TITLE: VISCORESISTIVE G-MODES AND BALLOONING

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Viscoresistive G-Modes and Ballooning

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Abstract: The resistive G-mode and its particular form, the resistive ballooning mode, are treated as limits of a single simple model. MHD theory including parallel and perpendicular viscosity, finite shear, and finite beta is employed to study their linear stability.

1. **Model:** The usual resistive interchange with parallel wavenumber $k_{\parallel} \sim 0$ and its special form, the ballooning mode, with $k_{\parallel} \sim 1/L$ are derived as limits of a single model. Advantage is taken of the simplicity of the magnetized plane slab geometry to simultaneously study the effects of both parallel and perpendicular viscosity, finite shear and finite beta. Magnetic curvature effects are simulated by a gravitational acceleration¹

$$G = G_0 \left(-\alpha + \omega_s \frac{2\pi}{L} z \right), \quad G_0 \equiv \frac{v_{th}^2 a}{v_r v_h R_c} = \beta_0 S \frac{a}{R_c} \quad (1)$$

where x, y, z are Cartesian coordinates, L is the connection length along a magnetic field line, v_{th} , v_r , v_h are the thermal, resistive, and Alfvén speeds defined with respect to a characteristic length a . R_c is the simulated radius of curvature. $S \equiv v_h/v_r$, $\beta_0 \equiv 2p_0/B_0^2$, p_0 , B_0 are characteristic values of the kinetic pressure and of the strength of the confining field respectively. α is a constant representing average curvature.

We employ Maxwell's equations along with the usual resistive MHD set with an anisotropic pressure tensor. We normalize velocity to v_r , time to a/v_h , p to $2p_0$, E to $v_r B_0$, ρ to ρ_0 , B to B_0 , and r to a . We let $B^{(0)} = (0, B_y^{(0)}(x), B_z^{(0)}(x))$ and $U \equiv v_v/v_h = \frac{3}{2} \beta_0 S Q^{-1}$, $Q \equiv \frac{3}{2} \beta_0 v_{th}^2/v_r v_v$, $U_{\perp} \equiv \mu_{\perp}/\mu_{\parallel}$, $v_v \equiv \mu_{\parallel}/a\rho_0$ where $\mu_{\parallel}$ and $\mu_{\perp}$ are the parallel and perpendicular coefficients of viscosity respectively. We consider "quasi-modes" of the form²

$$f^{(1)} = f(\delta x, z) \exp[iK(B_y^{(0)}z - B_z^{(0)}y) + \omega t] \text{ with } \delta \ll 1, K \gg 1.$$

We derive the equation of motion perpendicular to the line of force

$$\frac{\partial}{\partial z} \frac{1 + \hat{s}^2 z^2}{1 + \hat{s}^2 z^2 + \hat{\omega}/\hat{K}^2 B_z^{(0)} 2\partial z} \frac{\partial}{\partial z} v_x - \hat{\omega} \frac{\hat{K}^2 p^{(0)}}{S^2} (1 + \hat{s}^2 z^2) v_x - \frac{M_{\perp}}{S} \hat{K}^4 B_z^{(0)2} (1 + \hat{s}^2 z^2)^2 v_x + \frac{2\hat{K}^2}{S} \frac{\hat{\Gamma}}{\hat{\Omega}} \gamma p^{(0)} G(z) \hat{p} = 0 \quad (2)$$

Here the shear parameter $\hat{s}(x) \equiv (B_z^{(0)} B_y^{(0)} - B_z^{(0)} B_y^{(0)})/B_z^{(0)} B_z^{(0)}$

$$\text{Also, } \hat{\Omega} = \hat{Q} + \hat{\omega}, \quad \hat{Q} \equiv \gamma p^{(0)} Q, \quad \hat{\omega} \equiv \omega S, \quad \hat{K} = K B^{(0)}/B_z^{(0)}$$

$$M_{\perp} \equiv U U_{\perp}, \quad \hat{\Gamma} \equiv -p^{(0)}/\gamma p^{(0)} - \frac{3}{2}, \quad \hat{p} \equiv v_{\perp} v_{\perp}/v_x, \hat{p} = \frac{\hat{\Omega}}{\gamma p^{(0)} \hat{\Gamma}} p^{(1)}$$

Parallel viscosity is neglected in Eq. (2) as it can only introduce oscillation. However, this quantity plays an important role in the equation describing parallel motion

$$\frac{3}{2} \left(1 + \frac{B_y^{(0)2}}{B_z^{(0)2}}\right) \frac{p^{(0)} \hat{\omega}}{U \hat{\Gamma} S} \left[(\hat{\Gamma} + 5/2) v_x - \frac{\hat{\omega} \hat{\Gamma} \hat{p}}{\hat{\Omega}} \right] = - \frac{\partial^2}{\partial z^2} \hat{p} + \frac{\partial^2}{\partial z^2} v_x \quad (3)$$

2. Stationary Modes; $\omega=0$: A critical pressure gradient is obtained for the onset of ballooning ($\nu_{\parallel} = 0$): $\hat{\Gamma}\beta_0|_c \approx (Q/\kappa^2)B_y^{(0)2}/(B_y^{(0)2} + B_z^{(0)2})$

Parallel viscosity is seen to be responsible for the existence of a threshold which is not possible when resistivity is the only source of dissipation.

3. Growing Modes; $\omega \neq 0$: A. No Shear; $\hat{s}=0$: This is appropriate for devices like Field Reversed Theta Pinches and multipoles particularly in view of the fact that our treatment is valid for finite beta. We define

$$\hat{q} \equiv 4\hat{R}^2\hat{g}(\hat{K}^2B_z^{(0)2} + \hat{\omega})/\hat{\Omega}, \quad \hat{g} = G_0\hat{\Gamma}\gamma p^{(0)}, \quad \hat{R} = L/2\pi B_z^{(0)}$$

a. $\hat{q} \ll 1$: For $\hat{R}^2\hat{g} < \alpha$ we find a viscoresistive ballooning root with $\omega \sim S^{-1}\hat{K}^2\hat{g}\hat{R}^2$, for $\hat{R}^2\hat{g} = \alpha$, a "resonant" mode with $\omega \sim S^{-1/2}\hat{K}^2\hat{g}\hat{R}$, and for $\hat{R}^2\hat{g} > \alpha$ an ideal interchange being acted on by dissipation, $\omega \sim \hat{g}/M_i\hat{K}^2$,

b. $\hat{q} \gg 1$. For this case, the dispersion becomes a quintic in $\hat{\omega}$. Simple formulae are not very accurate. Using MACSYMA only two of the roots are found to be growing. Large perpendicular viscosity combined with small parallel viscosity can stabilize a resistive ballooning-like root for high and an ideal ballooning-like root for intermediate K .

In general $\nu_{\parallel}$ is found to be effective in stabilizing these modes for very small values of $\hat{g}$, viz. Sec. 2, however, for larger $\hat{g}$ it is destabilizing. $\nu_{\perp}$ is always stabilizing and becomes most effective as $K \rightarrow \infty$.

B. Finite Shear; $\hat{s} \neq 0$: This case is important for shear stabilized devices (Reversed Field Pinch), but also for tokamaks with rather peaked current profiles. We obtain simple analytic results using the "disconnected mode

approximation"³ and we make use of an expansion in terms of Hermite functions to treat the general problem.

a. Neglecting parallel inertia and electromagnetic effects we find ($\mu_{\perp} = 0$)

$$\tilde{\omega} = 1 - \frac{(1+\tilde{Q})}{(3+\tilde{Q})} \Lambda + \dots \text{ for } \Lambda \ll 1, \quad \tilde{\Omega} = \Lambda^{-1} + \frac{\tilde{Q}}{(1+\tilde{Q})^2} \Lambda^{-3} - \dots \text{ for } \Lambda \gg 1$$

where the tilde denotes normalization to ω_g , the growth rate for the "twisted slicing mode"² ($\mu_{\perp} \neq 0$). $\Lambda \equiv \hat{s}\omega_g S / (4\hat{g}(1-\alpha)^2 \hat{K}^2 \hat{R}^2 B_z^{(0)2})$ The first of Eq. (4) represents the G-mode limit, while the second is the ballooning limit.

b. Letting $\mu_{\perp} \neq 0$ and using the same approximation as above as well as an expansion in terms of Hermite functions, we derive a secular infinite matrix determinant for the problem. Using just the first term of this determinant we find very roughly (as $L \rightarrow \infty$)

$$\omega = -\hat{Q} + \frac{\omega_g + \hat{Q}}{KB^{(0)}} \left(\frac{\rho^{(0)} \omega_g}{2M_1} \right)^{1/2}, \text{ predicting a cutoff } K.$$

c. Utilizing the expansion in terms of Hermite functions, the general system of Eqs. (8) and (9) is solved also. Successive disconnected mode approximations are performed for calculating the eigenfunctions. The particular iterative scheme involved displays fast convergence. Growth rates are found to be greatly suppressed with respect to the $\hat{s} = 0$ case.

4. References

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