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OPTIMIZING THE AVERAGE LONGITUDINAL PHASE OF THE BEAM IN THE SLC LINAC*

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1. INTRODUCTION

The luminosity of the SLC depends on the energy spectrum of each beam at the final focus. In turn, the energy spectrum is determined by the beam's average linac phase ϕ_0 . For good luminosity the spectrum needs to be sufficiently narrow to fit well within an energy aperture of $\pm 5\%$. By riding the beam at the proper phase ahead of the rf crest the contribution to the energy spread due to the rf wave can be made to compensate that due to the longitudinal wakefield, resulting in a narrowed spectrum.¹ For example, with a bunch population of 5×10^{10} and a bunch length of 1.5 mm almost all the beam can be located within $\pm 3\%$ in this way.²

Besides its direct effect on the SLC luminosity, ϕ_0 also affects the population of the energy tails. And particles with energies far from the nominal value can undergo large orbit excursions in the linac. Both energy and orbit tail particles can reduce the effective luminosity, if they reach the final focus, by increasing the detector background. But if we assume that the collimators are adjusted properly, then both types of tail particles are clipped before the beam ever reaches the detector. Normally, therefore, backgrounds should not be an important consideration in choosing ϕ_0 .

If we choose ϕ_0 to minimize the rms deviation of the final energy spectrum δ_{rms} , then we will end up with a spectrum with a minimal tail population and a somewhat narrow core size. In a procedure meant to approximate this calculation the SLC operators were - at one time - told to adjust the overall phase of the linac to yield the energy distribution which, when fit to a gaussian, gives the gaussian with the smallest width. (The rms energy deviation itself cannot be measured since the energy tails tend to be below the noise of the measurement.) One problem with

this procedure is that the energy distribution is normally not gaussian. Another is that it tends to minimize the width of the beam core, which, as we shall see, does not in general result in the maximum luminosity.

The relation of ϕ_0 to the final energy spectrum in the SLC linac has been studied by many people over the years (see, for example, Refs. 1-6), with much of the work left unpublished. In this note we perform a somewhat thorough investigation of the problem. First we describe the calculation method, and discuss some common features of the energy spectrum. Then we calculate the value of ϕ_0 that minimizes δ_{rms} for the conceivable range of bunch population and bunch lengths of the SLC linac. This is followed by luminosity calculations, including the sensitivity of luminosity to variations in ϕ_0 . Finally we suggest a practical method of implementing the proper phase setting on the real machine.

2. THE ENERGY SPECTRUM AT THE END OF THE LINAC

The final linac energy, at position z within the bunch, is given by

$$E(z) = E_0 + E_a \cos(k_{rf}z + \phi_0) - e^2 NW(z) \quad , \quad (1)$$

with E_0 the initial energy (in this paper we ignore the beam's initial energy deviation), E_a the total peak rf energy gain, k_{rf} the rf wave number, N the total bunch population and $W(z)$ the longitudinal wake of the bunch for the entire linac. A negative phase means a position toward the front of the crest. Note that $W(z)$ depends on the bunch shape. If the final desired energy is $\langle E \rangle$ then the total peak energy gain needed is

$$E_a = (\langle E \rangle - E_0 + e^2 N \langle W \rangle) / \cos \phi_0 \quad , \quad (2)$$

if the charge distribution is symmetric about $z = 0$. The average of the wakefield

weighted by the charge distribution is denoted by $\langle W \rangle$. The quantity $\langle W \rangle$ is sometimes called the loss factor; the total beam induced energy loss E_{wake} is given by the product $e^2 N \langle W \rangle$.

The energy distribution, when normalized to unity, is given by

$$\frac{dn}{d\delta} = \frac{\langle E \rangle}{N} \sum_{z(E)} \frac{dN/dz}{|dE/dz|} \quad , \quad (3)$$

with $\delta \equiv E/\langle E \rangle - 1$, the energy deviation. The numerator in the sum is the charge distribution, the denominator is the derivative of Eq. (1). To evaluate these functions we first need to solve Eq. (1) for $z(E)$. Then if there is more than one solution ($E(z)$ is not, in general, monotonic) the sum must be performed over all of them. We note from Eq. (3) that the energy distribution becomes infinite wherever $dE/dz = 0$; its integral, however, is always finite. (We could include the effect of the incoherent energy spread by convolving its distribution with the results of our calculation. The new spectrum would then be everywhere finite. But since the incoherent contribution is small - in the SLC its rms width is .02% - and not important for our purposes, we will not include it in our calculations.)

We have written a computer program to calculate the final energy distribution of a gaussian bunch in the SLC linac. For all the results to be presented in this paper the parameter E_a has been adjusted so that $\langle E \rangle = 47$ GeV; the initial energy $E_0 = 1.153$ GeV; the total length of accelerating structure (needed for calculating $W(z)$) is 2744 m. The bunch wake is found by convolving the charge distribution with the delta function wake of the SLAC linac.⁷ Fig. 1 displays $W(z)$, over the whole linac, for gaussian bunches with $\sigma_z = .5, 1.0, 1.5$ mm. Thus, for example, we see that at $N = 5 \times 10^{10}$ and $\sigma_z = 1$ mm the peak of the induced loss for the

whole linac is 2 GeV, and is experienced by particles at 1 mm behind bunch center.

The expression

$$\left(\frac{E_{wake}}{\text{GeV}} \right) = .5 \left(\frac{N}{10^{10}} \right) \exp \left[- .6 \left(\frac{\sigma_z}{\text{mm}} \right)^{.55} \right] \quad \sigma_z \leq 5 \text{ mm} \quad , \quad (4)$$

approximates the total beam induced energy loss to good accuracy.

Fig. 1. The bunch wake, over the whole SLC linac, for gaussian bunches with $\sigma_z = .5, 1.0$ and 1.5 mm.

Let us first consider the case when $N = 3 \times 10^{10}$ and $\sigma_z = .75$ mm. We find that there is a range of values $\phi_0 < 0$ for which the linear part of the wakefield and of the rf wave tend to cancel each other to some extent, resulting in a somewhat narrow, "double horned" distribution¹ - i.e. one having two infinite peaks. Within this range, as the bunch is moved forward - away from the rf crest - the horn spacing narrows and the tail population grows; as it is moved backward - toward the crest - the opposite happens. The rms energy deviation δ_{rms} is minimized, and equals .28%, if we set ϕ_0 to -10.7 degrees. The top left plot of Fig. 2 gives the

Fig. 2. The energy distribution (top frames) and the function $\delta(z)$ (bottom frames) at the end of the SLC linac, when $\phi_0 = -10.7$ degrees (left) and -18.0 degrees (right). In both cases $N = 3 \times 10^{10}$ and $\sigma_z = .75$ mm.

energy distribution. For this figure, in order to show the heights of the peaks, we have convolved our calculation with a gaussian of .02%, which represents the contribution of the incoherent energy spread. The bottom plot displays the function $\delta(z)$; the dotted lines give the solutions of $\delta'(z) = 0$, which we call the horn edges. The horns coalesce when the beam is moved forward to $\phi_0 = -17.4$ degrees. The right two frames of Fig. 2 show the energy distribution and $\delta(z)$ when the beam is moved just beyond this point, to -18.0 degrees. The rms energy deviation is

.67%. We note that the core has narrowed while the tail population has increased. A measurement technique that minimizes the core and is insensitive to the tails will find -10.7 degrees a worse choice of phase than -18.0 degrees. However, in the former case only 3% of the beam is outside the $\pm 5\%$ aperture; in the latter case 24% is outside this window.

3. MINIMIZING THE RMS ENERGY SPREAD

We have calculated the final energy spectrum of gaussian bunches in the SLC, for bunch populations of $N = 1, 2, 3, 4, 5, 7 \times 10^{10}$ and lengths $\sigma_z = .5, .75, 1, 1.25, 1.5$ mm. In each case the parameter ϕ_0 was adjusted to minimize the rms energy deviation δ_{rms} . Fig. 3a displays the value of ϕ_0 that minimizes δ_{rms} as a function of σ_z for several values of bunch population. As we increase the current or decrease the bunch length the beam needs to move farther off crest for optimal energy spread compensation. For bunches short compared to the rf wavelength the optimal phase is found when the linear part of the rf cancels the average slope of the wake; i.e. when⁶

$$\sin \phi_0 = -\frac{e^2 N \langle W' \rangle}{k_{rf} E_a} \quad , \quad (5)$$

with $\langle W' \rangle$ the average value of the slope of the wake weighted by the charge distribution. In our case the curves can be approximated by

$$\left(\frac{\phi_0}{\text{deg}} \right) \approx -2.25 \left(\frac{N}{10^{10}} \right) \left(\frac{\sigma_z}{\text{mm}} \right)^{-1.4} \quad (6)$$

(see the dotted curves in Fig. 3a).

Fig. 3b gives the resulting figure of merit δ_{rms} . We see that in order to keep the rms energy spread small it is desirable to increase the bunch length as we

increase the current.² The dashed curve shows the rms deviation due to the rf curvature alone, which is given by $\delta_{rms} = k_{rf}^2 \sigma_z^2 / \sqrt{2}$. We note that in some cases the minimum energy spread is less when the wakefield is included than when it is not.

Whenever the current is sufficiently high and the bunch length sufficiently short the optimal spectrum will have a double horned shape. In Fig. 3c we display the horn separation $\Delta\delta_h$ of the optimal distribution. In all cases approximately 95% of the beam is found between the horns. The $N = 1, 2, 3 \times 10^{10}$ curves do not reach, respectively, to bunch length values of 1, 1.25, 1.5 mm since at these lengths the optimal distribution is no longer double horned. We note that these curves are very similar to those of Fig. 3b, with $\Delta\delta_h \approx 2\delta_{rms}$, due to the fact that a major portion of the charge is located in the vicinity of the horns. The curves are almost straight lines; they can be approximated by

$$\left(\frac{\Delta\delta_h}{\%} \right) \approx \left(\frac{N}{10^{10}} \right) \left[1.11 - .38 \left(\frac{\sigma_z}{\text{mm}} \right) \right] \quad , \quad (7)$$

(see the dotted lines in the plot).

4. OPTIMIZING THE LUMINOSITY

The luminosity depends on both beams' energy spectra at the final focus. After the linac the bunches traverse the arcs on the way to the final focus. Due to synchrotron radiation in the arcs each beam loses 1 GeV of energy and its spectrum is smeared by .08%.² Since these effects are small we will ignore them in all calculations that follow. We will further assume that the energy aperture of the arcs is at least as large as that of the energy collimator/final focus system, and that the energy spectrum at the final focus is essentially the same as at the end

Fig. 3. The average linac phase that minimizes δ_{rms} (a), the resulting rms energy deviation (b), and the separation of the horns (c), as functions of bunch length, for several values of bunch population.

of the linac. In addition to the energy spectrum, the luminosity depends on each beam's emittance at the final focus, which in turn depends on the orbit and optics errors in the linac, arcs and final focus.

Let us assume that the beams are transversely unperturbed gaussians, that the luminosity is not degraded by detector background and that we can ignore the luminosity enhancement of the beam-beam effect. Then by knowing the energy spectrum of each beam at the final focus and the energy response of the final focus - i.e. the final spot size as function of energy deviation - we can easily calculate an ideal luminosity. For example, a luminosity factor for two identical beams colliding head-on can be written as

$$r_{lum} = 2\sigma_x(0)\sigma_y(0) \int_{\delta_{min}}^{\delta_{max}} \frac{dn}{d\delta}(\delta) d\delta \int_{\delta_{min}}^{\delta_{max}} \frac{dn}{d\delta'}(\delta') \frac{d\delta'}{\bar{\sigma}_x(\delta, \delta')\bar{\sigma}_y(\delta, \delta')} , \quad (8)$$

with $\sigma_x(\delta)$, $\sigma_y(\delta)$ the spot sizes; with $\bar{\sigma}_x^2(\delta, \delta') = \sigma_x^2(\delta) + \sigma_x^2(\delta')$, and similarly for $\bar{\sigma}_y$. The limits of integration δ_{min} , δ_{max} , represent the acceptance limits of the energy collimator/final focus system. We note that r_{lum} gives the relative luminosity reduction due to only the longitudinal effects (it becomes 1 for monoenergetic beams). As such it represents an upper bound on the achievable luminosity, since any emittance growth will reduce the luminosity further. When comparing the luminosities for different currents, assuming the emittances are the same, we need to compare the quantities $r_{lum}N^2$.

If we suppose that the energy response of the final focus is flat within the aperture $\Delta\delta_a (\equiv \delta_{max} - \delta_{min})$, then the luminosity is just proportional to the square of the charge found within it, assuming also that the two beams are equal and collide head-on. Setting ϕ_0 to minimize δ_{rms} will then result in almost the

optimal luminosity if the distribution is double horned with $\Delta\delta_h < \Delta\delta_a$, since then at least 95% of the beam will be within the aperture. This situation can be improved upon slightly if we increase the horn separation (up to a maximum of $\Delta\delta_a$) by moving the beam nearer the rf crest. If the horn separation that minimizes δ_{rms} is greater than $\Delta\delta_a$, then the horns need to be brought within the aperture (by moving the beam farther off crest) for maximum luminosity.

The energy response of the SLC final focus does depend on energy deviation, as well as on the emittance. Fig. 4 shows calculation results, produced by K. Brown (see also Ref. 8), of the final spot size dependence on energy deviation for the SLC, for beams with typical normalized emittances. These are 5 and 3×10^{-5} nm in x and y respectively at the final focus (corresponding to 3×10^{-5} nm in each plane at the end of the linac). It is also assumed that the beams have come through the North Arc and that the final focus is configured for the Mark II detector. Although σ_y is rather flat over the whole $\pm.5\%$ range, σ_x is flat only over about $\pm.3\%$. The energy aperture of the SLC energy collimator/final focus system can be taken to be $\pm.5\%$.

We have numerically solved Eq. (8) for the beam distributions portrayed in Fig. 2 for $N = 3 \times 10^{10}$ and $\sigma_s = .75$ mm. We find the luminosity factor is .86 and .54 for respectively the distribution that minimizes δ_{rms} and the one which has one narrow peak. Clearly, minimizing the core width does not, in general, maximize the luminosity. We have repeated the calculation for beams with various lengths and currents, adjusting ϕ_0 each time to maximize r_{lum} . The results are shown in Fig. 5. The top frame gives the phase ϕ_0 which optimizes the luminosity. The curves for $N = 1, 2, 3 \times 10^{10}$ are almost identical to those of Fig. 3a. For the higher currents the beam needs to move farther off crest, resulting in a narrower horn

Fig. 4. The final spot size as function of energy deviation in the SLC when x and y emittances in the final focus are, respectively, 5 and 3×10^{-5} nm; the beams have come through the North Arc; the final focus is configured for the Mark II. (Produced by K. Brown.)

spacing, if we want to maximize the luminosity rather than minimize δ_{rms} . The maximum luminosity is given in Fig. 5b. We see that, although the dependence on bunch length is weak, as the current is increased the bunch should be lengthened to maximize the luminosity.

Given a value of bunch population, what is the optimal choice of bunch length? The orbit tolerances of the beam in the SLC linac tend to increase with decreasing bunch length. However, as we have just seen, for a given current if the bunch is too short we will need to move very far off crest to compensate the longitudinal wakefield, and we will also lose some luminosity. As a compromise between these considerations we suggest at, respectively, $N = 1, 2, 3, 4, 5, 7 \times 10^{10}$ choosing the bunch length .5, .75, .75, 1.0, 1.0, 1.25 mm.

Fig. 6a displays r_{lsm} as function of ϕ_0 , for the suggested current/bunch length

Fig. 5. The average linac phase that maximizes r_{lum} (a), and the resulting luminosity factor (b), as functions of bunch length for several values of bunch population. The spot functions of Fig. 4 were used in the calculations.

combinations, assuming the spot size functions of Fig. 4; Fig. 6b gives the horn spacing of the corresponding energy distribution. We clearly see that near its maximum value the luminosity is rather insensitive to the average rf phase and to the horn spacing. We also note that the curves are not symmetric about their maxima. Beginning somewhere just under $\Delta\delta_h = 1\%$, as the horn spacing is increased the luminosity drops quickly.

Fig. 6. The luminosity factor (a) and the horn separation (b) over a range of values of ϕ_0 , for the suggested current/bunch length combinations. The currents are $N = 1, 2, 3, 4, 5, 7 \times 10^{10}$, and the corresponding bunch lengths are .5, .75, .75, 1.0, 1.0, 1.25 mm. The spot functions of Fig. 4 were used in the calculations.

Table 1 gives the maximum values of r_{lum} shown in Fig. 6. We see that by going from 1 to 7×10^{10} the luminosity can increase by the factor $49(.68/.98) = 34$, if the emittance is unchanged. Also shown for the optimal situation are the fraction of the beam within the energy acceptance $n_{\pm .5\%}$, the phase ϕ_0 , and the horn spacing $\Delta\delta_h$. The pairs of parenthetical values are the increments of the given parameter

which result in a reduction of r_{lum} of .05; the first of each pair by going more off crest, the second by moving toward the rf crest.

Table 1. The optimum luminosity for typical bunch properties using the final focus response functions given in Fig. 4. The parenthetical values are the increments which result in a reduction of r_{lum} by .05.

$N/10^{10}$	σ_z/mm	r_{lum}	$n_{\pm 5\%}$	$-\phi_0/\text{deg}$	$\Delta\delta_h/\%$
1	.50	.98	1.00 (-.02/+0.00)	5.9 (+2.7/-3.0)	.25 (-.20/+ .34)
2	.75	.94	.99 (-.03/+0.01)	6.8 (+1.9/-1.8)	.43 (-.22/+ .27)
3	.75	.86	.96 (-.04/+0.03)	10.7 (+2.5/-1.9)	.59 (-.30/+ .27)
4	1.00	.82	.95 (-.04/+0.03)	9.7 (+2.1/-1.5)	.64 (-.30/+ .28)
5	1.00	.75	.92 (-.06/+0.03)	12.7 (+2.4/-1.5)	.71 (-.35/+ .26)
7	1.25	.68	.88 (-.06/+0.03)	13.6 (+1.7/-1.3)	.72 (-.30/+ .27)

The results of Figs. 5-6 and Table 1 are valid for typical current SLC operation. If the spot size functions change, for example due to a change in the beam's emittance, or because the final focus is reconfigured for the SLD detector, the results will be somewhat different.

5. FINDING THE DESIRED RF PHASE ON THE SLC

Ideally, to find the optimal SLC linac phase the luminosity should be measured and then the phase adjusted to maximize it. This being impractical we suggest that the calculated results given in Table 1 be used for finding the optimal phase setting. In the SLC linac the phase of each beam is adjusted by the phase ramp, a knob which changes the relative phase of the beam with respect to the linac by an amount $\Delta\phi_{ramp}$. What is the value $\Delta\phi_{ramp}$ that achieves an average linac phase

ϕ_0 ? If initially the phases of all the klystrons are identical, with the beam riding on crest, then $\Delta\phi_{ramp} = \phi_0$. Normally, however, the phases of the klystrons are not all lined up, primarily in order to take advantage of BNS damping. In such a case we can approximate $\Delta\phi_{ramp} \approx \phi_0 - \langle\phi_b\rangle$, with $\langle\phi_b\rangle$ the average value of the phase of the beam with respect to the klystrons, as long as the beam is near the crest in all sectors (within, say, 30 degrees). However, rather than setting the overall linac phase to achieve the desired value of ϕ_0 (as given in Table 1) directly, we suggest measuring the spectrum width and using it to determine the proper phase setting.

In the SLC the energy spectrum is obtained from an image of the beam at a high dispersion point at the end of the linac. In the set-up procedure the beam's average phase, initially near 0, should be reduced (by using the phase ramp) until a double horned distribution with the desired horn separation is found (see Table 1). At higher currents it is important that the linac orbit be good, so that the measurement is not spoiled by emittance growth or orbit tails. In addition, measurements far beyond the optimal phase should be avoided, since as the beam is moved to more negative values of phase the BNS damping is continually weakened, which will also tend to lead to emittance growth.

One practical problem with this procedure is that the resolution of the measurement has an rms value of .15%,⁹ which will tend to obscure the horns and make it impossible to measure spots with a full-width-at-half-maximum (FWHM) of less than .3%. Therefore, when $N = 1.25 \times 10^{10}$ we suggest adjusting the phase until the FWHM of the measured distribution is .5%, which is large compared to the resolution. From Table 1 we see that the luminosity will still be nearly optimal. At higher currents, where we are no longer resolution limited, the phase should be adjusted to give a FWHM equal to the value of $\Delta\delta_h$ given in Table 1.

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