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Effect of Experimental Variables on Flash Thermal Diffusivity Data Analysis

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ABSTRACT

Flash thermal diffusivity data is usually analyzed with a thermal model which assumes axial heat conduction and uniform illumination of the flashed surface. For high accuracy data reduction, it becomes important to bound the errors caused by radial heat flow and by non-uniform laser beam profiles. These effects are examined analytically for a case in which the incident laser beam is confined to a radius smaller than the sample radius. The dependence of the output of an averaging detector on the magnitude of the radial heat transfer coefficient is presented and the linear dependence of radial and axial loss sensitivity coefficients is discussed. From this discussion, we conclude that inclusion of radial loss effects in analysis of the thermal response of multilayer structures is not important unless the radial loss factor is very large. Analytical results are presented for the temperature vs. time response of a two layer composite sample with interfacial thermal resistance and high thermal losses at the sample faces. The use of these results to reduce data for two multilayer samples is presented to show the utility of new data reduction techniques.

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INTRODUCTION

The flash or pulse method of measuring the thermal diffusivity of liquid and solid materials is now widely used, although commercial equipment for making this measurement is just beginning to become available. The method has been discussed extensively in the literature and reviewed relatively recently by Taylor[1] and Taylor and Maglic[2]. Despite the relative maturity of the technique, there are still questions about its accuracy and its sensitivity to departures of the actual experimental conditions from those postulated in the theory used to reduce the temperature vs. time data. The purposes of this paper are to reexamine the effects of some experimental variables on the accuracy of flash diffusivity measurements and to present examples of the use of the method in determination of the diffusivity or thermal contact resistance of one layer in a multilayered composite sample.

In a critical examination of the flash method, Taylor[3] identified three sources of non-measurement error: (1) finite pulse time effects, (2) heat losses or gains from the sample surfaces during the measurement, and (3) non-uniform heating by the incident laser pulse. Finite pulse time effects become important when the half rise time of the sample back-surface temperature, $t_{1/2}$, is significant relative to the pulse time of the laser, τ_p . Methods of correcting for finite pulse time effects are well documented and have been implemented in numerous calculations and computer programs[1,2,4]. The major uncertainty arises from imprecise knowledge of the laser pulse shape or from random shifts in the pulse shape from or pulse position from flash to flash. If $t_{1/2} > 2\tau_p$, the exact shape of the pulse is unimportant, and various approximate shapes can be used[4].

Heat losses are usually taken into account in an approximate way by assuming that a linear or Newton's law of cooling heat transfer condition applies at the sample surfaces[5]. A schematic diagram of a laser pulse measurement with a cylindrical sample is shown in Fig. 1. The axial and radial heat transfer coefficients, h_z and h_r , are presumed to characterize both radiative losses from the sample to the surrounding furnace structure as well as conductive losses to sample supports. In the radiative case, the usual $h \propto (T^4 - T_0^4)$ radiation law is linearized, using the assumption that the deviation of the sample temperature, T , from the ambient temperature, T_0 , satisfies the condition, $|T - T_0| \ll T_0$. In a recent study by Degiovanni, Sinicki, and Laurent, numerical heat transfer analysis techniques were used to show that the linear approximation of radiative losses did not introduce significant error (<1%), for any reasonable values of T_0 and maximum front surface temperature, T_{max} [6].

SAMPLE GEOMETRY FOR ANALYTICAL STUDIES

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Fig. 1 Sample geometry for flash thermal diffusivity measurements. The incident laser flux, $\Phi(\vec{r}_\perp, t)$, may vary in the plane of the sample face, as described by the in-plane vector $\vec{r}_\perp$. The heat transfer from the front and back face is described by a heat transfer coefficient h_z and the heat transfer from the radial boundary is described by a heat transfer coefficient h_r .

The effects produced by a non-uniform laser beam profile are harder to quantify. Non-uniformities can be caused not only by illumination variations over the sample surface, but also by some of the incident energy striking the sample support structure. It has been noted by ourselves and others[3] that significant errors can be introduced by laser misalignment even though the temperature vs. time data agree well with a theoretically calculated curve. As we shall discuss in the next section, beam non-uniformities over the sample surface will not cause errors if a detector is used which responds linearly to the average temperature of the sample back surface. Another potential source of error is misalignment of the detector viewing optics such that the detector is not focused solely on the sample back surface.

Beam Shape and Detector View Effects

Ideally, we desire an experimental configuration in which variations in incident beam intensity over the sample front surface do not produce significant variations in the experimental data. The cylindrical symmetry of the flash diffusivity sample, as shown in Fig. 1, plays an important role in minimizing the sensitivity to flux inhomogeneities. Beedam and Dalrymple[7] and McKay and Schriempf[8] have theoretically analyzed the

effects produced by radial beam inhomogeneities for the situation in which a point temperature detector such as a thermocouple is utilized. They find that significant errors can be introduced in the diffusivity determination unless information about the laser beam shape is used in the temperature calculation, even for cases in which the non-uniformity is moderate. Recently, Degiovanni et al.[6] showed that when a detector is used which measures the average surface temperature, the data are invariant to radial beam variations. The only assumption necessary to prove this result from the heat conduction equation and linear boundary conditions is that of zero heat flux at the radial boundary. As we shall show, this is not a critical assumption for usual radial heat transfer conditions, and thus the Degiovanni result can be considered quite general in scope for incident beams with radial variations and averaging detectors.

It is easy to show from the linear heat conduction equation that azimuthal as well as radial variations in the incident beam do not effect the results if a temperature averaging detector is used. Using the geometry shown in Fig. 1, we consider the time and position dependent temperature, $T(\vec{r}, t) = T(r, \theta, z, t)$ in the sample after the arrival of the laser flash. It can be shown that the temperature produced by a laser pulse with a finite pulse length and specified pulse shape can be derived from the response to an instantaneous pulse[4,5], so we shall consider the laser pulse to be a zero time-width impulse. The mathematical form for the incident pulse is thus taken as,

$$\Phi(\vec{r}_\perp, t) = \Phi_0 \bar{\tau} \delta(t) f(\vec{r}_\perp), \quad (1)$$

where Φ_0 —average pulse energy flux, $\bar{\tau}$ —pulse length, and the product $\Phi_0 \bar{\tau}$ = pulse energy absorbed by the sample. $\delta(t)$ = unit impulse or Dirac delta function, and $f(\vec{r}_\perp)$ is the pulse energy distribution function over the front surface of the sample as characterized by the in-plane radius vector, $\vec{r}_\perp$. $f(\vec{r}_\perp)$ is assumed to be normalized over the sample surface, $\int f(\vec{r}_\perp) d^2 \vec{r}_\perp = 1$, so that the pulse energy is independent of f . The solution for the temperature can then be written as,

$$T(r, \theta, z, t) = T_\infty \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{k=0}^{\infty} \left[\frac{\exp[-t/\tau_{mn}] R_k(y_m r/a) Z(x_n z/l) \Theta(k\theta)}{N_R(y_m, k) N_Z(x_n) N_\Theta(k)} \right] \quad (2)$$

$$* \int_{\pi}^{\pi} d\theta' \int_0^a r' dr' \left\{ f(\vec{r}'_\perp) \Theta(k\theta') R_k(y_m r'/a) \right\}$$

The various expressions in Eq.(2) are fully defined in Appendix A. R , Z , and Θ are the radial, axial, and azimuthal eigenfunctions, respectively and the corresponding N functions in the denominator of Eq.(2) are the normalizing factors for these functions. The effect of the incident pulse

intensity function is contained in the double integral on the second line of Eq.(2).

It is easy to see that any θ variation in $f(\vec{r})$ does not affect the form of the back-surface temperature if an averaging detector is used. The average back-surface temperature is given by,

$$\langle T(t) \rangle_{BS} = (1/\pi a^2) \int_{-\pi}^{\pi} d\theta \int_0^a r dr T(r, \theta, l, t) \quad (3)$$

Since $\theta(\theta)$ in Eq.(2) is $\cos(k\theta)$ (see App. A), the θ integral in Eq.(3) is zero for $k \neq 0$, so only the $k=0$ term in Eq.(2) is left after the average is performed. Thus, any angular dependence in the intensity function, f , is washed out when the average temperature is measured. This conclusion depends critically on the assumption that the detector centerline is coincident with the sample centerline.

Any effect of pulse non-uniformity must arise from the radial integral in Eq.(2). As stated above, Degiovanni et al.[6] show that radial averaging removes these inhomogenieties if the radial heat flux on the sample circumference, $k\partial T(a, \theta, z, t)/\partial r|_{r=a} = 0$. If this quantity is non-zero, it can be shown that the Degiovanni result is still valid if radial losses are small but that it becomes increasingly less valid as h_r increases. Rather than presenting detailed analysis to support this conclusion, we show results of a calculation for the back-face temperature vs. reduced temperature, $t_r = \alpha t / l^2$, for a one layer sample with various degrees of restriction of the incident beam into a radius $a_l \leq a$. Fig. 2(a) shows the average back-face temperature relative to the maximum value of that temperature for a sample with an axial loss parameter, $b_z = 0.2$, a thickness/radius ratio of $1/3$, and equal radial and axial heat transfer coefficients, $h_r = h_z$, with illumination to sample radius ratios, $a_l/a = 0.2, .4, .6, .8$, and 1.0 . The laser beam has a constant intensity for $r < a_l$ and zero intensity for $r > a_l$. We see that restricting the laser beam to a radius smaller than that of the sample has only a small effect on the temperature history, in agreement with the Degiovanni prediction. In addition, the leading edge data ($t_r < 0.4$) are practically invariant to the illumination radius, indicating that leading edge data reduction methods are not influenced much by radial asymmetries. In contrast to this behavior, Fig. 2(b) shows the result of a calculation for a similar sample but with enhanced radial heat transfer such that $h_r/h_z = 10$. In this case, fairly substantial changes occur in the average back-face temperature.

Fig. 2(a) Theoretical temperature excursions for a homogeneous or single layer sample with the incident laser flux constant in magnitude but confined to a radius $a_l \leq a$, where a —sample radius. For this calculation, $h_r = h_z$, $b_z = h_z \ell / k = 0.2$, and $\ell / a = 0.3$.

(b) A calculation similar to that in Fig. 2(a) except that $h_r = 10h_z$.

Even in this case, however, data analysis techniques which concentrate on early time data should still yield relatively accurate results.

It is interesting to note that the effect produced by an aperture in the incident beam is the same as the effect produced when a similar aperture with a radius $a_d \leq a$ is placed in the field of view of the back-face temperature detector. From Eqs. (2) and (3), it can be shown that with the appropriate averages, the radial or m sum in Eq.(2) becomes,

$$\sum_m \rightarrow \sum_m \left[\frac{4a^2 J_1(y_m a_l / a) J_1(y_m a_d / a)}{a_l a_d J_0^2(y_m) (y_m^2 + b_r^2)} \right] \quad (4)$$

shows the complete symmetry between the detector and incident laser radii, a_d and a_l . In Eq.(4) J_0 and J_1 are ordinary Bessel functions.

A remaining question relates to the variation of the back face temperature with variations in the radial losses. Although the radial and axial heat transfer coefficients, h_r and h_z , enter separately in the theory, Eq.(2), it is not necessary to know both of them, and in fact, they cannot be determined separately in a flash diffusivity experiment. The insensitivity of the temperature to variations in h_r was first pointed out by Cape and Lehman[9]. These authors used various approximations to show that the radial and axial loss parameters, b_z and b_r , entered the theory as the linear combination, $b = b_z + (\ell/a)^2 b_r$. These loss factors are related to h_r and h_z through the relations, $b_z = h_z \ell / k$ and $b_r = h_r a / k$, respectively, where k —thermal conductivity of the sample. Many calculations indicate that b_r and b_z are linearly dependent parameters; variations in either affect the average back-face temperature in exactly the same way. This is shown in Figs. 3(a) and 3(b) for calculations associated with a two-layer sample composed of a front layer of graphite and a back layer of CP explosive. Fig 3(a) shows the normalized sensitivity coefficients $\partial T / \partial \beta$ [10], vs time for various parameters $\beta = \alpha_2$, T_∞ , b_2 , and b_r , where b_2 is the loss parameter of the second layer (see App. B). These coefficients are useful for showing how a given fractional parameter change, $\delta \beta / \beta$, affects the temperature. Similar results have been shown by Koski[4] for a single layer sample. The similar shapes of the sensitivity curves for b_2 and b_r in Fig. 3(a) indicate that these parameters are close to being linearly dependent. This means that the parameters cannot be determined independently in a least-squares data fitting process[10]. The other parameters, α_2 and T_∞ , have associated sensitivity coefficients which are not linearly dependent with each other or with b_2 . Fig. 3(b) shows the results of a temperature calculation which demonstrates the linear dependence of b_r and b_2 . The solid curve is for $b_2 = 1.33$ and $h_r / h_z = 1$, while the dashed curve has $b_2 = 1.20$ and $h_r / h_z = 10$. It is evident that both sets of parameters yield nearly identical results.

The above results indicate that the accuracy of flash diffusivity property measurements should not be critically affected by uncertainties in the laser intensity profile unless the radial losses are very high. It is also evident that uncertainties about the magnitude of the radial loss factor should not affect the value of a thermal diffusivity or contact resistance determined by least-squares data fitting techniques. A potential cause of experimental results being at variance with these predictions is the deviation of an actual infrared detector from an ideal thermal averager. If the sample is thin and the pulse asymmetry is large,

Fig. 3(a) Sensitivity coefficients for a two layer sample described in the text. The front layer is graphite with $\alpha_s=0.89$ cm²/s and $l_1=0.1$ cm. The radial loss sensitivity coefficient has been multiplied by a factor of 10 for ease in comparison with the second layer axial loss coefficient.

(b) Transient calculation using Fig. 3(a) parameters. The solid curve corresponds to the Fig 3(a) conditions, while the dashed curve is for $h_r/h_z=10$ and $b_2=h_z l_2/k_2=1.20$.

then there could be a significant difference in temperature across the sample back surface, especially early in the transient. If this temperature difference is large enough to produce a nonlinear response in the detector, the hottest parts of the sample will be too heavily weighted in the measurement, possibly resulting in significant errors. Thus, the results in this section must be accompanied with the assumption that radial variations in the pulse shape do not produce back-surface temperature variations large enough to cause the detector to depart from linearity.

Two Layer Sample Property Measurements

There has been increasing interest in using flash diffusivity to measure the thermal diffusivity of one layer in a planar multilayer structure with the restriction that the thermal properties of the other layers in the structure are known. Taylor has reviewed past work in this area[1,2], much of which was originated by him. Koski has recently extended the analysis of layered sample data by presenting the sensitivity coefficients for a number of cases[11], utilizing the analytical solutions developed by Lee[12] for the temperature vs. time history of two and three layer samples with no thermal losses. For measurements on thick, low conductivity samples, the magnitude of a layer loss parameter, $b_i = h l_i / k_i$, can be appreciable, even at low temperatures. In this light, we have extended Lee's two layer analysis to include both thermal losses at the boundaries and a thermal contact resistance between the layers. The solution is given in Appendix B in a form suitable for programming. This solution had been previously derived by Pratt[13], but left in a complicated form which proved difficult to compare directly with Lee's solution in the no-loss limit. For that reason, we have reanalyzed the loss case, following Lee's notation as much as possible.

A. AXM9Q Graphite-CP Explosive

Measurements on explosives will usually require a high conductivity front layer to reduce the maximum temperature seen by the explosive. This type of measurement has been discussed previously by Shoemaker, Stark, and Taylor[14] and by Shoemaker, Stark, Kosigoe, and Taylor[15]. Our samples consisted of a front layer of Poco AXM9Q graphite ≈ 0.1 cm thick pressed onto a disk of CP explosive powder of approximately the same thickness. Theoretical curves characterizing one of our samples have been presented in Fig. 3(b). A comparison between a least squares fit and data derived from primary transient recorder data is shown in Fig. 4(solid curve). The data in Fig. 4 were derived from the primary data by low-pass filtering and then averaging of adjacent channels to reduce the amount of data to a manageable size. The least squares parameters were $\alpha_{CP} = 5.9 \times 10^{-4} \text{ cm}^2/\text{s}$ and $b_2 = 1.32$. Also shown for interest are the theoretical single layer solutions found by our single layer analysis program described previously by Koski[4]. The single layer curve which best matches the leading edge data, $t < 5000$ ms, was found using our PULSE analysis, based on the original work by Clark and Taylor[16](dashed line). It predicted a diffusivity, $\alpha_{CP} = 5 \times 10^{-4} \text{ cm}^2/\text{s}$ and loss parameter $b_2 = 0.12$. The dot-dash curve came from our COWAN program, based on the trailing edge analysis of Cowan[17], yielding $\alpha_{CP} = 4 \times 10^{-4} \text{ cm}^2/\text{s}$ and $b_2 = 1.36$. The PULSE curve predicts a CP diffusivity which is within

$\approx 20\%$ of the least-squares result, while the COWAN curve predicts a loss parameter which is reasonably close to the correct value. The CP diffusivity predicted by a leading edge analysis and CP loss parameter predicted by a trailing edge analysis provide good starting values for a non-linear least squares solution. Shoemaker[14] has presented a similar comparison of single-layer vs. two-layer temperature profiles in his study on explosives.

Several points of interest may be noted in regard to the CP data. First, the measured thermal diffusivity for the data shown in Fig. 4 is a very low value, $0.00059 \text{ cm}^2/\text{s}$, corresponding to a conductivity, $k_{\text{CP}}=0.093 \text{ W/m-K}$. This is a conductivity more typical of a powder than a pressed material, but conductivities almost this low were found by Shoemaker et al. for some pressed powder samples[14,15]. The data are quite insensitive to the unknown contact resistance, a desirable feature. We used a nominal value of $0.1 \text{ cm}^2\text{-K/W}$ for the contact resistance. Changing this by an order of magnitude to $1 \text{ cm}^2\text{-K/W}$ produced a change in the diffusivity to $6.013 \times 10^{-4} \text{ cm}^2/\text{s}$, about 2%.

Fig. 4. Data(+ signs) and curve fits to CP explosive data at $T=23.6^\circ\text{C}$. Solid curve-least squares curve-fit giving $\alpha_2=\alpha_{\text{CP}}=0.00059 \text{ cm}^2/\text{s}$ and $b_2=1.32$, with $h_1/h_2=50$. Dash curve-fit using a single layer leading edge or Clark & Taylor analysis, yielding $\alpha=0.00050 \text{ cm}^2/\text{s}$. Dot-dash curve-fit using a single layer Cowan's or trailing edge analysis, yielding $\alpha=0.00040 \text{ cm}^2/\text{s}$.

Another point of interest is that the shot-to-shot "random" variation in the CP thermal diffusivity is larger in magnitude than the uncertainty in that parameter estimated from a single least-squares data analysis. For the sample in Fig. 4, the 95% confidence limits for α_{CP} derived from the diagonal elements of the covariance matrix and the sum of residual squares was $(5.884 \pm 0.099) \times 10^{-4} \text{ cm}^2/\text{s}$, corresponding to an uncertainty $\approx \pm 1.7\%$. Other measurements on this sample at the same temperature, however, led to α values outside this range; the difference between the largest and average α

Fig. 5(a) Data(+ signs) for a composite AXM9Q graphite-colloidal graphite-AXM9Q graphite sample at 209°C and least squares curve-fit yielding $R=0.32 \text{ cm}^2 \cdot \text{K}/\text{W}$ and $b_2=0.003$.

(b) Sensitivity coefficients for the sample in Fig. 5(a). The R sensitivity coefficient has a large magnitude only at short times, $t \leq t_{\max}$.

was $0.17 \times 10^{-4} \text{ cm}^2/\text{s}$, or about 2.9%. These apparently random shot-to-shot variations presently provide the major limitation on overall system accuracy, assuming that the critical sample parameters, such as the CP thickness, are known to an accuracy sufficient to reduce measurement errors below those associated with shot-to-shot variations.

B. AXM9Q-Colloidal Graphite-AXM9Q

As an example of measurement of the contact resistance of a reasonably low resistance layer, we chose a sample with a colloidal graphite layer between two high conductivity pieces of AXM9Q graphite. The sample was fabricated by spreading a thin layer of commercial colloidal graphite liquid on a disk of AXM9Q graphite and then placing a similar disk on top of the liquid layer to form a sandwich structure. The sample was then loaded with a weight, dried with a heat lamp and mounted in the flash diffusivity apparatus. A least squares fit to some typical data is shown in Fig. 5(a) and the sensitivity coefficients for R , α_1 , α_2 , and T_∞ are shown in Fig. 5(b). It is evident that this is a low loss case, as would be expected from the low temperature and high conductivity of the sample. The sensitivity coefficient curve for R in Fig. 5(b) indicates the high sensitivity at times slightly longer than $t_{1/2}$. A fit to the Lee no-loss theory at $t_{1/2} = 44 \text{ ms}$ yields a result, $R = 0.332 \text{ cm}^2\text{-W/K}$ which can be compared to the least-squares fit result, $R_{LS} = 0.317 \text{ cm}^2\text{-W/K}$, a shift of $\approx 5\%$. The sensitivity coefficients in Fig 5(b) show that for a resistance measurement of this type, a single fit between theory and experiment at or near the half-time is as good as a least squares fit because the R sensitivity coefficient peaks near $t_{1/2}$, while the magnitude of the α_2 coefficient is small near $t_{1/2}$. In this case, the no-loss theory yields a good result because of the insensitivity of the temperature to R at long times.

To examine the ability to measure the resistance in a system with greater overall thermal resistance, a sandwich of Pyroceram 9606-Colloidal graphite-Pyroceram 9606 was constructed in a manner similar to that used in the case with AXM9Q graphite. Since the Pyroceram sandwich had a much larger thermal resistance, the measurement of the resistance of the colloidal graphite layer would be expected to be more difficult. Examination of the sensitivity coefficients bore out this conclusion; the measurement was much more sensitive to the Pyroceram 9606 diffusivity than to the resistance of the layer, assuming that it was still in the range, $R < 1 \text{ cm}^2\text{-W/K}$. The measurements supported this reasoning, although the data did reveal the presence of the resistive layer. The resistance increased as the furnace temperature was increased from 23°C until about 175°C and then decreased again until it could not be resolved at 300°C . We

hypothesize that thermal expansion first opened up the joint, but subsequent high temperature sintering reduced the resistance. When the sample was returned to room temperature, the joint opened and a very high resistance was obtained.

Data reduction for the sample at low temperature showed that the resistance was approximately the same as the previous case with AXM9Q graphite. However, the sensitivity to the Pyroceram diffusivity was so high that, within the limits to which that quantity is known, the resistance could only be bounded, $R < 0.5 \text{ cm}^2\text{-W/K}$.

CONCLUSIONS

In the first part of this paper we have presented arguments which indicate that laser beam non-uniformities should not affect the accuracy of flash diffusivity measurements if an averaging detector is used. The question of whether or not an infrared detector which is viewing the entire sample back-surface truly averages over this surface is an open one which needs to be tested using known asymmetric pulses together with absolute temperature measurements. Studies of this type are planned for the future. The insensitivity to radial losses first predicted by Cape and Lehman[9] appears to be a general result and is consistent with the near linear dependence of these parameters over the entire transient time for all situations which we have examined. Thus, uncertainties in radial losses caused by unknown amounts of heat conduction to sample supports should not affect the accuracy of a thermal diffusivity or contact resistance measurement.

Data for typical multilayer samples have been presented to illustrate the current accuracy of the flash diffusivity technique. These data show that variations in parameters derived from transient data determined under supposedly identical conditions limit the accuracy of the method. Typically, the least-squares fit of a single data set to theory predicts a smaller parameter uncertainty than is actually observed in the shot-to-shot variations of least-squares-values. Future improvements in the measurement accuracy of the technique will require better knowledge of the sources of this variation.

Appendix A.

Single Layer Temperature vs. Time Formulas

In this appendix, the various expressions in the single layer temperature vs. time expression, Eq.(2), are defined. The dimensional symbols are

defined in Fig. 1. These expressions follow the notation of Ozisik[18]. The prefactor, T_∞ , is the long-time, $t \rightarrow \infty$, temperature rise, $T_\infty = \Phi_0 \bar{\tau} / \rho c_p$, where ρ -sample density and c_p -sample specific heat. The radial eigenfunction in Eq.(1) is defined by,

$$R_k(y_m r/a) = J_k(y_m r/a) \quad (A-1)$$

where J_k is the ordinary Bessel function of the first kind of order k and the y_m are solutions of the radial eigenvalue equation,

$$y_m J'_k(y_m) + b_r J_k(y_m) = 0 \quad (A-2)$$

In Eq.(A-2), J'_k is the derivative of J_k with respect to its argument. The radial loss factor is defined by $b_r = h_r a/k$, where k -thermal conductivity, $k = \alpha \rho c_p$. The axial eigenfunction, Z , is defined by,

$$Z(x_n z/l) = \left(\frac{x_n}{l}\right) \cos(x_n z/l) + \left(\frac{b_1}{l}\right) \sin(x_n z/l) \quad (A-3)$$

The axial eigenvalue equation is,

$$\tan(x_n) = \frac{x_n (b_1 + b_2)}{x_n^2 - b_1 b_2} \quad (A-4)$$

The loss factors in Eqs.(A-3,A-4) are given by, $b_1 = h_1 l/k$ and $b_2 = h_2 l/k$. Usually the assumption is made that the heat transfer constants, h_1 and h_2 , are equal, yielding $b_1 = b_2 = b_z$, the axial Biot number.

The azimuthal eigenfunction is given by,

$$\Theta(k\theta) = \cos(k\theta) \quad (A-5)$$

The normalization functions in the denominator of Eq.(2) are defined by,

$$N_R(y_m, k) = \frac{J_k(y_m)}{2} \left[\frac{b_r^2 + y_m^2 - k^2}{y_m^2/a^2} \right] \quad (A-6)$$

$$N_Z(x_n) = \frac{1}{2l} \left[(x_n^2 + b_1^2) \left(1 + \frac{b_2}{x_n^2 + b_2^2} + b_1 \right) \right]$$

$$N_\Theta(k) = \begin{cases} 2\pi, & k=0 \\ \pi, & k \neq 0 \end{cases}$$

The time constants, τ_{mn} , are defined by the relation,

$$\tau_{mn} = \frac{\ell^2}{\alpha} \left[y_m^2 (\ell/a)^2 + x_n^2 \right] \quad (A-7)$$

Appendix B

Two Layer Temperature vs. Time Solution, with Contact Resistance and Thermal Losses

In this review of two layer solutions for the axial problem in the presence of thermal losses at the boundaries, we follow the notation of Lee[12] as much as possible. Lee defines the parameters, $H_j = A\rho_j c_j \ell_j$ and $\eta_j = (\ell_j^2/\alpha_j)^{1/2}$, where A =sample area, and ρ_j , c_j , ℓ_j , and α_j are the density, specific heat, thickness, and diffusivity, respectively, of layer j . In our context, $j=1$ is the front layer onto which the pulse is incident and $j=2$ is the back layer. From the fundamental quantities H_j and η_j , Lee defines the dimensionless ratios,

$$\begin{aligned} \eta_{i/j} &= \eta_i / \eta_j \text{ and,} \\ H_{i/j} &= H_i / H_j \end{aligned} \quad (B-1)$$

Lee then defines the auxiliary quantities,

$$\begin{aligned} X_1 &= H_{1/2} \eta_{2/1} + 1 \\ X_2 &= X_1 - 2, \end{aligned} \quad (B-2)$$

and,

$$\begin{aligned} \omega_1 &= \eta_{1/2} + 1 \\ \omega_2 &= \omega_1 - 2, \end{aligned} \quad (B-3)$$

together with the dimensionless contact resistance parameter, R_c , which is related to the actual contact resistance, R , by,

$$R_c = R \rho_1 c_1 (\alpha_1 \alpha_2)^{1/2} / \ell_2 \quad (B-4)$$

The Lee solution for the no-loss back-surface temperature, relative to the maximum adiabatic temperature rise, T_∞ , can be written,

$$V(t) - T(t)/T_{\infty} = 1 + 2 \sum_{k=1}^{\infty} \frac{(1+PU)Q(x_k, \eta_2, t)}{d_0(\vec{\omega}, \vec{X}, x_k) + d_R(\vec{\omega}, \vec{X}, x_k)} \quad (B-5)$$

where $P=X_2/X_1$, $U=\omega_2/\omega_1$, $\vec{\omega}=(\omega_1, \omega_2)$, and $\vec{X}=(X_1, X_2)$. The quantity $Q(x_k, \eta_2, t)$ is the convolution integral, as defined by,

$$Q(x, \eta, t) = \int_0^t f(t') \exp \left[-\frac{x^2(t-t')}{\omega_1^2 \eta^2} \right] dt' \quad (B-6)$$

In Eq. (B-6), $f(t)$ is the laser pulse shape function. The terms in the denominator of Eq. (B-5) are defined by,

$$d_0 = \cos(x_k) + PU \cos(Ux_k), \quad (B-7)$$

$$d_R = q \left[\cos(x_k) - \cos(Ux_k) - x_k [\sin(x_k) - U \sin(ux_k)] \right],$$

where the quantity x_k is the k th root of the eigenvalue equation,

$$\sin(x_k) + P \sin(Ux_k) + qx_k [\cos(x_k) - \cos(Ux_k)] = 0, \quad (B-8)$$

and $q=R_c/\omega_1 X_1$. d_0 in Eq. (B-5) represents the no-loss, no-resistance contribution, while d_R represents the change produced by contact resistance between the two layers. This method of classifying denominator terms and terms in the eigenvalue equation remains in multilayer problems with interface resistance and thermal losses and provides an easy method of keeping track of the contributions of the various processes.

In the case where thermal losses occur at the sample boundaries, the layer loss parameters, $b_1=h_1\ell_1/k_1$ and $b_2=h_2\ell_2/k_2$ are introduced. The eigenvalue equation becomes,

$$f_0(x_k) + f_R(x_k) + f_L(x_k) + f_{LR}(x_k) = 0, \quad (B-9)$$

where f_0 and f_R are the previous no-loss and contact resistance terms, respectively from Eq. (B-8),

$$f_0(x_k) = \sin(x_k) + P \sin(Ux_k),$$

$$f_R(x_k) = qx_k [\cos(x_k) - \cos(ux_k)] \quad (B-10)$$

The loss term in the eigenvalue equation can be written,

$$f_L(x_k) = -\frac{\omega_1}{x_k} \left\{ Y_1 \cos(x_k) + PY_2 \cos(Ux_k) + \frac{Z\omega_1}{x_k} \left[\sin(x_k) - P \sin(Ux_k) \right] \right\}, \quad (B-11)$$

where $Y_{1,2} = b_2 \left[(b_{1/2}/\eta_{1/2}) \pm 1 \right]$ and $Z = b_2^2 b_{1/2}/\eta_{1/2}$, with $b_{1/2} = b_1/b_2$. The loss-resistance coupling term is given by,

$$f_{LR}(x_k) = q\omega_1 \left\{ [Y_1 \sin(x_k) - Y_2 \sin(ux_k)] - \frac{Z\omega_1}{x_k} [\cos(x_k) + \cos(ux_k)] \right\} \quad (B-12)$$

The temperature solution no longer has a first term of unity because the Laplace transform does not have a pole at $s=0$ when b_1 and/or b_2 are non-zero, representing the fact that there is no non-zero steady state component of $V(t)$. $V(t)$ can be written in the form,

$$V(l,t) = (1+PU) \sum_{k=1}^{\infty} \frac{2Q(x_k, \eta_2, t)}{d_0(x_k) + d_R(x_k) + d_L(x_k) + d_{RL}(x_k)} \quad (B-13)$$

d_0 and d_R are given by Eq.(B-6), while d_L is given by,

$$\begin{aligned} d_L = & \frac{\omega_1}{x_k} \left[Y_1 \sin(x_k) + PUY_2 \sin(Ux_k) \right] \\ & + \frac{\omega_1}{x_k^2} \left[Y_1 \cos(x_k) + PY_2 \cos(Ux_k) \right] \\ & + \frac{Z\omega_1^2}{x_k^3} \left[2[\sin(x_k) - P \sin(Ux_k)] - x_k [\cos(x_k) - PU \cos(Ux_k)] \right] \end{aligned}$$

and,

$$\begin{aligned} d_{LR} = & \frac{Zq\omega_1^2}{x_k^2} \left[\cos(x_k) + \cos(Ux_k) + x_k \sin(x_k) + U \sin(ux_k) \right] \\ & + q\omega_1 \left[Y_1 \cos(x_k) - UY_2 \cos(Ux_k) \right] \end{aligned}$$

It should be remarked that the solution of the eigenvalue equation, Eq.(B-9), by numerical techniques is not completely straightforward because of the presence of near double-roots for some values of the parameters.

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