PERTURBATIVE QUANTUM CHROMODYNAMICS*

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1. Introduction

From the perspective of the hadronic physics of a decade ago it seems incredible that there now exists a viable, fundamental theory of the strong interactions. In fact, quantum chromodynamics is radically different from the picture of hadronic phenomena which was envisioned in the 1960's. In contrast to the hadronic bootstrap, the quark and gluon quanta of QCD represent fundamental hadronic constituents, the elementary carriers of the electromagnetic and weak currents. In contrast to a strong-coupling model, the quark and gluon interactions of QCD approach scale-invariance at short distances and can be computed as a perturbative expansion in an asymptotically small coupling constant.^{1,2} As a consequence, the large momentum-transfer strong, electromagnetic, and weak interactions of hadrons are patterned after elementary short-distance gluon and quark subprocesses. The contrast between the old and new pictures is especially vivid in the reaction³ $\gamma\gamma \rightarrow$ hadrons: In QCD, one predicts a large scale-invariant cross section for the production of two $(q\bar{q})$ hadronic jets at large transverse momentum:

$$d\sigma(\gamma\gamma \rightarrow \text{jet} + \text{jet}) = 3 \sum_q e_q^4 d\sigma(\gamma\gamma \rightarrow q^+q^-) \\ \times [1 + O(\alpha_s(p_T^2), m^2/p_T^2)] \quad (1.1)$$

[The factor of 3 is from color; the sum is over quark flavors with $m_q^2 \ll p_T^2$.] From the historical perspective of vector-meson dominance, photon-photon reactions would have been expected to resemble hadron-hadron collisions; a multi-particle production reaction where all the hadrons emerge at large angles from the beam would certainly be extraordinary.

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There is now a huge arena of QCD applications in which predictions can be made with various degrees of reliability. It is clearly crucial to find critical unassailable tests of QCD. If there is even one bonafide failure in any area of hadronic phenomena, the theory is wrong.

In quantum chromodynamics the fundamental degrees of freedom of hadrons and their interactions are the quanta of quark and gluon fields which obey an exact internal SU(3) symmetry.^{2,4} It is possible (but by no means certain!) that quantum chromodynamics is the theory of the strong interactions in the same sense that quantum electrodynamics accounts for electromagnetic interactions. The fact that we now understand a fundamental parameter of nature such as the electron's gyromagnetic ratios to 10 significant figures encourages our optimism that there is an analogous local gauge field theoretic basis for hadrons. It is well known that the general structure of QCD meshes remarkably well with the facts of the hadronic world, especially quark-based (especially charm) spectroscopy, current algebra, the approximate parton-model structure of large momentum transfer reactions, logarithmic scale violations, the scaling and magnitude of $\sigma(e^+e^- \rightarrow \text{hadrons})$, jet-production, as well as the narrowness of the ψ and χ .⁶

In these lectures, we will concentrate on the application of QCD to hadron dynamics at short distances, where asymptotic freedom allows a systematic perturbative approach.⁷ The main theme of our approach^{8,9} will be to systematically incorporate the effects of the hadronic wave function in large momentum transfer exclusive and inclusive reactions. Although it is conventional to treat the hadron as a classical source of on-shell quarks, there are important dynamical effects due to hadronic

constituent structure which lead to a broader testing ground for QCD. We will especially discuss QCD predictions for exclusive processes and form factors at large momentum transfer in which the short-distance behavior and the finite compositeness of the hadronic wave functions play crucial roles. In addition we will review many of the standard tests of QCD including the predictions for $R = \sigma_{e^+e^- \rightarrow \text{had}} / \sigma_{e^+e^- \rightarrow \mu^+\mu^-}$, the structure functions of hadrons and photons, jet phenomena, and the QCD corrections to deep inelastic processes. We will also discuss how the exclusive-inclusive connection works in QCD, the effects of power-law scale-breaking contributions, and the important role of the available energy W in controlling logarithmic scale violations.

Despite the fact that they describe such different physical systems there are many similarities between QED and QCD, at least in perturbation theory. The renormalization procedure and infrared cancellation calculations of QCD parallel those of electrodynamics. Many of the results which we discuss here for inelastic lepton scattering and meson bound state effects are, with minor modification, applicable to QED positronium problems. The similarities are possibly not coincidental since it seems natural that QCD, QED, and the weak interactions are joined together in a unified gauge theory such as $SU(5)$:¹⁰

$$[SU_C(3) \times SU(2) \times U(1) \subset SU(5)]$$

The most remarkable postulate of QCD is the assumption of exact $SU(3)$ -color symmetry; there is no way to distinguish absolute color. The spin-1/2 quarks are in the fundamental (triplet) representation of $SU(3)_C$, the spin-1 gluons are in the adjoint (octet) representation, and hadrons are identified with singlet states; e.g., $|H\rangle \sim \sum_{i=1}^3 |q_i \bar{q}_i\rangle$.

$|B\rangle \sim \sum c_{ijk} |q_i q_j q_k\rangle$. In addition, gluonium (color-singlet bound states of 2 and 3 gluon quanta) should exist. Different quark representations are distinguishable by a flavor label. However, $SU(3)_C$ is an exact local symmetry: rotations in color space can be made independently at any space-time point. The mathematical realization of this is the (Yang-Mills) non-Abelian gauge field theory.¹¹

The Lagrangian density of QCD is

$$\begin{aligned} \mathcal{L}_{QCD}(x) = & \bar{q}_i(x) \gamma^\mu \left(i \frac{\partial}{\partial x_\mu} \delta_{ij} + \frac{g}{2} A_\mu^a(x) \lambda_{ij}^a \right) q_j(x) \\ & - \frac{1}{4} \left(\frac{\partial}{\partial x_\mu} A_\nu^a(x) - \frac{\partial}{\partial x_\nu} A_\mu^a(x) + g f_{abc} A_\mu^b A_\nu^c \right)^2 \\ & i, j = 1, 2, 3 \quad ; \quad a = 1, 2, \dots, 8 \end{aligned} \quad (1.2)$$

(A quark mass term and sum over flavors is understood.) Here the λ^a are the eight $SU(3)$ matrices with $\text{Tr}[\lambda^a \lambda^b] = 2\delta^{ab}$ (conventional normalization). We can contrast this with

$$\begin{aligned} \mathcal{L}_{QED} = & \bar{q}(x) \gamma^\mu \left(i \frac{\partial}{\partial x_\mu} + e A_\mu(x) \right) q(x) \\ & - \frac{1}{4} \left(\frac{\partial}{\partial x_\mu} A_\nu(x) - \frac{\partial}{\partial x_\nu} A_\mu(x) \right)^2 \end{aligned} \quad (1.3)$$

We can also use the more compact notation

$$\mathcal{L}_{QCD} = \bar{q}(x) \not{D} q(x) - \frac{1}{4} \text{Tr} F_{\mu\nu}^2 \quad (1.4)$$

where

$$\begin{aligned} D_\mu &= i \partial_\mu - g A_\mu \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - ig [A_\mu, A_\nu] \end{aligned} \quad (1.5)$$

where $A_\mu \equiv \sum_a \frac{\lambda_a}{2} A_\mu^a$, D_μ and $F_{\mu\nu}$ are 3×3 $SU(3)$ color matrices. Local gauge invariance and color symmetry follows from the invariance of $\mathcal{L}_{QCD}$ under the general gauge transformation

$$A_\mu(x) \rightarrow U(x) A_\mu(x) U^{-1}(x) + \frac{i}{g} U(x) \partial_\mu U^{-1}(x)$$

$$q(x) \rightarrow U(x) q(x) \quad (1.6)$$

where U is any unitary matrix $U = \exp i \sum_a \theta_a(x) \frac{\lambda_a}{2}$. Note that F is in general not invariant: $F_{\mu\nu}(x) \rightarrow U(x) F_{\mu\nu}(x) U^{-1}(x)$ since the field strength, like the gluon field, is in the adjoint representation of $SU(3)_{\text{color}}$.

The Feynman rules of QCD are analogous to QED, but have the added complication of the tri-gluon and quartic-gluon couplings contained in (1.4). The color algebra can be done automatically using the graphical method given by Cvitanovic.¹² The main rules are

- (1) A closed quark loop is equivalent to $\text{Tr}[I] = n_c = 3$.
- (2) A gluon propagator is equivalent to minus $1/n_c$ times the identity (to remove the U_3 singlet). Thus

$$\textcircled{2} = \textcircled{1} \textcircled{1} - \frac{1}{n_c} \textcircled{0} = n_c^2 - 1 = 8$$

(times the coupling constant $(g/2)^2 \text{Tr}[\lambda^8 \lambda^8] = g^2/2$.)

Additional rules allow the graphical reduction of the tri-gluon vertex. In typical perturbative calculations (e.g., soft radiation) we have the simple replacement

$$\alpha_{\text{QED}} \rightarrow \begin{cases} C_F \alpha_s = \frac{4}{3} \alpha_s & \text{quark current} \\ C_A \alpha_s = 3\alpha_s & \text{gluon current} \end{cases} \quad (1.7)$$

Effectively " $e_g^2 = 9/4 e_q^2$ ".

In addition to the color algebra, the non-Abelian couplings of QCD in general lead to the introduction of "ghost" (negative metric) scalar quanta which contribute to amplitudes which contain closed gluon loops. The crucial point is that the $gq + gq$ Compton amplitude is not transverse:

$$\mathcal{M}_{\mu\nu} = \text{diagram 1} + \text{diagram 2} + \text{diagram 3} \quad (1.8)$$

$$k_\mu^a \mathcal{M}_{\mu\nu} = k_\nu^b [\lambda_a, \lambda_b] \neq 0$$

unless we contract $\mathcal{M}_{\mu\nu}$ with ϵ_b^ν . Thus in typical gauges a gluon loop alone is not unitary:

$$\text{Im} \left(\text{diagram} \right) \neq \left| \text{diagram} \right|^2$$

but unitarity is restored when the scalar ghost loop is included.¹³ Ghost contributions occur in covariant gauges, in the Coulomb gauge, but not in axial gauges [$\eta \cdot A = 0$]. The axial gauge has many other advantages (see Chapter 4), but the introduction of a fixed Lorentz vector η_μ complicates the renormalization procedure.

Renormalization in QCD can be carried out in parallel with QED, but to retain explicit gauge-invariance one normalizes all of the 3 and 4-point vertices with their legs at a common off-shell (space-like) mass:

$p_1^2 = -\nu^2$. The choice of ν^2 is arbitrary. As a simplification, one can use dimensional regularization and identify the subtraction term with the pole terms in the loop integrals as the number of dimensions $n \rightarrow 4$.

Despite the parallels with QED perturbation theory, the postulated absence of asymptotic colored states implies that a perturbative expansion in terms of free - or even dressed - quark and gluon states does not exist in QCD. However, we shall assume that amplitudes with off-shell quark and gluon external legs - corresponding to processes which occur within the hadronic boundaries - do have a perturbative expansion, i.e., we shall suppose that for probes with $|q^2| \gg m_H^2$ non-perturbative effects can be numerically neglected. In the case of R for $e^+e^- \rightarrow$ hadrons, the effect of instanton vacuum fluctuations is of the form¹⁴

$$\frac{\Delta R}{R} \left(\text{instantons, size } \rho \sim O\left(\frac{1}{Q}\right) \right) \sim \left(\frac{1 \text{ GeV}}{Q} \right)^{12} \log^2 Q^2, \quad (1.9)$$

Similarly, non-zero vacuum expectation values of the field operating such as $\langle 0 | F_{\mu\nu} F^{\mu\nu} | 0 \rangle$ lead to power-suppressed contributions.¹⁵ All of this argues that at large Q^2 only perturbative contributions to R need to be considered and a straightforward description in terms of quark and gluon quanta is valid.

However, in the case of hadron-induced reactions such as the Drell-Yan process $H_1 + H_2 \rightarrow \nu^+ \nu^- X$ or large p_T hadron production $H_1 + H_2 \rightarrow H_3 + X$ there could be important non-perturbative effects. For example, the simplified calculation of instanton effects given by Ellis, Gaillard and Zakrewski¹⁶ implies that the pair production cross section does not in general factorize into contributions from deep inelastic hadronic

structure functions, as would be expected by perturbative or parton model arguments. Furthermore, in the case of massive pair production in a nuclear target, Glauber effects (e.g., upstream hadron production) would be expected to modify the state of the incoming hadron, again in a way that destroys the applicability of a naive factorized cross section. It is clearly of interest to test predictions based on perturbation theory for these processes as far as possible, and to attempt to understand possible non-perturbative effects.

The fundamental dimensionless coupling constant of QCD is $\alpha_s(\mu^2) = g^2(\mu^2)/4\pi$. Once $\alpha_s(\mu^2)$ is given at any normalization point μ^2 , the theory determines α_s at all other values through a renormalization group equation. In analogy to QED, one can write¹⁷

$$\alpha_s(q^2) = \frac{\alpha_s(\mu^2)}{[1 - \pi(q^2, \mu^2, \alpha_s(\mu^2))]} \quad (1.10)$$

where π is the irreducible gluon self-energy insertion. The lowest order diagrams give ($|q^2|, |\mu^2| \gg m_q^2$)

$$\pi(q^2) = \frac{\alpha_s(\mu^2)}{4\pi} \log\left(\frac{-q^2}{\mu^2}\right) \left[\frac{2}{3} a_f + 5 - 16 \right] \quad (1.11)$$

where (in the Coulomb gauge) the three terms correspond to the intermediate states, $q\bar{q}$, 2 transverse gluons, and 1 transverse gluon + 1 instantaneous gluon, respectively. Although the $q\bar{q}$ term must be positive (it is related by unitarity to $e^+e^- \rightarrow q\bar{q}$) the crucial Coulomb plus transverse gluon term does not correspond to the production of physical quanta and can indeed be negative.¹⁷

Thus to lowest order, $\alpha_s(q^2)$ decreases logarithmically (if $n_f \leq 16$)

$$\alpha_s(q^2) = \frac{\alpha_s(u^2)}{\left[1 + \frac{\alpha_s(u^2)}{4\pi} \cdot \left(11 - \frac{2}{3}n_f \right) \cdot \log \frac{-q^2}{u^2} \right]} \quad (1.12)$$

We shall assume that this is the correct asymptotic limit and verify that the result is self-consistent to all orders. The next order diagrams

give, as in QED, $\pi^{(4)} \sim O[\alpha_s^2(u^2) \log -q^2/u^2]$. However, we can include the effects of the self-energy and vertex insertions by utilizing $\alpha_s(k^2)$:¹⁸ we then have the effective replacement:

$$\begin{aligned} \alpha_s^2(u^2) \log \frac{-q^2}{u^2} &\longrightarrow \alpha_s(u^2) \int_u^{-q^2} \frac{dk^2}{k^2} \alpha_s(k^2) \\ &\sim \alpha_s(u^2) \log \log -q^2 \end{aligned} \quad (1.13)$$

assuming $\alpha_s(k^2) \sim 1/\log k^2$ asymptotically. Thus

$$\frac{4\pi}{\alpha_s(q^2)} = \frac{4\pi}{\alpha_s(u^2)} + \left(11 - \frac{2}{3}n_f \right) \log \frac{-q^2}{u^2} + O(\log \log -q^2) \quad (1.14)$$

It is easy to see that higher order insertions grow even less strongly with q^2 , and the original ansatz is indeed self-consistent if $11 - 2/3n_f > 0$. The logarithmic decrease of the "running coupling constant" $\alpha_s(q^2)$ indicates that the effective force due to gluon exchange becomes weak at short distance when vertex and self-energy insertions of all orders are accounted for. The effect of these insertions is to weaken the

ultraviolet growth of all loop calculations compared to lowest order perturbation theory.

Unlike QED where α can be fixed directly by Coulomb scattering, the empirical determination of α_g at any renormalization point is non-trivial. It is conventional to use the form

$$\alpha_g(q^2) = \frac{4\pi}{\left(11 - \frac{2}{3}n_f\right) \log \frac{-q^2}{\Lambda^2}} \quad (1.15)$$

and attempt to determine Λ^2 phenomenologically. However this form can only be used for $|q^2| \gg \Lambda^2$ and $\log|q^2/\Lambda^2| \gg \log \log |q^2|$; in particular, the pole at $-q^2 = \Lambda^2$ is incorrect. Many analyses unfortunately tend to determine Λ^2 by fitting to the rapid rise of α_g at $q^2 = -\Lambda^2$.

A fascinating question is what is the behavior of α_g as one approaches the low-momentum-transfer, non-perturbative domain. The recent lattice model evaluation by Creutz¹⁹ indicates that α_g changes continuously but rather abruptly from the perturbative to strong coupling domain at a distance where α_g is still relatively small ($\alpha_g \sim 0.16$ in the SU(2) gauge theory example of Ref. 19). At large distances, the effective potential approaches the linear confining form expected in string models. It is also interesting to observe that the value of Λ , in units of the string constant, turns out to be extremely small. It is possible that this is a consequence of the fact that quarks are not included in these pure gauge field theory calculations or the complication of different normalization conventions. Nevertheless, in these lectures we shall investigate the interesting, very real possibility that Λ is indeed very small compared to typical hadronic scales, and

thus for typical phenomenological applications that $\alpha_s(q^2)$ is slowly varying.

As a first example of an asymptotic freedom calculation, let us consider the perturbative corrections to $R = \sigma(e^+e^- \rightarrow \text{had}) / \sigma(e^+e^- \rightarrow \mu^+\mu^-)$. The hadronic contribution to vacuum contributions is

$$\begin{aligned} \pi^{\mu\nu} &= -ie^2 \int d^4x e^{iq \cdot x} \langle 0 | T(J^\mu(x) J^\nu(0)) | 0 \rangle \\ &= (q^\mu q^\nu - q^2 g^{\mu\nu}) \pi(q^2) \end{aligned} \quad (1.16)$$

By unitarity, $\sigma_{e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons}} = (4\pi\alpha/s) \text{Im } \pi(s)$ and $R = (3/\alpha) \text{Im } \pi(s)$. In the case of QED perturbation theory, the renormalized photon self-energy is ($|q^2| \gg m^2$)

$$\pi_{\text{QED}}(q^2) = -\frac{\alpha}{3\pi} \log\left(\frac{-q^2}{m^2}\right) - \frac{\alpha^2}{4\pi^2} \log\left(\frac{-q^2}{m^2}\right) + O(\alpha^2) \quad (1.17)$$

In the case of the QCD contributions, this becomes

$$\pi_{\text{QCD}}(q^2) = -e_q^2 \frac{\alpha}{3\pi} n_c \log\left(\frac{-q^2}{m^2}\right) - e_q^2 \frac{\alpha}{4\pi^2} \int \frac{dk^2}{k^2} \frac{n_c^2 - 1}{2} \alpha_s(k^2) \quad (1.18)$$

where $\frac{1}{2}(n_c^2 - 1) = 4$ is the color factor, and the running coupling constant $\alpha_s(k^2)$ accounts for self-energy and vertex insertions in the large k^2 region.¹⁸ Continuing to time-like s : $-q^2 \rightarrow -s = se^{-i\pi}$, and taking the absorptive part, we then obtain

$$R = 3 \sum_q e_q^2 \left[1 + \frac{\alpha_s(s)}{4\pi} + O\left(\frac{\alpha_s^2(s)}{\pi^2}\right) \right] \quad (1.19)$$

Recently the order α_s^2 correction has been computed by numerical²⁰ and analytic²¹ methods. The numerical result²⁰ is $(1.98 - 0.16 n_f) \alpha_s^2(s)/\pi^2$ using a standard dimensional regularization normalization of the coupling constant.

Thus, remarkably, the parton model result $R \rightarrow \sum_q e_q^2$ emerges as a rigorous QCD prediction for $s \rightarrow \infty$ with no correction²² in normalization. This result is a unique feature of renormalizable field theories which are asymptotically free.

The fact that the QCD correction to R can be written as a simple expansion in powers of $\alpha_s(s)$ without additional logarithms can be traced to the fact that the proper self-energy insertion, π_{QED} only contains a single power of $\log(-q^2/m^2)$ to any order in α . More generally, we note that factors of $\log s$ could never occur in $R = \text{Im } \pi$ since (a) infrared (gluon-mass) singularities always cancel in inclusive cross sections (Block-Nordsieck theorem), and (b) (quark) mass singularities cannot occur because of the Kinoshita-Lee-Nauenberg theorem.²³ [This theorem states that cross sections summed over initial and final states which become degenerate as $m_q \rightarrow 0$ are always finite. In the case of $\gamma^* \rightarrow$ hadrons there are no degenerate initial states, so the inclusive cross section will be finite for $m_q, m_g \rightarrow 0$.] Thus the only s dependence allowed in the asymptotic cross section is that which appears in $\alpha_s(s)$. Of course at small s we may obtain a dependence on $m^2(s)/s$ where $m(s)$ is the QCD running mass appearing in the renormalized quark propagator.

A rigorous derivation of these results can also be given using the operator product expansion at short distances and the renormalization group.^{1,2,7} Again one finds that, despite the complexities of confinement and the extraordinary mechanisms which lead to the evolution of quarks into hadrons, the asymptotic prediction for R is identical to that for free-quark-antiquark pairs.

Bjorken²⁴ has given a graphic description of the utility of asymptotic freedom. If we consider the interactions of hadrons at short distances less than a radius $r \sim 10^{-15}$ cm, i.e., processes involving momentum transfers $Q > r^{-1} \sim 10$ GeV then the effective coupling constant $\alpha_s(Q^2)$ is less than ~ 0.2 . Accordingly, within this "femto-universe" we can calculate perturbatively in terms of quark and gluon subprocesses. Beyond the radius of the femto-universe the processes which transform the quark and gluon precursors into hadrons takes place.²⁵ How such non-perturbative processes occur is perhaps the key question in QCD, but it is evident that they are soft-processes involving low momentum transfer and that they are probably short-range in rapidity. Thus the emerging hadronic jets must follow closely the energy and momentum flow of each gluon and quark emitted in the femto-universe subprocess, assuming they are well-separated in momentum space. It is this argument that gives perturbative QCD its predictive power for quark- and gluon-jet angular and momentum distributions. In particular, distributions calculated at the quark and gluon level which are insensitive²⁶ to infrared parameters (e.g., thrust, sphericity distributions) should give a reasonably accurate representation of the corresponding hadronic jet distributions. Such results cannot be exact, however, since one expects smearing from

non-perturbative effects, as well as ordinary hadronic and weak decay processes. In addition, the coherence effects which occur as jets overlap in phase-space are not as yet understood.

As we have emphasized, the evolution into the final state in QCD inclusive processes is not understood rigorously. However, proceeding heuristically, we can use perturbation theory and electrodynamics as a guide to anticipate some features of the hadronic state of the femto-universe-hadron universe boundary:

- (a) The final state in $e^+e^- \rightarrow \gamma^* \rightarrow X$, consists predominantly of q and $\bar{q}$ jets with gluon radiation and $q\bar{q}$ pairs filling the rapidity interval between y_q and $y_{\bar{q}}$.^{25,27}
- (b) The gluon energy distribution follows an "antenna" pattern, as described in Ref. 24.
- (c) The height of the rapidity distribution - i.e., the multiplicity density of soft radiation increases with energy.^{27,29}
- (d) The transverse momentum fall-off of gluons relative to the quark axis is $dN/dk_{1\perp}^2 \sim \alpha_g(k_{1\perp}^2)/k_{1\perp}^2$ for large $k_{1\perp}^2$ and $k_{1\perp}^2 \ll s$. Thus $\langle k_{1\perp}^2 \rangle_g \sim O(s/\log s)$, and QCD jets tend to expand in transverse momentum with available energy s .³⁰
- (e) The evolution in space-time corresponds to an "inside-outside" cascade - the slowest quanta are created first.²⁵
- (f) There is a finite probability proportional to $\alpha_g(Q^2)$ for 3 distinct well-separated $q\bar{q}g$ jets to be produced in e^+e^- annihilation.³¹

One of the most interesting questions in QCD is how to distinguish quark from gluon jets. Within the femto-universe one sees that the perturbative evolution of quark and gluons are controlled by their

Casimir color charges $C_A = 3$ for gluons versus $C_F = 4/3$ for quarks. The increased ratio of emitted collinear gluons should translate into a correspondingly larger central region energy or multiplicity density for gluon versus quark jets.^{27,32} Similarly, perturbative calculations predict that the average opening angle will be $\sim \sqrt{9/4}$ larger for gluon versus quark jets.³³ More speculatively, the azimuthal shape of quark and gluon jets may be different. For example, gluon jets may have a characteristic oblateness³⁴ in processes in which the femto-universe gluon is linearly polarized (e.g., in $e^+e^- \rightarrow q\bar{q}g$). Further, as discussed in Sections 4 and 5, the particle spectrum of gluon jets is expected to fall by an extra power of $(1-z)$ relative to the $z = p/p_{\max}$ distribution of hadrons in a quark jet.³⁵

We can also readily show that the hadrons which dominate the $z \sim 1$ fragmentation region in the jet must have the parent quark or gluon in its minimal Fock state.⁸

It can also be argued that the mean total charge of hadrons in a jet reflects the charge of the underlying quark or gluon.³⁶ In a simple model where the emitted q and $\bar{q}$ in the sea recombine with the original quark to form mesons, the mean total charge of the mesons in the jet fragmentation region equals the charge of the parent quark plus a constant equal to the mean charge of antiquark in the sea:³⁷

$$\langle Q_{\text{jet}} \rangle = Q_q + \langle Q_{\bar{q}} \rangle_{\text{sea}}$$

where phenomenologically³⁶ $\langle Q_{\bar{q}} \rangle_{\text{sea}} \sim 0.06$. This is consistent with a sea where

$$s\bar{s} : u\bar{u} : d\bar{d} \sim \frac{1}{2} : 1 : 1$$

A similar picture for gluon jets predicts $\langle Q_{jet} \rangle = 0$. If this ansatz is correct, then jets of different quantum numbers could be distinguished in a statistical average, although the absolute value of the quark charge is not determined. This analysis, however, can be complicated by possible energy dependence of $\langle Q_q \rangle_{sea}$, and the effects of baryons in the fragmentation region.

The femto-universe description makes it clear how the parton model prescription for calculating hadronic reactions at large transverse momentum and in deep inelastic processes emerges from perturbative QCD. The traditional parton-model assumption of strong convergence in transverse momentum is, however, incorrect. The effective transverse momentum integrations due to gluon emission are of the form

$$\int \frac{dk_{\perp}^2}{k_{\perp}^2} \alpha_s(k_{\perp}^2) \sim \log \log Q^2/\Lambda^2$$

and thus lead to logarithmic violation of Bjorken and parton-model scaling laws. However, we reemphasize that possible non-perturbative effects in the interactions of hadrons (which occur outside the femto-universe) could modify the perturbative predictions for processes involving more than one incident hadron.

In the case of large transverse momentum jet production in hadron-hadron collisions, only the interactions of quarks and gluons appear in the femto-universe. In deep inelastic lepton reactions, virtual photons and weak bosons directly participate in the QCD subprocess. Even real on-shell photons³⁰ can penetrate to the femto-universe level, as in deep inelastic Compton scattering ($\gamma q \rightarrow \gamma' q$) and two photon jet processes

($\gamma\gamma + q\bar{q}$, etc.) at large transverse momentum. These reactions are discussed in detail in Chapter 10.

In some processes, elements of the hadronic wave functions themselves appear directly inside the femto-universe hard scattering reaction. This occurs in the calculations of hadronic form factors at large momentum transfer, structure functions at the edge of phase $x \rightarrow 1$, and wherever one probes the far-off-shell tail of the hadronic wave function. The expectation that such reactions can be computed using perturbative QCD is confirmed in detail in Chapters 7 and 8 where we consider exclusive processes at large momentum transfer and the problem of the exclusive-inclusive connection in QCD.

2. Deep Inelastic Lepton Scattering and QCD Perturbation Theory

One of the most important areas for testing quantum chromodynamics is deep inelastic lepton scattering on nucleon targets.^{1,2,6,7} In these lectures we will particularly focus on the role of the hadronic wave function in testing QCD, particularly in the behavior of the structure functions at the edge of phase-space ($x_{Bj} = -q^2/2p \cdot q \sim 1$) and the effects of "high twist" operators, including processes which involve more than one quark in the target. It is now recognized that such high twist terms can seriously complicate the analysis of classic QCD scale-violation effects.

There are numerous exclusive channels which contribute to inelastic lepton scattering, but it is only the inclusive sum that contributes to the leading scaling behavior. It is simplest to consider the virtual Compton amplitude $T_{\mu\nu}(q^2, 2p \cdot q)$ and compute the structure function from its absorptive part. The contributions to $T_{\mu\nu}$ can be classified into

two classes as shown in Fig. 2.1. Diagrams in which interactions connect the quark propagator between the two photons to the remainder of the hadronic system do not contribute to the leading scaling behavior provided we choose an appropriate physical gauge (such as light-cone gauge), as we discuss below. Vertex and self-energy corrections are indicated by closed circles in the figure. In the case of the leading terms, a simple diagram such as Fig. 2.2 would yield exact scaling, as in the parton model, since the nucleon wave function is sufficiently convergent at large transverse momentum (see Chapter 7). Diagrams such as Fig. 2.3, however, contain loop integrals of the form

$$\int_{Q_0^2}^{Q^2} \frac{dk_1^2}{k_1^2} a_B(k_1^2) \sim \log \frac{\log Q^2/\Lambda^2}{\log Q_0^2/\Lambda^2} \quad (2.1)$$

and continued iteration in the t-channel leads to a sum of terms of order $[\log \log Q^2/\Lambda^2]^n$. After summation, these contributions exponentiate $[\alpha_s \log \log Q^2/\Lambda^2 + (\log Q^2/\Lambda^2)^\gamma]$ and lead to anomalous logarithmic dependence of the structure functions and their moments.^{1,2,6,7} We treat this in detail in Chapter 4. The dk_1^2/k_1^2 logarithmic integration corresponds in a massless theory to a collinear divergence, which arises when the gluon and quark momenta are parallel. The infrared (soft gluon momenta) divergences cancel in the inclusive sum.

What is the general criteria for whether a given hadronic process involving large momentum transfer can be computed using perturbative QCD methods? In the past we have had to be guided strictly by the formal operator product method⁴⁰ which required an explicit proof that an operator product of currents $J_\mu(x/2)J_\nu(-x/2)$ is dominated by the short

Fig. 2.1. Decomposition of the forward Compton amplitude.

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Fig. 2.2. Simple parton-model contribution to the forward Compton amplitude.

Fig. 2.3. Leading contributions to the deep inelastic scattering structure functions.

distance regime $x_1^2 \sim O(1/Q^2)$. However, this criteria is too rigid. More generally, we can proceed by examining the set of perturbative Feynman amplitudes which propagate the large momentum transfer invariant Q^2 (or p_T^2 , $m^2(1-x_B)$, etc.). Using power counting, one can readily catalog those diagrams which contribute to the leading power in Q^2 (the leading twist contribution). [This is particularly easy using light-cone perturbation theory and the light-cone gauge (see Chapter 4).] After renormalization of the vertex and self-energy insertions, each loop integration of a leading twist contribution can be written in terms of the running constant $\alpha_s(k^2)$ and running quark mass $m_q(k^2)$ with upper limits extending to order Q^2 . The basic question is whether the loop integration variable k^2 ($\sim k_1^2$ in time-ordered perturbation theory) of the radiative correction is dominated by the high Q^2 region.

To test this, we hold the running coupling constant at a fixed value $\alpha_s(\bar{k}^2)$ inside the loop integration. If the resulting integral is finite when all mass scales other than Q^2 are set to zero, then the propagators are indeed dominated by high momentum transfer and the correction is of order $\alpha_s(Q^2)$. As we have seen, the corrections to the annihilation ratio R are of this form.

More typically, one finds that the loop integrations depend logarithmically on Q^2 if α_s is held fixed, corresponding to contributions of order $\alpha_s \log Q^2/m^2$ or possibly $\alpha_s \log^2 Q^2/m^2$. The scale m^2 of the logarithms can depend either on the quark mass or the momentum scale set by the hadronic wave functions. [Note that ultraviolet divergences have already been eliminated and infrared (gluon-mass) divergences cancel in inclusive cross sections and color singlet matrix elements.]

Fortunately, the contributions of order $[\alpha_s \log Q^2]^n$ can usually be calculated and summed. This can be done formally through the operator product expansion and the renormalization group equations,^{1,2,40} or by summing sets of diagrams with an iterative structure^{7,41,42} (e.g., planar ladder diagrams in physical axial gauges^{41,8} such as the light-cone gauge), or equivalently, by deriving "evolution" equations^{41,43,44,8} which reproduce this iterative structure. This is done for quark and gluon distribution functions⁴⁴ in Chapters 4 and 5 and for the hadronic wave function⁸ (the quark "distribution amplitude") in Chapter 7.

As yet there is no rigorous treatment of loop integrals which have an order $\alpha_s \log^2 Q^2$ structure, although in some cases they can be organized into Sudakov form factors. Examples are discussed in Chapters 4-8. Of course even if a QCD radiative correction is of order $\alpha_s(Q^2)$ it may not be small. In fact, in the case of annihilation processes $q\bar{q} \rightarrow \ell\bar{\ell}$ ⁴⁵ and $q\bar{q} \rightarrow g g$ ⁴⁶ (heavy quarkonium decay) the coefficient is of order π^2 , and leading order calculations are misleading. On the other hand, we find that it is possible to sum terms of order $[\alpha_s(Q^2) \log^2(1/(1-x))]^n$ which are important at $x \rightarrow 1$ in deep inelastic scattering. This is discussed in detail in Chapter 4.

3. Perturbation Theory on the Light-Cone

One of the most convenient and physical formalisms for dealing with bound states and their interactions at large transverse momentum is time-ordered perturbation theory in the infinite momentum frame, or its equivalent, quantization on the light-cone.⁴⁷⁻⁴⁹ Defining $p^+ \equiv p^0 + p^3$, the total 4-momentum of a bound state directed along the 3-axis can be

parametrized as $p^\mu \equiv (p^+, p^-, \vec{p}_\perp) = (p^+, M^2/p^+, \vec{0}_\perp)$, i.e.: $p^0 = \frac{1}{2}(p^+ + M^2/p^+)$, $p^3 = \frac{1}{2}(p^+ - M^2/p^+)$. The momentum space wave function for a Fock state with n on-mass-shell constituents can be written as $\Psi_n(\vec{k}_{i(1)}, x_{(1)})$, where for each constituent's 4-momentum we parametrize

$$k^\mu \equiv (k^+, k^-, \vec{k}_\perp) = \left(x p^+, \frac{k_\perp^2 + m^2}{x p^+}, \vec{k}_\perp \right). \quad (3.1)$$

The wave function in time-ordered perturbation theory is frame dependent. It describes the state with each constituent at equal time as observed in a specific Lorentz frame, which we choose here to be $p^+ \rightarrow \infty$, i.e., the "infinite momentum frame." Equivalently, in a general frame (p^+ finite) Ψ_n corresponds to specifying the state at equal "time" $\tau = (z+t)$ on the light-cone. The renormalized wave functions Ψ_n exist for neutral or color singlet states in gauge theories because of the cancellation of infrared divergences. Momentum conservation requires

$$\sum_{i=1}^n x_i = 1, \quad \sum_{i=1}^n \vec{k}_{i(1)} = 0. \quad (3.2)$$

furthermore $x_i > 0$ since the constituent energies are positive. Polarization indices in $\Psi_{(n)}$ have been suppressed.

It is straightforward to work out the rules for time-ordered perturbation theory in the infinite momentum frame; i.e.: τ -ordered graphs. Only graphs with forward-moving quanta contribute. The rules for QED are^{49,50}

- (1) Assign momenta $k_{i(1)}^{(1)}$ to each line such that
 - (a) $k^+, \vec{k}_\perp$ are conserved at each vertex, and
 - (b) $k^2 = m^2$, i.e.: $k^- = (k_\perp^2 + m^2)/k^+$.

- (2) For the creation and annihilation of each fermion, anti-fermion or scalar, assign a factor $\theta(k^+)/k^+$. For the photon one assigns the factor $d_{\mu\nu}^{(k)} \theta(k^+)/k^+$ where $d_{\mu\nu}$ is the (gauge-dependent) polarization sum. In the Feynman gauge one takes $d_{\mu\nu} = -g_{\mu\nu}$. In an axial gauge where $\eta \cdot A = 0$ (η is an arbitrary fixed vector),

$$\begin{aligned} d_{\mu\nu}^{(k)} &= \sum_{\lambda=1,2} \epsilon_{\mu}(k,\lambda) \epsilon_{\nu}(k,\lambda) \\ &= -g_{\mu\nu} + \frac{\eta_{\mu} k_{\nu} + \eta_{\nu} k_{\mu}}{\eta \cdot k} \end{aligned} \quad (3.3)$$

where $k \cdot \epsilon = \eta \cdot \epsilon = 0$. The light-cone gauge where $\eta \equiv (\eta^+, \eta^-, \vec{\eta}_\perp) = (0, 2, \vec{0}_\perp)$, is particularly convenient, as we shall see.

- (3) The photon-fermion vertices are

$$e_0 \bar{u} \gamma^\mu u, \quad e_0 \bar{u} \gamma^\mu v, \quad -e_0 \bar{v} \gamma^\mu v, \quad -e_0 \bar{v} \gamma^\mu u.$$

- (4) The OPTh₀ energy denominators give a factor

$$\frac{i}{\sum_{\text{inc.}} k^- - \sum_{\text{interm.}} k^- + i\epsilon} \quad (3.4)$$

for each intermediate state.

- (5) The instantaneous part of the fermion propagator is $\gamma^+/2k^+$ where k^+ is specified by momentum conservation ($-\infty < k^+ < \infty$). (This contribution is the remnant of the backward moving fermion lines ("Z-graphs") which persist as $p^+ \rightarrow \infty$.) The instantaneous part of the photon propagator in $\eta \cdot A = 0$ gauge is $\eta_\mu \eta_\nu / k_+^2$. [Instantaneous photon and fermion propagators cannot occur at the same vertex in the light-cone gauge since $\gamma^+ \gamma^+ = 0$.]

(6) Integrate $\int_0^\infty dk^+ \int \frac{d^2 k_\perp}{2(2\pi)^3}$ for each independent k and sum over spins and polarizations.

[Despite the fact that all intermediate state particles are on the mass-shell in time-ordered perturbation theory, one still has the concept of off-shellness or virtuality. For example, the fermion self-energy correction in Fig. 3.1 is evaluated with "energy" denominator $D_A - D_C$ (where $D_A = \sum_{i \in A} k_i^-$, etc.) rather than $D_B - D_C$ which would reconstruct the self-energy for an on-shell quark. The difference $D_A - D_B$ is the measure of virtuality of the quark line; in fact $(D_A - D_B)(k^+) = (q+k)^2 - m^2$ (in a frame where $q^+ = 0$, so there are no other time orderings).

Subtracting for mass and wave function renormalization, the renormalized amplitude correspond to Fig. 3.1, then has the simple denominator structure

$$\frac{1}{(D_A - D_B)^2} \left[\frac{1}{D_A - D_C} - \frac{1}{D_B - D_C} \right] + \frac{1}{D_A - D_B} \frac{1}{(D_B - D_C)^2} = \frac{1}{(D_B - D_C)^2} \frac{1}{(D_A - D_C)} \quad (3.5)$$

which gives a finite integral over the mass spectrum in the intermediate state C. The complete renormalization program can be carried out in parallel with the standard covariant method.^{49]}

As a simple example of time-ordered perturbation theory calculations, the contribution to the electron charge renormalization from the vertex diagrams of Fig. 3.2 is

$$\bar{u}_p \gamma^\mu u_p + e_0^2 \int \frac{d^2 k_\perp}{2(2\pi)^3} \int_0^1 \frac{dx}{x^2(1-x)} \frac{d^{ab} \sum_{s,s'} \bar{u}_p \gamma^\alpha u_{k,s} \bar{u}_{k,s'} \gamma^\mu u_{k,s} \bar{u}_{k,s} \gamma^\beta u_p}{\left[m^2 - \frac{k_\perp^2 + m^2}{x} - \frac{k_\perp^2}{1-x} + i\epsilon \right]^2} \quad (3.6)$$

Fig. 3.1. Example of fermion self-energy correction to quark Compton scattering. The time-ordered perturbation theory energy denominators are indicated by dotted lines.

Fig. 3.2. Lowest order contributions to the $\gamma q \bar{q}$ vertex.

The simplest matrix element to evaluate is $\gamma^u + \gamma^+ = \gamma^0 + \gamma^3$ for which $\bar{u}_{k,s} \gamma^+ u_{k,s} = 2k^+ \delta_{s,s}$ and instantaneous fermion lines do not contribute. In the case of the Feynman gauge, the numerator is proportional to

$$\begin{aligned} \bar{u}_p \gamma_\alpha (k+m) \gamma^+ u_p &= \bar{u}_p [-4m + 2k] u_p \\ &= -8m^2 + k^+ u_p \gamma^- u_p + k^- u_p \gamma^+ u_p \\ &= -8m^2 + 2xm^2 + 2(k_1^2 + m^2)/x \end{aligned} \quad (3.7)$$

Thus

$$Z_1^F = 1 + \frac{g_0^2}{4\pi} \int d\vec{k}_1^2 \int_0^1 dx \frac{(1-x)[2m^2(1-4x+x^2) + 2k_1^2]}{[k_1^2 + (1-x)^2 m^2]^2} \quad (3.8)$$

(Feynman Gauge)

in agreement with the standard Feynman diagram calculation. A Pauli-Villars regulator can be used to define the ultraviolet integration. Notice that since $-d_0^2 = 4$, four rather than two-zero mass quanta are propagated in the Feynman gauge, so this calculation does not represent a physical wave function normalization. In the axial (and light-cone) gauge only the two physical photon states contribute. For the light-cone gauge calculation of Z_1 , we can use Eq. (3.3) for the polarization sum and easily find

$$Z_1^F = 1 + \frac{g_0^2}{2\pi} \int d\vec{k}_1^2 \int_0^1 dx \frac{\left[k_1^2 \left(\frac{1+x^2}{1-x} \right) + m^2(1-x)^3 \right]}{[k_1^2 + (1-x)^2 m^2]^2} \quad (3.9)$$

(L. C. Gauge)

The infrared divergence at $x \rightarrow 1$ can be regulated by defining $\frac{1}{n \cdot k} \rightarrow \frac{\eta \cdot k}{(n \cdot k)^2 + \epsilon^2}$ in the polarization sum.

The striking difference between Eq. (3.8) and Eq. (3.9) for Z_1^F illustrates the strong gauge dependence and even infrared sensitivity

of the wave function normalization for charged particles. Despite the fact that Z_1^F is unmeasurable, the integrand in Eq. (3.9) for $x \neq 1$ can be interpreted in terms of a probability distribution $P(x, \vec{k}_1)$ to find a "bare" electron (of any spin) in the dressed electron to order α_0 at $p_x \rightarrow \infty$. Including the Z_{2F}^{-1} external line renormalization, we have

$$1 = \int d^2 k_1 \int_0^1 dx P(x, \vec{k}_1) \quad (3.10)$$

where

$$P(x, \vec{k}_1) = \delta(1-x) \delta^{(2)}(\vec{k}_1) + \frac{\alpha_0}{2\pi^2} \left\{ \frac{\left[\frac{+2}{k_1^2} \left(\frac{1+x^2}{(1-x)} \right) + m^2 (1-x)^3 \right]}{\left[k_1^2 + (1-x)^2 m^2 \right]^2} \right. \\ \left. - \delta(1-x) \delta^{(2)}(\vec{k}_1) \int d^2 k_1 \int_0^1 dz \frac{\left[\frac{+2}{k_1^2} \left(\frac{1+z^2}{(1-z)} \right) + m^2 (1-z)^3 \right]}{\left[k_1^2 + (1-z)^2 m^2 \right]^2} \right\} + O(\alpha_0^2).$$

It is straightforward to extend these rules to QCD by including the trigluon and quartic gluon couplings. Ghost states are not required in the light-cone gauge.

4. Calculations of Deep Inelastic Scattering

Let us return to the calculation of the structure functions W_1 and W_2 for deep inelastic scattering as computed from the imaginary part of the forward virtual spin-averaged Compton amplitude $T_{\mu\nu}(q^2, q \cdot p)$. We shall choose the Lorentz frame ($q_1^2 = Q^2 \equiv -q^2$, $q \cdot p \equiv M\nu$)⁵:

$$p = \left(p^+, \frac{M^2}{p^+}, \vec{0}_1 \right), \quad q = \left(0, \frac{2M\nu}{p^+}, \vec{q}_1 \right) \quad (4.1)$$

which has the attractive feature that the interaction of the virtual

photon does not change the light-cone fraction x of the struck quark.

In particular $F_2(x_{Bj}, Q^2) = vW_2(q^2, 2p \cdot q) = (1/\pi)(p \cdot q) \text{Im} T_2$ where $T_2 = T^{++}/p^+p^+$, and $x_{Bj} = -q^2/2p \cdot q$.

Let us first consider the contribution to $T^{\mu\nu}$ from the graph of Fig. 4.1(a). Using light-cone perturbation theory, we have (spin indices are suppressed)

$$T^{++} = \sum_q e_q^2 \int d^2k_\perp \int_0^1 \frac{dx}{x} \int d\mathcal{M}^2 \left| \psi_{q/p}(x, \vec{k}_\perp, \mathcal{M}^2) \right|^2 \times \frac{\bar{u}(k) \gamma^+ (\not{k}' + m) \gamma^+ u(k)}{M^2 + 2q \cdot p - \frac{(\vec{k}_\perp + \vec{q}_\perp)^2 + m^2}{x} - \frac{\vec{k}_\perp^2 + \mathcal{M}^2}{1-x} + i\epsilon} \quad (4.2)$$

The integral over $\mathcal{M}^2$ represents the mass spectrum of the spectators in the hadronic wave function. For $(\vec{k}_\perp^2 + x\mathcal{M}^2)/(1-x) < Q^2$, the energy denominator can be approximated as $2q \cdot p - Q^2/x + i\epsilon$. (Note that $x_{Bj} \mathcal{M}^2 = x_{Bj}(q+p)^2 = (1-x_{Bj})Q^2$, rather than Q^2 sets the upper limit of phase space for the Fock state; we return to the consequences of this for the case $\mathcal{M}^2 \ll Q^2$ below.) The numerator matrix element, $\bar{u}(k) \gamma^+ (\not{k}' + m) \gamma^+ u(k)$, is simply $4k^+k^+$ independent of the quark helicity.

The calculation is, however, not complete since we must also take into account the quark self-energy and vertex insertions as indicated in Fig. 4.1(b). The self-energy correction $d_F(k^2) = Z_2^F(k^2)/Z_2^F(\nu^2)$ to the struck quark leg is a function of

$$k^2 = x \left[M^2 - \frac{\vec{k}_\perp^2 + \mathcal{M}^2}{1-x} - \frac{\vec{k}_\perp^2}{x} \right] \quad (4.3)$$

The $\gamma_{q\bar{q}}$ vertex correction $\Gamma_{\gamma FF}$ and the final state quark self-energy

Fig. 4.1. (a) Leading contribution to the forward Compton amplitude. The spectral mass of the spectator system is M . (b) Vertex and self-energy insertions in the forward Compton amplitude.

correction (which controls the quark jet evolution), are, in leading order, again controlled by the available energy W^2 (see Section 5). The ultraviolet renormalization thus yields the facts

$$d_F(k^2) \Gamma_{YFF}^2(W^2) d_F(W^2) = d_F(k^2)/d_F(W^2) \quad (4.4)$$

where we have used the Ward identity of QED. $\Gamma_{YFF}(W^2)d_F(W^2) = [Z_1^{-1}(W^2)/Z_1^{-1}(u^2)] \cdot [Z_2^F(W^2)/Z_2^F(u^2)] = 1$. Note that the square of the renormalized wave function $|\varphi_R|^2$ includes, by definition, one factor of $d_F(k^2)$. Thus for large Q^2 and W^2 ,

$$F_2(x, Q^2) \approx \frac{1}{x} d_F^{-1}(W^2) \sum_q e_q^2 x \int d^2 k_1 \int d\mathcal{M}^2 d_F(k^2) |\varphi_{q/p}^R(x, \vec{k}_1, \mathcal{M}^2)|^2 \times (\vec{k}_1^2 + x\mathcal{M}^2) \ll W^2 x \quad (4.5)$$

with $x \approx x_{Bj}$. The $\vec{k}_1$ and $\mathcal{M}$ integrals must be sensitive to the upper limit $x_{Bj} W^2 = (1 - x_{Bj}) Q^2$; otherwise $F_2(x, Q^2)$ would depend on the gauge and regularization dependent quantity $d_F(W^2)$. Accordingly, we are interested in the contributions from QCD which depend logarithmically on xW^2 . In light-cone gauge, the leading $\log W^2$ corrections all arise from planar ladder graphs, as shown in Fig. 2.3, where the transverse momentum integrations along the legs are strongly ordered $W^2 \gg \vec{k}_1^2 \gg \vec{k}_1^2 \dots$, such that a succession of logarithms can build up. Notice that in a crossed graph, the coupling of the loop variables leads to a transverse momentum integral insufficiently singular to yield a collinear logarithm. In the ladder graph of Fig. 4.2 the $\vec{k}_1, \vec{k}_1, \vec{j}_1$ integrals must decouple in order that the leading logarithms build up. It is easy to check that the $\vec{j}_1$ and $\vec{k}_1$ terms decouple from the denominator D_A if $\vec{j}_1^2 \ll \vec{k}_1^2 \ll \vec{k}_1^2$

Fig. 4.2. Calculation of ladder graph QCD contributions to the forward Compton amplitude. Energy denominators in light-cone perturbation theory are indicated by dotted lines.

$$-D_{(A)} \approx +\frac{\vec{k}_1^2}{z} + \frac{(\vec{k}_1 - \vec{l}_1)^2}{z\left(\frac{1-y}{y}\right)} + \frac{\vec{l}_1^2}{\frac{z}{y}\left(\frac{1-y'}{y'}\right)} \approx \frac{\vec{k}_1^2}{z(1-y)} \quad (4.6)$$

To set the limits on the $\vec{k}_1$ integration, we must examine denominator $D_{(B)}$:

$$-D_{(B)} = -2q \cdot p + \frac{(\vec{q}_1 + \vec{k}_1)^2}{z} + \frac{\vec{k}_1^2}{z\left(\frac{1-y}{y}\right)} \quad (4.7)$$

This becomes independent of $\vec{k}_1$ for

$$\vec{k}_1^2 \ll (1-y)Q^2 \quad (4.8)$$

Similarly, if we allow vacuum polarization contributions to gluon propagators, the gluon mass spectrum is limited by $\lambda^2 \leq \vec{k}_1^2/y \ll \frac{1-y}{y}Q^2$ at the top rung, and $\lambda^2 \leq \vec{l}_1^2/y'$ for the gluon carrying $(\vec{j}_1 - \vec{l}_1)^2$, etc. These conditions are equivalent to the limit $(\vec{k}_1^2 + \vec{l}_1^2) < xW^2$ of Eq. (4.3).

Let us now define the quantity $\tilde{q}(z, Q^2)$ as the ladder graph series for the hadronic wave function integrated over $\vec{k}_1^2 \leq Q^2$ (including full wave function renormalization of both quark legs). In the leading logarithm approximation $\tilde{q}$ has a simple repeated structure, as indicated in Fig. 4.3. In every rung, the ultraviolet renormalization leads to a factor

$$\frac{g_0^2}{4\pi} d_F^2\left(\frac{\vec{k}_1^2}{1-y}\right) \Gamma_{FFG}^2\left(\frac{\vec{k}_1^2}{1-y}\right) \cdot \frac{\vec{j}_1^2}{1-y'} \sim 0, \quad \frac{\vec{k}_1^2}{y} d_G\left(\frac{\vec{k}_1^2}{y}\right) \approx \alpha_s(\vec{k}_1^2) \quad (4.9)$$

[To see that this is the correct result, note that Γ_{FFG} is dependent on the leg which is furthest off-shell. Thus for $y \geq 1/2$, we have $d_F^2(\vec{k}_1^2/(1-y)) \Gamma_{FFG}^2(\vec{k}_1^2/(1-y)) = 1$ (where we used the Ward identity $Z_1^F = Z_2^F$ in the axial gauge). At small z , y is unconstrained and $\alpha_s(\vec{k}_1^2)$ is correct in leading logarithm approximation.]

Fig. 4.3. Ladder graph contributions to the quark distribution function $q(z, Q^2)$ including self-energy and vertex insertions. The leading order contribution in $\alpha_s(Q^2)$ comes from the transverse momentum integrations ordered as indicated in the figure.

The same algebra which leads to Eq. (3.10), then yields the simple integral equation

$$\left(C_F = \frac{n_c^2 - 1}{2n_c} = \frac{4}{3} \right)$$

$$\tilde{q}(z, Q^2) = \tilde{q}(z, Q_0^2) + 2C_F \int_{Q_0^2}^{Q^2} \frac{dk_1^2}{k_1^2} \frac{\alpha_s(k_1^2)}{4\pi} \int \frac{dy}{y} \frac{1+y^2}{1-y} \tilde{q}\left(\frac{z}{y}, k_1^2\right). \quad (4.10)$$

where Q_0^2 is an arbitrary division point such that α_s is still small. In order to have a gauge-invariant, physical quantity, we will define $q(z, Q^2) \equiv d_F^{-1}(Q^2) \tilde{q}(z, Q^2)$ (where $d_F^{-1}(Q^2)$ arises from the quark-line calculation of $T^{\mu\nu}$ as in (4.11)). We thus derive the "evolution" equation^{41,44}

$$\frac{\partial}{\partial \log Q^2} q(z, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int \frac{dy}{y} P_{q/q}(y) q\left(\frac{z}{y}, Q^2\right) \quad (4.11)$$

where

$$P_{q/q}(y) = C_F \left(\frac{1+y^2}{1-y} \right)_+ \equiv C_F \left[\frac{1+y^2}{1-y} - \delta(1-y) \int_0^1 dx \frac{1+x^2}{1-x} \right] \quad (4.12)$$

is the distribution (or "splitting") function for $q \rightarrow q+g$ at large k_1^2 . Note that the formal infrared divergence at $y=1$ which appears in Eq. (4.10) is canceled in the equation for $q(z, Q^2)$ by the infrared divergent terms in $d_F^{-1}(Q^2)$.

We may interpret $q(x, Q^2)$ as the "non-singlet" probability distribution⁵² for finding a quark of specific flavor i in the hadron target at a resolving momentum Q^2 specifically

$$q_i^{NS}(x, Q^2) \equiv q_i(x, Q^2) - \bar{q}_i(x, Q^2) (= q(x, Q^2)) \quad (4.13)$$

Similarly, we can define the gluon probability distribution function

$g(x, Q^2)$ and the singlet quark distribution

$$q^s(x, Q^2) = \sum_{i=1}^n q_i(x, Q^2) + \bar{q}_i(x, Q^2) \quad (4.14)$$

The complete evolution equations to leading order in $\alpha_s(Q^2)$ are then of the form⁴⁴

$$\frac{\partial q_1^{NS}(x, Q^2)}{\partial \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int \frac{dy}{x} P_{q/q}\left(\frac{x}{y}\right) q_1^{NS}(y, Q^2) \quad (4.15)$$

and

$$\frac{\partial g(x, Q^2)}{\partial \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int \frac{dy}{x} \tilde{P}\left(\frac{x}{y}\right) g(y, Q^2) \quad (4.16)$$

where we have used matrix notation⁴⁴

$$\tilde{q} = \begin{pmatrix} q_s \\ g \end{pmatrix}, \quad \tilde{P} = \begin{pmatrix} P_{q/q} & 2n_f P_{q/g} \\ P_{g/q} & P_{g/g} \end{pmatrix} \quad (4.17)$$

The off-diagonal splitting functions for $q \rightarrow gq$, $g \rightarrow q\bar{q}$ are ($N_c = 3$)

$$P_{g/q}(z) = \frac{4}{3} \left[\frac{1 + (1-z)^2}{z} \right] \quad (q \rightarrow g+q) \quad (4.18)$$

and

$$P_{q/g}(z) = \frac{1}{2} \left[z^2 + (1-z)^2 \right] \quad (g \rightarrow q+\bar{q}) \quad (4.19)$$

and yield the mixing between the gluon and singlet quark distributions which arise from the planar graphs of Fig. 2.3. These splitting functions can be obtained by substitution or a crossing relation from

$$P_{q/q}(z) = \frac{4}{3} \left[\frac{1+z^2}{(1-z)_+} - \frac{3}{2} \delta(1-z) \right] \quad (q \rightarrow q+g), \quad (4.20)$$

which is equivalent to (4.12). The gluon splitting function is completely

crossing symmetric (for $z \neq 1$):⁴⁴

$$P_{g/g}(z) = 6 \left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) + \frac{11-n_f}{12} \delta(1-z) \right] \quad (g \rightarrow g+g) \quad (4.21)$$

The $\delta(1-z)$ renormalization term in $P_{g/g}$ insures momentum conservation:

$$\int_0^1 dz \, z \left[P_{g/q} + P_{q/q} \right] = 0 \quad (4.22)$$

$$\int_0^1 dz \, z \left[2n_f P_{q/g} + P_{g/g} \right] = 0 \quad .$$

The moments of the quark and gluon distribution functions, e.g., $q_{(n)}^{NS}(Q^2) = \int_0^1 dx \, x^n q^{NS}(x, Q^2)$ are easily computed from the evolution Eq. (4.15) and (4.16) since the x/y and y integrations factorize. It is useful to define the integral of the running coupling constant⁴³

$$\xi(Q^2) - \xi(Q_0^2) = \frac{1}{4\pi} \int_{Q_0^2}^{Q^2} \frac{d\vec{k}_\perp^2}{\vec{k}_\perp^2} \alpha_s(\vec{k}_\perp^2) \quad . \quad (4.23)$$

For QCD, at large Q^2 , $\xi(Q^2) = 1/\beta \cdot \log \log Q^2/\Lambda^2$ where $\beta = 11 - (2/3)n_f$.

The non-singlet moments satisfy

$$\begin{aligned} \frac{\partial}{\partial \xi} q_{(n)}(Q^2) &= 2C_F \int_0^1 dy \, \frac{1-y^2}{1-y} (y^n - 1) q_{(n)}(Q^2) \\ &= -\gamma_{(n)} q_{(n)}(Q^2) \quad . \end{aligned} \quad (4.24)$$

Thus

$$q_{(n)}(Q^2) = q_{(n)}(Q_0^2) e^{-\gamma_{(n)} \xi} \approx q_{(n)}(Q_0^2) \left[\frac{\log Q^2/\Lambda^2}{\log Q_0^2/\Lambda^2} \right]^{-\gamma_{(n)}} \quad (4.25)$$

where

$$d_n = \gamma_n / \beta = \frac{C_F}{\beta} \left[1 + 4 \sum_{j=2}^{n+1} \frac{1}{j} - \frac{2}{(n+1)(n+2)} \right] \quad (4.26)$$

Notice that the anomalous logarithm dimensions d_n which control the Q^2 dependence of the moments $q_{(n)}(Q^2)$ are universal in the sense that they are independent of the physics of the hadronic target. The d_n can also be computed directly from the operator product expansion of current $J^\mu(x/2)J^\nu(-x/2)$ or quark fields $\bar{\psi}(x/2)\psi(-x/2)$. The target dependence is contained solely in the $q_{(n)}(Q_0^2)$, i.e., the initial condition $q(x, Q_0^2)$ of the evolution equation.

The fact that $d_0^{NS} = 0$, reflects the fact that flavor and charge are conserved by the splitting function, $\int_0^1 P_{q/q}(y) dy = 1$. The moments of the gluon and singlet quark distributions can be similarly computed after diagonalizing the coupled moment equations.⁴⁴ The quark and gluon distribution functions thus can be consistently normalized to the total flavor and momentum of the target.

Let us now consider in more detail the relationship between the structure function $F_2(x, Q^2)$ and the quark and gluon distribution functions which satisfy the evolution equation. In the case of a non-singlet structure function (such as $F_{2p} - F_{2n}$), we can identify to leading order in $\alpha_s(Q^2)$:

$$F_2^{NS}(x_{Bj}, Q^2) = \sum_i e_i^2 x_{Bj} \left[q_1^{NS}(x_{Bj}, Q^2) + \delta q_1^{NS}(x_{Bj}, Q^2) \right] \quad (4.27)$$

where⁵⁰

$$\delta q_1^{NS}(z, Q^2) = -2C_F \int_0^1 dy \frac{1+y^2}{1-y} \int_{(1-y)Q^2}^{Q^2} \frac{dk_1^2}{k_1^2} \frac{\alpha_s(k_1^2)}{4\pi} \left\{ q_1^{NS}\left(\frac{z}{y}, k_1^2\right) \frac{\theta(y > z)}{y} - q_1^{NS}(z, k_1^2) \right\} \quad (4.28)$$

corrects for the fact that the top loop in the calculation of $T^{\mu\nu}$ is constrained to $k_1^2 < (1-y)Q^2$, not $k_1^2 < Q^2$, as in the definition of q_1^{NS} . The correction term δq_1^{NS} is crucially important at $z \sim 1$. Strictly speaking, the structure function moments and quark distribution function moments coincide only for low n .

The evolution equations (and moments) predict that as Q^2 increases, the quark and gluon distributions become depleted at large x and increase at small x while conserving the total flavor and momentum of the target. The logarithmic dependence on Q^2 is set by the scale constant Λ^2 of $\alpha_s(Q^2)$. In order to understand the effects at large x in more detail let us reexamine the $x \sim 1$ behavior of the evolution equation for $q^{NS}(x, Q^2) = q(x, Q^2)$. Equation (4.28) can be written

$$\begin{aligned} \frac{\partial}{\partial \ln Q^2} q(x, Q^2) = & -2C_F q(x, Q^2) \int_0^x dy \frac{1+y^2}{1-y} \\ & + 2C_F \int_x^1 dy \frac{1+y^2}{1-y} \left[\frac{q(z/y, Q^2)}{y} - q(z, Q^2) \right] \end{aligned} \quad (4.29)$$

For $x \rightarrow 1$ the last term vanishes linearly faster than the first term;

$$\frac{\partial}{\partial \ln Q^2} q(x, Q^2) \approx C_F \{ 4 \log(1-x) + 3 \} q(x, Q^2) + \mathcal{O}[(1-x) q(x, Q^2)] \quad (4.30)$$

$$\begin{aligned}
q(x, Q^2) &= q(x, Q_0^2) (1-x)^{\frac{4C_F[\xi(Q^2) - \xi(Q_0^2)]}{e} \frac{3C_F[\xi(Q^2) - \xi(Q_0^2)]}{e}} \\
&\approx q(x, Q^2) (1-x)^{\left(\frac{4C_F}{8} \log \frac{\log Q^2/\Lambda^2}{\log Q_0^2/\Lambda^2} \right) \left(\frac{\log Q^2/\Lambda^2}{\log Q_0^2/\Lambda^2} \right)^{3C_F/8}} \quad (4.31)
\end{aligned}$$

This graphically displays how the $(1-x)$ power of the quark distribution function increases as the structure of the quark is probed at shorter and shorter distances. This is a universal feature of theories with spin-1 gluons. The corresponding gluon distribution power has $C_F = 4/3$ replaced by $C_A = 3$; i.e., the fast gluon radiates and loses momentum by a factor $C_A/C_F = 2/[1 - (1/n_c^2)] = 9/4$ relative to the quark. A graph of the power index $\tilde{\xi} = 4C_F\xi$ is given in Section 8.

However, as $x \rightarrow 1$ the connection between the structure function $F_2(x, Q^2)$ and quark distribution function $q(x, Q^2)$ starts to break down since the internal transverse momentum integrations are limited by $\vec{k}_1^2 < (1-y)Q^2 < (1-x_B)Q^2 = x_B W^2$. Qualitatively, the structure function evolves to $x_B W^2$, not Q^2 . For $x \sim 1$, we have from Eq. (4.28),

$$\delta q(x, Q^2) \approx 2C_F \int_0^x dy \frac{1+y^2}{1-y} \int_{(1-y)Q^2}^{Q^2} \frac{d\vec{k}_1^2}{\vec{k}_1^2} \frac{\alpha_s(\vec{k}_1^2)}{4\pi} q(x, \vec{k}_1^2) \quad (4.32)$$

Combining this with the $x \sim 1$ behavior of $q(x, Q^2)$, we find the correction factor for $F_2(x, Q^2)$,

$$\begin{aligned}
\frac{\delta q(x, Q^2) + q(x, Q^2)}{q(x, Q^2)} &= \frac{\log Q^2/\Lambda^2}{\frac{4C_F}{8} \log^2(1-x)} \left[\left(1 + \frac{\log(1-x)}{\log Q^2/\Lambda^2} \right)^{\frac{4C_F}{8} \log(1-x)} - 1 \right] \\
&\approx 1 + \frac{2C_F}{8} \frac{\log^2(1-x)}{\log Q^2/\Lambda^2} + \dots \quad (4.33)
\end{aligned}$$

as $x \rightarrow 1$. The correction from δq thus becomes appreciable when

$2C_F \log^2(1-x) \alpha_s(Q^2)/4\pi$ becomes large. Similarly, the correction to the n th moment of $F_2(x, Q^2)$ is

$$\frac{dq_n(Q^2)}{q_n(Q^2)} \sim 2C_F \log^2 n \frac{\alpha_s(Q^2)}{4\pi} + \dots \quad (4.34)$$

This, in fact, precisely reproduces the order $\alpha_s(Q^2) \log^2 n$ correction to the structure function moments computed using the operator product method.⁵⁴ Here we account for these terms simply by taking into account the kinematic constraint; Eq. (4.33) contains the sum of the $[\alpha_s(Q^2) \log^2 n]^j$ terms. Unless one takes into account this kinematic effect, there can be no expansion in $\alpha_s(Q^2)$ at large n .

The $[\log^2 n \cdot \alpha_s(Q^2)]$ terms are usually taken into account phenomenologically by rewriting the lowest order moments in terms of a variable scale constant Λ_n^2 which grows monotonically with n .⁵⁴ This can be understood qualitatively since we expect the structure functions to be a function of $\log Q^2(1-x)/\Lambda^2 \approx \log Q^2/n\Lambda^2$.

It is interesting to examine the distribution function $F_2(x, Q^2) \sim x[q(x, Q^2) + \delta q(x, Q^2)]$ in the fixed $Q^2 = (1-x)Q^2/x$, $Q^2 \rightarrow \infty$ limit. Using Eq. (4.33) (and ignoring powers of $\log Q^2$)

$$F_2(x, Q^2) \underset{\substack{\text{fixed } W^2 \\ Q^2 \gg W^2}}{\sim} q\left(1-x - \frac{W^2}{Q^2} \cdot Q_0^2\right) \left(\frac{W^2}{Q^2}\right)^{-\frac{4C_F}{8} \log \frac{\log W^2/\Lambda^2}{\log Q_0^2/\Lambda^2}} \quad (4.35)$$

The last factor, which gives a power law suppression in Q^2 , represents the probability of the struck quark to scatter to a final quark jet of fixed mass. It is analogous to the QED radiative correction for charged particle scattering with finite energy resolution.⁵⁵

$$d\sigma = d\sigma_0 e^{-\frac{\alpha}{\pi} \log \frac{Q^2}{m^2} \log \frac{s}{\Delta s}} \quad (4.36)$$

We emphasize here that taking into account the correct phase space limit $\vec{k}_1^2 < (1-y)Q^2$ is crucial to the result (4.35). If we had used $\vec{k}_1^2 < Q^2$ instead, then $F_2(x, Q^2) \sim xq(x, Q^2)$ where

$$q(x, Q^2) \underset{\substack{\text{fixed } W^2 \\ Q^2 \gg W^2}}{\approx} q\left(1-x = \frac{W^2}{Q^2}, Q_0^2\right) \left(\frac{W^2}{Q^2}\right)^{-\frac{4C_F}{B} \log \frac{\log Q^2/\Lambda^2}{\log Q_0^2/\Lambda^2}} \quad (4.37)$$

i.e., a much stronger suppression. The last factor is the QCD analog of the square of the QED Sudakov form factor

$$S^2(Q^2) \sim e^{-\pi/\pi \cdot \log^2 Q^2/Q_0^2} \quad (4.38)$$

which is the probability for an off-shell fermion to scatter elastically. We thus see that the scale-breaking effects from the QCD evolution equations are actually controlled by W^2 in the large x domain, and accordingly are more modest than what might be expected from formulae based on Q^2 evolution.

Detailed comparisons of the QCD predictions for the structure functions and moments with experiment are given in the contributions of Atwood and Barnett in this volume.⁷

5. Jet Fragmentation in Deep Inelastic Inclusive Reactions

Let us now consider the perturbative QCD feature of the final state quark jet in deep inelastic scattering. The lowest order gluon correction to the top rung of the Compton amplitude as illustrated in Fig. 5.1 is a prototype of the planar graphs which contribute to the leading logarithmic contributions to the cross section.

Ignoring mass terms, energy and momentum conservation for the final state indicated in Fig. 5.1 implies

$$w^2 = (q+p)^2 = \frac{\vec{k}_1^2}{xz(1-z)} + \frac{\vec{k}_1^2}{1-x} + \frac{(\vec{q}_1 + \vec{k}_1)^2}{x} - Q^2 \quad (5.1)$$

The $\vec{k}_1$ integration is accordingly limited by phase space to $\vec{k}_1^2 < xw^2z(1-z)$. The argument of α_s in the loop integration is effectively $\vec{k}_1^2/z$. Thus the maximal value of $\vec{k}_1^2/z$, which can lead to a factorized logarithmic evolution of the quark or gluon jet fragmentation distributions $D_{q'}/q$ and $D_{g/q}$, is $\vec{k}_1^2/z \ll (1-z)xw^2 = (1-z)(1-x)Q^2$. After renormalization, i.e., removing the $d_p((1-x)Q^2)$ factor incorporated into the structure functions, and summing all planar graphs with ordered transverse momenta, the resulting evolution equations of the fragmentation functions are identical to those of q^{NS} , q^S and g . The quark jet fragmentations satisfy the "initial conditions" $D_{q'}/q(z, \mu^2) = \delta(1-z)\delta_{qq'}$, $D_{g/q}(z, \mu^2) = 0$. Thus $D_{q'}/q(z, Q^2)$ represents⁵² the distribution of quarks q' at the resolution scale Q^{-1} in a quark q defined at resolution μ^{-1} . Despite their change of shape with Q^2 , the integral over the q' and g distributions conserve jet flavor and momentum:

Fig. 5.1. Example of a leading contribution to the evolution of the quark fragmentation function.

$$\int_0^1 dz D_{q'/q}(z, Q^2) = \delta_{q',q} \quad (5.2)$$

$$\sum_{q'} \int_0^1 dz z [D_{q'/q}(z, Q^2) + D_{g/q}(z, Q^2)] = 1 \quad (5.3)$$

The moments of $D_{q'/q}$ and $D_{g/q}$ are identical to those of the hadronic quark and gluon distributions. Since the final transfer $q' \rightarrow$ hadrons is a simple convolution at finite relative transverse momentum, the moments of the hadron fragmentation functions $D_{H/q}(z, Q^2)$ also have the same anomalous dimensions. Thus in leading logarithm approximation, the deep inelastic fragmentation cross section $l+p \rightarrow l+H+X$ has a factorized form which we write symbolically,

$$\frac{dN}{dx dz} \sim q(z, (1-x)Q^2) D_{H/q}(z, (1-z)(1-x)Q^2) \quad (5.4)$$

since the quark distribution in the target evolves up to $k_1^2 \ll (1-x)Q^2$ and the quark fragmentation in the final state quark jet evolves up to $k^2 \ll (1-z)(1-x)Q^2$. As expected, in leading logarithm approximation the jet fragmentation functions are universal functions, independent of the process.

In deep inelastic lepton scattering, the fragments of the final quark jet have their momenta directed along $(q+xp)^\mu$. Quark or gluon fragments at large transverse momentum $\vec{k}_\perp \sim O(\vec{q}_\perp)$ relative to the jet direction give a contribution to the cross section which (together with renormalization terms) is order $\alpha_s(Q^2)$ smaller than the $\vec{k}_\perp^2 \ll W^2$ leading contributions. Such contributions correspond to a multijet final state, $lq \rightarrow lqg$.

It is important to observe that the kinematic coupling of the x and z dependence in the $D_{R/q}$ fragmentation function destroys factorization of the $x^n z^m$ moments in Q^2 of the inclusive cross section (5.4), even before taking into account multijet processes. The breakdown of factorization and the importance of W^2 in controlling jet evolution has been seen phenomenologically in the data of Ref. 56.

More detailed discussion of the theory and phenomenology of QCD jet may be found in the contribution⁶ of G. Schierholz in this volume and in Ref. 7.

6. General QCD Short-Distance Subprocesses

It is now clear that for any hard-scattering subprocess we can sum sets of planar diagrams (ordered in transverse momentum) associated with each incoming or outgoing quark or gluon line, such that in leading logarithm approximation we obtain universal distribution and fragmentation functions $q(x, Q^2)$, $g(x, Q^2)$ which control the quark and gluon evolution at short distances.⁵⁷ The kinematic limits of the transverse momentum integrals associated with each radiative correction control the evolution and have to be individually examined.

For example, let us consider the gluon radiative corrections to the Drell-Yan subprocess $q\bar{q} \rightarrow l\bar{l}$ in the reaction $H_A + H_B \rightarrow l\bar{l} + X$. As usual we can choose the gauge such that only planar graphs (where the emitted gluon is reabsorbed on the same quark line in the squared diagram)-with ordered transverse moments-contribute to the leading "collinear" logarithms.

The kinematics of the leading logarithmic gluon corrections to the Drell-Yan process are illustrated in Figs. 6.1 and 6.2. We choose the

Fig. 6.1. Schematic representation of $q\bar{q} \rightarrow l\bar{l}$ contribution to the Drell-Yan process $H_A + H_B \rightarrow l\bar{l}X$.

Fig. 6.2. Radiative corrections to the $q\bar{q} \rightarrow l\bar{l}$ subprocess leading to structure function evolutions.

Lorentz frame in which the incident hadrons are oppositely directed:

$p_A = (P, m_A^2/P, \vec{0}_\perp)$, $p_B = (m_B^2/P, P, \vec{0}_\perp)$. Ignoring all masses, we then have $(x_a = k_a^+/P_A^+, x_b = k_b^-/P_B^-)$

$$Q^\mu = \left(x_a P - \frac{\vec{k}_{1b}^2 z_b}{x_b (1-z_b) P}, x_b P - \frac{\vec{k}_{1a}^2 z_a}{x_a (1-z_a) P}, \vec{k}_{1a} + \vec{k}_{1b} \right) \quad (6.1)$$

$$Q^2 = x_a x_b P^2 + \frac{\vec{k}_{1b}^2 z_b}{x_b (1-z_b)} \frac{\vec{k}_{1a}^2 z_a}{x_a (1-z_a) P^2} - \frac{\vec{k}_{1a}^2}{1-z_a} - \frac{\vec{k}_{1b}^2}{1-z_b} - 2\vec{k}_{1a} \cdot \vec{k}_{1b}$$

with $P^2 = s$. Thus the $\vec{k}_{1a}$ and $\vec{k}_{1b}$ integrals decouple if $\vec{k}_{1a}^2 \ll Q^2(1-z_a)$, $\vec{k}_{1b}^2 \ll Q^2(1-z_b)$; the parton model constraints $x_a x_b s = Q^2$, $x_a - x_b = Q_2/2P = x_F$ then become valid if $x_F > Q^2/s$. The leading-log summation and kinematic limits for the q and $\bar{q}$ distributions are then identical to those for the $l p \rightarrow l X$ structure function at the corresponding kinematics. To leading order in $\alpha_s(Q^2)$:

$$\frac{d\sigma}{dx_a dx_b} (H_A + u_B + i\bar{l} + X) = q(x_a, (1-x_a)Q^2) \bar{q}(x_b, (1-x_b)Q^2) \sigma_{q\bar{q} \rightarrow l\bar{l}}(Q^2) \quad (6.2)$$

where we have used the symbolic notation $q(x_a, (1-x_a)Q^2) = q(x_a, Q^2) + \delta q(x_a, Q^2)$ to indicate the $\vec{k}_\perp^2$ kinematic limits (see Chapter 4).

If one looks at the radiative corrections of order $\alpha_s(Q^2)$ beyond the leading logarithm approximation, it is now known that severe corrections to the standard factorized form arise. For example, the order $\alpha_s(Q^2)$ correction to the time-like vertex $q\bar{q} \rightarrow l\bar{l}$ leads to a factor^{45,56}

$$1 + \frac{\alpha_s(Q^2)}{2\pi} C_F \left(\frac{4\pi^2}{3} + \frac{11}{2} \right) \sim 1 + 4\alpha_s(Q^2) \quad (6.3)$$

Such corrections are numerically large, and it clearly is important to understand their higher order structure. Notice that part of the order $\alpha_s(Q^2)$ corrections to the annihilation cross section can be identified with higher order hard scattering subprocesses, e.g., $gq \rightarrow \gamma^* q$ and virtual corrections to the lowest order subprocesses. In fact, since the $\bar{q}$ distribution in the proton could be numerically smaller than $\alpha_s(Q^2)$ times the gluon distribution, the ordering of contributions to $pp \rightarrow \ell\bar{\ell}X$ according to their order in $\alpha_s(Q^2)$ can be misleading.

Similar perturbative analyses are possible for other reactions such as deep-inelastic Compton scattering, and large p_T hadron production processes. Again in leading logarithmic approximation one obtains the parton model structure for large transverse momentum jet production as a summation over hard scattering subprocesses⁵⁷

$$\begin{aligned} \frac{d\sigma(H_1 + H_2 \rightarrow h_1 + h_2 + X)}{dx_1 dx_2 dz_1 dz_2} &= q_1(x_1, Q^2(1-x_1)) q_2(x_2, Q^2(1-x_2)) \\ &\times \sigma_{q_1 q_2} + q_1 q_2 (\alpha_s(Q^2), Q^2) D_{h_1/q_1}(z, Q^2(1-x_1)(1-x_2)) \\ &\times D_{h_2/q_2}(z_2, Q^2(1-x_2)(1-x_2)) + (\text{higher order in } \alpha_s(Q^2)) \end{aligned} \quad (6.4)$$

plus the analogous contributions from the $qg \rightarrow qg$, $gg \rightarrow gg$, etc. leading 2-2 subprocesses. Here $Q^2 = 4p_T^2$ sets the large transverse momentum scale. As yet, phenomenological analyses have not taken into account the kinematic limits on the evolution functions indicated in (6.4).

As we have discussed in Chapter 1, there is no convincing proof which rules out soft hadronic interactions between the incoming hadrons in the Drell-Yan and high p_T hadron collision processes. For example,

a simplified model¹⁶ for the effect of instanton vacuum fluctuations appears to destroy the factorized structure proved in perturbative QCD. In the case of a nucleus-nucleus collision, $A_1 + A_2 \rightarrow \ell^+ + \ell^- + X$. It is difficult to believe that absorptive or Glauber processes do not rearrange the initial states and thus fundamentally destroy elementary factorization.

We will discuss the contributions of high twist terms to the Drell-Yan process in Chapter 8.

7. Large Momentum Transfer Exclusive Processes in QCD

The most detailed information on the structure of QCD may well come from analyses of exclusive processes at large momentum transfer. Channel by channel, each exclusive process cross section must depend in detail on the scaling and spin-dependence of quark and gluon scattering amplitudes and the form of the hadronic wave function at short distances. On the other hand, exclusive processes are clearly more difficult to analyze than inclusive reactions since one must deal with the features of the hadronic bound state. Despite this complexity, perturbative QCD can make definitive predictions; as an example, the asymptotic magnetic form factor of the nucleon in QCD has the form¹

$$G_M(Q^2) = \left[\frac{\alpha_s(Q^2)}{Q^2} \right]^2 \left| \sum_{n=0}^{\infty} b_n \left(\log \frac{Q^2}{\Lambda^2} \right)^{-\gamma_n^N/B} \right|^2 \left[1 + O\left(\alpha_s(Q^2), \frac{m^2}{Q^2} \right) \right] \quad (7.1)$$

where the γ_n^N are the (logarithmic) anomalous dimensions of the nucleon wave function.

Hadronic wave functions at short distance

Until recently, the physics associated with the hadronic wave function was generally considered to be off-limits to perturbative QCD. For example, the input values for the structure functions $q(x, Q_0^2)$, $g(x, Q_0^2)$ for the QCD evolution equations have been relegated to the domain of essentially uncalculable non-perturbative physics. However, this assumption is unduly pessimistic. The critical point is that the far-off-shell behavior of the hadronic wave function - the part of the wave function which controls the $x \rightarrow 1$ behavior of the structure functions as well as the large momentum transfer fall-off of form factors and other exclusive processes - is computable using the calculus of perturbative QCD. As we shall see, given the wave function in the "soft", near-on-shell domain, we can calculate the far-off-shell behavior using a new type of evolution equation derivable from perturbative QCD or the operator product expansion.² The net result is that the power-law fall of exclusive cross sections at large momentum transfer, and the $x \rightarrow 1$ dependence of structure functions agree with simple counting rules;^{4,5} however, QCD predicts additional logarithm dependence due to explicit powers of α_s and the evolution of the wave functions.

The counting rules were derived upon the premise that the power-law scaling behavior of wave functions can be derived from the first iteration of the Bethe-Salpeter kernel in a renormalizable theory.⁴ In brief, they are:

(a) (Spin-averaged) form factors at $|t| \gg M^2$

$$F_H(t) \sim \frac{1}{t^{n-1}} \quad (7.2)$$

where n = number of constituent fields in H .

(b) Large angle scattering at $n \gg M^2$, t/s fixed

$$\frac{d\sigma}{dt} (AB \rightarrow CD) \sim \frac{1}{s^{n-2}} f(t/s) \quad (7.3)$$

where n = total number of constituent fields in A, B, C and D .

(c) Structure functions at $Q^2 \gg M^2$, $x \ll 1$, Q^2/x fixed

$$F_{2H}(x, Q^2) \sim (1-x)^{2n_s-1} \quad (7.4)$$

where n_s = number of spectator fields in H .⁶

In each case the minimum number of fields dominates the asymptotic behavior. The rules follow simply from tree graphs in any renormalizable field theory if the four-momentum of each hadron is partitioned among its constituents.

The counting rules appear to summarize a wide range of large momentum transverse phenomena; we will compare with experiment below.

A very convenient framework for analyzing bound states in QCD is time-ordered perturbation theory on the light-cone, as discussed in Chapter 3. In general, in time-ordered perturbation theory the wave function describes a Fock state of positive energy, on-mass-shell constituents. The amplitude is evaluated at equal time for each constituent. In the infinite momentum limit $p_3 \rightarrow \infty$, this is equivalent to equal "time" $z^+ = z^0 + z^3$ on the light-cone. Thus in QCD, the meson state Ψ can be represented as a column vector of wave functions - one for each of the Fock states $q\bar{q}, q\bar{q}g, \dots$ in the meson. Fock states having a finite number of constituents exist for color singlet states because of the cancellation of all infrared divergences.⁷ In general, Ψ satisfies

the bound state equation $\Psi = SK\Psi$ - an infinite set of coupled equations where the matrix K is the completely irreducible kernel, $S_n^{-1} = M^2 - \sum_{j=1}^n (k_j^2 + m^2)_j / x_j + i\epsilon$ is the n -particle propagator, and $x_j = (k^0 + k^3)_j / (p_\pi^0 + p_\pi^3) \geq 0$ are the constituents' fractional longitudinal momenta $\left(\sum_{j=1}^n x_j = 1 \right)$. See Fig. 7.1. We can rigorously separate "hard" from "soft" components of the wave function by defining a propagator $S^{(\lambda)}$ which vanishes for virtual states near the energy shell

$$S^{(\lambda)} = \begin{cases} S & \text{if } |M^2 - \sum_j (k_j^2 + m^2)_j / x_j| \geq \lambda^2 & \text{('hard')} \\ 0 & \text{otherwise} & \text{('soft')} \end{cases} \quad (7.5)$$

We can then write

$$\begin{aligned} \Psi &= (S - S^{(\lambda)})K\Psi + S^{(\lambda)}K\Psi \\ &= \Psi_\lambda + S^{(\lambda)}K\Psi \end{aligned} \quad (7.6)$$

where the wave function $\Psi_\lambda \equiv (S - S^{(\lambda)})K\Psi$ is non-zero only when its constituents are near energy shell. The full wave function can be expressed in terms of Ψ_λ : [see Fig. 7.2]

$$\begin{aligned} \Psi &= \Psi_\lambda + G^{(\lambda)}K\Psi_\lambda \\ G^{(\lambda)} &\equiv S^{(\lambda)} + S^{(\lambda)}KG^{(\lambda)} \\ &= S^{(\lambda)} + S^{(\lambda)}KS^{(\lambda)} + S^{(\lambda)}KS^{(\lambda)}KS^{(\lambda)} + \dots \end{aligned} \quad (7.7)$$

By the definition in Eq. (7.6), the Green's function $G^{(\lambda)}$ contains only hard loop momenta, and thus it has a sensible perturbative expansion, at least in asymptotically free theories. In particular, bound state poles cannot develop in $G^{(\lambda)}$ since only far off-shell, propagation occurs

$$\Psi = S K \Psi$$

$$\Psi = \begin{bmatrix} \text{diagram 1} \\ \text{diagram 2} \\ \text{diagram 3} \\ \vdots \end{bmatrix} \quad S = \begin{bmatrix} \text{diagram 4} & \text{diagram 5} & \text{diagram 6} \\ \text{diagram 7} & \text{diagram 8} & \text{diagram 9} \\ \vdots & \vdots & \vdots \end{bmatrix} \quad K = \begin{bmatrix} \text{diagram 10} & (\text{diagram 11} + \text{diagram 12}) & \dots \\ (\text{diagram 13} + \text{diagram 14}) & \text{diagram 15} & \dots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

(a)

(b)

+

(c)

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Fig. 7.1. (a) The meson bound state equation for Fock states in time-ordered perturbation theory. (b) Lowest Fock state contribution to the meson form factor. (c) Higher Fock state contribution to the meson form factor.

$$\begin{aligned}
 \text{Diagram 1} &= \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} + \dots \\
 \text{Diagram 5} &= \text{Diagram 6} + \text{Diagram 7} + \text{Diagram 8}
 \end{aligned}$$

$k_1 > \lambda$

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Fig. 7.2. The $q\bar{q}$ Fock state wave function of the meson using Eq. (7.7). The two particle reducible amplitudes in the kernel only contribute for $k_1 > \lambda$.

in intermediate state (e.g., $V(r) = \arctan(r < 1/\lambda)$ does not bind for λ sufficiently large). The soft wave function Ψ_λ contains all intrinsically non-perturbative effects. Given Ψ_λ , Eq. (7.7) determines the far-off-shell structure of the full hadronic wave functions Ψ from perturbation theory.

The meson form factor $F_M(Q^2)$ can now be represented as a sum of matrix elements between initial and final Ψ_λ states:

$$F_M = \Psi_\lambda(q\bar{q})\Gamma^{(\lambda)}(q\bar{q}, q\bar{q})\Psi_\lambda(q\bar{q}) + \Psi_\lambda(q\bar{q})\Gamma^{(\lambda)}(q\bar{q}, q\bar{q}g)\Psi_\lambda(q\bar{q}g) + \dots \quad (7.8)$$

The amplitudes $\Gamma^{(\lambda)}(q\bar{q}, q\bar{q})$, etc. consist of all connected diagrams (reducible and irreducible), but with all loop momenta hard as in $G^{(\lambda)}$ (see Fig. 7.3). In light-cone⁹ gauge ($A^+ = 0$) (see Chapter 3), the nominal power law contribution to $F_M(Q^2)$ as $Q^2 \rightarrow \infty$ is $F_M(Q^2) \sim 1/(Q^2)^{n-1}$ if n quark or gluon constituents are forced to change direction. Thus only the $q\bar{q}$ component of Ψ_λ contributes to the leading $(1/Q^2)$ behavior of $F_M(Q^2)$. (Higher Fock states in which constituents annihilate before the exchange of the hard momentum q^μ can be treated as corrections to the $q\bar{q}$ components of Ψ_λ .)

The leading logarithmic corrections to this power-law behavior are readily identified in each order of perturbation theory. They are order $(\alpha_s \log Q^2)^n$ in n th order; double logarithmic terms, $(\alpha_s \log Q^2 \log Q^2)^n$, do not appear due to infrared cancellations in the color singlet state. As in Section 4, we choose a frame where q^μ is transverse to the direction of the incident meson ($-q^2 = Q^2 = q_1^2$). To leading order: the dominant momentum flow then occurs through the minimal exchange graphs T_B ; only planar ladder graphs are required (in light-cone gauge); and the trans-

$$\begin{aligned}
 & \text{Diagram 1: } p \text{ --- } (\lambda) \text{ --- } (\text{shaded circle}) \text{ --- } (\lambda) \text{ --- } p+q \text{ with } \gamma \text{ emission from the shaded circle} \\
 & + \text{Diagram 2: } \text{---} (\lambda) \text{ --- } (\text{wavy line}) \text{ --- } (\text{shaded circle}) \text{ --- } (\lambda) \text{ ---} \text{ with } \gamma \text{ emission from the shaded circle} + \dots \\
 \\
 & \text{Diagram 3: } (\text{shaded circle}) = \text{Diagram 4: } \text{---} \text{---} \text{---} + \text{Diagram 5: } \text{---} \text{---} \text{---} + \text{Diagram 6: } \text{---} \text{---} \text{---} + \text{Diagram 7: } \text{---} \text{---} \text{---} + \dots \\
 & \text{where Diagrams 6 and 7 are labeled } k_{\perp} > \lambda
 \end{aligned}$$

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Fig. 7.3. The meson form factor in QCD using Eq. (7.6).

verse momentum integrations are ordered, as indicated in Fig. 7.4. Up to neglected terms of order $\alpha_s(Q^2)$ and m/Q , the meson form factor in QCD now takes the form

$$F_H(Q^2) = \int_0^1 dx_1 dx_2 \delta\left(1 - \sum_j x_j\right) \int_0^1 dy_1 dy_2 \delta\left(1 - \sum_j y_j\right) \\ \times \phi^\dagger(y_1, Q) T_B(y_1, x_1, Q) \phi(x_1, Q) \quad (7.9)$$

where

$$T_B = 16\pi C_F \frac{\alpha_s(Q^2)}{Q^2} \frac{1}{x_2 y_2} \quad \left(C_F = \frac{4}{3}\right) \quad (7.10)$$

and

$$\alpha_s(Q^2) = \frac{4\pi}{\beta \log Q^2/\Lambda^2} \quad \left(\beta = 11 - \frac{2}{3} n_{\text{flavors}}\right) \quad (7.11)$$

for mesons with zero helicity, and

$$\phi(x_1, Q) = \left(\log \frac{Q^2}{\Lambda^2}\right)^{-C_F/\beta} \int_0^{Q^2} \frac{dk_1^2}{16\pi^2} \psi(x_1, \vec{k}_1) \\ \equiv x_1 x_2 \tilde{\phi}(x_1, Q) \quad (7.12)$$

is the two-body wave function integrated over transverse momenta $k_1^2 \leq Q^2$. The factor $(\log Q^2)^{-C_F/\beta}$ in ϕ is due to vertex and fermion self-energy corrections in T_B . The integrated wave function $\phi(x_1, Q^2)$ is gauge-invariant; physically, it represents the impact space wave function at $\vec{b}_1^2 \sim O(1/Q^2)$. Note that the integration over transverse momentum includes the fermion self-energy corrections on both legs of the wave function. The ultraviolet renormalization is identical to that of the quark distribution described in Chapter 4.

Fig. 7.4. Leading logarithm contributions to the meson form factor. The dominant momentum flow is through T_B . The ladder graphs and self-energy insertions yield the evolution Eq. (7.16) and anomalous dimensions of the wave function at short-distances.

We now proceed to calculate the kernel which controls the behavior of the meson wave function at short distance. The one-gluon exchange kernel greatly simplifies¹ because of the ordering of the transverse momentum integrations. The calculation can be readily carried out using the light-cone perturbation theory rules of Chapter 3. We can avoid explicitly calculating the instantaneous gluon exchange term by simply replacing the standard transverse polarization with

$$-g_{\mu\nu} + \frac{\eta_\mu \tilde{k}_\nu + \eta_\nu \tilde{k}_\mu}{\eta \cdot k} \quad (7.13)$$

where $\tilde{k}^+ = k^+$, $\tilde{k}_\perp = k_\perp$ but $\tilde{k}^-$ is computed using conservation of the (-) component between the initial and intermediate state, rather than $k^- = (\vec{k}_\perp^2 + m^2)/k^+$. [A similar trick¹⁰ for the fermion polarization sum can be used to avoid explicit calculations of instantaneous fermion exchange contributions.]

The single gluon exchange contribution corresponding to Fig. 7.5 for $\vec{k}_\perp^2 \gg \vec{k}_\perp^2$ is

$$K_{1g}(x, \vec{k}_\perp, y, \vec{k}_\perp) = \begin{cases} \frac{-2C_F g^2}{1-x^2} \left\{ \frac{1-x}{1-y} \left(1 + \frac{2}{x-y} \right) \theta(x > y) + \frac{1+x}{1+y} \left(1 - \frac{2}{x-y} \right) \theta(x < y) \right\} & \text{for } ++ \rightarrow ++ \\ \frac{-2C_F g^2}{1-x^2} \left\{ \frac{1-x}{1-y} \frac{2}{x-y} \theta(x > y) - \frac{1+x}{1+y} \frac{2}{x-y} \theta(x < y) \right\} & \text{for } ++ \rightarrow ++ \end{cases} \quad (7.14)$$

Although most of the contribution of the fermion self-energy is incorporated into the renormalization of the coupling constant, an infrared divergent part survives (in $A_+ = 0$ gauge) which exactly cancels a similar infrared divergence due to transverse gluon exchange at $x = y$ in Eq. (7.14).

Fig. 7.5. Lowest order contributions to the $q\bar{q} + q\bar{q}$ kernel computed in light-cone perturbation theory. The last diagram corresponds to instantaneous gluon exchange.

The wave function $\phi(x, Q^2)$ thus can be written in leading logarithm approximation as $\phi(x, Q_0^2)$ plus an integral over $Q_0^2 < k_1^2 < Q^2$ from the gluon exchange kernel. As in Chapter 4 we define the variable

$$\xi(Q^2) - \xi(Q_0^2) = \frac{1}{4\pi} \int_{Q_0^2}^{Q^2} \frac{dk_1^2}{k_1^2} a_s(k_1^2) \sim \frac{1}{\beta} \log \frac{\log Q^2/\Lambda^2}{\log Q_0^2/\Lambda^2} \quad (7.15)$$

Then $\tilde{\phi}$ satisfies the evolution equation^{1,2,11}

$$\begin{aligned} & x_1 x_2 \left\{ \frac{\partial}{\partial \xi} \tilde{\phi}(x_1, Q) + C_i \tilde{\phi}(x_1, Q) \right\} \\ &= \int_0^1 dy_1 dy_2 \delta\left(1 - \sum_j y_j\right) V(x_1, y_1) \tilde{\phi}(y_1, Q) \end{aligned} \quad (7.16)$$

where

$$\begin{aligned} V(x_1, y_1) &= 2C_F \left\{ y_1 x_2 \theta(y_2 - x_2) \left(\delta_{h_1 h_2} + \frac{\Delta}{y_2 - x_2} \right) + (1 \leftrightarrow 2) \right\} \\ &= V(y_1, x_1) \quad \left(\Delta \tilde{\phi} \equiv \tilde{\phi}(y_1, Q) - \tilde{\phi}(x_1, Q) \right) \end{aligned} \quad (7.17)$$

represents the one-gluon exchange interaction. The quantity $\delta_{h_1 h_2}$ is defined to be $O(1)$ when the $q\bar{q}$ helicities are parallel (anti-parallel). The terms cancelling the infrared divergences at $x_1 = y_1$ are due to self-energy corrections to the q and $\bar{q}$ legs.

The evolution equation has a general solution

$$\phi(x_1, Q) = x_1 x_2 \sum_{n=0}^{\infty} a_n C_n^{3/2}(x_1 - x_2) e^{-\gamma_n \xi} \quad (7.18)$$

where the Gegenbauer polynomials $C_n^{3/2}$ are eigenfunctions of $V(x_1, y_1)$. The corresponding eigenvalues are identical to the standard non-singlet and structure function moments

$$\gamma_n = C_F \left\{ 1 + 4 \sum_2^{n+1} \frac{1}{k} - \frac{2\delta_{h_1 h_2}}{(n+1)(n+2)} \right\} \geq 0 \quad (7.19)$$

The coefficients a_n can be determined from the soft wave function:

$$a_n \left(\log \frac{\lambda^2}{\Lambda^2} \right)^{-\gamma_n/\beta} = \frac{2(2n+3)}{(2+n)(1+n)} \int_{-1}^1 d(x_1 - x_2) C_n^{3/2}(x_1 - x_2) \phi(x_1, \lambda^2) \quad (7.20)$$

(If we assume isospin symmetry for the pion wave function, $\phi(x_1, x_2) = \phi(x_2, x_1)$ so that only $n = \text{even}$ terms contribute). Notice that as $Q^2 \rightarrow \infty$

$$\phi(x_1, Q) \rightarrow \begin{cases} a_0 x_1 x_2 & h_1 + h_2 = 0 \\ a_0 x_1 x_2 \left(\log \frac{Q^2}{\Lambda^2} \right)^{-C_F/\beta} & |h_1 + h_2| = 1 \end{cases} \quad (7.21)$$

where a_0 is 6 times the wave function at the origin (by Eqs. (7.12) and (7.20)). For pions this constant can be determined from the weak decay amplitude for $\pi \rightarrow \mu \nu$:

$$a_0 = \frac{3}{\sqrt{n_{\text{colors}}}} f_\pi \quad (f_\pi = 93 \text{ MeV}) \quad (7.22)$$

An analogous result is obtained for the kaon. The decay $\rho \rightarrow \pi \pi$ can be used to normalize the asymptotic ρ wave function. If we define $\langle 0 | J_\mu | \rho(p, \epsilon) \rangle = m f_\rho \epsilon_\mu$, then a_0 is $3f_\rho / \sqrt{n_c}$. The convergence of the series and the evolution equation is assured if the $q\bar{q}$ wave function satisfies the boundary condition

$$\phi(x_1, Q) < k x_1^\epsilon \quad \text{as} \quad x_1 \rightarrow 0 \quad (7.23)$$

for some $\epsilon > 0$. The fact that the converge of the kinetic energy

$\left\langle \sum_1 (\vec{k}_1^2 + m^2) / x_1 \right\rangle$ must be finite for composite systems automatically guarantees this condition. In theories with an elementary field

representing (or mixing strongly with) the meson; the bound state equation has a source term corresponding to the bare coupling $\bar{\psi}\gamma_5\psi$, and consequently ϕ tends to a constant as $x_1 \rightarrow 0$. Precisely this type of behavior occurs in the case of photon structure functions and photon transition form factors in QCD (see Chapter 10).

Because of the boundary condition (7.23), the singularity in T_B at $x_2 = 0$, $y_2 = 0$ (Eq. (7.10)) does not result in additional factors of $\log Q^2$. The behavior of $F_M(Q^2)$ is thus determined by T_B and the short distance behavior of the wave function $\psi(x_1, \vec{k}_1)$ (i.e., $k_1^2 \rightarrow \infty$, $x_1 \neq 0$). Since the wave function is essentially $\langle 0 | T(\psi(r)\bar{\psi}(0)) | M \rangle$, the anomalous dimensions γ_n of $\phi(x_1, Q^2)$ are those associated with the twist two operators appearing in the operator product expansion of $\psi(r)\bar{\psi}(0)$. Furthermore, the usual renormalization group arguments imply that the leading logarithms summed by the evolution equation are in fact the dominant contribution as $Q^2 \rightarrow \infty$. Of course non-leading terms may be relevant at present energies, but these too may be computed in the framework described above.

Combining Eqs. (7.9), (7.10) and (7.18), we find the QCD predictions for helicity zero mesons ($\pi, K, \rho_L, \dots$)^{1,2,12}

$$F_M(Q^2) = \frac{4\pi C_F \alpha_s(Q^2)}{Q^2} \left| \sum_{n=0}^{\infty} a_n \left(\log \frac{Q^2}{\Lambda^2} \right)^{-\gamma_n/8} \right|^2 \times [1 + O(\alpha_s(Q^2), m/Q)] \quad (7.24)$$

Asymptotically the a_0 term dominates and from Eq. (7.22)

$$F_\pi(Q^2) \rightarrow 16\pi \alpha_s(Q^2) \frac{f_\pi^2}{Q^2} \quad \text{as } Q^2 \rightarrow \infty \quad (7.25)$$

Identical results follow for F_K and F_{ρ_L} if f_π is replaced by f_K and f_ρ respectively.

Although Eqs. (7.24) and (7.25) agree asymptotically, the $n \neq 0$ terms in (7.24) can result in sizeable corrections to both the normalization and shape of $F_H(Q^2)$ until Q^2 is quite large. In general these terms tend to compensate for the fall-off in $a_n(Q^2)$. If we assume that $\phi(x_1, \lambda)$ is sharply peaked at $x_1 \sim \frac{1}{2}$, as is characteristic of non-relativistic bound states, then the evolution equation causes $\phi(x_1, Q)$ to broaden, as Q^2 increases, out to its asymptotic form $x_1 x_2$. Since T_B is maximum at $x_2 = 0$, this effect enhances the form factor. Figure 7.6 illustrates predictions for $Q^2 F_\pi$ assuming that $\phi(x_1, \lambda)$ is either strongly peaked at $x_1 = \frac{1}{2}$, or has a smooth $x_1 x_2$ dependence (no evolution). In neither case is the normalization arbitrary; both tend to the form given in Eq. (7.25) as $Q^2 \rightarrow \infty$. The value $\Lambda^2 = 1 \text{ GeV}^2$ (equivalent to $\Lambda_{\text{eff}}^2 \sim .25 \text{ GeV}^2$) is used here.

For mesons with helicity ± 1 (e.g., ρ_T) or for transition between mesons of differing helicities (e.g., $\gamma^* \rho_L \rightarrow \rho_T$), T_B vanishes as a power of Q faster than Eq. (7.10).¹⁴ One significant consequence of this is the suppression of reactions $e^+ e^- \rightarrow \rho_T \rho_T, \rho_L \rho_T, \pi \rho$ by a factor m^2/Q^2 (in the cross section) relative to $e^+ e^- \rightarrow \pi^+ \pi^-, \rho_L \rho_L, K\bar{K}$.¹⁵ Furthermore, each of the leading processes has a positive form factor at large Q^2 relative to its sign at $Q^2 = 0$.¹⁶ These are all non-trivial consequences of QCD dynamics. By way of comparison, $\pi^+ e^- \rightarrow \rho_T \rho_T$ is not suppressed in theories with scalar gluons. Furthermore, F_π, F_K and F_{ρ_L} in scalar theories become relatively negative for large Q^2 and thus must vanish at some finite Q^2 . Current data for F_π already rules out the scalar theory.

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Fig. 7.6. QCD prediction for the meson form factor for two extreme cases: (a) $\tilde{\phi}(x_i, \lambda) = \delta(x_i - 1/2)$ or (b) $\tilde{\phi}(x_i, \lambda) = x_1 x_2$. In the latter case the wave function is unchanged under evolution. The asymptotic predictions are absolutely normalized, according to Eq. (7.25). The bands correspond to $\pm a_s(Q^2)/\pi$. In this figure $\Lambda^2 = 1 \text{ GeV}^2$; notice that because of momentum sharing the natural argument of a_s is $\sim Q^2/4$ so this value is equivalent to $\Lambda_{\text{eff}}^2 \sim .25$. The determination of a value for Λ^2 requires the computation of the order $a_s(Q^2)$ terms in Eq. (7.24). The data are from the analysis of electroproduction $e^-p \rightarrow e^- + \pi^+ + n$; C. Bebek *et al.*, Phys. Rev. D13, 25 (1976).

The analysis of baryon form factors in QCD is similar to that for mesons to leading order in $\alpha_s(Q^2)$.¹ The leading power-law terms involve only the three-quark component of the baryon's wave function (in light-cone gauge, $A^+ = 0$). When the leading logarithms in each order of perturbation theory (i.e., $(\alpha_s \log Q^2)^n$) are summed, the form factor has the form $(-q^2 \equiv Q^2)$:

$$F_B(Q^2) = \int_0^1 [dx_1] \int_0^1 [dy_1] \phi^\dagger(x_1, Q) T_H(x_1, y_1, Q) \phi(y_1, Q). \quad (7.26)$$

Here $[C_B = (n_{\text{color}} + 1)/2n_{\text{color}} = 2/3]$

$$T_H = \left(\frac{C_B \alpha_s(Q^2)}{Q^2} \right)^2 f(x_1, y_1) \quad (7.27)$$

is the minimally-connected amplitude for $\gamma^* 3q \rightarrow 3q$, and

$$[dx_1] \equiv dx_1 dx_2 dx_3 \delta\left(1 - \sum_i x_i\right). \quad (7.28)$$

The effective wave function $\phi(x_1, Q)$ is the three-body qqg Fock state wave function integrated over transverse momenta $|k_1^{(1)}|^2 < Q^2$

$[C_F = (\alpha_c^2 - 1)/2n_c = 4/3]$:

$$\begin{aligned} \phi(x_1, Q) &= \left(\log \frac{Q^2}{\Lambda^2} \right)^{-(3/2)C_F/B} \int_0^Q \prod_{i=1}^3 \left(\frac{d^2 k_i^{(1)}}{16\pi^3} \right) 16\pi^3 \delta^2\left(\sum_i \vec{k}_i^{(1)}\right) \psi(x_1, \vec{k}_i^{(1)}) \\ &\equiv x_1 x_2 x_3 \tilde{\phi}(x_1, Q) \end{aligned} \quad (7.29)$$

Only baryon states with $L_z = 0$ contribute to the leading power. The factor $(\log Q^2)^{-(3/2)C_F/B}$ is due to vertex and fermion self-energy corrections in T_H which are more conveniently associated with ϕ rather

than T_H . As in the meson case, the leading behavior of ϕ for large Q^2 is determined (in $A^+ = 0$ gauge) by planar ladder diagrams with the transverse momenta in successive loops strongly ordered $\lambda^2 \ll (k_1^1)^2 \ll (k_1^2)^2 \ll \dots \ll Q^2$. Three- and four-gluon couplings play no role in this order (other than in standard vertex renormalization) since they destroy the strong ordering. Consequently, defining ξ as in (7.15), we can derive an evolution equation¹ for $\tilde{\phi}(x_1, Q)$ (relating it to $\tilde{\phi}(x_1, \lambda)$ for some $\lambda < Q$):

$$x_1 x_2 x_3 \left\{ \frac{\partial}{\partial \xi} \tilde{\phi}(x_1, Q) + \frac{3}{2} C_F \tilde{\phi}(x_1, Q) \right\} = \int_0^1 [dy_1] V(x_1, y_1) \tilde{\phi}(y_1, Q) \quad (7.30)$$

where¹⁷

$$V(x_1, y_1) = x_1 x_2 x_3 \sum_{i \neq j} 2C_B \delta(x_i - y_i) \delta(x_k - y_k) \frac{y_1}{x_1} \left(\frac{\delta_{h_i \tilde{h}_j}}{x_1 + x_j} - \frac{\Delta}{y_1 - x_1} \right) \\ = V(y_1, x_1) \quad \left(\tilde{\Delta} \equiv \tilde{\phi}(y_1, Q) - \tilde{\phi}(x_1, Q) \right) \quad (7.31)$$

is the interaction between each pair of quarks due to exchange of a single gluon. The Kronecker delta $\delta_{h_i \tilde{h}_j}$ is 1(0) when quark helicities are anti-parallel (parallel). As in the meson case, the infrared singularity at $y_1 = x_1$ is cancelled because the baryon is a color singlet.

Any solution of the evolution equation can be expressed in terms of the eigenfunctions of V

$$\phi(x_1, Q) = x_1 x_2 x_3 \sum_{n=0}^{\infty} a_n \tilde{\phi}_n(x_1) e^{-\gamma_n \xi} \quad (7.32)$$

$$\left(\frac{3}{2} C_F - \gamma_n \right) x_1 x_2 x_3 \tilde{\phi}_n = V \tilde{\phi}_n \quad (7.33)$$

The coefficients a_n may be determined from the soft wave function:

$$a_n \left(\log \frac{\lambda^2}{\Lambda^2} \right)^{-\gamma_n/\beta} = \int_0^1 [dx_1] \cdot \tilde{\phi}_n(x_1) \phi(x_1, \lambda) \quad (7.34)$$

The leading eigenvalues γ_n and eigenfunctions $\tilde{\phi}_n(x_1)$ for helicity 1/2 and 3/2 baryons are given in Refs. 1 and 18. The asymptotic wave function for very large Q^2 is

$$\phi(x_1, Q) \rightarrow C x_1 x_2 x_3 \begin{cases} (\log Q^2/\Lambda^2)^{-2/3B} & |h| = 1/2 \\ (\log Q^2/\Lambda^2)^{-2/B} & |h| = 3/2 \end{cases} \quad (7.35)$$

where C is determined by the qqq wave function at the origin, and h is the total helicity. Since asymptotically ϕ is symmetric under interchange of the x_i 's, Fermi statistics demand that the corresponding flavor-helicity wave functions must be completely symmetric under particle exchange - i.e., identical to those assumed in the symmetric SU(6) quark model.¹⁹

The magnetic form factor $G_M(Q^2)$ for nucleons is given by Eq. (7.36) where T_B is computed from the sum of all minimally connected diagrams for $\gamma^* 3q \rightarrow 3q$ ($h_1 = h_3 = -h_2 = h$)²⁰

$$T_B = 64\pi^2 \left(\frac{C_B \alpha_s(Q^2)}{Q^2} \right)^2 \left\{ \sum_{j=1}^3 e_j T_j(x_1, y_1) + (x_1 \leftrightarrow y_1) \right\} \quad (7.36)$$

where

$$T_1 = T_3(1 \leftrightarrow 3) = \frac{1}{x_2 x_3 (1-x_3)} \frac{1}{y_2 y_3 (1-y_1)} - \frac{1}{x_3 (1-x_1)^2} \frac{1}{y_3 (1-y_1)^2} \\ - \frac{1}{x_2 (1-x_1)^2 y_2 (1-y_1)^2} \quad ; \quad (7.37a)$$

$$T_2 = - \frac{1}{x_1 x_3 (1-x_1)} \frac{1}{y_1 y_3 (1-y_3)} \quad (7.37b)$$

and e_j is the electromagnetic charge (in units of e) of particle j .

Convoluting with wave function (7.33), we obtain the QCD prediction for the large Q^2 behavior of G_M :

$$G_M(Q^2) \sim \frac{32\pi^2}{9} \frac{\alpha_s^2(Q^2)}{Q^4} \sum_{n,m} b_{n,m} \left(\log \frac{Q^2}{\Lambda^2} \right)^{-\gamma_n/8 - \gamma_m/8} \left[1 + O(\alpha_s(Q^2), m/Q) \right] \quad (7.38)$$

For very large Q^2 , the $n=m=0$ term dominates and we find

$$G_M(Q^2) \sim \frac{32\pi^2}{9} C^2 \frac{\alpha_s^2(Q^2)}{Q^4} \left(\log \frac{Q^2}{\Lambda^2} \right)^{-4/3\beta} (-e_{||}) \quad (7.39)$$

where $e_{||}$ ($e_{-||}$) is the mean total charge of quarks with helicity parallel (anti-parallel) to the nucleon's helicity (in the fully symmetric flavor-helicity wave function). For protons and neutrons we have

$$e_{||}^p = 1 \quad e_{-||}^p = 0 \quad e_{||}^n = -e_{-||}^n = -1/3 \quad (7.40)$$

The constants C are generally unknown for baryons; however, by isospin symmetry $C_p = C_n$ and thus QCD predicts the ratio of form factors as $Q^2 \rightarrow \infty$. The ratio $G_M^p(Q^2)/G_M^n(Q^2)$ is a sensitive measure of the nucleon wave function. For the initial condition $\phi(x_1, \lambda) = \delta(x_1 - 1/3) \delta(x_2 - 1/3)$, the ratio $-G_M^p(Q^2)/G_M^n(Q^2) \approx 1$ at $Q^2 = \lambda^2$, and decreases asymptotically to zero as

$$\left(\log Q^2/\Lambda^2 \right)^{(\gamma_0 - \gamma_3)/8} = \left(\log Q^2/\Lambda^2 \right)^{-32/9\beta} \quad (7.41)$$

Both the sign and magnitude of the ratio are non-trivial consequences of QCD; they depend upon the detailed behavior of T_B and $\phi(x_1, Q)$ as $Q^2 \rightarrow \infty$. For comparison, note that in a theory with scalar or pseudo-scalar gluons, diagrams in which the struck quark has anti-parallel helicity

vanish. Thus scalar QCD predicts a ratio $G_N^p/G_M^p + e_0^h/e_0^p = -1/3$.

The predictions for $G_M(Q^2)$ in the subasymptotic domain depend on the $n, m \neq 0$ terms in Eqs. (7.33) and (7.38). Figure 7.7 illustrates the predictions for $Q^4 G_M^p(Q^2)$ assuming an initial wave function condition $\phi(x_1, \lambda) = \delta(x_1 - 1/3) \delta(x_2 - 1/3)$ with $\lambda^2 = 2 \text{ GeV}^2$ and various QCD scale parameters $\Lambda^2 = 1, 0.1, 0.01, \text{ and } 0.001 \text{ GeV}^2$. Because of the factors of α_s in Eq. (7.36), it is difficult to fit the data²¹ for $\Lambda^2 > 0.1 \text{ GeV}^2$. In principle, non-leading terms could be important for $Q^2 \lesssim 25 \text{ GeV}^2$. These corrections can be computed using the formalism outlined above.

As is the case for mesons, form factors for processes in which the baryon's helicity is changed ($\Delta h \neq 0$), or in which the initial or final baryon has $h > 1$, are suppressed by factors of m/Q , where m is an effective quark mass. (Crossing and the $\Delta h = 0$ rule imply that form factors for particles with opposite helicity dominate for q^2 timelike.) Thus the helicity-flip nucleon form factor is predicted to fall roughly as $F_2 \sim m/Q^6$, and the elastic ep and en cross sections become ($-t = Q^2 + m^2$)

$$\frac{d\sigma}{dt} \rightarrow \frac{4\pi\alpha^2}{t^2} \left[\frac{s^2 + t^2}{2s^2} \right] G_M^2(-t) \quad (7.42)$$

Cross sections for transitions such as $ep \rightarrow e\Delta$ ($|h_\Delta| = 1/2$) are also given by Eq. (7.42) (with G_M as in (7.38) and (7.39) but with $C^2 \rightarrow C_p C_\Delta$ in the latter); quark charges are still those given in (7.40). Cross sections with $|h_\Delta| = 3/2$ are suppressed (by m^2/t). The reaction $e^+e^- \rightarrow \Delta^+\Delta^-$ is dominated by baryons with $|h_\Delta| = 1/2$; the cross section for production of $|h_\Delta| = 3/2$ pairs or deltas with $|h_\Delta| = 3/2$ and $1/2$ is suppressed.

Fig. 7.7. Prediction for $Q^4 G_M^p(Q^2)$ for various QCD scale parameters Λ^2 (in GeV^2). The data are from Ref. 21. The initial wave function is taken as $\phi(x, \lambda) \propto \delta(x_1 - 1/3) \delta(x_2 - 1/3)$ at $\lambda^2 = 2 \text{ GeV}^2$. The factor $(1 + m_0^2/Q^2)^{-2}$ is included in the prediction as a representative of mass effects. From Ref. 1.

Most of these predictions test the vector nature of the gluon. For example, transitions $ep \rightarrow e\Delta$ ($|h_\Delta| = 3/2$) are not suppressed in scalar QCD.

It should be noticed that the integration over the light-cone momentum fractions x_1 and y_1 in Eq. (7.26) would diverge linearly if the nucleon wave function were replaced by a constant. Compositeness only insures that $\phi(x_1) \sim (1-x_1)^\epsilon$ as $x_1 \rightarrow 1$ for $\epsilon > 0$; thus the possible end-point singularities make the proof of the short-distance dominance of the nucleon form factor more subtle. However, each eigenfunction of the evolution equation (corresponding to a leading twist contribution to the operator product expansion for $\psi\psi\psi$) leads to a contribution to $\phi_B(x_1, Q^2)$ which is of the form $x_1 x_2 x_3$ times a polynomial. The sum of each terms is convergent and yields a wave function $\phi_B(x_1, Q^2)$ which vanishes as $(1-x_1)^{2-\delta(Q^2)}$ where $\delta(Q^2)$ vanishes monotonically as $Q^2 \rightarrow \infty$. Thus the region of finite x_1 yields a contribution to the form factor which is dominated by the short-distance domain. There remains, however, the potentially dangerous region where some of the x_i are infinitesimally small, e.g., $x_2, x_3 \sim O(m/Q)$. However, in this region T_H itself is suppressed by a Sudakov form factor since a close to on-shell quark ($k^2 \sim O(mQ)$) is scattered at large momentum transfer Q^2 .²²

Thus the baryon form factor in QCD like the meson form factor is not dominated by the end point region in the x_1 integration, and the short distance domain of the operator products controls the asymptotic behavior.

It is easy to see that the same singularities²³ that appear in the operator product expansion which control the leading twist contributions

to the non-singlet structure function moments at large Q^2 , also determine the behavior of the meson wave function at short distances, and this in turn determines the behavior of the form factor of helicity-zero mesons at large momentum transfer. The distinguishing characteristic between the structure function moment and wave function calculations is essentially just the difference between "forward" $\langle p | \phi_{(n)} | p \rangle$ and "non-forward" $\langle 0 | \phi_{(n)} | p \rangle$ matrix elements, respectively, where the $\phi_{(n)}$ are the local operators that appear in the short-distance expansions. In the latter case, one has additional operators which involve extra factors of "outside" derivatives ∂_a . However, the "inside" derivatives $\vec{\partial}_a$ acting between the field operators determine the anomalous dimension since these are related to the internal integrations. Thus each anomalous dimension for the non-singlet structure function corresponds to a corresponding logarithmic correction factor to the nominal $\alpha_s(Q^2)/Q^2$ fall-off of the helicity zero meson form factor.

The form factor analysis discussed here is also applicable to the deuteron and other nuclei. The nominal asymptotic prediction^{4,24} for the spin-averaged form factor of the deuteron is $F_D(t) \sim \alpha_s^5(t)/t^5$, modulo anomalous dimension terms (see Fig. 7.8). The comparison with the prediction²⁵ $f_D(t) \equiv F_D(t)/F_N^2(t/4) \rightarrow c/(-m^2)$ is shown in Fig. 7.9. A more detailed discussion of the application of QCD to large momentum transfer nuclear physics problems is given in Ref. 26.

It should be emphasized that the specific integral powers Q^{-4} , predicted for the baryon form factor, and Q^{-2} , predicted for the meson form factor, reflect both the scale invariance of internal quark-quark interaction and the fact that the minimal color singlet wave functions

Fig. 7.8. Hadronic form factors multiplied by $(Q^2)^{n-1}$. From Ref. 26.

Fig. 7.9. The "reduced" form factor $f_D(Q^2)$ of the deuteron multiplied by $(1 + Q^2/m_0^2)$ with $m_0^2 = 0.5 \text{ GeV}^2$. The dimensional counting prediction at large Q^2 is $(1 + Q^2/m_0^2)f_D(Q^2) \rightarrow \text{constant}$.

of hadrons contain 3 quarks or quark plus antiquark. Thus both the dynamics and symmetry properties of QCD are directly tested. As we have seen, the spin dependence of quark-quark interactions can be tested at short distances by studying the helicity dependence of elastic and transition form factors. The QCD "selection rules" imply that hadrons tend to be produced at large momentum transfer with minimal helicity, and directly check the gluon spin.

It should be possible to relate the normalization and structure of the wave functions $\phi(x_1, Q_0^2)$ at large distances to the wave functions used in the study of baryon spectroscopy. We emphasize that the evolution equations completely determine the hadronic wave functions at short-distances:

$$\begin{aligned} \phi_M(x_1, x_2, Q^2) &\xrightarrow{Q^2 \rightarrow \infty} C_M x_1 x_2 \quad (\text{helicity zero}) \\ \phi_B(x_1, x_2, x_3, Q^2) &\rightarrow C_B x_1 x_2 x_3 \left(\log \frac{Q^2}{\Lambda^2} \right)^{-\gamma_0^N/\beta} \end{aligned} \quad (7.43)$$

The spin and flavor wave functions are then determined by [SU(6)] symmetry. In the case of the mesons the constants C_M are normalized by the leptonic decay amplitudes.²⁷

The large momentum transfer fall-off of the hadronic wave functions $\psi(x, \vec{k}_1)$ can be immediately determined from $\partial\phi(x, Q^2)/\partial Q^2$. Modulo logarithms,²

$$\psi_M(x, \vec{k}_1^2) \sim \frac{1}{\vec{k}_1^2} \quad (7.44)$$

Thus the fall-off of hadrons relative to a quark jet axis is $dN/d\vec{k}_1^2 \sim O(\vec{k}_1^{-4})$ compared to $dN/d\vec{k}_1^2 = \alpha_s(\vec{k}_1^2)/\vec{k}_1^2$ for gluons ($\vec{k}_1^2 \ll Q^2$). The high momentum tail of the hadron distribution has a power-law fall-off,

not Gaussian or exponential as usually assumed. Nevertheless, the mean value of $\langle k_1^2 \rangle$ is still essentially constant for the hadronic distribution versus the strong growth $\langle k_1^2 \rangle_g \sim a_s(W^2)W^2$ expected due to hard gluon radiation.

Fixed angle scattering^{2,9}

The techniques which we have discussed for obtaining asymptotic results for form factors can be extended to the computations of any exclusive process involving large momentum transfer between color singlets.² Here we shall focus on fixed angle hadronic scattering $d\sigma/dt(A+B \rightarrow C+D)$ as $s \rightarrow \infty$ at fixed t/s or θ_{cm} . In general, each hadron is represented by its Fock state decomposition; the leading power-law dependence as $s \rightarrow \infty$ is obtained from the Fock state with the minimum number of interacting components. The analysis of fixed angle scattering is complicated by pinch singularities, so we must consider two different scattering mechanisms.

A. Hard subprocesses

In this case the momentum transfer between constituents occurs through a single hard scattering amplitude T_H with all internal legs off-shell and proportional to $\Gamma_{\mu}^2 = tu/s$. The fixed angle amplitude is then to leading order in $a_s(p_T^2)$ (see Fig. 7.10)^{2,3}

$$\begin{aligned} \cdot M_{AB \rightarrow CD} = & \int \prod_i dx_i \phi_C^+(x_c, p_T^2) \phi_D^+(x_d, p_T^2) \\ & \times T_H(x_1, p_T^2) \phi_A(x_a, p_T^2) \phi_B(x_b, p_T^2) \end{aligned} \quad (7.45)$$

The amplitude T_H yields the power-law fall-off $p_T^{-(n-4)}$ where n is the total number of constituents, in agreement with the dimensional counting

Fig. 7.10. Fixed angle meson-baryon scattering contribution in QCD for hard scattering subprocesses.

rule.^{4,5} Examples of the leading contributions to lowest order in $\alpha_s(p_T^2)$ for meson-baryon scattering are shown in Fig. 7.11. Single gluon exchange between color singlet hadrons is of course zero. The constituent interchange graphs,²⁹ Fig. 7.11 are among the dominant contributions and lead to large flavor-exchange amplitudes. In general, there are an enormous number of such hard scattering tree graphs in QCD.

As in the form factor calculation, the evolution of the wave functions $\phi(x, \lambda^2)$ to $\phi(x, p_T^2)$ yields a series of terms with anomalous powers of $\alpha_s(p_T^2)$. The asymptotic cross section at $p_T^2 \rightarrow \infty$ has the form

$$\begin{aligned} \frac{d\sigma}{dt} (A+B \rightarrow C+D) &\rightarrow \frac{\left[\alpha_s(p_T^2)\right]^{n-2+\frac{\gamma_I}{2}}}{\left[p_T^2\right]^{n-2}} f(\theta_{cm}) \\ &= \frac{\alpha_s^2(p_T^2)}{p_T^2} F_A(p_T^2) F_B(p_T^2) F_C(p_T^2) F_D(p_T^2) f(\theta_{cm}) \end{aligned} \quad (7.46)$$

where γ_I is the leading anomalous dimension and $F_I(t)$ is the asymptotic form factor of hadron I. (The non-leading anomalous dimensions are not expected to be given correctly by Eq. (7.46) because of the different weighting of the x_i integrations.) The anomalous logarithms from each wave function is specific to each hadron; we thus have a "factorization theorem" for the logarithmic corrections to dimensional counting analogous to the factorization theorems for scale-violations to high p_T inclusive reactions.

B. Soft subprocesses

As first emphasized by Landshoff,³⁰ amplitudes with "pinch" singularities, analogous to Glauber scattering amplitudes may provide an alternative and possibly phenomenologically important source of

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Fig. 7.11. Examples of hard scattering processes for meson-baryon elastic scattering.

hadrons at large p_T . The classic example is pp scattering, which can proceed via three successive (nearly on-shell) $qq \rightarrow qq$ elastic collisions each at $\theta_i \sim \theta_{cm}$ (see Fig. 7.12). The hadronic amplitude has the form ($\hat{s} \sim 1/3s$, $\hat{t} \sim 1/3t$)

$$\mathcal{M}_{pp \rightarrow pp} \sim \left[\frac{1}{\sqrt{stu\lambda^2}} \right]^2 \left[\mathcal{M}_{qq \rightarrow qq}(\hat{s}, \hat{t}) \right]^3 \quad (7.47)$$

which gives nominally for spin-1 exchange

$$\frac{d\sigma}{dt}(pp \rightarrow pp) \sim \frac{1}{\lambda^4 t^8} f(s/t) \sim \frac{1}{t^8} \text{ for } s \gg |t| \quad (7.48)$$

The $1/\sqrt{stu\lambda^2}$ factor in Eq. (7.47) represents the probability amplitude for the scattered quarks to overlap with the final state hadrons. As noted by Donnachie and Landshoff,³⁰ data from the ISR and FNAL at $s > 800 \text{ GeV}^2$, $4 < |t| < 10 \text{ GeV}^2$ is compatible with Eq. (7.48) (see Fig. 7.13).

The anomalous power law of the pinch singularities arises from exclusive $qq \rightarrow qq$ amplitudes where each intermediate state is nearly on-shell. Thus Fock state amplitudes Ψ_λ in the soft domain are required. Since gluon radiation is excluded in this exclusive reaction, $\mathcal{M}_{qq \rightarrow qq}$ is suppressed by a Sudakov-like quark form factor $S(\hat{t})$. Thus the leading pinch singularity contribution to pp scattering is³¹

$$\frac{d\sigma}{dt} \sim \frac{\alpha_s^6(\hat{t})}{t^8} S^6(\hat{t}) f(s/t) \quad (7.49)$$

where in this case $S(\hat{t})$ falls faster than any power. The pinch contribution thus eventually becomes negligible compared to the hard-scattering contributions. Nevertheless, it is possible that such multiple scattering

Fig. 7.12. Landshoff "pinch" singularity contribution to elastic baryon-baryon scattering. The elastic $qq + qq$ amplitudes are nearly on-shell.

Fig. 7.13. Comparison of ISR and FNAL data at $s \gg |t|$ with the pinch singularity contribution (7.48). From Ref. 30.

processes do play an important role in the $s \gg |t|$ domain of Fig. 7.13, especially considering the fact that $\bar{t} \sim 1/9t$ is of only of order ~ 1 GeV^2 . However, it is not clear why the ISR and FNAL data do not indicate a faster fall-off than t^{-8} considering the strong variation of $\alpha_s(\bar{t})$ and the Sudakov form factor in Eq. (7.49) - unless Λ is very small.

The phenomenology of large angle scattering

The overall features of $d\sigma/dt$ ($pp+pp$) are sketched schematically in Fig. 7.14. The data at central angles $30^\circ \lesssim \theta_{\text{cm}} \lesssim 150^\circ$ fall rather uniformly as $s^{-10} f(\theta_{\text{cm}})$ and merge with a t^{-8} energy-independent "envelope" in the small θ_{cm} $s \gg |t|$ region. This s -independent small θ_{cm} envelope is consistent with multiple gluon exchange mechanisms which produce fixed $J=1$ Regge behavior. The overall behavior of the central angle data is consistent with the quark interchange²⁹ QCD diagrams; the observed shape of $f(\theta_{\text{cm}}) \sim (\sin\theta_{\text{cm}})^{-10}$ to -14 is compatible with this ansatz. Furthermore, the roughly symmetrical shape of $d\sigma/dt$ ($np+np$) around 90° and the large $pp/p\bar{p}$ ratio at 90° are all consistent with quark exchange or interchange hard scattering mechanisms.^{28,29} The observed normalization of the fixed angle nucleon-nucleon cross section is enormous compared to the single estimate $d\sigma/dt \sim G_H^2(t)(4\pi\alpha_s^2/t^2)$. However, this could be accounted for by (in fact, demands) the large number of hard scattering QCD amplitudes which contribute here.

The prediction $d\sigma/dt \sim s^{-10} f(\theta_{\text{cm}})$ seems to be consistent with data over a large range of energies and angles (see Fig. 7.15). The overall best fit is $s^{-9.7 \pm 0.5} f(\theta_{\text{cm}})$. A recent measurement of $d\sigma/dt$ ($pp+pp$) and $d\sigma/dt$ ($np+np$) at $\theta_{\text{cm}} = 90^\circ$ gives best fits $s^{-9.8 \pm 0.5}$ and

Fig. 7.14. Qualitative features of elastic pp scattering.

Fig. 7.15. Proton-proton elastic scattering at fixed θ_{cm} . From P. V. Landshoff and J. C. Polkinghorne, *Phys. Lett.* **44B**, 293 (1973).

$s^{-10.4 \pm 0.34}$, respectively.³²

However, the hard scattering mechanisms fail to account for either:

- (a) the apparent slow oscillation of the 90° cross section about the s^{-10} prediction,²⁸ nor
- (b) the striking, strongly varying spin correlation recently measured at Argonne.³³ (See Fig. 7.16.) At $s = 24 \text{ GeV}^2$, $\theta_{\text{cm}} = 90^\circ$, it is ~ 4 times more likely for two protons to scatter with their spins aligned normal and parallel to the scattering plane than anti-parallel. In contrast, the quark interchange amplitude predicts this ratio should be close to 2 independent of angle.^{34,35}

The possibility that non-perturbative (instanton) effects could be interfering with the quark interchange amplitude to produce large spin correlations has been investigated by Farrar, Gottlieb, Sivers, and Thomas.³⁴ More recently Brodsky, Carlson, and Lipkin³⁵ have considered another possibility: since the triple scattering pinch singularity contribution to pp scattering requires the qq + qq amplitude at $\hat{s} = 1/9s$ and the relatively low momentum transfer $\hat{t} \sim 1/9t \sim -1.1 \text{ GeV}^2$, it is not unlikely that the qq scattering is dominated here by simple meson exchange or Reggeon diagrams. In fact a low energy pinch contribution, computed with triple π or π exchange and interfering with the quark interchange amplitude, can produce a spin-correlation with the observed magnitude and dependence on θ_{cm} . This model leads to a number of testable predictions including s-dependence, np/pp ratios, and the spin correlation for longitudinally polarized nucleons. A large double-helicity flip amplitude is predicted due to scalar and the pseudoscalar meson exchange. An ever more intriguing possibility is that there is actually a high

Fig. 7.16. Comparison of $pp \rightarrow pp$ data of Ref. 33 at $p_{lab} = 11.75$ GeV for the ratio $r_{nn} = d\sigma(\uparrow\uparrow)/d\sigma(\uparrow\downarrow)$ for protons polarized normal to the scattering plane with the calculation of Ref. 35. The prediction for protons polarized longitudinally (or sidewise) is also shown in the figure.

mass dibaryon resonance which contributes to the $s = 24 \text{ GeV}^2$ $pp + pp$ amplitude, which by interfering with the conventional QCD mechanisms produces strong spin polarization. High mass dibaryon resonances might occur due to internal color polarization states.

In the case of photoproduction at large angles, one predicts $d\sigma/dt (\gamma p + \pi^+ n) \sim s^{-7} f(\theta_{cm})$, since the photon is an elementary field. The best fit to the 90° data³⁶ (Fig. 7.17a) is $s^{-7.3 \pm 0.4}$. The expectation that $d\sigma/dt (\gamma p + \gamma p) / d\sigma/dt (\gamma p + \pi^0 p)$ increases with energy at fixed angle is borne out by the data³⁷ shown in Fig. 7.17b. Meson-baryon scattering data³⁸ at 90° is not inconsistent with the counting rule $s^{-8} f(\theta_{cm})$ prediction.

Throughout this comparison of experiment and theory for exclusive processes there seems to be one central fact: the leading logarithm modifications predicted by QCD to simple dimensional counting scaling behavior are not evident in the data. A definitive comparison will require a calculation of the order α_s corrections. A very interesting possibility is that the scale parameter Λ^2 in $\alpha_s(Q^2)$ is effectively very small so that α_s is effectively constant. For example, the interesting sum rule analysis of Novikov et al.³⁹ implies $\Lambda \approx 70 \text{ MeV}$, $\Lambda^2 \approx 0.005 \text{ GeV}^2$. The factor $[\alpha_s(t)]^{10}$ in $d\sigma/dt (pp + pp)$ then changes by a factor of 4.2 from $|t| = 8$ to $|t| = 25 \text{ GeV}^2$, raising the effective power by only about 1 unit. A small value for Λ^2 could perhaps be a problem in the case of the normalization of the pion form factor if we assume that the wave function has evolved almost completely to its lowest eigenfunction in the presently accessible range of Q^2 ($\leq 4 \text{ MeV}^2$). The data shown in Fig. 7.6 for $F_\pi(Q^2)$ is taken from the extrapolation

Fig. 7.17. (a) Pion photoproduction data at 90° , from Ref. 36. (b) Ratio of 90° Compton scattering and π^0 photoproduction, from Ref. 37. The best fit to the ratio is s^n with $n = 2.1 \pm 0.6$.

of the $ep + e\pi^+n$ cross section. It will be very useful to have carefully normalized high Q^2 , $e^+e^- \rightarrow \pi^+\pi^-$ and $e^+e^- \rightarrow K^+K^-$ data to see whether this is an intractable problem. Calculations of the higher order $\alpha_s(Q^2)$ corrections will also be important for a definitive comparison of theory and experiment.

8. Structure Functions at $x \rightarrow 1$ and the Exclusive-Inclusive Connection

As we have seen in Chapter 7, exclusive processes involving large momentum transfer test both the internal dynamics of hadrons and the detailed structure of hadronic wave functions at short distances. In this section we shall show the behavior of structure functions in the $x_{Bj} \rightarrow 1$ region can also be calculated using perturbative quantum chromodynamics methods.⁴⁰ As $x_{Bj} \rightarrow 1$, essentially all of the hadron's momentum must be carried by one quark, and thus each quark and gluon propagator which transfers this momentum becomes far off-shell: $k^2 \sim O\left(-\frac{k^2 + m^2}{1-x}\right) \rightarrow -\infty$. The main power-law dependence at $x \rightarrow 1$ is thus determined by the minimal number of gluon exchanges required to stop the hadronic spectators. For example, in the case of ϕ^4 field theory, one immediately obtains the spectator counting rule⁶ $G_{q/H}(x) \sim (1-x)^{2n_g-1}$ where n_g is the number of momentum-carrying spectator fields forced to stop as $x \rightarrow 1$; spin effects can lead to further suppression. In general, the hadronic wave functions are integrated over large transverse momentum $k_{\perp}^2 \lesssim O\left(\frac{m^2}{1-x}\right)$, and can be again analyzed at short distances using evolution equations or the operator product expansion.

In the case of the nucleon's structure function, the leading power behavior at $x \rightarrow 1$ is determined (in the light-cone gauge) by its minimal

3 quark Fock state. The large momentum flow is through the hard scattering amplitude (or operator) T_H given by the $3q + \gamma^* + 3q$ tree graphs (see Fig. 8.1). T_H must then be convoluted with the hadronic wave functions $\phi_B(y_1, k_x^2)$ or $\phi_M(y_1, k_x^2)$, where the upper limit of the internal transverse momentum integrations k_1^2 is set by $|k_x^2| \sim O(m^2/(1-x))$. In this domain the internal transverse momentum integrations factorize from the kinematics of the hard momentum transfer process and build up the hadronic anomalous dimensions. In addition, gluon radiation from the struck quark (labelled x_1 in the Fig. 8.1a) yields the usual Q^2 evolution of the quark structure function. We return to this effect later.

The leading $(1-x)$ power-law contribution to the $F_2(x, Q^2)$ baryon structure function in light-cone perturbation theory arises from the configurations such as that of Fig. 8.1b where the intermediate fermion leg is instantaneous. The amplitude has the form

$$T_H \sim \frac{[\alpha_s(-k_x^2)]^2}{[k_1^2/(1-x_1)]^3} \frac{k_1^2}{\sqrt{x_2}\sqrt{x_3}} \left(\frac{1}{\sqrt{x_2}} \frac{1}{\sqrt{x_3}} \right) \quad (8.1)$$

which after integration over phase space, yields the nominal power, $F_2(x) \sim (1-x)^3 \alpha_s^4(k_x^2)$. [In contrast, the contribution of Fig. 8.1c gives $F_2(x) \sim (1-x)^5$]. Including the wave function anomalous dimension and the gluon radiative correction, as discussed in Chapter 7, we have

$$F_{2N}(x, Q^2) \xrightarrow{x \rightarrow 1} (1-x)^3 \alpha_s^4(-k_x^2) \left| \sum_{j=0} b_j \left(\log \frac{-k_x^2}{\Lambda^2} \right)^{-Y_j^N/B} \right|^2 F(x, Q^2) + O(1/Q^2, \alpha_s(-k_x^2)) \quad (7.2)$$

Fig. 8.1. (a) The leading tree-graph contribution to the hard Scattering amplitude $T_H(3q + \gamma^* + 3q)$. (b) Light-cone perturbation theory contribution to T_H which yields the leading $(1-x)^3$ power-law contribution to the nucleon structure function as $x \rightarrow 1$. (c) Light-cone perturbation theory term giving a $(1-x)^5$ contribution to the nucleon structure function.

where $P(x, Q^2)$ controls the Q^2 evolution and is computed in detail in Chapter 4. For example, if we consider the kinematical domain with $(1-x)$ fixed, but small, and $Q^2 \rightarrow \infty$, then the δq contribution in Eq. (4.27) becomes relatively negligible and we have

$$P(x, Q^2) \sim (1-x)^{\left(\frac{4C_F}{8} \log \frac{\log Q^2/\Lambda^2}{\log Q_0^2/\Lambda^2} \right) \left(\frac{\log Q^2/\Lambda^2}{\log Q_0^2/\Lambda^2} \right)^{3C_F/8}} \quad (8.3)$$

Accordingly, the power of $(1-x)$ (which is plotted in Fig. 8.2) increases as $\log \log Q^2$. The parameter Q_0 is set by the hadronic scale for off-shell momenta of quarks and gluons within the hadron. The extra suppression of $P(x, Q^2)$ at $x \rightarrow 1$ reflects the occurrence of a Sudakov form factor for a near-one-shell quark in a region where the gluon radiation is kinematically constrained.

It should be emphasized that the specific shape and behavior of the structure function at large x provides an important check of SU(3) color and QCD. In particular, the specific value of 3 in Eq. (8.2) reflects both the asymptotic scale-invariance of the quark-quark interactions and the fact that the minimal Fock state in the baryon has 3 quarks. Experimentally, the effective values of the $(1-x)$ power of the proton structure function range from ~ 3 to 4.⁴¹ Careful checks of the variation of the power from low to high Q^2 can further constrain the value of Λ and determine empirically the effective value of Q_0 . It clearly would be very useful to extend the standard evolution equation analyses⁴¹ of the data by including the δq contributions, and using Eq. (8.2) as a constraint on the initial quark distribution function.

Fig. 8.2. Increase $\Delta\xi = (4C_F/3) \log(\log Q^2/\Lambda^2/\log Q_0^2/\Lambda^2)$ of the structure function power $(1-x)^{\Delta\xi}$ from QCD evolution.

It is also interesting to note that the leading $(1-x)^3$ contribution from Fig. 8.1b only occurs if the spectator quarks with momenta y_2 and y_3 have opposite helicities.⁴² Thus for $x \rightarrow 1$ the struck quark must have the same helicity as the target nucleon. We find

$$G_{q^+/p^+} \sim (1-x)^3 \quad ; \quad G_{q^+/p^+} \sim (1-x)^5 \quad . \quad (8.4)$$

This implies

$$\frac{F_{2n}(x, Q^2)}{F_{2p}(x, Q^2)} \xrightarrow{x \rightarrow 1} \frac{\sum_q e_q^2 n_{q^+/n^+}}{\sum_q e_q^2 n_{q^+/p^+}} = \frac{3}{7} \quad (8.5)$$

where we have used the (asymptotic) SU(6) symmetry of the nucleon wave function. It also implies that the leading quark in the proton is 5 times more likely to be an up quark than a down quark.⁴² These predictions seem to be in agreement with experiment. In particular the π^+/π^- ratio in pp collisions at large x_F seems to be approaching a 5:1 ratio.⁴³ These results also should apply to quark + baryon fragmentation functions. In particular $q \rightarrow \Delta$ should be dynamically suppression if the Δ has helicity 3/2.

Thus far we have only considered the diagrams where the photon 4-momentum is routed to give the minimal powers of $(Q^2)^{-1}$. Other contributions to T_H for $(qqq) + \gamma^* \rightarrow (qqq) + \gamma^*$ [including diagrams where gluons interact between the photon interactions, and contributions where the photons interact on different quark lines as in Fig. 2.1] yield a series of terms:⁴⁴

$$F_2(x, Q^2) = \left[F_2(x, Q^2) \right]_{\text{leading twist}} \\ = \left[1 + \frac{A\nu^2}{Q^2(1-x)} + \frac{B\nu^4}{Q^4(1-x)^2} \frac{1}{\alpha_s(m^2/(1-x))} + \dots \right] \quad (8.6)$$

where $A\nu^2 \sim O(m_p^2)$ is set by the wave function scale. Each of these terms contribute roughly equally at fixed W^2 and have the same Q^2 -evolution as the leading twist $(1-x)^3$ terms. (The extra $\alpha_s^{-1}(k_x^2)$ factor arises from a logarithmic integration over the final state transverse momentum.) In addition, we can consider these same diagrams in the region where $(\vec{k}_1)_1 \sim O(x_1 \vec{q}_1)$, e.g., where $\vec{q}_1$ is routed through two quark legs. This contribution corresponds physically to a diquark recoiling against the lepton; it gives a contribution of order

$$F_2(x, Q^2) \sim D \frac{(1-x)^2}{Q^4} \mu^4 \alpha_s^2(Q^2) \alpha_s^2(k_x^2) \quad , \quad (8.7)$$

with corrections from the wave function anomalous dimensions and the Q^2 -evolution due to gluon emission from the coherent $\bar{3}_c$ diquark. The diquark term contributes in leading order to F_L , as could be expected for a boson current.⁴⁵ All of the power-law terms are examples of "high twist" contributions, e.g., the diquark term corresponds to a local 4-quark term in the operator product expansion of $J_\mu(\pi/2)J_\nu(-\pi/2)$. It is important to note that a simple form such as

$$F_2(x, Q^2) = C_0(1-x)^3 + C_1 \frac{(1-x)}{Q^2} \quad (8.8)$$

where $C_1/C_0 \sim O(m_g^2)$ can simulate some of the effects normally attributed to logarithmic scale violations.^{46,47} In particular this form predicts

that the structure function (at moderate Q^2) will fall rapidly with Q^2 for x near 1. The corresponding structure function moments have the form

$$M_N(Q^2) = k_N \left[1 + \frac{C_1}{C_0 Q^2} \frac{(N+3)}{3} \right] \quad (8.9)$$

which yields a strong dependence on Q^2 at large N . [The mass scale (C_1/C_0) must be of the same magnitude as $Q^2 C_N^h(Q^2)$ since the same physics controls both normalizations.] It should be emphasized that these power-law corrections are an integral part of QCD which arise in part because of the compositeness of the hadrons. The high twist terms can only be neglected if

$$N \ll Q^2/M_p^2 \quad (8.10)$$

Since the QCD logarithm and power law terms act in the same direction, it is clear that analyses of scale-violation which do not include the high twist terms lead to an overestimate of the logarithmic corrections. In particular, if the only scale-violation allowed in a QCD parametrization is via $\alpha_s(Q^2)$ then the scale parameter Λ^2 (where $\alpha_s(Q^2)$ is rapidly varying) will have to be set at a value which simulates the power-law term.⁴⁸ Again we note that the actual value of Λ could be considerably smaller than the conventional values $\Lambda \sim 400$ to 500 MeV usually assumed. This is really an incredible ambiguity; in fact it has been shown in Ref. 47 that high twist contributions alone could account for all of the observed scale-violations in deep inelastic scattering.

It is important that the high twist terms in the nucleon structure function be computed as carefully as possible in order to resolve this ambiguity. A simultaneous calculation and parametrization of the struc-

ture functions and the elastic and transition form factors should restrict the normalization and form of the power law terms.

It is also fascinating to see how the hadron structure functions join with the elastic and resonance contributions as one approaches the exclusive limit. The $x \rightarrow 1$ limit has to be carefully specified. We have already derived the form for the nucleon structure function when $Q^2 \rightarrow \infty$, and then $1-x$ becomes small (see Eq. (8.2)). In this case the leading twist contribution is of the form $(1-x)^3 + [\tilde{\xi}(Q^2) - \tilde{\xi}(Q_0^2)]$ where $\tilde{\xi}(Q^2)$ increases as $(4C_F/3) \log \log Q^2$. The fixed $W^2 = [(1-x)/x]Q^2$, $Q^2 \rightarrow \infty$ limit is quite different. First, the leading twist contribution is modified by the δq term in Eq. (4.28) induced by the W^2 kinematic limit on the gluon transverse momentum integrations. In leading order the leading twist contribution is then of the form $(1-x)^3 + \tilde{\xi}(W^2) - \tilde{\xi}(Q_0^2)$. Second, the high twist contributions $(1-x)/Q^2$ etc. are of comparable importance at fixed W^2 . These terms again have their power increased by $\tilde{\xi}(W^2) - \tilde{\xi}(Q_0^2)$ due to the gluon radiation from the struck quark lines. Finally, there are the contributions where q is routed through all three quark lines in T_H . This should match with the elastic and resonance transition form factor contributions, which yield a spectral contribution of the form

$$F_2(x, Q^2) \sim \int d\alpha_R^2 \delta(\alpha_R^2) \delta(W^2 - m_R^2) F^2(Q^2) \alpha^2 \\ \sim \frac{1}{1-x} \frac{\alpha_s^4(Q^2)}{Q^8} \left| \sum a_n \log^{-d_n} Q^2/\Lambda^2 \right|^4. \quad (8.11)$$

if the spectrum $\delta(\alpha_R^2) \sim 1/m_R^2$. At fixed W^2 each of the contributions have the same nominal power fall off in Q^2 .⁴³ However, the single quark and

double quark recoil terms each contain the extra $(1-x)^{\tilde{\zeta}(W^2)}$ suppression factor from gluon radiation. Thus, the leading contribution at fixed W^2 is actually from the 3 quark (elastic and transition form factor) contributions, since in this case hard gluon radiation from the internal quark legs is power-law suppressed. In a sense, the $\tilde{\zeta}(W^2)$ mismatch implies the inclusive-exclusive connection fails in QCD, since the contributions which dominate the Bjorken limit of the structure functions are non-leading at fixed W^2 compared to the exclusive channels.⁵⁰

Meson structure functions

The structure function of mesons at $x \sim 1$ can also be readily computed in QCD. A novel feature is that the leading twist contribution is suppressed by an extra factor of $(1-x)$ in the transverse structure function compared to what one expects from spectator counting.⁵¹ This suppression can be attributed to the mismatch between the quark spin and the spin of the meson. Proceeding as in the calculation of the nucleon structure function, one finds that the meson structure function has the form^{40,51}

$$F_2(x, Q^2) \underset{x \rightarrow 1}{\sim} (1-x)^2 \alpha_s^2(-k_x^2) \left| \sum_{j=0} a_j \left(\log - \frac{k_x^2}{\Lambda^2} \right)^{-\gamma_j/8} \right|^2 P(x, Q^2) + O\left(1/Q^2, \alpha_s(-k_x^2)\right) \quad (8.12)$$

where $P(x, Q^2) \sim (1-x)^{\tilde{\zeta}(W^2) - \tilde{\zeta}(Q_0^2)}$ represents the QCD evolution.

In addition, if one looks at the high twist terms, one finds that the longitudinal structure function has an anomalous (non-scaling) component which is nearly flat at large x :^{52,53,40}

$$F_2(x, Q^2) \approx F_L(x, Q^2) \sim \frac{(1-x)^0}{x+1} \frac{1}{Q^2} a_g(-k_x^2) \quad (\text{high twist}) \quad (8.13)$$

The leading contribution to (8.13) in fact comes from the photon scattering off an instantaneous fermion line in light-cone perturbation theory. The dominance of the longitudinal structure function in the fixed Q^2 limit for mesons is an essential prediction of perturbative QCD. It can be tested directly⁵² in the $e^+e^- \rightarrow \pi X$ angular distribution near the kinematic boundary ($x \rightarrow 1$). Perhaps the most dramatic consequence is in the Drell-Yan process $\pi p \rightarrow \ell^+ \ell^- X$ (Fig. 8.3). In this case one predicts^{53a} that for fixed Q^2 pair-mass, the angular distribution of the ℓ^+ (in the pair rest frame) will change from the conventional $(1 + \cos^2 \theta_+)$ distribution to $\sin^2 \theta_+$ for pairs produced at large longitudinal momentum, $x_L(\ell^+ \ell^-) \rightarrow 1$. Again this is due to the dominance of the meson's longitudinal structure function at large x and fixed Q^2 (see Fig. 8.4). In fact the recent analysis by the Chicago-Princeton group⁵⁴ of this angular distribution in $\pi N \rightarrow \mu^+ \mu^- X$ appears to confirm the QCD high twist prediction (see Figs. 8.4 and 8.5). A calculation^{51b} of the conventional order $a_g(Q^2)$ logarithmic QCD corrections does not give very strong corrections to the angular distribution (at low transverse momentum). Thus this lepton pair experiment appears to yield the first strong evidence for high twist contributions to the structure functions.

It is of course extremely important to check whether the meson structure function has the predicted $(1-x)^2 + C/Q^2$ form (modulo logarithms). Figure 8.4a shows that this form is not far different from the usual $(1-x)$ parametrization, so detailed fits over a large range of Q^2 will be required. The corresponding analyses of the $q \rightarrow \pi$ fragmentation function

Fig. 8.4. (a) The pion structure function at large x . Comparison of a standard $(1-x)^1$ fit (dotted line) with the $F_2^{\pi} \sim (1-x)^2 + C/Q^2$ prediction of QCD. (b) QCD high twist prediction for the ν^+ angular distribution $1 + a(x_F) \cos^2 \theta_+$ where θ_+ is measured relative to the incident pion in the $\nu^+ \mu^-$ rest frame. From Ref. 53.

Fig. 8.5. Comparison of the data of Ref. 54 for the virtual photon polarization in $\pi N \rightarrow \mu^+ \mu^- X$ with the high twist QCD prediction of Ref. 53. The μ^+ angular distribution is parametrized as $1 + \alpha(x_1) \cos^2 \theta_+$ where x_1 is the $\bar{q}$ momentum fraction.

may be somewhat complicated by the effects of resonance decays. Thus structure functions and fragmentation distributions can be predicted in detail by perturbative quantum chromodynamics in the $x \rightarrow 1$ kinematic region. We also note that in the same region, the transverse momentum distributions are controlled by the hadronic wave functions. The resulting distributions $dN/d^2\vec{k}_\perp dz$ in a quark jet thus have a computable power-law fall off at large $\vec{k}_\perp$ (see Chapter 7), in contrast to the Gaussian distribution usually assumed for non-perturbative distributions in the jet.

9. High Twist Terms and p_T Reactions

As we have seen, perturbative QCD predicts two types of breaking of scale-invariance (1) logarithmic, via $\alpha_s(Q^2)$ and the evolution equations, and (2) power-law breaking via high twist subprocesses - short distance subprocesses which involve more than the minimal number of interacting fields. Despite their power-law suppression at high momentum transfer, the high twist terms usually have a compensating lower power-law fall-off at the edge of phase space. In the case of pion-induced lepton pairs, the high twist contribution corresponding to the subprocess $nq \rightarrow \ell\bar{\ell}q$ is even predicted to dominate the $q\bar{q} \rightarrow \ell\bar{\ell}$ leading twist term at large x_T .^{53a}

In the case of the production of hadrons at large p_T , $A+B \rightarrow C+X$, the high twist QCD subprocesses are further enhanced due to a kinematic effect, referred to as "trigger bias" in the literature.⁵⁵ The essential point is that if C is produced at a given p_T by quark (or gluon) fragmentation then the parent particle must be produced at even larger transverse momentum. Since the production cross section falls rapidly

with transverse momentum, the hadron triggered cross section is strongly suppressed compared to the quark (or gluon) jet cross section at the same p_T . Very roughly, if $D_{C/q}(z) \sim (1-z)^n$, the suppression factor is $(\epsilon)^{n+1}$ where ϵ is typically of order 1/10. Thus, cross sections for $A+B \rightarrow C+X$ are extremely sensitive to the $z \sim 1$ parametrization of the structure functions. Further, if any hard scattering subprocess can produce the hadron C directly without the necessity of fragmentation, then there is no trigger bias suppression. This in fact occurs in the case of direct photon production, via the subprocess $g+q \rightarrow \gamma+q$. In the case of hadron production, higher twist subprocesses such as $g+q \rightarrow M+q$, $(q\bar{q})+q \rightarrow M+q$, $(qqq)+q \rightarrow B+q$, $q\bar{q} \rightarrow M+(\bar{q}q)$, etc. produce hadrons directly at large p_T without any trigger-bias suppression, but of course at the expense of extra powers of p_T . These types of high twist QCD subprocesses correspond to the contributions incorporated into the "constituent interchange"⁵⁶ and "quark fusion" models.⁵⁷ An example of the high twist contribution $(q\bar{q})+q \rightarrow \pi+q$ is shown in Fig. 9.1. Similarly, the high twist contribution to the meson structure function produces the term $D(z, Q^2) \sim (1-z)^0/Q^2$, which has a relatively small trigger bias suppression.

In the case of the high p_T inclusive process $pp \rightarrow \pi X$, one obtains cross section contributions with the nominal powers,

$$\frac{E d\sigma}{d^3p} \sim A \frac{(1-x_T)^{10}}{p_T^4} \epsilon^3 + B \frac{(1-x_T)^8}{p_T^6} \epsilon + C \frac{(1-x_T)^8}{p_T^6} + D \frac{(1-x_T)^9}{p_T^8} + \dots \quad (9.1)$$

corresponding to subprocesses (a) $q\bar{q} \rightarrow q\bar{q}$, etc. with $D_{\pi/q} \sim (1-z)^2$ and $G_{q/p}(x) \sim (1-x)^3$, (b) $q\bar{q} \rightarrow q\bar{q}$, etc. with $D_{\pi/q} \sim (1-z)^0/p_T^2$, (c) $gq \rightarrow \pi q$ with $G_{g/p}(x) \sim (1-x)^4$, (d) $(q\bar{q})+q \rightarrow \pi q$, assuming $G_{(q\bar{q})/p} \sim (1-x)^5$. In

Fig. 9.1. Example of a QCD-CIM high twist contribution $(q\bar{q})+q \rightarrow \pi+q$ to $pp \rightarrow \pi X$ production at large transverse momentum.

addition, QCD predicts that each term will have logarithm modifications from powers of $\alpha_s(p_T^2)$, $\alpha_s(m^2/(1-x_T))$, and the evolution of the hadronic structure functions and wave functions. Since α_s is small, the minimal twist 2+2 subprocess contributions will not dominate until p_T becomes quite large; in the CIM the cross-over point between the p_T^{-8} and p_T^{-4} terms was estimated to occur at about $p_T \sim 8$ to 10 GeV.⁵⁸ In fact, this type of transition has been observed for the recent $pp \rightarrow \pi^0 X$ data from the ISR (see Fig. 9.2).

There are a number of experimental indications which also point to the dominance of high twist terms in high p_T hadron production cross sections below $p_T \sim 8$ GeV:

(1) The best power-law fit to the Chicago-Princeton⁵⁹ FNAL data for $p_T \lesssim 8$ GeV is

$$E \frac{d\sigma}{d^3p} (pp \rightarrow \pi^+ X) = \frac{1}{p_T^{8.2 \pm .5}} (1-x_T)^{9.0 \pm 0.5}$$

in agreement with high twist term expectations.

(2) The best fit to the angular distribution of the subprocess in $pp \rightarrow \pi X$ is $d\sigma/dt \propto 1/su^3$ in agreement with predicted high twist forms.⁵⁶

(3) The $q\bar{q} \rightarrow q\bar{q}$, $gq \rightarrow Mq$, and $q\bar{q} \rightarrow M\bar{M}$ subprocesses imply that flavor is generally exchanged in the hard scattering reaction.⁶⁰ For example, consider the quark interchange and $q\bar{q} \rightarrow M\bar{M}$ fusion contributions to $pp \rightarrow K^+ X$ shown in Fig. 9.3. The average charge of the recoil quark is slightly positive for the K^+ trigger and $> +1/3$ for the case of the K^- trigger. Thus the charge and flavor of the away-side jet in the CIM can be correlated with the flavor quantum numbers of the trigger. In contrast, gluon exchange diagrams predict very small⁶¹ flavor correlations

Fig. 9.2. Comparison of CIM high twist plus QCD (p_T^{-4}) contributions to the $pp \rightarrow \pi^0 X$ cross section with high p_T data. The QCD contribution is computed with $\alpha_s = .15$ and without scale-breaking. From Jones and Gunion, Ref. 58.

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Fig. 9.3. Analyses of charge flow in CIM high twist diagrams for $pp + K^2X$. Quark exchange in the subprocess implies charge correlations between the trigger and away side jet. See Ref. 60.

between the away-side systems. The data from the BSF-ISR group⁶² (see Fig. 9.4) for various charge triggers at 90° show striking flavor correlations, especially for K^- and $\bar{p}$ triggers, in general agreement with the above expectations for the quark exchange processes of the CIM model.

(4) In the case of $pp \rightarrow pX$, one expects a dominant high twist contribution from the $qB \rightarrow qp$ subprocess.⁵⁶ The theoretical prediction is $E d\sigma/d^3p(pp \rightarrow pX) \propto p_T^{-12} (1-x_T)^7$. The Chicago-Princeton⁵⁹ fit at 90° in fact gives $p_T^{-11.7} (1-x_T)^{6.8}$ at FNAL energies, $p_T < 7$ GeV, with uncertainties in the exponents of order ± 0.5 . We emphasize that a successful model for single particle production must account for both high p_T meson and baryon data. There does not seem any way to account for the $pp \rightarrow pX$ scaling behavior in terms of $2 \rightarrow 2$ QCD subprocesses without enormous scale-breaking in the $q+p$ distribution function; we note that data for $e^+e^- \rightarrow \bar{p}X$ appears to be reasonably consistent with scale-invariance.

On the other hand, recent data for the ratio of $\pi^+p \rightarrow \pi^-X$ cross sections from the Chicago-Princeton group⁶³ is in severe disagreement with dominance of the $(q\bar{q})+q \rightarrow \pi^-q$ high twist subprocesses, using the CIM parametrization of Ref. 58. Thus the normalization of this subprocess must be overestimated,⁶⁴ and a recalculation utilizing all the QCD high twist terms is required, including the $D_{\pi/q}(z) \sim (1-z)^0/Q^2$ and the $gq \rightarrow Mq$ contribution.

On the other hand, the high twist contributions seem to give a natural explanation for the observed p_T^{-8} and p_T^{-12} scaling behavior of the meson and proton production cross sections at moderate p_T . There is accordingly no compelling reason to attribute this observed strong

Fig. 9.4. Average number of fast positive and negative particles on the side away from a 90° trigger for various trigger types, from Ref. 62. The gluon exchange QCD diagrams lead to negligible away side jet charge correlation with the trigger charge. See Ref. 61.

scale-breaking to anomalous scale-violating effects (large k_T , etc.) associated with the leading twist $q\bar{q} \rightarrow q\bar{q}$, $qg \rightarrow qg$, and $gg \rightarrow gg$ sub-processes.^{61,65} In fact, calculations of the effect of the parton transverse momentum which retain the correct off-shell kinematics do not lead to large scale-violation corrections.⁶⁶

It is clearly extremely important to have careful measurements of the fixed x_T and θ_{cm} scaling behavior of direct photon, quark and gluon jet cross sections, because of the absence of trigger-bias suppression of the leading twist contributions. Recent data for the γ/π^0 ratio at high p_T from FNAL and ISR appear to be consistent with the prediction of different p_T scaling laws for direct photon and π^0 cross sections.

Because of the trigger-bias effect, it may be a mistake to identify the system on the same side of the trigger hadron with the remnants of quark or gluon jet fragmentation; the trigger side jet can be a single hadron of resonance in the case of the high twist subprocess.

A systematic parametrization of the power-law scaling and logarithm scale-violations for high twist and low twist QCD contributions to inclusive cross sections is given in Appendix A.

10. Two-Photon Collisions and Tests of Quantum Chromodynamics

The production of hadrons in the collision of two photons via the process $e^+e^- \rightarrow e^+e^-X$ (see Fig. 10.1) can provide an ideal laboratory for testing many of the features of the photon's hadronic interactions, especially its short distance aspects. We will review here that part of two-photon physics which is particularly relevant to tests of perturbative QCD.

Fig. 10.1. Two-photon annihilation into hadrons in e^+e^- collisions.

Large p_T jet production²⁻⁷

Perhaps the most interesting application of two photon physics to QCD is the production of hadrons and hadronic jets at large p_T . The elementary reaction $\gamma\gamma \rightarrow q\bar{q} \rightarrow \text{hadrons}$ yields an asymptotically scale-invariant two-jet cross section at large p_T proportional to the fourth power of the quark charge. The $\gamma\gamma \rightarrow q\bar{q}$ subprocess⁷ implies the production of two non-collinear, roughly coplanar high p_T (SPEAR-like) jets, with a cross section nearly flat in rapidity. Such "short jets" will be readily distinguishable from $e^+e^- \rightarrow q\bar{q}$ events due to missing visible energy, even without tagging the forward leptons. It is most useful to determine the ratio,

$$R_{\gamma\gamma} \equiv \frac{d\sigma(e^+e^- \rightarrow e^+e^-q\bar{q} + e^+e^- + \text{jets})}{d\sigma(e^+e^- \rightarrow e^+e^- \mu^+\mu^-)} \quad (10.1)$$

since experimental uncertainties due to tagging efficiency tend to cancel.⁸ In QCD, with 3-colors, one predicts²⁻⁴

$$R_{\gamma\gamma} = 3 \sum_{q=u,d,s,c,\dots} e_q^4 \left(1 + O\left[\frac{\alpha_s(p_T^2)}{\pi}\right] \right) \quad (10.2)$$

where p_T is the total transverse momentum of the jet (or muon) and $\alpha_s(Q^2) \rightarrow 4\pi/(\beta \log Q^2/\Lambda^2)$, $\beta = 11 - 2/3n_f$ for n_f flavors. Measurements of the two-jet cross section and $R_{\gamma\gamma}$ will directly test the scaling of the quark propagator p^{-1} at large momentum transfer, check the color factor and the quark fractional charge. The QCD radiative corrections are expected to depend on the jet production angle and acceptance. Such corrections are of order $\alpha_s(p_T^2)$ since there are neither infrared singularities in the inclusive cross section, nor quark mass singularities

at large p_T which could give compensating logarithmic factors. The onset of charm and other quark thresholds can be studied once again from the perspective of $\gamma\gamma$ -induced processes. The cross section for the production of jets with total hadronic transverse momentum ($p_T > p_{Tmin}$) from the $\gamma\gamma + q\bar{q}$ subprocess alone can be estimated from the convenient formula,^{2,4}

$$\begin{aligned} \sigma_{e^+e^- \rightarrow e^+e^- \text{ Jet} + X} (s, p_T^{\text{jet}} > p_{Tmin}) &\equiv R_{\gamma\gamma} \sigma_{e^+e^- \rightarrow e^+e^- u^+u^-} (s, p_T^{u^+} > p_{Tmin}) \\ &\approx R_{\gamma\gamma} \frac{32\pi\alpha^2}{3} \left(\frac{\alpha}{2\pi} \log \frac{s}{m_e^2} \right)^2 \frac{\left(\log \frac{s}{p_{Tmin}^2} - \frac{19}{6} \right)}{p_{Tmin}^2} \\ &\approx \frac{0.5 \text{ nb GeV}^2}{p_{Tmin}^2} \quad \text{at} \quad \sqrt{s} = 30 \text{ GeV} \quad . \quad (10.3) \end{aligned}$$

where we have taken $R_{\gamma\gamma} = 3 \sum_q e_q^4 = 34/27$ above the charm threshold. For $p_{Tmin} = 4 \text{ GeV}$, $\sqrt{s} = 30 \text{ GeV}$, this is equivalent to 0.3 of unit of R ; i.e., 0.3 times the $e^+e^- \rightarrow \mu^+\mu^-$ rate. We note that at $\sqrt{s} = 200 \text{ GeV}$, the cross section from the $e^+e^- \rightarrow e^+e^- q\bar{q}$ subprocess with $p_{Tmin} = 10 \text{ GeV}$ is 0.02 nb, i.e., about 9 units of R ! At such energies e^+e^- colliding beam machines are more nearly laboratories for $\gamma\gamma$ scattering than they are for e^+e^- annihilation! A useful graph⁴ of the increase in R from the $\gamma\gamma + q\bar{q}$ process for various $x_{Tmin} = 2p_{Tmin}/\sqrt{s}$ is shown in Fig. 10.2. The $\log s/p_{Tmin}^2 \sim 19/6$ in Eq. (10.3) arises from integration over the nearly flat rapidity distribution of the $\gamma\gamma$ system. The final state in high p_T $\gamma\gamma + q\bar{q}$ events in the $\gamma\gamma$ center-of-mass should be similar in multiplicity and other hadronic properties as $e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q}$, although

Fig. 10.2. The contribution to R from $\gamma\gamma + e^+e^-$ two jet processes at $\sqrt{s} = 30$ and 140 GeV (from Ref. 4).

$u\bar{u}$ and $c\bar{c}$ events should be enhanced relative to $d\bar{d}$ and $s\bar{s}$ due to the e_q^4 dependence. Monte Carlo studies of SPEAR events at $s = 4p_T^2$ distributed uniformly in rapidity would be useful in order to learn how to identify and trigger $\gamma\gamma \rightarrow q\bar{q}$ events.

Although the above prediction for $R_{\gamma\gamma}$ is one of the most straightforward consequences of perturbative QCD, from the perspective of photon physics of 10 years ago, the occurrence of events with the structure $\gamma\gamma \rightarrow \text{jet} + \text{jet}$ at high p_T could only be regarded as revolutionary. From the VMD standpoint, a real photon acts essentially as a sum of vector mesons; however, it is difficult to imagine an inelastic collision of two hadrons producing two large p_T jets without hadronic energy remaining in the beam direction!

On the other hand, if the $\gamma\gamma \rightarrow \text{two jet}$ events are not seen at close to the predicted magnitude with an approximately scale invariant cross section, then it would be hard to understand how the perturbative structure of QCD could be applicable to hadronic physics. In particular, unless the pointlike couplings of real photons to quarks are confirmed, then the analogous predictions for perturbative high p_T processes, involving gluons such as $gg \rightarrow q\bar{q}$ are probably meaningless.

In fact, preliminary results from the Pluto group at PETRA give indications that high p_T jet events with a single electron tag do exist. "Triplexity" analyses seem to be well suited to finding the $q\bar{q}$ jet axes and total momenta.⁹

In addition to $\gamma\gamma \rightarrow q\bar{q}$ one also expects gluon jet production $\gamma\gamma \rightarrow gg$ at order $\alpha_s^2(p_T^2)$ via a quark loop box diagram.¹⁰ Calculations predict the $gg/q\bar{q}$ ratio should be of order 20%.

Multi-jet processes and the photon structure function

In addition to the two-jet processes, QCD also predicts 3- and 4-jet events^{2,3,4} from subprocesses such as $\gamma q \rightarrow gq$ (3 jet production where one photon interacts with the quark constituent of the other photon) as well as the conventional high p_T QCD subprocesses $qq \rightarrow qq$ and $q\bar{q} \rightarrow gg$ (which lead to jets down the beam direction plus jets at large p_T). The structure of these events are very similar to that for hadron-hadron collisions. The cross section for $E d\sigma/d^3p_j$ ($\gamma\gamma \rightarrow \text{jet} + X$ or $ee \rightarrow ee \text{ jet} + X$) can be computed in the standard way from the hard scattering expansion ($\hat{s} = x_a x_b s$, etc.)

$$E \frac{d\sigma}{d^3p} (AB \rightarrow CX) \approx \sum_{abd} \int_0^1 dx_a \int_0^1 dx_b G_{a/A}(x_a) G_{b/B}(x_b) \frac{d\sigma}{d\hat{t}} (ab \rightarrow cd) \Big|_{\hat{s}, \hat{t}, \hat{u}} \frac{\hat{s}}{\pi} \delta(\hat{s} + \hat{t} + \hat{u}) \quad (10.4)$$

where the hard scattering occurs in $ab \rightarrow cd$ and the fragmentation function $G_{a/A}(x_a)$ gives the probability of finding constituent a with light-cone fraction $x_a = (p_a^0 + p_a^3)/(p_A^0 + p_A^3)$. In general, $G_{a/A}$ has a scale-breaking dependence on $\log W^2$ which arises from the constituent transverse momentum integration when gluon bremsstrahlung or pair production is involved.

However, there is an extraordinary difference between photon- and hadron-induced processes. In the case of proton-induced reactions, $G_{q/p}(x, Q^2)$ is determined from experiment, especially deep inelastic lepton scattering. In the case of the photon, the $G_{q/\gamma}$ structure function required in Eq. (10.4) has a perturbative component which can be predicted from first principles in QCD. This component, as first computed by Witten,¹¹ has the asymptotic form at large probe momentum Q^2

$$G_{q/\gamma}(x, Q^2) \Rightarrow \frac{\alpha}{\alpha_s(Q^2)} f(x) + O(\alpha^2) \quad (10.5)$$

i.e., aside from an overall logarithmic factor, the $\gamma \rightarrow q$ distribution Bjorken scales; $f(x)$ is a known, calculable function. Unlike the proton structure function which contracts to $x=0$ at infinite probe momentum $Q^2 \rightarrow \infty$, this component of the photon structure function increases as $\log Q^2$ independent of x . This striking fact is of course due to the direct $\gamma \rightarrow q\bar{q}$ perturbative component in the photon wave function. (The apparent violation of momentum conservation when $\alpha_s(Q^2) < \alpha$ should be cured when higher order terms in α are taken into account.) In addition to the perturbative component, one also expects a nominal hadronic component due to intermediate vector meson states.

Returning to the high p_T jet cross sections, we note the following striking fact: in each contribution to the four-jet cross section the two factors of $\alpha_s(p_T^2)$ from the subprocess cross section, e.g.,

$$\frac{d\sigma}{dt}(qq + q\bar{q}) \sim \frac{4\pi(\alpha_s(t))^2}{t^2} \quad (10.6)$$

(see Fig. 10.3a) actually cancel (in the asymptotic limit) the two inverse powers of $\alpha_s(p_T^2)$ from the two $G_{q/\gamma}(x, p_T^2)$ structure functions.² Similarly the single power of $\alpha_s(p_T^2)$ in $d\sigma/dt(\gamma q + q\bar{q})$ cancels the single inverse power of $\alpha_s(p_T^2)$ structure function in the 3-jet cross section (see Fig. 10.3b). Thus miraculously all of these jet trigger cross sections obey exact Bjorken scaling

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Fig. 10.3. Contributions from QCD subprocesses to (a) 4-jet and (b) 3-jet final states.

$$E \frac{d\sigma}{d^3p} (\gamma\gamma \rightarrow \text{Jet} + X) \xrightarrow{p_T^2 \rightarrow \infty} \frac{1}{p_T} f(x_T, \theta_{cm}) \quad (10.7)$$

When the leading QCD perturbative corrections to all orders are taken into account. Furthermore, the asymptotic cross sections are even independent of $\alpha_s(p_T^2)$. The asymptotic prediction thus has essentially zero parameters.

To leading order in $\alpha_s(Q^2)$ the $G_{q/\gamma}$ structure function of the photon can be obtained via the convolution of the $G_{q/q}$ distribution with the Born term for the $\gamma \rightarrow q\bar{q}$ coupling (as schematically indicate in Fig. 10.4). In terms of the evolution equations, the direct $\gamma \rightarrow q\bar{q}$ coupling provides a driving term:^{11,13}

$$\begin{aligned} \frac{\partial}{\partial \log Q^2} G_{q/\gamma}(x, Q^2) &= \frac{\alpha}{2\pi} \frac{q_{em}}{q} [x^2 + (1-x)^2] \\ &+ \frac{\alpha_s(Q^2)}{2\pi} \int \frac{dy}{y} P_{q/q}(x/y) G_{q/\gamma}(y, Q^2), \end{aligned} \quad (10.8)$$

Taking moments, then leads to the the anti-scaling form (10.5). At large Q^2 and $x \sim 1$ one finds¹⁴

$$G_{q/\gamma}(x, Q^2) \underset{x \rightarrow 1}{\sim} \frac{3}{2\pi} e_q^2 \frac{\alpha}{\alpha_s(Q^2)} \frac{4}{8 - (3/4\gamma_E)C_F + 4C_F \log \frac{1}{1-x}}, \quad (10.9)$$

much flatter than the power-law-damped meson structure functions. Here $C_F = 4/3$, $\beta = 11 - 2/3 n_F$, and $\gamma_E = 0.577...$ is Euler's constant. An illustration of the photon structure function is shown in Fig. 10.5. Corrections to the next order in $\alpha_s(Q^2)$ have also been computed by Buras and Bardeen;¹⁵ the main result is to increase the fall off at x close to 1. Measurements of the photon structure function, $F_{2\gamma}(x, Q^2) =$

Fig. 10.4. Representation of the QCD photon structure function in deep inelastic scattering on a photon target. Real and virtual gluon corrections to all orders are included in the analytic results.

Fig. 10.5. The valence photon structure function $G_{q/\gamma}(x)$ as calculated in (a) Born approximation, (b) to all orders in QCD, and (c) the $x \rightarrow 1$ limit (Eq. (10.9)). An overall factor proportional to $\log Q^2/\Lambda^2$ is factored out (from Ref. 2).

$\sum_q e_q^2 x G_{q/\gamma}(x, Q^2)$ and the σ_L/σ_T ratio will clearly provide one of the most important tests of perturbative QCD. Detailed calculations are given in Ref. 11.

Figure 10.6 illustrates the magnitude of the QCD 2, 3, and 4 jet cross section at an e^+e^- center of mass energy of 30 GeV, compared to a simple vector meson dominance estimate.² The contrast between the scale-breaking hadron-induced jet production cross sections with scale-invariant p_T^{-4} two-photon induced jets is remarkable. In addition, one can estimate the single particle large p_T production cross sections for $\gamma\gamma \rightarrow \pi X$, etc. expected from QCD leading twist and high twist sub-processes such as $\gamma q \rightarrow \pi q$. Again, the analysis is much simpler and cleaner than the corresponding hadron induced reactions; measurements will lead to important constraints on the theory and the parametrization of the high twist terms p_T^{-6} which are expected to dominant at moderate p_T .

It also may be possible to measure two photon reactions with each of the initial photons virtual. As shown by Chase,¹⁶ the $\gamma^* + \gamma^* \rightarrow$ Jet+Jet cross section and angular distribution can be predicted using perturbative QCD for the far-off-shell q_1^2, q_2^2 region with $s = (q_1 + q_2)^2$ fixed. The calculation to leading order in $\alpha_s(Q^2)$ involves the evolution equation¹⁷ which determines the meson wave function at short distances.

Exclusive channels in two-photon reactions

Another very important area for testing QCD are large momentum exclusive $\gamma\gamma$ reactions. For example, the same techniques used to compute the pion form factor¹⁷ can also be used to determine the large Q^2 $\gamma \rightarrow \pi^0$ transition form factor. We find, to leading order in $\alpha_s(Q^2)$,^{17,18}

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Fig. 10.6. QCD (and VMD) contributions to the $e^+e^- \rightarrow e^+e^- \text{ Jet } X$. The 4-jet cross section includes the contributions from $qq \rightarrow qq$, $q\bar{q} \rightarrow q\bar{q}$, and $q\bar{q} \rightarrow gg$ (from Ref. 2).

$$\begin{aligned}
 F_{\pi\gamma}(Q^2) &= \frac{2(e_u^2 - e_d^2)\sqrt{n_c}}{Q^2} \sum_{n=0}^{\infty} a_n \left(\log \frac{Q^2}{\Lambda^2}\right)^{-\gamma_n} \\
 &= \frac{1}{Q^2} 2 \left[\frac{Q^2 F_{\pi}(Q^2)}{16\pi \alpha_B(Q^2)} \right]^{1/2} \\
 &\rightarrow 2 \frac{f_{\pi}}{Q^2} \quad \text{as} \quad Q^2 \rightarrow \infty
 \end{aligned} \tag{10.10}$$

where the $\gamma^* \gamma \pi^0$ vertex is defined as $i e^2 F_{\pi\gamma}(Q^2) \epsilon_{\mu\nu\sigma\rho} p_{\pi}^{\nu} q^{\rho} \epsilon^{\sigma}$.

The two-photon process will also allow a detailed test of the application of perturbative QCD to large angle exclusive scattering. The processes $\gamma\gamma \rightarrow \pi^+ \pi^-$ and $\gamma\gamma \rightarrow K^+ K^-$ at fixed θ_{cm} and large $s_{\gamma\gamma}$ (with nearly on-mass-shell photon) are the simplest non-trivial, 2-body scattering reactions which can be computed in QCD. The Landshoff-type pinch singularity contributions are power-law suppressed compared to the leading hard scattering amplitudes. One thus predicts the fixed angle scaling behavior¹⁹

$$R_{\gamma\gamma \rightarrow \pi^+ \pi^-} = \frac{\frac{d\sigma}{dt}(\gamma\gamma \rightarrow \pi^+ \pi^-)}{\frac{d\sigma}{dt}(\gamma\gamma \rightarrow \mu^+ \mu^-)} \approx F_{\pi}^2(s) f(\theta_{cm}) \tag{10.11}$$

The absolute normalization and angular dependence of the $\pi^+ \pi^-$ and $K^+ K^-$ cross sections also are predictable in QCD. We also note that the fixed angle $\gamma^* \gamma \rightarrow \pi^+ \pi^-$ QCD amplitude has negligible dependence²⁰ in q^2 if $s \ll |q^2|$, in strong contrast to what would be expected by vector meson dominance.²⁰

In summary, it becomes evident that two photon collisions can provide a clean and elegant testing ground for perturbative quantum

chromodynamics. The occurrence of $\gamma\gamma$ reactions at an experimentally observable level implies that the entire range of hadronic physics which can be studied, for example, at the CERN-ISR can also be studied in parallel in e^+e^- machines. Although low p_T $\gamma\gamma$ reactions should strongly resemble meson-meson collisions, the elementary field nature of the photon implies dramatic differences at large p_T . We have especially noted the sharp contrasts between hadron-and photon-included reactions due to the photon's pointlike coupling to the quark current and the ability of a photon to give nearly all of its momentum to a quark. The large momentum transfer region can be a crucial testing ground for QCD since not only are a number of new subprocesses predicted ($\gamma\gamma \rightarrow q\bar{q}$, $\gamma q \rightarrow gq$, $\gamma q \rightarrow Mq$, deep inelastic scattering on a photon target) with essentially with no free parameters, but most important, one can make predictions for a major component of the photon structure function directly from QCD. We also note that there are open questions in hadron-hadron collisions, e.g., whether non-perturbative effects (instantons, wee parton interactions) are important for large p_T reactions. Such effects are presumably absent for the perturbative, pointlike interactions of the photon. We also note that the interplay between vector-meson-dominance and pointlike contributions to the hadronic interactions of photon is not completely understood in QCD, and $\gamma\gamma$ processes may illuminate these questions.

11. Conclusions

As we have discussed in these lectures, large momentum transfer exclusive processes and the short-distance structure of hadronic wave functions can now be rigorously analyzed within the context of perturbative QCD. The basic framework (see Chapter 7) has been the development of evolution equations which determine the behavior of Fock components of the hadronic wave function at small distances from their structure at large distance. The eigensolutions of the evolution equations determine the form of the meson and nucleon wave functions and are directly related to terms in the operator product expansions of the wave functions; the eigenvalues then appear as exponents in anomalous logarithmic corrections to the nominal (dimensional-counting) power-law fall-off of exclusive amplitudes at large momentum transfer. Thus the dimensional counting rules for form factors, photoproduction, and hadron-hadron scattering at large momentum transfer become rigorous predictions of QCD up to calculable powers of the running coupling constant $\alpha_s(Q^2)$ or $\ln g^{-1} Q^2/\Lambda^2$. The power-law behavior of the wave functions at large $k_{\perp}$ is also explicitly determined.

In general, the power-laws predicted by QCD for exclusive reactions appear to be consistent with data. It should be emphasized that the specific integral power Q^{-4} predicted for the baryon form factors G_M at large Q^2 reflects the essential scale-invariance of the off-shell quark and gluon propagators - i.e., the internal quark-quark interactions, as well as the prediction that the minimal spin- $\frac{1}{2}$ color singlet wave function contains 3 quarks. Thus both the dynamics and symmetry properties of QCD are directly tested in a non-trivial way by the form factor scaling

behavior. The spin-dependence of quark-quark interactions at short-distances can be tested by studying the helicity-dependence of elastic and transition form factors - i.e., the QCD "selection rules" discussed in Chapter 7.

It is certainly of crucial importance that experiments test the logarithmic modifications to large momentum transfer exclusive process amplitudes predicted by QCD. Definitive tests will require computation of the order $\alpha_s(Q^2)$ corrections; at the present time there is no convincing signal for the logarithmic anomalous dimension corrections. This seems to be strong evidence that $\alpha_s(Q^2)$ is slowly varying for $Q^2 \gtrsim 2 \text{ GeV}^2$ and thus that the fundamental scale parameter Λ is probably quite small.

We have also emphasized that the behavior of the quark and gluon distributions in hadrons for $x \rightarrow 1$ provides a rigorous test of the short-distance dynamical structure of hadrons predicted by perturbative QCD. Thus, in addition to the usual $\log Q^2$ evolution of the structure functions, the theory also predicts the shapes of the structure functions themselves at large x . Perhaps the most critical experimental check of QCD in this area is the test of the prediction (modulo logarithm corrections) $G_{q/M}(x, Q^2) \sim (1-x)^2 + C/Q^2$ for the form of the meson structure function (and fragmentation functions) at large x . [See Chapter 9.] The dominance of longitudinal photon polarization observed at large x_F for pion-induced lepton pairs is an important first-step towards confirmation of the QCD prediction. It is crucial to check that this form represents the meson structure function, and that the $d\sigma/didz$ ($e^+e^- \rightarrow MX$) cross section becomes pure longitudinal ($\sin^2\theta$) at large z .

Throughout these lectures we have emphasized that QCD predicts two important sources of Q^2 -variation of the structure functions

- (i) The familiar (target-independent) $\log Q^2$ (or more accurately, $\log W^2$) evolution of the quark and gluon distribution functions and their moments.
- (ii) The power-law corrections ("high twist" terms) derived from the dynamics of the multiquark hadronic wave functions.

Despite their completely different physical origin, the scale-breaking effects of (i) and (ii) on the observed structure functions are sufficiently similar that they can be confused in phenomenological analyses. If particular, if one only allows for logarithmic QCD scale-breaking in the fits to structure functions, then the scale constant Λ^2 will inevitably be chosen to simulate the value of Q^2 where power-law scale-breaking occurs. The actual value of Λ^2 could be quite different.

The high twist contributions to the structure functions arise from the same T_H hard scattering amplitudes which determine the $x \rightarrow 1$ behavior; their form and normalization is controlled by the same dynamics which determines the asymptotic form factors. For example, in the case of the nucleon structure function at large x and large Q^2 , $F_2(x, Q^2)$ has the power-law structure (see Chapter 9)

$$F_2(x, Q^2) \approx A(1-x)^3 + B(1-x)^2 \frac{\nu^2}{Q^2} + C(1-x) \frac{\nu^4}{Q^4} + D(1-x)^0 \frac{\nu^6}{Q^6} + E \frac{1}{1-x} \frac{\nu^8}{Q^8} \quad (11.1)$$

corresponding to terms which represent a kinematic trade-off between powers of $(1-x)$ and powers of $(Q^2)^{-1}$, and terms where the lepton recoils against more than one quark. In addition, each of these terms have

logarithmic corrections from explicit powers of $\alpha_g(Q^2)$ or $\alpha_g(m^2/(1-x))$ in T_H , the evolution of the hadronic wave function, and (except for exclusive channels), the Q^2 -evolution due to gluon radiation from the struck lines. The scale of these terms is set by $\nu^2 \sim m_0^2$ since this is the scale of the elastic contribution. Terms of order $(1-x)^2/Q^4$ from diquark recoil contribute to the longitudinal cross section and could account in part for the large values of σ_L/σ_T measured in the low Q^2 SLAC domain. We emphasize that if elastic and resonance contributions - which are inherently high twist terms - are included in fits to structure functions or their moments, then the contributions A-E must all be included in the QCD parametrization.

We have also shown in these lectures that because of the kinematic limit on gluon radiation, the evolution of the observed structure functions is controlled by W^2 rather than Q^2 . [See Chapter 4.] This accounts in a simply way for the $O[\alpha_g(Q^2) \log^2 N]$ correction terms which appear in the operator product analyses for moments at large N , and makes the calculation of higher order terms of this type tractable. We have also given extended evolution equations which account for this kinematic constraint. It is clear that a careful analysis of the kinematic effects, together with the high twist contributions will be necessary before a definitive test of QCD and precise determination of α_g will be possible in deep inelastic scattering.

Another fascinating area of QCD is the question of the role of higher Fock states in the hadronic wave functions. By necessity, there are "intrinsic" gluon and $q\bar{q}$ sea contributions in the wave function which are not generated by the Q^2 -evolution equations. The higher Fock

components give higher powers of Q^2 or $(1-x)$ in form factors or structure functions, but may play an important role in momentum sum rules, heavy flavor production, fast forward reactions, etc. Many of the techniques discussed in these lectures can be applied to these components.

Even without direct confrontation with its enticing anomalous logarithmic structure there are several first rank predictions of the theory which are essentially unique features of QCD:

- (a) The prediction that R asymptotically approaches the sum of squares of charges can only occur in a renormalizable theory which is asymptotically free.
- (b) The basic Born structure of QCD is tested in considerable detail by exclusive processes. In addition, large p_T jet production $H_1 + H_2 + \text{Jet} + X$ gives an important check of the scaling and angular behavior of the $2 \rightarrow 2$ subprocess amplitudes $qq \rightarrow qq$, $gq \rightarrow gq$, $gg \rightarrow gg$, etc. As yet there has been no real check of the QCD scaling predictions in these reactions, although there is strong evidence for final state jet production.
- (c) The 3-jet process of e^+e^- annihilation corresponding to the subprocess $e^+e^- \rightarrow q\bar{q}g$ in principle yields the cleanest and most straightforward determination of $\alpha_s(Q^2)$ and the gluon-quark coupling. Recent measurements of 3-jet events at PETRA appear to be in agreement with these expectations although definite value of $\alpha_s(Q^2)$ will require high statistics, and careful attention and control of the quark and gluon jet structure and separation of higher jet processes.

- (d) Hard-photon processes involving large transfer of momentum, such as large- p_T direct-photon production, deep inelastic Compton scattering; two photon processes $\gamma\gamma \rightarrow \text{Jet} + X$, and the photon structure functions all lead to critical tests of QCD. Photons, as well as gluons, couple directly to quarks in the underlying hard processes at large momentum transfer. The most remarkable feature is that the photon structure function is inversely proportional to $\alpha_s(Q^2)$. [See Chapter 10.]

Thus, in summary, the testing ground of perturbative QCD where rigorous, definite tests of the theory can be made can now be extended throughout the whole domain of large momentum transfer exclusive and inclusive hadronic and leptonic reactions. At this point the study of these reactions is just beginning, but there is a real possibility that an exact microscopic description of hadron dynamics is at hand. The next step will be to unify the short and long distance physics and make contact with hadronic spectroscopy, low momentum transfer reactions, and non-perturbative physics.

APPENDIX A

QCD Parametrizations of Inclusive Reactions

In this appendix we give a simplified analytic approach to large transverse momentum reactions which incorporates logarithmic corrections from the evolution equations for the structure functions and hadronic wave functions, as well as the contribution of high twist, power-law suppressed QCD subprocesses. The main goal will be to obtain simple formulae²¹ for the inclusive cross section at large transverse momentum and the edge of phase space $x_R = p/p_{\max} \rightarrow 1$. Only the leading terms in order $\alpha_s(p_T^2)$ will be retained.

Let us consider a given reaction $A+B \rightarrow C+X$ where C is detected at large CM angles with transverse momentum p_T and momentum fraction $x_R = |\vec{p}|/p_{\max}$. Each hadron A, B , and C is represented by its Fock state components Ψ_λ .

We then identify²² all hard scattering processes T_B ($a+b \rightarrow c+d$) which can contribute to the final state high p_T trigger by the scattering of incident constituents. The simplest subprocesses are $qq \rightarrow qq$, $qg \rightarrow qg$, $gg \rightarrow gg$, etc., but for some kinematic regions we should also consider "high twist" subprocesses such as $q\bar{q} \rightarrow M\bar{M}$, $Mq \rightarrow Mq$, $Bq \rightarrow Bq$, etc. As in exclusive scattering, the nominal power-law fall off of T_B in p_T^2 reflects the number (n_{active}) of fundamental fields forced to change directions. By definition every intermediate state associated with T_B and its radiative corrections is off shell by at least λ^2 ; otherwise its contribution is already included in Ψ_λ .

As $x_R \rightarrow 1$, the propagation of each constituent a, b, c (assuming $c \neq C$) requires the far-off-shell behavior of the Fock state wave functions for A, B, C . The leading power law behavior as $(1-x_R) \rightarrow 0$ is given by diagrams with the minimal number of gluons exchanged between the Fock state constituents which can transfer all of the hadronic momentum to the hard scattering subprocess. The evolution equations for the Fock state wave functions then gives anomalous logarithms $a_i [a_s(k_{x_i}^2)]^{Y_i}$ where $k_{x_i}^2 = (\vec{k}_{i1}^2 + \vec{m}_i^2)/(1-x_i)$, as in the structure function calculations. [See Chapters 7 and 9.] The asymptotic inclusive cross section in QCD then takes the factorized form ($\hat{s} = x_a x_b s$, etc.)²³

$$E \frac{d\sigma}{d^3p} (AB \rightarrow CX) = \sum_{ab \rightarrow cd} \int_0^1 dx_a G_{a/A}(x_a, \xi_a) \int_0^1 dx_b G_{b/B}(x_b, \xi_b) \int_0^1 \frac{dx_c}{x_c^2} G_{c/C}(x_c, \xi_c) \frac{d\hat{\sigma}}{dt} (ab \rightarrow cd) \frac{\hat{s}}{\pi} \delta(\hat{s} + \hat{t} + \hat{u}) \quad (A.1)$$

where

$$G_{a/A}(x_a, \xi_a) \underset{x_a \rightarrow 1}{\sim} (1-x_a)^{8_{\pi/A} + \xi_a(p_T^2, x_a)} \left[a_s(k_{x_a}^2) \right]^{\Gamma_{a/A}} \quad (A.2)$$

is the structure function for finding constituent a in A with light-cone momentum fraction x_a and

$$\xi_a = \frac{C_a}{\pi} \int_{Q_0^2}^{p_T^2(1-x_a)} \frac{d\vec{k}_1^2}{\vec{k}_1^2} a_s(\vec{k}_1^2) \quad (A.3)$$

controls the structure function evolution. (The upper limit approximately reflects the phase-space limits of the gluon transverse momenta: see Chapter 4 for the more precise formulae.) The C_a are the color $SU(3)$

charges:

$$C_g = \begin{cases} 0 & \text{color singlet} \\ 4/3 & 3 \text{ or } \bar{3} \\ 3 & \text{color octet} \end{cases} \quad (\text{A.4})$$

Note that there is no p_T^2 -evolution in the case of color singlet constituents inside hadrons. [This cancellation²⁴ of gluon bremsstrahlung corrections occurs as long as we exclude $x \sim 1$, low transverse momentum regions in the wave functions as in the baryon form factor calculation.] The power $g_{g/A}$ can be computed from the leading $T_H^{g/A}$ contribution to the structure functions as in Chapter 8; e.g., $g_{g/A} = 2$ for q/π , $= 3$ for parallel-helicity quarks in the nucleon. The powers are obtained simply from the order in α_s of the tree graph $T_H^{g/A}$ plus the exponent $\gamma_{g/A}$ representing the anomalous dimensions of the Fock state wave function.

The distribution function $G_{g/p}$ and $G_{q/p}$ can be computed by convoluting the g/q and q/g distribution with the $G_{q/p}$ distribution as dictated by the structure function evolution equations. In addition one expects "intrinsic" gluon and anti-quark components in the proton from the higher particle number Fock states in the hadron wave function.

Excluding pinch singularities, the leading hard scattering diagram $T_H^{a+b \rightarrow c+d}$ for each subprocess give cross sections of the form

$$\frac{d\sigma}{dt}(a+b \rightarrow c+d) \sim \left[\frac{\alpha_s(p_T^2)}{2} \right]^{n_{\text{active}} - 2} f_{ab+cd}(0_{\text{cm}}) \quad (\text{A.5})$$

where n_{active} is the total number of fields $n_a + n_b + n_c + n_d$ which exchange large transverse momenta. Calculations of the intrinsic distributions can

be summarized by the simple formula²⁵

$$G_{a/A}(x_a, \xi_a) \sim \frac{(1-x_a)^{2n_a-1+2|\Delta S|+\xi_a}}{x_a+1} \left[\alpha_s \left(\frac{m^2}{1-x_a} \right) \right]^{\Gamma_{a/A}} \quad (A.6)$$

where n_a is the number of "spectator" fields in the Fock state, and $\Delta S = |S_z^a - S_z^A|$ gives the suppression factor due to helicity mismatch.

The inclusive cross section for $A+B \rightarrow C+D$ corresponding to the $a+b \rightarrow c+d$ subprocess then has the form

$$E \frac{d\sigma}{d^3p} \sim \left[\frac{\alpha_s(p_T^2)}{2} \right]^{n_{\text{active}}-2} (1-x_T)^F \left[\alpha_s \left(\frac{m^2}{1-x_T} \right) \right]^{\Gamma} f(\theta_{cm}) \quad (A.7)$$

where $F = (g_{a/A} + \xi_a) + (g_{b/B} + \xi_b) + (g_{c/C} + \xi_c) + 2$ and $\Gamma = \Gamma_{a/A} + \Gamma_{b/B} + \Gamma_{c/C}$. In addition, if a, b, c or d is a color singlet composite system, then we also obtain anomalous logarithm factors from the evolution of the hadronic wave functions:

$$\prod_{i=a,b,c,d} \left| \sum_n a_n^{(i)} \left(\frac{\log p_T^2/\Lambda^2}{\log Q_0^2/\Lambda^2} \right)^{-\gamma_n^{(i)}/\beta} \right|^2 \quad (A.8)$$

In particular, the leading anomalous dimension γ_0 is zero for helicity zero mesons (see Chapter 7).

This result should be a useful guide to large angle inclusive cross section contributions to leading order in $\alpha_s(p_T^2)$. To this order the effect of transverse momentum integrations due to both gluon radiation and the fall-off of the hadronic wave function is already included. Note that a three jet amplitude such as $qq \rightarrow qqg$ contributes to both the $qq \rightarrow qq$ and $qg \rightarrow qg$ subprocesses depending on whether the extra

gluon's or quark's transverse momentum is integrated over. The region of phase space where all three jets emerge at large p_T in separate directions is of higher order in $\alpha_s(p_T^2)$. The off-shell kinematics eliminates the possibility of singular contributions from the integration over a gluon or quark pole.²²

APPENDIX B

Origin of the Nachtmann Variable

It is interesting to see how the kinematic Nachtmann²⁶ variable which is introduced formally in the operator product analyses enters naturally into the perturbative analysis of deep inelastic lepton scattering.

Let us look in more detail at the simple parton model diagram (Fig. B.1):

We choose a general Lorentz frame with

$$\begin{aligned} P &= (p^0, p_1, p^3) = \left(P + \frac{M^2}{4P}, \hat{0}_1, P - \frac{M^2}{4P} \right) \\ k &= \left(xP + \frac{k^2 + k_1^2}{4xP}, \hat{k}_1, xP - \frac{k^2 + k_1^2}{4xP} \right) \end{aligned} \quad (B.1)$$

where $x \equiv \frac{k^0 + k^3}{p^0 + p^3} = \frac{k^+}{p^+}$ is the light-cone fraction. The parameter $P = p^0 + p^3$ is arbitrary, reflecting boost invariance along the z-direction. The spectator system has timelike mass $(p-k)^2 = \tilde{m}^2$, which implies for the struck quark,

$$k^2 = x \left[M^2 - \frac{k_1^2 + \tilde{m}^2}{1-x} - \frac{k_1^2}{x} \right]. \quad (B.2)$$

For $x \rightarrow 1$, k^2 thus becomes infinitely spacelike. The choice of photon 4-momentum is still largely arbitrary. If we choose q along the 1-axis then

$$q = \left(-zP - \frac{q^2}{4zP}, \hat{0}_1, -zP + \frac{q^2}{4zP} \right). \quad (B.3)$$

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Fig. B.1. Parton model diagram for deep inelastic scattering corresponding to the parametrization of Eq. (B.1).

where the photon's "light-cone" variable is $z = -(q^0 + q^3)/(p^0 + p^3)$,

$$2q \cdot p = -zM^2 - \frac{q^2}{z} \quad (B.4)$$

Thus

$$z = -\frac{q^2}{2p \cdot q} \frac{2}{1 + \sqrt{1 - M^2 q^2 / (q \cdot p)^2}} \quad (B.5)$$

which is in fact the kinematic variable introduced by Nachtmann. The condition that the struck quark is on the mass shell, $(k+q)^2 = m^2$ implies

$$\underbrace{x \left[M^2 - \frac{\vec{k}_1^2 + m^2}{x} - \frac{\vec{k}_1^2 + \tilde{m}^2}{1-x} \right]}_{k^2 - m^2} - \underbrace{\frac{xq^2}{z} - z \left[M^2 - \frac{\vec{k}_1^2 + \tilde{m}^2}{1-x} \right]}_{2k \cdot q} + q^2 = 0 \quad (B.6)$$

which can be solved in closed form for x .

Notice that if we ignore that $\vec{k}_1^2 + m^2$ term, then $x \rightarrow z$, i.e., the Nachtmann variable would precisely specify the light-cone variable of the struck quark. In general we have

$$x = z \left[1 + \frac{\vec{k}_1^2 + m^2}{-q^2} + \dots \right] \quad (B.7)$$

which gives a significant corrections in the case of large transverse momentum partons of heavy quark production. Notice, however, that the corrections to the $x=z$ relation are independent of the target, spectator, and struck-quark masses. The leading QCD corrections arise from the region $\vec{k}_1^2 \ll W^2$ (see chapter 4).

The Nachtmann variable is kinematically more accurate than the Bjorken variable in that correctly incorporates target mass corrections.

The use of the Nachtmann variable cannot, however, justify including the elastic scattering contribution or resonance contributions when comparing with structure function moments computed using only the leading twist contribution. A discussion of high twist terms is given in Chapter 9.

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 g_0^2 r_{\gamma q \bar{q}}^2(k^2) d_F^4(k^2) r_{g q \bar{q}}^2(k^2) d_G(k^2) &= g_0^2 d_F^2(k^2) r_{g q \bar{q}}^2(k^2) d_G(k^2) \\
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