plot for RS-2315 taken from the fit to the data shown in figure 39.

Figure 41: A $\delta t - x$ plot for experiment RS-2315. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 6.5$ mm and the second break point is at 7.5 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 42: A U_s versus x plot for RS-2315 taken from the fit to the data shown in figure 41.

Figure 43: A $\delta t - x$ plot for experiment RS-2306. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 5.5$ mm and the second break point is at 6.5 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 44: A U_s versus x plot for RS-2306 taken from the fit to the data shown in figure 43.

Figure 45: A $\delta t - x$ plot for experiment RS-2306. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 6.5$ mm and the second break point is at 7.5 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 46: A U_s versus x plot for RS-2306 taken from the fit to the data shown in figure 45.

Figure 47: A $\delta t - x$ plot for experiment RS-2306. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 7.5$ mm and the second break point is at 8.5 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 48: A U_s versus x plot for RS-2306 taken from the fit to the data shown in figure 47.

Figure 49: A $\delta t - x$ plot for experiment RS-2306. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 8.5$ mm and the second break point is at 9.5 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 50: A U_s versus x plot for RS-2306 taken from the fit to the data shown in figure 49.

Figure 51: A $\delta t - x$ plot for experiment RS-2308. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 4.8$ mm and the second break point is at 5.8 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 52: A U_s versus x plot for RS-2308 taken from the fit to the data shown in figure 51.

Figure 53: A $\delta t - x$ plot for experiment RS-2308. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 5.8$ mm and the second break point is at 6.8 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 54: A U_s versus x plot for RS-2308 taken from the fit to the data shown in figure 53.

Figure 55: A $\delta t - x$ plot for experiment RS-2308. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 6.8$ mm and the second break point is at 7.8 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 56: A U_x versus x plot for RS-2308 taken from the fit to the data shown in figure 55.

Figure 57: A $\delta t - x$ plot for experiment RS-2308. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 8.8$ mm and the second break point is at 9.8 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 58: A U_s versus x plot for RS-2308 taken from the fit to the data shown in figure 57.

Figure 59: A $\delta t - x$ plot for experiment RS-2308. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 7.8$ mm and the second break point is at 8.8 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 60: A U_s versus x plot for RS-2308 taken from the fit to the data shown in figure 59.

Figure 61: A $\delta t - x$ plot for experiment RS-2314. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 5.5$ mm and the second break point is at 6.5 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 52: A U_s versus x plot for RS-2314 taken from the fit to the data shown in figure 61.

Figure 68. A $\delta t - x$ plot for experiment RS-2314. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 6.5$ mm and the second break point is at 7.5 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 64: A U_x versus x plot for RS-2314 taken from the fit to the data shown in figure 63.

Figure 65: A $\delta t - x$ plot for experiment RS-2314. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 7.5$ mm and the second break point is at 8.5 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 66: A U_s versus x plot for RS-2314 taken from the fit to the data shown in figure 65.

Figure 67: A $\delta t - x$ plot for experiment RS-2314. The dashed line is a fit to the adjusted δt data. The first break point is at $x = 8.5$ mm and the second break point is at 9.5 mm. Quadratic polynomials have been used for the fits in the second and third zones.

Figure 68: A U_s versus x plot for RS-2314 taken from the fit to the data shown in figure 67.

Figure 69: Detailed diagram of the different regions that occur when a simple-centered release wave attenuates a shock.

Figure 70: Comparison of the δt data for RS-2302 and calculational results from the code SHKSIM. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 13.

Figure 71: Comparison of the δt data for RS-2302 and calculational results from the code SHKSIM. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 15.

Figure 72: Comparison of the δt data for RS-2302 and calculational results from the code SHKSIM. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 16.

Figure 73: Comparison of the δt data for RS-2302 and calculational results from the code SHKSIM. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 17.

Figure 74: Comparison of the δt data for RS-2315 and calculational results from the code SHKSIM. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 18.

Figure 75: Comparison of the δt data for RS-2315 and calculational results from the code SHKSIM. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 19.

Figure 76: Comparison of the δt data for RS-2315 and calculational results from the code SHKSIM. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 20.

Figure 77: Comparison of the δt data for RS-2315 and calculational results from the code SHKSIM. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 21.

Figure 78: Comparison of the δt data for RS-2309 and calculational results from the code SHKSIM. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 22.

Figure 79: Comparison of the δt data for RS-2309 and calculational results from the code SHKSIM. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 23.

Figure 80: Comparison of the δt data for RS-2309 and calculational results from the code SHKSIM. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 24.

Figure 81: Comparison of the δt data for RS-2309 and calculational results from the code SHKSIM. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 25.

Figure 82: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for PS-2302. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 70 (see also table 13).

Figure 83: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2315. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 74 (see also table 18).

Figure 84: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2309. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 78 (see also table 22).

Figure 85: Comparison of the δt data for RS-2302 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 26.

Figure 86: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2302. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 85 (see also table 26).

Figure 87: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2302. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 71 (see also table 15).

Figure 88. Comparison of the δt data for RS-2302 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 27.

Figure 89: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2302. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 88 (see also table 27).

Figure 90: Comparison of the δt data for RS-2315 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 28.

Figure P1: Comparison of the δt data for RS-2309 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 29.

Figure 92: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2315. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 90 (see also table 28).

Figure 93: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2309. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 91 (see also table 29).

Figure 94: Comparison of the δt data for RS-2302 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 30.

Figure 95: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2302. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 94 (see also table 30).

Figure 96: Comparison of the δt data for RS-2315 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 31.

Figure 97: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2315. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 96 (see also table 31).

Figure 98: Comparison of the δt data for RS-2309 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 32.

Figure 99: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2309. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 98 (see also table 32).

Figure 100: Comparison of the δt data for RS-2302 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 33.

Figure 101: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2302. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 100 (see also table 33).

Figure 102: Comparison of the δt data for RS-2306 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 34.

Figure 103: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2306. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 102 (see also table 34).

Figure 104: Comparison of the δt data for RS-2306 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 35.

Figure 105: Comparison of the δt data for RS-230C and calculational results from the code SPKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 36.

Figure 106: Comparison of the δt data for RS-2306 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 37.

Figure 107: Comparison of the δt data for RS-2306 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 38.

Figure 108: Comparison of the δt data for RS-2306 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 39.

Figure 109: Comparison of the δt data for RS-2314 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 40.

Figure 110: Comparison of the δt data for RS-2314 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 41.

Figure 111: Comparison of the δt data for RS-2314 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 42.

Figure 112: Comparison of the δt data for RS-2314 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 43.

Figure 113: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2314. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 109 (see also table 40).

Figure 114: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2314. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 110 (see also table 41).

Figure 115: Comparison of the δt data for RS-2308 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 44.

Figure 116: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2308. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 115 (see also table 44).

Figure 117: Comparison of the δt data for RS-2308 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 45.

Figure 118: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2308. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 117 (see also table 45).

Figure 119: Comparison of the δt data for RS-2308 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 46.

Figure 120: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2308. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 119 (see also table 46).

Figure 121: Comparison of the δt data for RS-2308 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 47.

Figure 122: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2308. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 121 (see also table 47).

Figure 123: Comparison of the δt data for RS-2302 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 48.

Figure 124: Comparison of the δt data for RS-2302 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 49.

Figure 125: Comparison of the δt data for RS-2302 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 50.

Figure 126: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2302. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 123 (see also table 48).

Figure 127: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2302. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 124 (see also table 49).

Figure 128: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2302. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 125 (see also table 50).

Figure 129: Comparison of the δt data for RS-2315 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 51.

Figure 130: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2315. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 129 (see also table 51).

Figure 131: Comparison of the δt data for RS-2309 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 52.

Figure 152: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2309. The solid line is the velocity curve taken from the fit to the data. The corresponding dt plots are in figure 131 (see also table 52).

Figure 133: Comparison of the δt data for RS-2302 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 53.

Figure 134: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2302. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 133 (see also table 53).

Figure 135: Comparison of the δt data for RS-2315 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 54.

Figure 136: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2315. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 135 (see also table 54).

Figure 137: Comparison of the δt data for RS-2309 and calculational results from the code SHKSJB. The dashed line is the δt curve from the calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 55.

Figure 138: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2309. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 137 (see also table 55).

Figure 139: Comparison of the δt data for RS-2302 and calculational results from the code SHKSJB. The dashed line is the δt curve from the code calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 56.

Figure 140: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2302. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 139 (see also table 56).

Figure 141: Comparison of the δt data for RS-2315 and calculational results from the code SHKSJB. The dashed line is the δt curve from the code calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 57.

Figure 142: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2315. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 141 (see also table 57).

Figure 143: Comparison of the δt data for RS-2309 and calculational results from the code SHKSJB. The dashed line is the δt curve from the code calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 58.

Figure 144: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2309. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 143 (see also table 58).

Figure 145: Comparison of the δt data for RS-2302 and calculational results from the code SHKSJB. The dashed line is the δt curve from the code calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 59.

Figure 146: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2302. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 145 (see also table 59).

Figure 147: Comparison of the δt data for RS-2315 and calculational results from the code SHKSJB. The dashed line is the δt curve from the code calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 60.

Figure 148: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2315. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 147 (see also table 60).

Figure 149: Comparison of the δt data for RS-2309 and calculational results from the code SHKSJB. The dashed line is the δt curve from the code calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 61.

Figure 150: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2309. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 149 (see also table 61).

Figure 151: Comparison of the δt data for RS-2302 and calculational results from the code SHKSJB. The dashed line is the δt curve from the code calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 62.

Figure 152: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2302. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 151 (see also table 62).

Figure 153: Comparison of the δt data for RS-2315 and calculational results from the code SHKSJB. The dashed line is the δt curve from the code calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 63.

Figure 154: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2315. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 153 (see also table 63).

Figure 155: Comparison of the δt data for RS-2309 and calculational results from the code SHKSJB. The dashed line is the δt curve from the code calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 64.

Figure 156: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2309. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 155 (see also table 64).

Figure 157: Comparison of the δt data for RS-2306 and calculational results from the code SHK3.3. The dashed line is the δt curve from the code calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 65.

Figure 158: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2306. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 157 (see also table 65).

Figure 159: Comparison of the δt data for RS-2314 and calculational results from the code SHKSJB. The dashed line is the δt curve from the code calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 66.

Figure 160: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2314. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 159 (see also table 66).

Figure 161: Comparison of the δt data for RS-2308 and calculational results from the code SHKSJB. The dashed line is the δt curve from the code calculation. Parameters for the calculational model and parameters for the data analysis are listed in table 67.

Figure 162: A comparison of the shock velocity from the code calculation and the shock velocity curve from a fit to the data for RS-2308. The solid line is the velocity curve taken from the fit to the data. The corresponding δt plots are in figure 161 (see also table 67).

Figure 163. Comparison of numerical results from SHKSIM and an analytical solution for shock attenuation. The initial zoning for SHKSIM was 0.05 mm. The initial flyer plate velocity in the calculation was 3.7 km/s.

Figure 164. Comparison of numerical results from SHKSJB and an analytical solution for shock attenuation. The initial zoning for SHKSJB was 0.05 mm. The initial flyer plate velocity in the calculation was 3.7 km/s.

Figure 165. Comparison of numerical results from SHKSJB and an analytical solution for shock attenuation. The initial zoning for SHKSJB was 0.025 mm. The initial flyer plate velocity in the calculation was 4.65 km/s.

Figure 166. Comparison of the shock velocities from SHKSJB and an analytical solution plotted as a function of shock position. The corresponding δt - x plots are shown in figure 165.

Figure 167. Temperature along the principal Hugoniot for copper taken from the integration of equations (d3)–(d5). Electronic effects are not explicitly taken into account.

Figure 168. Temperature along the principal Hugoniot for stainless-steel taken from the integration of equations (d3)-(d5). Electronic effects are not explicitly taken into account.

Figure 169. The Gruneisen parameter for copper plotted as a function of η from different formulas.

Figure 170. Pressure as a function of η along zero-degree isotherms for copper. The dashed line is calculated using the Mie-Gruneisen equation of state. The dotted line is calculated using the Rice equation of state. The solid line is from the integration of equations (d3)-(d5).

Figure 171. The Gruneisen parameter for stainless-steel plotted as a function of η from different formulas.

Figure 172. Pressure as a function of η along zero-degree isotherms for stainless-steel. The dashed line is calculated using the Mie-Gruneisen equation of state. The dotted line is calculated using the Rice equation of state. The solid line is from the integration of equations (d3)-(d5).

Figure 173. The Hugoniot for stainless-steel in the P - ρ plane as calculated from the Mie-Grüneisen equation of state. The dashed line is the calculated release isentrope from 150 GPa. The dotted line is the isentrope from 80 GPa.

Figure 174. The Hugoniot for stainless-steel in the P - ρ plane as calculated from the Rice equation of state. The dashed line is the calculated release isentrope from 150 GPa. The dotted line is the isentrope from 80 GPa.

Figure 175. The Hugoniot for stainless-steel in the P - ρ plane as calculated from the Tillotson equation of state. The dashed line is the calculated release isentrope from 150 GPa. The dotted line is the isentrope from 80 GPa.

Figure 176. The Hugoniot for stainless-steel in the P - ρ plane as calculated from the Gray equation of state. The dashed line is the calculated release isentrope from 150 GPa. The dotted line is the isentrope from 80 GPa.

Figure 177. The Hugoniot for copper in the P - ρ plane as calculated from the Mie-Gruneisen equation of state. The dashed line is the calculated release isentrope from 150 GPa. The dotted line is the isentrope from 60 GPa.

Figure 178. The Hugoniot for copper in the P - ρ plane as calculated from the Rice equation of state. The dashed line is the calculated release isentrope from 150 GPa. The dotted line is the isentrope from 80 GPa.

Figure 179. The Hugoniot for copper in the P - ρ plane as calculated from the Tillotson equation of state. The dashed line is the calculated release isentrope from 150 GPa. The dotted line is the isentrope from 80 GPa.

Figure 180. The Hugoniot for copper in the P - ρ plane as calculated from the Gray equation of state. The dashed line is the calculated release isentrope from 150 GPa. The dotted line is the isentrope from 80 GPa.

Figure 181. The Hugoniot for stainless-steel in the E - P plane as calculated from the Mie-Grüneisen equation of state. The dashed line is the calculated release isentrope from 150 GPa. The dotted line is the isentrope from 80 GPa.

Figure 182. The Hugoniot for stainless-steel in the E - P plane as calculated from the Rice equation of state. The dashed line is the calculated release isentrope from 150 GPa. The dotted line is the isentrope from 80 GPa.

Figure 183. The Hugoniot for stainless-steel in the E - P plane as calculated from the Tillotson equation of state. The dashed line is the calculated release isentrope from 150 GPa. The dotted line is the isentrope from 80 GPa.

Figure 184. The Hugoniot for stainless-steel in the E - P plane as calculated from the Gray equation of state. The dashed line is the calculated release isentrope from 150 GPa. The dotted line is the isentrope from 80 GPa.

Figure 185. The Hugoniot for copper in the E - P plane as calculated from the Mie-Grüneisen equation of state. The dashed line is the calculated release isentrope from 150 GPa. The dotted line is the isentrope from 80 GPa.

Figure 186. The Hugoniot for copper in the E - P plane as calculated from the Rice equation of state. The dashed line is the calculated release isentrope from 150 GPa. The dotted line is the isentrope from 80 GPa.

Figure 187. The Hugoniot for copper in the E - P plane as calculated from the Tillotson equation of state. The dashed line is the calculated release isentrope from 150 GPa. The dotted line is the isentrope from 80 GPa.

Figure 188. The Hugoniot for copper in the E - P plane as calculated from the Gray equation of state. The dashed line is the calculated release isentrope from 150 GPa. The dotted line is the isentrope from 80 GPa.

Figure 189. The sound speeds for stainless-steel along the Hugoniot as calculated from the various equations of state.

Figure 190. The sound speeds for copper along the Hugoniot as calculated from the various equations of state.

Figure 191. The sound speeds for stainless-steel along the 150-GPa isentrope as calculated from the various equations of state.

Figure 192. The sound speeds for copper along the 150-GPa isentrope as calculated from the various equations of state.

Figure 193. The sound speeds for stainless-steel along the 80-GPa isentrope as calculated from the various equations of state.

Figure 194. The sound speeds for copper along the 80-GPa isentrope as calculated from the various equations of state.

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