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STABILITY ANALYSIS OF A HOT ELECTRON EBT PLASMA

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ABSTRACT

Previous theoretical studies have predicted that an EBT-confined plasma will have a stable equilibrium for beta values of the toroidal core plasma comparable to the beta value for the hot electron annulus. We show that, when the ring is allowed to be perturbed, there is a new physical effect, enhanced compression of the core plasma; that in the limit of negligible β_{core} , the interchange mode is stable for β_{ring} greater than about 20%, as was found before; but that when the core pressure is increased, the plasma goes unstable again at values of β_{core} significantly smaller than the value for β_{ring} .

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1. INTRODUCTION

One proof-of-principle objective for the proposed EBT-P device is to demonstrate bulk plasma stability at high values for the plasma temperature and beta: specifically, for $T_i > 1$ keV and $\beta_i > 1\%$. The reason for this interest is that theory has predicted that, at high β , an EBT plasma can have a stable equilibrium for beta values of the toroidal core plasma comparable to the beta value for the hot electron annulus and on the order of 20-40%. Such a feature would make an EBT confinement device very attractive from the reactor scaling point of view.

This theoretical result was obtained from two somewhat differing approaches. Nelson and Hedrick [1] used a modified MHD approach, treating the hot electron annulus as a rigid non-interacting current ring and modeling the EBT as a bumpy cylinder. Nelson [2] used a slab-model kinetic approach and allowed the hot electron component to be perturbed.

In this paper, we also use low-frequency kinetic theory and include the perturbation behavior of the hot electron ring, as well as that of the warm core plasma. The model we study is that of a bumpyless torus. We find that the response of the hot electron species leads to a marked enhancement of the compression of the core plasma. This is a new physical effect, overlooked in the analysis of Ref. 2. In the limit of

negligible core beta, the plasma is found to be interchange stable when the annulus beta value exceeds 20% (or approximately four times the ratio of the annulus width to the local radius of curvature). This is the same result as before, although we will show that it comes from slightly different physics. Our most significant result is that as the core pressure is increased, the plasma will again become interchange unstable, at values for the core plasma beta which are substantially smaller than that for the ring beta, in contrast to earlier predictions.

2. PHYSICAL BEHAVIOR OF PLASMA AND RINGS

In the EBT device, macrostability is provided by the hot electron rings, created through microwave heating. These have a large dipolar diamagnetic current and produce a local magnetic well, which can lead to average minimum-B stability. The experimentally observed transition from the cold, noisy C-mode of operation to the relatively quiet T-mode which occurs when the ring beta is increased beyond a value of 10-15% is generally associated with this stability mechanism. Previous theories [1,2] calculated this beta threshold to be 10-20%, in good agreement. The other theoretical prediction, that the plasma is still stable for the core beta comparable to the ring beta above threshold, is difficult to check since current EBT experiments cannot achieve core beta values above 0.5-1.0%.

Ideally, the bulk plasma profile in EBT is nearly flat topped, with the magnetic well due to the annulus located where the pressure falls off [cf. Fig. 4 of Ref. 1]. In the region of interest, defined as the region where the radial gradient of the magnetic field strength is non-negative, we will make the crude simplifying assumption that the hot electrons and the warm core plasma have the same density profile. (The spatial dependence of their temperatures will be ignored, also for simplicity.) For later reference, we note that the density scale length (or ring half width), r_p , divided by the radius of curvature, R_c , is considered to be about 0.05 in current EBT experiments. The hot electron to core plasma temperature ratio is $T_h/T_i \approx 10^3$; the density ratio, $\delta = N_h/N_i$, is about 0.1 or smaller.

The toroidal core plasma behaves like a warm fluid, for which the diamagnetic frequency, $\omega_* = -i(v_{th}^2/\Omega)(\hat{b} \times \nabla \ln N) \cdot \nabla_{\perp}$, and the magnetic drift frequency, $\omega_d = -i\Omega^{-1} \left[v_{\parallel}^2 (\hat{b} \times \hat{b} \cdot \nabla \hat{b}) + \frac{1}{2} v_{\perp}^2 (\hat{b} \times \nabla \ln B) \right] \cdot \nabla_{\perp}$, may be taken to be smaller than the fluctuation frequencies, ω , of interest here. The core plasma basically responds to $E \times B$ motion. Hence, its pressure perturbation (for an isotropic distribution) is given by the MHD expression

$$\tilde{p} = -\xi_{\perp} \cdot \nabla p + \frac{\gamma p}{B^2} \left[\xi_{\perp} \cdot (B \cdot \nabla B) + \xi_{\perp} \cdot \nabla (\frac{1}{2} B^2) + B \cdot \tilde{B} \right]. \quad (1)$$

Perturbed quantities are denoted by a tilde. Kinetic theory, for $\omega \gg \omega_*, \omega_d$, yields an equivalent expression:

$$\tilde{p} = -eN_i (\omega_{*i}/\omega) (1+\tau) \tilde{\Phi} + eN_i (\omega_{di}/\omega) (1+\tau) \tilde{\Phi} + (\beta/4\pi) B \tilde{B}_{||} , \quad (2)$$

with $\tau = T_e/T_i$. The first term in either Eq. (1) or (2) is the convective pressure change; the remaining terms represent the change in pressure due to compression. Now recall the quasi-static condition for transverse perturbed pressure balance,

$$B \tilde{B}_{||} + 4\pi \tilde{p}_{\perp} = 0 , \quad (3)$$

which is generally valid for low frequency modes whose perpendicular wavelength is much smaller than its parallel wavelength or any equilibrium scale length. The condition (3) may be interpreted as eliminating the fast Alfvén wave from consideration. Taken together with either Eq. (1) or (2), it implies that the longitudinal magnetic field perturbation is proportional to the electrostatic part of the pressure fluctuation divided by $1 + \beta$. This is the usual MHD finite-beta effect.

The hot electrons in the annulus, on the other hand, due to their large perpendicular energy, respond poorly to $E \times B$

drifts and basically exhibit adiabatic behavior, i.e., $\omega \ll \omega_{*h}, \omega_{dh}$. Their drift motion follows the field lines

$$\tilde{v}_{dh} \cdot \nabla \tilde{p}_{\perp h} + \tilde{v}_{dh} \cdot \nabla p_{\perp h} = 0. \quad (4)$$

If the magnetic field is perturbed, the hot electrons will have their trajectories deformed accordingly. Thus, the ring current satisfies $\nabla \cdot \tilde{J}_{\perp h} \cong 0$, since it is mostly due to the perturbed ∇B -drift. Furthermore, because the surfaces of constant pressure are the same as the const- B contours in the adiabatic limit, we may express the hot electron pressure perturbation as

$$\tilde{p}_{\perp h} \cong \left(\frac{\partial p_{\perp h}}{\partial B} \right) \tilde{B}_{\parallel} = \frac{1}{2} \beta_h \left(\frac{\omega_{*h}}{\omega_{bh}} \right) B \tilde{B}_{\parallel}, \quad (5)$$

with ω_{bh} the velocity-averaged ∇B -drift frequency for the hot electrons. The expression in Eq. (5) is consistent with the statement of Eq. (4) since we may assume $T_{\perp h} \gg T_{\parallel h}$ from anisotropic heating.

Equations (2) and (5) combined with the quasi-static condition (3) now yield the longitudinal field perturbation as

$$B \tilde{B}_{\parallel} = -4\pi \tilde{p}_{\perp}^{e.s.} / [1 + \beta_c + \frac{1}{2} \beta_h (\omega_{*h} / \omega_{bh})]. \quad (6)$$

Here, $\beta_c = \beta_i + \beta_e$ is the total core beta, and $\tilde{p}_1^{e.s.}$ is the electrostatic part of the total perpendicular pressure fluctuation. If, for example, we use the slab equilibrium pressure balance relation to write $\omega_{*h}/\omega_{bh} \approx \omega_{*i}/\omega_{bi} = -2/\beta$ (since there is no curvature in the slab model), we find a near cancellation between the first and last terms in the denominator of Eq. (6). More precisely, we have

$$D \equiv 1 + \beta_c + \frac{1}{2}\beta_h \left(\frac{\omega_{*h}}{\omega_{bh}} \right) \rightarrow \frac{\beta_c}{\beta} (1 + \beta) , \quad (7)$$

which for current EBT parameters is very small compared to unity. Clearly, the magnetic field perturbation is greatly enhanced by the behavior of the hot electron population. This interesting new physical effect does not occur in a rigid-ring analysis, nor was it noticed in Ref. 2. Its significance is that it produces a considerable enhancement of the bulk plasma compression. (specifically, the inductive part), which will be shown to influence the stability analysis.

3. DISCUSSION OF STABILITY ($\delta = 0$)

We confine our attention to low-frequency ($\omega \ll \Omega_i$) flute interchange modes. These are described by the equation

$$\frac{c^2}{2(\omega - \omega_{*i})} \nabla_{\perp}^2 \tilde{\Phi} - \left(\frac{4\pi ic}{B} \right) [\hat{b} \times (\hat{b} \cdot \nabla \tilde{b})] \cdot \nabla_{\perp} (\tilde{p}_{\parallel} + \tilde{p}_{\perp}) = 0 , \quad (8)$$

which is equivalent to charge quasi-neutrality and is valid in general geometry for modes with small transverse wavelength.

Equation (8) may be readily derived, for instance, from the

zeroth moment of the drift-kinetic equation with magnetic moment $\mu = Mv_{\perp}^2/2B$ and longitudinal invariant $J = v_{\parallel}R$ as constants of the motion [3]. The first term in Eq. (8) arises from the ion polarization drift and the finite gyroradius remnant of the ion $E \times B$ drift, the third term from the field line curvature drift of all species.

In order to discuss as simply as possible how the annulus stabilizes the bulk plasma, we will consider the hot electron density fraction $\delta = N_h/N_i$ to be vanishingly small even though its beta value is still finite. For current EBT parameters, this is not an unreasonable approximation. Also, we will employ a large-poloidal-mode-number local approximation, ignoring azimuthal variation in the equilibrium quantities. The enhanced compression effect described in Sec. 2 implies that $\tilde{p}_{\perp} = \tilde{p}_{\perp c} + \tilde{p}_{\perp h} = \tilde{p}_{\perp c}/D$. Then, take an isotropic, convective pressure perturbation for the warm core plasma: $\tilde{p}_{\parallel c} = \tilde{p}_{\perp c} = -eN_i(\omega_{*i}/\omega)(1+\tau)\tilde{\phi}$. Under these assumptions, Eq. (8) turns into a local dispersion relation,

$$\omega(\omega - \omega_{*i})k_{\perp}^2\rho_i^2 = -\omega_{Ki}\omega_{*i}(1+\tau)(1+\frac{1}{D}), \quad (9)$$

with ω_{Ki} the velocity-averaged ion curvature-drift frequency. We will analyze Eq. (9) in various limits.

First, suppose that the core beta is negligibly small. When the hot electron ring is absent ($\beta_h = 0$), we have $D \approx 1$ and immediately obtain the usual interchange instability with $\omega^2 = c_s^2/r_p R_c$, purely growing if we neglect finite Larmor radius stabilization (i.e., $\omega \gg \omega_{*i}$). In this case, the driving term

in Eq. (9) involves pure curvature only, which merely shows that the bulk plasma cannot be stabilized by its own diamagnetic well.

However, with the annulus present, we have $D \cong \omega_{ki}/\omega_{bi}$, if we temporarily overlook the detail of anisotropy. Then one obtains $\omega^2 = -\omega_{ki}\omega_{*i}(1+\tau)[1+\omega_{bi}/\omega_{ki}] = -\omega_{*i}\omega_{di}(1+\tau)$, and the mode becomes stable when the magnetic drift frequency ω_{di} changes sign relative to ω_{*i} . This requires a local minimum in the field strength such that the gradient is of order $|\nabla B/B| \cong 1/R_c$, which in turn implies that, for stability, the ring beta must be at least as large as the threshold value

$$\beta_h = 4(r_p/R_c) \cong 20\% . \quad (10)$$

This result is the same as that obtained by using a rigid-ring analysis[1]. The reason is that the hot electron species is incompressible in the limit $N_h/N_i \rightarrow 0$ and, as long as $\beta_c \ll \beta_h$, therefore enters the problem only through the equilibrium pressure balance.

For satisfactory reactor performance, it is desirable to boost the core plasma pressure and also to have a narrow annulus (i.e., reduce the ring power input). Under these conditions, it is possible to have $\frac{1}{2}\beta_c |\nabla p/p| > 1/R_c$, which means that curvature may be ignored in the equilibrium pressure relation (as in a slab model). Then one finds that interchange modes are no longer stable, even though mod-B may still have a local minimum.

A slightly more detailed explanation for this transition back to instability follows from the observation that, generally speaking, the enhanced compressional terms (enhancement factor = $1/D$) can contribute to stability only when D is negative. Such is indeed the case when $\beta_c = 0$. But, as β_c increases in value, the sign of D can become positive. This can be seen from writing

$$D \cong \omega_{ki}/\omega_{bi} + \beta_c (1 - \omega_{*i}/2\omega_{bi}) \quad (11)$$

$$\propto (\epsilon - \frac{1}{2}\beta_c)(1 + \beta_c) - \frac{1}{2}\beta_c \beta_h ,$$

with $\epsilon = r_p/R_c$. Equation (11) yields an upper limit on β_c for stability against interchange:

$$\beta_c \leq 2\epsilon/(1 + \beta_h) . \quad (12)$$

For β_h in the 20-40% range, this limit requires $\beta_c \leq 7-8\%$.

Figure 1 displays the regime of values for β_i and β_h (shown shaded) within which the enhanced compression terms can contribute to stabilization. (Setting $T_e/T_i = 0$ does not materially affect the net result.) The upper boundary of this region corresponds to where $D = 0$, the left-hand boundary to where $\omega_{di} = 0$. For purposes of comparison, we also plot in Fig. 1 the three curves labeled $\delta = 0, 0.05$, and 0.10 . These are the marginal stability curves derived by Nelson [2]: interchange modes are claimed to be "stable" for all values of β_i below a given such curve. Qualitatively similar curves were obtained by Nelson and Hedrick [1], with annulus elongation instead of δ as the parameter. Both of these theories predicted

substantially higher values of β_i to be stably attainable than does our theory. The difference becomes quite clear (for $\delta = 0$) when β_h is larger than 35% and β_i is greater than about 7%. We contend that the diminished stable region obtained in our theory follows from properly taking into account the adiabatic response of the annular hot electrons.

4. STABILITY ANALYSIS FOR FINITE δ

In what follows, we will retain space-charge effects associated with finite $\delta = N_h/N_i$. Also, we will not begin from Eq. (8), but will instead use a different, though equivalent approach in order to compare directly with the analysis of Ref. 2.

For flute modes, only the electrostatic potential $\tilde{\phi}$ and the parallel component $\tilde{B}_{||}$ are needed to describe the perturbed electromagnetic fields. These two field quantities are related by the condition of quasi-neutrality, $\sum q\tilde{n} = 0$ (with the summation being over all species), and by the quasi-static condition, Eq.(3). The partial charge density is given by

$$q\tilde{n} = -\frac{k_D^2}{4\pi} \left[\tilde{\phi} - (\omega - \omega_*) \left\langle \frac{1}{\omega - \omega_d} \left(1 - \frac{k_{\perp}^2 v_{\perp}^2}{2\Omega^2} \right) \left(\tilde{\phi} + \frac{v_{\perp}^2}{2\Omega} \tilde{B}_{||} \right) \right\rangle \right], \quad (13)$$

and the total transverse pressure perturbation by

$$\tilde{p}_{\perp} = \sum \left(\frac{q_{MN}}{T} \right) (\omega - \omega_*) \left\langle \frac{v_{\perp}^2/2}{\omega - \omega_d} \left(1 - \frac{k_{\perp}^2 v_{\perp}^2}{2\Omega^2} \right) \left(\tilde{\phi} + \frac{v_{\perp}^2}{2\Omega} \tilde{B}_{||} \right) \right\rangle. \quad (14)$$

If, as before, we assume $\omega \gg \omega_{*i,e}, \omega_{di,e}$, for the bulk plasma ions and electrons and reverse the inequality for the hot electrons, we can easily derive a local dispersion relation: $a\omega^2 + b\omega + c = 0$.

The coefficients of the quadratic equation are:

$$a = (k_{\perp} \rho_i)^2 + \delta^2 \left(\frac{\beta_i}{2D} \right) \left(1 - \frac{\omega_{*h}}{\omega_{bh}} \right)^2, \quad (15)$$

$$b = \delta \left[\omega_{*i} - \omega_{di} - \left(\frac{\beta_c}{D} \right) \left(1 - \frac{\omega_{*h}}{\omega_{bh}} \right) (\omega_{*i} - \omega_{di} - \omega_{bi}) \right], \quad (16)$$

$$c = \left[(\omega_{*i} - \omega_{di}) \omega_{di} - \omega_{bi}^2 + \left(\frac{\beta_c}{2D} \right) (\omega_{*i} - \omega_{di} - \omega_{bi})^2 \right] (1 + \tau), \quad (17)$$

with $\beta_c = \beta_i + \beta_e$ and $\tau = (1 - \delta)(T_e/T_i)$. The implicit neglect of hot electron gyroradius effects and the rather cavalier handling of temperature anisotropy in this derivation may be suspect. However, with Eqs. (15)-(17), the local dispersion relation has the same form as that derived in Ref. 2. For simplicity, we will again take $T_e = 0$.

We now analyze this local dispersion relation using two different models. The first such is the slab model, referred to previously and also used exclusively in the work of Ref. 2. In this model, we have $D \propto \beta_i$ from Eq. (7). A curious consequence is that then β_i and β_h no longer appear separately in

the dispersion relation, rather only in combination as the total beta $\beta = \beta_i + \beta_h$. The other free parameter is δ . The condition for stability, namely, $b^2 - 4ac \geq 0$, is never satisfied when $\delta = 0$. In Fig. 2 is plotted the density fraction required for marginal stability (with $k_{\perp} \rho_i = 0.1$). At high beta values, δ must be greater than roughly 5% to have stability. The result shown in Fig. 2 is what a slab-model treatment, such as that of Ref. 2, should find. A stability plot like that of Fig. 1 cannot be obtained in this model.

However, because the effect of the curvature cannot be ignored, we will adopt instead a local model such that $\omega_{*i}/\omega_{bi} = -(2/\beta)(1 - \omega_{ki}/\omega_{bi})$, with the curvature and various gradients replaced by their local values, e.g., $|\omega_k| \propto 1/R_c$. A marginal stability analysis then yields β_i as a function of β_h , as in Fig. 1 earlier. The complete results (for $k_{\perp} \rho_i = 0.1$) are shown in Fig. 3

We see that for finite values of δ , there are now two stability regions. The lower one is nearly the same as the fundamental stability region for $\delta = 0$ (shown shaded) which was discussed previously. The only difference is that the threshold for β_h , when β_i is zero, is shifted from 20% down to 10% as δ increases from zero. For $\delta \geq 0.05$, the left-hand boundary of this lower region approximates the diagonal line beyond which $|B|$ no longer has a minimum. The upper boundary is still determined by the strong effect at $D = 0$. The rather singular behavior when $D \rightarrow 0$ appears to indicate conversion to a new electromagnetic mode carried by the hot electrons.

The upper stability region shown in Fig. 3 allows larger values for β_i but is probably unrealistic unless the ion temperature can be dramatically increased. To appreciate this, consider that the ratio of ring and ion plasma pressures is given by

$$\beta_h/\beta_i = \delta(T_h/T_i) . \quad (18)$$

For current EBT parameters, this ratio is about 100; even for the proposed EBT-P device, it is still an order of magnitude. Furthermore, if one tries to decrease the hot density fraction, the upper stability region disappears.

Thus, the relevant stability region is the lower one. We should append the qualification that in the vicinity of the vertical axis in Fig. 3, the stability results are probably not creditable since $\beta_h \approx 0$, $\delta \neq 0$ implies an unphysical cold but yet adiabatic response for the annulus electrons.

Nevertheless, our overall conclusion remains essentially unchanged: viz., that the maximum value for the toroidal core pressure allowed for stability against interchange modes is rather less than the annulus pressure and substantially smaller than predicted in other theories. Whether this somewhat pessimistic result for EBT stability will be confirmed by a non-local calculation in a realistic geometry and with a proper equilibrium remains to be seen.

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NOTED ADDED IN PROOF: Curves similar to those of Fig. 3, with two stability regions, were recently obtained also at Oak Ridge National Laboratory by D. A. Spong (private communication).

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FIGURE 1. Toroidal core plasma and annulus pressures, β_i and β_h , for interchange stability when $\delta = 0$, as derived in our theory (shaded region). Reference #2 predicted "stability" below and "instability" above the marginal curves ($\delta = 0, 0.05, 0.10$), shown here for comparison.

FIGURE 2. Hot electron density fraction, $\delta = N_h/N_i$, required for marginal stability for a given total beta, $\beta = \beta_i + \beta_h$, in a slab model (no curvature); $k_\perp \rho_i = 0.1$.

FIGURE 3. Marginal stability boundaries for β_i vs. β_h , with δ as parameter and $k_1 \rho_i = 0.1$. Interchange stable both below the common lower boundary (top of the fundamental region for $\delta = 0$, shown shaded) and also above the upper curve (labeled by the appropriate δ value).