Pseudoscalar Decay Constants and $\eta \rightarrow 3\pi$
in Chiral and $1/N$ Perturbation Theory

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Abstract

The normal and anomalous chiral Ward identities are re-examined from the point of view of chiral and $1/N$ perturbation theory. Constraints of $\psi \rightarrow \eta\gamma$ and $\psi \rightarrow \eta'\gamma$ and pseudoscalar two photon decays are imposed, yielding a system of six simultaneous equations. A solution for the pseudoscalar decay constants is found which is relatively insensitive to the unknown gluonic bound states. The decay $\eta \rightarrow 3\pi$ is then calculated using these results, with good agreement with recent measurements.

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The U(1) problem,¹ at the crossroads of current algebra and QCD, of chiral perturbation theory and the $1/N$ expansion,² has received recent theoretical interest, leading to new experimental predictions.³ In particular, the saturation of the anomalous Ward identities by gluonic states and pseudoscalar mesons provides relations among matrix elements of the strong anomaly and pseudoscalar decay constants measured in $\eta \rightarrow 3\pi$, $\psi \rightarrow \eta\gamma$, $\psi \rightarrow \eta'\gamma$, and photonic decays of η , η' , and the pion.⁴ In this article we subject these relations to experimental test and find excellent agreement with the data.

Let us begin by commenting on the nature of the approximations we shall make.

Saturation of the normal Ward identities by one particle states, it has long been realized, is an uncontrolled approximation without appeal to chiral perturbation theory, an expansion in powers of the $SU(N) \times SU(N)$ symmetry breaking parameters, the quark masses $\sim \delta$. We learn here that 2 particle states are suppressed by a power of δ compared to one particle poles in channels with Goldstone behavior

$$m_\pi^2, m_K^2, m_\eta^2 = 0(\delta)$$

$$f_\pi, f_K, f_\eta = 0(1)$$

$$f_\pi - f_K, \text{etc.}, = 0(\delta)$$

as described in Dashen,⁵ at fixed N =number of colors.

For the anomalous Ward identities, involving the η' and the ninth axial current, the situation is not so clear. As the η' is not a Goldstone in the chiral limit $\delta \rightarrow 0$, according to the QCD resolution to the U(1) problem, there is no apparent justification for pole saturation and no suppression of 2 particle intermediate states by a power of δ . However, if Witten⁶ is correct, the

Goldstone character of the η' is restored in the $N \rightarrow \infty$ limit, with, at $\delta = 0$

$$F_{\eta'}, F_{\eta}, F_{\pi}, F_K \sim \sqrt{N}$$

$$F_{\eta'} - F_{\pi} \sim \frac{1}{\sqrt{N}}$$

$$m_{\eta'}^2 \sim \frac{1}{N}$$

Then with $\frac{1}{N}$ playing a role similar to δ , the 2 particle continuum contributions to matrix elements of $F\tilde{F}$ is suppressed in the chiral limit by a two powers of $\left(\frac{1}{N}\right)$, one because $m_{\eta'}^2 \propto \frac{1}{N}$, another because it is a one particle state. With this counting there is a consistent perturbation theory, with the η' a Goldstone as $N \rightarrow \infty$, $\delta \rightarrow 0$. It is interesting to note that, from the standpoint of $1/N$ perturbation theory, 2 particle states are always down by $\frac{1}{N}$ from one particle pole contributions, even if the pole is not a Goldstone.

Thus it appears that one should attempt simultaneously to do chiral and $1/N$ perturbation theory when saturating the normal and anomalous Ward identities, following powers of δ and N to leading order. To this end, let us define the various decay constants that enter the problem as

$$\langle 0 | \partial_{\mu} A_{\mu}^3 | \pi^0 \rangle = m_{\pi}^2 F_{\pi}$$

$$\langle 0 | \partial_{\mu} A_{\mu}^6 | K^0 \rangle = m_K^2 F_K$$

$$\langle 0 | \partial_{\mu} A_{\mu}^8 | \eta \rangle = m_{\eta}^2 F_{\eta}$$

$$\langle 0 | \partial_{\mu} A_{\mu}^0 | \eta' \rangle = m_{\eta'}^2 F_{\eta'}$$

$$\langle 0 | \partial_{\mu} A_{\mu}^8 | \eta' \rangle = m_{\eta'}^2 F_{\eta'8}$$

$$\langle 0 | \partial_{\mu} A_{\mu}^0 | \eta \rangle = m_{\eta}^2 F_{\eta 0}$$

$$\langle 0 | \partial_{\mu} A_{\mu}^0 - \sqrt{\frac{2}{3}} \frac{3\alpha}{4\pi} \text{Tr } F\tilde{F} | \eta' \rangle = m_{\eta'}^2 \tilde{F}_{\eta'}$$

$$\langle 0 | \partial_{\mu} A_{\mu}^0 - \sqrt{\frac{2}{3}} \frac{3\alpha}{4\pi} \text{Tr } F\tilde{F} | \eta \rangle = m_{\eta}^2 \tilde{F}_{\eta}$$

where the tilded quantities refer to the symmetry current divergence. With these definitions we have the behavior^{6,7}

$$m_{\pi}^2, m_K^2, m_{\eta}^2 \sim O(\delta)$$

$$m_{\eta'}^2 \sim O(\delta) + O\left(\frac{1}{N}\right)$$

$$F_{\pi}, F_K, F_{\eta}, F_{\eta_0}, \tilde{F}_{\eta_0} \sim O(\sqrt{N})$$

$$F_{\eta'8}, \tilde{F}_{\eta'} \sim \left(\frac{O(\delta\sqrt{N})}{O(\delta) + O\left(\frac{1}{N}\right)} \right)$$

$$F_{\eta'} - \tilde{F}_{\eta'} \sim \left(\frac{O\left(\frac{1}{\sqrt{N}}\right)}{O(\delta) + O\left(\frac{1}{N}\right)} \right)$$

from which it is clear that the η' is a pseudo-Goldstone in the chiral limit but a genuine Goldstone in the combined chiral and $\frac{1}{N}$ limit.

Let us now examine the relevant identities for the ninth, third, and eighth axial currents. The contributing intermediate states are the nonet of pseudoscalar mesons and purely gluonic bound states. The anomalous Ward identities receive single particle contributions from the pseudoscalar nonet as well as gluonic bound states in the QCD surface term⁸ $\langle 0 | \tilde{F}\tilde{F}(x) \tilde{F}\tilde{F}(0) | 0 \rangle$; the normal Ward identities also have contributions from both but the gluonic poles are suppressed by a factor of δ/N and will be ignored. The result of this saturation is then⁴

$$\frac{4}{3} F_K^2 m_K^2 - \frac{1}{3} F_{\pi}^2 m_{\pi}^2 = F_{\eta}^2 m_{\eta}^2 + F_{\eta'8}^2 m_{\eta'}^2, \quad (1)$$

$$F_{\pi}^2 m_{\pi}^2 = m_{\eta}^2 F_{\eta} (F_{\eta} - \sqrt{2} \tilde{F}_{\eta_0}) + m_{\eta'}^2 F_{\eta'8} (F_{\eta'8} + \sqrt{2} \tilde{F}_{\eta'}) \quad (2)$$

$$3F_{\pi}^2 m_{\pi}^2 = 2S + m_{\eta}^2 (F_{\eta} + \sqrt{2} \tilde{F}_{\eta_0})^2 + m_{\eta'}^2 (F_{\eta'8} + \sqrt{2} \tilde{F}_{\eta'})^2 \quad (3)$$

where

$$g = \frac{2i}{3} \int d^4x \, T \langle 0 | \frac{3\alpha_s}{4\pi} \text{Tr} F \tilde{F}(x) \frac{3\alpha_s}{4\pi} \text{Tr} F \tilde{F}(0) | 0 \rangle.$$

The single particle contributions to g are

$$g = -m_\eta^2 (F_{\eta_0} - \tilde{F}_{\eta_0})^2 - m_{\eta'}^2 (F_{\eta'} - \tilde{F}_{\eta'})^2 - m_{G^2}^2$$

where $m_{G^2}^2$ is the contribution of gluonic bound states. Unlike Ref. 4 we do not take g to be zero. In fact, $-g$ is a matrix element of a positive semi-definite operator and only vanishes if the contribution of every intermediate state vanishes. The existence of the decays $\psi \rightarrow \eta \gamma, \eta' \gamma$ shows that this cannot occur [see Eq.(4) below]. It should be noted that the second derivative of the vacuum energy of QCD with respect to θ , the angle conjugate to the ninth axial charge, does vanish in the limit of massless quarks. However, this derivative differs from g by a contact term; a careful reading of Ref. 6 shows that this vanishing occurs in the $1/N$ expansion by a cancellation between this contact term, the η' contribution, and the glueball contribution, if it exists, which is consistent with the δ, N behavior of the masses and decay constants given above. In Table I we show this leading power behavior for the pole states. (Continuum states are suppressed by δ/N for octet channels, and by $\frac{1}{N^2}$ for singlet channels.) The behavior of $m_\eta^2 (F_{\eta_0} - \tilde{F}_{\eta_0})^2$, the η contribution to g , however, is difficult to establish. In the $1/N$ expansion, it surely is $O(\frac{1}{N})$. As for its dependence on δ , all that one can say is that it may vanish as $\delta \rightarrow 0$ or cancel with some other part of g or the contact term. (The naive estimate indicates it is $O(1/N\delta)$ which is nonsense as $\delta \rightarrow 0$, but does suggest that it is $O(1)$ in the chiral, $N \rightarrow \infty$ limit, the same order as the other terms in (3).) For our purposes here we will rely on the experimental fact that^{9,10}

$|F_{\eta_0} - \tilde{F}_{\eta_0}| / |F_{\eta'} - \tilde{F}_{\eta'}| \approx 1$ and treat the term on the same footing as the η' contribution to g .

In addition to the results of Eqs.(1), (2), and (3), there are, as explained in Ref. 4 three more constraints among the decay constants coming from the $\psi \rightarrow \eta' \gamma / \psi \rightarrow \eta \gamma$ ratio,¹¹ and the ratio of the rates $\eta \rightarrow \gamma \gamma / \eta' \rightarrow \gamma \gamma / \pi^0 \rightarrow \gamma \gamma$, yielding

$$\left| \frac{F_{\eta'} - \tilde{F}_{\eta'}}{F_{\eta_0} - \tilde{F}_{\eta_0}} \right| = .88 \quad (4)$$

$$2\sqrt{2} F_{\eta} F_{\pi} - F_{\eta_0} F_{\pi} = (1.41)\sqrt{3} (F_{\eta'} F_{\eta} - F_{\eta_0} F_{\eta'8}) \quad (5)$$

$$-2\sqrt{2} F_{\eta'8} F_{\pi} + F_{\eta'0} F_{\pi} = .78 (F_{\eta'} F_{\eta} - F_{\eta_0} F_{\eta'8}) \quad (6)$$

At this point it appears that we have a soluble system of six equations in the six unknowns ($m_G F_G$), $F_{\eta'}$, $\tilde{F}_{\eta'}$, $F_{\eta'8}$, F_{η_0} , and $\tilde{F}_{\eta_0}$, with $F_{\eta} = F_K = F_{\pi}$ as known input. However, this is not quite the case. Equation (1) may be solved for $F_{\eta'8}$, and then (5) and (6) appear to determine F_{η_0} , $F_{\eta'}$, but the symmetry breaking term F_{η_0} then turns out to be equal to $2\sqrt{2} F_{\eta}$, which is certainly unreasonably large, whereas $F_{\eta'}$ is then $2\sqrt{2} F_{\eta'8}$, also of the order of a symmetry breaking term. The resolution of this paradox is to take (5) and (6) to be parallel in the $F_{\eta'}$, F_{η_0} plane, which means that the determinant of that system vanishes:

$$0 = 1 - \sqrt{6} F_{\eta'8} - (.78)\sqrt{3} F_{\eta'} \quad (7)$$

Equations (1) and (7) overdetermine $F_{\eta'8}$, and it is nontrivial that there is, within errors, a common solution:

$$F_{\eta'8} = -.15 \quad (8)$$

(Alternatively, we could solve (1) and (7) for F_{η} , $F_{\eta'8}$ with the following reasonable solution:

$$F_{\eta} = 1.02, \quad F_{\eta'8} = -.15 \quad (9)$$

This may be taken as evidence that our solution is consistent with the $1/N$ expansion.) We are left with four equations for the remaining five unknowns. All we can do is determine four of the unknowns in terms of the fifth, which we take to be the bound state gluonic contribution. A numerically accurate solution to this set of equations, assuming $m_G^2 F_G^2 / m_{\eta}^2 F_{\eta}^2 < 20$, is

$$\begin{aligned}
\frac{F_{\eta'}}{F_{\pi}} &= 1.12 + .017 \frac{m_{FG}^2}{m_{\eta\pi}^2}, & \frac{\tilde{F}_{\eta'}}{F_{\pi}} &= .65 + .074 \frac{m_{FG}^2}{m_{\eta\pi}^2}, \\
\frac{\tilde{F}_{\eta^0}}{F_{\pi}} &= -.46 + .034 \frac{m_{FG}^2}{m_{\eta\pi}^2}, & \frac{F_{\eta^0}}{F_{\pi}} &= .07 - .031 \frac{m_{FG}^2}{m_{\eta\pi}^2}.
\end{aligned} \tag{10}$$

We learn here that the decay constants are quite insensitive to the gluonic contributions for a wide range of m_{FG}^2 scaled to $m_{\eta\pi}^2$, the natural scale of the octet contributions. Given the uncertainties, theoretical and experimental, in our original set of equations, it appears this is not the place to learn about gluonic states.

We note that singlet-octet mixing via the gauge invariant currents is small

$$\frac{F_{\eta'8}}{F_{\pi}} = -.15, \quad \frac{F_{\eta^0}}{F_{\pi}} = .07 - .031 \frac{m_{FG}^2}{m_{\eta\pi}^2}$$

whereas both η and η' have large matrix elements with $F\tilde{F}$, as measured by $F_{\eta'}$, $\tilde{F}_{\eta'}$, and $F_{\eta^0} - \tilde{F}_{\eta^0}$.

Finally, we are in a position to calculate $\eta \rightarrow \pi^+ \pi^- \pi^0$, using our determination of the decay constants, assuming $m_{FG}^2/m_{\eta\pi}^2$ is not unusually large, that is neglecting the bound gluonic contributions. The details of this calculation appear elsewhere.³ Here we review the calculation for the sake of completeness. This decay has long been a problem for current algebra, and is an aspect of the U(1) puzzle, but it is now recognized that this process proceeds via the strong anomaly.¹² We begin by reducing in a soft π^0 in the amplitude for $\eta \rightarrow \pi^+ \pi^- \pi^0$.⁸

$$A(\eta \rightarrow \pi^+ \pi^- \pi^0) = \left(1 - \frac{2E_0}{m_{\eta}}\right) \frac{m_d - m_u}{F_{\pi}} \langle \pi^+ \pi^- | \int d^4x (\bar{u}\gamma_5 u + \bar{d}\gamma_5 d) | \eta \rangle \tag{11}$$

where we have made explicit the current algebra zero at vanishing charged pion four momentum. We have here the zero momentum matrix element of the operator

$$\bar{u}\gamma_5 u + \bar{d}\gamma_5 d = \frac{m_d - m_u}{m_d + m_u} (\bar{u}\gamma_5 u - \bar{d}\gamma_5 d) - \frac{1}{m_d + m_u} \partial_\mu (\bar{u}\gamma_\mu \gamma_5 u + \bar{d}\gamma_\mu \gamma_5 d) + \frac{2i}{m_d + m_u} \frac{2\alpha_s}{8\pi} \text{Tr } F\tilde{F} . \quad (12)$$

We neglect the first two terms on the right-hand-side of (12) because the first is higher order in $m_d - m_u$ and has the wrong isospin and the second vanishes by Sutherland-Veltman-Adler arguments.¹³ Therefore we find approximately

$$\lim_{k \rightarrow 0} \langle \pi^+(p^+) \pi^-(p^-) \pi^0(k) | \eta(q) \rangle = \frac{2i}{F_\pi} \frac{m_d - m_u}{m_d + m_u} \int d^4 x \langle \pi^+(p^+) \pi^-(p^-) | \frac{2\alpha_s}{8\pi} \text{Tr } F\tilde{F} | \eta(q) \rangle . \quad (13)$$

The important step is to now recognize (this is the PCU₁C hypothesis⁷) that $\text{Tr } F\tilde{F}$ may be taken as an interpolating field for the η' . That is, since

$$\sqrt{\frac{2}{3}} \langle 0 | \frac{3\alpha_s}{4\pi} \text{Tr } F\tilde{F} | \eta' \rangle = m_{\eta'}^2 \langle F_{\eta'} - \tilde{F}_{\eta'} \rangle \quad (14)$$

it is reasonable to take

$$m_{\eta'}^2 \langle F_{\eta'} - \tilde{F}_{\eta'} \rangle \varphi_{\eta'} = \sqrt{\frac{2}{3}} \frac{3\alpha_s}{4\pi} \text{Tr } F\tilde{F} . \quad (15)$$

Using this hypothesis in (13) and reducing in the η with $q^2 = (p^+ + p^-)^2$ on the η mass shell, we have

$$\begin{aligned} \lim_{k \rightarrow 0} \langle \pi^+(p^+) \pi^-(p^-) \pi^0(k) | \eta(q) \rangle \\ = -\sqrt{\frac{2}{3}} \frac{F_{\eta'} - \tilde{F}_{\eta'}}{F_\pi} \frac{m_d - m_u}{m_d + m_u} \int d^4 x d^4 y e^{i(p^+ + p^-) \cdot y} \\ \times (q^2 + m_\eta^2) m_\eta^2 \langle \pi^+(p^+) \pi^-(p^-) | T(\varphi_\eta(x) \varphi_\eta(y)) | 0 \rangle . \end{aligned} \quad (16)$$

Appearing here is just the $\eta' \rightarrow \eta \pi^+ \pi^-$ amplitude at $k=0$:

$$\lim_{k \rightarrow 0} \langle \pi^+(p^+) \pi^-(p^-) \eta(q) | \eta'(k) \rangle = - \int d^4x d^4y e^{i(p^+ + p^-) \cdot y} m_{\eta'}^2 (q^2 + m_{\eta'}^2) \times \langle \pi^+(p^+) \pi^-(p^-) | T(\varphi_{\eta}(y) \varphi_{\eta'}(x)) | 0 \rangle, \quad (17)$$

and so we conclude

$$\lim_{k \rightarrow 0} \langle \pi^+(p^+) \pi^-(p^-) \pi^0(k) | \eta(q) \rangle \cong \sqrt{\frac{2}{3}} \frac{m_d - m_u}{m_d + m_u} \frac{F_{\eta'} - \tilde{F}_{\eta'}}{F_{\pi}} \lim_{k \rightarrow 0} \langle \pi^+(p^+) \pi^-(p^-) \eta(q) | \eta'(k) \rangle. \quad (18)$$

or

$$\frac{A(\eta \rightarrow \pi^+ \pi^- \pi^0)}{A(\eta' \rightarrow \eta \pi^+ \pi^-)} = \sqrt{\frac{2}{3}} \frac{m_d - m_u}{m_d + m_u} \frac{F_{\eta'} - \tilde{F}_{\eta'}}{F_{\pi}} \quad (19)$$

where the η' and the π^0 are at zero momentum. Taking the electromagnetic mass difference estimate of Gross, Treiman, and Wilczek¹⁴ for the isospin violation,¹⁵ $(m_d - m_u)/(m_d + m_u) \cong .29$, and our result for $F_{\eta'} - \tilde{F}_{\eta'} \cong .47 F_{\pi}$, we find¹⁶

$$\frac{A(\eta \rightarrow \pi^+ \pi^- \pi^0)}{A(\eta' \rightarrow \eta \pi^+ \pi^-)} = 0.11 \pm .03 \quad (20)$$

where the error reflects our estimate of theoretical and experimental uncertainty.

We now compare this prediction to experiment. Recently Binnie et al. and Abrams et al.¹⁷ have measured the η' full width. With the well-known branching ratios to $\eta \pi^+ \pi^-$ this determines $A(\eta' \rightarrow \eta \pi^+ \pi^-)$. (The Dalitz plot for this decay is nearly flat.¹⁸) The numerator is determined from the $\eta \rightarrow \pi^+ \pi^- \pi^0$ rate taking into account the current algebra variation of the matrix element with π^0 energy. The result, with the π^0 at zero momentum, is

$$\frac{A(\eta \rightarrow \pi^+ \pi^- \pi^0)}{A(\eta' \rightarrow \pi^+ \pi^-)} = 0.11 \pm .02, \quad (12)$$

where the error is based on the uncertainty of the experimental rates. The agreement of our result with experiment is excellent,¹⁹ supporting our determination of the decay constants from the normal and anomalous Ward identities, and our conclusion that the contribution of gluonic state is not large.

We would like to thank K. Lane for many helpful conversations, and acknowledge the receipt of a manuscript from P. Nath and R. Arnowitt²⁰ on this subject while this manuscript was in preparation.

Table I

$$\frac{4}{3} F_{KK}^2 - \frac{1}{3} F_{\pi\pi}^2 = F_{\eta\eta}^2 + F_{\eta',8}^2,$$

$N\delta$	$N\delta$	$N\delta$	$\frac{N\delta^2}{O(\delta) + O(\frac{1}{N})}$
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$$F_{\pi\pi}^2 = m_{\eta\eta}^2 (F_{\eta\eta} + \sqrt{2} \tilde{F}_{\eta\eta})^2 + m_{\eta',8}^2 (F_{\eta',8} + \sqrt{2} \tilde{F}_{\eta',8})^2$$

$N\delta$	$N\delta$	$\frac{N\delta^2}{O(\delta) + O(\frac{1}{N})}$
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$$3F_{\pi\pi}^2 + 2m_{\eta\eta}^2 (F_{\eta\eta} - \tilde{F}_{\eta\eta})^2 + 2m_{\eta',8}^2 (F_{\eta',8} - \tilde{F}_{\eta',8})^2 + 2m_G^2 F_G^2 = m_{\eta\eta}^2 (F_{\eta\eta} + \sqrt{2} \tilde{F}_{\eta\eta})^2 + m_{\eta',8}^2 (F_{\eta',8} + \sqrt{2} \tilde{F}_{\eta',8})^2$$

$N\delta$	See text	$\frac{1}{N}$ $O(\delta) + O(\frac{1}{N})$	1	$N\delta$	$\frac{N\delta^2}{O(\delta) + O(\frac{1}{N})}$
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