SINGLE PASS COLLIDER MEMO

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TITLE: Optical Transition Radiation from a Thin Carbon Foil--A Beam Profile Monitor for the SLC.

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Introduction

This memo considers placement of an ultra thin carbon foil into the SLC beam. Transition radiation light would be emitted from the surface of the foil. The optical spot from the foil could be viewed with a microscope objective lens and registered with an image detector.

Multiple scattering for the foil thicknesses necessary will not affect the beam emission. Calculations show that a thin carbon foil can withstand the electron beam if the electron beam is larger than $10\mu\text{m}$ in size.

There are many possible radiation mechanisms from a foil - bremsstrahlung, black body temperature radiation, Čerenkov light, scintillation light, and transition radiation. Transition radiation is apparently dominant. It is proposed to use thin carbon foils, 75-150 Å thick. Calculations indicate that 5×10^{10} beam electrons will radiate a useable number of optical photons. Specifically with 150 Å foils the fractional yield of "useful" optical photons is 10^{-3} photon per incident electron. 5×10^7 optical photons imaged upon an image plane. Spread these photons over a 32×32 pixel CCD and one has the readout system of a monitor.

The proposed geometries are shown in Fig. 1 and 2.

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MASTER

mf

FIGURE 1

Scale - mm

Transition and Radiation

The theory of transition radiation has been worked out extensively.¹⁻⁵ The DiNardo and Goland experiment gives the absolute yield of transition radiation photons for a 300 Å foil and confirms the theory for nonrelativistic electrons; this paper gives the optical constants for carbon foils. Ritchie and Eldridge derive the formula for a normal incidence foil; Ashley derives the transition radiation from a tilted foil. The tilted foil is a convenient geometry for light collection. The Wartski et al. paper discusses tilted foils, ultra relativistic incident electrons, and possible uses for beam diagnostics.

Transition radiation is a classical effect. The incident electron represents "white photons" incident upon a dielectric, metallic boundary. These photons are reflected, transmitted according to the classical laws of reflection, transmission given by Fresnel. Imagine an incident photon replacing the incident electron. Angle of incidence equals the angle of reflection. The incident photons create an electromagnetic surface field which in turn radiates photons at the angle of reflection. Transition radiation is the case of an incident electron replacing the incident photon. Kinematics forbids radiation at exactly the angle of reflection. Nevertheless the radiation is strongly peaked, just to the sides of the classical angle of reflection. The normal incident geometry case, Figure 1, loses this radiation peak due to the hole in the 45° mirror. Whether this loss of nearly on axis photons is harmful or whether diffraction optics spreads these photons too widely to be "useful" is discussed later. For reference the transition radiation formula for a normally incident electron is:

$$\begin{aligned} \frac{d^2n(\mu, \lambda)}{d\Omega d\lambda} &= \frac{\text{photons}}{\text{steradian cm incident electron}} \\ &= \frac{\alpha\beta^2}{\pi^2\lambda} \frac{\mu^2(1-\mu^2)|\epsilon-1|^2}{|\Delta|^2|(1-\mu^2\beta^2)|^2|1-\beta^2\sigma^2|^2} \\ &\quad \times [(1-\beta\sigma-\beta^2)(1+\beta\sigma)(\sigma+\mu\epsilon)e^{-it\sigma} \\ &\quad + (1+\beta\sigma-\beta^2)(1-\beta\sigma)(\sigma-\mu\epsilon)e^{it\sigma} \\ &\quad - 2\sigma(1-\beta\mu-\epsilon\beta^2)(1+\beta\mu)e^{-it/\beta}]^2 \end{aligned}$$

with

$$\sigma^2 = \epsilon - 1 + \mu^2, \beta = v/c, \mu = \cos\theta, t = \frac{av}{c}, \Delta = (\mu\epsilon - \sigma)^2 e^{it\sigma} - (\mu\epsilon + \sigma)^2 e^{-it\sigma}, \alpha = e^2/\hbar c.$$

The radiation is proportional to $|\epsilon - 1|^2$. Carbon foils have a large dielectric constant, $\epsilon = 5.21 + i3.55$. The factors Δ , $e^{-it\sigma}$, $e^{it\sigma}$, $e^{-it/\beta}$ contain the radiation from the back,

front surfaces of the foil and minor interference effects. The light is polarized in the plane of the incident electron and the outgoing photon. More transition radiation formulas as given in the Appendix.

At the ultrarelativistic energies of SLC the general formula exhibits a very, very strong forward and backward peak, due to the factor:

$$\frac{1 - \mu^2}{(1 - \beta^2 \mu^2)^2} = \frac{\theta^2}{[\frac{1}{\gamma^2} + \theta^2]^2}$$

This term is relativistic electric dipole radiation due to an incident electron plus its virtual image positron. Most of the light is contained within a few milliradians of forward, backward directions. With a 150 Å carbon foil at normal incidence one obtains for $\lambda = 4000$ Å:

$$\frac{d^2n}{d\Omega d\lambda} = 3.13 \frac{\theta^2}{[\frac{1}{\gamma^2} + \theta^2]^2} \frac{\text{photon}}{\text{steradian cm}}$$

which yields upon integration:

$$\int \int \frac{d^2n}{d\Omega d\lambda} d\Omega d\lambda = 0.84 \times 10^{-3} \text{ photon/beam electron}$$

using integration limits $3000 < \lambda < 6000$ Å, $0.02 < \theta < 0.10$ radian. 5×10^{10} beam electrons will yield 4.2×10^7 optical photons. The photons are emitted "off axis" where diffraction effects are not a major problem. The choice of the lower integration limit $\theta = 0.02$ radian minimizes the diffraction problem.

A CCD detector has .10-.50 quantum photon efficiency in the visible optical region. 4.2×10^7 optical photons are adequate but not ample if focused on a 32×32 pixel area CCD. Preferably the image detector should be UV sensitive.

The general transition radiation formula has been coded on a computer- Apple II - for both the normal incidence and tilted foil cases. Fig. 3 shows the light output as a function of foil thickness. Making the foil thinner reduces the light approximately as the thickness squared. Saturation in light output occurs for foil thicknesses that are greater than 1000 Å. One of course picks as thin a foil as practicable, to minimize multiple scattering. 75-150 Å is a reasonable choice.

FIGURE 3

Thin Foil Properties

Carbon foils are available in 10-1000 Å thicknesses.⁶ In composition they are partly graphite and partly glassy (amorphous) carbon. They may be mounted in a frame and are self supported over 5-10 mm. The foil carbon density is 1.82 g/cm³. The thin foil is nearly transparent to optical wavelengths. Multiple scattering for a 150 Å foil is relatively insignificant.

$$\langle \Delta\theta \rangle = \frac{21 \text{ MeV}}{50 \text{ GeV}} \sqrt{\frac{2.73 \times 10^{-6} \text{ g/cm}^2}{42.5 \text{ g/cm}^2}} = 1.06 \times 10^{-7} \text{ radians}$$

One may note that circular machines may not have ultra thin foils placed in the beam. Multiple scattering is only tolerable in a single pass collider.

Consider 5×10^{10} electrons passing through a foil with a $dE/dx = 2.7 \text{ MeV/g/cm}^2$ per electron. The central temperature rise is:

$$\begin{aligned} \Delta Q &= \frac{dE}{dx} \Delta x \frac{N_0}{2\pi\sigma_0^2} \exp\left(-\frac{(x^2+y^2)}{2\sigma_0^2}\right) \\ &= \tau_p \rho \Delta x \Delta T \quad \tau_p = .71 - 2.1 \frac{J}{gK} \end{aligned}$$

Table 1

$\sigma_x = \sigma_y$	τ_p	ΔT
10 $\mu\text{ m}$	1.9	1789 K
15 $\mu\text{ m}$	1.65	915 K
20 $\mu\text{ m}$	1.4	607 K
50 $\mu\text{ m}$	1.0	137 K
100 $\mu\text{ m}$	.80	40 K

The conclusion is that an electron beam would burn a hole in a carbon foil if the beam size is less than $\sigma_0 = 10\mu\text{m}$. One must limit the maximum temperature of the foil to 2500 K due to carbon sublimation

There are 180 pulses/second. Consider the heat diffusion equation:

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T = \frac{k}{\rho c_p} \nabla^2 T$$

which has a solution:

$$T = T_0 \frac{t}{(t + \tau)} e^{-\frac{r^2}{2\sigma_0^2} (\frac{t}{t+\tau})}$$

with:

$$\tau = \frac{\sigma_0^2}{2\alpha} \quad , \quad k = 1.2 \frac{W}{cmK}$$

Table II

σ_0	τ = Time Constant
10 μm	5.4×10^{-7} s.
20 μm	2.1×10^{-6}
50 μm	1.3×10^{-5}
100 μm	5.4×10^{-5}

The conclusion is that rapid cooling occurs between pulses. The single pulse limitation is the limitation, not multiple pulses. Cooling by black body radiation is not a major cooling mechanism.

Diffraction Limited Optics

On Page 3 the angular distribution of the light emitted is given as:

$$\frac{d^2n}{d\Omega d\lambda} = 3.13 \frac{\theta_1^2}{[\frac{1}{\gamma^2} + \theta_1^2]^2} \frac{\text{photon}}{\text{steradian cm}}$$

In such a formula the light emission peak occurs at $\theta_{max} = 1/\gamma = 10^{-5}$ radian. *

An optical imaging system will have some difficulty in imaging such light. This is due to the circular aperture diffraction limit.

$$\Delta\theta_2 = 1.22 \frac{\lambda}{a'} \quad \text{with } a' = \text{aperture} = 2d_1\theta_{max}$$

Nearly on axis light is strongly diffracted and forms a wide halo on the image plane.

* This theory expects a lateral coherence length $\rho = \gamma\lambda/2\pi = 8mm$. Practical coherence lengths are likely to be less. Screening in the foil, beam divergence, and collective beam effects should reduce θ_{max} - probably to $10^{-4} - 10^{-3}$ radian. Nevertheless the diffraction optics problem remains.

Quantitative calculation is necessary. One may analyze the circular aperture diffraction pattern for a transition radiation point source, specifically the unnormalized angular distribution.

$$I(\theta_2) = \left| \int_{0.2a}^{1.0a} \frac{\rho^2 d\rho}{[d_1^2/\gamma^2 + \rho^2]} \int_0^{2\pi} d\phi \cos\phi e^{-ik\rho \sin\theta_2 \cos\phi} \right|^2$$

Without the presence of the extra angular factor,

$$\frac{\rho}{[d_1^2/\gamma^2 + \rho^2]}$$

and the polarization factor, $\cos\phi$, the above formula reduces to normal circular diffraction.

A plot of the transition radiation diffraction pattern is shown in Figure 4. When there is no stop, $\rho_{min} = 0$, then there is a long tail with many photons in a halo, photons which represent noise. With a 20% stop, $\rho_{min} = 0.2a$, the halo is reduced to the order of 0.2%. Such a 20% stop yields diffraction resolution of $5 \mu\text{m}$ assuming times one magnification*. The ratio of the diffraction resolution to the image size is an invariant, independent of the optical magnification or lens placement.

Obviously the image spot is degraded by adding $5 \mu\text{m}$ in quadrature with the source beam σ_0 . This is not very significant for $\sigma_0 \geq 10 \mu\text{m}$.

* For comparison normal textbook circular aperture diffraction resolution would be $2.5 \mu\text{m}$.

Conclusions

1. The beams at the SLC are microscopic. Viewing a microscopic optical spot with an optical microscope appears "appropriate." Microscope optics are not essential.
2. If material must be placed in the beam then it is best that it be a low Z material with a high melting point. Carbon is such a strong, durable material.
3. An intercepting monitor works for a single pass machine, and not in a multi-pass machine.
4. Transition radiation yields 10^{-3} optical photon per incident beam electron for a 150 Å foil. This yield is confirmed by experiment.
5. Diffraction limited optics degrades the image slightly.

Figure Captions

1. A proposed foil with normal incidence electrons. Light is emitted in the backward direction. There is a 1 mm hole in the 45° 200 Å Aluminum foil mirror, resulting in the loss of much of the backward light peak. Light acceptance is down by a factor of 15 due to the hole compared with Figure 2 which accepts the full peak.
2. A proposed foil with electrons incident at 30° . Light is collected at the peak, occurring at the angle of reflection.
3. Typical light output curve as a function of foil thickness. $\gamma = 10^5$ is for SLC, $\gamma = \sqrt{10}$ refers to the experimental work on carbon foil by DiNardo and Goland.
4. The transition radiation diffraction pattern assuming a point source. The magnification is one. Higher magnifications magnify both the image and diffraction pattern error with no net relative change.

References

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6. The Arizona Carbon Foil Co., 2239, E. Kleindale Rd. Tucson, Arizona 85719. Foils in the range $2 \times 10^{-7} \text{ g/cm}^2 < \Delta x < 2 \times 10^{-5} \text{ g/cm}^2$ may be purchased.

APPENDIX

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$$\frac{d^2 n}{d\Omega d\lambda} = \frac{e^2 \beta^2}{\pi^2 \lambda} \frac{\mu^2(1-\mu^2)|\epsilon-1|^2}{|\Delta|^2(1-\mu^2\beta^2)^2|1-\beta^2\sigma^2|^2} \times \left\{ (1-\beta\sigma-\beta^2)(i+\beta\sigma)(\sigma+\mu\epsilon)e^{i\psi} + (1+\beta\sigma-\beta^2)(1-\beta\sigma)(\sigma-\mu\epsilon)e^{i\psi} - 2\sigma(1-\beta\mu-\epsilon\beta^2)(1+\beta\mu)e^{-i\psi/2} \right\} \frac{\text{photon}}{\text{steradian cm electron}}$$

and $\epsilon = (\epsilon - 1 + \mu^2)^{1/2}$, $\beta = v/c$, $\mu = \cos\theta$, $i = \omega/c$, $\Delta = (\mu\epsilon - \sigma)^2 e^{i\psi} - (\mu\epsilon + \sigma)^2 e^{-i\psi}$ and $\alpha = e^2/\hbar c$ is the fine structure constant. One may show that the equation for the radiation into the half-space $z < 0$ is nearly identical with that given by Eq. (14); it is only necessary to replace μ by $|\cos\theta|$, and to put $\beta \rightarrow -\beta$ in Eqs. (14) and (15). In the general case $\epsilon = \epsilon_1 + i\epsilon_2$, and one must put $\sigma = R(\cos\phi + i\sin\phi)$ with $R = [(\epsilon_1 - 1 + \mu^2)^2 + \epsilon_2^2]^{1/2}$ and $\phi = \frac{1}{2} \tan^{-1}[\epsilon_2/(\epsilon_1 - 1 + \mu^2)]$. $\sigma^2 = \epsilon^{-1} + \mu^2$

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$$\begin{aligned} \frac{d^2 N}{d\Omega d\omega} = \frac{\alpha^2 \mu^2 \epsilon^2}{\pi^2 \omega} \left| \frac{\epsilon-1}{\gamma+\gamma_-} \right|^2 \frac{\sin^2 \theta e^{i\psi/2}}{d_1 \delta_+ \delta_-} & \left[m_+ \gamma_+ (\rho^2 - \beta\sigma + \rho - \beta^2 \gamma^2) e^{-i\psi} + m_- \gamma_- (\rho^2 + \beta\sigma + \rho - \beta^2 \gamma^2) e^{i\psi} \right. \\ & - 2\sigma \delta_- (\rho^2 - \beta\mu + \rho - \epsilon^2 \gamma^2) e^{-i\psi/2} \Big] + \frac{\beta(1-\mu^2) \cos\psi \sin\theta}{d_1 d_1} \left\{ \frac{e^{i\psi/2}}{\delta_+} [n_-^2 m_- \gamma_- e^{i\psi} - n_+^2 m_+ \gamma_+ e^{-i\psi}] \right. \\ & + n_-^2 m_- \gamma_- - n_+^2 m_+ \gamma_+ - 4\epsilon^2 \beta \tau (\epsilon-1) \Big\} + (2\epsilon \delta_-) \left\{ \left[\beta m_+ n_- + \frac{n_-}{\delta_+} (\rho^2 - \mu\sigma \beta^2 \gamma^2) \right] n_+ e^{-i\psi} \right. \\ & + \left[\beta m_- n_+ - (n_- \delta_+ (\rho^2 + \mu\sigma \beta^2 \gamma^2)) n_- e^{i\psi} - 2\epsilon \beta \tau \rho (\epsilon-1) / \delta_+ \right] \Big\} \\ & + \frac{\beta^2 \tau \mu \sin\theta \cos\psi e^{i\psi/2}}{d_1 \delta_+ \delta_-} (n_- \gamma_- e^{i\psi} + n_+ \gamma_+ e^{-i\psi} - 2\sigma \delta_+ e^{-i\psi/2}) \Big\}^2 \quad (5) \end{aligned}$$

$$\frac{d^2 N}{d\Omega d\omega} = \frac{\alpha^2 \mu^2 \epsilon^2 (1-\gamma^2) \sin^2 \psi}{\pi^2 \omega} \left| \frac{\epsilon-1}{d_1 \delta_+ \delta_- \gamma_+ \gamma_-} \right|^2 \left[n_- \gamma_- e^{i\psi} + n_+ \gamma_+ e^{-i\psi} - 2\sigma \delta_+ e^{-i\psi/2} \right]^2, \quad (6)$$

where $\sigma = e^2/\hbar c$, $\beta = v/c$, $\mu = \cos\theta$, $\gamma = \cos\theta$, $\rho = 1 - \beta \cos\psi \sin\theta \sin\theta$, $i = \omega/c$, $\sigma^2 = \epsilon - 1 + \mu^2$ ($\text{Im}\sigma > 0$), $d_1 = (\mu\epsilon - \sigma)^2 e^{i\psi} - (\mu\epsilon + \sigma)^2 e^{-i\psi}$, $d_2 = (\mu - \sigma)^2 e^{i\psi} - (\mu + \sigma)^2 e^{-i\psi}$, $\delta_{\pm} = \rho \pm \beta\sigma\tau$, $\gamma_{\pm} = \sigma \pm \mu\epsilon$, $n_{\pm} = \sigma \pm \mu$. These results derived for $0 \leq \theta \leq \frac{1}{2}\pi$ (i.e., $z > 0$), may be transformed to the other hemisphere of direction, i.e., $\frac{1}{2}\pi \leq \theta \leq \pi$, by the prescription $\mu \rightarrow -\mu$, $\beta \rightarrow -\beta$, and $\cos\psi \rightarrow -\cos\psi$.

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